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BSQ110

DEVOIR #2

#1

a) $\langle 0|+\rangle = (1\ 0)\,1/\sqrt{2}\begin{pmatrix}1\\1\end{pmatrix} = 1/\sqrt{2}(1+0) = 1/\sqrt{2}$

4/4 b) $\langle -|+\rangle = 1/\sqrt{2}^*(1^*\ -1^*)\,1/\sqrt{2}\begin{pmatrix}1\\1\end{pmatrix} = \frac{1}{2}(1-1) = 0$

c) $\langle -|0\rangle = 1/\sqrt{2}(1\ -1)\begin{pmatrix}1\\0\end{pmatrix} = 1/\sqrt{2}(1+0) = 1/\sqrt{2}$

d) $\langle -|1\rangle = 1/\sqrt{2}(1\ -1)\begin{pmatrix}0\\1\end{pmatrix} = 1/\sqrt{2}(0-1) = -1/\sqrt{2}$

#2

1/2 a) $|\psi\rangle = \sqrt{p}\,|0\rangle + \sqrt{1-p}\,|1\rangle$ ✓

b) $|\psi'\rangle = \frac{1}{\sqrt{3}}|0\rangle + \frac{1}{\sqrt{3}}|1\rangle$ ✗ Ex: $|\psi'\rangle = \sqrt{p}|0\rangle + i\sqrt{1-p}|1\rangle$

#3 - a) $|\psi_a'\rangle = \frac{1}{\sqrt{\langle\psi_A|\psi_A\rangle}}|\psi_A\rangle$ $\quad \langle\psi_A|\psi_A\rangle = 1/2^*(1^*\ 1^*)\,1/2\begin{pmatrix}1\\1\end{pmatrix} = 1/4(1+1) = 1/2$ ✓

$= \frac{1}{\sqrt{1/2}}|\psi_A\rangle = 1/\sqrt{1/2}\cdot 1/2(|0\rangle + |1\rangle) = \sqrt{1/2}|0\rangle + \sqrt{1/2}|1\rangle$

$|\sqrt{1/2}|^2 = P_0 = 1/2 = |\sqrt{1/2}|^2 = P_1$

4/4

b) $|\psi_B'\rangle = \frac{1}{\sqrt{\langle\psi_B|\psi_B\rangle}}|\psi_B\rangle$ $\quad \langle\psi_B|\psi_B\rangle = (1^*\ -3^*)\begin{pmatrix}1\\-3\end{pmatrix} = (1+9) = 10$

$= \frac{1}{\sqrt{10}}|\psi_B\rangle = \frac{1}{\sqrt{10}}|0\rangle - \frac{1}{\sqrt{10}}\cdot 3|1\rangle = \frac{1}{\sqrt{10}}|0\rangle - \frac{3}{\sqrt{10}}|1\rangle$ ✓

$\left|\frac{1}{\sqrt{10}}\right|^2 = P_0 = \frac{1}{10}$ $\quad \left|\frac{3}{\sqrt{10}}\right|^2 = P_1 = \frac{9}{10}$

#4 - 4/4 a) $\langle\psi|\psi\rangle = \left(\cos\frac{\theta}{2}^* \quad e^{i\varphi}\sin\frac{\theta}{2}^*\right)\begin{pmatrix}\cos\frac{\theta}{2}\\ e^{i\varphi}\sin\frac{\theta}{2}\end{pmatrix} = \cos^2\left(\frac{\theta}{2}\right) + 1\cdot\sin^2\left(\frac{\theta}{2}\right) = 1$

l'identité trigonométrique $\sin^2(\theta) + \cos^2(\theta) = 1$ ✓
est vraie pour n'importe quelle valeur θ

b) i) $\cos\left(\frac{\theta}{2}\right) = 1$ $\quad e^{i\varphi}\sin\frac{\theta}{2} = 0$

$\cos\left(\frac{\theta}{2}\right) = 1$ $\quad e^{i\varphi}\sin\frac{\theta}{2} = 0$

État $|0\rangle$: $\theta = 0 \quad \varphi \in \mathbb{R}$

ii) $\cos\frac{\theta}{2} = 0$ $\quad e^{i\varphi}\sin\frac{\theta}{2} = 1$

$\cos\frac{\pi}{2} = 0$ $\quad \sin\frac{\pi}{2} = 1$

$\theta = \pi$ $\quad \theta = \pi$

État $|1\rangle$: $\theta = \pi \quad \phi = \cancel{0}$

$e^{i\phi} = 1$

$\phi = 0 \rightarrow$ comment savoir algébriquement que ϕ peut prendre n'importe quelle valeur?

iii) $|+\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$

$\cos\frac{\theta}{2} = \frac{1}{\sqrt{2}}$

$\arccos\frac{1}{\sqrt{2}} = \frac{\theta}{2} = \frac{\pi}{4}$

$\theta = \frac{\pi}{2}$

$e^{i\phi}\sin\left(\frac{\theta}{2}\right) = \frac{1}{\sqrt{2}}$

$e^{i\phi}\sin\frac{\frac{\pi}{2}}{2} = \frac{1}{\sqrt{2}}$

$e^{i\phi} = \frac{\frac{1}{\sqrt{2}}}{\sin(\pi/4)} = \frac{\frac{1}{\sqrt{2}}}{\frac{\sqrt{2}}{2}} = 2$

formule d'Euler

$e^{i\varphi} = \cos\varphi + i\sin\varphi = 1$

$\varphi = 0$

État $|+\rangle$: $\theta = \frac{\pi}{2}$
$\phi = 0$

iv) $|-\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$

$\cos\frac{\theta}{2} = \frac{1}{\sqrt{2}}$

$\theta = \frac{\pi}{2}$

$e^{i\phi}\sin\left(\frac{\theta}{2}\right) = -\frac{1}{\sqrt{2}}$

$\frac{e^{i\phi}\sin\left(\frac{\frac{\pi}{2}}{2}\right)}{\sin(\pi/4)} = \frac{-\frac{1}{\sqrt{2}}}{\sin(\pi/4)}$

$e^{i\phi} = \cos\phi + i\sin\phi = -1$

$\phi = \pi$

État $|-\rangle$: $\theta = \frac{\pi}{2}$
$\phi = \pi$

V) $|{+i}\rangle$: $\cos\frac{\theta}{2} = 1/\sqrt{2}$

$\theta = \frac{\pi}{2}$

$e^{i\phi}\sin\left(\frac{\theta}{2}\right) = i\,1/\sqrt{2}$

$e^{i\phi} = i\,\frac{1/\sqrt{2}}{\sin(\pi/4)}$

$e^{i\varphi} = i\cdot 1$

$e^{i\varphi} = i = \pi/2$

État $|{+i}\rangle$: $\theta = \frac{\pi}{2}$

$\varphi = \frac{\pi}{2}$

III) $|{-i}\rangle$ $\cos\frac{\theta}{2} = 1/\sqrt{2}$

$\theta = \frac{\pi}{2}$

$e^{i\varphi}\sin\left(\frac{\theta}{2}\right) = -i\,1/\sqrt{2}$

$e^{i\varphi} = -i\,\frac{1/\sqrt{2}}{\sin(\pi/4)}$

$e^{i\varphi} = -i = -\pi/2$

État $|{-i}\rangle$: $\theta = \frac{\pi}{2}$

$\varphi = -\frac{\pi}{2}$