#1 - a) $\hat{H}\hat{X}\hat{H} = \hat{Z}$

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$$\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\cdot\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\cdot\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$

$$\frac{1}{2}\begin{pmatrix}1 & 1\\ -1 & 1\end{pmatrix}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$

$$\frac{1}{2}\begin{pmatrix}2 & 0\\ 0 & -2\end{pmatrix}=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$

$$\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$ ✓

b) $\hat{S}\hat{X}\hat{S}^{\dagger} = \hat{Y}$

$$\begin{pmatrix}1 & 0\\ 0 & i\end{pmatrix}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & -i\end{pmatrix}=\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}$$

$$\begin{pmatrix}0 & 1\\ i & 0\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & -i\end{pmatrix}=\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}$$

$$\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}=\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}$$ ✓

c) $\hat{S}\hat{Z}\hat{S}^{+} = \hat{Z}$

$$\begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -i \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & -i \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -i \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \checkmark$$

d) $\hat{X}\hat{Z}\hat{X} = -\hat{Z}$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \checkmark$$

#2-a) $\hat{X} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\hat{X}^T = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $(\hat{X}^T)^* = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

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$\hat{X} = \hat{X}^\dagger$ ✓

$\hat{Y} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ $\hat{Y}^T = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$ $(\hat{Y}^T)^* = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

$\hat{Y} = \hat{Y}^\dagger$ ✓

$\hat{Z} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ $\hat{Z}^T = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ $(\hat{Z}^T)^* = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$\hat{Z} = \hat{Z}^\dagger$ ✓

b) $\hat{X}\hat{Y} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} \neq \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = i\hat{Z}$ Il faut quand même garder les phases globales!

-1 $\hat{Z}\hat{X} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} = \hat{Y}^T = i\hat{Y}$

$\hat{Y}\hat{Z} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = i\hat{X}$

$\hat{Y}\hat{X} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} = -i\hat{Z}$

$\hat{X}\hat{Z} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = -i\hat{Y}$

$\hat{Z}\hat{Y} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} = -i\hat{X}$

le produit de deux matrices de Pauli différentes peut toujours s'exprimer à l'aide de la troisième matrice de Pauli non utilisée

-1 $\pm i$ en fonction de l'ordre X, Y, Z ou Z, Y, X

#3 - a) $\hat{Y}\hat{S}\hat{H}\hat{Y}\hat{S}$ ok

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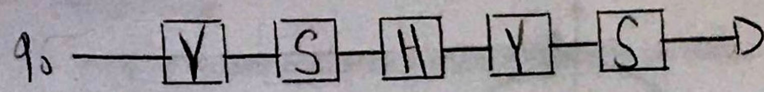

b) $\hat{G} = \hat{S}\hat{Y}\hat{H}\hat{S}\hat{Y}$ ok.

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$$\hat{S}\hat{Y}\hat{H}\hat{S}\hat{Y}|0\rangle = |+i\rangle$$

$$\hat{S}\hat{Y}\hat{H}\hat{S}\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = |+i\rangle$$

$$\hat{S}\hat{Y}\hat{H}\begin{pmatrix}1 & 0\\ 0 & i\end{pmatrix}\begin{pmatrix}0\\i\end{pmatrix} = |+i\rangle$$

$$\hat{S}\hat{Y}\cancel{\tfrac{1}{\sqrt{2}}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\begin{pmatrix}0\\i^2\end{pmatrix} = |+i\rangle$$

$$\frac{1}{\sqrt{2}}\hat{S}\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}\begin{pmatrix}i^2\\-i^2\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\i\end{pmatrix}$$

$$\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 0\\ 0 & i\end{pmatrix}\begin{pmatrix}i^3\\i^3\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}i^3\\-i\end{pmatrix}$$

$$\begin{pmatrix}i^3\\i^4\end{pmatrix} = i^3\begin{pmatrix}1\\i\end{pmatrix} = i^3|+i\rangle$$

Autres états?