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BSQ-110

DEVOIR #4

#1-a) $\hat{I} \otimes \hat{H} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{pmatrix}$

$\hat{P} = (\hat{I} \otimes \hat{H}) C\hat{X} = 1/\sqrt{2} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} = \boxed{\begin{pmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & -1 & 1 & 0 \end{pmatrix} 1/\sqrt{2}}$ ✓

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b) $|\Phi^{+}\rangle = \begin{pmatrix} 1/\sqrt{2} \\ 0 \\ 0 \\ 1/\sqrt{2} \end{pmatrix} \longrightarrow 1/\sqrt{2} \begin{pmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & -1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} \\ 0 \\ 0 \\ 1/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 2/\sqrt{2} \\ 0 \\ 0 \\ 0 \end{pmatrix} 1/\sqrt{2}$

$= \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ ✓

$$|\Phi^-\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\0\\-1\end{pmatrix}$$

$$\rightarrow \frac{1}{\sqrt{2}}\begin{pmatrix}1&0&0&1\\1&0&0&-1\\0&1&1&0\\0&-1&1&0\end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\0\\-1\end{pmatrix} = \frac{1}{2}\begin{pmatrix}0\\2\\0\\0\end{pmatrix} = \begin{pmatrix}0\\1\\0\\0\end{pmatrix}$$ ✓

$$|\Psi^+\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\1\\0\end{pmatrix}$$

$$\rightarrow \frac{1}{\sqrt{2}}\begin{pmatrix}1&0&0&1\\1&0&0&-1\\0&1&1&0\\0&-1&1&0\end{pmatrix}\begin{pmatrix}0\\1\\1\\0\end{pmatrix}\frac{1}{\sqrt{2}} = \frac{1}{2}\begin{pmatrix}0\\0\\2\\0\end{pmatrix} = \begin{pmatrix}0\\0\\1\\0\end{pmatrix}$$ ✓

$$|\psi^-\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\-1\\0\end{pmatrix}$$

$$\rightarrow \frac{1}{\sqrt{2}}\begin{pmatrix}1&0&0&1\\1&0&0&-1\\0&1&1&0\\0&-1&1&0\end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\-1\\0\end{pmatrix} = \frac{1}{2}\begin{pmatrix}0\\0\\0\\-2\end{pmatrix} = \begin{pmatrix}0\\0\\0\\-1\end{pmatrix}$$

$$|00\rangle = \begin{pmatrix}1\\0\\0\\0\end{pmatrix}$$

$$\rightarrow \frac{1}{\sqrt{2}}\begin{pmatrix}1&0&0&1\\1&0&0&-1\\0&1&1&0\\0&-1&1&0\end{pmatrix}\begin{pmatrix}1\\0\\0\\0\end{pmatrix} = \begin{pmatrix}1\\1\\0\\0\end{pmatrix}\frac{1}{\sqrt{2}}$$

$$P_{00} = \left|\frac{1}{\sqrt{2}}\right|^2 = \frac{1}{2}$$

$$P_{01} = \left|\frac{1}{\sqrt{2}}\right|^2 = \frac{1}{2}$$

#2 - a) $\hat{C}\hat{H}_{10}|00\rangle = |00\rangle$ b)

$$\hat{C}\hat{H}_{10}|01\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\0\\1\end{pmatrix} = \frac{1}{\sqrt{2}}|01\rangle + \frac{1}{\sqrt{2}}|11\rangle$$ ✓

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$$\hat{C}\hat{H}_{10}|10\rangle = |10\rangle$$ ✓

$$\hat{C}\hat{H}_{10}|11\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\0\\-1\end{pmatrix} = \frac{1}{\sqrt{2}}|01\rangle - \frac{1}{\sqrt{2}}|11\rangle$$

c) $$\hat{C}\hat{H} = \begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1/\sqrt{2} & 0 & 1/\sqrt{2}\\ 0 & 0 & 1 & 0\\ 0 & 1/\sqrt{2} & 0 & -1/\sqrt{2}\end{pmatrix}$$ ✓

#3- $\hat{H} \otimes \hat{H} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \otimes \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

a)

$$= \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix} \otimes \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix}$$

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$$= \begin{pmatrix} 1/2 & 1/2 & 1/2 & 1/2 \\ 1/2 & -1/2 & 1/2 & -1/2 \\ 1/2 & 1/2 & -1/2 & -1/2 \\ 1/2 & -1/2 & -1/2 & 1/2 \end{pmatrix} = \hat{P}$$ ✓

$$\rightarrow \hat{P}|00\rangle = \begin{pmatrix} 1/2 \\ 1/2 \\ 1/2 \\ 1/2 \end{pmatrix} \rightarrow \begin{aligned} P_{00} &= |1/2|^2 = 1/4 \\ P_{01} &= |1/2|^2 = 1/4 \\ P_{10} &= |1/2|^2 = 1/4 \\ P_{11} &= |1/2|^2 = 1/4 \end{aligned}$$ ✓

$$\rightarrow \hat{p}|01\rangle = \begin{pmatrix} 1/2 \\ -1/2 \\ 1/2 \\ -1/2 \end{pmatrix} \qquad \hat{p}|10\rangle = \begin{pmatrix} 1/2 \\ 1/2 \\ -1/2 \\ -1/2 \end{pmatrix}$$

$$\hat{p}|11\rangle = \begin{pmatrix} 1/2 \\ -1/2 \\ -1/2 \\ 1/2 \end{pmatrix}$$

les états obtenus sont similaires car la probabilité pour chaque état de base possible ne change pas mais les signes changent de.

phases relatives.

3|a

$$\hat{CX}_{10}\,|{-}{+}\rangle = \hat{CX}_{10}\left(- \otimes + \right) = \hat{CX}_{10} \left(\begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix} \otimes \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} \right)$$

$$= \hat{CX}_{10} \begin{pmatrix} 1/2 \\ 1/2 \\ -1/2 \\ -1/2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1/2 \\ 1/2 \\ -1/2 \\ -1/2 \end{pmatrix}$$

$$= \begin{pmatrix} 1/2 \\ -1/2 \\ -1/2 \\ 1/2 \end{pmatrix}$$

non ce n'est pas surprenant comme résultat vu que comme le nom du phénomène l'indique, la phase du sous-programme ? est obtenue par rapport à l'état du contrôleur

-1 $= |{-}{-}\rangle$