Supervised Learning Project Report

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1 Problem 1

1.1 KNN with k = 1, 5, 50

The algorithm is very intuitive. For any test data x_i, we compute the distance between x_i and all training data $\tilde{x}$ and store these distance into an array A, and sort array A in an ascending order. For different k, we get the first k element from array A and their corresponding label. The majority label decides the label of our test data x_i.

We implement the KNN algorithm in both Matlab and Java. The file name of the k-nearest neighbor algorithm is "knnForDigit.m". The test file of the k-nearest neighbor algorithm is "testknnForDigit.m". We use 3437 instances in the dataset to train the knn model, and use 1000 instances to test the model. The test results are as follows
When k = 1, the correct rate is 99.3%.
When k = 5, the correct rate is 98.1%.
When k = 50, the correct rate is 92.2%.

1.2 Support vector machine

From the class, we learned that the soft-margin support vector machine has a prime problem as follows:

$$\begin{aligned} &\min_{\beta,\beta_0} \frac{1}{2}||\beta|| + C\sum_{i=1}^{N}\xi_i \\ &\text{subject to } y_i(x_i^T\beta + \beta_0) \geq 1 - \xi_i,\ \xi_i \geq 0,\ i = 1, ..., N \end{aligned} \tag{1}$$

and its dual

$$\begin{aligned} \min L_D &= \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\alpha_i\alpha_j y_i y_j x_i^T x_j^T - \sum_{i=1}^{N}\alpha_i \\ \text{subject to } & 0 \le \alpha_i \le C \\ & \sum_{i=1}^{N}\alpha_i y_i = 0 \end{aligned} \tag{2}$$

We use Matlab quadratical problem solver 'quadprog' to solve dual problem and the problem is formalized as follows to plug into 'quadprog'.

$$\begin{aligned} \min L_D &= \frac{1}{2}\alpha^T H\alpha - f^T\alpha \\ \text{subject to } & A\alpha \le \mathbf{C} \\ & Aeq\alpha = beq \end{aligned} \tag{3}$$

where $\alpha = (\alpha_1, ..., \alpha_N)^T, H(i,j) = y_i y_j x_i^T x_j^T, H \in N*N, f = (1, ..., 1)^T, A = \text{diag}(1, ..., 1) \in N*N, Aeq = (y_1, ..., y_N)^T, beq = 0 \in N*1$. And also α has lower bound 0 and upper bound C. Then we use matlab command 'quadprog(H,f,A,b,Aeq,beq,LB,UB)' to solve α_i and then solve β and β_0 using

$$\begin{aligned} \beta &= \sum_{i=1}^{N}\alpha_i y_i x_i \\ \alpha_i(y_i(x_i^T\beta + \beta_0) - (1 - \xi_i)) &= 0. \end{aligned} \tag{4}$$

We also use an average of all the solutions for β_0 due to numerical stability. Since we have ten digit to classify, we build ten decision function to decide which digit our tests fall. Thus we have ten different α vectors, denoted by

α_{iR} which lead to ten different β_{iR} and $\beta_{0iR}, i = 0, ..., 9$.

$$\begin{aligned}
f_{0R} &= x^T\beta_{0R} + \beta_{00R} \\
f_{1R} &= x^T\beta_{1R} + \beta_{01R} \\
f_{2R} &= x^T\beta_{2R} + \beta_{02R} \\
f_{3R} &= x^T\beta_{3R} + \beta_{03R} \\
f_{4R} &= x^T\beta_{4R} + \beta_{04R} \\
f_{5R} &= x^T\beta_{5R} + \beta_{05R} \\
f_{6R} &= x^T\beta_{6R} + \beta_{06R} \\
f_{7R} &= x^T\beta_{7R} + \beta_{07R} \\
f_{8R} &= x^T\beta_{8R} + \beta_{08R} \\
f_{9R} &= x^T\beta_{9R} + \beta_{09R}
\end{aligned} \tag{5}$$

And the maximal of f_{iR} decides the digit of the test x.
The file name of the support vector machine algorithm is "SVMDigitClassifier_1.m". The test file of the support vector machine algorithm is "TestSVMClassifier_1.m". We implement three different kernels. They are implemented separately in the "GKernel.m", "DegreePolyKernel.m" and "NeuralNetworkKernel.m" files. Their equations are as follows

$$\text{Gaussian} \ : K(x, x') = exp(-|x - x'|^2/2\sigma^2); \tag{6}$$

$$\text{polynomial} : K(x, x') = (1+ < x, x' >)^d; \tag{7}$$

$$\text{neural network} : K(x, x') = tanh(k_1 < x, x' > +k_2); \tag{8}$$

where $< x, x' >= x^T * x'$. And the process of solving kernel SVM is a little different from original SVM. Instead of solving β, we solve kernel function. The other process are the same of original SVM. For the result of kernel SVM and original SVM, please check Problem 4.

2 Problem 2

The step for ten-fold cross validation: first, we choose 10% of the train data randomly by using 'randsample'; then, the remaining 90% of the data are used for training data while this 10% are used for testing; finally, we run ten times and get an average confusion matrix. We have generated confusion matrix for knn, $k = 1, k = 5$, and $k = 50$. And row i and column j means that the true label for this digit is i and our algorithm predicts it as

j. Therefore, it is a diagonally dominate matrix and exactly what we get.

C_1 is the confusion matrix computed by KNN for k = 1 and mean error is 0.64%:

$$C_1 = \begin{pmatrix} \frac{131}{1250} & 0 & 0 & 0 & \frac{1}{1250} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{188}{1875} & \frac{1}{1250} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{750} & \frac{131}{1250} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{3750} & 0 & \frac{121}{1250} & 0 & 0 & 0 & \frac{1}{1875} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{383}{3750} & 0 & 0 & 0 & 0 & \frac{1}{3750} \\ 0 & 0 & 0 & 0 & 0 & \frac{191}{1875} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{373}{3750} & 0 & 0 & 0 \\ 0 & \frac{1}{1875} & 0 & \frac{1}{3750} & 0 & 0 & 0 & \frac{373}{3750} & 0 & 0 \\ \frac{1}{3750} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{341}{3750} & 0 \\ 0 & \frac{1}{1875} & 0 & \frac{1}{3750} & \frac{1}{3750} & \frac{1}{3750} & 0 & 0 & 0 & \frac{349}{3750} \end{pmatrix}.$$

C_2 is the confusion matrix computed by KNN for k = 5 and mean error is 0.83%:

$$C_2 = \begin{pmatrix} \frac{41}{375} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{383}{3750} & \frac{3}{1250} & \frac{4}{1875} & 0 & 0 & 0 & \frac{1}{3750} & 0 & 0 \\ 0 & 0 & \frac{187}{1875} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{3750} & \frac{1}{3750} & \frac{347}{3750} & 0 & \frac{1}{3750} & 0 & \frac{1}{3750} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{13}{125} & 0 & 0 & \frac{1}{3750} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{188}{1875} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{121}{1250} & 0 & 0 & 0 \\ 0 & \frac{1}{1875} & 0 & 0 & 0 & 0 & 0 & \frac{181}{1875} & 0 & 0 \\ \frac{1}{1250} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3750} & \frac{182}{1875} & 0 \\ 0 & 0 & 0 & \frac{1}{1875} & 0 & 0 & 0 & 0 & 0 & \frac{7}{75} \end{pmatrix}.$$

C_3 is the confusion matrix computed by KNN for k = 50 and mean error

is 6.43%:

$$C_3 = \begin{pmatrix} \frac{377}{3750} & 0 & 0 & 0 & \frac{3}{1250} & 0 & \frac{1}{625} & 0 & 0 & 0 \\ 0 & \frac{107}{1250} & \frac{2}{125} & \frac{7}{1875} & 0 & \frac{1}{1875} & \frac{1}{1875} & \frac{7}{3750} & 0 & 0 \\ 0 & \frac{1}{1250} & \frac{123}{1250} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{625} & \frac{1}{1875} & \frac{61}{625} & 0 & 0 & 0 & \frac{1}{1250} & 0 & 0 \\ 0 & \frac{1}{1875} & 0 & 0 & \frac{64}{625} & 0 & 0 & \frac{1}{1875} & 0 & 0 \\ 0 & 0 & 0 & \frac{3}{1250} & 0 & \frac{178}{1875} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{176}{1875} & 0 & 0 & 0 \\ 0 & \frac{13}{3750} & 0 & \frac{13}{3750} & 0 & 0 & 0 & \frac{347}{3750} & 0 & 0 \\ \frac{1}{1250} & \frac{1}{625} & \frac{1}{1875} & \frac{7}{3750} & 0 & \frac{1}{750} & \frac{1}{750} & \frac{1}{375} & \frac{323}{3750} & \frac{1}{1250} \\ \frac{1}{625} & \frac{11}{3750} & 0 & \frac{3}{1250} & \frac{2}{1875} & \frac{17}{3750} & 0 & 0 & 0 & \frac{157}{1875} \end{pmatrix}.$$

We only computer SVM using Gaussian kernel with $C = 0.05$ and $\sigma = 59$ due to the huge amount of time. Mean error is 4.55%.

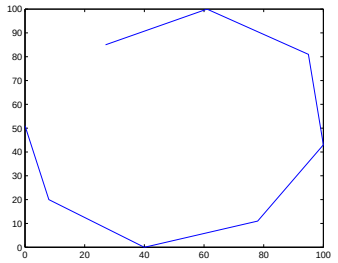
$$C_4 = \begin{pmatrix} 0.1093 & 0.0107 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.1147 & 0.0080 & 0.0027 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.0053 & 0.1200 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.0960 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.0027 & 0 & 0 & 0.0960 & 0 & 0 & 0 & 0 & 0.0027 \\ 0 & 0 & 0 & 0 & 0 & 0.0933 & 0 & 0 & 0 & 0.0027 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.0693 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0827 & 0 & 0 \\ 0 & 0.0053 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0827 & 0 \\ 0 & 0.0027 & 0 & 0 & 0 & 0.0027 & 0 & 0 & 0 & 0.0907 \end{pmatrix}.$$

3 Problem 3

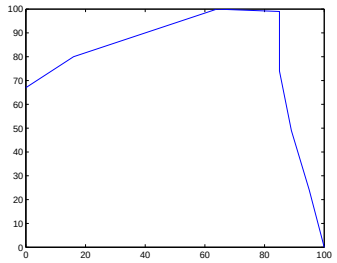
For Guassian SVM formulation, we have each digit as follows. We use train data and the error is less than 0.05, $|f_{iR} - 1| \leq 0.05$. Please check (Figure 1) and (Figure 2).

4 Problem 4

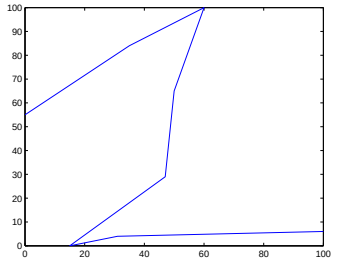
SVM is beneficial from generalization power because it is capable of moving the entire problem into a representation of greater dimension, enabling it to separate more complex problems. While KNN method is very intuitive, it computes the similarity of two digits and using majority vote to decide which label it is.



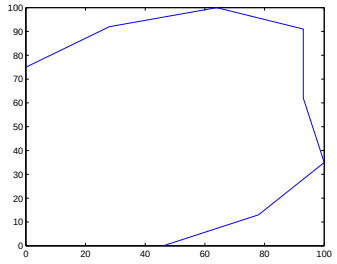
(a) digit 0



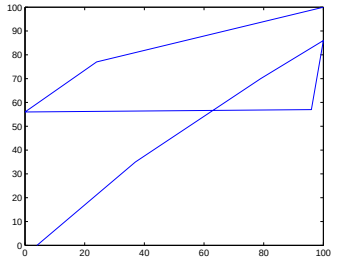
(b) digit 1



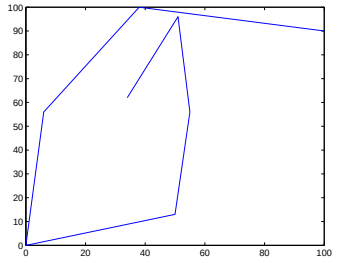
(c) digit 2



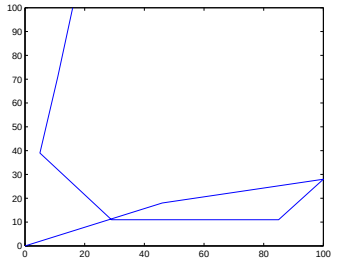
(d) digit 3



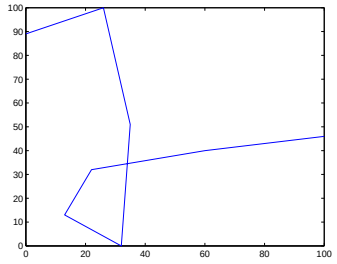
(e) digit 4



(f) digit 5

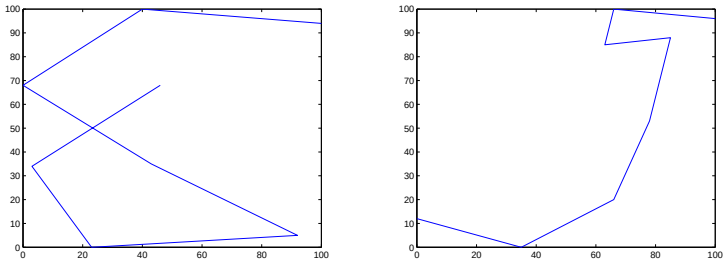


(g) digit 6



(h) digit 7

Figure 1: These figures present the support vector for digit 0 to 7



(a) digit 8 (b) digit 9

Figure 2: These figures present support vector for 8 and 9

The following table gives the overall correct rate of our methods. And as you can see, KNN-1 method has a very high correct rate because this problem is more like a local consistency problem.

label	Correct Rate	Parameter
KNN-1	99.3%	$k = 1$
KNN-5	98.1%	$k = 5$
kNN-50	92.2%	$k = 50$
SVM	99%	$C = 0.05, \xi = 5e - 5$
SVM-Gaussian	96.4%	$C = 0.05, \sigma = 59$
SVM-Polynomial	98.4%	$C = 0.05, d = 4$
SVM-Neural Network	87.03%	$C = 0.05, k_1 = 0.2, k_2 = -2$

We also compare the SVM and 1NN method for every digit by using the confusion matrix in the problem 2. As we can see, 1NN predict some 9 as 1 and SVM does not. SVM predict some 5 to 10 and 1NN does not. This is because KNN is local consistency and SVM is global consistency. If it is global consistency, it can be separated by a hyperplane. When it is local consistency, majority vote is very efficient.

5 Problem 5

We predict the digit using KNN-1 algorithm, the result is in the "3_predictResult.csv" file.

6 Problem 6

The transfer SVM algorithm is in the "TransferSVM.m" file. The algorithm is as follows. First, solve source data set and get solution of β_s and β_{s0}. Then solve the dual problem of target set and get solution of β_t. After that, we solve β_{t0} using equation

$$\alpha_i(y_i(x_i^T\beta_s + \beta_{s0}) + y_i(x_i^T\beta_t + \beta_{t0})) = 1 - \xi_i \tag{9}$$

where we already solved β_s, β_{s0} and β_t. Finally, we compute the f_{08} like f_{iR} in the problem 1. And when $f_{08} \geq 0$, we label the digit as 0. When $f_{08} < 0$, we label the digit as 8. The result is in the "6_0vs8TestResult.csv" file.

7 Code

All the codes are inside the "code" folder.