Time Series Analysis

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Importing the required bibs for work

```
library(tseries)
```

```
## Registered S3 method overwritten by 'quantmod':
##   method             from
##   as.zoo.data.frame zoo
```

```
require(urca)
```

```
## Loading required package: urca
```

```
library(tidyverse)
```

```
## -- Attaching packages --------------------------------------- tidyverse 1.3.1 --
```

```
## v ggplot2 3.3.6     v purrr   0.3.4
## v tibble  3.1.7     v dplyr   1.0.9
## v tidyr   1.2.0     v stringr 1.4.0
## v readr   2.1.2     v forcats 0.5.1
```

```
## -- Conflicts ------------------------------------------ tidyverse_conflicts() --
## x dplyr::filter() masks stats::filter()
## x dplyr::lag()    masks stats::lag()
```

```
library(fpp2)
```

```
## -- Attaching packages ---------------------------------------------- fpp2 2.4 --
```

```
## v forecast  8.16     v expsmooth 2.3
## v fma       2.4
```

```
##
```

```
library(vars)
```

```
## Loading required package: MASS
```

```
## 
## Attaching package: 'MASS'

## The following objects are masked from 'package:fma':
## 
##     cement, housing, petrol

## The following object is masked from 'package:dplyr':
## 
##     select

## Loading required package: strucchange

## Loading required package: zoo

## 
## Attaching package: 'zoo'

## The following objects are masked from 'package:base':
## 
##     as.Date, as.Date.numeric

## Loading required package: sandwich

## 
## Attaching package: 'strucchange'

## The following object is masked from 'package:stringr':
## 
##     boundary

## Loading required package: lmtest
```

```r
library(forecast)
require(lmtest)
library(readxl)
suppressWarnings(expr)
```

```
## function (expr)
## {
##     enexpr(expr)
## }
## <bytecode: 0x7f8b22c63408>
## <environment: namespace:rlang>
```
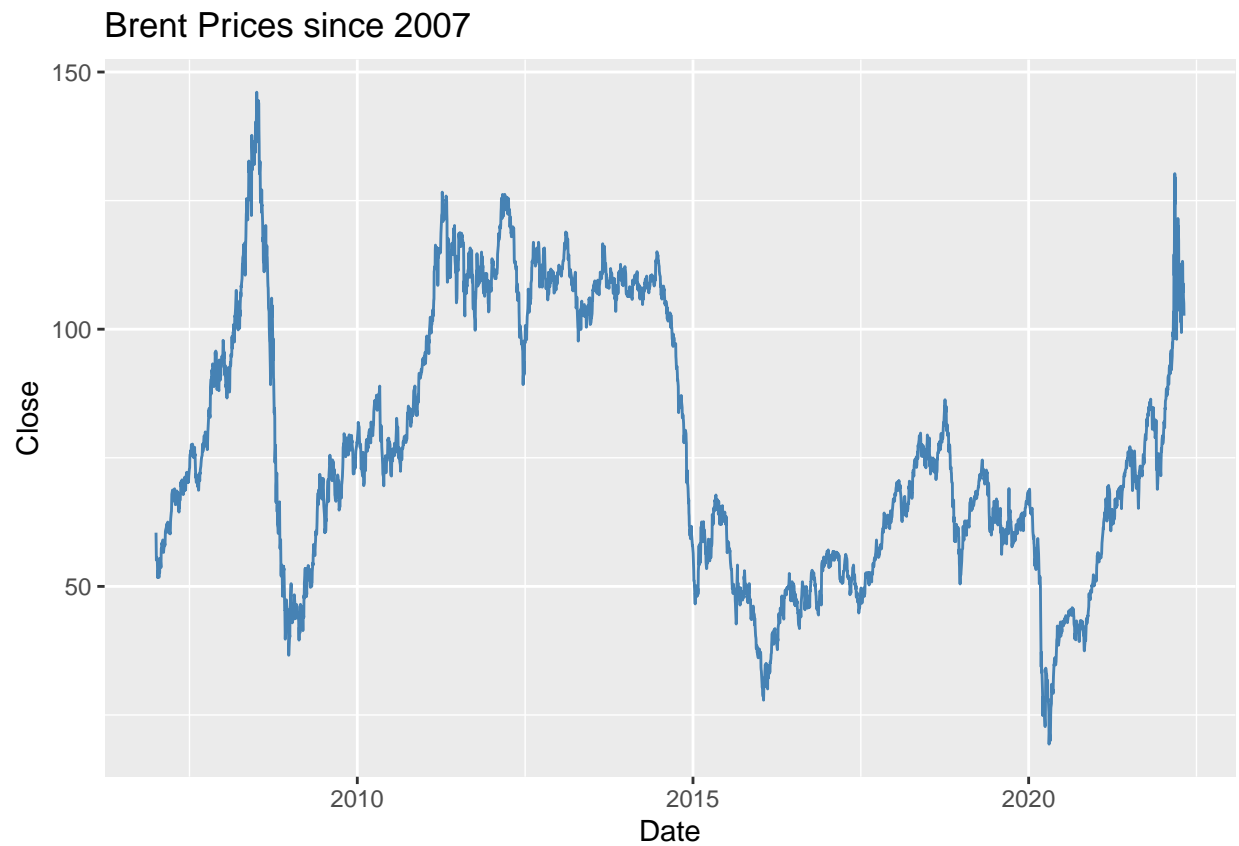
Importing the data

```r
brent_prices <- read_xlsx("brent_prices.xlsx")
Petr4_prices <- read_xlsx("Petr4_prices.xlsx")
```
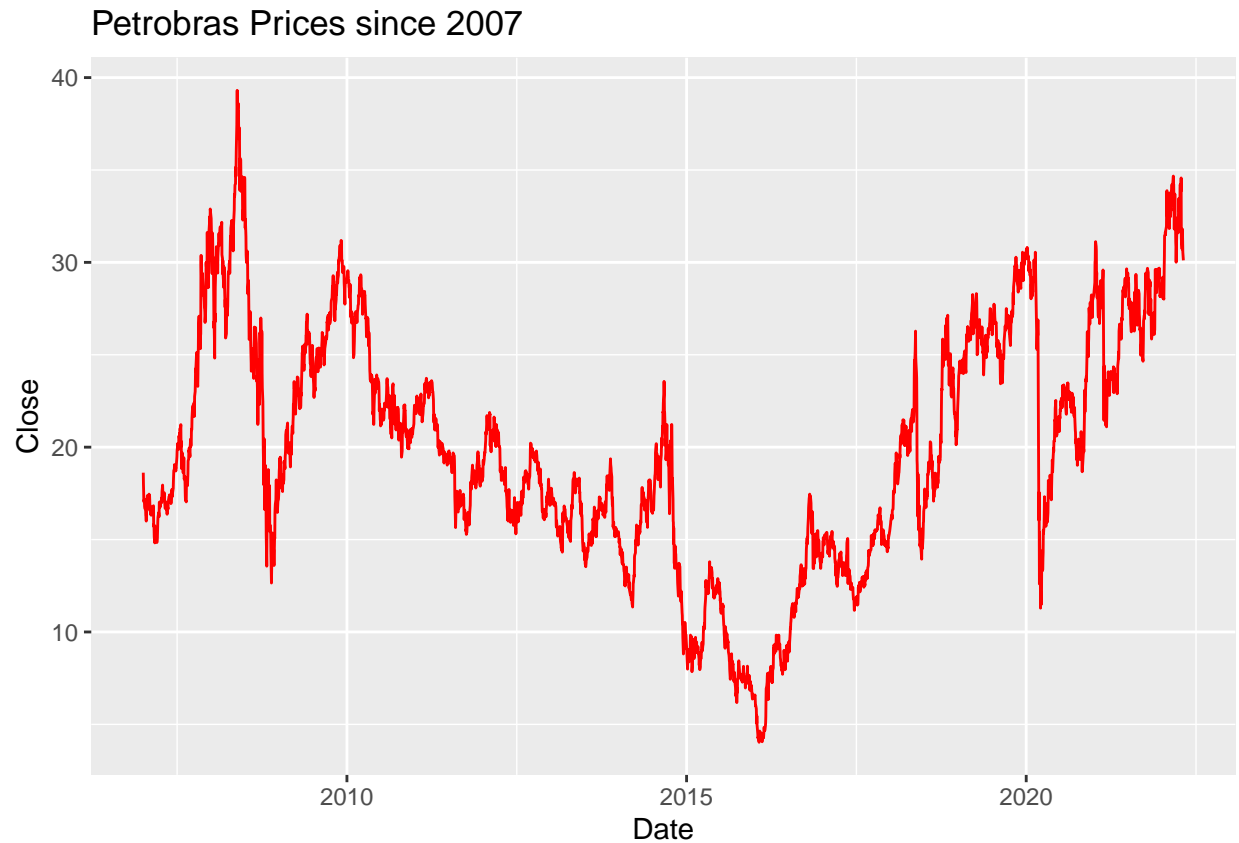
Brent Oil Prices Graph

```
ggplot(brent_prices, aes(x=Date)) +
  geom_line(aes(y = Close), color = "steelblue") +
  labs(title = 'Brent Prices since 2007')
```

Brent Prices since 2007

Petrobrás stock prices graph

```
ggplot(Petr4_prices, aes(x=Date)) +
  geom_line(aes(y = `Close`), color = "red") +
  labs(title = 'Petrobras Prices since 2007')
```

Petrobras Prices since 2007

Managing the data

```
final_data <- merge(Petr4_prices, brent_prices, by = 'Date')
names(final_data) <- c('Date','Petrobras', 'Brent Oil Prices')
normalized_final_data <- data_frame(final_data$Date,final_data[,2] / final_data[1,2],final_data[,3]/fina
```

```
## Warning: 'data_frame()' was deprecated in tibble 1.1.0.
## Please use 'tibble()' instead.
## This warning is displayed once every 8 hours.
## Call 'lifecycle::last_lifecycle_warnings()' to see where this warning was generated.
```

```
names(normalized_final_data) <- c('Date','Petrobras', 'Brent Oil Prices')
```

Brent x Petrobrás Graph in normalized scale

```
ggplot(normalized_final_data, aes(x = Date)) +
  geom_line(aes(y = Petrobras, color = "Petrobras")) +
  geom_line(aes(y = `Brent Oil Prices`, color = "Brent Oil Prices")) +
  labs(x = "Year",
       y = "(%)",
       color = "Legend")
```
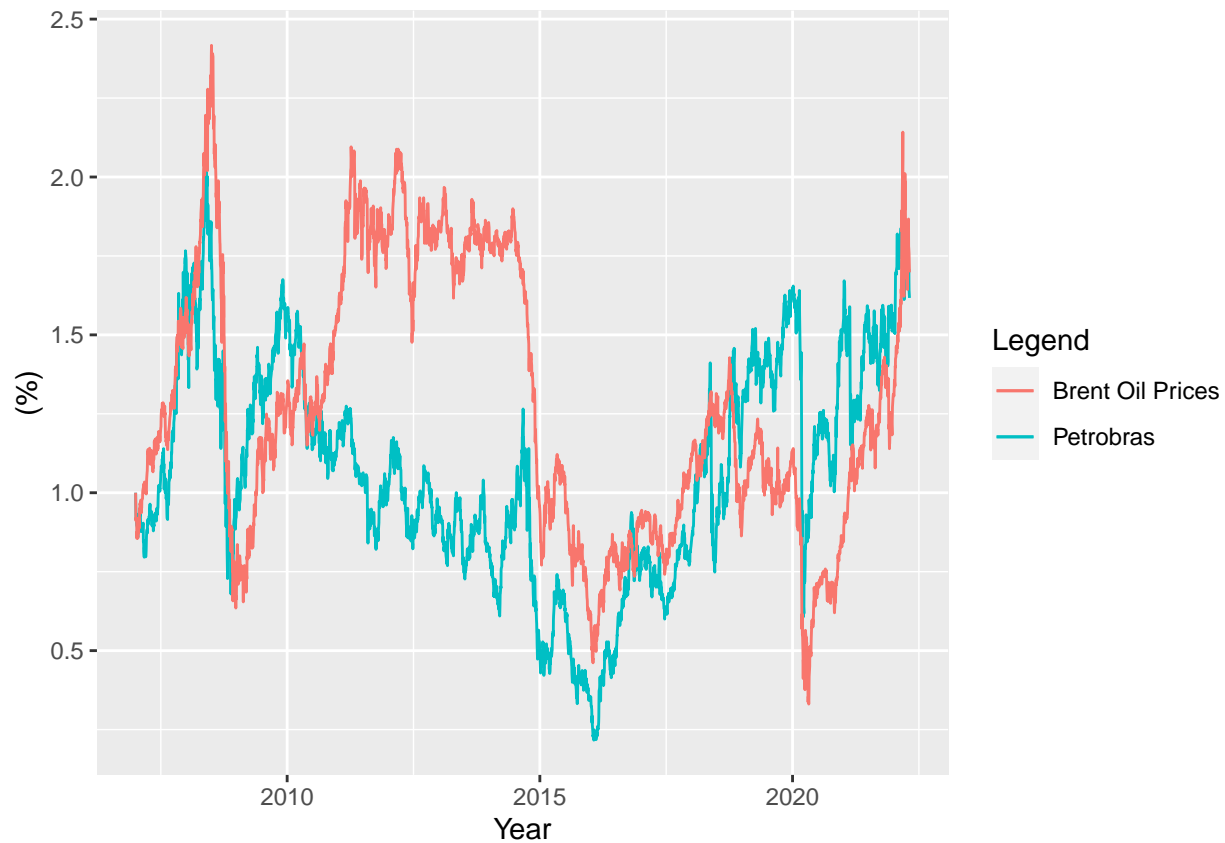
Transforming the data to the logarithm scale and plotting Petr4 x Brent Graph

```
log_data <- log(final_data[,2:3])
log_final_data <- data_frame(final_data$Date,log(final_data[,2:3]))
log_data <- data_frame(final_data$Date,log(final_data[,2:3]))
names(log_final_data) <- c('Date','Petrobras', 'Brent Oil Prices')
names(log_data) <- c('Date','Petrobras', 'Brent Oil Prices')

ggplot(log_final_data, aes(x = Date)) +
  geom_line(aes(y = Petrobras, color = "Petrobras")) +
  geom_line(aes(y = `Brent Oil Prices`, color = "Brent Oil Prices")) +
  labs(x = "Year",
       y = "(%)",
       color = "Legend",
       title = 'PETR4 x Brent Logarithm')
```

PETR4 x Brent Logarithm
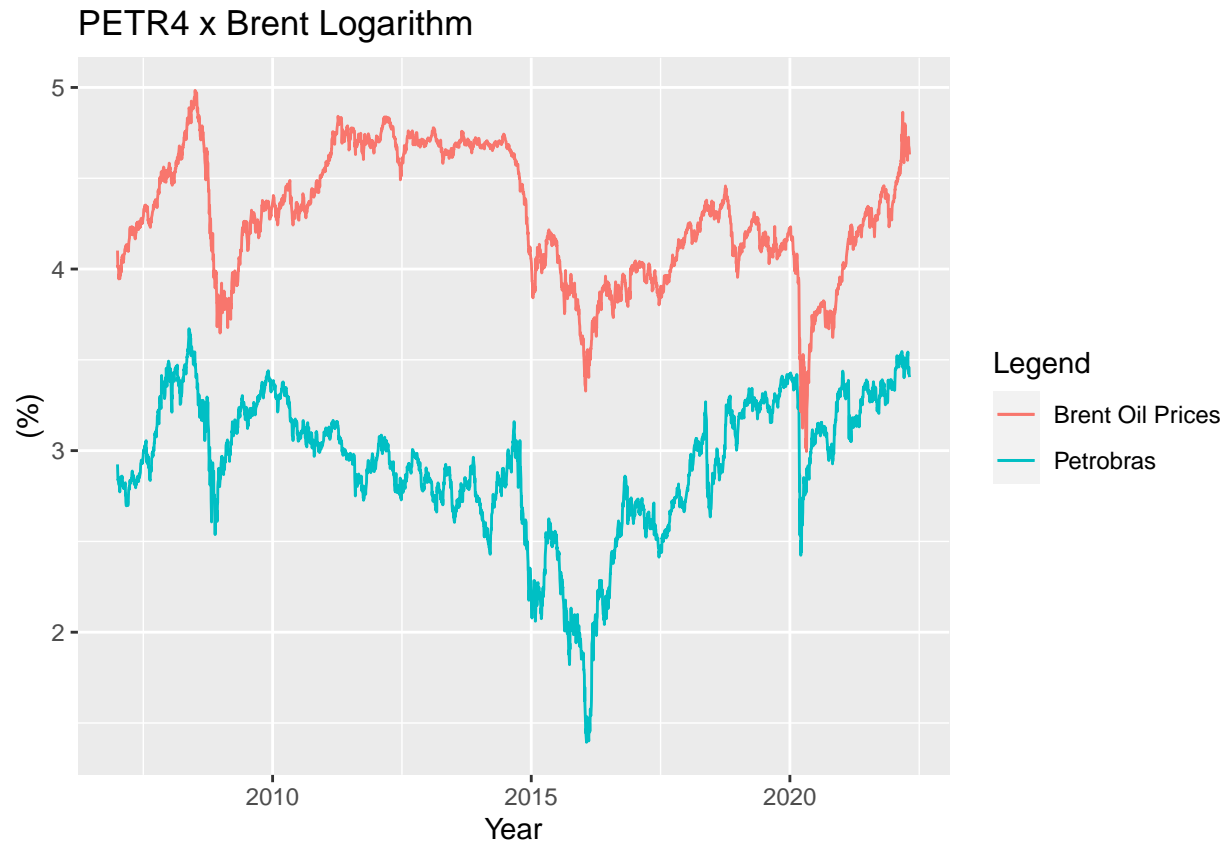
Augmented dickey fuller test for Brent Prices

```
adf_test_brent <- ur.df(log_data$`Brent Oil Prices`,selectlags = "BIC")
summary(adf_test_brent)
```

```
##
## ###############################################
## # Augmented Dickey-Fuller Test Unit Root Test #
## ###############################################
##
## Test regression none
##
##
## Call:
## lm(formula = z.diff ~ z.lag.1 - 1 + z.diff.lag)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.277534 -0.010297  0.000714  0.010998  0.192156
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## z.lag.1    2.063e-05  9.136e-05   0.226    0.821
## z.diff.lag -1.718e-02  1.627e-02  -1.056    0.291
##
## Residual standard error: 0.02415 on 3778 degrees of freedom
```

```
## Multiple R-squared:  0.0003075,   Adjusted R-squared:  -0.0002218
## F-statistic: 0.581 on 2 and 3778 DF,  p-value: 0.5594
##
##
## Value of test-statistic is: 0.2258
##
## Critical values for test statistics:
##      1pct  5pct 10pct
## tau1 -2.58 -1.95 -1.62
```

Augmented dickey fuller test for Brent Prices with drift

```
adf_test_brent <- ur.df(log_data$`Brent Oil Prices`,type="drift",selectlags = "BIC")
summary(adf_test_brent)
```

```
##
## ###############################################
## # Augmented Dickey-Fuller Test Unit Root Test #
## ###############################################
##
## Test regression drift
##
##
## Call:
## lm(formula = z.diff ~ z.lag.1 + 1 + z.diff.lag)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.278572 -0.010430  0.000732  0.011012  0.189506
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.009944   0.004846   2.052   0.0402 *
## z.lag.1     -0.002284   0.001127  -2.027   0.0427 *
## z.diff.lag  -0.015984   0.016269  -0.982   0.3259
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.02414 on 3777 degrees of freedom
## Multiple R-squared:  0.001382,   Adjusted R-squared:  0.0008529
## F-statistic: 2.613 on 2 and 3777 DF,  p-value: 0.07345
##
##
## Value of test-statistic is: -2.027 2.1311
##
## Critical values for test statistics:
##      1pct  5pct 10pct
## tau2 -3.43 -2.86 -2.57
## phi1  6.43  4.59  3.78
```

Augmented dickey fuller test for Brent Prices with trend

```
adf_test_brent <- ur.df(log_data$`Brent Oil Prices`,type="trend",selectlags = "BIC")
summary(adf_test_brent)
```

```
##
## ###############################################
## # Augmented Dickey-Fuller Test Unit Root Test #
## ###############################################
##
## Test regression trend
##
##
## Call:
## lm(formula = z.diff ~ z.lag.1 + 1 + tt + z.diff.lag)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.278335 -0.010410  0.000744  0.011055  0.189507
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  1.188e-02  5.583e-03   2.128   0.0334 *
## z.lag.1     -2.616e-03  1.223e-03  -2.139   0.0325 *
## tt          -2.731e-07  3.905e-07  -0.699   0.4844
## z.diff.lag  -1.576e-02  1.627e-02  -0.969   0.3328
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.02414 on 3776 degrees of freedom
## Multiple R-squared:  0.001511,   Adjusted R-squared:  0.0007177
## F-statistic: 1.905 on 3 and 3776 DF,  p-value: 0.1266
##
##
## Value of test-statistic is: -2.1394 1.5836 2.2987
##
## Critical values for test statistics:
##       1pct  5pct 10pct
## tau3 -3.96 -3.41 -3.12
## phi2  6.09  4.68  4.03
## phi3  8.27  6.25  5.34
```

Augmented dickey fuller test for Petrobrás stock prices

```
adf_test_petr <- ur.df(log_data$Petrobras,type="none", selectlags = "BIC")
summary(adf_test_petr)
```

```
##
## ###############################################
## # Augmented Dickey-Fuller Test Unit Root Test #
## ###############################################
##
## Test regression none
##
```

```
##
## Call:
## lm(formula = z.diff ~ z.lag.1 - 1 + z.diff.lag)
##
## Residuals:
##      Min        1Q    Median        3Q       Max
## -0.35544 -0.01446   0.00077   0.01486   0.19378
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## z.lag.1     1.575e-06  1.638e-04   0.010   0.9923
## z.diff.lag -3.002e-02  1.626e-02  -1.846   0.0649 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.02972 on 3778 degrees of freedom
## Multiple R-squared:  0.0009016,  Adjusted R-squared:  0.0003727
## F-statistic: 1.705 on 2 and 3778 DF,  p-value: 0.182
##
##
## Value of test-statistic is: 0.0096
##
## Critical values for test statistics:
##      1pct  5pct 10pct
## tau1 -2.58 -1.95 -1.62
```

Augmented dickey fuller test for Petrobrás stock prices with drift

```
adf_test_petr <- ur.df(log_data$Petrobras,type="drift", selectlags = "BIC")
summary(adf_test_petr)
```

```
##
## ###############################################
## # Augmented Dickey-Fuller Test Unit Root Test #
## ###############################################
##
## Test regression drift
##
##
## Call:
## lm(formula = z.diff ~ z.lag.1 + 1 + z.diff.lag)
##
## Residuals:
##      Min        1Q    Median        3Q       Max
## -0.35487 -0.01459   0.00065   0.01476   0.19289
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.008233   0.003760   2.189   0.0286 *
## z.lag.1     -0.002764   0.001274  -2.170   0.0301 *
## z.diff.lag  -0.028561   0.016266  -1.756   0.0792 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
##
## Residual standard error: 0.0297 on 3777 degrees of freedom
## Multiple R-squared:  0.002147,   Adjusted R-squared:  0.001618
## F-statistic: 4.063 on 2 and 3777 DF,  p-value: 0.01727
##
##
## Value of test-statistic is: -2.1699 2.3964
##
## Critical values for test statistics:
##       1pct  5pct 10pct
## tau2 -3.43 -2.86 -2.57
## phi1  6.43  4.59  3.78
```

Augmented dickey fuller test for Petrobrás stock prices with trend

```
adf_test_petr <- ur.df(log_data$Petrobras,type="trend", selectlags = "BIC")
summary(adf_test_petr)
```

```
##
## ###############################################
## # Augmented Dickey-Fuller Test Unit Root Test #
## ###############################################
##
## Test regression trend
##
##
## Call:
## lm(formula = z.diff ~ z.lag.1 + 1 + tt + z.diff.lag)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.35511 -0.01455  0.00066  0.01472  0.19265
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  7.882e-03  3.874e-03   2.035   0.0419 *
## z.lag.1     -2.753e-03  1.275e-03  -2.160   0.0308 *
## tt           1.674e-07  4.429e-07   0.378   0.7055
## z.diff.lag  -2.861e-02  1.627e-02  -1.759   0.0787 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.02971 on 3776 degrees of freedom
## Multiple R-squared:  0.002185,   Adjusted R-squared:  0.001392
## F-statistic: 2.756 on 3 and 3776 DF,  p-value: 0.04095
##
##
## Value of test-statistic is: -2.1599 1.6449 2.425
##
## Critical values for test statistics:
##       1pct  5pct 10pct
## tau3 -3.96 -3.41 -3.12
## phi2  6.09  4.68  4.03
## phi3  8.27  6.25  5.34
```

Log Returns for the data

```
log_ret_petr <- diff(log_data$Petrobras)
log_ret_brent <- diff(log_data$`Brent Oil Prices`)
```
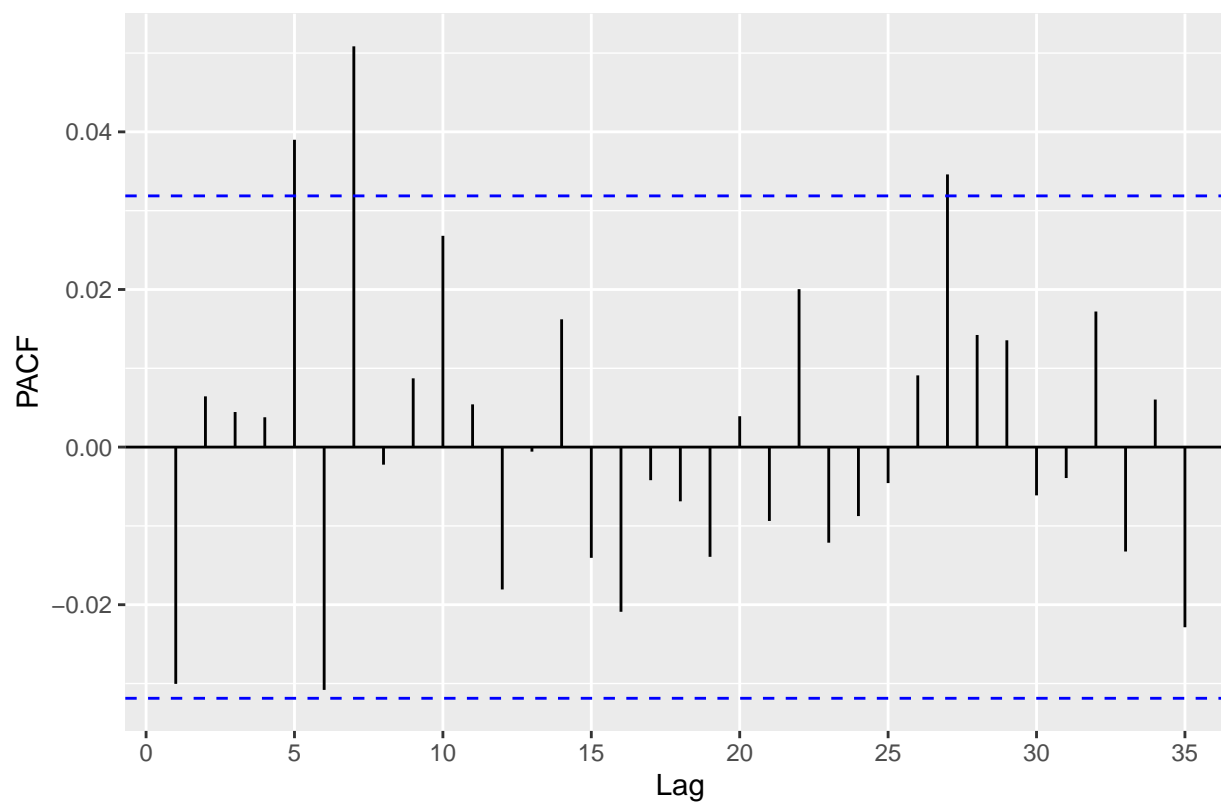
ACF, PACF and series of returns for Petrobras

```
ggAcf(log_ret_petr) + labs(title = "AutoCorrelation for Petrobras Log Diff")
```
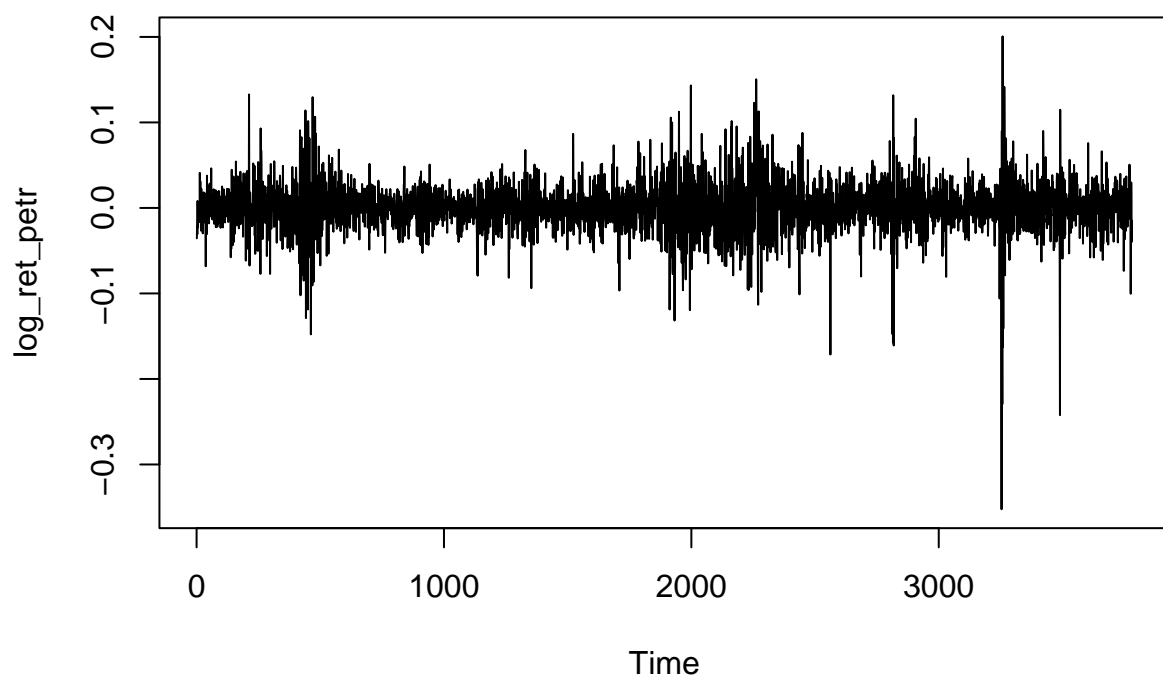
AutoCorrelation for Petrobras Log Diff
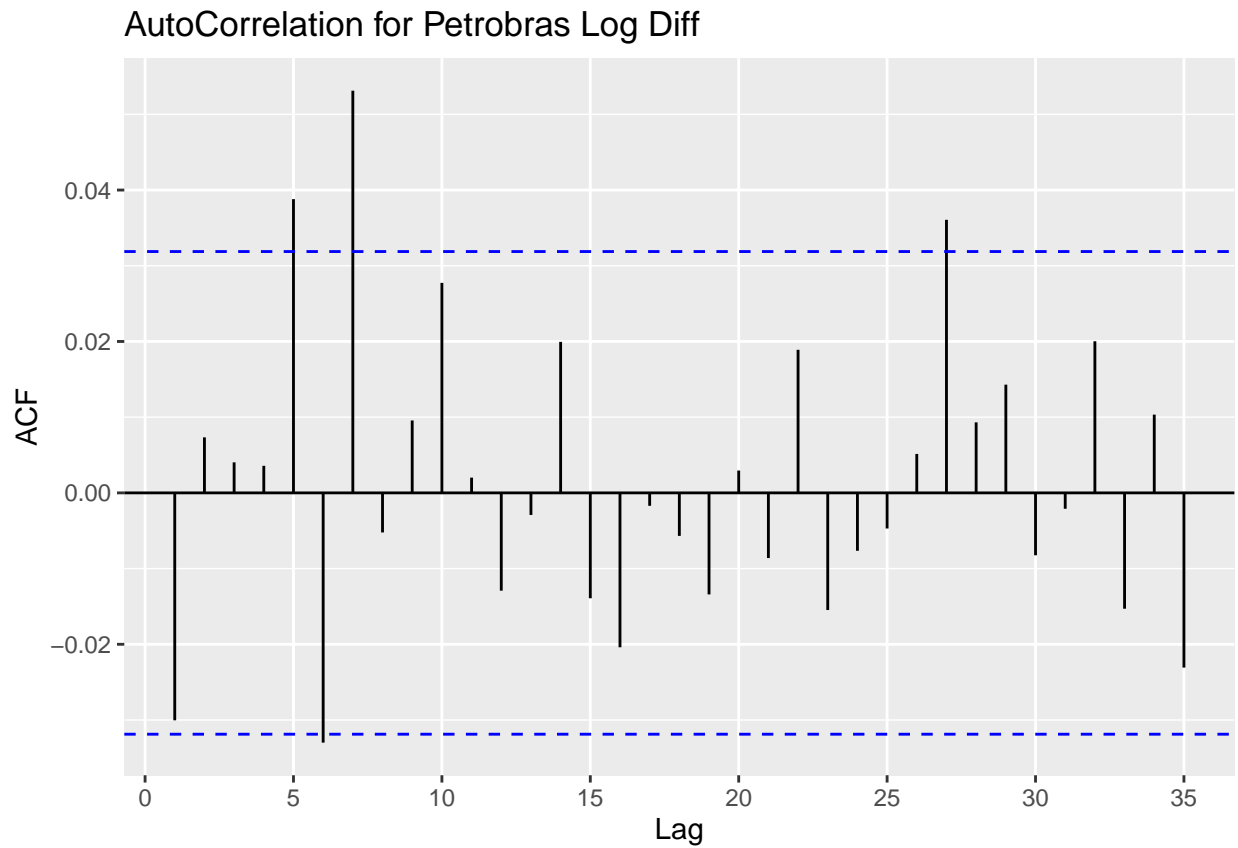
```
ggPacf(log_ret_petr) + labs(title = "Partial AutoCorrelation for Petrobras Log Diff")
```
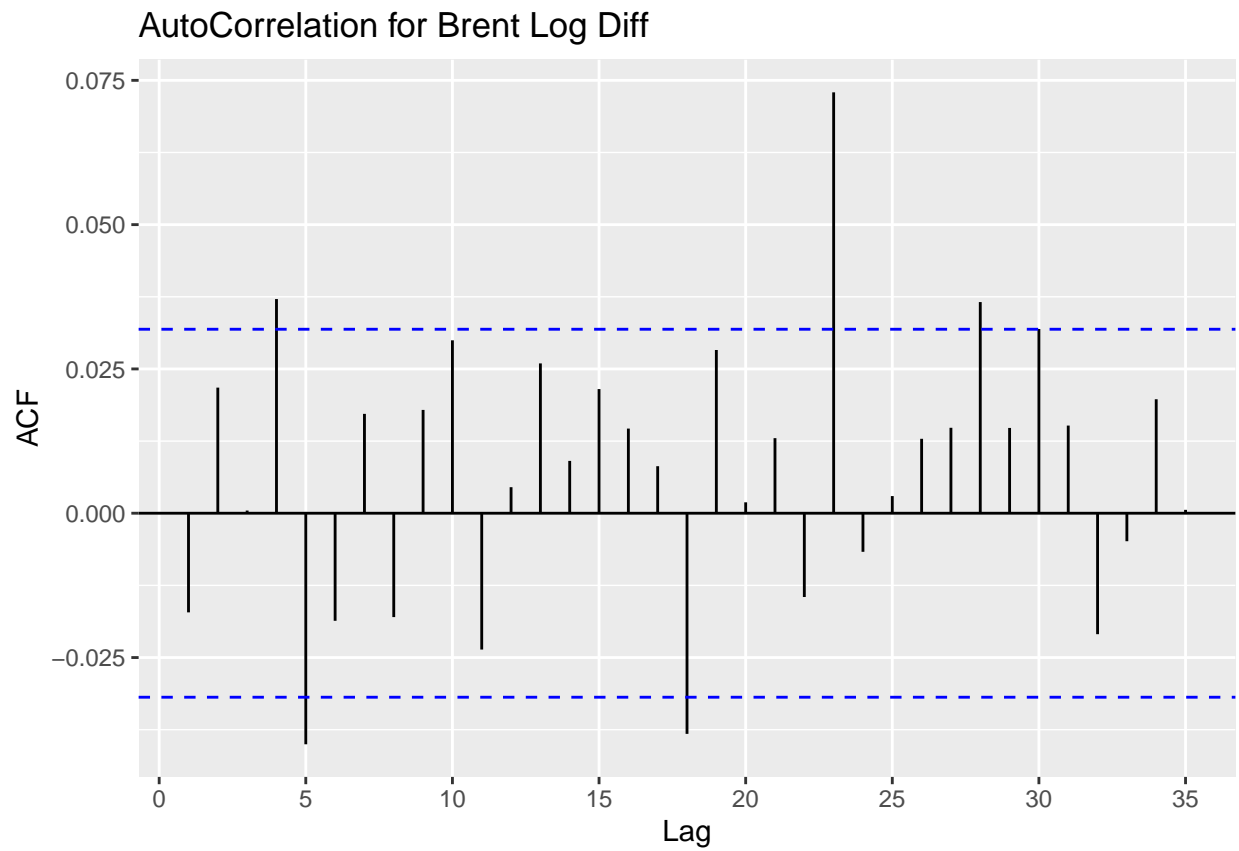
Partial AutoCorrelation for Petrobras Log Diff

```
ts.plot(log_ret_petr)
```

ACF, PACF and series of returns for Petrobras

```
log_ret_brent <- diff(log_data$`Brent Oil Prices`)
ggAcf(log_ret_brent) + labs(title = "AutoCorrelation for Brent Log Diff")
```

AutoCorrelation for Brent Log Diff


```
ggPacf(log_ret_brent) + labs(title = "Partial AutoCorrelation for Brent Log Diff")
```

Partial AutoCorrelation for Brent Log Diff
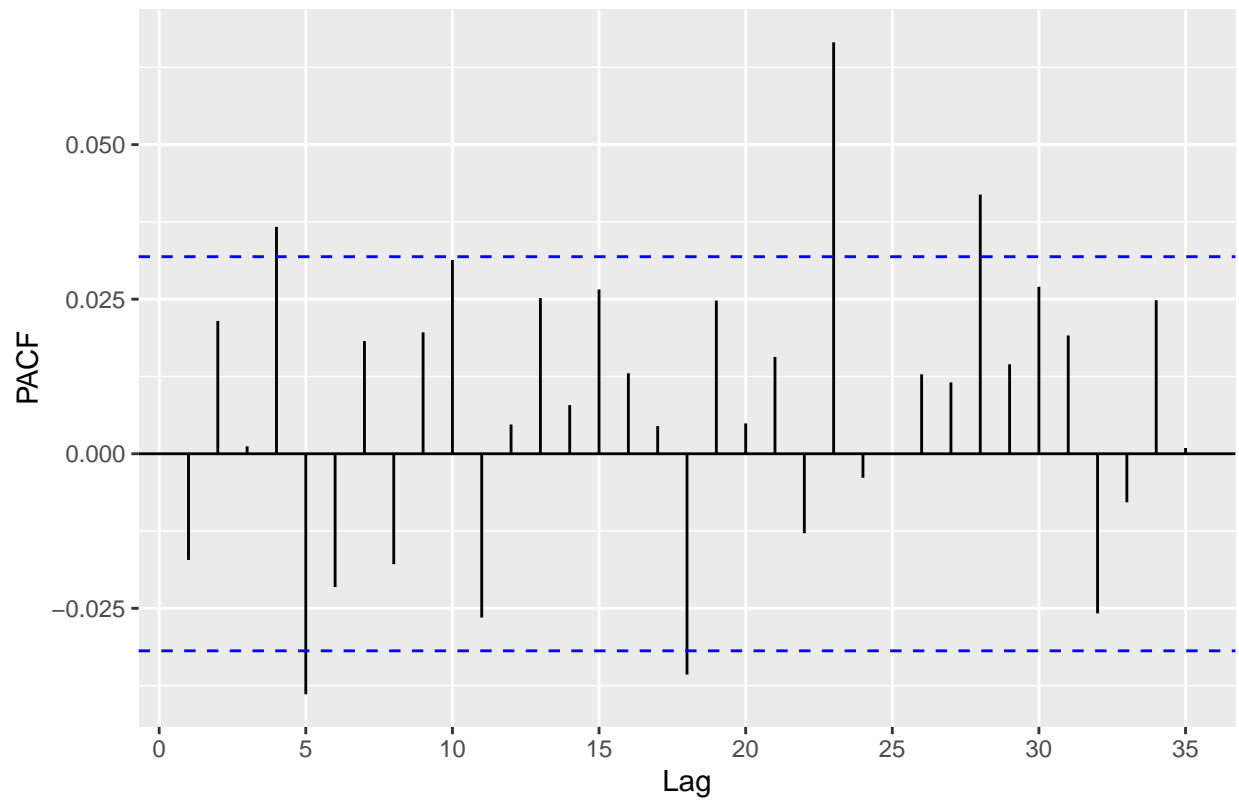

```
ts.plot(log_ret_brent)
```
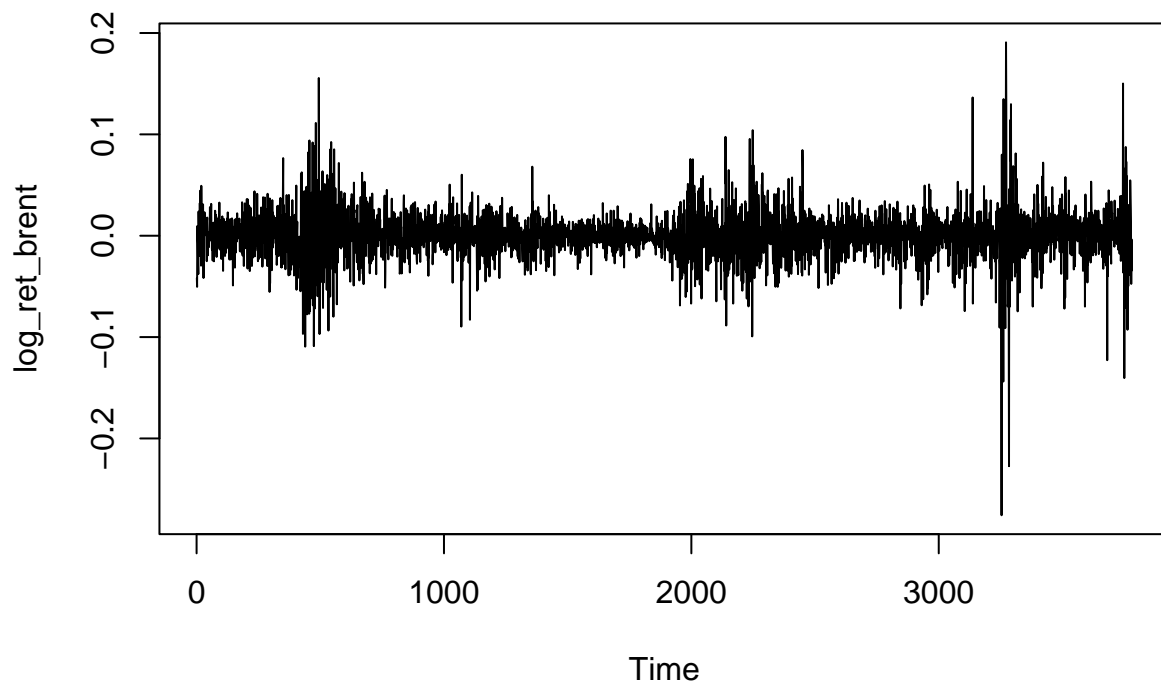
After the transformation of data, we need to test the new data (log return scaled) for stationarity.

Augmented dickey fuller test for Log Returns of Petrobrás stock prices with drift

```
adf_test_petr <- ur.df(log_ret_petr,type="drift", selectlags = "BIC")
summary(adf_test_petr)
```

```
##
## ###############################################
## # Augmented Dickey-Fuller Test Unit Root Test #
## ###############################################
##
## Test regression drift
##
##
## Call:
## lm(formula = z.diff ~ z.lag.1 + 1 + z.diff.lag)
##
## Residuals:
##      Min       1Q    Median       3Q       Max
## -0.35520 -0.01451  0.00063  0.01472  0.19428
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.0001457  0.0004835    0.301    0.763
## z.lag.1     -1.0236688  0.0233588  -43.824   <2e-16 ***
```

```
## z.diff.lag  -0.0064274  0.0162734  -0.395     0.693
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.02972 on 3776 degrees of freedom
## Multiple R-squared:  0.5152, Adjusted R-squared:  0.5149
## F-statistic:  2006 on 2 and 3776 DF,  p-value: < 2.2e-16
##
##
## Value of test-statistic is: -43.8237 960.2569
##
## Critical values for test statistics:
##       1pct  5pct 10pct
## tau2 -3.43 -2.86 -2.57
## phi1  6.43  4.59  3.78
```

Augmented dickey fuller test for Log Returns of Petrobrás stock prices with trend

```
adf_test_petr <- ur.df(log_ret_petr,type="trend", selectlags = "BIC")
summary(adf_test_petr)
```

```
##
## ###############################################
## # Augmented Dickey-Fuller Test Unit Root Test #
## ###############################################
##
## Test regression trend
##
##
## Call:
## lm(formula = z.diff ~ z.lag.1 + 1 + tt + z.diff.lag)
##
## Residuals:
##      Min      1Q   Median       3Q      Max
## -0.35545 -0.01456  0.00062  0.01472  0.19402
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -1.937e-04  9.677e-04  -0.200    0.841
## z.lag.1     -1.024e+00  2.336e-02 -43.821   <2e-16 ***
## tt           1.795e-07  4.433e-07   0.405    0.686
## z.diff.lag  -6.372e-03  1.628e-02  -0.392    0.695
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.02972 on 3775 degrees of freedom
## Multiple R-squared:  0.5152, Adjusted R-squared:  0.5148
## F-statistic:  1337 on 3 and 3775 DF,  p-value: < 2.2e-16
##
##
## Value of test-statistic is: -43.8206 640.0842 960.1262
##
## Critical values for test statistics:
```

```
##         1pct  5pct 10pct
## tau3 -3.96 -3.41 -3.12
## phi2  6.09  4.68  4.03
## phi3  8.27  6.25  5.34
```

Augmented dickey fuller test for Log Returns of Petrobrás stock prices

```
adf_test_petr <- ur.df(log_ret_petr,type="none", selectlags = "BIC")
summary(adf_test_petr)
```

```
##
## ###############################################
## # Augmented Dickey-Fuller Test Unit Root Test #
## ###############################################
##
## Test regression none
##
##
## Call:
## lm(formula = z.diff ~ z.lag.1 - 1 + z.diff.lag)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.35505 -0.01437  0.00077  0.01486  0.19443
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## z.lag.1    -1.023621   0.023355 -43.828   <2e-16 ***
## z.diff.lag -0.006451   0.016271  -0.396    0.692
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.02972 on 3777 degrees of freedom
## Multiple R-squared:  0.5152, Adjusted R-squared:  0.5149
## F-statistic:  2007 on 2 and 3777 DF,  p-value: < 2.2e-16
##
##
## Value of test-statistic is: -43.8279
##
## Critical values for test statistics:
##       1pct  5pct 10pct
## tau1 -2.58 -1.95 -1.62
```

Augmented dickey fuller test for Log Returns of Brent prices with drift

```
adf_test_petr <- ur.df(log_ret_brent,type="drift", selectlags = "BIC")
summary(adf_test_petr)
```

```
##
## ###############################################
## # Augmented Dickey-Fuller Test Unit Root Test #
## ###############################################
```

```
## 
## Test regression drift
## 
## 
## Call:
## lm(formula = z.diff ~ z.lag.1 + 1 + z.diff.lag)
## 
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.277196 -0.010377  0.000572  0.010925  0.192128
## 
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.0001641  0.0003927   0.418    0.676
## z.lag.1     -0.9963238  0.0231957 -42.953   <2e-16 ***
## z.diff.lag  -0.0214655  0.0162584  -1.320    0.187
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## 
## Residual standard error: 0.02414 on 3776 degrees of freedom
## Multiple R-squared:  0.5095, Adjusted R-squared:  0.5092
## F-statistic:  1961 on 2 and 3776 DF,  p-value: < 2.2e-16
## 
## 
## Value of test-statistic is: -42.9529 922.477
## 
## Critical values for test statistics:
##       1pct  5pct 10pct
## tau2 -3.43 -2.86 -2.57
## phi1  6.43  4.59  3.78
```

Augmented dickey fuller test for Log Returns of Brent prices with trend

```
adf_test_petr <- ur.df(log_ret_brent,type="trend", selectlags = "BIC")
summary(adf_test_petr)
```

```
## 
## ###############################################
## # Augmented Dickey-Fuller Test Unit Root Test #
## ###############################################
## 
## Test regression trend
## 
## 
## Call:
## lm(formula = z.diff ~ z.lag.1 + 1 + tt + z.diff.lag)
## 
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.277234 -0.010379  0.000589  0.010915  0.192090
## 
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
```

```
## (Intercept)   1.114e-04  7.859e-04   0.142    0.887
## z.lag.1      -9.963e-01  2.320e-02 -42.947   <2e-16 ***
## tt            2.791e-08  3.600e-07   0.078    0.938
## z.diff.lag   -2.146e-02  1.626e-02  -1.320    0.187
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.02414 on 3775 degrees of freedom
## Multiple R-squared:  0.5095, Adjusted R-squared:  0.5091
## F-statistic:  1307 on 3 and 3775 DF,  p-value: < 2.2e-16
##
##
## Value of test-statistic is: -42.9471 614.8248 922.2369
##
## Critical values for test statistics:
##       1pct  5pct 10pct
## tau3 -3.96 -3.41 -3.12
## phi2  6.09  4.68  4.03
## phi3  8.27  6.25  5.34
```

Augmented dickey fuller test for Log Returns of Brent prices

```
adf_test_petr <- ur.df(log_ret_brent,type="none", selectlags = "BIC")
summary(adf_test_petr)
```

```
##
## ###############################################
## # Augmented Dickey-Fuller Test Unit Root Test #
## ###############################################
##
## Test regression none
##
##
## Call:
## lm(formula = z.diff ~ z.lag.1 - 1 + z.diff.lag)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.277026 -0.010214  0.000735  0.011092  0.192289
##
## Coefficients:
##            Estimate Std. Error t value Pr(>|t|)
## z.lag.1    -0.99623    0.02319 -42.956   <2e-16 ***
## z.diff.lag -0.02151    0.01626  -1.323    0.186
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.02414 on 3777 degrees of freedom
## Multiple R-squared:  0.5094, Adjusted R-squared:  0.5092
## F-statistic:  1961 on 2 and 3777 DF,  p-value: < 2.2e-16
##
##
## Value of test-statistic is: -42.9556
```

```
## 
## Critical values for test statistics:
##      1pct  5pct 10pct
## tau1 -2.58 -1.95 -1.62
```

Specifying ARIMA model for log returns of Petrobras

```
petr_model <- arima(log_ret_petr, order=c(1,0,1),  include.mean = FALSE, method = "ML")
summary(petr_model)
```

```
## 
## Call:
## arima(x = log_ret_petr, order = c(1, 0, 1), include.mean = FALSE, method = "ML")
## 
## Coefficients:
##          ar1      ma1
##      -0.0152  -0.0146
## s.e.   0.4903   0.4868
## 
## sigma^2 estimated as 0.0008827:  log likelihood = 7929.89,  aic = -15853.77
## 
## Training set error measures:
##                        ME       RMSE        MAE MPE MAPE      MASE
## Training set 0.0001310876 0.02971103 0.02052066 NaN  Inf 0.6882743
##                       ACF1
## Training set -4.157921e-05
```

```
coeftest(petr_model)
```

```
## 
## z test of coefficients:
## 
##      Estimate Std. Error z value Pr(>|z|)
## ar1 -0.015218   0.490292  -0.031   0.9752
## ma1 -0.014590   0.486827  -0.030   0.9761
```

```
par(mfrow = c(2,1))
ggAcf(residuals(petr_model))
```

Series: residuals(petr_model)
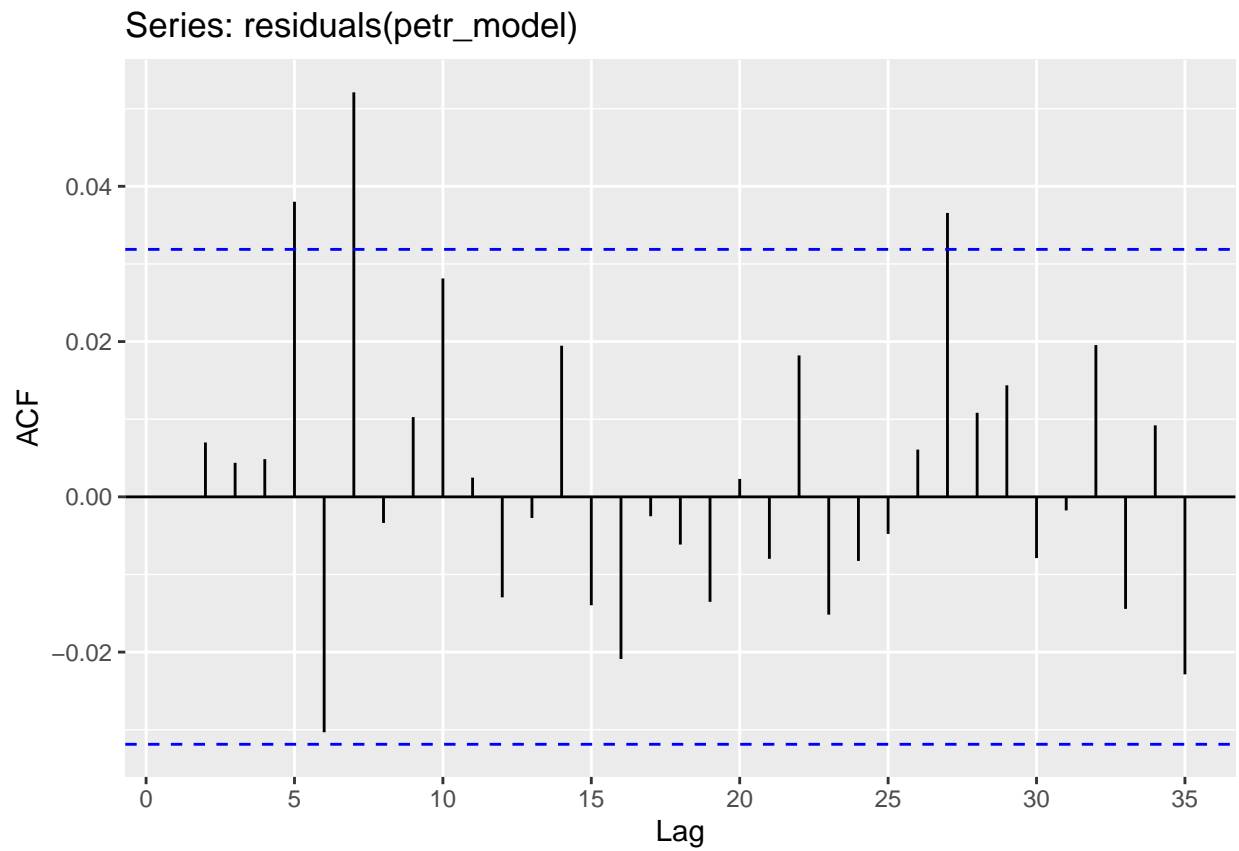
```
ts.plot(residuals(petr_model))
hist(residuals((petr_model)))
```

Histogram of residuals((petr_model))
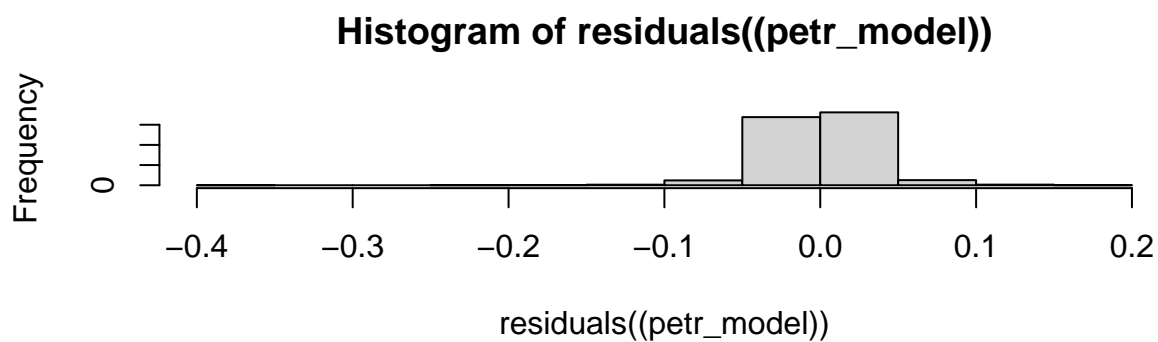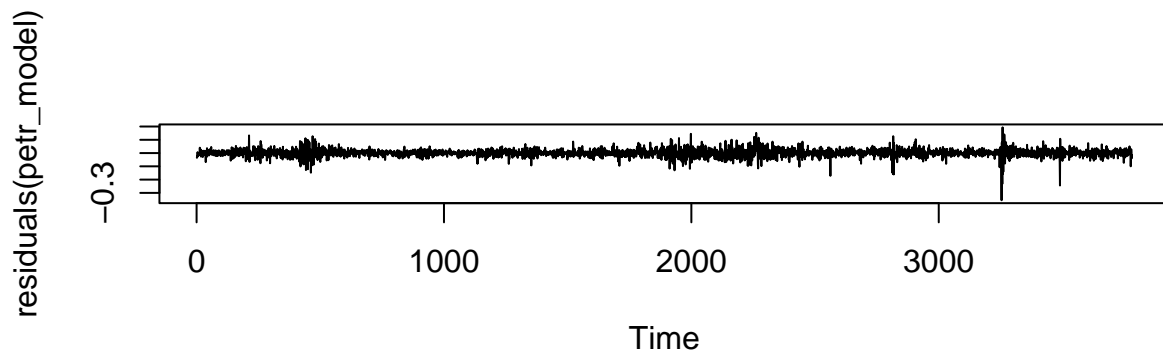
Specifying ARIMA model for log returns of Brent

```
brent_model <- arima(log_ret_brent, order=c(2,0,0),  include.mean = FALSE, method = "ML")
summary(brent_model)
```

```
##
## Call:
## arima(x = log_ret_brent, order = c(2, 0, 0), include.mean = FALSE, method = "ML")
##
## Coefficients:
##          ar1     ar2
##      -0.0168  0.0215
## s.e.   0.0163  0.0163
##
## sigma^2 estimated as 0.0005831:  log likelihood = 8713.72,  aic = -17421.44
##
## Training set error measures:
##                        ME       RMSE        MAE MPE MAPE      MASE
## Training set 0.0001392721 0.02414819 0.01613219 NaN  Inf 0.6792112
##                       ACF1
## Training set -7.694423e-05
```

```
coeftest(brent_model)
```

```
##
```

```
## z test of coefficients:
##
##       Estimate Std. Error z value Pr(>|z|)
## ar1 -0.016759   0.016268 -1.0302   0.3029
## ar2  0.021542   0.016275  1.3236   0.1856
```

```
par(mfrow = c(2,1))
ggAcf(residuals(brent_model))
```
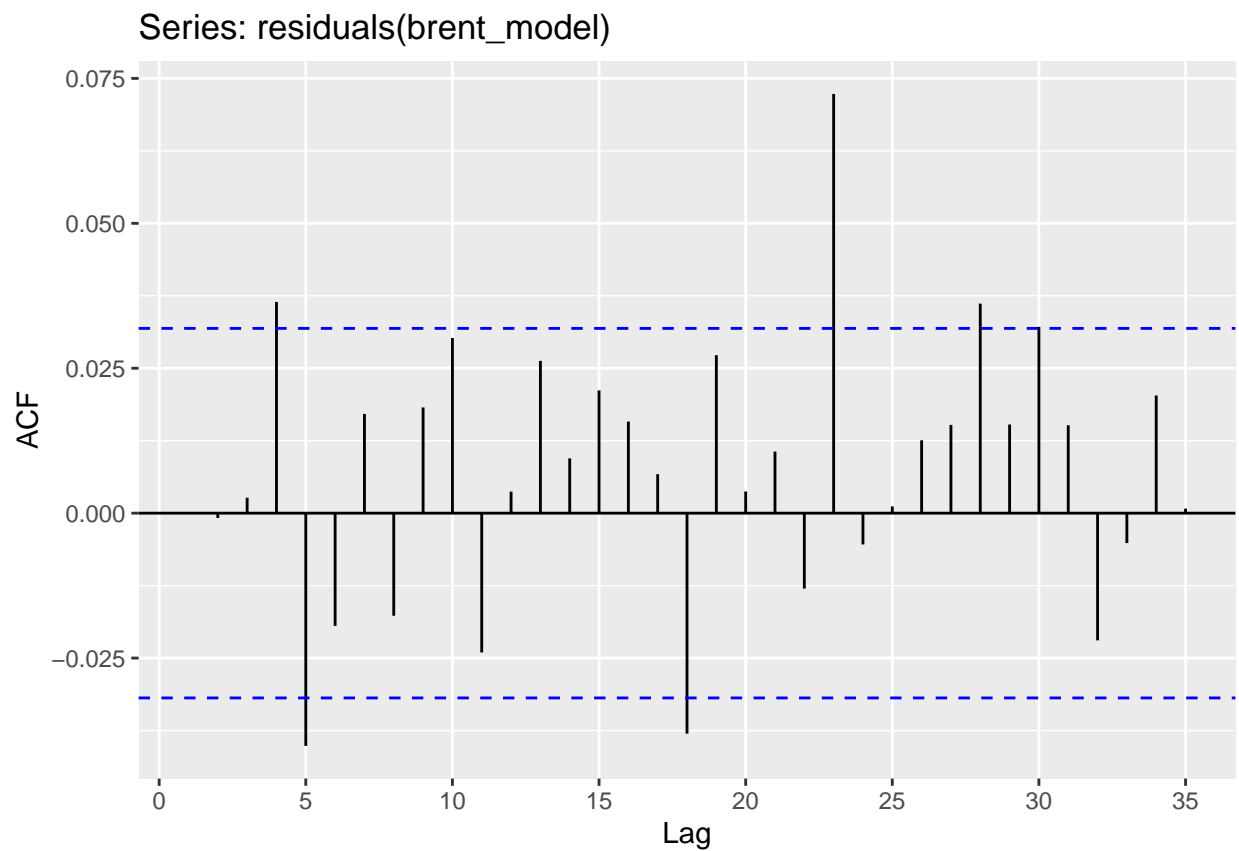


```
ts.plot(residuals(brent_model))
hist(residuals((brent_model)))
```

Histogram of residuals((brent_model))
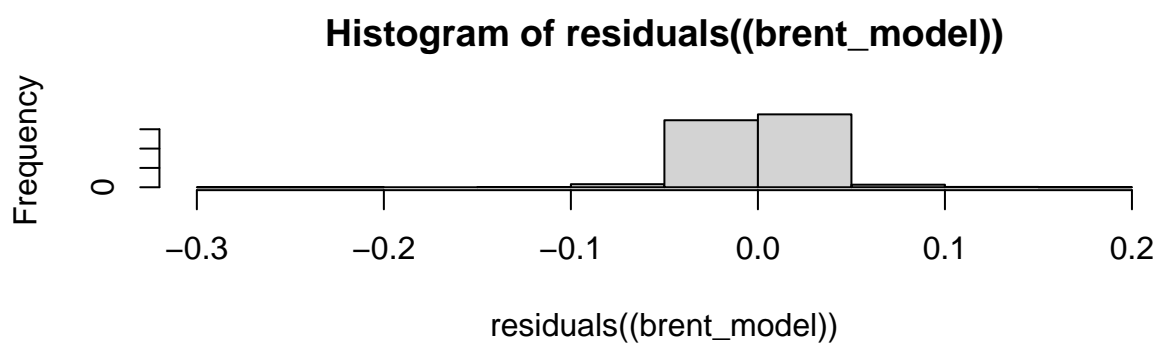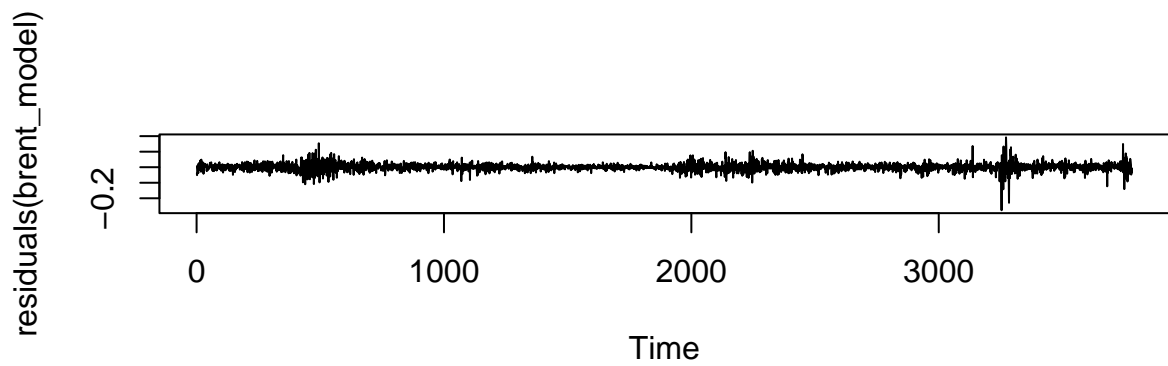
Real time series x Fitted Values for the model specified for Log Returns of Petrobras

```
ts.plot(log_ret_petr, fitted(petr_model), col=c("red", "black"), ylab = "Real x Model Brent")
```
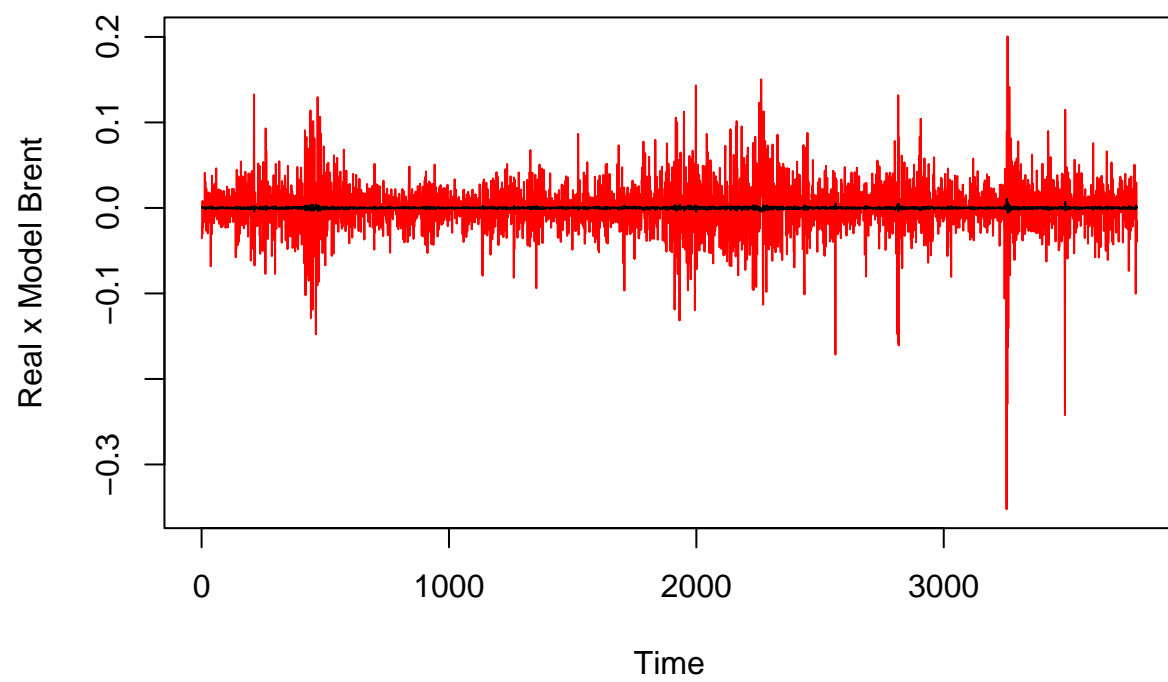
```
ts.plot(residuals(petr_model), col="blue", ylab="Residuals")
```
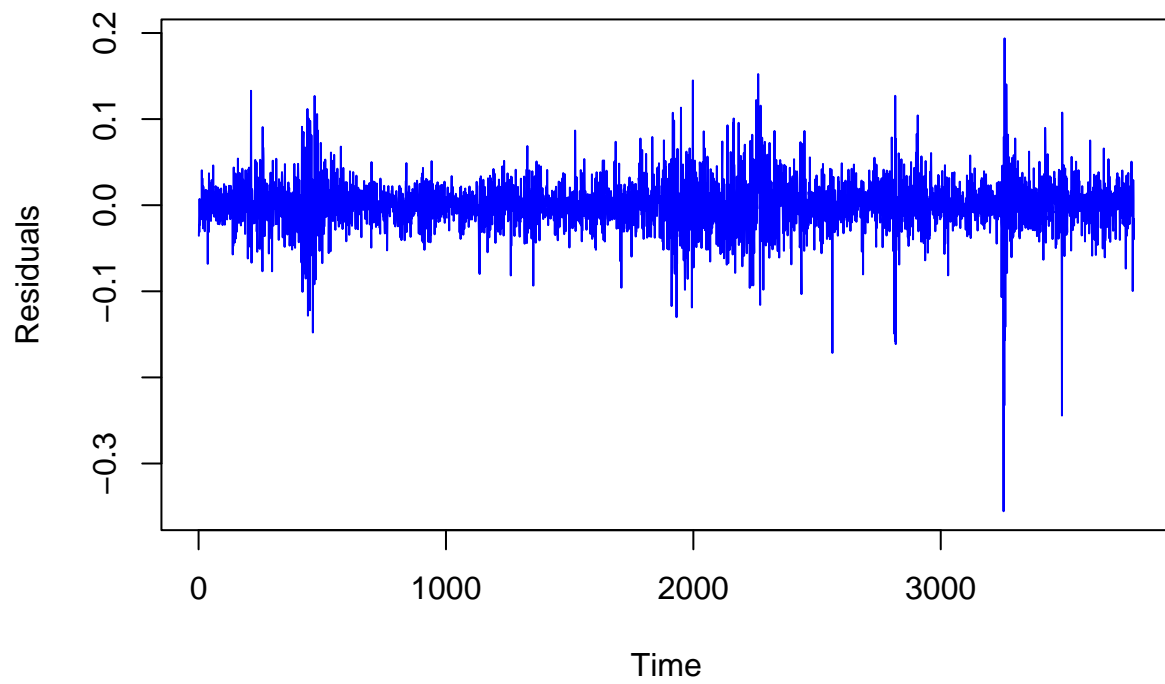
Real time series x Fitted Values for the model specified for Log Returns of Brent

```
ts.plot(log_ret_brent, fitted(brent_model), col=c("red", "black"), ylab = "Real x Model Brent")
```
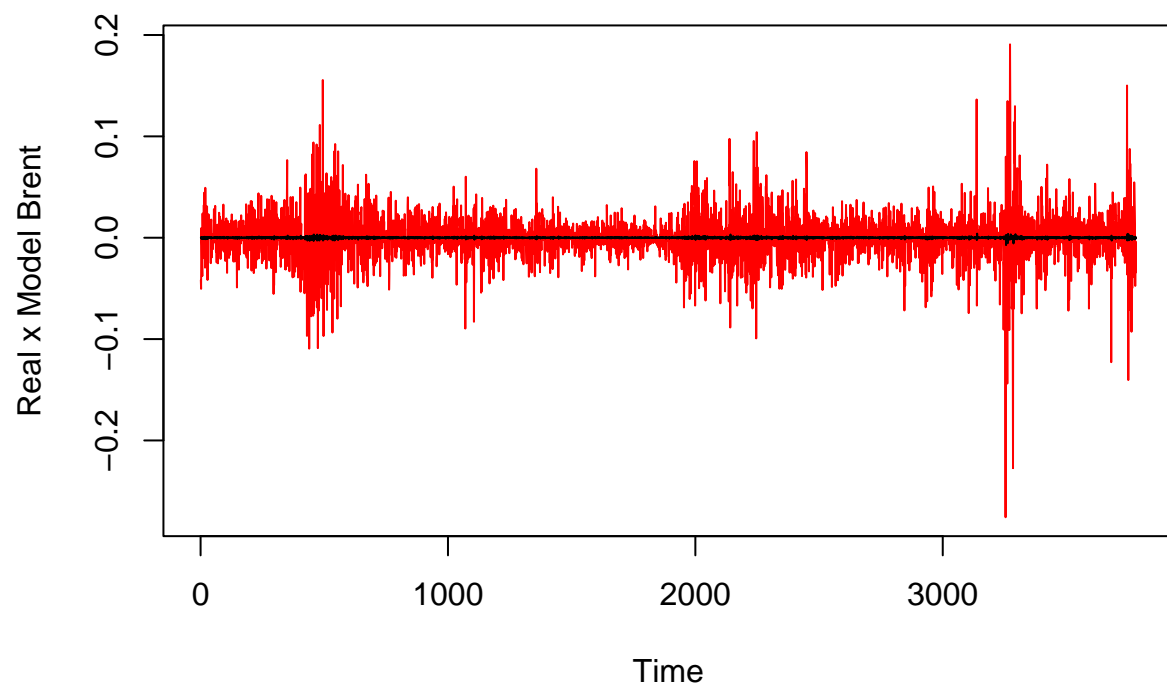
```
ts.plot(residuals(brent_model), col="blue", ylab="Residuals")
```
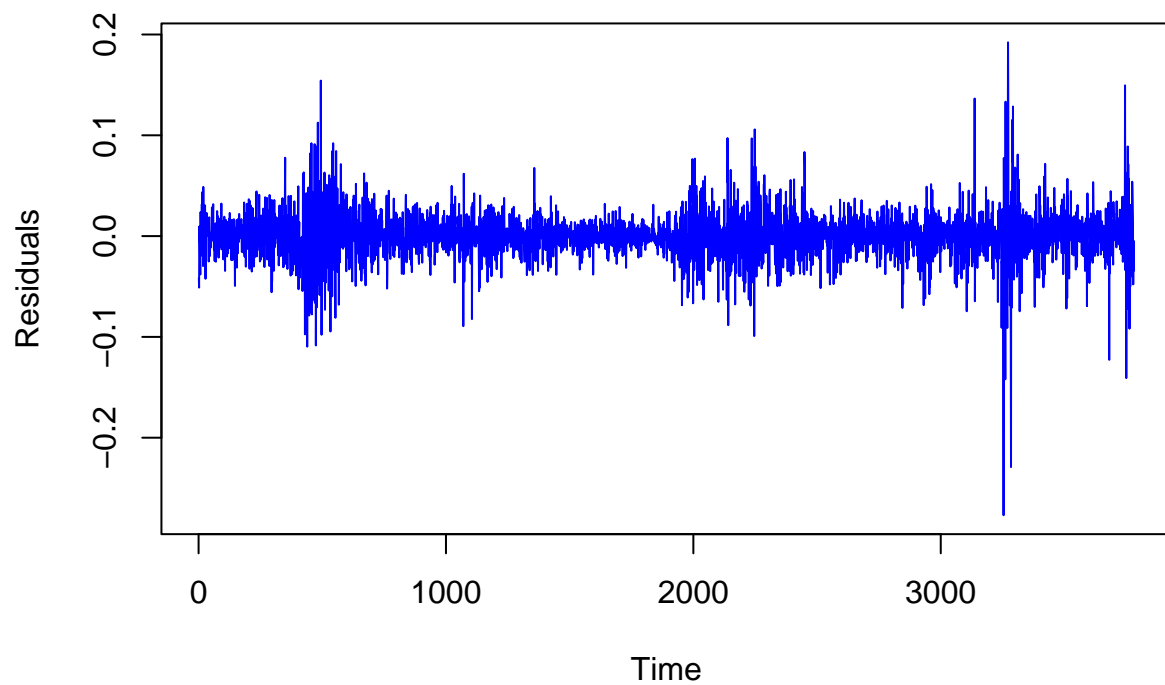
Specifying and model to predict Petrobras prices with Brent Prices

```r
model_arimax <- Arima(log_ret_petr, order = c(0,0,1), xreg =log_ret_brent, include.constant = FALSE)
ts.plot(log_ret_petr, fitted(model_arimax), col=c("red", "green4"), ylab = "Real x Modelo Petrobrás")
```
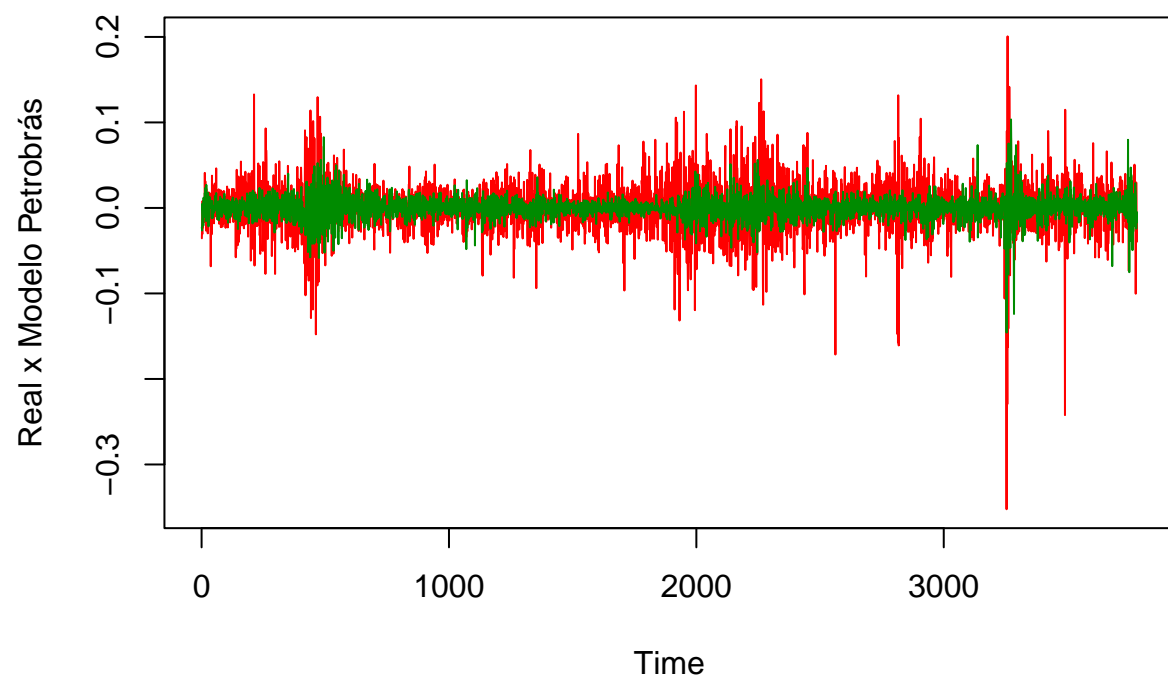
```
ts.plot(residuals(model_arimax), col="black", ylab="resíduo")
```

```
checkresiduals(model_arimax)
```
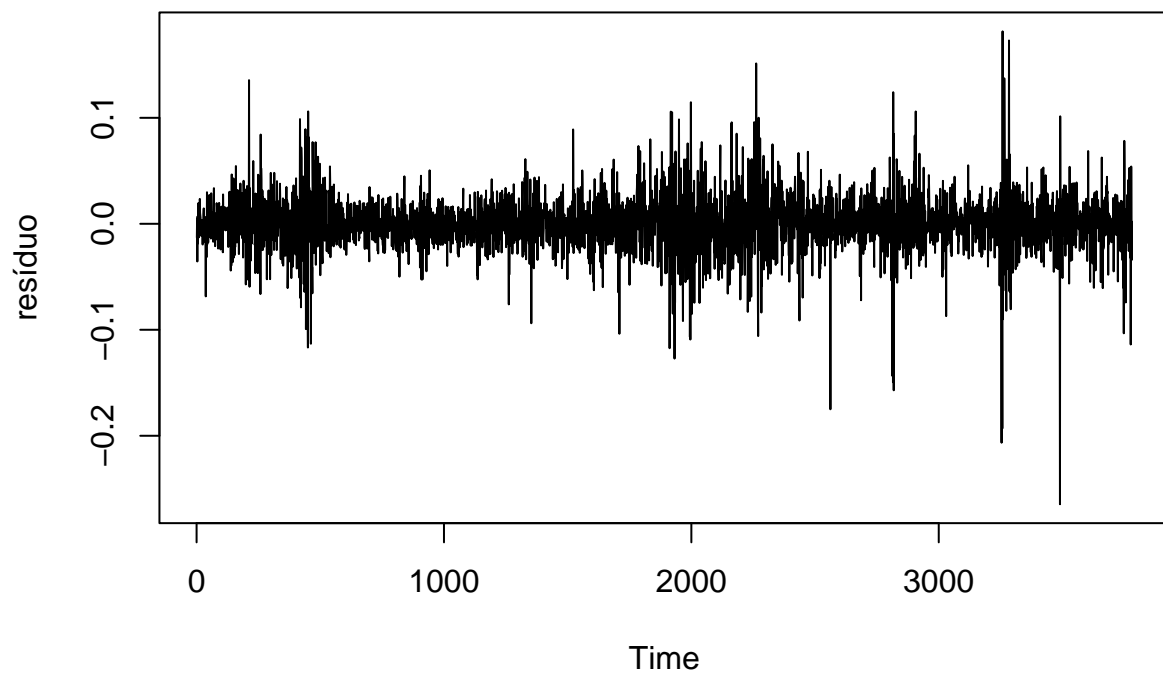
Residuals from Regression with ARIMA(0,0,1) errors
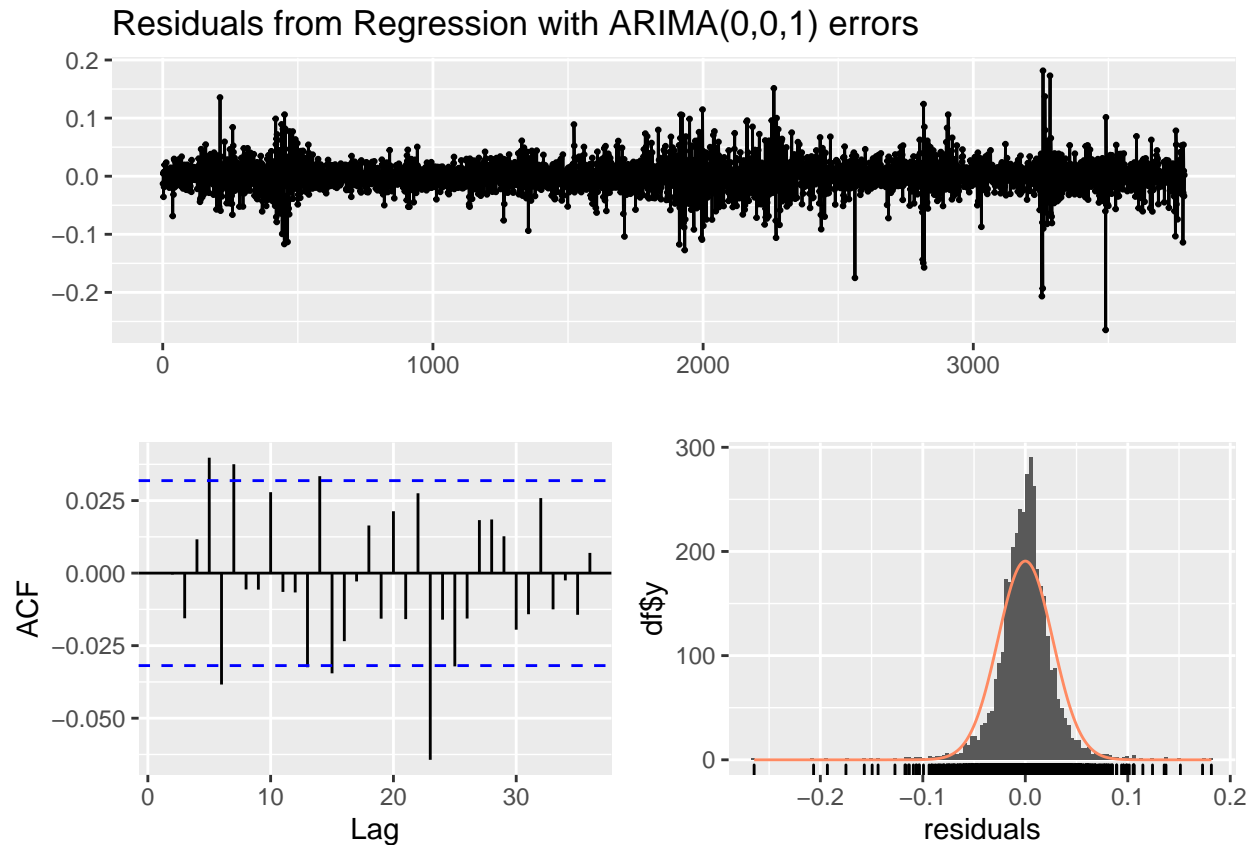

```
##
##  Ljung-Box test
##
## data:  Residuals from Regression with ARIMA(0,0,1) errors
## Q* = 21.545, df = 8, p-value = 0.005832
##
## Model df: 2.    Total lags used: 10
```

For the forecast produce, we gonna get the J.P Morgan, Reuters and Barclays forecasts for Brent Oil prices for 2022. The information are avaible at the following links:

- J.P Morgan: 'https://www.thestreet.com/investing/oil-will-hit-125-a-barrel-in-2022-150-in-2023-jpmorgan'

- Reuters: 'https://www.reuters.com/business/energy/oil-prices-drop-demand-worries-rising-supplies-2021-11-16/'

- Barclays: 'https://boereport.com/2022/01/24/barclays-hikes-2022-oil-price-view-by-5-bbl-on-depressed-inventories/'

```
jpm_forecast <- forecast(model_arimax, h = 1, xreg = forecast_jpm, level = 0.95)
reuters_forecast <- forecast(model_arimax, h = 1, xreg = forecast_reuters, level = 0.95)
barclays_forecast <- forecast(model_arimax, h = 1, xreg = forecast_barclays, level = 0.95)

plot(jpm_forecast, col = c("red"), xlab = "J.P. Morgan")
```
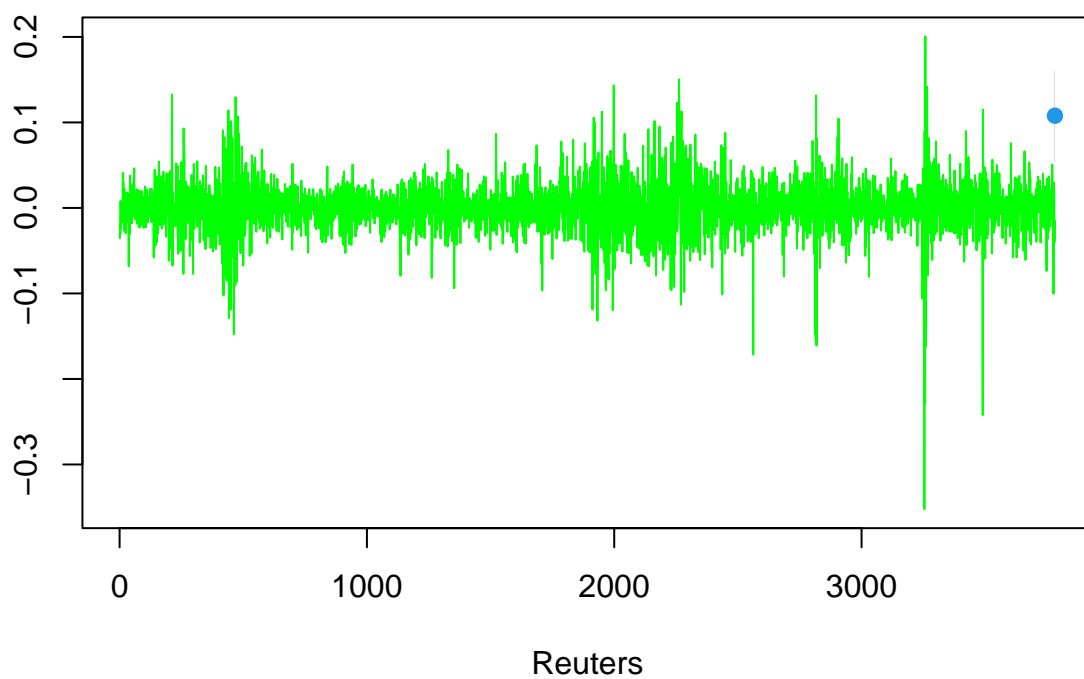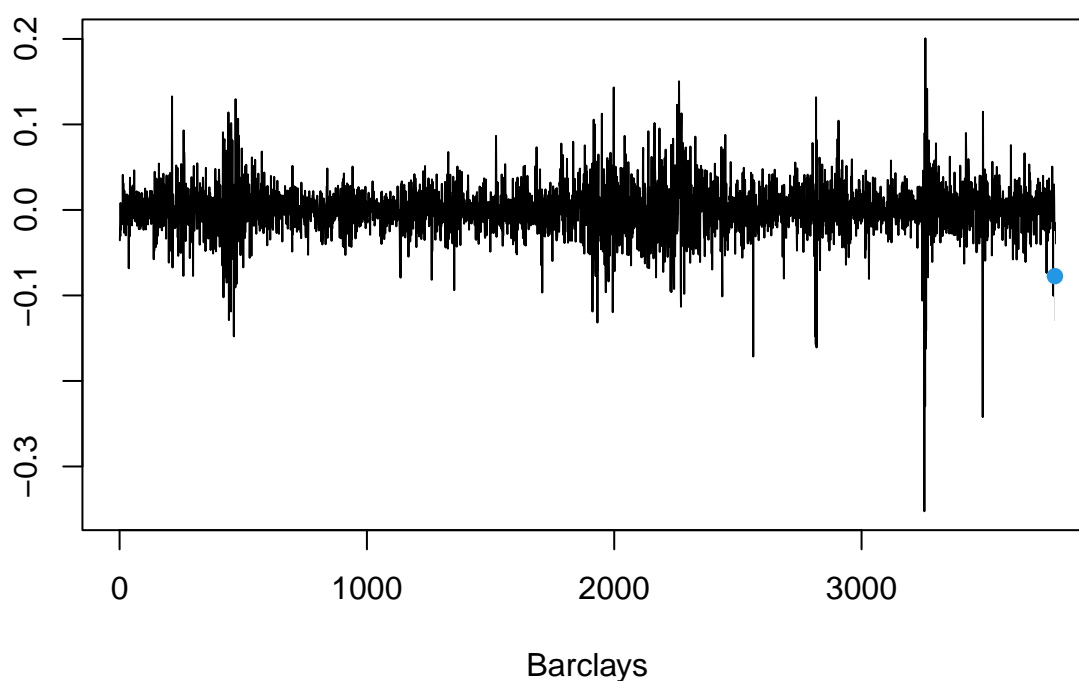
Forecasts from Regression with ARIMA(0,0,1) errors

J.P. Morgan
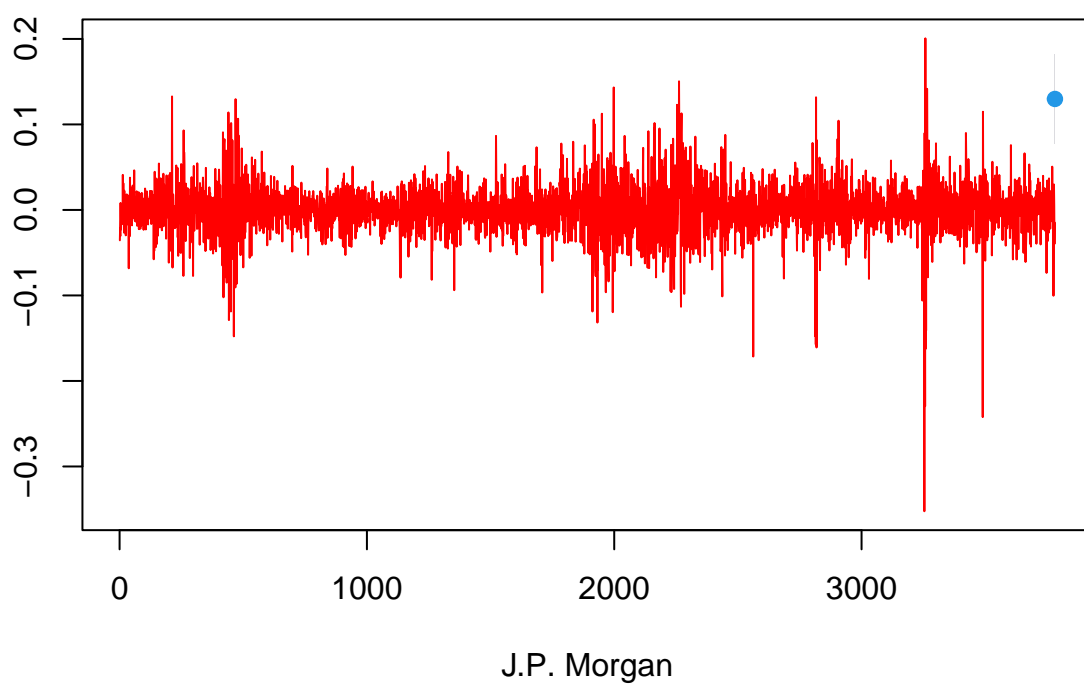
```
plot(reuters_forecast, col = c("green"), xlab = 'Reuters')
```

Forecasts from Regression with ARIMA(0,0,1) errors

Reuters

```
plot(barclays_forecast, xlab = "Barclays")
```

Forecasts from Regression with ARIMA(0,0,1) errors

Barclays

```
## The stock price to Petrobrás prediction according to JPM in Brent prices is
##   33.47248
```
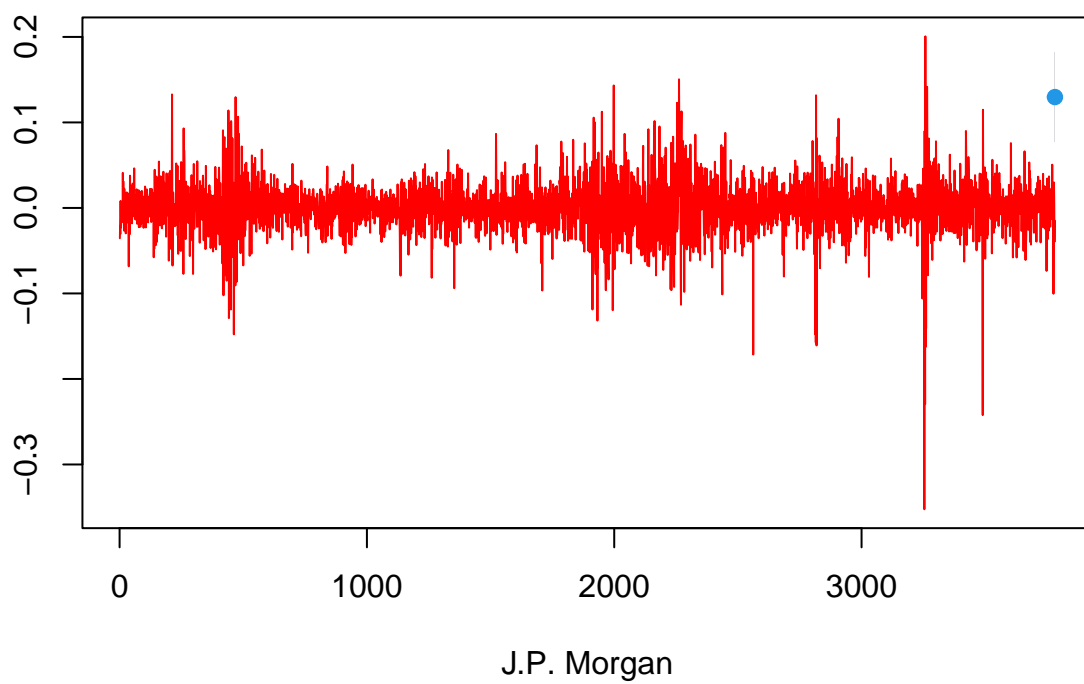
```
## The stock price to Petrobrás prediction according to Reuters in Brent prices is
##   32.74623
```

```
## The stock price to Petrobrás prediction according to Barclays in Brent prices is
##   27.20733
```

Since Brazil is going throught an election year, we decide to add a auto-regressive component, since the PETR4 stock price can pass to depend to previous values.
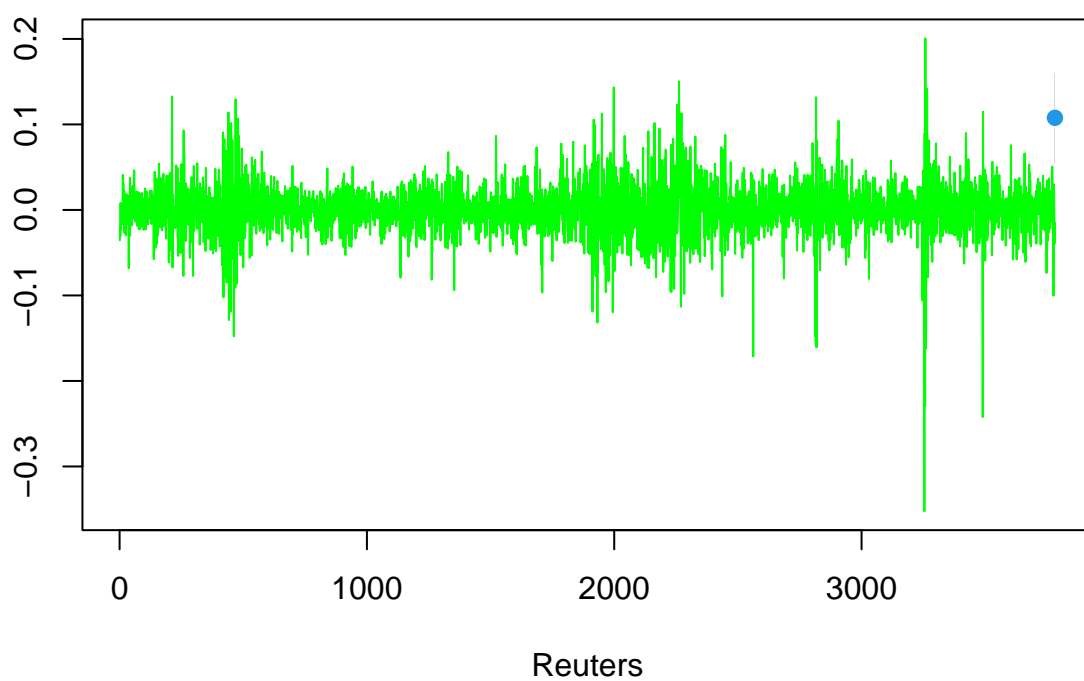
```r
plot(jpm_forecast_elections, col = c("red"), xlab = "J.P. Morgan")
```
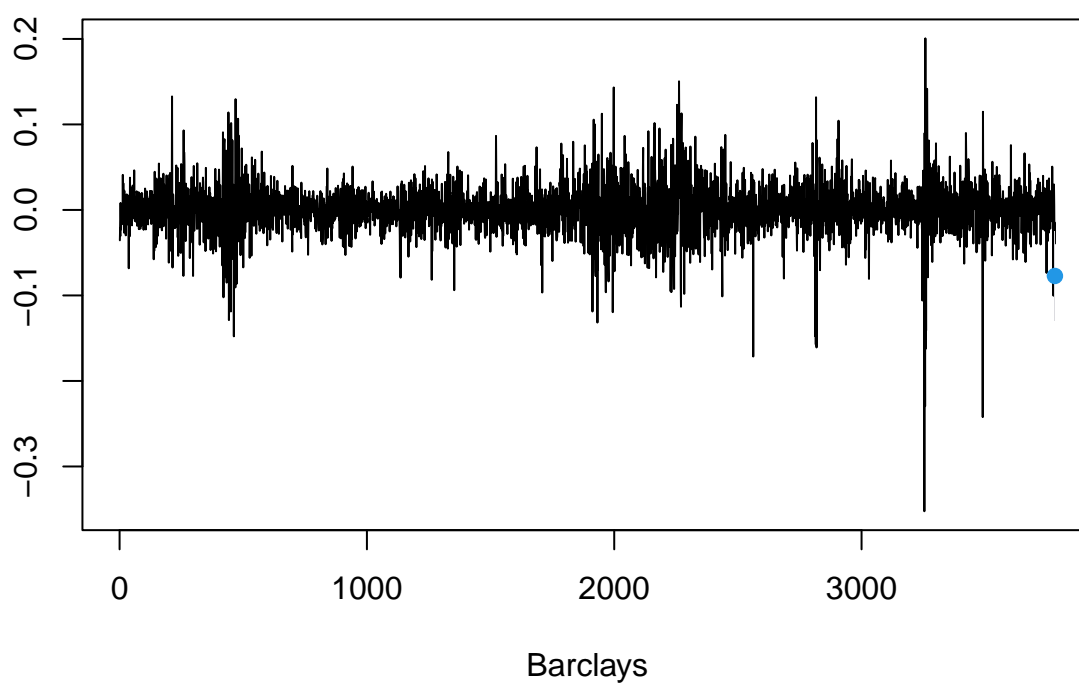
Forecasts from Regression with ARIMA(0,0,2) errors

J.P. Morgan

```
plot(reuters_forecast_elections, col = c("green"), xlab = 'Reuters')
```

Forecasts from Regression with ARIMA(0,0,2) errors

Reuters

```
plot(barclays_forecast_elections, xlab = "Barclays")
```

Forecasts from Regression with ARIMA(0,0,2) errors

Barclays

```
## The stock price to Petrobrás prediction according to JPM in Brent prices is
##  33.46923
```

```
## The stock price to Petrobrás prediction according to Reuters in Brent prices is
##  32.74434
```

```
## The stock price to Petrobrás prediction according to Barclays in Brent prices is
##  27.21483
```