

Time Series Analysis

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Importing the required bibs for work

```
library(tseries)
```

```
## Registered S3 method overwritten by 'quantmod':  
##   method           from  
##   as.zoo.data.frame zoo
```

```
require(urca)
```

```
## Loading required package: urca
```

```
library(tidyverse)
```

```
## -- Attaching packages ----- tidyverse 1.3.1 --
```

```
## v ggplot2 3.3.6      v purrr   0.3.4  
## v tibble  3.1.7      v dplyr  1.0.9  
## v tidyr   1.2.0      v stringr 1.4.0  
## v readr   2.1.2      v forcats 0.5.1
```

```
## -- Conflicts ----- tidyverse_conflicts() --  
## x dplyr::filter() masks stats::filter()  
## x dplyr::lag()     masks stats::lag()
```

```
library(fpp2)
```

```
## -- Attaching packages ----- fpp2 2.4 --
```

```
## v forecast 8.16      v expsmooth 2.3  
## v fma       2.4
```

```
##
```

```
library(vars)
```

```
## Loading required package: MASS
```

```
##
## Attaching package: 'MASS'

## The following objects are masked from 'package:fma':
##
##   cement, housing, petrol

## The following object is masked from 'package:dplyr':
##
##   select

## Loading required package: strucchange

## Loading required package: zoo

##
## Attaching package: 'zoo'

## The following objects are masked from 'package:base':
##
##   as.Date, as.Date.numeric

## Loading required package: sandwich

##
## Attaching package: 'strucchange'

## The following object is masked from 'package:stringr':
##
##   boundary

## Loading required package: lmtest

library(forecast)
require(lmtest)
library(readxl)
suppressWarnings(expr)

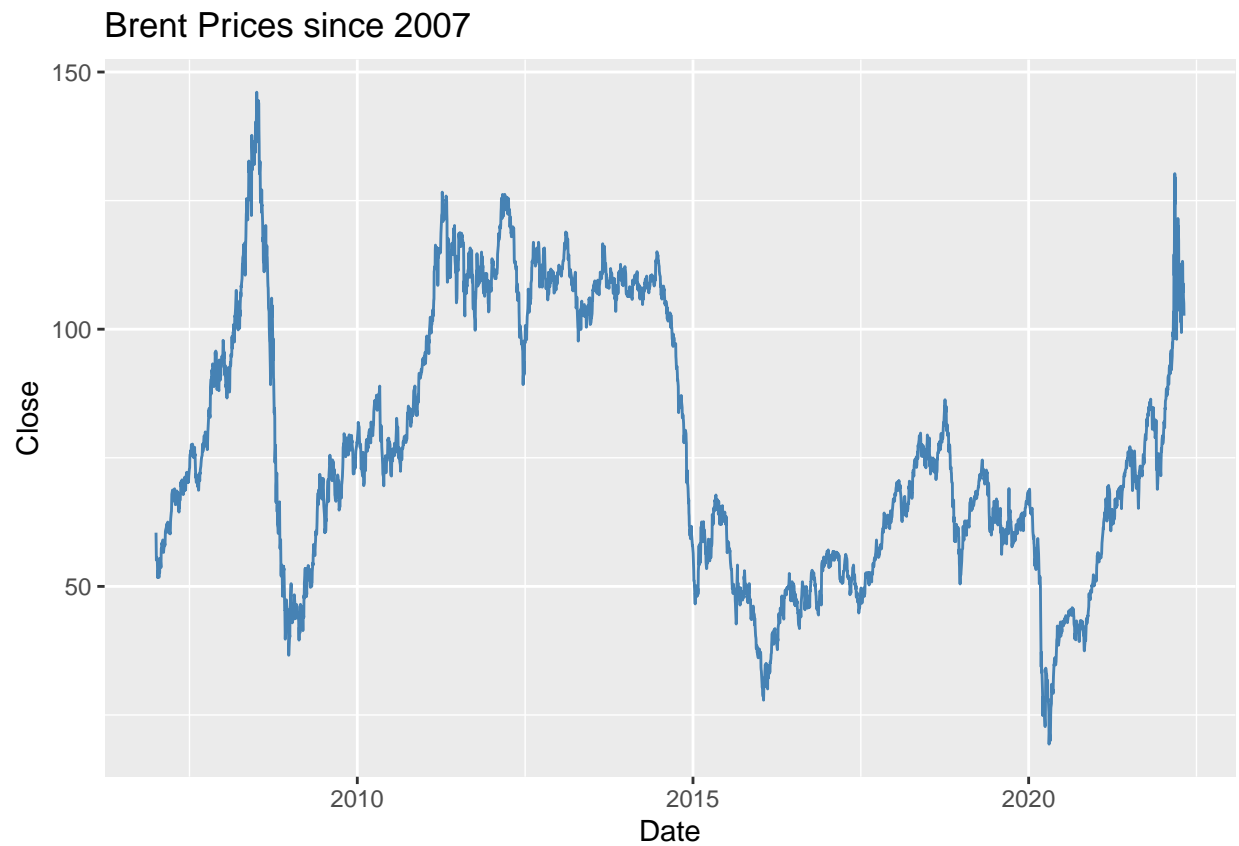
## function (expr)
## {
##   enexpr(expr)
## }
## <bytecode: 0x7f9203ad2120>
## <environment: namespace:rlang>
```

Importing the data

```
brent_prices <- read_xlsx("brent_prices.xlsx")
Petr4_prices <- read_xlsx("Petr4_prices.xlsx")
```

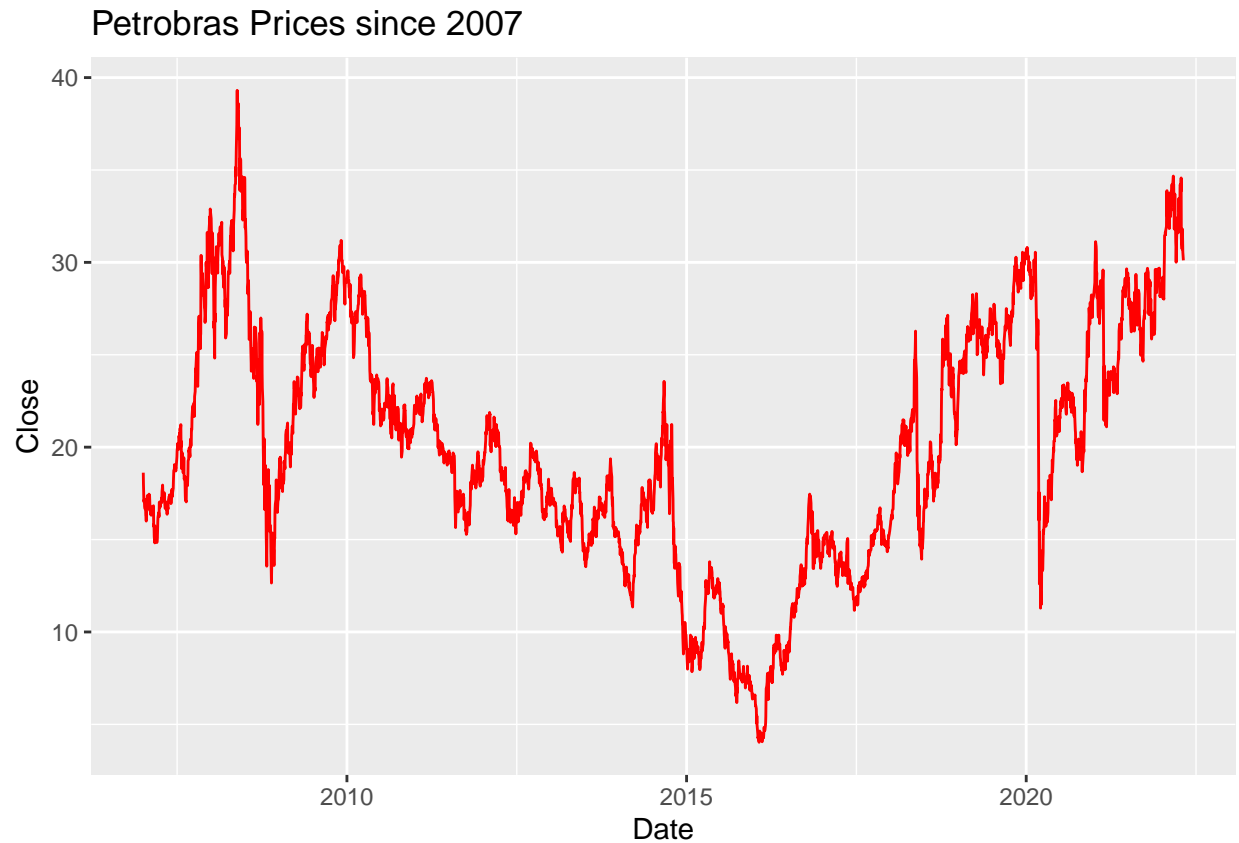
Brent Oil Prices Graph

```
ggplot(brent_prices, aes(x=Date)) +
  geom_line(aes(y = Close), color = "steelblue") +
  labs(title = 'Brent Prices since 2007')
```



Petrobrás stock prices graph

```
ggplot(Petr4_prices, aes(x=Date)) +
  geom_line(aes(y = `Close`), color = "red") +
  labs(title = 'Petrobras Prices since 2007')
```



Managing the data

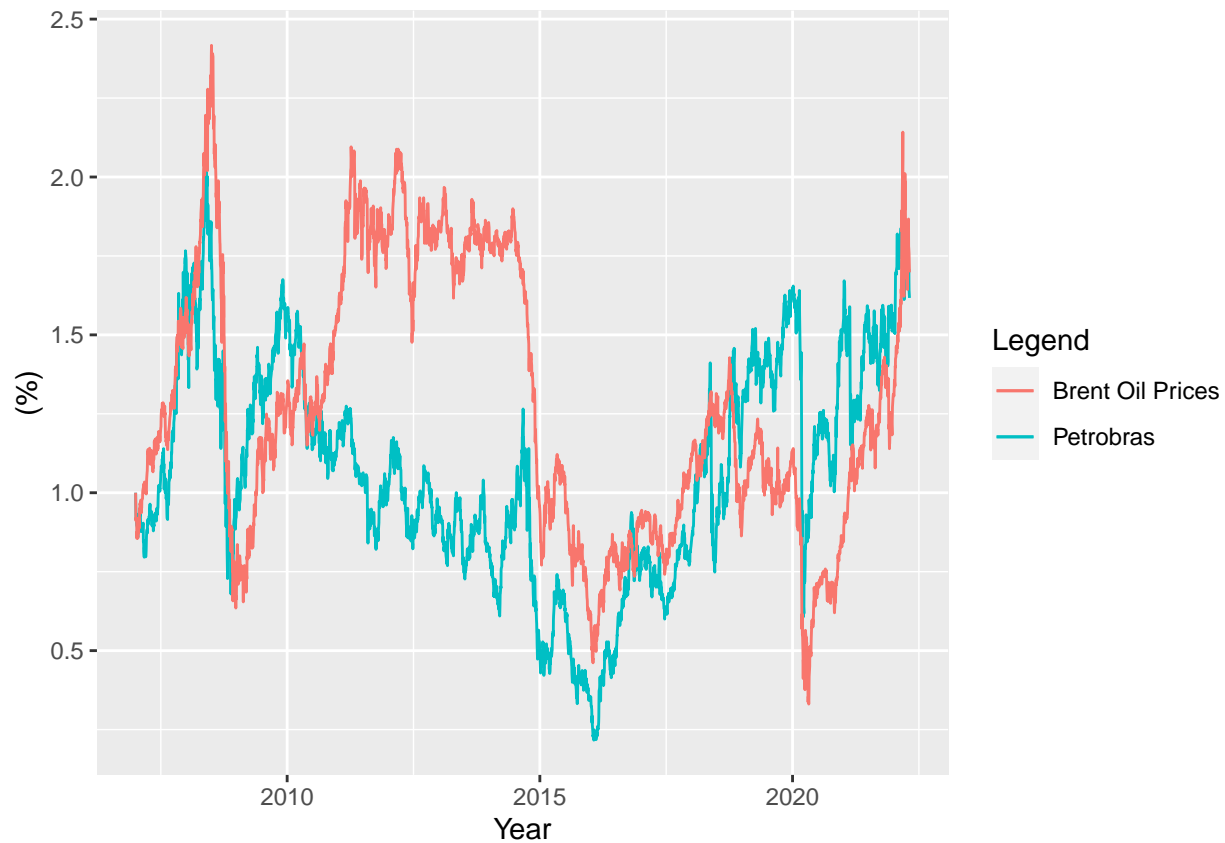
```
final_data <- merge(Petr4_prices, brent_prices, by = 'Date')
names(final_data) <- c('Date', 'Petrobras', 'Brent Oil Prices')
normalized_final_data <- data_frame(final_data$Date, final_data[,2] / final_data[1,2], final_data[,3] / final_data[1,3])
```

```
## Warning: 'data_frame()' was deprecated in tibble 1.1.0.
## Please use 'tibble()' instead.
## This warning is displayed once every 8 hours.
## Call 'lifecycle::last_lifecycle_warnings()' to see where this warning was generated.
```

```
names(normalized_final_data) <- c('Date', 'Petrobras', 'Brent Oil Prices')
```

Brent x Petrobrás Graph in normalized scale

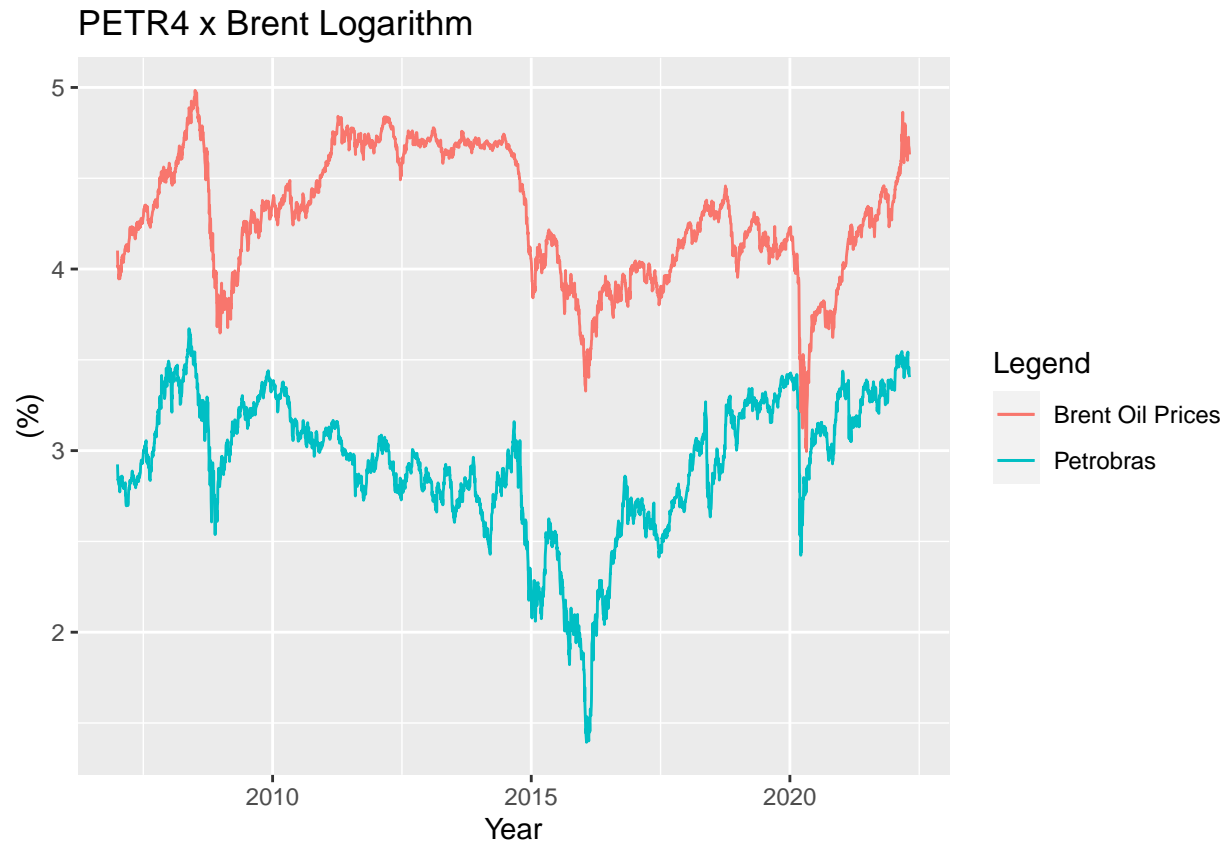
```
ggplot(normalized_final_data, aes(x = Date)) +
  geom_line(aes(y = Petrobras, color = "Petrobras")) +
  geom_line(aes(y = `Brent Oil Prices`, color = "Brent Oil Prices")) +
  labs(x = "Year",
       y = "%",
       color = "Legend")
```



Transforming the data to the logarithm scale and plotting Petr4 x Brent Graph

```
log_data <- log(final_data[,2:3])
log_final_data <- data_frame(final_data$Date, log(final_data[,2:3]))
log_data <- data_frame(final_data$Date, log(final_data[,2:3]))
names(log_final_data) <- c('Date', 'Petrobras', 'Brent Oil Prices')
names(log_data) <- c('Date', 'Petrobras', 'Brent Oil Prices')

ggplot(log_final_data, aes(x = Date)) +
  geom_line(aes(y = Petrobras, color = "Petrobras")) +
  geom_line(aes(y = `Brent Oil Prices`, color = "Brent Oil Prices")) +
  labs(x = "Year",
       y = "%)",
       color = "Legend",
       title = 'PETR4 x Brent Logarithm')
```



Augmented dickey fuller test for Brent Prices

```
adf_test_brent <- ur.df(log_data$`Brent Oil Prices`,selectlags = "BIC")
summary(adf_test_brent)
```

```
##
## #####
## # Augmented Dickey-Fuller Test Unit Root Test #
## #####
##
## Test regression none
##
##
## Call:
## lm(formula = z.diff ~ z.lag.1 - 1 + z.diff.lag)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.277534 -0.010297  0.000714  0.010998  0.192156
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## z.lag.1      2.063e-05  9.136e-05   0.226   0.821
## z.diff.lag -1.718e-02  1.627e-02  -1.056   0.291
##
## Residual standard error: 0.02415 on 3778 degrees of freedom
```

```
## Multiple R-squared:  0.0003075, Adjusted R-squared:  -0.0002218
## F-statistic: 0.581 on 2 and 3778 DF,  p-value: 0.5594
##
##
## Value of test-statistic is: 0.2258
##
## Critical values for test statistics:
##      1pct  5pct 10pct
## tau1 -2.58 -1.95 -1.62
```

Augmented dickey fuller test for Brent Prices with drift

```
adf_test_brent <- ur.df(log_data$`Brent Oil Prices`,type="drift",selectlags = "BIC")
summary(adf_test_brent)
```

```
##
## #####
## # Augmented Dickey-Fuller Test Unit Root Test #
## #####
##
## Test regression drift
##
##
## Call:
## lm(formula = z.diff ~ z.lag.1 + 1 + z.diff.lag)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.278572 -0.010430  0.000732  0.011012  0.189506
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.009944   0.004846   2.052   0.0402 *
## z.lag.1      -0.002284   0.001127  -2.027   0.0427 *
## z.diff.lag   -0.015984   0.016269  -0.982   0.3259
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.02414 on 3777 degrees of freedom
## Multiple R-squared:  0.001382, Adjusted R-squared:  0.0008529
## F-statistic: 2.613 on 2 and 3777 DF,  p-value: 0.07345
##
##
## Value of test-statistic is: -2.027 2.1311
##
## Critical values for test statistics:
##      1pct  5pct 10pct
## tau2 -3.43 -2.86 -2.57
## phi1  6.43  4.59  3.78
```

Augmented dickey fuller test for Brent Prices with trend

```
adf_test_brent <- ur.df(log_data$`Brent Oil Prices`,type="trend",selectlags = "BIC")
summary(adf_test_brent)
```

```
##
## #####
## # Augmented Dickey-Fuller Test Unit Root Test #
## #####
##
## Test regression trend
##
##
## Call:
## lm(formula = z.diff ~ z.lag.1 + 1 + tt + z.diff.lag)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.278335 -0.010410  0.000744  0.011055  0.189507
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  1.188e-02  5.583e-03   2.128  0.0334 *
## z.lag.1      -2.616e-03  1.223e-03  -2.139  0.0325 *
## tt           -2.731e-07  3.905e-07  -0.699  0.4844
## z.diff.lag   -1.576e-02  1.627e-02  -0.969  0.3328
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.02414 on 3776 degrees of freedom
## Multiple R-squared:  0.001511, Adjusted R-squared:  0.0007177
## F-statistic: 1.905 on 3 and 3776 DF, p-value: 0.1266
##
##
## Value of test-statistic is: -2.1394 1.5836 2.2987
##
## Critical values for test statistics:
##      1pct  5pct 10pct
## tau3 -3.96 -3.41 -3.12
## phi2  6.09  4.68  4.03
## phi3  8.27  6.25  5.34
```

Augmented dickey fuller test for Petrobrás stock prices

```
adf_test_petr <- ur.df(log_data$Petrobras,type="none", selectlags = "BIC")
summary(adf_test_petr)
```

```
##
## #####
## # Augmented Dickey-Fuller Test Unit Root Test #
## #####
##
## Test regression none
##
```



```
##
## Call:
## lm(formula = z.diff ~ z.lag.1 - 1 + z.diff.lag)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.35544 -0.01446  0.00077  0.01486  0.19378
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## z.lag.1      1.575e-06  1.638e-04   0.010   0.9923
## z.diff.lag -3.002e-02  1.626e-02  -1.846   0.0649 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.02972 on 3778 degrees of freedom
## Multiple R-squared:  0.0009016, Adjusted R-squared:  0.0003727
## F-statistic: 1.705 on 2 and 3778 DF, p-value: 0.182
##
##
## Value of test-statistic is: 0.0096
##
## Critical values for test statistics:
##      1pct  5pct 10pct
## tau1 -2.58 -1.95 -1.62
```

Augmented dickey fuller test for Petrobrás stock prices with drift

```
adf_test_petr <- ur.df(log_data$Petrobras,type="drift", selectlags = "BIC")
summary(adf_test_petr)
```

```
##
## #####
## # Augmented Dickey-Fuller Test Unit Root Test #
## #####
##
## Test regression drift
##
##
## Call:
## lm(formula = z.diff ~ z.lag.1 + 1 + z.diff.lag)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.35487 -0.01459  0.00065  0.01476  0.19289
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.008233   0.003760   2.189   0.0286 *
## z.lag.1     -0.002764   0.001274  -2.170   0.0301 *
## z.diff.lag  -0.028561   0.016266  -1.756   0.0792 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
##
## Residual standard error: 0.0297 on 3777 degrees of freedom
## Multiple R-squared: 0.002147, Adjusted R-squared: 0.001618
## F-statistic: 4.063 on 2 and 3777 DF, p-value: 0.01727
##
##
## Value of test-statistic is: -2.1699 2.3964
##
## Critical values for test statistics:
##      1pct  5pct 10pct
## tau2 -3.43 -2.86 -2.57
## phi1  6.43  4.59  3.78
```

Augmented dickey fuller test for Petrobrás stock prices with trend

```
adf_test_petr <- ur.df(log_data$Petrobras,type="trend", selectlags = "BIC")
summary(adf_test_petr)
```

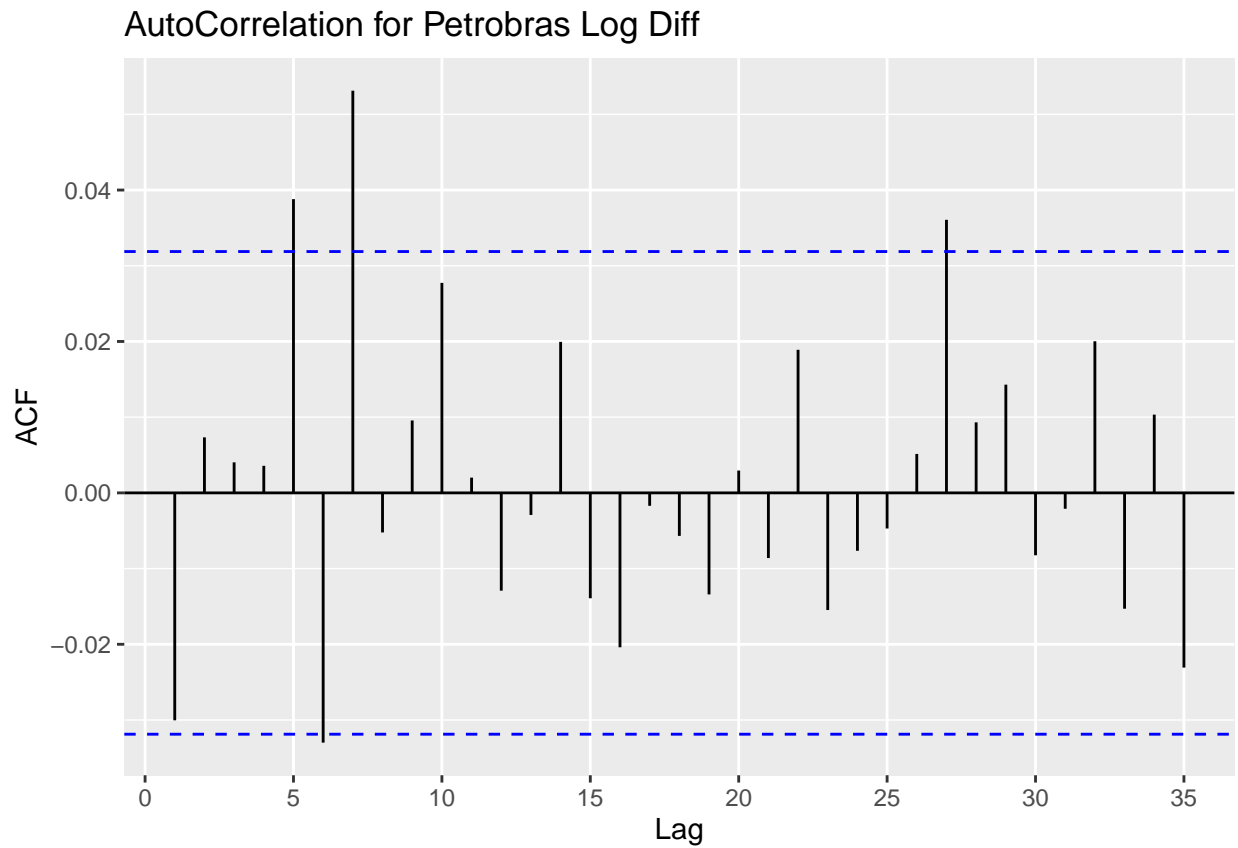
```
##
## #####
## # Augmented Dickey-Fuller Test Unit Root Test #
## #####
##
## Test regression trend
##
##
## Call:
## lm(formula = z.diff ~ z.lag.1 + 1 + tt + z.diff.lag)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.35511 -0.01455  0.00066  0.01472  0.19265
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  7.882e-03  3.874e-03   2.035  0.0419 *
## z.lag.1      -2.753e-03  1.275e-03  -2.160  0.0308 *
## tt           1.674e-07  4.429e-07   0.378  0.7055
## z.diff.lag   -2.861e-02  1.627e-02  -1.759  0.0787 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.02971 on 3776 degrees of freedom
## Multiple R-squared: 0.002185, Adjusted R-squared: 0.001392
## F-statistic: 2.756 on 3 and 3776 DF, p-value: 0.04095
##
##
## Value of test-statistic is: -2.1599 1.6449 2.425
##
## Critical values for test statistics:
##      1pct  5pct 10pct
## tau3 -3.96 -3.41 -3.12
## phi2  6.09  4.68  4.03
## phi3  8.27  6.25  5.34
```

Log Returns for the data

```
log_ret_petr <- diff(log_data$Petrobras)
log_ret_brent <- diff(log_data$`Brent Oil Prices`)
```

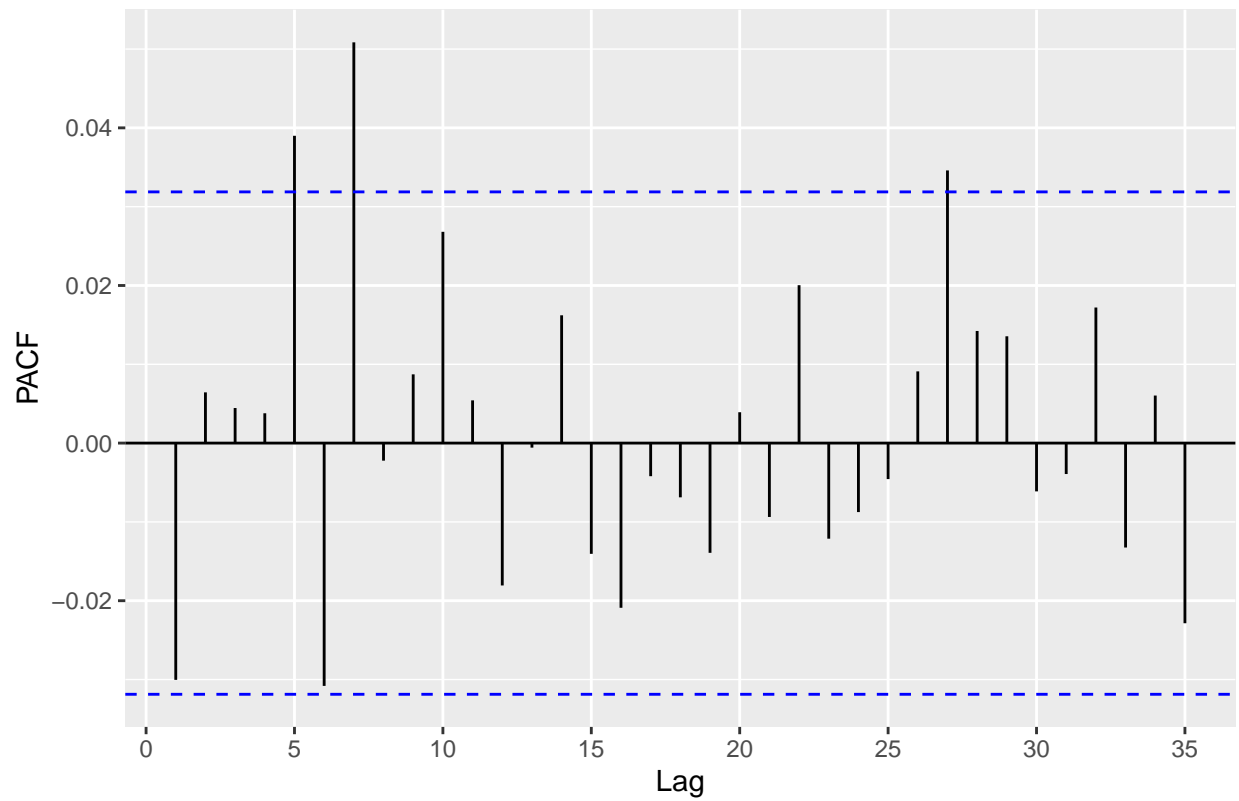
ACF, PACF and series of returns for Petrobras

```
ggAcf(log_ret_petr) + labs(title = "AutoCorrelation for Petrobras Log Diff")
```

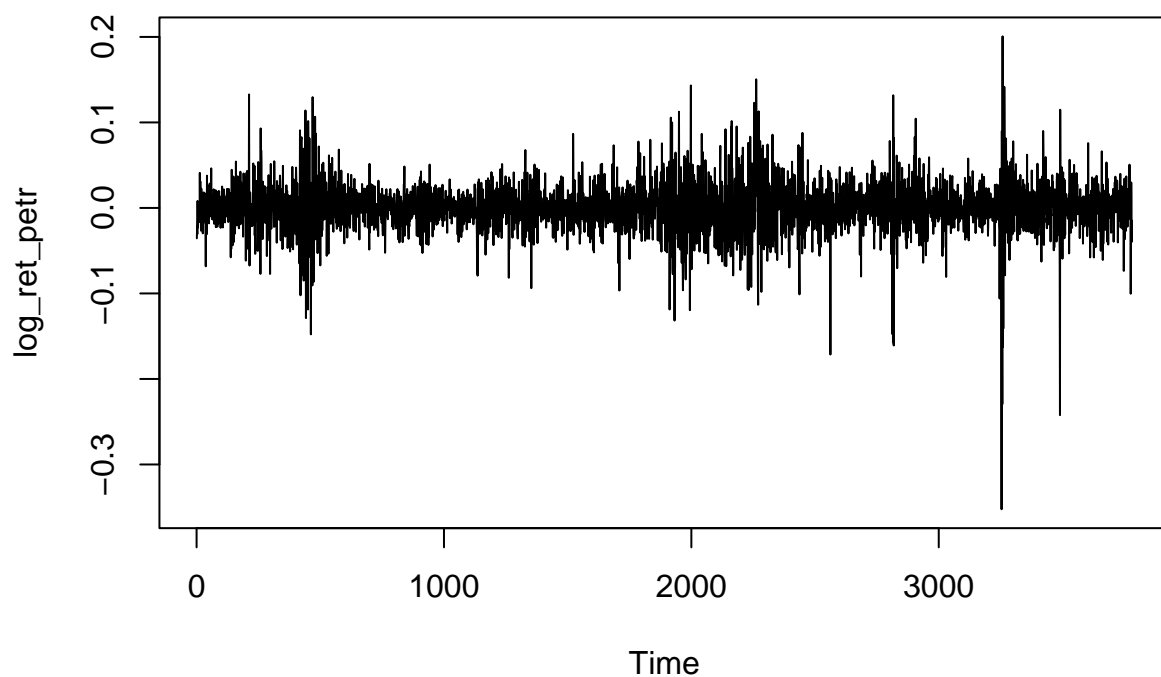


```
ggPacf(log_ret_petr) + labs(title = "Partial AutoCorrelation for Petrobras Log Diff")
```

Partial AutoCorrelation for Petrobras Log Diff



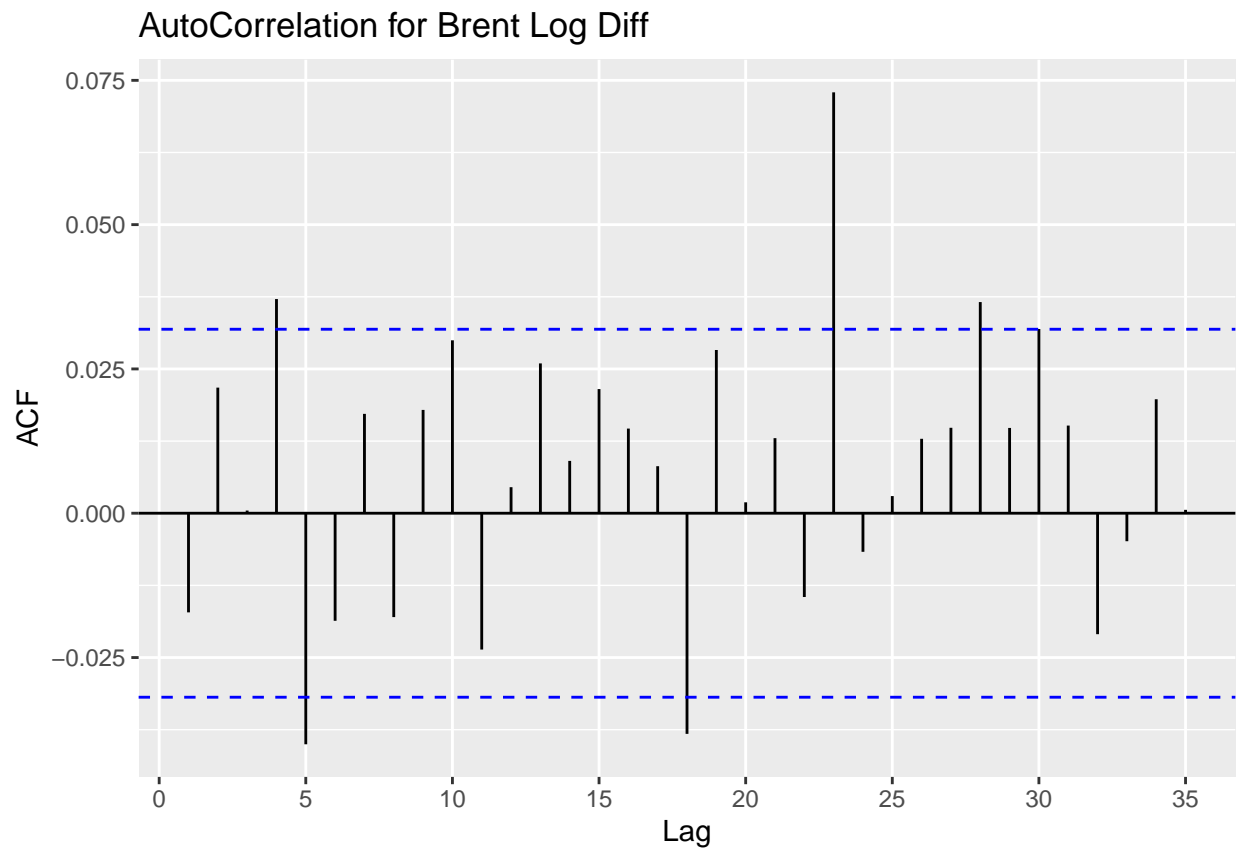
```
ts.plot(log_ret_petr) %>%  
  labs(title = "Log of returns for Petrobras")
```



```
## [[1]]
## NULL
##
## $title
## [1] "Log of returns for Petrobras"
##
## attr("class")
## [1] "labels"
```

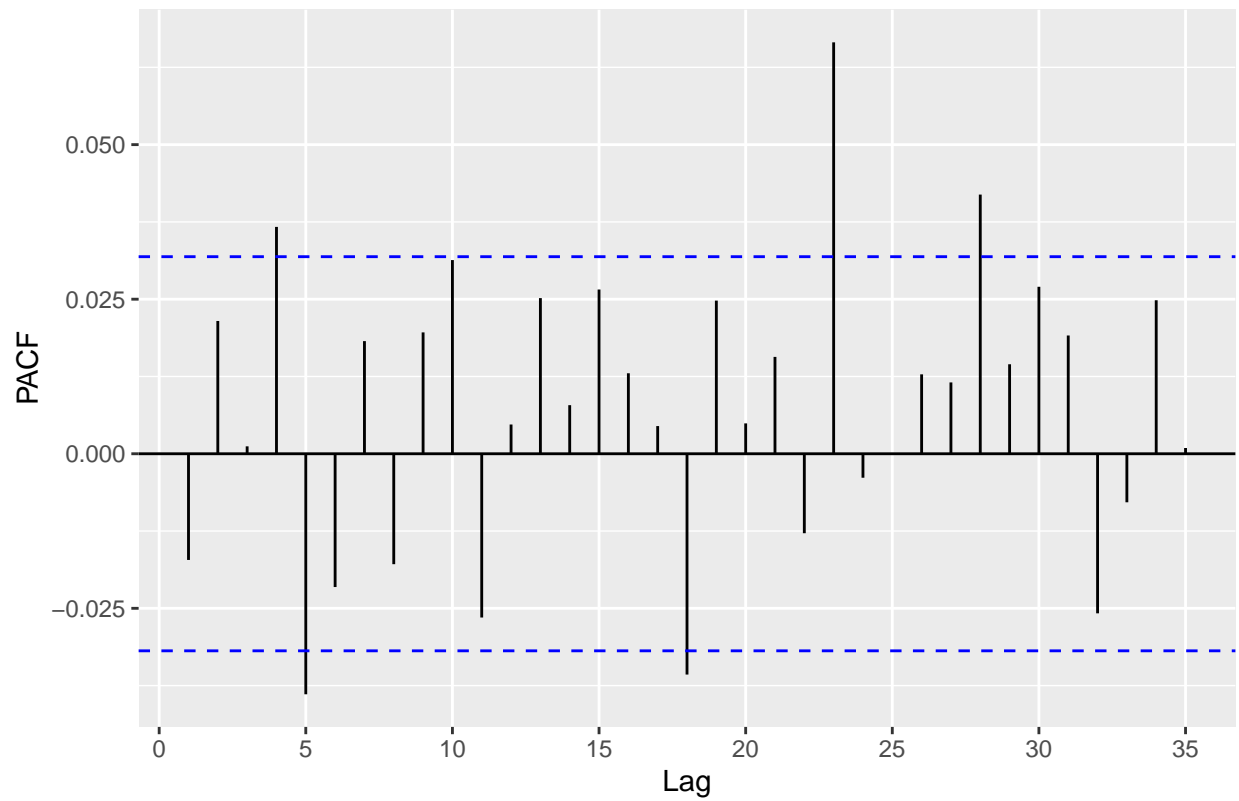
ACF, PACF and series of returns for Petrobras

```
log_ret_brent <- diff(log_data$`Brent Oil Prices`)
ggAcf(log_ret_brent) + labs(title = "AutoCorrelation for Brent Log Diff")
```

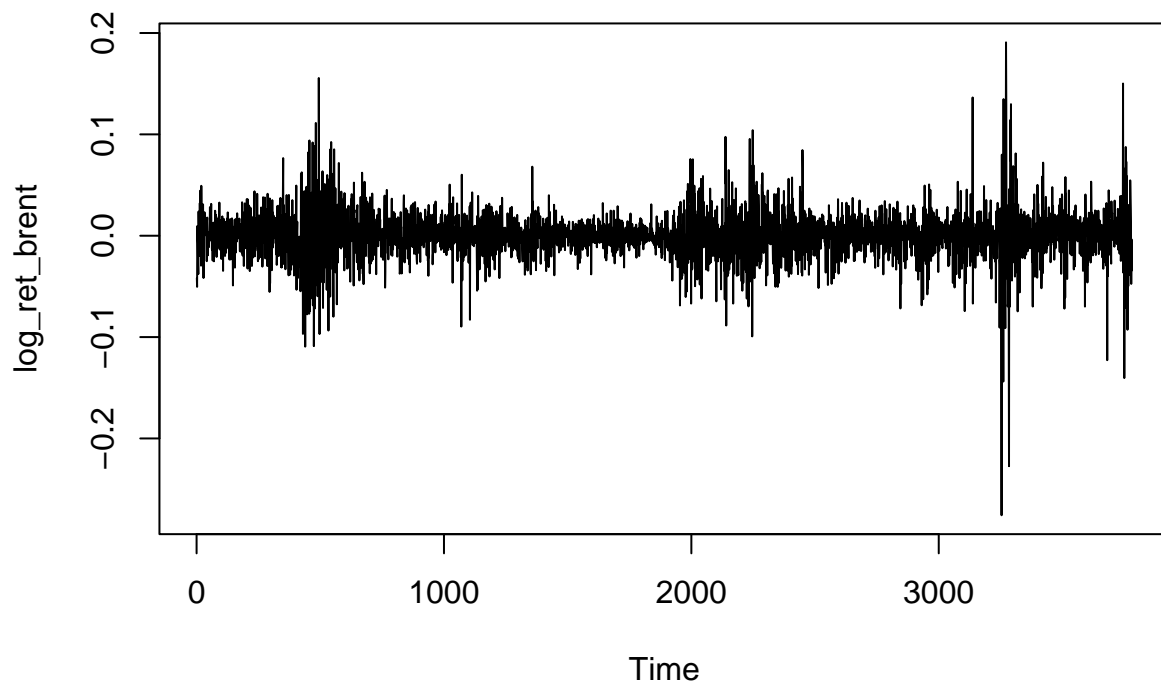


```
ggPacf(log_ret_brent) + labs(title = "Partial AutoCorrelation for Brent Log Diff")
```

Partial AutoCorrelation for Brent Log Diff



```
ts.plot(log_ret_brent) %>%  
  labs(title = "Log of returns for Brent Oil Prices")
```



```
## [[1]]
## NULL
##
## $title
## [1] "Log of returns for Brent Oil Prices"
##
## attr("class")
## [1] "labels"
```

After the transformation of data, we need to test the new data (log return scaled) for stationarity.

Augmented dickey fuller test for Petrobrás stock prices with drift

```
adf_test_petr <- ur.df(log_ret_petr,type="drift", selectlags = "BIC")
summary(adf_test_petr)
```

```
##
## #####
## # Augmented Dickey-Fuller Test Unit Root Test #
## #####
##
## Test regression drift
##
##
## Call:
```



```
## lm(formula = z.diff ~ z.lag.1 + 1 + z.diff.lag)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.35520 -0.01451  0.00063  0.01472  0.19428
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.0001457  0.0004835   0.301   0.763
## z.lag.1      -1.0236688  0.0233588 -43.824 <2e-16 ***
## z.diff.lag   -0.0064274  0.0162734  -0.395   0.693
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.02972 on 3776 degrees of freedom
## Multiple R-squared:  0.5152, Adjusted R-squared:  0.5149
## F-statistic: 2006 on 2 and 3776 DF, p-value: < 2.2e-16
##
##
## Value of test-statistic is: -43.8237 960.2569
##
## Critical values for test statistics:
##      1pct  5pct 10pct
## tau2 -3.43 -2.86 -2.57
## phi1  6.43  4.59  3.78
```

Augmented dickey fuller test for Petrobrás stock prices with trend

```
adf_test_petr <- ur.df(log_ret_petr,type="trend", selectlags = "BIC")
summary(adf_test_petr)
```

```
##
## #####
## # Augmented Dickey-Fuller Test Unit Root Test #
## #####
##
## Test regression trend
##
##
## Call:
## lm(formula = z.diff ~ z.lag.1 + 1 + tt + z.diff.lag)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.35545 -0.01456  0.00062  0.01472  0.19402
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -1.937e-04  9.677e-04  -0.200   0.841
## z.lag.1      -1.024e+00  2.336e-02 -43.821 <2e-16 ***
## tt           1.795e-07  4.433e-07   0.405   0.686
## z.diff.lag   -6.372e-03  1.628e-02  -0.392   0.695
## ---
```

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.02972 on 3775 degrees of freedom
## Multiple R-squared:  0.5152, Adjusted R-squared:  0.5148
## F-statistic: 1337 on 3 and 3775 DF,  p-value: < 2.2e-16
##
##
## Value of test-statistic is: -43.8206 640.0842 960.1262
##
## Critical values for test statistics:
##      1pct  5pct 10pct
## tau3 -3.96 -3.41 -3.12
## phi2  6.09  4.68  4.03
## phi3  8.27  6.25  5.34
```

Augmented dickey fuller test for Petrobrás stock prices

```
adf_test_petr <- ur.df(log_ret_petr,type="none", selectlags = "BIC")
summary(adf_test_petr)
```

```
##
## #####
## # Augmented Dickey-Fuller Test Unit Root Test #
## #####
##
## Test regression none
##
##
## Call:
## lm(formula = z.diff ~ z.lag.1 - 1 + z.diff.lag)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.35505 -0.01437  0.00077  0.01486  0.19443
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## z.lag.1      -1.023621   0.023355 -43.828  <2e-16 ***
## z.diff.lag  -0.006451   0.016271  -0.396   0.692
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.02972 on 3777 degrees of freedom
## Multiple R-squared:  0.5152, Adjusted R-squared:  0.5149
## F-statistic: 2007 on 2 and 3777 DF,  p-value: < 2.2e-16
##
##
## Value of test-statistic is: -43.8279
##
## Critical values for test statistics:
##      1pct  5pct 10pct
## tau1 -2.58 -1.95 -1.62
```

Augmented dickey fuller test for Brent prices with drift

```
adf_test_petr <- ur.df(log_ret_brent,type="drift", selectlags = "BIC")
summary(adf_test_petr)
```

```
##
## #####
## # Augmented Dickey-Fuller Test Unit Root Test #
## #####
##
## Test regression drift
##
##
## Call:
## lm(formula = z.diff ~ z.lag.1 + 1 + z.diff.lag)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.277196 -0.010377  0.000572  0.010925  0.192128
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.0001641  0.0003927   0.418   0.676
## z.lag.1      -0.9963238  0.0231957 -42.953 <2e-16 ***
## z.diff.lag   -0.0214655  0.0162584  -1.320   0.187
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.02414 on 3776 degrees of freedom
## Multiple R-squared:  0.5095, Adjusted R-squared:  0.5092
## F-statistic: 1961 on 2 and 3776 DF,  p-value: < 2.2e-16
##
##
## Value of test-statistic is: -42.9529 922.477
##
## Critical values for test statistics:
##      1pct  5pct 10pct
## tau2 -3.43 -2.86 -2.57
## phi1  6.43  4.59  3.78
```

Augmented dickey fuller test for Brent prices with trend

```
adf_test_petr <- ur.df(log_ret_brent,type="trend", selectlags = "BIC")
summary(adf_test_petr)
```

```
##
## #####
## # Augmented Dickey-Fuller Test Unit Root Test #
## #####
##
## Test regression trend
##
##
## Call:
```

```
## lm(formula = z.diff ~ z.lag.1 + 1 + tt + z.diff.lag)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.277234 -0.010379  0.000589  0.010915  0.192090
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  1.114e-04  7.859e-04   0.142   0.887
## z.lag.1      -9.963e-01  2.320e-02 -42.947 <2e-16 ***
## tt           2.791e-08  3.600e-07   0.078   0.938
## z.diff.lag   -2.146e-02  1.626e-02  -1.320   0.187
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.02414 on 3775 degrees of freedom
## Multiple R-squared:  0.5095, Adjusted R-squared:  0.5091
## F-statistic: 1307 on 3 and 3775 DF, p-value: < 2.2e-16
##
##
## Value of test-statistic is: -42.9471 614.8248 922.2369
##
## Critical values for test statistics:
##      1pct  5pct 10pct
## tau3 -3.96 -3.41 -3.12
## phi2  6.09  4.68  4.03
## phi3  8.27  6.25  5.34
```

Augmented dickey fuller test for Brent prices

```
adf_test_petr <- ur.df(log_ret_brent,type="none", selectlags = "BIC")
summary(adf_test_petr)
```

```
##
## #####
## # Augmented Dickey-Fuller Test Unit Root Test #
## #####
##
## Test regression none
##
##
## Call:
## lm(formula = z.diff ~ z.lag.1 - 1 + z.diff.lag)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.277026 -0.010214  0.000735  0.011092  0.192289
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## z.lag.1      -0.99623    0.02319 -42.956 <2e-16 ***
## z.diff.lag   -0.02151    0.01626  -1.323   0.186
## ---
```

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.02414 on 3777 degrees of freedom
## Multiple R-squared:  0.5094, Adjusted R-squared:  0.5092
## F-statistic: 1961 on 2 and 3777 DF,  p-value: < 2.2e-16
##
##
## Value of test-statistic is: -42.9556
##
## Critical values for test statistics:
##      1pct  5pct 10pct
## tau1 -2.58 -1.95 -1.62
```

Specifying ARIMA model for log returns of Petrobras

```
petr_model <- Arima(log_data$Petrobras, order=c(0,1,1), include.constant = TRUE)
summary(petr_model)
```

```
## Series: log_data$Petrobras
## ARIMA(0,1,1) with drift
##
## Coefficients:
##          ma1  drift
##       -0.0296 1e-04
## s.e.    0.0161 5e-04
##
## sigma^2 = 0.0008832:  log likelihood = 7929.91
## AIC=-15853.82  AICc=-15853.81  BIC=-15835.11
##
## Training set error measures:
##              ME      RMSE      MAE      MPE      MAPE      MASE
## Training set 8.625723e-07 0.02970694 0.02051251 -0.006977304 0.741714 0.998765
##              ACF1
## Training set -0.0002519786
```

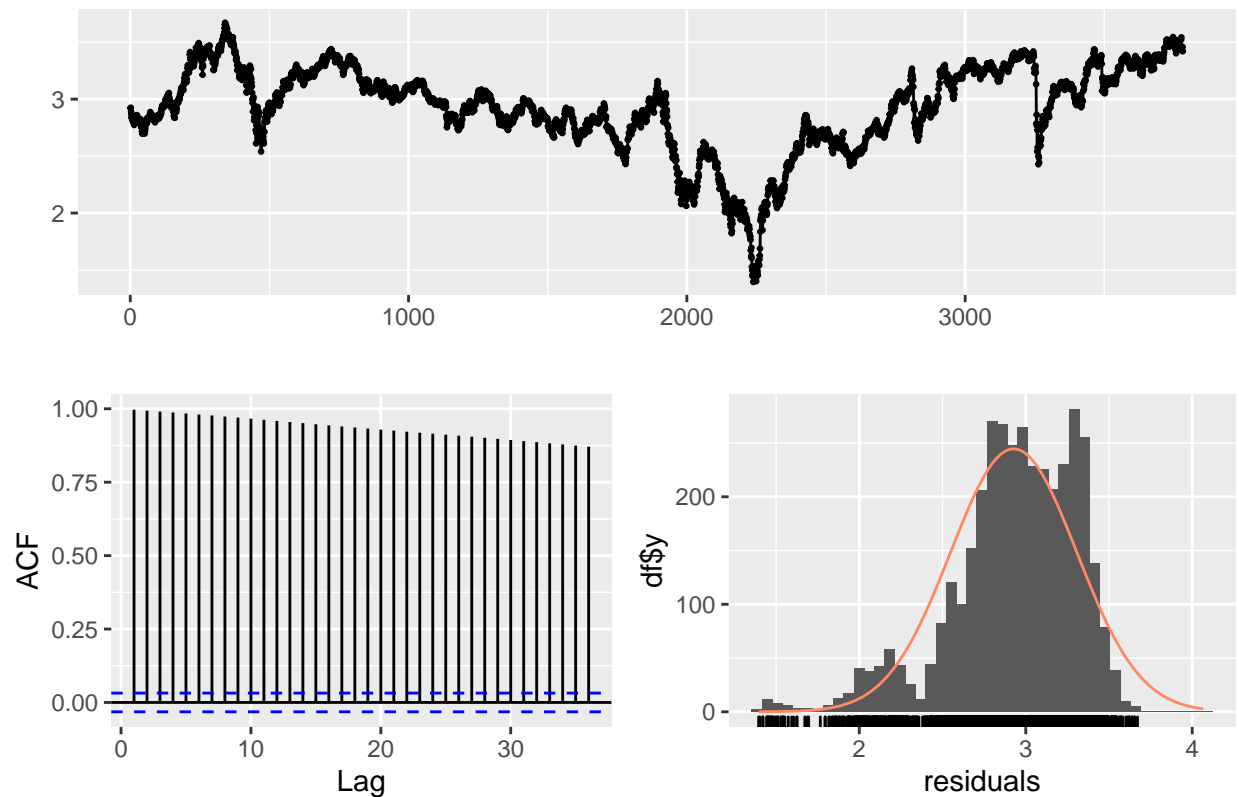
```
coeftest(petr_model)
```

```
##
## z test of coefficients:
##
##      Estimate Std. Error z value Pr(>|z|)
## ma1  -0.02962846  0.01613361 -1.8364  0.06629 .
## drift  0.00012716  0.00046915  0.2710  0.78636
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
checkresiduals(fitted(petr_model))
```

```
## Warning in modeldf.default(object): Could not find appropriate degrees of
## freedom for this model.
```

Residuals



Specifying ARIMA model for log returns of Brent

```
brent_model <- Arima(log_data$`Brent Oil Prices`, order=c(2,1,0), include.constant = TRUE)
summary(brent_model)
```

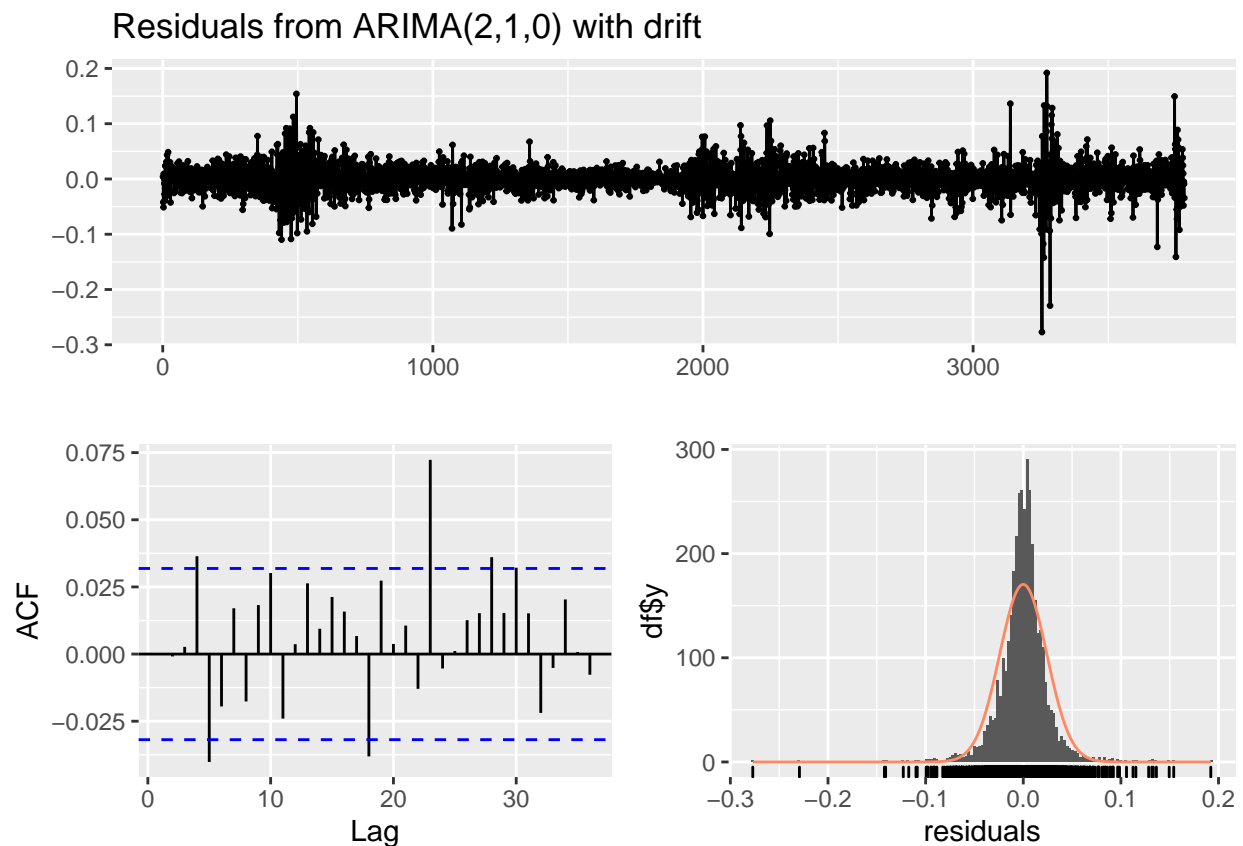
```
## Series: log_data$`Brent Oil Prices`
## ARIMA(2,1,0) with drift
##
## Coefficients:
##      ar1      ar2  drift
##      -0.0168  0.0215  1e-04
## s.e.    0.0163  0.0163  4e-04
##
## sigma^2 = 0.0005836:  log likelihood = 8713.78
## AIC=-17419.56   AICc=-17419.55   BIC=-17394.61
##
## Training set error measures:
##              ME          RMSE          MAE          MPE          MAPE
## Training set -1.441514e-06  0.02414469  0.01612419 -0.001970277  0.3894332
##              MASE          ACF1
## Training set  0.9985879 -0.0001247242
```

```
coeftest(brent_model)
```

```
##
```

```
## z test of coefficients:
##
##      Estimate Std. Error z value Pr(>|z|)
## ar1  -0.01678977 0.01626836 -1.0321 0.3020
## ar2   0.02149753 0.01627547  1.3209 0.1865
## drift  0.00014248 0.00039490  0.3608 0.7183
```

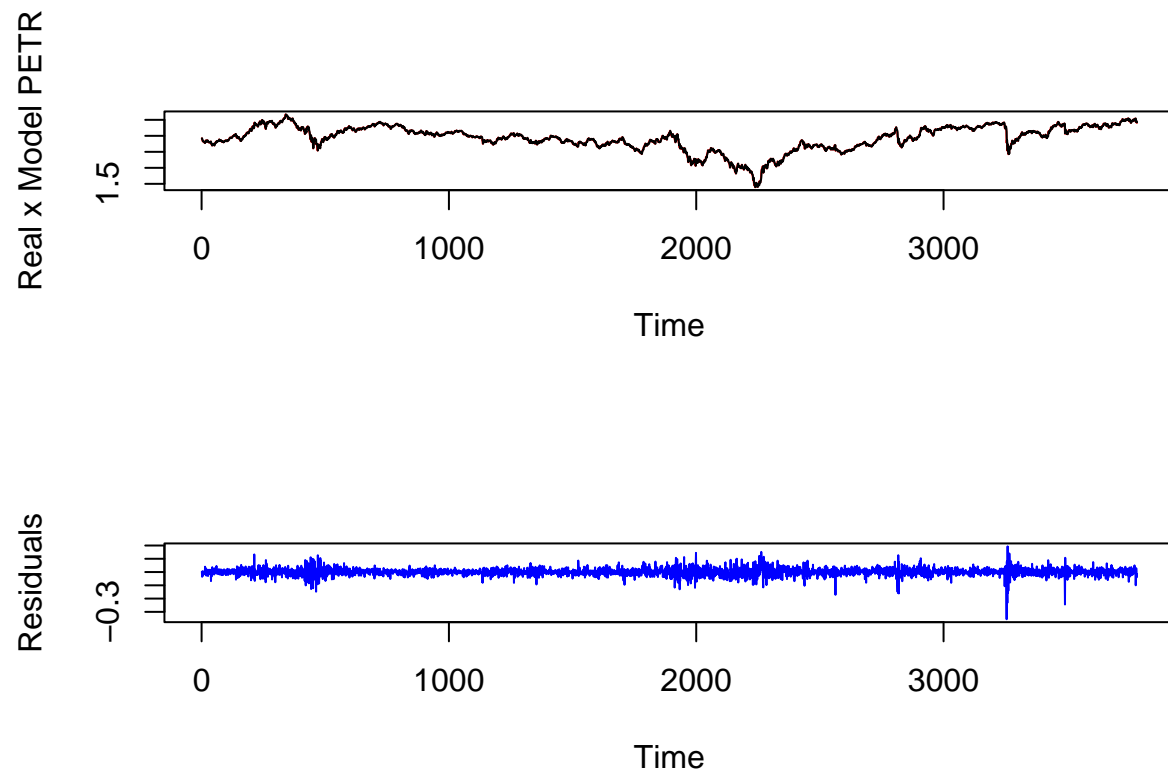
```
checkresiduals((brent_model))
```



```
##
## Ljung-Box test
##
## data: Residuals from ARIMA(2,1,0) with drift
## Q* = 19.614, df = 7, p-value = 0.006467
##
## Model df: 3. Total lags used: 10
```

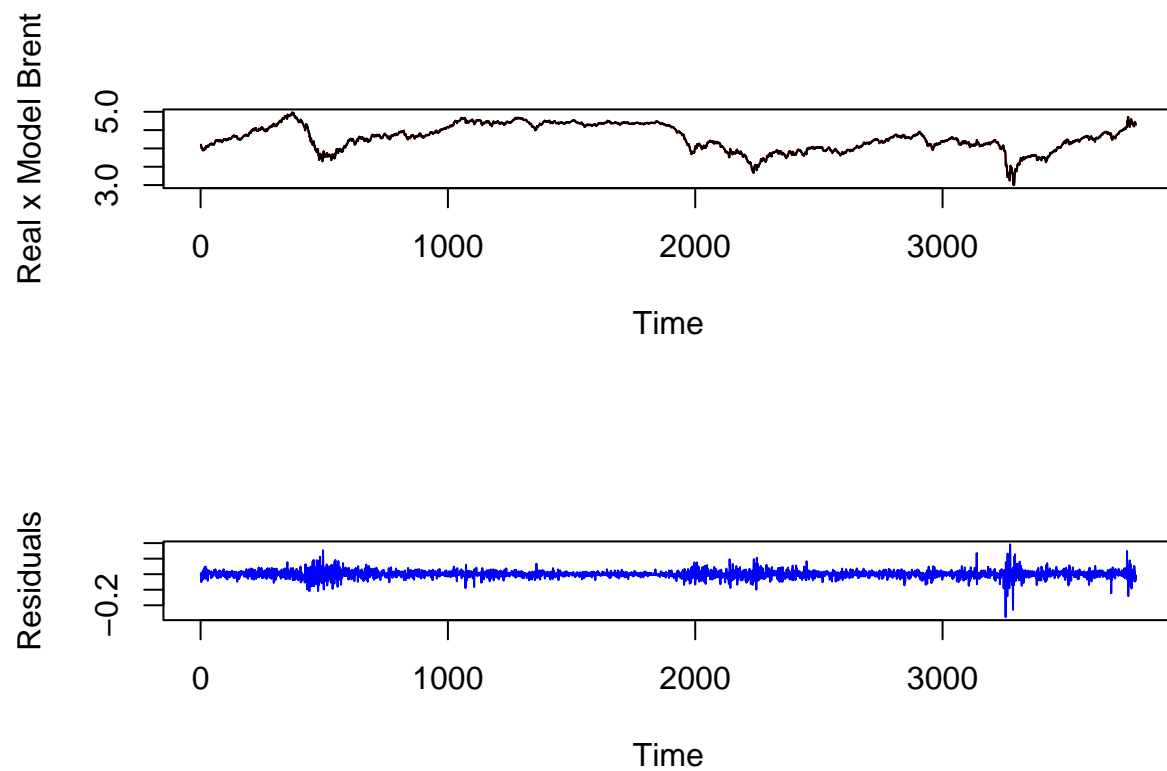
Real time series x Fitted Values for the model specified for Petrobras

```
par(mfrow = c(2,1))
ts.plot(as.ts(log_data$Petrobras), fitted(petr_model), col=c("red", "black"), ylab = "Real x Model PETR")
ts.plot(residuals(petr_model), col="blue", ylab="Residuals")
```



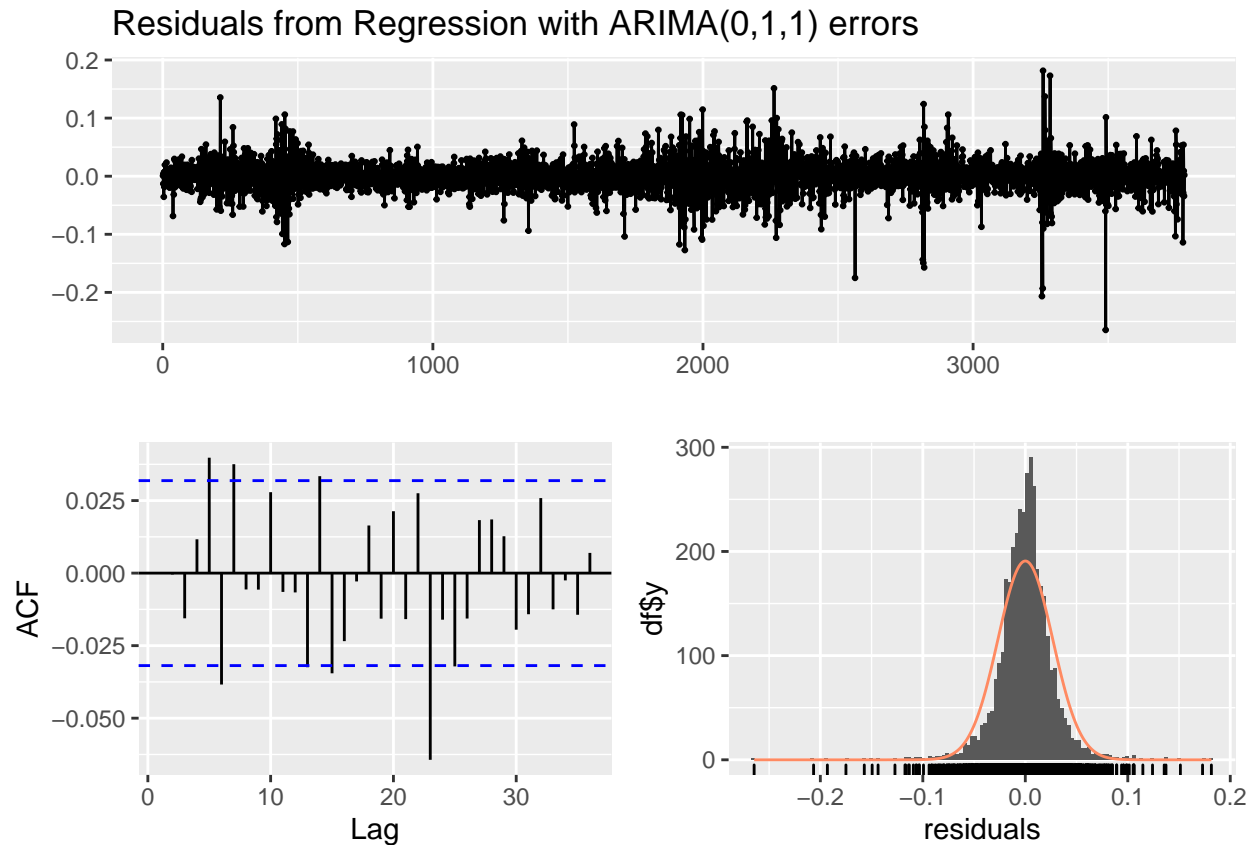
Real time series x Fitted Values for the model specified for Brent

```
par(mfrow = c(2,1))
ts.plot(as.ts(log_data$`Brent Oil Prices`), fitted(brent_model), col=c("red", "black"), ylab = "Real x Fitted")
ts.plot(residuals(brent_model), col="blue", ylab="Residuals")
```

Specifying and model to predict Petrobras prices with Brent Prices

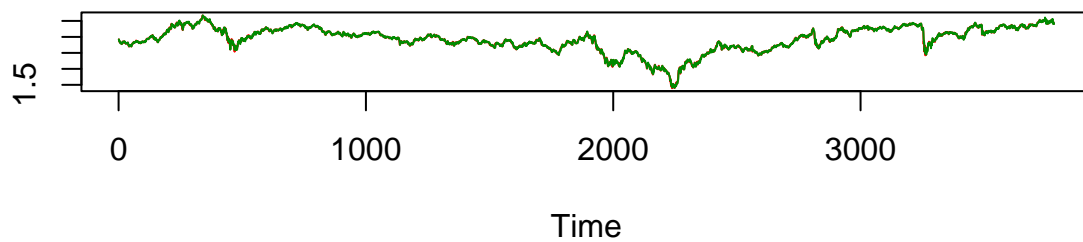
```
model_arimax <- Arima(log_data$Petrobras, order = c(0,1,1), xreg = log_data$`Brent Oil Prices`)  
checkresiduals(model_arimax)
```



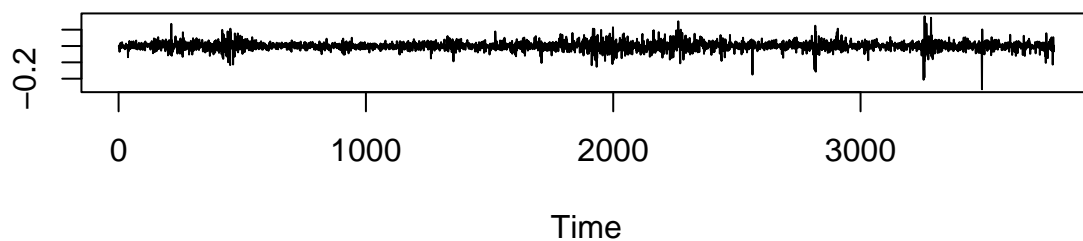
```
##
##  Ljung-Box test
##
## data:  Residuals from Regression with ARIMA(0,1,1) errors
## Q* = 21.551, df = 8, p-value = 0.005819
##
## Model df: 2.   Total lags used: 10
```

```
par(mfrow=c(2,1))
ts.plot(log_data$Petrobras, fitted(model_arimax), col=c("red", "green4"), ylab = "Real x Modelo Petrobras")
ts.plot(residuals(model_arimax), col="black", ylab="resíduo")
```

Real x Modelo Petrobrás

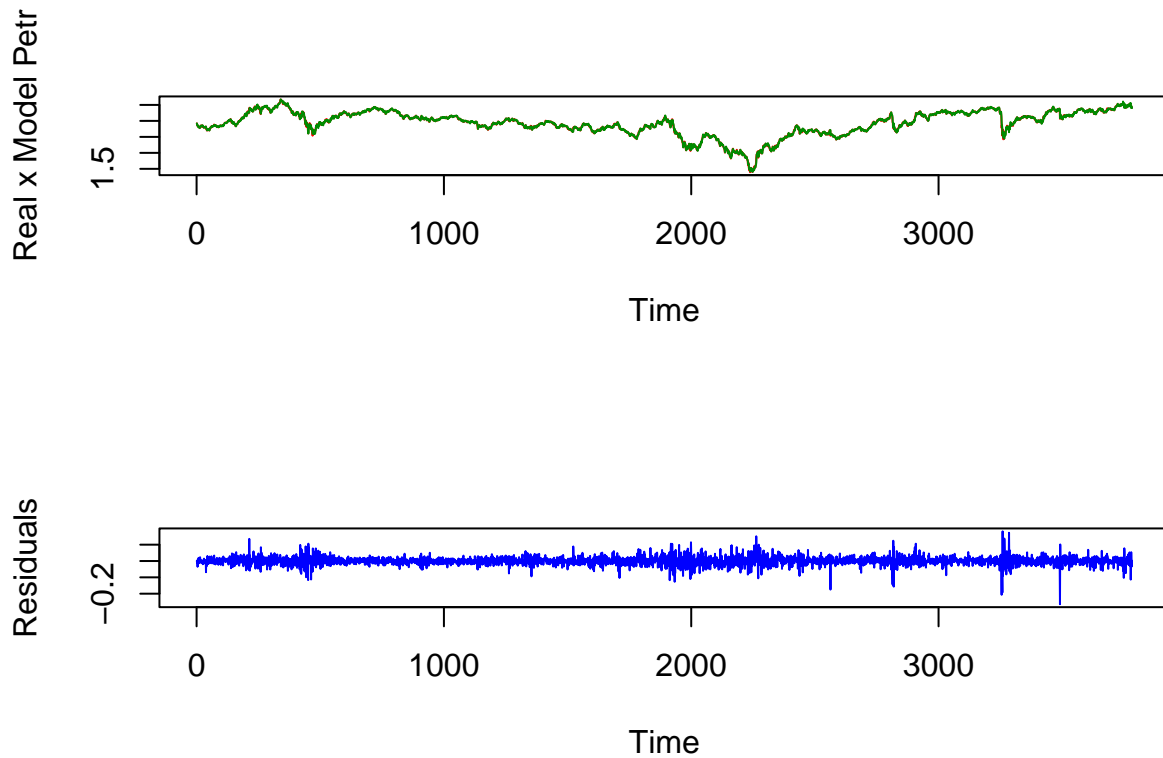


resíduo



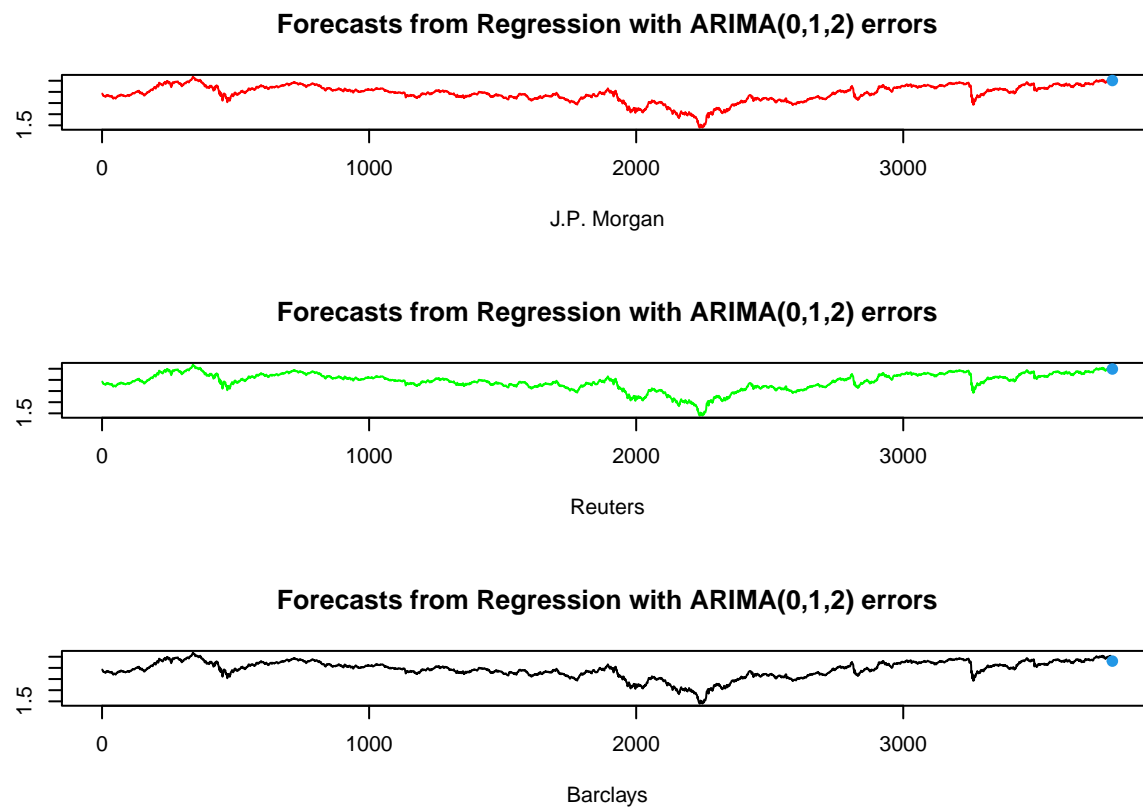
The model with 2 moving averages components seems to fit better

```
par(mfrow=c(2,1))
model_arimax <- Arima(log_data$Petrobras, order = c(0,1,2), xreg = log_data$`Brent Oil Prices`)
ts.plot(log_data$Petrobras, fitted(model_arimax), col=c("red", "green4"), ylab = "Real x Model Petr")
ts.plot(residuals(model_arimax), col="blue", ylab="Residuals")
```



For the forecast produce, we gonna get the J.P Morgan, Reuters and Barclays forecasts for Brent Oil prices for 2022. The information are avaiable at the following links:

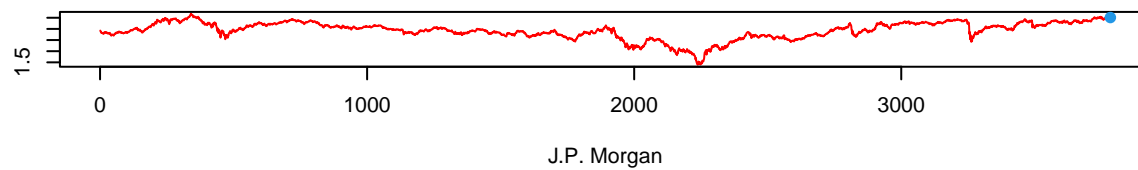
- J.P Morgan: '<https://www.thestreet.com/investing/oil-will-hit-125-a-barrel-in-2022-150-in-2023-jpmorgan>'
- Reuters: '<https://www.reuters.com/business/energy/oil-prices-drop-demand-worries-rising-supplies-2021-11-16/>'
- Barclays: '<https://boereport.com/2022/01/24/barclays-hikes-2022-oil-price-view-by-5-bbl-on-depressed-inventories/>'



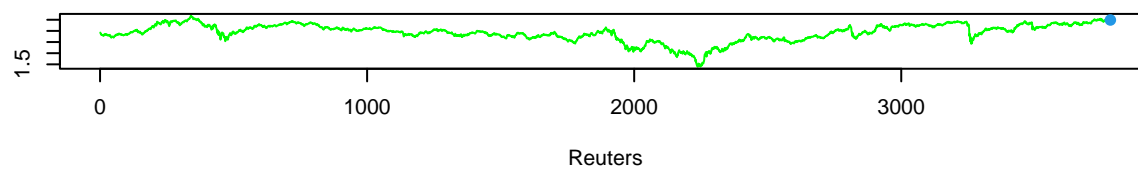
Since Brazil is going through an election year, we decide to add an auto-regressive component, since the PETR4 stock price can pass to depend on previous values.

```
par(mfrow=c(3,1))
plot(jpm_forecast_elections, col = c("red"), xlab = "J.P. Morgan")
plot(reuters_forecast_elections, col = c("green"), xlab = 'Reuters')
plot(barclays_forecast_elections, xlab = "Barclays")
```

Forecasts from Regression with ARIMA(1,1,2) errors



Forecasts from Regression with ARIMA(1,1,2) errors



Forecasts from Regression with ARIMA(1,1,2) errors

