1er Parcial Matemática Superior

Curso K3011 – 1er Cuatrimestre 2020

30/05/2020

1.

$$Z^2 + Zj + 3j = -Z + 6$$

$$Z^2 + Zj + 3j + Z - 6 = 0$$

$$\underbrace{1}_{a} * Z^2 + Z * \underbrace{(j+1)}_{b} + \underbrace{(3j-6)}_{c} = 0$$

$$Z_1, Z_2 = \frac{-b \pm \sqrt{b^2 - 4\,ac}}{2a} = \frac{-(j+1) \pm \sqrt{(j+1)^2 - 4\,(3j-6)}}{2}$$

$$Z_1, Z_2 = \frac{-j - 1 + \sqrt{\cancel{-1} + 2j \cancel{+1} - 12j + 24}}{2a} = \frac{-j - 1 + \sqrt{24 - 10j}}{2}$$

$$\boxed{Z_1 = \frac{-j - 1 + 5 - j}{2} = \frac{4 - 2j}{2} = 2 - j}$$

$$\boxed{Z_2 = \frac{\cancel{-j} - 1 - 5 \cancel{+j}}{2} = -3}$$

$$Z = (2 - j)^{-3}$$

$$\ln(Z) = -3 \ln(2 - j)$$

$$Z = e^{-3 \ln(2-j)}$$

$$Z = e^{-3\,(\ln \rho + \varphi j)} = e^{-3\,(\ln(2{,}235) + 5{,}82j)}$$

$$Z = \underbrace{e^{-3\,\ln(2{,}235)}}_{\rho} * e^{-17{,}46j}$$

$$\boxed{\rho = 0{,}09 \approx 0{,}0896}$$

C. A

$$Z = 24 - 10J$$

$$|Z| = \sqrt{24^2 + (-10)^2}$$

$$|Z| = 26$$

$$X = \pm\sqrt{\frac{26 + 24}{2}} = \pm 5$$

$$Y = \pm\sqrt{\frac{26 - 24}{2}} = \pm 1$$

$$b < 0 \;\therefore\; Sg\ distintos$$

$$5 - j \quad \vee \quad -5 + j$$

$$\rho * \cos(\varphi) = 2$$

$$\rho * \operatorname{sen}(\varphi) = -1$$

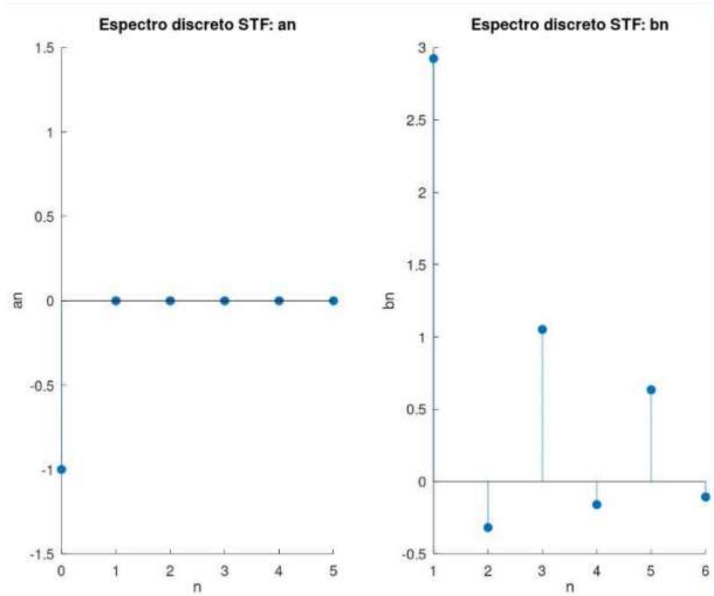
$$tg(\varphi) = -\frac{1}{2}$$

$$\varphi = 5{,}820$$

$$\rho = 2{,}235$$

2.

$$f(t)\begin{cases} t^2+1 & 0 \le t \le 1 \\ \dots & 1 < t < 2 \end{cases}$$



Del gráfico se puede interprestar:

$n = 0 \rightarrow a_0 = -1$

$n = 1 \rightarrow a_1 = 0$

$n = 2 \rightarrow a_2 = 0$

$$C_n = \frac{\cancel{a_n} - b_n j}{2} \longrightarrow$$

- *Los coeficientes* C_n *son imaginarios*
- *No posee simetría de media onda ya que* $b_{2k} \neq 0 \vee a_0 \neq 0$
- C_n los coeficientes son imaginarios puros excepto C_0

$\boxed{-(t-2)^2 - 3}$ → para que tenga continuidad

3.

$$\int_0^{\infty} \underbrace{t * \cos(3t)}_{f(t)} * \underbrace{a^t}_{e} = 0$$

$$L[t * \cos(3t)] = \frac{s^2-9}{(s^2+9)^2} = 0$$

$$s^2 - 9 = 0$$
$$s^2 = 9$$
$$|s| = \pm 3$$

C.A

$$\int_0^{\infty} f(t) * e^{-st}$$

Por tabla:

$$t * \cos(\omega t) = \frac{s^2-\omega^2}{(s^2+\omega^2)^2}$$

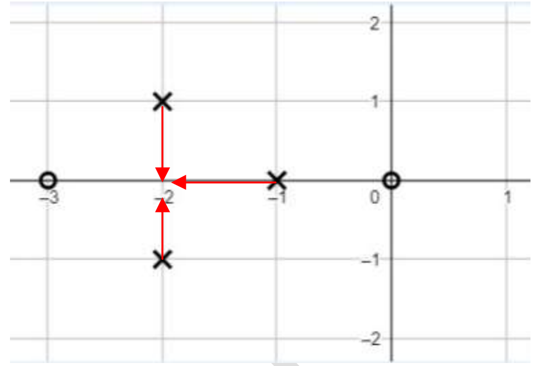
Para que me de cero:

$$a^t = e^{-3t} = \Bigg(\underbrace{\frac{1}{e^3}}_{a}\Bigg)^t$$

$\boxed{a = 0{,}0498}$

4.

$$|G(-2)| = 6, X(t) = \underbrace{f(t) = \delta(t)}_{F(s)=1}$$



Ceros:
$Z_1 = -3$
$Z_2 = 0$

Polos:
$p_1 = -1$
$p_2 = -2 + j$
$p_3 = -2 - j$

$$G(s) = \frac{k * (s+3) * s}{(s+1) * ((s+2)^2 + 1)}$$

Módulos:
$|Z_1| = \sqrt{1^2 + 0^2} = 1$
$|Z_2| = \sqrt{(-2)^2 + 0^2} = 2$
$|p_1| = \sqrt{(-1)^2 + 0^2} = 1$
$|p_2| = \sqrt{0^2 + 1^2} = 1$
$|p_3| = \sqrt{0^2 + (-1)^2} = 1$

$$|G(-2)| = \frac{k * 1 * 2}{1 * 1 * 1} = 6$$

$$\boxed{k = 3}$$

$$G(s) = 3\frac{s * (s+3)}{(s+1) * ((s+2)^2 + 1)}$$

$$Y(s) = F(s) * G(s) = 1 * \frac{3 * s * (s+3)}{(s+1) * ((s+2)^2 + 1)} = \frac{3s^2 + 9s}{(s+1) * ((s+2)^2 + 1)}$$

$$Y(s) = \frac{3 * s * (s+3)}{(s+1) * ((s+2)^2 + 1)} = \frac{A}{(s+1)} + \frac{Bs + C}{(s+2)^2 + 1}$$

$$A * (s^2 + 4s + 5) + Bs(s+1) + C(s+1) = 3s^2 + 9s$$

$$\textit{Haciendo los despejes quedan:} \begin{cases} A = -3 \\ B = 6 \\ C = 15 \end{cases}$$

$$Y(s) = \frac{-3}{(s+1)} + \frac{6s + 15}{(s+2)^2 + 1^2}$$

Por definición:

$$Y(s) = \frac{Bs + C}{(s-\alpha)^2 + \omega^2}$$

$$y(t) = Ae^{\alpha t}\cos(\omega t) + \frac{B + \alpha A}{\omega} e^{\alpha t} sen(\omega t)$$

$$y(t) = -3e^{-t} + 6e^{-2t}\cos(t) + 3e^{-2t}\text{sen}(t)$$

$$y(\pi) = -3e^{-\pi} + 6e^{-2\pi}\cos(\pi) + 3e^{-2\pi}\text{sen}(\pi)$$

$$\boxed{y(\pi) = -0{,}1408}$$

5.

$$X(n)\begin{cases} -(3)^n & n\ par \\ (-1)^n & n\ impar \end{cases}$$

$$Z[X(n)] = \sum_{k=0}^{\infty} -(3)^{2k} * Z^{-2k} + \sum_{k=0}^{\infty} (-1)^{2k+1} * Z^{-(2k+1)}$$

$$Z[X(n)] = -\sum_{k=0}^{\infty} 9^k * \frac{1}{Z^{2k}} + \sum_{k=0}^{\infty} (-1)^{2k} * (-1) * \frac{1}{Z^{2k}} * \frac{1}{Z}$$

$$Z[X(n)] = -\sum_{k=0}^{\infty} \left(\frac{9}{Z^2}\right)^k + \frac{(-1)}{Z} \sum_{k=0}^{\infty} \left(\frac{1}{Z^2}\right)^k \rightarrow Aplicando\ S.G: \sum_{n=0}^{\infty} r^n = \frac{1}{1-r}\ , |r| < 1$$

$$Z[X(n)] = -\frac{1}{1-\frac{9}{z^2}} - \frac{1}{Z} * \frac{1}{1-\frac{1}{z^2}}$$

$$\boxed{Z[X(n)] = -\frac{Z^2}{Z^2-9} - \frac{Z}{Z^2-1}}$$

$$X(4) = -\frac{4^2}{4^2-9} - \frac{4}{4^2-1}$$

$$\boxed{X(4) = -\frac{268}{105} \approx -2{,}55}$$