Property Graphs

The Property Graph Data Model

- Born in the database community
  - Meant to be queried and processed
  - **<u>THERE IS NOTHING SUCH A STANDARD!</u>**
- Two main constructs: nodes and edges
  - Nodes represent entities,
  - Edges relate pairs of nodes, and may represent different types of relationships
- Nodes and edges might be labeled,
- and may have a set of properties represented as attributes (key-value pairs)***
- Further assumptions:
  - Edges are directed,
  - Multi-graphs are allowed

**** Note: in some definitions (the least) edges are not allowed to have attributes*

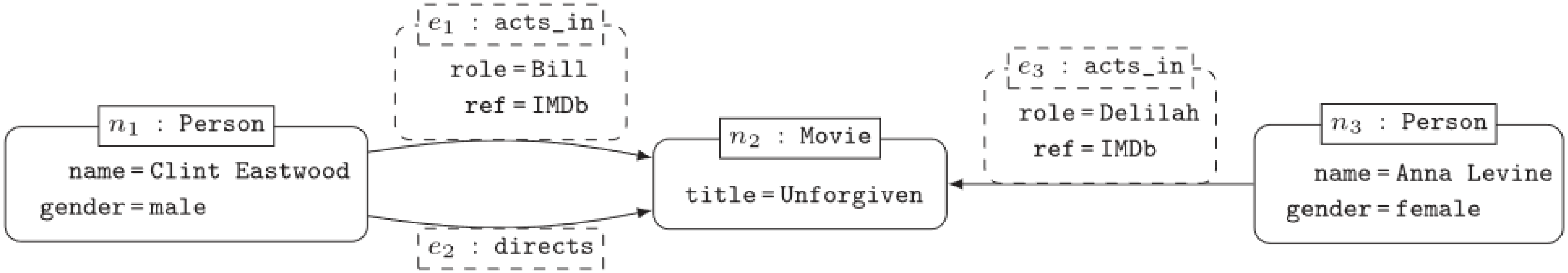
Formal Definition

Definition 2.3 (Property graph). A property graph G is a tuple $(V, E, \rho, \lambda, \sigma)$, where:

(1) V is a finite set of *vertices* (or *nodes*).
(2) E is a finite set of *edges* such that V and E have no elements in common.
(3) $\rho : E \to (V \times V)$ is a total function. Intuitively, $\rho(e) = (v_1, v_2)$ indicates that e is a directed edge *from* node v_1 *to* node v_2 in G.
(4) $\lambda : (V \cup E) \to Lab$ is a total function with Lab a set of labels. Intuitively, if $v \in V$ (respectively, $e \in E$) and $\lambda(v) = \ell$ (respectively, $\lambda(e) = \ell$), then ℓ is the label of node v (respectively, edge e) in G.
(5) $\sigma : (V \cup E) \times Prop \to Val$ is a partial function with $Prop$ a finite set of properties and Val a set of values. Intuitively, if $v \in V$ (respectively, $e \in E$), $p \in Prop$ and $\sigma(v, p) = s$ (respectively, $\sigma(e, p) = s$), then s is the value of property p for node v (respectively, edge e) in the property graph G.

Extracted from: R. Angles et al. Foundations of Modern Query Languages for Graph Databases

Example of Property Graph



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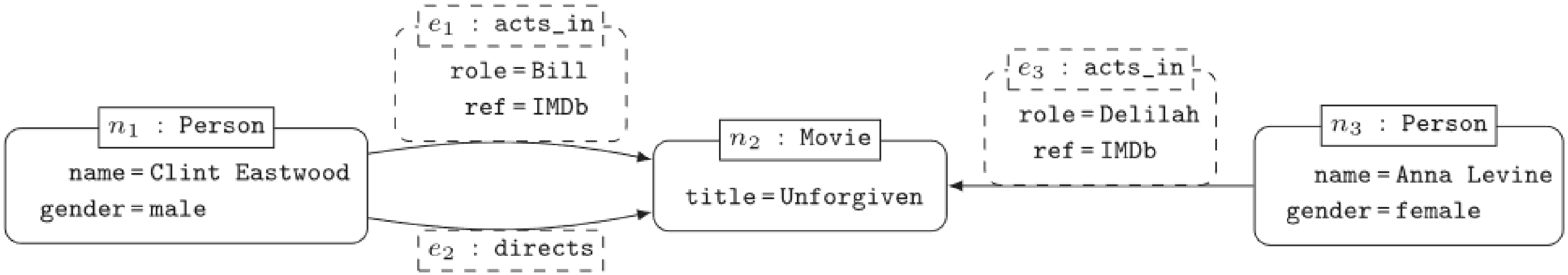

Formal definition:

$$V = \{n_1, n_2, n_3\} \qquad E = \{e_1, e_2, e_3\}$$

$$\rho(e_1) = (n_1, n_2) \qquad \rho(e_2) = (n_1, n_2)$$
$$\rho(e_3) = (n_3, n_2)$$

$$\lambda(n_1) = \texttt{Person} \qquad \lambda(n_2) = \texttt{Movie}$$
$$\lambda(n_3) = \texttt{Person} \qquad \lambda(e_1) = \texttt{acts\_in}$$
$$\lambda(e_2) = \texttt{directs} \qquad \lambda(e_3) = \texttt{acts\_in}$$

$$\begin{aligned}
\sigma(n_1, \texttt{name}) &= \texttt{Clint Eastwood} \\
\sigma(n_1, \texttt{gender}) &= \texttt{male} \\
\sigma(n_2, \texttt{title}) &= \texttt{Unforgiven} \\
\sigma(n_3, \texttt{name}) &= \texttt{Anna Levine} \\
\sigma(n_3, \texttt{gender}) &= \texttt{female} \\
\sigma(e_1, \texttt{role}) &= \texttt{Bill} \\
\sigma(e_1, \texttt{ref}) &= \texttt{IMDb} \\
\sigma(e_3, \texttt{role}) &= \texttt{Delilah} \\
\sigma(e_3, \texttt{ref}) &= \texttt{IMDb}
\end{aligned}$$

Traversal Navigation

- We define the graph traversal pattern as: *“the ability to rapidly traverse structures to an arbitrary depth (e.g., tree structures, cyclic structures) and with an arbitrary path description (e.g. friends that work together, roads below a certain congestion threshold)”* [Marko Rodriguez]

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- Totally opposite to set theory (on which relational databases are based on)
  - Sets of elements are operated by means of the relational algebra

Traversing Data in a RDBMS

- In the relational theory, it is equivalent to joining data (schema level) and select data (based on a value)

User	Address	Phone	Email	Alternate
Alice	123 Foo St.	12345678	alice@example.org	alice@neo4j.org
Bob	456 Bar Ave		bob@example.org	
...	...	...	...	...
Zach	99 South St.		zach@example.org	

User	OrderId
Alice	1234
Alice	5678
Alice	9012
...	...
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OrderId	LineItemId
1234	abcd
1234	efab
...	...
5588	bafd

LineItemId	Description	Handling
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```
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orders o, items i
WHERE u.user = uo.user AND
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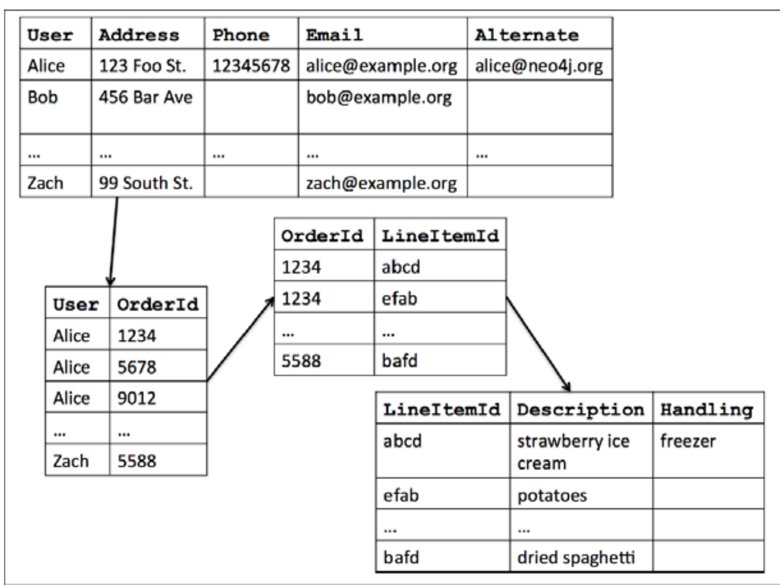

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Cardinalities:

|User|: 5.000.000
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|Orders|: 1.000.000.000
|Item|: 35.000

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Query Cost?!

Activity

- *Wear your data steward hat and discuss in pairs the database tuning that would guarantee the most efficient access plan for this query*
  - *What join algorithm would you take? Why?*

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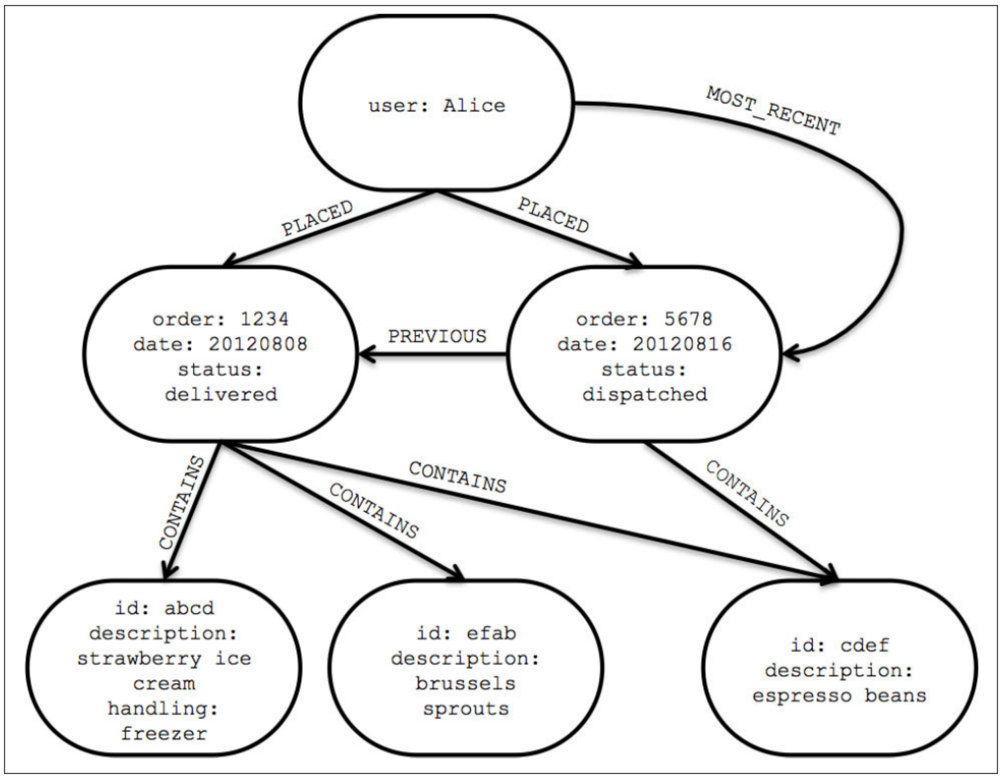

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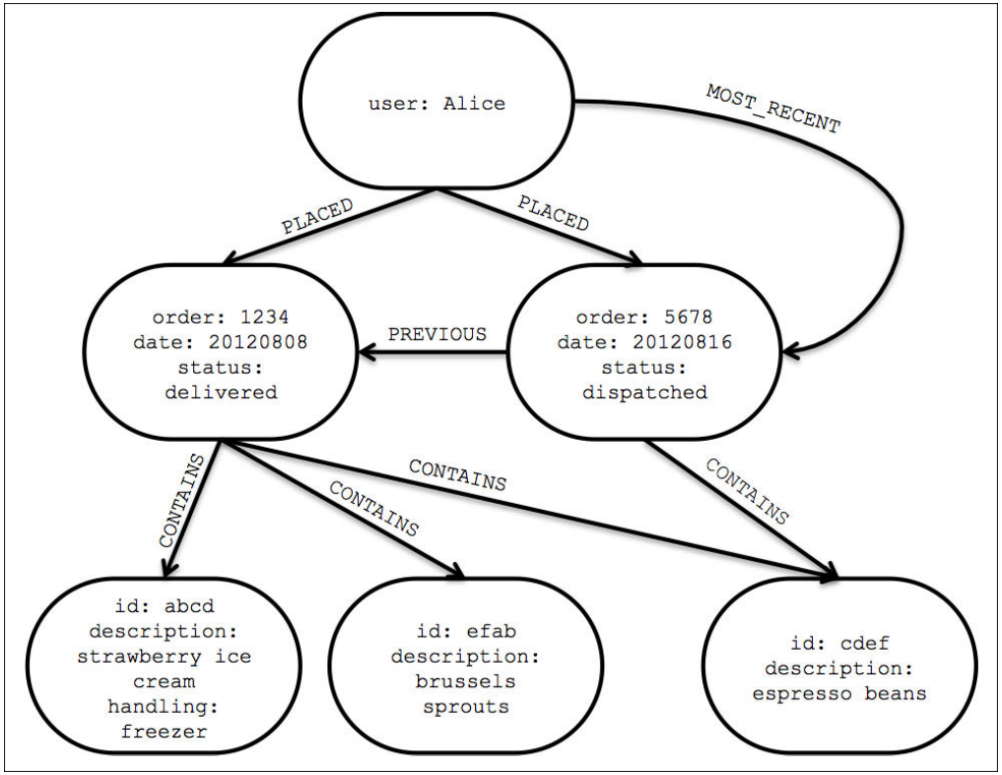

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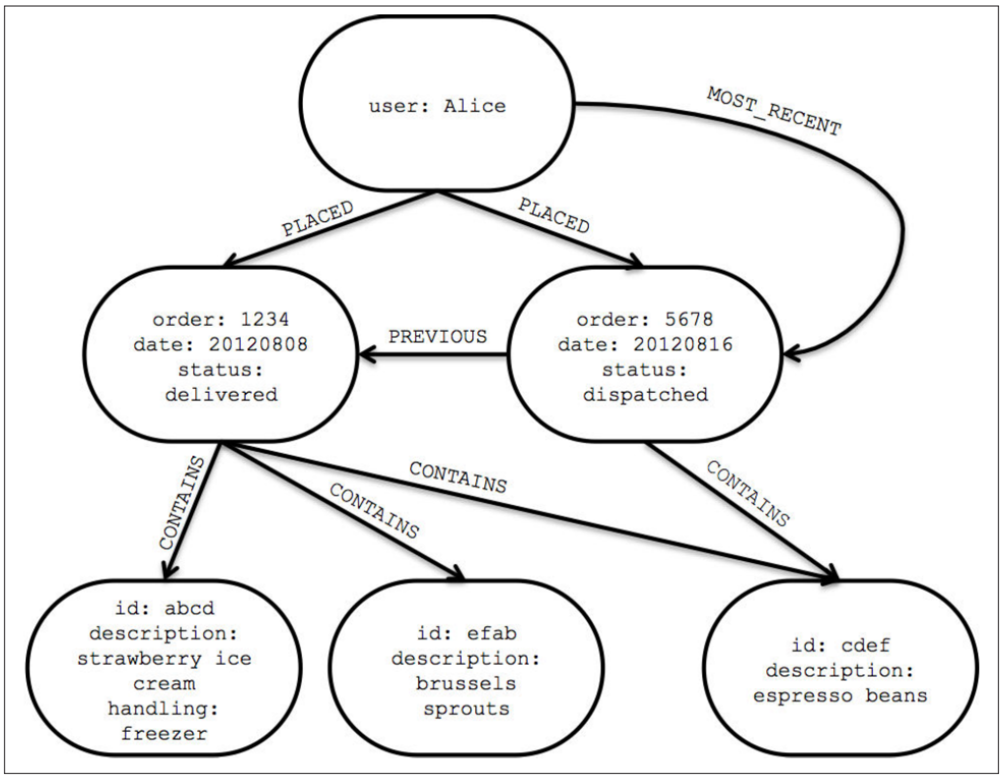
Activity

- *What would be the cost of this query in a graph database?*



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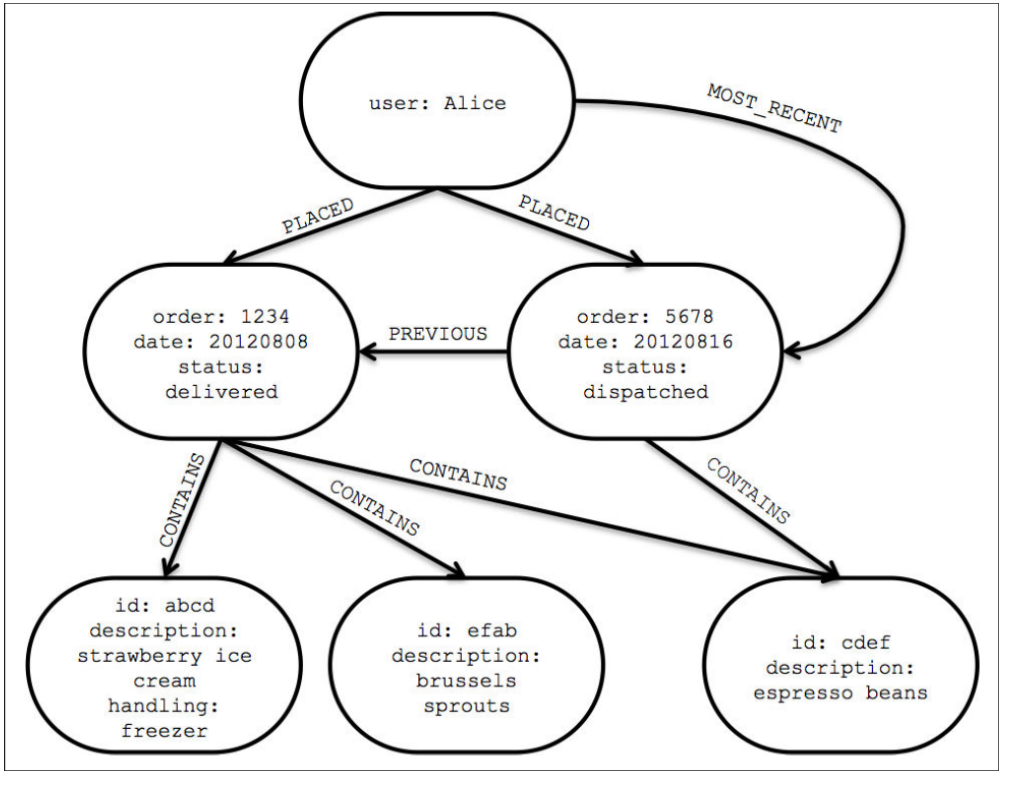

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Query Cost?!

Traversing Property Graphs

- Traversing graph data depends on two main variables
  - The size of the graph (i.e., #edges),
  - The topology of the graph,
  - The query topology

REFRESHING SOME BASICS ON GRAPHS

GRAPH OPERATIONS

Graph Operations

- Content-based queries
  - The value is relevant
    - Get a node, get the value of a node / edge attribute, etc.
    - A typical case are summarization queries (i.e., aggregations)
- Topological queries
  - Only the graph topology is considered
  - Typically, several business problems (such as fraud detection, trend prediction, product recommendation, network routing or route optimization) are solved using graph algorithms exploring the graph topology
    - Computing the betweenness centrality of a node…
      - in a social network, an analyst can detect influential people or groups for targeting a marketing campaign audience.
      - in a telecommunication operator, an analyst may detect central nodes of an antenna network and optimize the routing and load balancing across the infrastructure accordingly
- Hybrid approaches

Topological Queries

- Divided in three (four) main categories
  - Adjacency,
  - Reachability,
  - Pattern Matching,
  - [Graph metrics]

Adjacency Queries

- Formalized as node adjacency or edge incidence
  - Node adjacency
  - Edge incidence (node degree, out-degree, in-degree)
  - K-neighbourhood of a node
- <u>Computational cost</u>: linear cost on the number of edges to visit
- <u>Examples</u>:
  - Find all friends of a person
  - Airports with a direct connection
  - Movies watched by a person
  - Products bought by a customer
  - ...

Reachability Queries

- Formalized as traversal queries
  - Fixed-length paths (fixed #edges and nodes)
  - Regular simple paths (restrictions as regular expressions)
    - Hybrid if the restriction is in the *content*
  - Shortest path
- <u>Computational cost</u>: hard to compute for large graphs
  - Shortest-path (Dijkstra's algorithm): **O(|V²|)**
    - Smarter implementations based on priority queues yield **O(|E|*|V|log|V|)** complexity
- <u>Examples</u>:
  - Friend-of-a-friend
  - Flight connections
  - Logistics (goods distribution)
  - Items bought in a user orders
  - ...

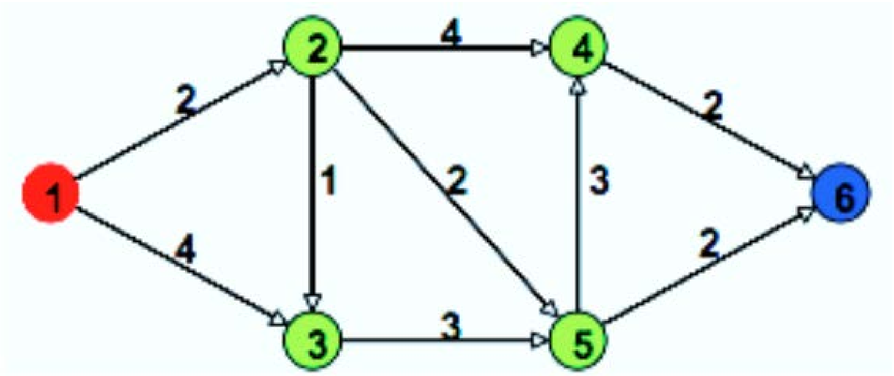
Single-Source Shortest-Path

- Dijkstra's algorithm
  - Main idea:
    - Optimal substructure: The subpath of any shortest path is itself a shortest path
    - Triangle inequality: $\delta(u,v) \leq \delta(u,x) + \delta(x,v)$, if u,v is the shortest path
  - Input:
    - A weighted graph **G = (V,E),**
    - A source vertex $\mathbf{V_s} \in \mathbf{V}$
  - Internal structures:
    - **S**: Set of vertices whose shortest paths from the source have already been determined,
    - **Q (V-S)**: Remaining vertices,
    - **d**: current estimated shortest paths to each vertex,
    - [**pre**: array de predecessors for each vertex] – traceback
  - Output:
    - The graph representing all the paths from one vertex to all the others must be a spanning tree
    - There will be no cycles as a cycle would define more than one path from the selected vertex to at least one other vertex

Pseudocode

```
shortest_paths( Graph g, Node s )
    initialise_single_source( g, s )
    S := { 0 }          /* Make S empty */
    Q := Vertices( g ) /* Put the vertices in a PQ */
    while not Empty(Q)
        u := ExtractCheapest( Q );
        AddNode( S, u ); /* Add u to S */
        for each vertex v in Adjacent( u )
            relax( u, v, w )

 relax( Node u, Node v, double w[][] )
     if d[v] > d[u] + w[u,v] then
        d[v] := d[u] + w[u,v]
        pi[v] := u
```



Label-constrained Reachability

- Definition:

$$G_L^* = \{(s,t) \mid \text{there is a path in } G \text{ from } s \text{ to } t \text{ using only edges with labels in } L\}$$

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- It is equivalent to determine whether or not there is a path in G from s to t such that the concatenation of the edge labels along the path forms a string in the language denoted by the regular expression $(\ell_1 \cup \cdots \cup \ell_n)^*$

where: $L = \{\ell_1, \ldots, \ell_n\}$, U *disjunction and * the Kleene star*

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- Typically, the allowed topology and labels involved are expressed as a regular expression
  - In general, the cost of regular label-constrained queries is known to be NP-complete

Pattern Matching

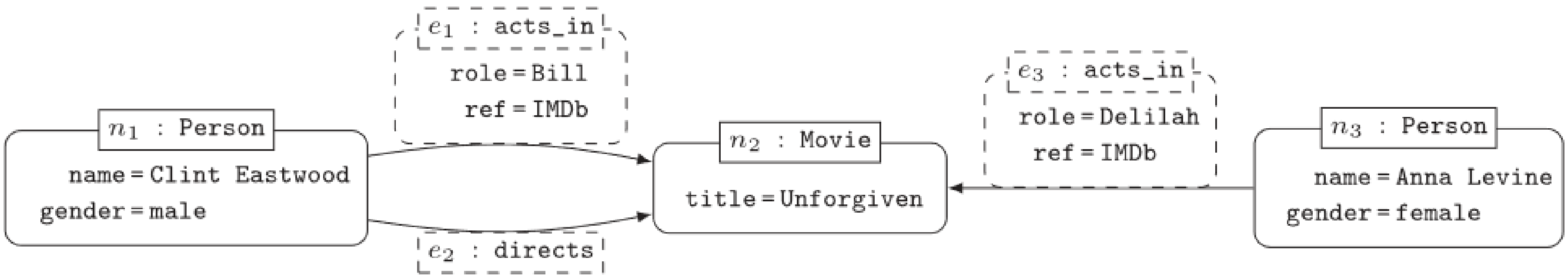
- Formalized as the graph isomorphism problem
  - Input: property graph G, and a pattern graph P
  - Output: all sub-graphs of G that are isomorphic to P
- <u>Computational cost</u>: hard to compute, in general, NP-complete
- Examples:
  - Group of cities all of them directly connected by flights
  - People without telephone
  - People who have watched sci-fi movies in the last month
  - …

Property Graph Patterns

- Based on *basic graph patterns* (bgps)
  - Equivalent to conjunctive queries
- A *bgp* for querying property graphs is a property graph where variables can appear in place of any constant (labels / properties)
- A **match** for a *bgp* is a mapping from variables to constants such that when the mapping is applied to the *bgp*, the result is *contained* within the original graph
- The **results** for a *bgp* are then all mappings from variables in the query to constants that comprise a match

Example of Graph Pattern

Graph:



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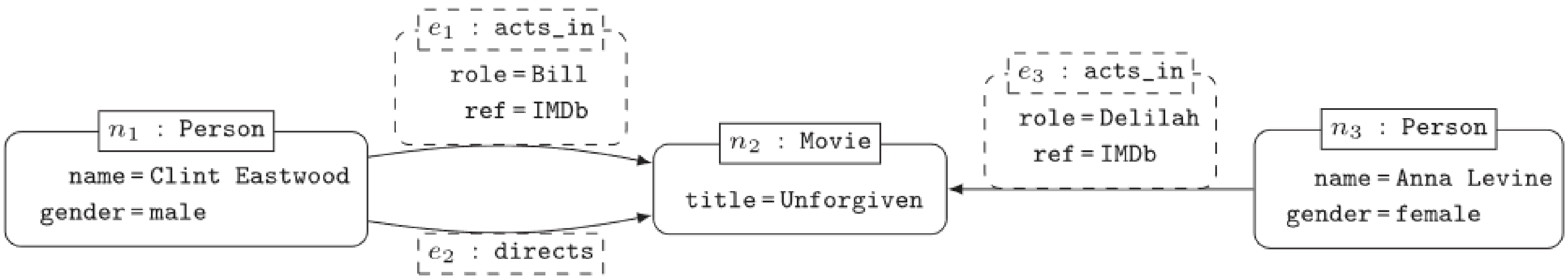
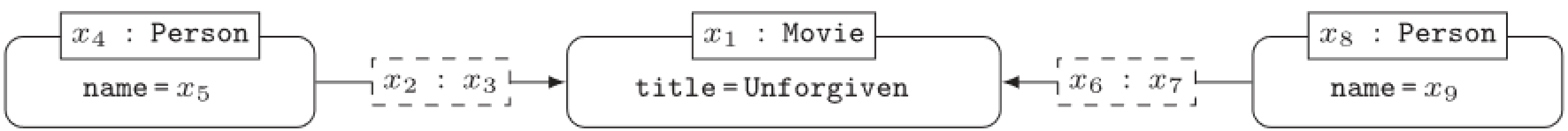


BGP:



Evaluating Graph Patterns

- Evaluating a *bgp* **Q** against a graph database **G** corresponds to listing all possible matches of **Q** with respect to **G**
- Formally:

Definition 3.5 (Match). Given an edge-labelled graph $G = (V, E)$ and a bgp $Q = (V', E')$, a *match* h of Q in G is a mapping from $Const \cup Var$ to $Const$ such that:

(1) for each constant $a \in Const$, it is the case that $h(a) = a$; that is, the mapping maps constants to themselves; and
(2) for each edge $(b, l, c) \in E'$, it holds that $(h(b), h(l), h(c)) \in E$; this condition imposes that (a) each edge of Q is mapped to an edge of G, and (b) the structure of Q is preserved in its image under h in G (that is, when h is applied to all the terms in Q, the result is a sub-graph of G).

Extracted from: R. Angles et al. Foundations of Modern Query Languages for Graph Databases

Semantics of a Match

- **Homomorphism-based semantics**: the previous definition maps to a homomorphism from **Q** to **G**
  - Multiple variables in **Q** can map to the same term in **G**
  - Corresponds to the familiar semantics of select-from-where queries (conjunctive queries) in relational databases
- **Isomorphism-based semantics**: an additional constraint is added, the match function *h* must be injective. Still, different semantics can be applied:
  - Strict isomorphism (no-repeated-anything): *h* is injective
  - No repeated-node semantics: *h* is only injective for nodes
  - No repeated-edge semantics: h is only injective for edges

Activity

- Objective: Understand the differences between graph matching isomorphism-based and homomorphism-based semantics
  - Given the following graph, bgp and potential results...

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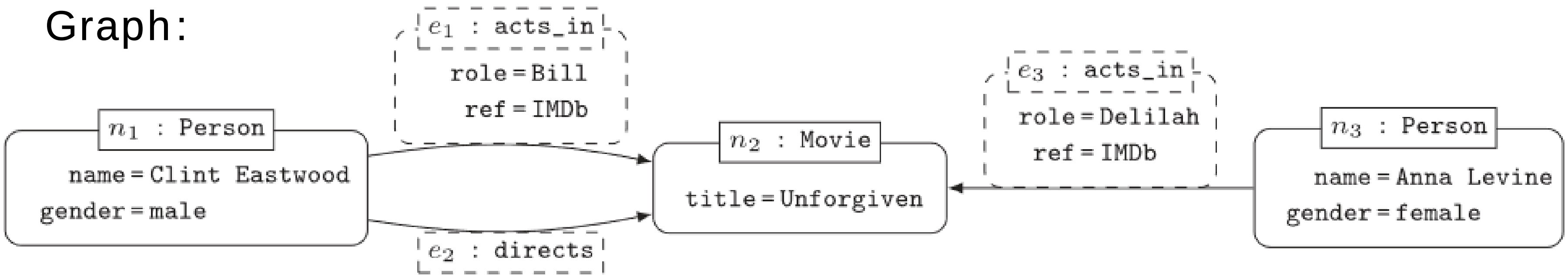

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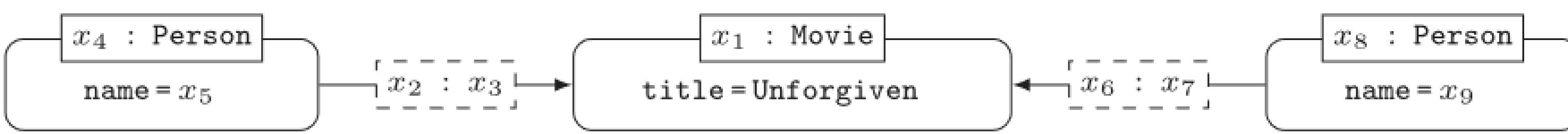

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Results:

x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
n_2	e_2	directs	n_1	Clint Eastwood	e_3	acts_in	n_3	Anna Levine
n_2	e_3	acts_in	n_3	Anna Levine	e_2	directs	n_1	Clint Eastwood
n_2	e_1	acts_in	n_1	Clint Eastwood	e_3	acts_in	n_3	Anna Levine
n_2	e_3	acts_in	n_3	Anna Levine	e_1	acts_in	n_1	Clint Eastwood
n_2	e_2	directs	n_1	Clint Eastwood	e_1	acts_in	n_1	Clint Eastwood
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n_2	e_1	acts_in	n_1	Clint Eastwood	e_1	acts_in	n_1	Clint Eastwood
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n_2	e_3	acts_in	n_1	Anna Levine	e_3	acts_in	n_1	Anna Levine

Circle what results would be obtained if applying isomorphism-based or homomorphism-based semantics

From Intractable to Tractable Matching

- These results apply to property graphs:
  - Graph isomorphism is known to be NP-complete in the worst case
  - Graph homomorphism is also known to be NP-complete in the worst case
  - However, *graph simulation and bi-simulation,* a relaxed form of graph homomorphism, can be computed within polynomial time
    - It might still hard to compute for large graphs
    - New iterative algorithms allow to scale-well

Fan et al. Graph Pattern Matching: From Intractable to Polynomial Time

Graph Metrics

- They can be formalized either as adjacency, reachability or pattern matching
  - Thus, the cost depends on how the metric is formalized
- Given their relevance, they are typically provided as built-in functions
- <u>Examples</u>:
  - Graph node order,
  - the min / max degree in the graph,
  - the length of a path,
  - the graph diameter,
  - the graph density,
  - closeness / betwenness of a node,
  - the pageRank of a node,
  - ...

Topological Queries

- Support provided by current GBDs

	Adjacency		Reachability				
Graph Database	Node/edge adjacency	k-neighborhood	Fixed-length paths	Regular simple paths	Shortest path	Pattern matching	Summarization
Allegro	•		•			•	
DEX	•		•	•	•	•	
Filament	•		•			•	
G-Store	•		•	•	•	•	
HyperGraph	•					•	
Infinite	•		•	•	•	•	
Neo4j	•		•	•	•	•	
Sones	•					•	
vertexDB	•		•	•		•	

R. Angles. A Comparison of Current Graph Database Models (as of 2012)

Implementation of the Operations

- Note that the operations presented are conceptual: agnostic of the technology
- The implementation of the ops depends on:
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Summary

- Property graphs do not have a defined standard, but there is a de-facto standard
  - Nodes / edges may have labels (equivalent to the concept of typing) and / or properties (to the concept of attributes)
- The basic operations on graphs are:
  - Adjacency queries,
  - Reachability queries and
  - Pattern matching
- However, their traditional definition needs to be redefined for property graphs, yielding different computational complexities
  - It basically depends on the graph topology, graph pattern and internal structures the graph database is implemented on