UNIVERSIDADE FEDERAL DO MARANHÃO

CENTRO DE CIÊNCIAS EXATAS E TECNOLOGIA

CURSO DE C COMPUTAÇÃO

6,0

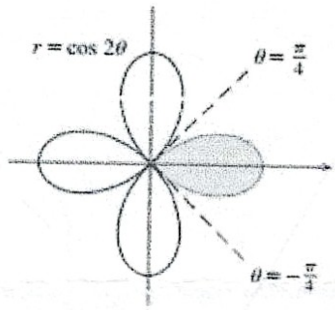
ALUNO(A):

Avaliação 1 de Cálculo Diferencial e Integral II

A

01) (1,5) Calcule a integral $\int \frac{\text{sen}\, x\, dx}{\cos^2 x}$.

02) (1,5) Calcule a área delimitada por uma rosácea de quatro pétalas cujo gráfico mostramos abaixo.



03) (1,5) Determine o volume do sólido gerado pela rotação da curva $x^2 + y^2 = r^2$, $r > 0$ em torno do eixo-x.

04) (1,5) Calcule a integral $\int \sqrt{x^2 + 5}\, dx$.

05) (1,5) Calcule o limite $\lim_{x \to 0^+} x^2 \ln x$.

06) (1,5) Calcule a integral $\int_0^{+\infty} x e^{-x}\, dx$.

07) (1,0) Determinar os quatro primeiros termos do polinômio de Taylor, em torno de $a = 4$, da função $f(x) = x^{3/2}$.

Boa Sorte!

1) $\int \frac{\operatorname{sen} x \, dx}{\cos^2 x} = \int \frac{du}{u^2} = -\frac{1}{u} + c = -\frac{1}{\cos x} + c$ //

$u = \cos x$

$du = \operatorname{sen} x \, dx$

2) $r = \cos 2\theta$, resultado . 4

$A = \frac{1}{2}\int_{-\pi/4}^{\pi/4} (\cos 2\theta)^2 d\theta = \frac{1}{2}\int_{-\pi/4}^{\pi/4} (\cos u)^2 \frac{du}{2} = \frac{1}{2}\cdot\frac{1}{2}\int_{-\pi/4}^{\pi/4} (\cos u)^2 du$

$\frac{1}{2}\operatorname{sen} 2\theta$

$u = 2\theta$

$du = 2 d\theta$

$d\theta = \frac{du}{2}$

$\operatorname{sen}^2\theta + \cos^2\theta = 1$

$\cos^2\theta = 1 - \operatorname{sen}^2\theta$

$= \frac{1}{4}\int_{-\pi/4}^{\pi/4} 1 - \int_{-\pi/4}^{\pi/4} -\operatorname{sen}^2 u \, du$

$\frac{1}{4}\left(2\theta + \operatorname{sen}^2 2\theta\right)\Big|_{-\pi/4}^{\pi/4}$

$A = \frac{2\frac{\pi}{4}}{4} + \frac{\operatorname{sen}^2 2\frac{\pi}{4}}{4} - \left(\frac{2\cdot\frac{-\pi}{4}}{4} + \frac{\operatorname{sen}^2 2\cdot\frac{-\pi}{4}}{4}\right) = \frac{\frac{\pi}{2}}{4} + \frac{\operatorname{sen}^2\frac{\pi}{2}}{4} + \frac{\frac{\pi}{2}}{4} - \frac{\operatorname{sen}^2 -\frac{\pi}{2}}{4} = 2\frac{\frac{\pi}{2}}{4} + \frac{\operatorname{sen}^2 90^\circ}{4} - \frac{\operatorname{sen}^2}{4}$

$\frac{\pi}{4}$

$A = \frac{\pi}{4} + \frac{1}{4} + \frac{1}{4} \rightarrow A = \frac{2+\pi}{4} \rightarrow$ área total $= 4 \cdot \frac{2+\pi}{4}$

$AT = 2 + \pi$ m //

3) $x^2+y^2=r^2$; $r>0$; eixo x $(y=0)$

$y^2=r^2-x^2$

$y=\sqrt{r^2-x^2}=0 \rightarrow r^2-x^2=0 \rightarrow r^2=x^2 \rightarrow r=x$

$V=\pi\int_a^b [f(x)]^2\,dx$

$y^2=x^2-x^2 \rightarrow y=0$
$y=1$

$V=\pi\int_0^1 r^2-x^2\,dx=\pi\left(\int_0^1 r^2dx-\int_0^1 x^2\,dx\right)$

$V=\pi\left(\frac{r^3}{3}-\frac{x^3}{3}\right)\Big|_0^1=\frac{\pi r^3}{3}-\frac{\pi x^3}{3}\Big|_0^1=\frac{\pi r^3}{3}-\frac{\pi}{3}-\left(\frac{\pi r^3}{3}-\frac{\pi 0}{3}\right)=\frac{\pi}{3}$

$V=\frac{\pi}{3}$ //

4) $\int\sqrt{x^2+5}\,dx=\int(x^2+5)^{1/2}\,dx=2\int(u)^{1/2}\,du$

$\frac{1}{2}+1=\frac{3}{2}$

$u=x^2+5$ $\quad=2\cdot u^{3/2}$

$du=2\,dx$ $\quad=2\sqrt{(x^2+5)^3}+c$ //

5) $\lim_{x\to0^+}x^2\ln x=\cancel{\lim_{x\to0^+}2x\cdot\frac{1}{x}}=\lim_{x\to0^+}\frac{\ln x}{\frac{1}{x^2}}=\lim_{x\to0^+}\frac{\frac{1}{x}}{-\frac{2}{x^3}}=\lim_{x\to0^+}-\frac{x^3}{2x}=\lim_{x\to0^+}-\frac{3x^2}{2}=-\frac{3\cdot0^2}{2}=0$ //

$\left(\frac{1}{x^2}\right)'=(x^{-2})'=-2x^{-3}=-\frac{2\cdot1}{x^3}$

6) $\int_0^{+\infty} x e^{-x} dx = -x e^{-x} - \int -e^{-x} dx = -x e^{-x} + \int e^{-x} dx = x e^{-x} - e^{-x} \Big|_0^{+\infty}$

$u = x \quad dv = e^{-x}$

$du = dx \quad \cancel{v = e^{x}}$

$v = -e^{-x}$

$= \lim_{a \to +\infty} x e^{-x} - e^{-x} \Big|_0^{a} = \lim_{a \to +\infty} a e^{-a} - e^{-a} - (0 \cdot e^{0} - e^{0})$

$e^{-a} = \frac{1}{e^{a}}$

$= \lim_{a \to +\infty} 1 + a e^{-a} - e^{-a} = 1 - \lim_{a \to +\infty} a e^{-a} - e^{-a}$

$= 1 - \lim_{a \to +\infty} a \cancel{\frac{1}{e^{a}}}^{0} - \cancel{\frac{1}{e^{a}}}^{0} = 1 - 0 = 1$ //

7) quatro primeiros termos, $a = 4$, $f(x) = x^{3/2}$

$P(x) = f(a) + \frac{f'(a)}{1!} + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 \to P(x) = 4^{3/2} + \frac{3}{2} \cdot 4^{1/2} + \frac{\frac{3}{4} \cdot 4^{-1/2}}{2}(x-4)^2 + \frac{-\frac{3}{8} \cdot 4^{-3/2}}{6}(x-4)^3$

$f'(x) = 3/2\, x^{1/2}$

$\frac{3}{2} - 1 = \frac{1}{2}$

$f''(x) = \frac{3}{4} x^{-1/2}$

$\frac{1}{2} - 1 = -\frac{1}{2}$

$f'''(x) = -\frac{3}{8} x^{-3/2}$

$-\frac{1}{2} - 1 = \frac{-3}{2}$

$P(x) = \sqrt[2]{4^3} + \frac{3\sqrt[2]{4}}{2} + \frac{\frac{3}{4} \cdot \sqrt{\frac{1}{4}}}{2} x^2 - 8x + 16 + \frac{-\frac{3}{8}\sqrt{\frac{1}{4^3}}}{6} x^3 - 8x^2 + 16x - 4x^2 + 32x - 64$

$2^6 = 2^2 \cdot 2^2 \cdot 2^3$

$P(x) = 8 + 3 + \frac{3}{16}(x^2 - 8x + 16) - \frac{3}{64 \cdot 6}(x^3 - 12x^2 + 48x - 64)$ //