

homework 4

1. Mean vectors

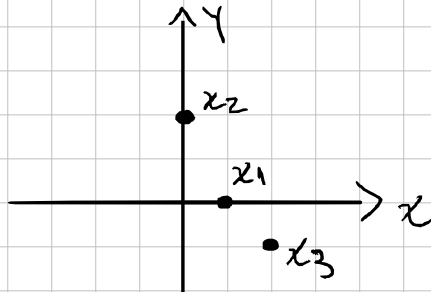
$$\mu_1 = \begin{bmatrix} 2 \\ -1 \end{bmatrix} \quad \mu_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Covariance matrices

$$\Sigma_1 = \begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix} \quad \Sigma_2 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

Mixing coefficients

$$\pi_1 = 0.5 \quad \pi_2 = 0.5$$



$$r_{ki} = \frac{\pi_k N(x_i | \mu_k, \Sigma_k)}{\sum_{i=1}^3 \pi_k N(x_i | \mu_k, \Sigma_k)}$$

$$r_{11} = \frac{\pi_1 N(x_1 | \mu_1, \Sigma_1)}{\pi_1 \left(\frac{1}{\sqrt{2\pi}|\Sigma_1|^{1/2}} e^{-\frac{1}{2}(x_1 - \mu_1)^T \Sigma_1^{-1} (x_1 - \mu_1)} + \frac{1}{\sqrt{2\pi}|\Sigma_2|^{1/2}} e^{-\frac{1}{2}(x_2 - \mu_2)^T \Sigma_2^{-1} (x_2 - \mu_2)} + \frac{1}{\sqrt{2\pi}|\Sigma_3|^{1/2}} e^{-\frac{1}{2}(x_3 - \mu_3)^T \Sigma_3^{-1} (x_3 - \mu_3)} \right)}$$

$$= 0.3221$$

Similarly

$$r_{21} = 0.6779 \quad r_{12} = 0.0916 \quad r_{22} = 0.9084$$

$$r_{13} = 0.7696 \quad r_{23} = 0.2304$$

M-Step

$$N_k = \sum_{i=1}^n r_{ki} \quad (\text{some dos pesos de cada cluster})$$

$$N_1 = r_{11} + r_{12} + r_{13} = 1.1832$$

$$N_2 = r_{21} + r_{22} + r_{23} = 1.8168$$

$$\pi_k = \frac{N_k}{N} \quad \pi_1 = \frac{N_1}{N} = 0.3944$$

$$\pi_2 = 0.6056$$

$$\mu_k = \frac{1}{N_k} \sum_{i=1}^n r_{ki} \cdot x_i$$

$$\mu_1 = \frac{1}{N_1} (r_{11} \cdot x_1 + r_{12} \cdot x_2 + r_{13} \cdot x_3) = \begin{bmatrix} 2.2233 \\ -0.4455 \end{bmatrix}$$

$$\mu_2 = \frac{1}{N_2} (r_{21} \cdot x_1 + r_{22} \cdot x_2 + r_{23} \cdot x_3) = \begin{bmatrix} 0.7537 \\ 0.5732 \end{bmatrix}$$

$$\Sigma_k = \frac{1}{N_k} \sum_{i=1}^n r_{ki} (x_i - \mu_k)(x_i - \mu_k)^T$$

$$\Sigma_1 = \begin{bmatrix} 1.1824 & -0.8494 \\ -0.8494 & 0.7145 \end{bmatrix}$$

$$\Sigma_2 = \begin{bmatrix} 0.9467 & -1.0386 \\ -1.0386 & 1.5645 \end{bmatrix}$$

Epoch 2:

E-Step

Similarly to before:

$$r_{11} = 0.3421 \quad r_{12} = 0.0036 \quad r_{13} = 0.9534$$

$$r_{21} = 0.6576 \quad r_{22} = 0.9964 \quad r_{23} = 0.0466$$

M-Step:

$$N_1 = r_{11} + r_{12} + r_{13} = 0.12994$$

$$N_2 = r_{21} + r_{22} + r_{23} = 1.7000$$

$$\mu_1 = \begin{bmatrix} 2.4646 \\ -0.7281 \end{bmatrix} \quad \mu_2 = \begin{bmatrix} 0.4689 \\ 1.1444 \end{bmatrix}$$

$$\Sigma_1 = \begin{bmatrix} 0.7425 & -0.4066 \\ -0.4066 & 0.2147 \end{bmatrix}$$

$$\Sigma_2 = \begin{bmatrix} 0.4136 & -0.6189 \\ -0.6189 & 1.0615 \end{bmatrix}$$