

The effect of Relativistic Aberration on Cosmological Distances

N. Cedola¹

Dipartimento di fisica e Astronomia "Augusto Righi", Università di Bologna, via Piero Gobetti 93/2, Bologna, Italia
e-mail: nicolo.cedola@studio.unibo.it

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ABSTRACT

Aims. We propose that the condition of relative motion between us and the objects that we observe in the Universe should generate relativistic aberration on the photons that such objects emit, varying the observed flux similarly to the cases of blazars and neutron stars, with instead a decrease of this radiative flux.

Methods. We follow some important papers and textbook used in modern cosmology, adding the effects of relativistic aberration in the theoretical description of the Cosmos.

Results. New definitions for the luminosity distance and the angular distance arise, with the consequence of changing the cosmological parameters measured in the last years. A qualitative description of all of this is also performed on the multipole analysis of the CMB.

Key words. Cosmological parameters - Cosmology: theory - Cosmology: observations - Relativistic processes

1. Introduction

Starting from the work of (Hubble 1929), a hundred years of observations showed that all the spectra measured from very far away objects appear redshifted by a factor, which is the same for all the wavelength as stated in (Ellis et al. 2012); in the same work it's underlined how this redshift may be interpreted as an effect of time dilation given by the condition of relative motion between us and any object as a result of the expansion of the Universe. From now on, we will call *Cosmological Object* (CO) anyone of this far away objects or events that produce radiation.

The presence of this redshift was immediately interpreted as a recession velocity and a consequence of the expansion of the Universe, thanks to the previous work of (Friedmann 1922), (Friedmann 1924), (Lemaître 1927) and the end of the static-model of the Universe, as in (Robertson 1933). (Pauli & Sparzani 2008) attribute to Von Laue in 1931¹ the result that in a RW metric the product between the momentum of a photon and the scale parameter of the Universe is constant; this was then expanded to derive the variation of the wavelength and direction of the photons with the expansion of the Cosmos as in (Ehlers 1961) and (Ellis et al. 1983) and to describe the evolution of the anisotropies of the CMB as a sum of covariant spherical multipoles as in (Thorne 1981) and (Ellis et al. 1983).

The interpretation of redshift as a recession velocity created also some problems with some observed velocities of COs which were apparently superluminal, like in the case of 3C 279; these problems were resolved in (Dishon & Weber 1977) and (Horák

1978) with the introduction of relativistic Doppler effect instead of the classical one. In the last years many papers about the interpretation of redshift as a recession velocity were published, like (Chodorowski 2007), (Bunn & Hogg 2009), (Chodorowski 2011) and (Ter-Kazarian 2022).

From the observations of the redshifted spectra of Supernovae Type Ia, such as in (Perlmutter et al. 1999) and the homogeneity and isotropy of the CMB, as measured in (Planck Collaboration et al. 2016), the current understanding of the cosmos describes an homogeneous, isotropic and expanding Universe mostly made of dark energy and dark matter in which all of the COs are following the expansion, increasing the distance between them as cosmological time passes.

The aim of this work is to find how the interpretation of the observed redshift as a recession velocity have some consequences on the photons emitted by the COs, moving at relativistic speeds: as the COs are in a condition of relative motion, we expect that relativistic aberration should affect the trajectories of the departing photons from these objects as observed in our reference frame. Recently, the work of (Maartens et al. 2023) also used relativistic aberration in order to explain the observed Hubble tension, but in a different manner than in this paper.

This work is divided as follows: in Sec. 2 we will give a full treatment on the effects of relativistic aberration on the observed distances and spectra of COs; in Sec. 3 we will talk about how the results of Sec. 2 may change our view of the Universe, with just a qualitative review of its effects on the CMB multipole expansion analysis, and in Sec. 4 we will give the conclusions of this paper.

¹ This work is practically unobtainable, as I searched for it for months and found nearly nothing. Instead of writing it in the bibliography, I decided to write the little informations I was able to find here; the note that (Pauli & Sparzani 2008) cited was some publication/lecture of the "Preußischen Akademie der Wissenschaften" in 1931 and was called "Die Lichtfortpflanzung in Raunien init zeitlieli veranderlicher Kriimmung nach der allgemeinen Relativitatstheorie. (Kl. 26. Febr. ; SB.)"

2. Computations

2.1. Relativistic Aberration

Everywhere in modern cosmological literature, the geometry of the Universe is described by the FLRW metric, in which the infinitesimal line element ds^2 connecting two close events is expressed as

$$ds^2 = dt^2 - R^2(t) \left(\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right) \quad (1)$$

we have clearly used natural unit $c = 1$. This metric was firstly proposed in (Friedmann 1922) to show the possibility of the existence of a non-static Universe even in the absence of the cosmological term in the Einstein equation, as stated in (Pauli & Sparzani 2008). The spectra of all the COs are redshifted by a factor z , which is related to the scale factor $R(t)$ of the metric by

$$1 + z = \frac{R(t_0)}{R(t_1)} \quad (2)$$

in which t_0 is the cosmological time of observation of a photon, and t_1 is the cosmological time of its emission, as in (Weinberg 1972). As in (Ehlers 1961) and (Ellis et al. 2012), we interpret this redshift as the sum of two contributions, one is a "Doppler" term given by the expansion of the Universe, and the second is a "gravitational" term given by the variation of the four-velocity of the observer. In theory, this redshift could be given by a lot of other different contribution, but in a FLRW Universe the only contribution will be the relativistic Doppler effect. Other models were described such as in (Ellis et al. 1978) dominated by the gravitational redshift, but as stated in a footnote by the same author in (Ellis et al. 2012) "it is not a realistic Universe model".

As a consequence of all of this, we suppose that the origin of redshift will only be the Doppler effect; if that's not the case, it's sufficient to suppose that the expansion of the Universe is not changing its velocity too fast², so that we can ignore the presence of gravitational redshift. In fact, the covariant component of the velocity v_i of an object in a Riemannian space-time described by the metric tensor g_{ik} in the position x^i can be expressed as in (Pauli & Sparzani 2008) as

$$v_i = \frac{dx_i}{d\tau} = \frac{d(g_{ik}x^k)}{d\tau} = \frac{dg_{ik}}{d\tau}x^k + g_{ik}\frac{dx^k}{d\tau} \quad (3)$$

so even if an object isn't changing its position, like galaxies in this description of the Universe, the evolution of the metric with the proper time of the said object can result in an observed relative velocity. All of this means that every CO is in free-fall following this expansion, and so that every one of them is in its own free-falling reference frame. The question now is how this will affect the observed trajectories of the emitted photons as seen in a different point of spacetime, which *also corresponds to different reference frames*.

(Chodorowski 2011) tried to link the observed redshifts with proper recession velocities, and the following arguments will give the same mathematical results as for what he called "Milne Universe", an empty Universe in which every observer has a constant 4-velocity; we will use a different approach to the matter to

try to find, between all of the possible ways to link the redshift to the observed recession velocity, what we think is the correct one.

As stated in a lot of sources in literature, such as (Weinberg 1972) or (Ellis et al. 2012), the Cosmological Principle imposes that any emitted photon following the expansion of the Universe will draw radially-directed geodesics with null length in the reference frame of the source: but different observers should see different initial directions for these geodesics as a consequence of the condition of relative motion in which they are with the COs.

One could think that we don't need any such treatment because relative to the Universe our instruments and all of the events that we observe are in first approximation points without dimensions, connected by one null-length geodesic: but the COs that we observe and the telescopes that we use have in fact physical dimensions even if they are extremely small compared to the Universe, and the difference of these dimensions as seen in their respective reference frame have an impact on measures, as we will soon show.

Thanks to the Equivalence Principle and the invariance of space we can treat this aberration: the Equivalence Principle was originally described in (Einstein 1916) and we can illustrate it as in (Pauli & Sparzani 2008): "For every space-time region infinitely small (...) there always exists a reference frame K_0 (...) in which every effect of gravity is absent on the motion of material points and every other physical phenomena.". Every K_0 is clearly a free-falling reference frame; we can individuate two free-fall reference frames in both our reference frame and in the reference frame in which any CO is, as both are following the expansion of the Universe (i.e. they move only under the effect of gravity so they are in free-fall). Then (Pauli & Sparzani 2008) continues, stating that "All the systems K_0 , obtainable one from another with a Lorentz transformation, are equivalents." If we suppose that the variation of the velocity of expansion of the Universe is not too fast (i.e. is negligible), then the Lorentz transformation relating our reference frame with the CO reference frame won't have any type of dependency on accelerations and it will be the usual Lorentz transformation of (Einstein 1905); in other words, *these two reference frame will be inertial and the transformation laws of Special Relativity can be applied*.

Clearly this isn't exactly valid, as in (Weinberg 1972) can be found that in order to change position between two given points in an FLRW metric we must do it through "quasi-translations" and they aren't Lorentz transformations; the argument presented above is a way of justifying the fact that the condition of motion between two points in a Universe which is evolving must be considered in order to obtain correct observations: with this we mean that performing a quasi-translation only changes the position in an FLRW Universe, but not the reference frame. (Ellis et al. 2012) also showed that usual Lorentz contraction and time dilation are contained in the FLRW metric, and their presence immediately implies the existence of relativistic aberration in cosmology.

We can analyze the effect of aberration as seen in an inertial reference frame with a null relative velocity respect to us, very close to the CO, in a condition of motion relative to it; let's call it auxiliary reference frame. We suppose that the auxiliary reference frame is so close to the CO that the effects of general relativity are negligible as the Equivalence Principle states. Clearly this third reference frame will be inertial with both the other two. This enables us to describe the variation of the *initial* directions of photons as the usual special relativistic aberration. Then we will move from this auxiliary reference system to our actual po-

² as a "proof" of this, think about the fact that the Hubble parameter nowadays is nearly a constant.

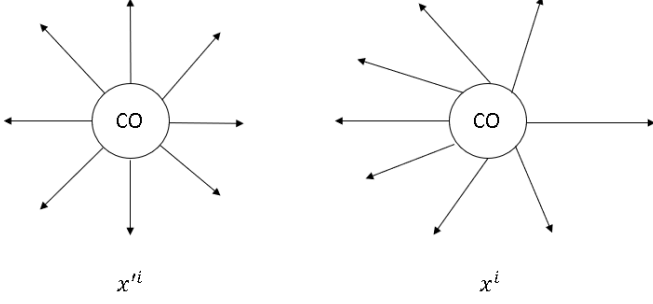


Fig. 1. Effect of aberration in two different reference frames.

The circles represents a generic CO, while each arrow represents an initial direction for a departing photon. In the image on the left x^{ti} , which correspond to the CO reference frame, the radiation is emitted isotropically. In the image on the right x^i , which correspond to the auxiliary reference frame (and so also our reference frame), the radiation is emitted not isotropically.

sition³, with a null relative velocity with it, and see how this impact the observed fluxes. This translation will affect the direction of the photons only geometrically, as in the canonical work of (Weinberg 1972): these photons will proceed in their journey following the geodesics defined by general relativity, but will appear more numerous in the direction of motion of the CO rather than in the direction towards us.

With all of this we can now say that a *decrease of the observed flux* of the CO will be seen in our reference system. Fig. 2.1 helps understand this effect.

First of all we can link the redshift z and the module of the velocity v^i of the reference frame in which the CO is relative to the auxiliary system thanks to the relativistic relation which, for example, can be found in (Einstein 1905) for the Doppler effect in the case of no transversal motion

$$z := \frac{\lambda_0 - \lambda}{\lambda} = \sqrt{\frac{1 + \beta}{1 - \beta}} \quad (4)$$

in which λ is the wavelength as observed in the CO reference frame, which is moving with a relative velocity $\|v\| = c\beta$, while λ_0 is the observed wavelength in the auxiliary system; clearly λ_0 is also the wavelength as observed by us. We can easily reverse this relationship in order to find the relative motion of the CO as a function of the observed redshift of the spectra

$$\beta = \frac{(1 + z)^2 - 1}{(1 + z)^2 + 1} \quad (5)$$

this relationship can be found in (Dishon & Weber 1977), in which it was derived to explain the apparent superluminal motion of the blazar 3C279, but also in (Chodorowski 2007) and (Ter-Kazarian 2022).

Equation 5 clearly doesn't hold if the expansion of the Universe between us and the CO is faster than c , but in that case we simply can't detect any light signal from the CO so the problem doesn't arise.

Let's now see how the initial direction of the trajectories for the photons changes as seen in our reference frame as a function

³ mathematically it means using the usual quasi-translation described in (Weinberg 1972)

of the redshift. As a result of the theorem of relativistic composition of velocities, an observer in our reference frame won't observe the radiation as emitted isotropically, but will see an increase of the radiative flux in the direction of motion of the CO, with a decrease of the flux in the opposite direction.

All of this affects *only the initial direction* of the departing photons, which will then travel through space along the usual null-length geodesics; with this we mean that each photon will travel along a *different* (relative to the canonical treatment) radially directed null-length geodesic centered at the CO, as no acceleration is present.

We will follow the treatment given in (Weinberg 1972), which in order to estimate the observed flux from any CO has considered just the quasi-translation in space and not the relative movement of the CO, so we will add this correction to understand how the number of observed photons change. Suppose that we have a telescope with a given mirror of radius b and we are observing a point-like CO. In (Weinberg 1972) it's shown that b is related to the angular dimension ϵ that the radius of the mirror implies in the sky as seen in the position of the CO as

$$b \approx R(t_0)r_1\epsilon \quad (6)$$

here r_1 is the position of CO as seen by us, which, thanks to the fact that photons travel along radially directed null-length geodesic, will be given as in (Weinberg 1972) by

$$\int_{t_1}^{t_0} \frac{dt}{R(t)} = \int_0^{r_1} \frac{dr}{\sqrt{1 - kr^2}} = f(r_1) \quad (7)$$

where

$$f(x) = \begin{cases} \sin^{-1} x & \text{if } k = +1 \\ x & \text{if } k = 0 \\ \sinh^{-1} x & \text{if } k = -1 \end{cases} \quad (8)$$

while $R(t_0)$ is defined as in equation 2. Equation 6 doesn't take account of the aberration generated by the relative motion of the said CO: it's a result in which the object isn't moving relative to us, so the CO is in the same inertial reference frame as we are. Let's call ϵ' the angular dimension of the telescope in the CO reference frame. We can easily found how this angle is related to ϵ , the angular dimension of the telescope as seen in the auxiliary reference frame: the transformation of a generic angle α under the change of reference frame is described by (Einstein 1905) with the formula

$$\tan \alpha = \frac{\sin \alpha' \sqrt{1 - \beta^2}}{\cos \alpha' + \beta} \quad (9)$$

in which the angle α is the angle formed by the direction of light and the positive direction of the x^1 (or x'^1) axis. In an expanding Universe, the CO is effectively receding from us, so the mirror will be placed in the opposite direction of the CO direction of motion. All of this means that instead of working with ϵ and ϵ' we have to work with the angles $\pi - \epsilon$ and $\pi - \epsilon'$:

$$\tan(\pi - \epsilon) = \frac{\sin(\pi - \epsilon') \sqrt{1 - \beta^2}}{\cos(\pi - \epsilon') + \beta} \quad (10)$$

Thanks to the symmetries of the trigonometric functions and the fact that the angles ϵ and ϵ' are very small, we can rewrite the expression 10 as

$$\epsilon = \frac{\sqrt{1-\beta^2}}{1-\beta} \epsilon' \quad (11)$$

which can be expressed as a function of the redshift z thanks to the equation 4

$$\epsilon = (1+z)\epsilon' \quad (12)$$

The physical meaning of this equation is that the photons captured by the telescope will fall in a solid angle with area $\pi\epsilon'^2$, which is smaller than the canonical $\pi\epsilon^2$: if the radiation is emitted isotropically in the CO reference frame, instead of capturing the fraction of photons emitted on the whole 4π surface

$$\frac{\pi\epsilon^2}{4\pi} = \frac{\pi b^2}{4\pi R^2(t_0)r_1^2}$$

our telescope will detect the fraction of the total number of photons

$$\frac{\pi\epsilon'^2}{4\pi} = \frac{\pi\epsilon^2}{4\pi(1+z)^2} = \frac{\pi b^2}{4\pi R^2(t_0)r_1^2(1+z)^2} \quad (13)$$

As we can see, we have a decrease by a factor $(1+z)^{-2}$ more than in the canonical treatment. The effect of aberration in an expanding Universe is, as expected, a decrease of the measured flux which may have been interpreted as an overestimation of the cosmological distances. We will now derive an expression for the luminosity distance which will be used later: if we continue to follow (Weinberg 1972) adding the effects of relativistic aberration, the power P received by the mirror of our telescope in our reference frame is linked to the luminosity L of the CO in its reference frame by

$$P = L \left(\frac{R^2(t_1)}{R^2(t_0)} \right) \left(\frac{A}{4\pi R^2(t_0)r_1^2(1+z)^2} \right) \quad (14)$$

Here t_0 and t_1 are defined as in equation 2, while $A = \pi b^2$ is the proper area of the mirror. The extra-factors $R^2(t_1)/R^2(t_0)$ are given by the effects of both time-dilation and wavelength increase with the expansion of the Universe (change of reference frame) as in the canonical treatment. The radiative flux F passing through the mirror of the telescope as observed in our reference frame will be

$$F = \frac{P}{A} = \frac{LR^2(t_1)}{4\pi R^4(t_0)r_1^2(1+z)^2} \quad (15)$$

and thanks to equation 2 which links z and $R(t)$, the luminosity distance will immediately be given by

$$d_L := \left(\frac{L}{4\pi F} \right)^{1/2} = \frac{R^3(t_0)}{R^2(t_1)} r_1 \quad (16)$$

we notice that it has a factor $R(t_0)/R(t_1)$ more than in the canonical treatment as a consequence of the presence of relativistic aberration on the direction of photons.

2.2. The distance module

The treatment will be done following that of (Ellis et al. 2012) and adding the correction of the relativistic aberration. Thanks to the expansion of the Universe (the change of reference frame), if the CO has a luminosity L in its reference frame, the observed flux F passing through a surface dS in our reference frame will be

$$FdS = \frac{L}{4\pi} \frac{d\Omega}{(1+z)^4} \quad (17)$$

two factors $(1+z)^{-1}$ come from the different energies and times measured by a receding observer, while the other $(1+z)^{-2}$ factor comes as a result of relativistic beaming, as the transformation law for the angular dimension of the telescope placed opposite to the direction motion of the CO, remembering that $d\Omega \propto \epsilon^2$, will be

$$d\Omega' = \frac{d\Omega}{(1+z)^2} \quad (18)$$

which is valid only for small solid angles. There is also another way of deriving equation 18: as in (Thorne 1981), the invariant integration element on the light cone dV_p in the local rest frame of the observer who is looking for photons with a given frequency ν can be expressed as

$$dV_p = \nu d\Omega d\nu \quad (19)$$

but in the rest frame of the CO emitting these same photons will have a different frequency ν' , so as widely acknowledge if we measure photons with a given frequency ν in the CO reference frame they will have a frequency such that

$$\nu' = (1+z)\nu \quad (20)$$

this equation with equation 19 and the invariance of dV_p gives the transformation law 18 for observing the same photons.

Let's now continue the treatment defining the galaxy area distance r_G by the relation

$$dS_G = r_G^2 d\Omega_G \quad (21)$$

in which the values dS_G and $d\Omega_G$ are respectively the cross-sectional area of the telescope as seen in the auxiliary reference frame and the solid angle implied by the telescope in the same condition. With this equation, we express equation 17 as

$$F = \frac{L}{4\pi} \frac{1}{(1+z)^4 r_G^2} \quad (22)$$

which we can rewrite as an *inverse square law*

$$F = \frac{F_G}{d_L^2} \quad ; \quad F_G := \frac{L}{4\pi} \quad (23)$$

Here the denominator is another way in which we can define the luminosity distance

$$d_L := (1+z)^2 r_G \quad (24)$$

Equation 23 will give us the same old module distance law, relating the apparent magnitude m of the CO, its absolute magnitude M and its luminosity distance d_L , but with a different relation between d_L and z

$$m - M = 5 \cdot \log_{10} \left(\frac{d_L(z)}{[Mpc]} \right) + 25 \quad (25)$$

Clearly this is an "ideal world" equation: in the real world when it's time to measure some luminosity distances, one should continue to include in equation 25 the terms for the k-correction, the extinction, ecc.

2.3. The Etherington Relation

At this point, an apparent problem for this model would be that the change in the definition of the luminosity distance destroys the Etherington relation, which is supported by a lot of observational data such as (Liao et al. 2016), (Rana et al. 2017), but there are a lot of other experimental evidences in literature; a closer look at the problem shows that these difficulties don't arise.

If we continue following (Ellis et al. 2012), we define the surface area dS_O of the CO as seen in our reference frame as

$$dS_O := r_O^2 d\Omega_O \quad (26)$$

in which $d\Omega_O$ is the solid angle implied by the CO as seen in our reference frame and r_O is the distance binding these two quantities; this solid angle will also be affected by relativistic aberration. Contrary to what one could expect from a quick look back at equation 12, the solid angle of the source $d\Omega_O$ in our reference frame is actually *bigger* than it would appear in a reference frame at rest with the CO.

The condition of relative motion, the invariance of c and the fact that photons coming from different points of an extended CO have different directions and screen spaces produce a bigger angle implied by the CO, as seen in Fig. 2.3. If we position our telescope orthogonal to the photons coming from the center of a galaxy, the photons coming from its edge will be slightly inclined relative to this direction; it may be an extremely small angle, but for high redshift, as we can see from equation 12 this effect becomes important even if the angles involved are small.

All of this could seem contradictory with what stated for the angular dimension of the telescope as seen in the CO reference frame, but it's not just the trajectories of the photons which are important, but also their directions: effectively, the dimension of the mirror in the CO reference frame will appear bigger, but the photons which will hit it will be of a lower number.

Let's see more into detail why we should see bigger angular sizes for the COs: as for the luminosity distance, we construct an inertial reference frame at rest with the CO, but this time we put it at our position; it will clearly move relative to us with a velocity given by equation 5, but it's important to remember that this reference frame is receding away from us; we will call it assistant reference frame in order to distinguish it from the other two and also from the auxiliary one. Again, it's a reference frame at our position which is inertial to us, so every effect given by general relativity is canceled as the Equivalence Principle states. This means that we can use the usual Lorentz transformations in order to work between these two reference frames. The assistant reference frame will travel relative to us with a velocity $\mathbf{v} = (-c\beta, 0, 0)$ where β is given by equation 5.

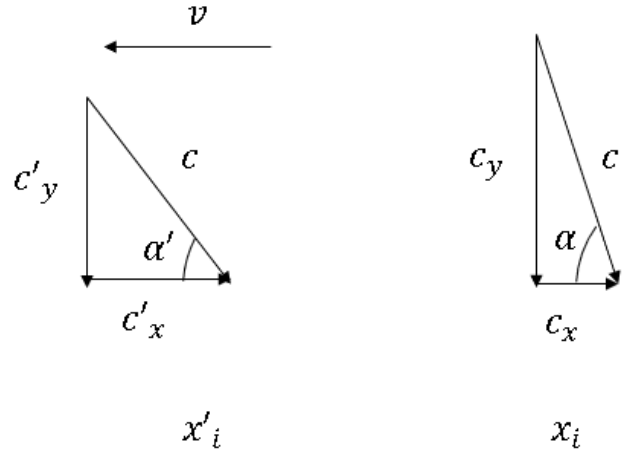


Fig. 2. Effect of aberration on angular dimensions of observed COs. The right reference frame x_i is our, while the left one x'_i is the assistant reference frame. The above arrow labeled with v reminds us that this reference frame is receding away from us. In each right triangle the hypotenuse is the direction of light in the two reference frames, while its cathetes represent its x and y components. We see that the same light ray will imply a bigger solid angle in the sky in our reference frame x_i relative to the assistant reference frame x'_i .

Now, let's consider a photon in the assistant reference frame which travels orthogonal to the $x' = 0$ plane: we may think of this as the CO covers half of the sky in this reference frame, with the center of the CO placed in the $x' = 0$ plane and the photon originated at the edge of it. We choose this photon to have a velocity $\mathbf{u}' = (0, -c, 0)$ in the assistant reference frame, but in our reference frame we will observe a different velocity for this photon as a result of the relativistic theorem of velocities composition for a purely radial motion: from (Einstein 1905)

$$u_x = \frac{u'_x + v_x}{1 + v_x u'_x / c^2} \quad ; \quad u_y = \frac{u'_y \sqrt{1 - \beta^2}}{1 + v_x u'_x / c^2} \quad (27)$$

we ignore the z spatial component because it's null in both the reference frames. In our reference frame we will see the photon hit the $x = 0$ plane with a velocity $\mathbf{u} = (-c\beta, -c\sqrt{1 - \beta^2}, 0)$. This is a qualitative way that helps us understand that the angle implied by the CO in the sky as measured by us will be *bigger* than in a reference frame at rest with it, because we measure it from the negative x axis. For small angular sized COs it can be easily checked that the angle transformation law is still given by equations 12 and 18, but with swapped superscripts.

If we see a bigger source for high redshifts, this doesn't necessarily mean that the object is closer to us because the angular distance has a minimum, as stated in (Sandage 1961), in agreement with the correction of the luminosity distance 16 and 24. All of this said, let's correct equation 12 with the observed quantity as seen in our reference frame, instead that in the assistant reference frame, with the use of equation 26

$$dS_O = r_O^2 (1 + z)^2 d\Omega_O \quad (28)$$

this yields a natural definition for the angular distance

$$d_A := r_O (1 + z) \quad (29)$$

From conservation of the number of photons in the space between us and the CO, we can prove as in (Ellis et al. 2012) that there exists a conserved quantity along the path of the photons

$$dS_G d\Omega_O = dS_O d\Omega_G (1+z)^2 \quad (30)$$

This relation won't be affected by aberration, but only because we have already taken it into account and we are now doing the computations in a fixed reference frame, we just have to choose one and use it. Substituting equations 21 and 28 in it and using the definitions for luminosity distance 24 and for the angular distance 29 we immediately obtain the Etherington relation, which *still holds*

$$\frac{d_L}{d_A(1+z)^2} = 1 \quad (31)$$

2.4. The shape of luminosity and angular distances

We can link the luminosity distance 24 and the angular distance 29 with the cosmological parameters appearing in the Friedmann equations, the usual matter density Ω_M , the vacuum density Ω_Λ and the curvature parameter Ω_k as expressed in (Carroll et al. 1992)

$$\Omega_M := \frac{8\pi G}{3H_0^2} \rho_{M0} \quad ; \quad \Omega_\Lambda := \frac{\Lambda}{3H_0^2} \quad ; \quad \Omega_k := -\frac{k}{R_0^2 H_0^2} \quad (32)$$

where ρ_{M0} is the present-day baryons+dark matter density, H_0 is the present-day Hubble constant, G is the gravitational constant, $R_0 = R(t_0)$, k is the normalized constant curvature and Λ is the cosmological constant.

We will follow the treatment given in (Carroll et al. 1992), expressing the computations in detail showing how and where we need to add the correction for relativistic aberration.

The null-length radially directed geodesic equation in the FLRW metric can be characterized as

$$\frac{dr}{dt} = \frac{\sqrt{1 - kr^2}}{R(t)} \quad (33)$$

with the infinitesimal cosmological time dt which can be linked to z , Ω_M and Ω_Λ through the Friedmann equation as in the canonical treatment offered in (Carroll et al. 1992) by

$$dt = \frac{1}{H_0} \frac{dz}{(1+z) \sqrt{(1+z)^2(1+\Omega_M z) - z(2+z)\Omega_\Lambda}} \quad (34)$$

using this value in equation 33, rearranging the terms, multiplying both sides by $R(t_0)$ and integrating yields

$$R(t_0)f(r_1) = \frac{1}{H_0} \int_0^z \frac{dz'}{\sqrt{H(z', \Omega_M, \Omega_\Lambda)}} \quad (35)$$

with

$$H(z', \Omega_M, \Omega_\Lambda) := (1+z')^2(1+\Omega_M z') - z'(2+z') \quad (36)$$

and $f(x)$ defined as in the equation 8. Using the definition 32 for Ω_k immediately allows us to express the proper distance r_1

between us and the CO as a function of redshift as in the canonical treatment

$$r_1(z, \Omega_k, \Omega_\Lambda, \Omega_M) = F \left(\sqrt{|\Omega_k|} \int_0^z \frac{dz'}{\sqrt{H(z', \Omega_M, \Omega_\Lambda)}} \right) \quad (37)$$

with

$$F(x) = \begin{cases} \sin x & \text{for a closed Universe } \Omega_k < 0 \\ x & \text{for a flat Universe } \Omega_k = 0 \\ \sinh x & \text{for an open Universe } \Omega_k > 0 \end{cases} \quad (38)$$

the final step will be to derive the dependence of luminosity distance d_L from redshift z inserting equation 37 and 2 into equation 16, which gives us

$$d_L = \frac{c(1+z)^2}{H_0 \sqrt{|\Omega_k|}} \cdot F \left(\sqrt{|\Omega_k|} \int_0^z \frac{dz'}{\sqrt{H(z', \Omega_M, \Omega_\Lambda)}} \right) \quad (39)$$

using the Etherington relation 31, we can also get the angular distance d_A as a function of z

$$d_A = \frac{c}{H_0 \sqrt{|\Omega_k|}} \cdot F \left(\sqrt{|\Omega_k|} \int_0^z \frac{dz'}{\sqrt{H(z', \Omega_M, \Omega_\Lambda)}} \right) \quad (40)$$

We have inserted in both the right hand sides of equations 39 and 40 a factor c in order to express d_L in Mpc and H_0 in Km/s/Mpc without the natural unit $c = 1$ used almost everywhere in this paper.

We can think of this correction in an easy way:

- for a given module distance, the CO will appear fainter than it is in the canonical treatment by a factor $(1+z)$ because we are detecting a lower number of photons; in order to correct for it, we added a factor $(1+z)$ in equation 39;
- for a given angular distance, the CO will appear bigger than it is in the canonical treatment by a factor $(1+z)$; considering the fact that $d_A(z)$ has a minimum, as said in (Sandage 1961), observing larger COs for high redshifts z makes us think that they are farther than in reality; in order to correct for it, we added a factor $(1+z)$ in equation 40.

If we compare the luminosity distance 39 with the canonical one used to measure the cosmological parameters with Supernovae type Ia in (Goobar & Perlmutter 1995), (Perlmutter et al. 1997), (Riess et al. 1998), (Perlmutter et al. 1999) we see that the cosmological parameters measured there may be wrong, including the curvature of the Universe. As we will see, following this treatment the measure of cosmological parameters through the CMB analysis used up until now should also be corrected for relativistic aberration.

2.5. The observed Fluxes and Intensities

It's easy to see that the shape of the spectra and the measures of fluxes and intensities are unaffected by this correction, with the only consequence of changing their relationship with cosmological parameters as stated in the last Subsect. .

We continue to follow the treatment of (Ellis et al. 2012), but little to no modifies are needed: the only things that change are the formulae for d_L and d_A , but this won't affect the shape of the spectra.

We define a function $\mathfrak{Z}(\nu)$ such that $L\mathfrak{Z}(\nu)d\nu$ is the rate at which the radiation is emitted by the CO between frequencies ν and $\nu + d\nu$. The function is normalized such that its integral over all frequencies is 1. We can then rewrite the inverse square law 23 as

$$\frac{Fd_L^2}{F_G} = \int_0^\infty \mathfrak{Z}(\nu_G)d\nu_G = (1+z) \int_0^\infty \mathfrak{Z}(\nu_O(1+z))d\nu_O \quad (41)$$

clearly here d_L is defined as in 39 and 24 instead of that in the canonical treatment. Then the observed flux between frequencies ν and $\nu + d\nu$ will be

$$F_\nu d\nu = \frac{F_G(1+z)}{d_L^2} \mathfrak{Z}(\nu(1+z))d\nu \quad (42)$$

again, with the different definition of d_L . A little precaution goes into the specific intensity I_ν , but the result is the same. An object with a surface area dS_O as defined in 28 will have an observed specific intensity

$$I_\nu d\nu := \frac{F_\nu d\nu}{d\Omega_O} = I_G \frac{\mathfrak{I}(\nu(1+z))}{(1+z)^3} d\nu \quad ; \quad I_G := \frac{F_\nu}{dS'_O} \quad (43)$$

where I_G is the surface brightness of the CO as seen in the assistant reference frame, which is also the one that we observe. This result still holds because the Etherington relation 31 remains valid. It's easy to see from 43 that any point-like CO that emits as a blackbody with a temperature T_G is seen by any observer as a blackbody but with an observed temperature T_O as in the canonical treatment

$$\frac{T_G}{T_O} = 1+z \quad (44)$$

The fact that we have modified the definition of luminosity distance with the experimental evidence that the CMB is still a blackbody in our reference frame is another justification of the fact that we also need to modify the angular distance.

3. Discussion

3.1. Gravitational waves

First of all, let us state that it's evident how all said up until now is also valid for the fluxes of gravitational waves. It's possible it will have an impact on the measures of masses of merging bodies, as a Fourier transform of the observed fluxes is needed in order to measure them. This article can be easily experimentally disproved if the correction proposed applied to the data analysis of a gravitational wave generated by two merging neutron stars will produce two measures of their original masses above the Volkov-Oppenheimer limit.

3.2. Qualitative effect on cosmological parameters

Let's now study, from a qualitative point of view, what happens at the measures of cosmological parameters 32, and in particular we start with the case of a flat Universe: we easily see from equation 25 that the numerical value of d_L taken from the measure of the distance modulus are correct, but the presence of the new factor $(1+z)$ in its expression means that until now we could have

possibly *overestimated* the integral in 39 and 37: in order for the integral to be smaller we should need a bigger denominator 36, which happens if we decrease Ω_Λ and we increase Ω_M . So for a flat Universe, up until now we could have *underestimated* the mass component and *overestimated* the dark energy component: we don't anymore need an acceleration of the expansion of the Universe as strong as measured in order to explain the increased distance between us and any CO: it's the aberration which does the trick of decreasing the observed flux, making us think that the COs are more distant from us than they really are.

All of this would be a beautiful result, but only measures will say if it's true; more importantly, we also have the effect of the curvature of the Universe to consider, which deeply impact the shape of $d_L(z)$.

In a curved space-time, if we measure a d_L and we increase by a factor $(1+z)$ the RHS of 39, it means that the function F should decrease and/or that $|\Omega_k|$ should be above 1: not only we have different matter density and cosmological constant, but also a *different value of the F function*, suggesting that *the curvature of the Universe is yet to be determined*.

All of this might have an impact on the age of the Universe and its end. Fortunately the age of Universe vary very little with the variation of cosmological parameters and depends strongly only on H_0 , as shown in (Carroll et al. 1992), but the values of Ω_Λ and Ω_M have an impact on the deceleration parameter q_0 as stated, in the same paper, and might produce a different end for the Universe.

3.3. A few notes on the CMB

It seems like the cosmological parameters observed through the CMB should be correct, but the fact that it's an extended object has an impact on its observation. As we said in Subsect. 2.5, a generic point-like source emitting as a blackbody will continue to appear as a blackbody with a lower temperature to every observer, but the CMB is an object that cover the whole sky so another effect will be present as equations 12 and 18 aren't exactly valid anymore.

Qualitative speaking, from equations 12 and 18 we see that the effect of relativistic aberration is in fact a *spread* of the CMB on the sky: from observations $z_{CMB} \approx 1000$, so in this small angle approximation a small anisotropy will imply a solid angle 10^6 larger in our reference frame than in the assistant reference frame. The problem is that this effect happen *for every point of the sky*, so this also spreads the points near the anisotropies. The overall result will be a "mixing" of the CMB spectra measured in a point of the sky with its adjacent regions; we don't have to confuse this mixing with the one described in literature as (Thorne 1981) and (Challinor 2000), because while there relativistic aberration is presented as a source that generates a dipole in the CMB and increases the intensity of high-order multipoles, here is exactly the opposite: every anisotropy of the CMB is mixed with its more uniform surrounding regions, making it appear more homogeneous and isotropic. It's interesting to notice that the effect on the surface area of every point of the CMB is increasing it by 10^6 , while the temperature anisotropies of the CMB are in the order of 10^{-6} K as measured in (Planck Collaboration et al. 2016).

This will have an impact on the measures of the cosmological parameters from the analysis of the high-order multipoles of the CMB, which should be corrected and cleaned by this effect. The problem presented is extremely complex, and we will eventually discuss it in another paper.

The CMB has an extremely homogeneous spectrum, as showed in the observations of (Planck Collaboration et al. 2016), but it's possible that this homogeneity may be the result of this mixing and the Universe may not be so much homogeneous and isotropic as we think. How much the intrinsic homogeneity and isotropy of the Universe and relativistic aberration are responsible for it, only time will tell, as for now we lack the complete mathematical description and analysis of it complete with the correction proposed in this paper.

As a result of this, there's even the possibility that the Cosmological Principle may, in fact, not be entirely true, as the CMB is one of the major clues for its validity, and breaking down its homogeneity will have a deep impact on Cosmology. But again, these last few lines are only speculation and thoughts for now.

3.4. The measures of the Hubble constant

One of the problems of modern cosmology is the so called Hubble Tension, as explained in great detail in (Di Valentino et al. 2021), with a discrepancy between late-Universe and early-Universe measurement of H_0 which is now around $4, 4\sigma$, as in (Riess et al. 2019), and isn't anymore negligible. As said in Sect. 1, relativistic aberration was also used in a (Maartens et al. 2023) to resolve this problem, but with a different approach, so there one can find different results than here.

So, what should we do with all said to try to correct the Hubble tension? If we compare the luminosity distance found in this paper with the canonical one used to measure the cosmological parameters with Supernovae type Ia in (Carroll et al. 1992), (Goobar & Perlmutter 1995), (Perlmutter et al. 1997), (Riess et al. 1998), (Perlmutter et al. 1999) it's easy to see that the measures of the Hubble constant for low redshifts $z \approx 0$ using the distance luminosity are essentially the same, while for what said in Subsect. 3.3 about the multipoles of the CMB, the indirect measures obtained from it should be corrected. With all of this in mind, it's also possible that the late Universe direct measures should be corrected if one looks too far in the past of the Universe.

Let's consider the example of the measure of the Hubble constant which involves the use of the luminosity distance; as described in Subsect. 2.2, the equation for the module distance 25 isn't modified by relativistic aberration, but only its dependency from the redshift z is changed. This means that the measures of the module distances are correct, so

$$d_L = 10^{m-M-25} = \text{const.} \quad (45)$$

is equal in both the canonical treatment and the luminosity distance derived in this paper; let's respectively call them $d_{L,can}$ and d_L . Imposing $d_{L,can} = d_L$, using equation 39 and the expression for the luminosity distance given in (Perlmutter et al. 1999) we get

$$\frac{c(1+z)^2}{H_0 \sqrt{|\Omega_k|}} r_1(z, \Omega_k, \Omega_\Lambda, \Omega_M) = \frac{c(1+z)}{H_{0,can}} r_1(z, 0, \Omega_{\Lambda,can}, \Omega_{M,can}) \quad (46)$$

in which we have called $\Omega_{\Lambda,can}$ and $\Omega_{M,can}$ the value of the cosmological parameters as measured in (Perlmutter et al. 1999), while r_1 is defined as in equation 37. The RHS describes a flat Universe as measured in the paper just cited. Rearranging the factors in this relation and eliminating common factors produces

$$\frac{H_{0,can}}{H_0} = \frac{\sqrt{|\Omega_k|}}{(1+z)} \frac{r_1(z, 0, \Omega_{\Lambda,can}, \Omega_{M,can})}{r_1(z, \Omega_k, \Omega_\Lambda, \Omega_M)} \quad (47)$$

clearly for low redshifts we won't observe any difference; this can be checked looking at this equation, remembering that at low redshift $z \approx 0$ and the curvature of the universe becomes negligible, just like the influence of cosmological parameters over r_1 and, as expected and as one could see from a quick look at equation 12, the effect of relativistic aberration also becomes negligible, so for $z \approx 0$

$$\frac{H_{0,can}}{H_0} \approx 1 \quad (48)$$

and the presence of relativistic aberration won't impact the measures of H_0 . But what the equation 47 tells us? It says that if we are looking too far in the past, and this too far is extremely small as a new term $\propto z^2$ is present in equation 39, in order to obtain the correct values of the cosmological parameters 32 we also have to ensure that the value that we use for H_0 is correct and vice versa. There are also a lot of other ways of measuring H_0 , as explained in (Di Valentino et al. 2021), but the aim of this article is not the solution of the Hubble tension problem so we rather stop here: this subsec. is just the proposal of something new for this question with little to no computations, but with all said about the measure of the cosmological parameters 32 it's also important to underlie the possibility of the existence of wrong measures of H_0 .

3.5. Other needed modifications on the measures

It's evident how all of this will have an impact on other aspects of cosmology, for example the theory and use of the Malmquist Bias and the Number of counts, which both need to be corrected in order to take into account relativistic aberration. If relativistic aberration will change these, it's possible that we will need to re-evaluate a lot of observational datas used up until now for our galaxy formation and evolutionary models.

4. Conclusions

In this paper we tried to argue that if the redshifts that we observe from Cosmological Objects, namely COs, can be described as an apparent 4-velocity of the sources relative to us, then a modification of the luminosity distance and angular distance are necessary in order to take relativistic aberration into account for the measurements of COs at high redshifts. In particular, this addition may have generated an overestimation of the cosmological distances, which may have led to the measures of wrong cosmological parameters both from Supernovae type Ia as in (Perlmutter et al. 1999). and from the CMB power spectrum analysis of the Planck Collaboration (Planck Collaboration et al. 2016). It's possible that the proposed correction may also have an impact on other measures, such as the Hubble constant H_0 and a lot of datas used in modeling galaxy formation and evolution.

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