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Multi-position two-phase person-position matching decision-making with intermediary participation

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ABSTRACT

With the rapid development in society, intermediaries play an increasingly important role in person-position matching decision-making. To obtain reasonable enterprise-intermediary matching and enterprise-candidate matching schemes under multiple positions, preferences of enterprises, headhunting companies and candidates should be fully considered from a two-phase perspective. Thus, a two-sided matching decision-making method under the probabilistic linguistic environment is proposed to solve the multi-position two-phase person-position matching problem. First, related theories of probabilistic linguistic term sets and two-sided matching are given. On this basis, the problem of multi-position two-phase person-position matching under the probabilistic linguistic environment is described. The decision-making process is divided into the following two phases: In the first phase, satisfactions of enterprises and headhunting companies are calculated considering subject expectations based on prospect theory. Then, considering the maximization of satisfactions of two-sided subjects, the many-to-many two-sided matching model between enterprises and headhunting companies is established and solved to obtain the optimal many-to-many matching scheme. In the second phase, according to evaluation matrices and expectation matrices, satisfactions of enterprises and candidates under multiple positions are calculated considering intermediary evaluations. Then, considering the maximization of satisfactions of enterprises and candidates, one-to-many matching models and one-to-one stable matching models under multiple positions are developed and solved to obtain the optimal person-position matching schemes under multiple positions. Finally, an example of the person-position matching with intermediary participation is used to illustrate the feasibility and effectiveness of the proposed method.

1. Introduction

With the change of social requirements and the development of technology, enterprises pay more attention to the improvement of product or service quality to cope with the change of customer needs. However, different positions have different requirements, and candidates with specific skills and work experience are needed to be matched with appropriate positions. Many enterprises tend to entrust intermediaries to recruit candidates, which help them to obtain candidates. Thus, intermediaries play an important role in person-position matching. However, there are few scholars studying person-position matching considering the participation of intermediaries. Therefore, the person-position matching problem with intermediary participation is worth studying.

In view of this, this paper considers to solve the multi-position two-phase person-position matching problem with intermediary participation, which is composed of enterprises, headhunting companies and candidates. The complex person-position matching includes the matching between enterprises and headhunting companies (first phase), and the matching between enterprises and candidates (second phase). Thus, the complex person-position matching is regarded as a two-phase person-position matching. In addition, enterprises often need many candidates for vacant positions in realistic life. The situation of multiple positions should be fully considered in person-position matching, which is more in line with the complex situation of human resource allocation. Therefore, the second phase matching of enterprises and candidates should be considered under multiple positions.

In the decision-making environment, due to the complexity of decision-making problems and differences of people's cognition, it is difficult for subjects to express their preferences information about alternatives/attributes with clear numerical values. Different types of evaluation information are often given by decision-makers, such as

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intuitionistic fuzzy sets (Liu et al., 2023), hesitant fuzzy sets (Qahtan et al., 2023) and linguistic term sets (Ma et al., 2023). Therefore, it is inevitable to combine theories of many kinds of fuzzy sets with two-sided matching decision-making. Zadeh (1975) proposed the concept of linguistic variables, which began to be applied to practical decision-making problems. Then, Rodriguez et al. (2011) proposed hesitant fuzzy linguistic term sets, which allow decision-makers to propose several possible values for linguistic variables. Furthermore, Pang et al. (2016) introduced the concept of the probabilistic linguistic term set (PLTS), allowing possible linguistic terms to have different probabilities. Considering that PLTSs can better express the fuzziness and uncertainty of decision-making information and the hesitation of decision-makers, many researchers apply PLTSs to decision-making matching problems (Zou et al., 2024; Wu & Zhang, 2024). Therefore, it is believed that PLTSs to express fuzzy preferences.

Based on the above analysis, the purpose of this paper is to propose a multi-position two-phase person-position matching decision-making method. In the proposed method, the evaluation information of trilateral subjects is provided in the form of PLTSs, which can effectively deal with the fuzziness and uncertainty of preferences of trilateral subjects. Then, the proposed method is divided into two phases: In the first phase, satisfactions of enterprises and headhunting companies are calculated based on prospect theory. Then, a many-to-many two-sided matching model is developed to obtain the matching scheme between enterprises and headhunting companies. In the second phase, satisfactions of enterprises and candidates are calculated considering subject expectations and intermediary evaluations. Then, one-to-many two-sided matching models between enterprises and candidates are built to obtain the optimal person-position matching schemes under multiple positions.

The rest of this paper is structured as follows: Section 2 summarizes the relevant literature. Section 3 introduces related concepts of PLTSs and two-sided matching. Section 4 is the description of multi-position two-phase person-position matching problem under the probabilistic linguistic environment. In section 5, the probabilistic linguistic decision-making method for multi-position two-phase person-position matching is proposed. Section 6 reveals the effectiveness and feasibility of the proposed method through an example analysis. Section 7 summarizes this paper.

2. Literature review

In this section, first, related studies of two-sided matching are reviewed. Second, the person-position matching methods and models are reviewed. Third, the application of PLTSs is demonstrated through recent research. Finally, motivations and contributions of this paper are discussed.

2.1. Two-sided matching

Two-sided matching decision-making refers to match subjects according to their individual preference information using some matching methods (Aouad & Saban, 2023). In real life, some scholars have studied the two-sided matching problem from different perspectives, such as, matching between students and projects in resource allocation (Liu et al., 2023), matching between customers and manufacturers in large-scale cloud manufacturing services (Tong & Zhu, 2022), matching between investors and suppliers (Liu et al., 2021), stable ride-sharing matching (Peng et al., 2020), matching between hospitals and patients (Wang et al., 2022). Thus, the related research on two-sided matching has important theoretical and realistic significance for many practical problems. For example, Chen et al. (2019) proposed a novel two-sided matching multi-attribute decision making method considering psychological behaviors of matching subjects. Liang et al. (2019) considered the preference heterogeneity of matching subjects and further explored the corresponding generalized models and methods for multi-criteria two-sided matching. Moreover, some scholars have studied different types of two-sided matching decision-making with different numbers of two-sided matching subjects: one-to-one two-sided matching (Yue, 2022), one-to-many two-sided matching (Noruzoliaee & Zou, 2022), many-to-many two-sided matching (Zhang et al., 2021).

The existing research provides a solid theoretical and methodological basis for solving two-sided matching problems. However, many researchers focus on a certain type of two-sided matching problem, ignoring complex two-sided matching problems that may contain multiple two-sided matching decision-makings and cannot be considered separately, such as the person-position matching with intermediary participation. Therefore, designing a multi-phase two-sided matching decision-making method is beneficial to achieve reasonable two-sided matching.

2.2. Person-position matching

Person-position matching is a typical reality of two-sided matching. The concept of Person-position matching was first proposed by Caplan (1987), which has attracted many scholars to carry out research on the field of person-position matching. Human resources are not only an increase in quantity and quality, but also the ability to make rational use of human resources. Therefore, scholars have proposed many models and methods based on analytic hierarchy process (AHP) (Golec & Kahya, 2007), artificial neural network (Drigas et al., 2004) and other methods. For example, Korkmaz et al. (2008) designed an AHP-based two-sided matching decision support system for person-position matching. To describe the correlation between evaluated attributes. Yu and Xu (2020) proposed a Choquet integral aggregation operator and an intuitionistic fuzzy two-sided matching model. Considering that it is impossible for two-sided subjects to achieve complete rationality. Liu and Wang (2021) developed a multiple criteria decision-making method based on TODIM for person-position matching considering psychological behaviors of decision-makers. Dai and Hu (2020) obtained weights of attributes by using AHP and evaluations of person-position matching of university managers by the BP neural network, and then a person-position matching evaluation model of university managers is constructed.

The existing research provides a rich application background for solving the person-position matching problem. However, it should be pointed out that the research on person-position matching decision-making with intermediary participation is limited. The influence of intermediaries on enterprises and candidates has not been fully considered in person-position matching decision-making. In addition, the problem of person-position matching is often limited to the matching between positions and candidates under a certain enterprise, ignoring the situation of multiple enterprises. Therefore, designing a satisfaction calculation method and multi-position person-position matching models with intermediary participation is significance.

2.3. Probabilistic linguistic term set

Due to the fuzziness of the decision-making environment, it is difficult for subjects to express their preferences with clear numerical values for objective things. They may tend to provide several linguistic terms with their probabilities as evaluations, and PLTSs can well express this type of subjects' preference (Pang et al., 2016). Therefore, many scholars applied theories of PLTSs to two-sided matching decision-making. For example, Jia et al. (2023) used the probabilistic linguistic aggregation operator based on Choquet integral to obtain the comprehensive evaluation, and then solved the two-sided matching decision-making problem by using TOPSIS method. Li et al. (2021) proposed a two-sided matching decision-making method based on regret theory under the probabilistic linguistic environment. Jia and Wang (2022) proposed a multi-participant two-sided matching decision-making method considering risk attitudes of subjects under the probabilistic linguistic environment based on prospect theory. You and Hou (2023) studied the renewable energy utilization mode selection problem with multiple stakeholders by using the probabilistic linguistic preference relation.

The existing research provides theoretical and methodological guidance for the two-sided matching problem under the probabilistic linguistic environment. However, the complex person-position matching problem under the probabilistic linguistic environment is rarely studied. In real life, trilateral subjects may encounter the challenges of ambiguity and uncertainty due to the uncertain environment. PLSTs allow trilateral subjects to express their subjective evaluations on a larger scope. Thus, it is necessary to study the complex person-position matching problem with intermediary participation based on PLTSs.

2.4. Motivations

In view of the research on person-position matching decision-making and PLTSs, there are the following deficiencies: (1) The research on person-position matching decision-making with intermediary participation is limited. (2) The matching between enterprises and candidates under multiple positions is rarely studied. (3) The application of PLTSs in person-position matching decision-making still needs further research. (4) In the decision-making of person-position matching, the calculation of satisfactions of enterprises considering the gains and losses of intermediaries is extremely rare. (5) The multi-position two-phase person-position matching models have not been established.

Therefore, motivations of this study are as follows: (1) It is a novel idea to introduce the intermediary into person-position matching decision-making, and it is more reasonable to identified the person-position matching with intermediary participation as a two-phase person-position matching. This study will fill the gap in the two-phase person-position matching with intermediary participation. (2) The second phase of the proposed method is the one-to-many matching between enterprises and candidates under multiple positions. The situation of multiple positions will be more in line with the reality of human resource allocation. (3) The theory of PLTSs is introduced into the process of person-position matching decision-making, which will enrich the research of person-position matching decision-making under the probabilistic linguistic environment. (4) Subject expectations and intermediary evaluations are important references for the satisfaction calculation of enterprisers and candidates. Therefore, the satisfaction calculation method considering double reference points of subject expectations and intermediary evaluations needs more attention. (5) The multi-position two-phase person-position matching decision-making models include the many-to-many matching model of enterprise and headhunting companies established in the first phase, the one-to-many matching models and the one-to-one stable matching models of enterprises and candidates established in the second phase. These matching models can reveal the uncertainty and ambiguity of person-position matching decision-making in a more reasonable and detailed way. Therefore, it is necessary to develop multi-position two-phase person-position matching models.

3. Basic knowledge

Before demonstrating the decision-making method for multi-position two-phase person-position matching, some basic conceptions on PLTSs and two-sided matching are discussed in this section.

3.1. Probabilistic linguistic term set

Definition 1. (Pang et al., 2016) Let $S = \{s_\tau | \tau = 0, 1, ..., 2\varsigma\}$ be a given set of linguistic terms, then the PLTS based on S is described as:

$$L(p) = \left\{ L^k(p^k) \middle| L^k \in S, p^k \geq 0, k = 1, 2, ..., |L(p)| \right\}$$

where $L^k(p^k)$ represents the linguistic term L^k with probability p^k, and $|L(p)|$ is the number of linguistic terms in $L(p)$. When $\sum_{k=1}^{|L(p)|} p^k < 1$, it indicates that the probability distribution of linguistic terms is incomplete; and when $\sum_{k=1}^{|L(p)|} p^k = 1$, it denotes that the PLTS $L(p)$ has complete probability distribution.

Definition 2. (Pang et al., 2016) Let $L(p)$ be a PLTS with incomplete probability distribution information, $\sum_{k=1}^{|L(p)|} p^k < 1$, the standardization of $L(p)$ is defined as:

$$\widehat{L}(p) = \left\{ L^k(\widehat{p}^k) \middle| L^k \in S, \widehat{p}^k \geq 0, k = 1, 2, ..., |\widehat{L}(p)| \right\}$$

where

$$\widehat{p}^k = p^k / \sum_{k=1}^{|L(p)|} p^k$$

Assum that $\widehat{L}_1(p)$ and $\widehat{L}_2(p)$ are two standardized PLTSs. When $|\widehat{L}_1(p)| \neq |\widehat{L}_2(p)|$, if $|\widehat{L}_1(p)| > |\widehat{L}_2(p)|$, add the smallest linguistic term of $\widehat{L}_1^k$ to $\widehat{L}_1(p)$ and let the probability be 0 until $|\widehat{L}_1(p)| = |\widehat{L}_2(p)|$; if $|\widehat{L}_1(p)| < |\widehat{L}_2(p)|$, add the smallest linguistic term of $\widehat{L}_2^k$ to $\widehat{L}_2(p)$ and let the probability be 0 until $|\widehat{L}_1(p)| = |\widehat{L}_2(p)|$.

Example 1. In employee performance appraisal, many companies will require employees to evaluate each other. Assume that there are 10 employees in a company's sales department for mutual evaluation, and the highest rating is 6 stars. For employee A_2, 6 employees evaluated A_2 as 2 stars, 2 employees evaluated A_2 as 3 stars and other employees did not express their opinions. In this case, the information can be denoted as a PLTS $L(p) = \{s_2(0.6), s_3(0.2)\}$. The evaluation of other employees can be similarly obtained. Therefore, PLTSs can be obtained in realistic problems.

Definition 3. (Pang et al., 2016) For a given linguistic term set $S = \{ s_\tau | \tau = 0, 1, ..., 2\varsigma\}$, let $L(p) = \left\{ L^k(p^k) \middle| L^k \in S, p^k \geq 0, k = 1, 2, ..., |L(p)| \right\}$ be a PLTS on S, and r^k be a subscript of the probabilistic linguistic term L^k, the score $E(L(p))$ can be defined as:

$$E(L(p)) = s_\tau \tag{1}$$

where the subscript $\tau = \sum_{k=1}^{|L(p)|} p^k r^k / \sum_{k=1}^{|L(p)|} p^k$.

Definition 4. (Pang et al., 2016) Let $L(p) = \left\{ L^k(p^k) \middle| L^k \in S, p^k \geq 0, k = 1, 2, ..., |L(p)| \right\}$ be a PLTS, and the deviation $\sigma(L(p))$ of $L(p)$ is defined as:

$$\sigma(L(p)) = \left(\sum_{k=1}^{|L(p)|} \left(p^k \left(r^k - \tau \right) \right) \right)^{\frac{1}{2}} / \sum_{k=1}^{|L(p)|} p^k \tag{2}$$

Definition 5. (Pang et al., 2016) Let $L_1(p)$ and $L_2(p)$ be two PLTSs, then the comparison rule between $L_1(p)$ and $L_2(p)$ is given as:

1) If $E(L_1(p)) > E(L_2(p))$, then $L_1(p) > L_2(p)$;
2) if $E(L_1(p)) < E(L_2(p))$, then $L_1(p) < L_2(p)$;
3) when $E(L_1(p)) = E(L_2(p))$, if $\sigma(L_1(p)) = \sigma(L_2(p))$, then $L_1(p) \sim L_2(p)$; if $\sigma(L_1(p)) < \sigma(L_2(p))$, then $L_1(p) < L_2(p)$; if $\sigma(L_1(p)) > \sigma(L_2(p))$, then $L_1(p) > L_2(p)$.

Definition 6. (Wu et al., 2018) Let $L_1(p)$ and $L_2(p)$ be two PLTSs, then the Hamming distance between $L_1(p)$ and $L_2(p)$ is calculated as:

$$d(L_1(p), L_2(p)) = \sum_{i=1}^{|L_1(p)|} \sum_{j=1}^{|L_2(p)|} \left(p_1^i \times p_2^j \right) \left| f(L_1^i) - f\left(L_2^j\right) \right| \tag{3}$$

where $f(L^k)$ is the transformation function, $f(s_\tau) = \frac{\tau}{2\varsigma}$.

3.2. Multi-type two-sided matching and stable matching

Let $\partial = \{\partial_1, \partial_2, ..., \partial_m\}$ be one subject set and $\delta = \{\delta_1, \delta_2, ..., \delta_n\}$ be the other subject set, where m and n represent numbers of two-sided subjects respectively, $m \geq n$; ∂_i is the $i-$th subject in set ∂ and δ_j is the $j-$th subject in set δ. Let $H = \{1, 2, ..., m\}$, $T = \{1, 2, ..., n\}$.

Definition 7. (Yue 2022) Assume that $\Re$: $\partial \cup \delta \to \partial \cup \delta$ is a one-to-one mapping. If the mapping $\Re$ meets these conditions: 1) $\Re(\partial_i) \in \delta \cup \partial_i$; 2) $\Re(\delta_j) \in \partial$; 3) $\Re(\partial_i) = \delta_j$ if and only if $\Re(\delta_j) = \partial_i$, then $\Re$ is called a one-to-one two-sided matching, where $\Re(\partial_i) = \delta_j$ (or (∂_i, δ_j)) represents ∂_i and δ_j are matched, and $\Re(\partial_i) = \partial_i$ (or (∂_i, ∂_i)) represents ∂_i is unmatched or single.

Definition 8. (Noruzoliaee & Zou, 2022) Assume that $\wp$: $\partial \cup \delta \to \partial \cup \delta$ is a one-to-many mapping. $\forall$ ∂_i and δ_j, if the mapping $\wp$ meets these conditions: 1) $\wp(\partial_i) \subset \delta \cup \partial_i$; 2) $\wp(\delta_j) \in \partial \cup \delta_j$; 3) if $\wp(\partial_i) = \left\{\delta^i_1, ..., \delta^i_{m_i}\right\}$, then $\Im(\delta^i_{j_i}) = \partial_i, j_i \in \{1, ..., m_i\}$; 4) if $\wp(\delta_j) = \partial_i$, then $\delta_j \in \left\{\delta^i_1, ..., \delta^i_{m_i}\right\}$; 5) $\wp(\partial_i) \cap \wp(\partial_l) = \varnothing$, $\forall \partial_l \in \partial$, $\partial_l \neq \partial_i$; then $\wp$ is called a one-to-many two-sided matching. Wherein $\wp(\partial_i) = \delta_j$ (or $\partial_i \leftrightarrow \delta_j$) represents that ∂_i and δ_j are matched, $\wp(\partial_i) = \left\{\delta^i_1, ..., \delta^i_{m_i}\right\}$ (or $\partial_i \leftrightarrow \left\{\delta^i_1, ..., \delta^i_{m_i}\right\}$) represents that ∂_i and $\left\{\delta^i_1, ..., \delta^i_{m_i}\right\}$ are matched, and $\wp(\delta_j) = \delta_j$ (or $\delta_j \leftrightarrow \delta_j$) represents that δ_j is unmatched or single.

Definition 9. (Zhang et al., 2021) Assume that $\Im$: $\partial \cup \delta \to \partial \cup \delta$ is a many-to-many mapping. $\forall$ ∂_i and δ_j, if the mapping $\Im$ meets these conditions: 1) $\Im(\partial_i) \subset \delta \cup \partial_i$; 2) $\Im(\delta_j) \subset \partial \cup \delta_j$; 3) if $\Im(\partial_i) = \left\{\delta^i_1, ..., \delta^i_{m_i}\right\}$, then $\Im(\delta^i_{j_i}) = \partial_i$, $j_i \in \{1, ..., m_i\}$; 4) if $\Im(\delta_j) = \left\{\partial^j_1, ..., \partial^j_{n_j}\right\}$, then $\Im(\partial^j_{i_j}) = \delta_j$, $i_j \in \{1, ..., n_j\}$; 5) if $\Im(\delta_j) = \partial_i$, $\delta_j \in \left\{\delta^i_1, ..., \delta^i_{m_i}\right\}$ 6) if $\Im(\partial_i) = \delta_j$, $\partial_i \in \left\{\partial^j_1, ..., \partial^j_{n_j}\right\}$; then $\Im$ is called a many-to-many two-sided matching. Wherein $\Im(\partial_i) = \delta_j$ (or $\partial_i \leftrightarrow \delta_j$) represents that ∂_i and δ_j are matched; $\Im(\partial_i) = \left\{\delta^i_1, ..., \delta^i_{m_i}\right\}$ (or $\partial_i \leftrightarrow \left\{\delta^i_1, ..., \delta^i_{m_i}\right\}$) represents that ∂_i and $\left\{\delta^i_1, ..., \delta^i_{m_i}\right\}$ are matched; and $\Im(\delta_j) = \delta_j$ (or $\delta_j \leftrightarrow \delta_j$) represents that δ_j is unmatched or single.

According to the idea of Zhang et al. (2021), this paper gives a definition of one-to-one stable matching.

Definition 10. (Zhang et al., 2021) Assume that $\Re$: $\partial \cup \delta \to \partial \cup \delta$ is a one-to-one mapping. Let α^∂_{ij} be the satisfaction of ∂_i to δ_j, β^δ_{ij} be the satisfaction of δ_j to ∂_i. $\forall \partial_i, \partial_l \in \partial$, $\delta_j, \delta_k \in \delta$, when $\alpha^\partial_{ij} \neq \alpha^\partial_{ik}$, $\beta^\delta_{ij} \neq \beta^\delta_{lj}$, if there are not the following three cases:

1) $\exists$ $\partial_l \in \partial$, $\delta_k \in \delta$, $\Re(\partial_i) = \delta_k$, $\Re(\delta_j) = \partial_l$; and $\alpha^\partial_{ij} > \alpha^\partial_{ik}$, $\beta^\delta_{ij} > \beta^\delta_{lj}$.
2) $\exists$ $\delta_k \in \delta$, $\delta_k \in \delta$, $\Re(\partial_i) = \delta_k$, $\Re(\delta_j) = \delta_j$; and $\alpha^\partial_{ij} > \alpha^\partial_{ik}$.
3) $\exists$ $\partial_l \in \partial$, $\Re(\partial_i) = \partial_i$, $\Re(\delta_j) = \partial_l$; and $\beta^\delta_{ij} > \beta^\delta_{lj}$.

Then, $\Re$ is called as a one-to-one stable matching.

Remark 1. In Definition 10, if one of the above three cases occurs, it is called an unstable matching pair. Condition (1) is for the case that ∂_i and δ_j are matched with other subjects. It means that ∂_i is more willing to match with δ_j compared with the matched subject δ_k, and δ_j is more willing to match with ∂_i compared with the matched subject ∂_l; condition (2) is for the case that ∂_i is matched with δ_k and δ_j is not matched. It indicates that ∂_i is more willing to match with δ_j compared with the matched subject δ_k; condition (3) is for the case that δ_j is matched with ∂_l and ∂_i is not matched. It indicates that δ_j is more willing to match with ∂_i compared with the matched subject ∂_l.

4. Description of the multi-position two-phase person-position matching problem

Based on the above conceptions, the multi-position two-phase person-position matching problem is discussed in this section.

A multi-position two-phase person-position matching problem composed of enterprises, headhunting companies and candidates is considered in this paper. In the first phase, enterprises may entrust multiple headhunting companies and headhunting companies may choose multiple companies, that is, the many-to-many two-sided matching problem between enterprises and headhunting companies is formed. In the second phase, based on the optimal matching scheme between enterprises and headhunting companies of the first stage, headhunting companies provides candidates under multiple positions for the matched enterprises. A position of each enterprise may require multiple candidates, but one candidate can only be matched with a position of an enterprise, that is, the one-to-many two-sided matching problems between enterprises and candidates under multiple positions are formed. The specific description of two-phase person-position matching is displayed in Fig. 1.

It can be seen from Fig. 1, in first phase, the dashed line (——) represents the possible matching pair, and the double arrow line ($\leftrightarrow$) indicates that the effective matching pairs of enterprise and headhunting company, such as $\partial_1 \leftrightarrow \delta_1$ ($\partial_1 \leftrightarrow \delta_2$) indicates that ∂_1 is matched with δ_1 (δ_2), $\delta_1 \leftrightarrow \partial_1$ ($\delta_1 \leftrightarrow \partial_2$) indicates that δ_1 is matched with ∂_1 (∂_2); in the second phase, the single arrow dashed line ($\leftarrow -$) indicates that candidate is recommended by headhunting company to enterprise; for example, $A^{h_1}_1 \leftarrow -\delta_1$, $\partial_1 \leftarrow -A^{h_1}_1$ and $\partial_2 \leftarrow -A^{h_1}_1$ indicate that candidate $A^{h_1}_1$ is recommended by headhunting company δ_1 to enterprise ∂_1 and ∂_2 under position h_1; and the double arrow line ($\leftrightarrow$) indicates the effective matching pairs of enterprises and candidates under each position, such as $\partial_m \leftrightarrow A^{h_1}_{f_{j_i}}$ ($\partial_m \leftrightarrow A^{h_1}_{f_q}$) indicates that ∂_m is matched with $A^{h_1}_{f_{j_i}}$ ($A^{h_1}_{f_q}$) under position h_1.

Moreover, the problem description of the multi-position two-phase person-position matching is given. The first phase: First, let $\partial = \{\partial_1, ..., \partial_m\}$ be the set of enterprises; $h = \{h_1, ..., h_l\}$ be the position set of all enterprises; and $\delta = \{\delta_1, ..., \delta_n\}$ be the set of headhunting companies. Assume that m_i is the maximum number of headhunting companies that an enterprise can be matched, and n_j is the maximum number of enterprises that a headhunting company can be matched.

Then, assumed that $U^\partial = \left[u^\partial_{ij}\right]_{m\times n}$ is the probabilistic linguistic evaluation matrix of enterprises to headhunting companies; $E^\partial = (e^\partial_1, ..., e^\partial_m)$ is the probabilistic linguistic expected vector of enterprises for all headhunting companies; $V^\delta = \left[v^\delta_{ij}\right]_{m\times n}$ is the probabilistic linguistic evaluation matrix of headhunting companies to enterprises; $E^\delta = (e^\delta_1, ..., e^\delta_n)$ is the probabilistic linguistic expected vector of headhunting companies for all enterprises. In addition, $S = \{s_0, s_1, ..., s_\theta\}$ is the considered linguistic term set, where s_τ indicates the $\tau-$th linguistic term, $\tau = 0, 1, ..., \theta$.

The second phase: First, assume that $\partial_i \leftrightarrow \left\{\delta^i_1, ..., \delta^i_{m_i}\right\}$ $(i = 1, ..., m)$ is the matching result of enterprises and headhunting companies in the first phase. Then, let $A^{h_q} = \left\{A^{h_q}_1, ..., A^{h_q}_{f_q}\right\}$ be the set of candidates under position h_q; $A^{\delta^i_{j_i}\leftrightarrow\partial_i}_{h_q} = \left\{A^{\delta^i_{j_i}\leftrightarrow\partial_i}_{h_q,1}, ..., A^{\delta^i_{j_i}\leftrightarrow\partial_i}_{h_q,n_{j_i}}\right\}$ be a set of partial candidates under position h_q recommended by headhunting company $\delta^i_{j_i}$ to enterprise ∂_i. Let $U^{h_q}_i = \left(u^{h_q}_{i1}, ..., u^{h_q}_{in_{j_i}}\right)$ $(i = 1, ..., m)$ be the probabilistic linguistic evaluation vector of enterprise ∂_i to candidate $A^{\delta^i_{j_i}\leftrightarrow\partial_i}_{h_q f_{j_i}}$ under position; $E^{h_q}_\partial = \left(e^{h_q}_1, ..., e^{h_q}_m\right)$ be the probabilistic linguistic expected

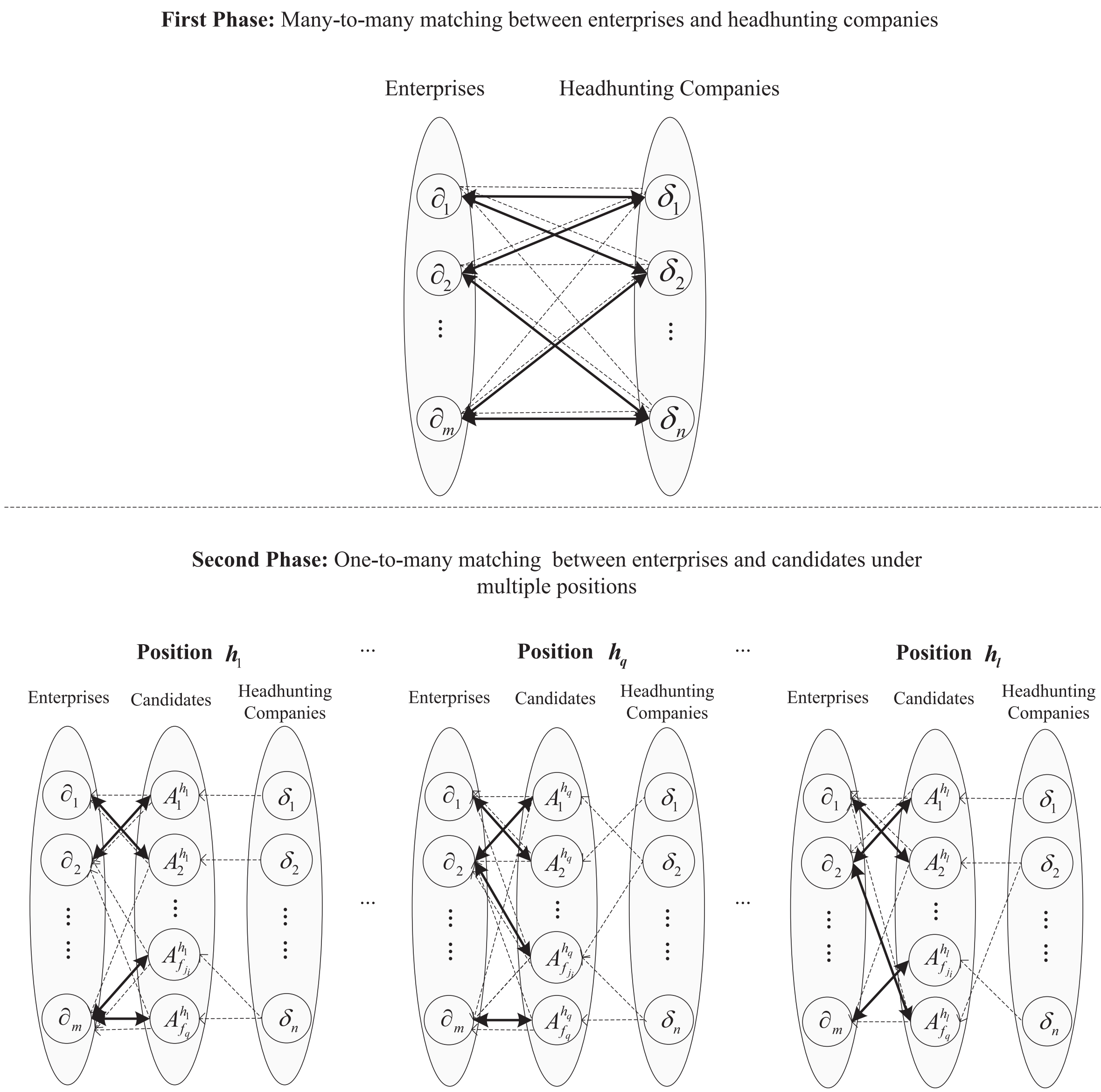

Fig. 1. Description of the multi-position two-phase person-position matching problem.

vector of enterprises for all candidates under position h_q. Let $V_{j_i}^{h_q} = \left(v_{j_i1}^{h_q}, \ldots, v_{j_in_{j_i}}^{h_q}\right)$ ($j_i = 1,\ldots,m_i$) be the probabilistic linguistic evaluation vector of headhunting company $\delta_{j_i}^i$ to candidate $A_{h_q f_{j_i}}^{\delta_{j_i}^i \leftrightarrow \partial_i}$ under position h_q. Let $Z_{f_{j_i}}^{h_q} = \left(z_{1f_{j_i}}^{h_q}, \ldots, z_{mf_{j_i}}^{h_q}\right)$ be the probabilistic linguistic evaluation vector of candidate $A_{h_q f_{j_i}}^{\delta_{j_i}^i \leftrightarrow \partial_i}$ to enterprise ∂_i under position h_q; $E_A^{h_q} = \left(e_1^{h_q}, \ldots, e_{n_{j_i}}^{h_q}\right)$ ($f_{j_i} = 1, \ldots, n_{j_i}$) be the probabilistic linguistic expected vector of candidates for all enterprises under position h_q. The assumed symbols are further explained in Table 1.

This paper considers solving the following problems: (1) How to obtain the optimal many-to-many two-sided matching scheme $\Im^*$ of enterprises and headhunting companies according to the probabilistic linguistic evaluation matrices $U^\partial = \left[u_{ij}^\partial\right]_{m\times n}$ and $V^\delta = \left[v_{ij}^\delta\right]_{m\times n}$, and the probabilistic linguistic expected vectors $E^\partial = \left(e_1^\partial, \ldots, e_m^\partial\right)$ and $E^\ell = \left(e_1^\delta, \ldots, e_n^\delta\right)$. (2) How to obtain the optimal one-to-many two-sided matching scheme $\wp_1^*, \wp_2^*, \ldots, \wp_l^*$ of enterprises and candidates under position all positions according to the probabilistic linguistic evaluation vectors $U_i^{h_q} = \left(u_{i1}^{h_q}, \ldots, u_{in_{j_i}}^{h_q}\right)$ ($i \in \{1, \ldots, m\}$) and $Z_{f_{j_i}}^{h_q} = \left(z_{1f_{j_i}}^{h_q}, \ldots, z_{mf_{j_i}}^{h_q}\right)$ ($f_{j_i} \in \{1, \ldots, n_{j_i}\}$), the probabilistic linguistic expected vectors $E_\partial^{h_q} = \left(e_1^{h_q}, \ldots, e_m^{h_q}\right)$ and $E_A^{h_q} = \left(e_1^{h_q}, \ldots, e_{n_{j_i}}^{h_q}\right)$, and the probabilistic linguistic evaluation vectors $V_{j_i}^{h_q} = \left(v_{j_i1}^{h_q}, \ldots, v_{j_in_{j_i}}^{h_q}\right)$ ($j_i \in \{1, \ldots, m_i\}$).

5. Decision-making method for multi-position two-phase person-position matching

To solve the considered complex person-position matching problem, a multi-position two-phase person-position matching decision-making method is exhibited in this section. The proposed method is divided into two phases, and matching mechanisms of the two phases are given as follows. First, the matching mechanism between enterprises and headhunting companies is designed. Second, the matching mechanism between enterprises and candidates under multiple positions are developed. Finally, steps of multi-position two-phase person-position matching decision-making are displayed.

Table 1
Meanings of symbols of the considered person-position matching problem.

Symbol	Meaning
$\partial = \{\partial_1, \ldots, \partial_m\}$	Set of enterprises, where ∂_i indicates the $i-$th enterprise, $\partial_i \in \partial, i = 1, \ldots, m$
$\delta = \{\delta_1, \ldots, \delta_n\}$	Set of headhunting companies, where δ_j indicates the $j-$th headhunting company, $\delta_j \in \delta, j = 1, \ldots, n$
$P = \{h_1, \ldots, h_l\}$	Position set of all enterprises, where h_q indicates the $q-$th position, $h_q \in P, q = 1, \ldots, l$
$A^{h_q} = \left\{A_1^{h_q}, \ldots, A_{f_q}^{h_q}\right\}$	Set of candidates under position h_q
$A_{h_q}^{\delta_{j_i}^i \leftrightarrow \partial_i} = \left\{A_{h_q,1}^{\delta_{j_i}^i \leftrightarrow \partial_i}, \ldots, A_{h_q,n_{j_i}}^{\delta_{j_i}^i \leftrightarrow \partial_i}\right\}$	Set of partial candidates under position h_q recommended by headhunting company $\delta_{j_i}^i$ to enterprise ∂_i, where $A_{h_q,f_{j_i}}^{\delta_{j_i}^i \leftrightarrow \partial_i}$ indicates the $f_{j_i}-$th candidate under the position $h_q, f_{j_i} \in Q_{j_i} = \{1, \ldots, n_{j_i}\}$
m_i	Maximum number of headhunting companies that enterprise ∂_i can be matched
n_j	Maximum number of enterprises that headhunting company δ_j can be matched
$t_i^{h_q}$	Maximum number of candidates that can be matched with enterprise ∂_i under position h_q
$\partial_i \leftrightarrow \left\{\delta_1^i, \ldots, \delta_{m_i}^i\right\}$	Matching result of enterprise ∂_i and headhunting companies, where $\delta_{j_i}^i$ indicates the j_i-th headhunting company, $\delta_{j_i}^i \in \left\{\delta_1^i, \ldots, \delta_{m_i}^i\right\}$
$U^\partial = \left[u_{ij}^\partial\right]_{m\times n}$	Evaluation matrix of enterprises to headhunting companies
$E^\partial = (e_1^\partial, \ldots, e_m^\partial)$	Expected vector of enterprises for all headhunting companies
$V^\delta = \left[v_{ij}^\delta\right]_{m\times n}$	Evaluation matrix of headhunting companies to enterprises
$E^\delta = (e_1^\delta, \ldots, e_n^\delta)$	Expected vector of headhunting companies for all enterprises
$U_i^{h_q} = \left(u_{i1}^{h_q}, \ldots, u_{in_{j_i}}^{h_q}\right)$	Evaluation vector of enterprise ∂_i to partial candidates under position h_q
$E_\partial^{h_q} = \left(e_1^{h_q}, \ldots, e_m^{h_q}\right)$	Expected vector of enterprises for all candidates under position h_q
$V_{j_i}^{h_q} = \left(v_{j_i1}^{h_q}, \ldots, v_{j_in_{j_i}}^{h_q}\right)$	Evaluation vector of headhunting company $\delta_{j_i}^i$ to partial candidates under position h_q
$Z_{f_{j_i}}^{h_q} = \left(z_{1f_{j_i}}^{h_q}, \ldots, z_{mf_{j_i}}^{h_q}\right)$	Evaluation vector of partial candidates to enterprises under position h_q
$E_A^{h_q} = \left(e_1^{h_q}, \ldots, e_{n_{j_i}}^{h_q}\right)$	Expected vector of partial candidates for all enterprises under position h_q

5.1. *First phase: Matching mechanism between enterprises and headhunting companies*

In the first phase, first, according to the probabilistic linguistic evaluation matrices and the probabilistic linguistic expected vectors, the expected profit and loss values of enterprises and headhunting companies are calculated based on the prospect theory. Second, the satisfactions of enterprises and headhunting companies are calculated by using the risk coefficients and standardization. Then, to maximize the satisfactions of enterprises and headhunting companies, the many-to-many two-sided matching model is established. Furthermore, the optimal many-to-many matching scheme $\Im^*$ between enterprises and headhunting companies is obtained by solving the model. The specific process is shown in Fig. 2.

5.1.1. *Calculation of satisfactions of enterprises and headhunting companies*

In this section, the satisfactions of enterprises and headhunting companies are calculated based on prospect theory. First, this paper takes the evaluation value as the actual value and the expected value as the reference point. Then, the distances between actual values and expected points are calculated to obtain the expected profit and loss values. Furthermore, the satisfactions of two-sided subjects are calculated by considering risk attitudes and using standardization. The calculation process for the satisfaction is given as follows.

First, for enterprises, distance $d\left(u_{ij}^\partial, e_i^\partial\right)$ between the actual value u_{ij}^∂ and the expected point e_i^∂ is calculated as:

$$d\left(u_{ij}^\partial, e_i^\partial\right) = \sum_{x=1}^{\left|u_{ij}^\partial\right|} \sum_{y=1}^{\left|e_i^\partial\right|} \left(p_{u_{ij}^\partial}^x \times p_{e_i^\partial}^y\right) \left| g\left(L_{u_{ij}^\partial}^x\right) - g\left(L_{e_i^\partial}^y\right) \right| \tag{4}$$

where Eq. (4) is extended by Eq. (3).

Second, let $\gamma_{ij}^\partial\left(u_{ij}^\partial, e_i^\partial\right)$ be the expected profit and loss value of enterprise ∂_i to headhunting company δ_j, then, $\gamma_{ij}^\partial\left(u_{ij}^\partial, e_i^\partial\right)$ is calculated as:

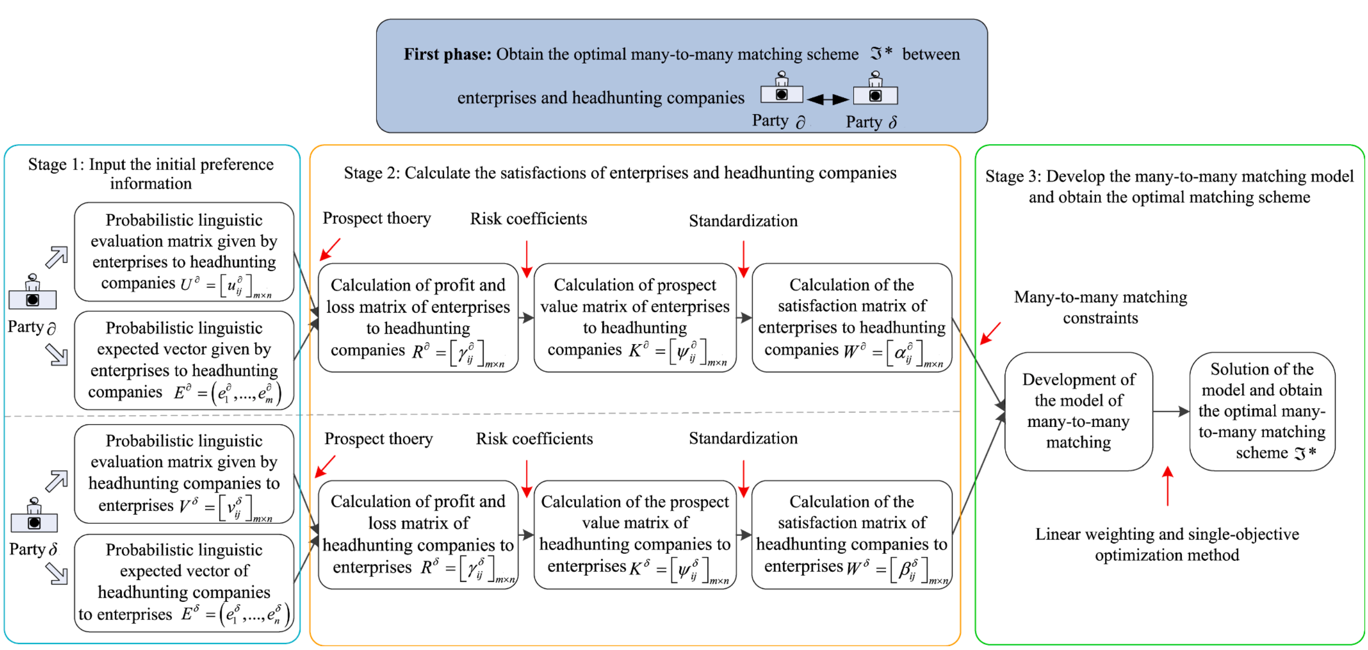

Fig. 2. Matching mechanism of enterprises and headhunting companies under the probabilistic linguistic environment in the first phase.

$$\gamma_{ij}^{\partial}\left(u_{ij}^{\partial}, e_i^{\partial}\right) = \begin{cases} d\left(u_{ij}^{\partial}, e_i^{\partial}\right), & u_{ij}^{\partial} \geq e_i^{\partial} \\ -d\left(u_{ij}^{\partial}, e_i^{\partial}\right), & u_{ij}^{\partial} < e_i^{\partial} \end{cases} \tag{5}$$

In Eq. (5), when γ_{ij}^{∂} is positive, it is profit; and when it is negative, it is loss; the values of u_{ij}^{∂}

and e_i^{∂} can be compared by Eqs. (1) and (2). The expected profit and loss value matrix $R^{\partial} = \left[\gamma_{ij}^{\partial}\right]_{m\times n}$ of enterprises is constructed by Eq. (5).

Third, considering the risk attitudes of enterprises towards the profit and loss, the prospect value $\psi_{ij}^{\partial}\left(u_{ij}^{\partial}, e_i^{\partial}\right)$ of enterprise ∂_i to headhunting company δ_j is calculated as:

$$\psi_{ij}^{\partial}\left(u_{ij}^{\partial}, e_i^{\partial}\right) = \begin{cases} \left(\gamma_{ij}^{\partial}\left(u_{ij}^{\partial}, e_i^{\partial}\right)\right)^{a}, & u_{ij}^{\partial} \geq e_i^{\partial} \\ -\lambda\left(-\gamma_{ij}^{\partial}\left(u_{ij}^{\partial}, e_i^{\partial}\right)\right)^{b}, & u_{ij}^{\partial} < e_i^{\partial} \end{cases} \tag{6}$$

where $0 < a < 1, 0 < b < 1, \lambda > 1$. The coefficient λ reflects the psychological behavior of two-sided subjects, and all subjects are more sensitive to loss compared with profit. The greater the coefficient λ is, the greater the degree of loss avoidance is. According to the idea of Ruggeri et al. (2020), the values of coefficients are determined, i.e., $a = b = 0.88$, $\lambda = 2.25$. By Eq. (6), the prospect value matrix $K^{\partial} = \left[\psi_{ij}^{\partial}\right]_{m\times n}$ of enterprises to headhunting companies is constructed.

Furthermore, the satisfaction α_{ij}^{∂} of enterprise ∂_i to headhunting company δ_j is calculated by standardization, where α_{ij}^{∂} is calculated as:

$$\alpha_{ij}^{\partial} = \frac{\psi_{ij}^{\partial} - \min\limits_{i\in H, j\in T}\left\{\psi_{ij}^{\partial}\right\} + \frac{1}{m}\max\limits_{i\in H, j\in T}\left\{\psi_{ij}^{\partial}\right\}}{\frac{m+1}{m}\max\limits_{i\in H, j\in T}\left\{\psi_{ij}^{\partial}\right\} - \min\limits_{i\in H, j\in T}\left\{\psi_{ij}^{\partial}\right\}}, i \in H, j \in T \tag{7}$$

By Eq. (7), the satisfaction matrix $W^{\partial} = \left[\alpha_{ij}^{\partial}\right]_{m\times n}$ of enterprises to headhunting companies is constructed, where $\alpha_{ij}^{\partial} \in [0, 1]$.

Based on the above analysis, Algorithm 1 for calculating satisfactions of enterprises to headhunting companies is displayed as follows:

Algorithm 1

Input: Evaluation matrix $U^{\partial} = \left[u_{ij}^{\partial}\right]_{m\times n}$ and expected vector $E^{\partial} = (e_1^{\partial}, \dots, e_m^{\partial})$

Output: Satisfaction matrix $W^{\partial} = \left[\alpha_{ij}^{\partial}\right]_{m\times n}$

Begin

Step 1.1: Construct expected profit and loss value matrix $R^{\partial} = \left[\gamma_{ij}^{\partial}\right]_{m\times n}$ by Eqs. (4) and (5);

Step 1.2: Structure prospect value matrix $K^{\partial} = \left[\psi_{ij}^{\partial}\right]_{m\times n}$ by Eq. (6);

Step 1.3: Obtain satisfaction matrix $W^{\partial} = \left[\alpha_{ij}^{\partial}\right]_{m\times n}$ by Eq. (7).

End

Similarly, for headhunting companies, the distance $d\left(v_{ij}^{\delta}, e_j^{\delta}\right)$ between the actual value v_{ij}^{δ} and the expected value e_j^{δ} of headhunting company δ_j to enterprise ∂_i is calculated as:

$$d\left(v_{ij}^{\delta}, e_j^{\delta}\right) = \sum_{x=1}^{\left|v_{ij}^{\delta}\right|}\sum_{y=1}^{\left|e_j^{\delta}\right|}\left(p_{v_{ij}^{\delta}}^{x} \times p_{e_j^{\delta}}^{y}\right)\left|g\left(L_{v_{ij}^{\delta}}^{x}\right) - g\left(L_{e_j^{\delta}}^{y}\right)\right| \tag{8}$$

Let $\gamma_{ij}^{\delta}\left(v_{ij}^{\delta}, e_j^{\delta}\right)$ be the expected profit and loss value of headhunting company δ_j to enterprise ∂_i, then, $\gamma_{ij}^{\delta}\left(v_{ij}^{\delta}, e_j^{\delta}\right)$ is calculated as:

$$\gamma_{ij}^{\delta}\left(v_{ij}^{\delta}, e_j^{\delta}\right) = \begin{cases} d\left(v_{ij}^{\delta}, e_j^{\delta}\right), & v_{ij}^{\delta} \geq e_j^{\delta} \\ -d\left(v_{ij}^{\delta}, e_j^{\delta}\right), & v_{ij}^{\delta} < e_j^{\delta} \end{cases} \tag{9}$$

In Eq. (9), when γ_{ij}^{δ} is positive, it is profit; and when it is negative, it is loss. By Eq. (9), the expected profit and loss value matrix $R^{\delta} = \left[\gamma_{ij}^{\delta}\right]_{m\times n}$ of headhunting companies is constructed.

Considering the risk attitudes of headhunting companies towards the profit and loss, the prospect value $\psi_{ij}^{\delta}\left(v_{ij}^{\delta}, e_j^{\delta}\right)$ of headhunting company δ_j to enterprise ∂_i is calculated as:

$$\psi_{ij}^{\delta}\left(v_{ij}^{\delta}, e_j^{\delta}\right) = \begin{cases} \left(\gamma_{ij}^{\delta}\left(v_{ij}^{\delta}, e_j^{\delta}\right)\right)^{a}, & v_{ij}^{\delta} \geq e_j^{\delta} \\ -\lambda\left(-\gamma_{ij}^{\delta}\left(v_{ij}^{\delta}, e_j^{\delta}\right)\right)^{b}, & v_{ij}^{\delta} < e_j^{\delta} \end{cases} \tag{10}$$

By Eq. (10), the prospect value matrix $K^{\delta} = \left[\psi_{ij}^{\delta}\right]_{m\times n}$ of enterprises to headhunting companies is constructed.

Furthermore, the satisfaction β_{ij}^{δ} of headhunting company δ_j to enterprise ∂_i is calculated by standardization, where β_{ij}^{δ} is calculated as:

$$\beta_{ij}^{\delta} = \frac{\psi_{ij}^{\delta} - \min\limits_{i\in H, j\in T}\left\{\psi_{ij}^{\delta}\right\} + \frac{1}{n}\max\limits_{i\in H, j\in T}\left\{\psi_{ij}^{\delta}\right\}}{\frac{n+1}{n}\max\limits_{i\in H, j\in T}\left\{\psi_{ij}^{\delta}\right\} - \min\limits_{i\in H, j\in T}\left\{\psi_{ij}^{\delta}\right\}}, i \in H, j \in T \tag{11}$$

By Eq. (11), the satisfaction matrix $W^{\delta} = \left[\beta_{ij}^{\delta}\right]_{m\times n}$ of headhunting companies to enterprises is constructed, where $\beta_{ij}^{\delta} \in [0, 1]$.

Then, Algorithm 2 for calculating satisfactions of headhunting companies to enterprises is displayed as follows:

Algorithm 2

Input: Evaluation matrix $V^{\delta} = \left[v_{ij}^{\delta}\right]_{m\times n}$ and expected vector $E^{\delta} = (e_1^{\delta}, \dots, e_n^{\delta})$

Output: Satisfaction matrix $W^{\delta} = \left[\beta_{ij}^{\delta}\right]_{m\times n}$

Begin

Step 2.1: Construct expected profit and loss value matrix $R^{\delta} = \left[\gamma_{ij}^{\delta}\right]_{m\times n}$ by Eqs. (8) and (9);

Step 2.2: Structure prospect value matrix $K^{\delta} = \left[\psi_{ij}^{\delta}\right]_{m\times n}$ by Eq. (10);

Step 2.3: Obtain satisfaction matrix $W^{\delta} = \left[\beta_{ij}^{\delta}\right]_{m\times n}$ by Eq. (11).

End

5.1.2. Construction of many-to-many two-sided matching model between enterprises and headhunting companies

First, introducing the two-sided matching matrix $X = [x_{ij}]_{m\times n}$, where $x_{ij} = \begin{cases} 1, \Im(\partial_i) = \delta_j \\ 0, \Im(\partial_i) \neq \delta_j \end{cases}$. Then, according to the two-sided matching matrix $X = [x_{ij}]_{m\times n}$ and satisfaction matrices $W^{\partial} = \left[\alpha_{ij}^{\partial}\right]_{m\times n}$ and $W^{\delta} = \left[\beta_{ij}^{\delta}\right]_{m\times n}$, the two-sided matching model (M−1) under many-to-many matching conditions are constructed as follows:

$$(\mathrm{M}-1)\begin{cases} \mathrm{Max}D_1 = \sum\limits_{i=1}^{m}\sum\limits_{j=1}^{n}\alpha_{ij}^{\partial}x_{ij} \\ \mathrm{Max}D_2 = \sum\limits_{i=1}^{m}\sum\limits_{j=1}^{n}\beta_{ij}^{\delta}x_{ij} \\ s.t.\ \sum\limits_{j=1}^{n}x_{ij} \leq m_i, i \in H \\ \sum\limits_{i=1}^{m}x_{ij} \leq n_j, j \in T \\ x_{ij} \in \{0, 1\}, i \in H; j \in T \end{cases}$$

where MaxD_1 means to maximize the sum of satisfactions of enterprisers, and MaxD_2 means to maximize the sum of satisfactions of headhunting companies.

For objective functions D_1 and D_2, considering that α_{ij}^{∂} and β_{ij}^{δ} are of the same dimension, model (M−1) can be transformed into a single objective model (M−2) by the linear weighting method:

$$
(\mathrm{M-2})\begin{cases}
\mathrm{Max}D = \sum_{i=1}^{m}\sum_{j=1}^{n}\left[\omega_1\alpha_{ij}^{\partial}+\omega_2\beta_{ij}^{\delta}\right]x_{ij} \\
\quad \mathrm{s.t.} \ \sum_{j=1}^{n} x_{ij} = 1, i \in H \\
\quad \sum_{i=1}^{m} x_{ij} \leq 1, j \in T \\
x_{ij} \in \{0,1\}, i \in H; j \in T
\end{cases}
$$

where ω_1 and ω_2 represent the weights of objective functions D_1 and D_2 respectively. By using the mathematical software such as Lingo to solve model (M−2), the optimal matching scheme is obtained.

5.2. Second phase: Matching mechanism between enterprises and candidates under multiple positions

In the second phase, first, according to the probabilistic linguistic evaluation vectors and the probabilistic linguistic expected vectors, the profit and loss values of enterprises and candidates are calculated considering subject expectations and intermediary evaluations. Second, the satisfactions of enterprises and candidates are calculated by considering risk attitudes and using standardization. Third, to maximize the sum of satisfactions of enterprises and candidates, one-to-many matching models and one-to-one stable matching models under multiple positions are developed. Finally, these developed models are solved to obtain the optimal person-position matching schemes $\wp_1^*, \wp_2^*, \ldots, \wp_l^*$ under multiple positions. The specific process of the second phase is displayed in Fig. 3.

5.2.1. Calculation of satisfactions of enterprises considering double reference points

Considering the bounded rationality of subjects, this paper believe that subject expectations and intermediary evaluations will act on the psychological behavior of subjects at the same time. Thus, enterprise expectations and intermediary evaluations are taken as reference points. Then, the distances between actual values and reference points are calculated to obtain the profit and loss values. Furthermore, the satisfactions of enterprises are calculated by considering risk attitudes and using standardization. The calculation of the satisfactions of enterprises is given as follows.

First, for enterprises, the distance $d\left(u_{if_{j_i}}^{h_q}, e_i^{h_q}\right)$ between the actual value $u_{if_{j_i}}^{h_q}$ and the expected reference point $e_i^{h_q}$ of enterprise ∂_i to candidates $A_{h_q f_{j_i}}^{\delta_{j_i}^i \leftrightarrow \partial_i}$ under position h_q is calculated as:

$$
d\left(u_{if_{j_i}}^{h_q}, e_i^{h_q}\right) = \sum_{x=1}^{\left|u_{if_{j_i}}^{h_q}\right|}\sum_{y=1}^{\left|e_i^{h_q}\right|}\left(p_{u_{if_{j_i}}^{h_q}}^{x} \times p_{e_i^{h_q}}^{y}\right)\left|g\left(L_{u_{if_{j_i}}^{h_q}}^{x}\right) - g\left(L_{e_i^{h_q}}^{y}\right)\right| \tag{12}
$$

Let $\gamma_{if_{j_i}}^{h_q}\left(u_{if_{j_i}}^{h_q}, e_i^{h_q}\right)$ be the expected profit and loss value of enterprise ∂_i to candidates $A_{h_q f_{j_i}}^{\delta_{j_i}^i \leftrightarrow \partial_i}$ under position h_q, then, $\gamma_{if_{j_i}}^{h_q}\left(u_{if_{j_i}}^{h_q}, e_i^{h_q}\right)$ is calculated as:

$$
\gamma_{if_{j_i}}^{h_q}\left(u_{if_{j_i}}^{h_q}, e_i^{h_q}\right) = \begin{cases} d\left(u_{if_{j_i}}^{h_q}, e_i^{h_q}\right), & u_{if_{j_i}}^{h_q} \geq e_i^{h_q} \\ -d\left(u_{if_{j_i}}^{h_q}, e_i^{h_q}\right), & u_{if_{j_i}}^{h_q} < e_i^{h_q} \end{cases} \tag{13}
$$

In Eq. (13), when $\gamma_{if_{j_i}}^{h_q}$ is positive, it is profit; and when it is negative, it is loss. The expected profit and loss value vectors $R_i^{h_q} = \left(\gamma_{i1}^{h_q}, \ldots, \gamma_{in_{j_i}}^{h_q}\right)$ $(i = 1, \ldots, m;\ q = 1, \ldots, l)$ of enterprises to candidates under all positions are constructed by Eq. (13).

Second, the distance $d\left(u_{if_{j_i}}^{h_q}, v_{jf_{j_i}}^{h_q}\right)$ between the actual value $u_{if_{j_i}}^{h_q}$ and

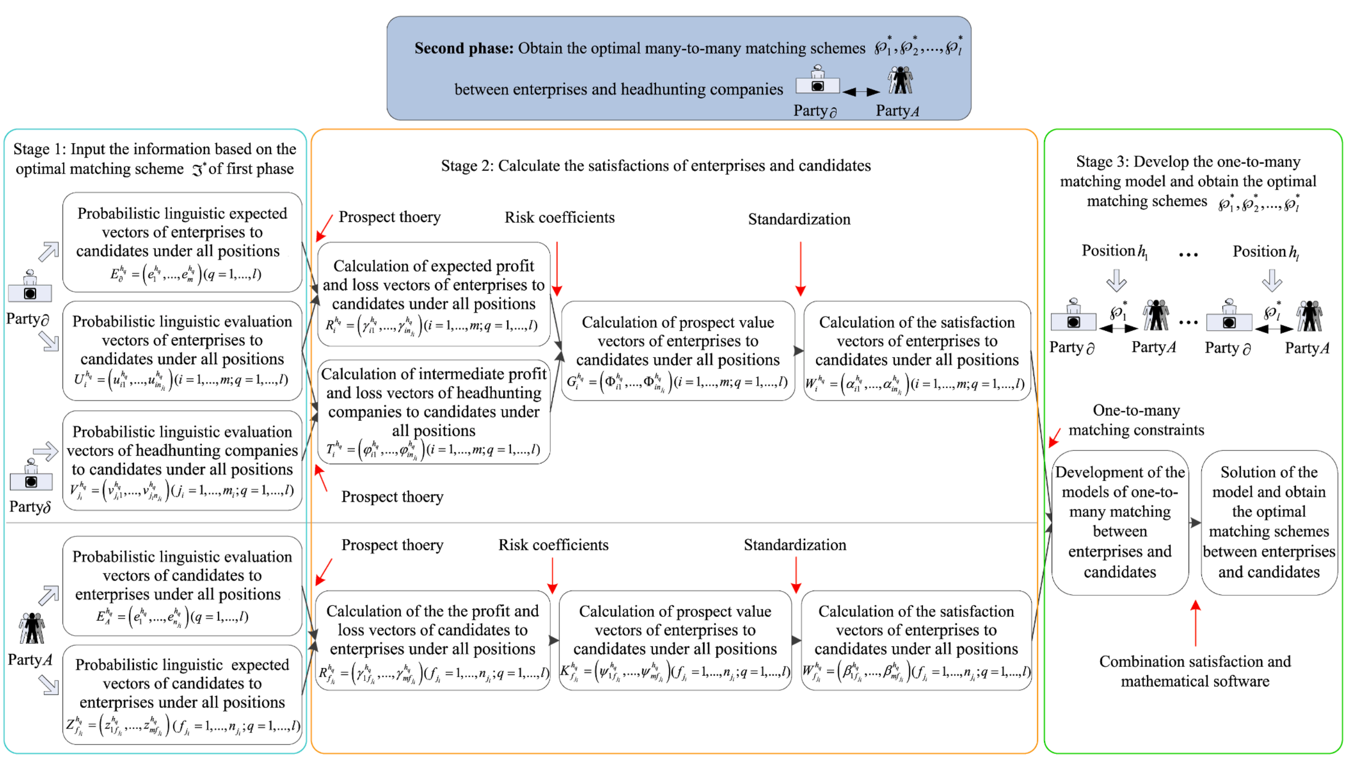

Fig. 3. Matching mechanism of enterprises and candidates under the probabilistic linguistic environment.

the intermediary reference point $v_{jf_{ji}}^{h_q}$ of enterprise ∂_i to candidates $A_{h_q f_{ji}}^{\delta_{ji}^i \leftrightarrow \partial_i}$ under position h_q is calculated as:

$$d\left(u_{if_{ji}}^{h_q}, v_{jf_{ji}}^{h_q}\right) = \sum_{x=1}^{\left|u_{if_{ji}}^{h_q}\right|} \sum_{y=1}^{\left|v_{jf_{ji}}^{h_q}\right|} \left(p_{u_{if_{ji}}^{h_q}}^{x} \times p_{v_{jf_{ji}}^{h_q}}^{y}\right) \left| g\left(L_{u_{if_{ji}}^{h_q}}^{x}\right) - g\left(L_{v_{jf_{ji}}^{h_q}}^{y}\right) \right| \tag{14}$$

Let $\varphi_{if_{ji}}^{h_q}\left(u_{if_{ji}}^{h_q}, v_{jf_{ji}}^{h_q}\right)$ be the intermediary profit and loss value of enterprise ∂_i to candidates $A_{h_q f_{ji}}^{\delta_{ji}^i \leftrightarrow \partial_i}$ under position h_q, then, $\varphi_{if_{ji}}^{h_q}\left(u_{if_{ji}}^{h_q}, v_{jf_{ji}}^{h_q}\right)$ is calculated as:

$$\varphi_{if_{ji}}^{h_q}\left(u_{if_{ji}}^{h_q}, v_{jf_{ji}}^{h_q}\right) = \begin{cases} d\left(u_{if_{ji}}^{h_q}, v_{jf_{ji}}^{h_q}\right), & u_{if_{ji}}^{h_q} \geq v_{jf_{ji}}^{h_q} \\ -d\left(u_{if_{ji}}^{h_q}, v_{jf_{ji}}^{h_q}\right), & u_{if_{ji}}^{h_q} < v_{jf_{ji}}^{h_q} \end{cases} \tag{15}$$

In Eq. (15), when $\gamma_{if_{ji}}^{h_q}$ is positive, it is profit; and when it is negative, it is loss. The expected profit and loss value vectors $T_i^{h_q} = \left(\varphi_{i1}^{h_q}, \ldots, \varphi_{in_{ji}}^{h_q}\right)$ $(i = 1, \ldots, m;\ q = 1, \ldots, l)$ of enterprise ∂_i to candidates under all position are constructed by Eq. (15).

Third, considering that enterprises have similar psychological feelings to the expected reference point and the intermediary reference point, the comprehensive profit and loss value vectors $G_i^{h_q} = \left(\Phi_{i1}^{h_q}, \ldots, \Phi_{in_{ji}}^{h_q}\right)$ $(i = 1, \ldots, m;\ q = 1, \ldots, l)$ under position h_q are obtained, where $\Phi_{if_{ji}}^{h_q}$ is calculated as:

$$\Phi_{if_{ji}}^{h_q} = \rho\gamma_{if_{ji}}^{h_q} + (1-\rho)\varphi_{if_{ji}}^{h_q} \tag{16}$$

In Eq. (16), ρ is defined as the psychological tendency coefficient, and $\rho \in [0, 1]$. $\rho = 0$ means that enterprises only consider $\varphi_{if_{ji}}^{h_q}$ to measure the profit and loss. $0 < \rho < 0.5$ means that enterprises consider $\varphi_{if_{ji}}^{h_q}$ can better reflect the profit and loss than $\gamma_{if_{ji}}^{h_q}$. $\rho = 0.5$ means that enterprises have the same tendency to $\varphi_{if_{ji}}^{h_q}$ and $\gamma_{if_{ji}}^{h_q}$. $1 > \rho > 0.5$ means that enterprises consider $\gamma_{if_{ji}}^{h_q}$ can better reflect the profit and loss than $\varphi_{if_{ji}}^{h_q}$. $\rho = 1$ means that enterprises only consider $\gamma_{if_{ji}}^{h_q}$ to measure the profit and loss.

Then, considering the different risk attitudes of enterprises towards the profit and loss, the prospect value $\psi_{if_{ji}}^{h_q}$ of enterprise ∂_i to candidates $A_{h_q f_{ji}}^{\delta_{ji}^i \leftrightarrow \partial_i}$ under position h_q is calculated as:

$$\psi_{if_{ji}}^{h_q} = \begin{cases} \left(\Phi_{if_{ji}}^{h_q}\right)^a, & \Phi_{if_{ji}}^{h_q} \geq 0 \\ -\lambda\left(-\Phi_{if_{ji}}^{h_q}\right)^b, & \Phi_{if_{ji}}^{h_q} < 0 \end{cases} \tag{17}$$

By Eq. (17), the prospect value vectors $K_i^{h_q} = \left(\psi_{i1}^{h_q}, \ldots, \psi_{in_{ji}}^{h_q}\right)$ $(i = 1, \ldots, m;\ q = 1, \ldots, l)$ of enterprise ∂_i to candidates under all position are constructed.

Furthermore, according to the prospect value vectors $K_i^{h_q} = \left(\psi_{i1}^{h_q}, \ldots, \psi_{in_{ji}}^{h_q}\right)$ $(i = 1, \ldots, m;\ q = 1, \ldots, l)$, the satisfaction vectors $W_i^{h_q} = \left(\alpha_{i1}^{h_q}, \ldots, \alpha_{in_{ji}}^{h_q}\right)$ $(i = 1, \ldots, m;\ q = 1, \ldots, l)$ of enterprise ∂_i to candidates under all positions are obtained by standardization, where $\alpha_{if_{ji}}^{h_q}$ $(\alpha_{if_{ji}}^{h_q} \in [0, 1])$ is calculated as:

$$\alpha_{if_{ji}}^{h_q} = \frac{\psi_{if_{ji}}^{h_q} - \min\limits_{i \in H, f_{ji} \in Q_{ji}}\left\{\psi_{if_{ji}}^{h_q}\right\} + \frac{1}{m}\max\limits_{i \in H, f_{ji} \in Q_{ji}}\left\{\psi_{if_{ji}}^{h_q}\right\}}{\frac{m+1}{m}\max\limits_{i \in H, f_{ji} \in Q_{ji}}\left\{\psi_{if_{ji}}^{h_q}\right\} - \min\limits_{i \in H, f_{ji} \in Q_{ji}}\left\{\psi_{if_{ji}}^{h_q}\right\}}, i \in H, f_{ji} \in Q_{ji} \tag{18}$$

Based on the above analysis, Algorithm 3 for calculating satisfactions of enterprise ∂_i to candidates under position h_q is displayed as follows:

Algorithm 3

Input: Evaluation vectors $U_i^{h_q} = \left(u_{i1}^{h_q}, \ldots, u_{in_{ji}}^{h_q}\right)$ and $Z_{f_{ji}}^{h_q} = \left(z_{1f_{ji}}^{h_q}, \ldots, z_{mf_{ji}}^{h_q}\right)$, and expected vector $E_\partial^{h_q} = \left(e_1^{h_q}, \ldots, e_m^{h_q}\right)$

Output: Satisfaction vector $W_i^{h_q} = \left(\alpha_{i1}^{h_q}, \ldots, \alpha_{in_{ji}}^{h_q}\right)$

Begin

Step 3.1: Calculate expected profit and loss value vector $R_i^{h_q} = \left(\gamma_{i1}^{h_q}, \ldots, \gamma_{in_{ji}}^{h_q}\right)$ by Eqs. (12) and (13);

Step 3.2: Calculate expected profit and loss value vector $T_i^{h_q} = \left(\varphi_{i1}^{h_q}, \ldots, \varphi_{in_{ji}}^{h_q}\right)$ by Eqs. (14) and (15);

Step 3.3: Construct comprehensive profit and loss value vector $G_i^{h_q} = \left(\Phi_{i1}^{h_q}, \ldots, \Phi_{in_{ji}}^{h_q}\right)$ by Eq. (16);

Step 3.4: Structure prospect value vector $K_i^{h_q} = \left(\psi_{i1}^{h_q}, \ldots, \psi_{in_{ji}}^{h_q}\right)$ by Eq. (17);

Step 3.5: Obtain satisfaction vectors $W_i^{h_q} = \left(\alpha_{i1}^{h_q}, \ldots, \alpha_{in_{ji}}^{h_q}\right)$ by Eq. (18).

End

5.2.2. Calculation of satisfactions of candidates considering the expected reference point

In this section, the satisfactions of candidates are calculated considering the expectations. First, the expected values of candidates are taken as the expected reference points. Second, the distances between the actual values and reference points are calculated to obtain the expected profit and loss values. Then, the satisfactions of candidates are calculated by considering risk attitudes and using standardization. The calculation of the satisfactions of candidates is given as follows.

First, for candidates, the distance $d\left(z_{if_{ji}}^{h_q}, e_{f_{ji}}^{h_q}\right)$ between the actual value $z_{if_{ji}}^{h_q}$ and the expected reference point $e_{f_{ji}}^{h_q}$ of candidate $A_{h_q f_{ji}}^{\delta_{ji}^i \leftrightarrow \partial_i}$ to enterprise ∂_i under position h_q is calculated as:

$$d\left(z_{if_{ji}}^{h_q}, e_{f_{ji}}^{h_q}\right) = \sum_{x=1}^{\left|z_{if_{ji}}^{h_q}\right|} \sum_{y=1}^{\left|e_{f_{ji}}^{h_q}\right|} \left(p_{z_{if_{ji}}^{h_q}}^{x} \times p_{e_{f_{ji}}^{h_q}}^{y}\right) \left| g\left(L_{z_{if_{ji}}^{h_q}}^{x}\right) - g\left(L_{e_{f_{ji}}^{h_q}}^{y}\right) \right| \tag{19}$$

Let $\gamma_{if_{ji}}^{h_q}\left(z_{if_{ji}}^{h_q}, e_{f_{ji}}^{h_q}\right)$ be the profit and loss value of candidate $A_{h_q f_{ji}}^{\delta_{ji}^i \leftrightarrow \partial_i}$ to enterprise ∂_i under position h_q, then, $\gamma_{if_{ji}}^{h_q}\left(z_{if_{ji}}^{h_q}, e_{f_{ji}}^{h_q}\right)$ is calculated as:

$$\gamma_{if_{ji}}^{h_q}\left(z_{if_{ji}}^{h_q}, e_{f_{ji}}^{h_q}\right) = \begin{cases} d\left(z_{if_{ji}}^{h_q}, e_{f_{ji}}^{h_q}\right), & z_{if_{ji}}^{h_q} \geq e_{f_{ji}}^{h_q} \\ -d\left(z_{if_{ji}}^{h_q}, e_{f_{ji}}^{h_q}\right), & z_{if_{ji}}^{h_q} < e_{f_{ji}}^{h_q} \end{cases} \tag{20}$$

In Eq. (20), when $\gamma_{if_{ji}}^{h_q}$ is positive, it is profit; and when it is negative, it is loss. Then, the expected profit and loss value vectors $R_{f_{ji}}^{h_q} = \left(\gamma_{1f_{ji}}^{h_q}, \ldots, \gamma_{mf_{ji}}^{h_q}\right)$ $(f_{ji} = 1, \ldots, n_{ji};\ q = 1, \ldots, l)$ of candidates to enterprises under all positions are constructed by Eq. (20).

Considering the different risk attitudes of candidates towards the profit and loss, the prospect value $\psi_{if_{ji}}^{h_q}\left(z_{if_{ji}}^{h_q}, e_{f_{ji}}^{h_q}\right)$ of candidate $A_{h_q f_{ji}}^{\delta_{ji}^i \leftrightarrow \partial_i}$ to enterprise ∂_i under position h_q is calculated as:

$$\psi_{if_{ji}}^{h_q}\left(z_{if_{ji}}^{h_q}, e_{if_{ji}}^{h_q}\right) = \begin{cases} \left(\gamma_{if_{ji}}^{h_q}\left(z_{if_{ji}}^{h_q}, e_{if_{ji}}^{h_q}\right)\right)^a z_{if_{ji}}^{h_q} \geq e_{if_{ji}}^{h_q}; \\ -\lambda\left(-\gamma_{if_{ji}}^{h_q}\left(z_{if_{ji}}^{h_q}, e_{if_{ji}}^{h_q}\right)\right)^b z_{if_{ji}}^{h_q} < e_{if_{ji}}^{h_q}; \end{cases} \tag{21}$$

By Eq. (21), the prospect value vectors $K_{f_{ji}}^{h_q} = \left(\psi_{1f_{ji}}^{h_q}, \ldots, \psi_{mf_{ji}}^{h_q}\right)$ ($f_{ji} = 1, \ldots, n_{ji}$; $q = 1, \ldots, l$) of candidate $A_{h_q f_{ji}}^{\delta_{ji}^{i} \leftrightarrow \partial_i}$ to enterprises under all positions are constructed.

Furthermore, according to the prospect value vector $K_{f_{ji}}^{h_q} = \left(\psi_{1f_{ji}}^{h_q}, \ldots, \psi_{mf_{ji}}^{h_q}\right)$ ($f_{ji} = 1, \ldots, n_{ji}$; $q = 1, \ldots, l$), the satisfaction vectors $W_{f_{ji}}^{h_q} = \left(\beta_{1f_{ji}}^{h_q}, \ldots, \beta_{mf_{ji}}^{h_q}\right)$ ($f_{ji} = 1, \ldots, n_{ji}$; $q = 1, \ldots, l$) of candidate $A_{h_q f_{ji}}^{\delta_{ji}^{i} \leftrightarrow \partial_i}$ to enterprises under all positions are constructed by standardization, where $\beta_{if_{ji}}^{h_q}$ ($\beta_{if_{ji}}^{h_q} \in [0,1]$) is calculated as:

$$\beta_{if_{ji}}^{h_q} = \frac{\psi_{if_{ji}}^{h_q} - \min\limits_{i \in H, f_{ji} \in Q_{ji}} \left\{\psi_{if_{ji}}^{h_q}\right\} + \frac{1}{n_{ji}} \max\limits_{i \in H, f_{ji} \in Q_{ji}} \left\{\psi_{if_{ji}}^{h_q}\right\}}{\frac{n_{ji}+1}{n_{ji}} \max\limits_{i \in H, f_{ji} \in Q_{ji}} \left\{\psi_{if_{ji}}^{h_q}\right\} - \min\limits_{i \in H, f_{ji} \in Q_{ji}} \left\{\psi_{if_{ji}}^{h_q}\right\}}, i \in H, f_{ji} \in Q_{ji} \quad (22)$$

Based on the above analysis, Algorithm 4 for calculating satisfactions of candidate $A_{h_q f_{ji}}^{\delta_{ji}^{i} \leftrightarrow \partial_i}$ to enterprises under position h_q is displayed as follows:

$$(M-3)\begin{cases} \mathrm{Max}L_1 = \sum\limits_{i=1}^{m} \sum\limits_{f_{ji}=1}^{f_q} \alpha_{if_{ji}}^{h_q} x_{if_{ji}}^{h_q} \\ \mathrm{Max}L_2 = \sum\limits_{i=1}^{m} \sum\limits_{f_{ji}=1}^{f_q} \beta_{if_{ji}}^{h_q} x_{if_{ji}}^{h_q} \\ s.t. \sum\limits_{f_{ji}=1}^{f_q} x_{if_{ji}}^{h_q} \le t_{qi},\ i \in H \\ \sum\limits_{i=1}^{m} x_{if_{ji}}^{h_q} \le 1, f_{ji} \in Q_{ji} \\ x_{if_{ji}}^{h_q} \in \{0,1\}, i \in H; f_{ji} \in Q_{ji} \end{cases}$$

where MaxL_1 means to maximize the sum of satisfactions of enterprisers, and MaxL_2 means to maximize the sum of satisfactions of candidates.

For objective functions L_1 and L_2, considering that $\alpha_{if_{ji}}^{h_q}$ and $\beta_{if_{ji}}^{h_q}$ are of the same dimension, model (M−3) can be transformed into a single objective model (M−4) by the combination satisfaction method (Li et al., 2021):

$$(\mathrm{M}-4)\begin{cases} \mathrm{Max}L = \sum\limits_{i=1}^{m} \sum\limits_{f_{ji}=1}^{f_q} \left[\widehat{\omega}_1 \left(\alpha_{if_{ji}}^{h_q} + \beta_{if_{ji}}^{h_q}\right) + \widehat{\omega}_2 \sqrt{\alpha_{if_{ji}}^{h_q} \times \beta_{if_{ji}}^{h_q}}\right] x_{if_{ji}}^{h_q} = \sum\limits_{i=1}^{m} \sum\limits_{f_{ji}=1}^{f_q} \eta_{if_{ji}}^{h_q} x_{if_{ji}}^{h_q} \\ s.t. \sum\limits_{f_{ji}=1}^{f_q} x_{if_{ji}}^{h_q} \le t_{qi}, i \in H \\ \sum\limits_{i=1}^{m} x_{if_{ji}}^{h_q} \le 1, f_{ji} \in Q_{ji} \\ x_{if_{ji}}^{h_q} \in \{0,1\}, i \in H; f_{ji} \in Q_{ji} \end{cases}$$

Algorithm 4

Input: Evaluation vector $Z_{f_{ji}}^{h_q} = \left(z_{1f_{ji}}^{h_q}, \ldots, z_{mf_{ji}}^{h_q}\right)$ and expected vector $E_A^{h_q} = \left(e_1^{h_q}, \ldots, e_{n_{ji}}^{h_q}\right)$

Output: Satisfaction vector $W_{f_{ji}}^{h_q} = \left(\beta_{1f_{ji}}^{h_q}, \ldots, \beta_{mf_{ji}}^{h_q}\right)$

Begin

Step 4.1: Construct expected profit and loss value vector $R_{f_{ji}}^{h_q} = \left(\gamma_{1f_{ji}}^{h_q}, \ldots, \gamma_{mf_{ji}}^{h_q}\right)$ is obtained by Eqs. (19) and (20);

Step 4.2: Structure prospect value vector $K_{f_{ji}}^{h_q} = \left(\psi_{1f_{ji}}^{h_q}, \ldots, \psi_{mf_{ji}}^{h_q}\right)$ by Eq. (21);

Step 4.3: Obtain satisfaction vector $W_{f_{ji}}^{h_q} = \left(\beta_{1f_{ji}}^{h_q}, \ldots, \beta_{mf_{ji}}^{h_q}\right)$ by Eq. (22).

End

5.2.3. Construction of one-to-many two-sided matching model between enterprises and candidates under multiple positions

First, introducing the two-sided matching vectors $X_{if_{ji}}^{h_q} = \left(x_{i1}^{h_q}, \ldots, x_{in_{ji}}^{h_q}\right)$ ($i = 1, \ldots, m$; $q = 1, \ldots, l$) of enterprises and candidates, where $x_{if_{ji}}^{h_q} = \begin{cases} 1, \wp(\partial_i) = A_{h_q f_{ji}}^{\delta_{ji}^{i} \leftrightarrow \partial_i} \\ 0, \wp(\partial_i) \ne A_{h_q f_{ji}}^{\delta_{ji}^{i} \leftrightarrow \partial_i} \end{cases}$. Then, according to the two-sided matching vectors $X_{if_{ji}}^{h_q} = \left(x_{i1}^{h_q}, \ldots, x_{in_{ji}}^{h_q}\right)$ and satisfaction vectors $W_i^{h_q} = \left(\alpha_{i1}^{h_q}, \ldots, \alpha_{in_{ji}}^{h_q}\right)$ and $W_{f_{ji}}^{h_q} = \left(\beta_{1f_{ji}}^{h_q}, \ldots, \beta_{mf_{ji}}^{h_q}\right)$ ($i = 1, \ldots, m$; $f_{ji} = 1, \ldots, n_{ji}$; $q = 1, \ldots, l$), the two-sided matching model (M−3) under one-to-many matching conditions under position h_q is constructed as follows:

where $\widehat{\omega}_1$ and $\widehat{\omega}_2$ represent the weights of the objective functions L_1 and L_2 respectively, $(0 \le \widehat{\omega}_1, \widehat{\omega}_2 \le 1, \widehat{\omega}_1 + \widehat{\omega}_2 = 1)$. $\widehat{\omega}_1 = 0$ represents that the decision-making goal satisfies the consistency; $\widehat{\omega}_2 = 0$ represents that the decision-making goal satisfies the complementarity. Moreover, considering that there may have some missing elements in the satisfaction matrix $\left[\eta_{if_{ji}}^{h_q}\right]_{m \times f_q}$, the missing elements as set as $-\varsigma$ to ensure the reliability of the results, where ς is a large positive number. Then, By using the mathematical software such as Lingo to solve model (M−4), the optimal one-to-many matching schemes $\wp_1^*, \wp_2^*, \ldots, \wp_l^*$ under all position are obtained.

5.2.4. Construction of one-to-one stable matching model between enterprises and candidates under multi-positions

To construct the stable matching model, the one-to-many two-sided matching problem is transformed into one-to-one two-sided matching problem, which is presented as follows: First, enterprise ∂_i is regarded as $t_i^{h_q}$ virtual subjects with the same preference, that is $\left\{\partial_1^i, \ldots, \partial_{t_i^{h_q}}^i\right\}$. Thus, $\partial = \{\partial_1, \ldots, \partial_m\}$ is transformed into the virtual subject set $\overline{\partial} = \left\{\partial_1^1, \ldots, \partial_{t_1^{h_q}}^1, \partial_1^2, \ldots, \partial_{t_2^{h_q}}^2, \ldots, \partial_1^m, \ldots, \partial_{t_m^{h_q}}^m\right\}$ of enterprises. To facilitate analysis, $\overline{\partial}$ is donated as $\widehat{\partial} = \{\widehat{\partial}_1, \ldots, \widehat{\partial}_d\}$, where $\widehat{\partial}_c$ is the $c-$th enterprise, $c \in C = \{1, 2, \ldots, d\}$, $d = \sum_{i=1}^{m} t_i^{h_q}$. Then, the satisfaction vectors $W_i^{h_q} = \left(\alpha_{i1}^{h_q}, \ldots, \alpha_{in_{ji}}^{h_q}\right)$ and $W_{f_{ji}}^{h_q} = \left(\beta_{1f_{ji}}^{h_q}, \ldots, \beta_{mf_{ji}}^{h_q}\right)$ are transformed into the expanded satisfaction vectors $\widehat{W}_c^{h_q} = \left(\alpha_{c1}^{h_q}, \ldots, \alpha_{cn_{ji}}^{h_q}\right)$ and $\widehat{W}_{f_{ji}}^{h_q} = \left(\beta_{1f_{ji}}^{h_q}, \ldots, \beta_{df_{ji}}^{h_q}\right)$.

Furthermore, the two-sided matching vectors $X^{h_q}_{cf_{j_i}} = \left(x^{h_q}_{c1}, \ldots, x^{h_q}_{cn_{j_i}}\right)$ $(c = 1, \ldots, d;\ q = 1, \ldots, l)$ of enterprises and candidates are introduced, where $x^{h_q}_{if_{j_i}} = \begin{cases} 1, \Re(\widehat{\partial}_c) = A^{\delta^i_{j_i} \leftrightarrow \partial_i}_{h_q f_{j_i}} \\ 0, \Re(\widehat{\partial}_c) \neq A^{\delta^i_{j_i} \leftrightarrow \partial_i}_{h_q f_{j_i}} \end{cases}$. Considering the expanded satisfactions may be equal, such as $\alpha^{h_q}_{ck_{j_i}} = \alpha^{h_q}_{cf_{j_i}}$ and $\beta^{h_q}_{lf_{j_i}} = \beta^{h_q}_{cf_{j_i}}$, stable matching constraints in Definition 10 are not suitable to the above situation. Thus, two more constraints are given to ensure the stability. Namely, if $\Re(\widehat{\partial}_c) \neq A^{\delta^i_{j_i} \leftrightarrow \partial_i}_{h_q f_{j_i}}$, then at least one of the following conditions should be satisfied: i) If $\Re(\widehat{\partial}_c) = A^{\delta^i_{j_i} \leftrightarrow \partial_i}_{h_q, k_{j_i}}$, then $A^{\delta^i_{j_i} \leftrightarrow \partial_i}_{h_q, k_{j_i}} \in A^{\delta^i_{j_i} \leftrightarrow \partial_i}_{h_q}$ and $\alpha^{h_q}_{ck_{j_i}} \geq \alpha^{h_q}_{cf_{j_i}}$. ii) If $\Re(A^{\delta^i_{j_i} \leftrightarrow \partial_i}_{h_q f_{j_i}}) = \widehat{\partial}_l$, then $\widehat{\partial}_l \in \widehat{\partial}$ and $\beta^{h_q}_{lf_{j_i}} \geq \beta^{h_q}_{cf_{j_i}}$. Otherwise, it constitutes an unstable matching pair.

According to the two-sided matching vectors $X^{h_q}_{cf_{j_i}} = \left(x^{h_q}_{c1}, \ldots, x^{h_q}_{cn_{j_i}}\right)$ and the expanded satisfaction vectors $\widehat{W}^{h_q}_c = \left(\alpha^{h_q}_{c1}, \ldots, \alpha^{h_q}_{cn_{j_i}}\right)$ and $\widehat{W}^{h_q}_{f_{j_i}} = \left(\beta^{h_q}_{1f_{j_i}}, \ldots, \beta^{h_q}_{df_{j_i}}\right)$ $(c = 1, \ldots, d;\ f_{j_i} = 1, \ldots, n_{j_i};\ q = 1, \ldots, l)$, the stable two-sided matching model (M−5) under position h_q is developed as follows:

$$(\mathrm{M}-5)\begin{cases} \mathrm{Max}G_1 = \sum\limits_{c=1}^{d}\sum\limits_{f_{j_i}=1}^{f_q} \alpha^{h_q}_{cf_{j_i}} x^{h_q}_{cf_{j_i}} \\ \mathrm{Max}G_2 = \sum\limits_{c=1}^{d}\sum\limits_{f_{j_i}=1}^{f_q} \beta^{h_q}_{cf_{j_i}} x^{h_q}_{cf_{j_i}} \\ s.t. \sum\limits_{f_{j_i}=1}^{f_q} x^{h_q}_{cf_{j_i}} \leq 1, c \in C \\ \sum\limits_{c=1}^{d} x^{h_q}_{cf_{j_i}} \leq 1, f_{j_i} \in Q_{j_i} \\ x^{h_q}_{cf_{j_i}} + \sum\limits_{\alpha^{h_q}_{ck_{j_i}} \geq \alpha^{h_q}_{cf_{j_i}}} x^{h_q}_{ck_{j_i}} + \sum\limits_{\beta^{h_q}_{lf_{j_i}} \geq \beta^{h_q}_{cf_{j_i}}} x^{h_q}_{lf_{j_i}} \geq 1, c, l \in C; f_{j_i}, k_{j_i} \in Q_{j_i} \\ x^{h_q}_{cf_{j_i}} \in \{0,1\}, c \in C; f_{j_i} \in Q_{j_i} \end{cases}$$

where $\mathrm{Max}G_1$ means to maximize the sum of satisfactions of enterprisers, and $\mathrm{Max}G_2$ means to maximize the sum of satisfactions of candidates.

For the objective functions G_1 and G_2, considering that $\alpha^{h_q}_{if_{j_i}}$ and $\beta^{h_q}_{if_{j_i}}$ are of the same dimension, model (M−5) can be transformed into a single objective model (M−6) by the combination satisfaction method:

$$(\mathrm{M}\text{ - }6)\begin{cases} \mathrm{Max}G = \sum\limits_{c=1}^{d}\sum\limits_{f_{j_i}=1}^{f_q}\left[w_1\left(\alpha^{h_q}_{cf_{j_i}} + \beta^{h_q}_{cf_{j_i}}\right) + w_2\sqrt{\alpha^{h_q}_{cf_{j_i}} \times \beta^{h_q}_{cf_{j_i}}}\right] x^{h_q}_{cf_{j_i}} = \sum\limits_{i=1}^{m}\sum\limits_{f_{j_i}=1}^{f_q} \zeta^{h_q}_{if_{j_i}} x^{h_q}_{if_{j_i}} \\ s.t. \sum\limits_{f_{j_i}=1}^{f_q} x^{h_q}_{cf_{j_i}} \leq 1, c \in C \\ \sum\limits_{c=1}^{d} x^{h_q}_{cf_{j_i}} \leq 1, f_{j_i} \in Q_{j_i} \\ x^{h_q}_{cf_{j_i}} + \sum\limits_{\alpha^{h_q}_{ck_{j_i}} \geq \alpha^{h_q}_{cf_{j_i}}} x^{h_q}_{ck_{j_i}} + \sum\limits_{\beta^{h_q}_{lf_{j_i}} \geq \beta^{h_q}_{cf_{j_i}}} x^{h_q}_{lf_{j_i}} \geq 1, c, l \in C; f_{j_i}, k_{j_i} \in Q_{j_i} \\ x^{h_q}_{cf_{j_i}} \in \{0,1\}, c \in C; f_{j_i} \in Q_{j_i} \end{cases}$$

where w_1 and w_2 represent weighs of objective functions G_1 and G_2 respectively, $(0 \leq w_1, w_2 \leq 1, w_1 + w_2 = 1)$. $w_1 = 0$ represents that the decision-making goal satisfies the consistency; $w_2 = 0$ represents that the decision-making goal satisfies the complementarity. Moreover, considering that there may be some missing elements in the satisfaction matrix $\left[\zeta^{h_q}_{if_{j_i}}\right]_{m \times f_q}$, the missing elements are set as $-\varsigma$ to ensure the reliability of the results, where ς is a large positive number. Then, By using mathematical software such as Lingo to solve model (M−6), the optimal stable matching schemes $\wp^*_1, \wp^*_2, \ldots, \wp^*_l$ under all positions are obtained.

5.3. Steps of multi-position two-phase person-position matching decision-making

The proposed multi-position two-phase person-position matching decision-making is divided into two phases. Matching mechanism of the first phase is the many-to-many two-sided matching decision-making between enterprises and headhunting companies, and the specific steps are given as follows.

Matching mechanism for the first phase:

Step 5.1: According to the probabilistic linguistic evaluation matrix $U^{\partial} = \left[u^{\partial}_{ij}\right]_{m \times n}$ and the probabilistic linguistic expected vector $E^{\partial} = \left(e^{\partial}_1, \ldots, e^{\partial}_m\right)$, the satisfaction matrix $W^{\partial} = \left[\alpha^{\partial}_{ij}\right]_{m \times n}$ of enterprises to headhunting companies is constructed by Algorithm 1.

Step 5.2: According to the probabilistic linguistic evaluation matrix $V^{\delta} = \left[v^{\delta}_{ij}\right]_{m \times n}$ and the probabilistic linguistic expected vector $E^{\delta} = \left(e^{\delta}_1, \ldots, e^{\delta}_n\right)$, the satisfaction matrix $W^{\delta} = \left[\beta^{\delta}_{ij}\right]_{m \times n}$ of headhunting companies to enterprises is constructed by Algorithm 2.

Step 5.3: According to the two-sided matching matrix $X = \left[x_{ij}\right]_{m \times n}$ and satisfaction matrices $W^{\partial} = \left[\alpha^{\partial}_{ij}\right]_{m \times n}$ and $W^{\delta} = \left[\beta^{\delta}_{ij}\right]_{m \times n}$, a many-to-many two-sided matching model (M−1) between enterprises and headhunting companies is established under the many-to-many matching constraints.

Step 5.4: By using the linear weighting method, model (M−1) is transformed into a single objective model (M−2), and the optimal many-to-many matching scheme $\Im^*$ between enterprises and headhunting companies is obtained by solving model (M−2).

Matching mechanism of the second phase is multiple one-to-many two-sided matching between enterprises and candidates, and the specific steps are given as follows.

Matching mechanism for the second phase:

Step 6.1: According to probabilistic linguistic evaluation vectors $U^{h_q}_i = \left(u^{h_q}_{i1}, \ldots, u^{h_q}_{in_{j_i}}\right)$ $(i = 1, \ldots, m)$ and expected vector $E^{h_q}_{\partial} = \left(e^{h_q}_1, \ldots, e^{h_q}_m\right)$

of enterprises, and $V_{j_i}^{h_q} = \left(v_{j_i1}^{h_q}, ..., v_{j_in_{j_i}}^{h_q}\right)$ ($j_i = 1, ..., m_i$) of headhunting companies under positions h_q, satisfaction vectors $W_i^{h_q} = \left(\alpha_{i1}^{h_q}, ..., \alpha_{in_{j_i}}^{h_q}\right)$ ($i = 1, ..., m$) of enterprises to candidates under position h_q are constructed by Algorithm 3.

Step 6.2: According to probabilistic linguistic evaluation vectors $U_i^{h_q} = \left(u_{i1}^{h_q}, ..., u_{in_{j_i}}^{h_q}\right)$ ($i = 1, ..., m$) of enterprises and $V_{j_i}^{h_q} = \left(v_{j_i1}^{h_q}, ..., v_{j_in_{j_i}}^{h_q}\right)$ ($j_i = 1, ..., m_i$) of headhunting companies under positions h_q, satisfaction vectors $W_{f_{j_i}}^{h_q} = \left(\beta_{1f_{j_i}}^{h_q}, ..., \beta_{mf_{j_i}}^{h_q}\right)$ ($f_{j_i} = 1, ..., m_i$) of candidates to enterprises under positions h_q are constructed by Algorithm 4.

Step 6.3: According to two-sided matching vectors $X_{if_{j_i}}^{h_q} = \left(x_{i1}^{h_q}, ..., x_{in_{j_i}}^{h_q}\right)$ ($i = 1, ..., m$) and satisfaction vectors $W_i^{h_q} = \left(\alpha_{i1}^{h_q}, ..., \alpha_{in_{j_i}}^{h_q}\right)$ ($i = 1, ..., m$) and $W_{f_{j_i}}^{h_q} = \left(\beta_{1f_{j_i}}^{h_q}, ..., \beta_{mf_{j_i}}^{h_q}\right)$ ($f_{j_i} = 1, ..., m_i$), a one-to-many two-sided matching model (M−3) between enterprises and candidates is established.

Step 6.4: By using the combination satisfaction method, model (M−3) is transformed into a single-objective model (M−4), and the optimal one-to-many matching schemes $\wp_1^*, \wp_2^*, ..., \wp_l^*$ between enterprises and candidates under positions $h_1, h_2, ..., h_l$ are obtained.

Step 6.5: By converting the enterprise set $\partial = \{\partial_1, ..., \partial_m\}$ into the virtual subject set $\widehat{\partial} = \{\widehat{\partial}_1, ..., \widehat{\partial}_d\}$ of enterprises, satisfaction vectors $W_i^{h_q} = \left(\alpha_{i1}^{h_q}, ..., \alpha_{in_{j_i}}^{h_q}\right)$ ($i = 1, ..., m$) and $W_{f_{j_i}}^{h_q} = \left(\beta_{1f_{j_i}}^{h_q}, ..., \beta_{mf_{j_i}}^{h_q}\right)$ ($f_{j_i} = 1, ..., m_i$) are converted into an extended satisfaction vectors $W_c^{h_q} = \left(\alpha_{c1}^{h_q}, ..., \alpha_{cn_{j_i}}^{h_q}\right)$ ($c = 1, ..., d$) and $W_{f_{j_i}}^{h_q} = \left(\beta_{1f_{j_i}}^{h_q}, ..., \beta_{df_{j_i}}^{h_q}\right)$ ($f_{j_i} = 1, ..., m_i$).

Step 6.6: According to two-sided matching vectors $X_{cf_{j_i}}^{h_q} = \left(x_{c1}^{h_q}, ..., x_{cn_{j_i}}^{h_q}\right)$ ($c = 1, ..., d$) and the extended satisfaction vectors $W_c^{h_q} = \left(\alpha_{c1}^{h_q}, ..., \alpha_{cn_{j_i}}^{h_q}\right)$ ($c = 1, ..., d$) and $W_{f_{j_i}}^{h_q} = \left(\beta_{1f_{j_i}}^{h_q}, ..., \beta_{df_{j_i}}^{h_q}\right)$ ($f_{j_i} = 1, ..., m_i$), a one-to-one two-sided matching model (M−5) between enterprises and candidates is established.

Step 6.7: By using the combination satisfaction method, model (M−5) is transformed into a single-objective model (M−6), and the optimal one-to-many matching schemes $\wp_1^{**}, \wp_2^{**}, ..., \wp_l^{**}$ between enterprises and candidates under positions $h_1, h_2, ..., h_l$ are obtained.

6. Example analysis

In this section, to verify the effectiveness and feasibility of the proposed method, a realistic example of person-position matching with intermediary participation is considered.

Assuming that a recruitment is carried out by enterprises and headhunting companies. There are four enterprises $\partial = \{\partial_1, \partial_2, \partial_3, \partial_4\}$ that need to recruit candidates for three vacant positions (h_1, h_2, h_3) and five headhunting companies $\delta = \{\delta_1, \delta_2, \delta_3, \delta_4, \delta_5\}$ want to undertake the recruitment business of enterprises. Considering that numbers of candidates that can be matched with enterprises under each position are limited, it is assumed that enterprises $\partial_1, \partial_2, \partial_3$ need two candidates and ∂_4 needs one candidate under position h_1; enterprises ∂_1, ∂_2 need two candidates and ∂_3, ∂_4 need one candidate under position h_2; enterprises ∂_1, ∂_3 need two candidates and ∂_2, ∂_4 need one candidate under position h_3. To obtain the appropriate matching schemes between enterprises and headhunting companies and between enterprises and candidates, the proposed method is used and the decision-making process is designed as two phases. Then, the optimal person-position matching schemes are made by the decision-maker of the recruitment based on preferences of tripartite subjects.

6.1. Probabilistic linguistic evaluation information of first phase

In the first phase, it is assumed that the maximum numbers of headhunting companies that enterprises can be matched are $m_1 = m_2 = m_3 = m_4 = 2$, and the maximum numbers of enterprises that headhunting companies can be matched are $n_1 = n_2 = n_3 = 2$ and $n_4 = n_5 = 1$. Enterprises and headhunting companies consider the ordered linguistic term set S={s_0: poor; s_1: middle-lower; s_2: middle; s_3: middle-upper; s_4: good }. Probabilistic linguistic evaluation matrix $U^\partial = \left[U_{ij}^\partial\right]_{4\times5}$ and expected vector $E^\partial = \left(E_1^\partial, ..., E_4^\partial\right)$ of enterprises and headhunting companies are shown in Table 2; probabilistic linguistic evaluation matrix $V^\delta = \left[V_{ij}^\delta\right]_{4\times5}$ and expected vector $E^\delta = \left(E_1^\delta, ..., E_5^\delta\right)$ of headhunting companies are shown in Table 3.

6.2. Solution process of first phase

Step 5.1: According to the probabilistic linguistic evaluation matrix $U^\partial = \left[u_{ij}^\partial\right]_{m\times n}$ and the probabilistic linguistic expected vector $E^\partial = \left(e_1^\partial, ..., e_m^\partial\right)$, the satisfaction matrix $W^\partial = \left[\alpha_{ij}^\partial\right]_{m\times n}$ of enterprises to headhunting companies is constructed by Algorithm 1, as shown in Table 4.

Step 5.2: According to the probabilistic linguistic evaluation matrix $V^\delta = \left[v_{ij}^\delta\right]_{m\times n}$ and the probabilistic linguistic expected vector $E^\delta = \left(e_1^\delta, ..., e_n^\delta\right)$, the satisfaction matrix $W^\delta = \left[\beta_{ij}^\delta\right]_{m\times n}$ of headhunting companies to enterprises is constructed by Algorithm 2, as shown in Table 5.

Step 5.3: According to the two-sided matching matrix $X = \left[x_{ij}\right]_{m\times n}$ and the satisfaction matrices $W^\partial = \left[\alpha_{ij}^\partial\right]_{m\times n}$ and $W^\delta = \left[\beta_{ij}^\delta\right]_{m\times n}$, a many-to-many two-sided matching model (M−1) between enterprises and headhunting companies is constructed under the many-to-many matching constraints.

Step 5.4: Let $\omega_1 = \omega_2 = 0.5$, then model (M−1) is transformed into a single objective model (M−2) by using the linear weighting method, and the optimal many-to-many matching matrix $X = \left[x_{ij}^*\right]_{m\times n}$ between enterprises and headhunting companies can be obtained by solving the model (M−2), as shown in Table 6.

It can be seen from Table 11 that the optimal matching scheme between enterprise and headhunting companies is $\wp^* = \{\partial_1 \leftrightarrow \{\delta_2, \delta_3\}, \partial_2 \leftrightarrow \{\delta_1, \delta_3\}, \partial_3 \leftrightarrow \{\delta_1, \delta_5\}, \partial_4 \leftrightarrow \{\delta_2, \delta_4\}\}$.

Table 2
Evaluation and expected information of enterprises to headhunting companies.

U_{ij}^∂	δ_1	δ_2	δ_3	δ_4	δ_5	E_i^∂
∂_1	$\{s_2(0.4), s_3(0.6)\}$	$\{s_3(0.5), s_4(0.5)\}$	$\{s_2(0.4), s_3(0.6)\}$	$\{s_0(0.2), s_2(0.4), s_3(0.4)\}$	$\{s_0(0.3), s_2(0.4), s_3(0.3)\}$	$\{s_1(0.3), s_3(0.7)\}$
∂_2	$\{s_1(0.3), s_2(0.4), s_4(0.3)\}$	$\{s_2(0.2), s_3(0.5), s_4(0.3)\}$	$\{s_2(0.4), s_3(0.6)\}$	$\{s_0(0.4), s_3(0.4), s_4(0.2)\}$	$\{s_0(0.3), s_1(0.4), s_3(0.3)\}$	$\{s_2(0.4), s_3(0.6)\}$
∂_3	$\{s_2(0.5), s_3(0.5)\}$	$\{s_2(0.6), s_3(0.2)\}$	$\{s_0(0.2), s_2(0.4), s_3(0.4)\}$	$\{s_2(0.4), s_3(0.6)\}$	$\{s_0(0.2), s_3(0.3), s_4(0.5)\}$	$\{s_2(0.6), s_4(0.4)\}$
∂_4	$\{s_1(0.6), s_2(0.4)\}$	$\{s_1(0.2), s_2(0.5), s_3(0.3)\}$	$\{s_2(0.2), s_3(0.8)\}$	$\{s_2(0.6), s_3(0.4)\}$	$\{s_2(0.4), s_3(0.6)\}$	$\{s_2(0.2), s_4(0.8)\}$

Table 3
Evaluation and expected information of headhunting companies to enterprises.

V_{ij}^{δ}	δ_1	δ_2	δ_3	δ_4	δ_5
∂_1	$\{s_1(0.3), s_3(0.7)\}$	$\{s_0(0.2), s_2(0.4), s_3(0.4)\}$	$\{s_1(0.2), s_2(0.5), s_3(0.3)\}$	$\{s_0(0.2), s_2(0.4), s_3(0.4)\}$	$\{s_0(0.2), s_2(0.4), s_3(0.4)\}$
∂_2	$\{s_1(0.2), s_2(0.5), s_3(0.3)\}$	$\{s_1(0.3), s_2(0.4), s_3(0.3)\}$	$\{s_2(0.4), s_3(0.6)\}$	$\{s_1(0.3), s_2(0.4), s_3(0.3)\}$	$\{s_1(0.3), s_2(0.4), s_3(0.3)\}$
∂_3	$\{s_2(0.8), s_3(0.2)\}$	$\{s_2(0.5), s_3(0.5)\}$	$\{s_0(0.2), s_2(0.4), s_3(0.4)\}$	$\{s_2(0.4), s_3(0.6)\}$	$\{s_2(0.8), s_3(0.2)\}$
∂_4	$\{s_1(0.3), s_2(0.4), s_3(0.3)\}$	$\{s_1(0.2), s_2(0.5), s_3(0.3)\}$	$\{s_2(0.5), s_3(0.5)\}$	$\{s_2(0.6), s_3(0.4)\}$	$\{s_0(0.2), s_2(0.3), s_3(0.5)\}$
E_j^{δ}	$\{s_1(0.5), s_3(0.5)\}$	$\{s_1(0.4), s_3(0.6)\}$	$\{s_3(0.7), s_4(0.3)\}$	$\{s_2(0.5), s_4(0.5)\}$	$\{s_2(0.4), s_4(0.6)\}$

Table 4
Satisfaction matrix $W^{\partial} = \left[\alpha_{ij}^{\partial}\right]_{m\times n}$ of enterprises to headhunting companies.

	δ_1	δ_2	δ_3	δ_4	δ_5
∂_1	0.7980	1.0000	0.7980	0.7980	0.7980
∂_2	0.8233	0.9017	0.7390	0.8634	0.5680
∂_3	0.5309	0.6461	0.5680	0.4947	0.8233
∂_4	0.0652	0.4947	0.1598	0.4243	0.2894

Table 5
Satisfaction matrix $W^{\delta} = \left[\beta_{ij}^{\delta}\right]_{m\times n}$ of headhunting companies to enterprises.

	δ_1	δ_2	δ_3	δ_4	δ_5
∂_1	0.7461	0.8595	0.1248	0.4356	0.3816
∂_2	0.9140	0.8595	0.1000	0.5051	0.4631
∂_3	1.0000	0.8841	0.1747	0.3025	0.5480
∂_4	0.8841	0.8841	0.0508	0.4356	0.3025

Table 6
Optimal many-to-many matching matrix $X = \left[x_{ij}^{*}\right]_{m\times n}$ between enterprises and headhunting companies.

	δ_1	δ_2	δ_3	δ_4	δ_5
∂_1	0	1	1	0	0
∂_2	1	0	1	0	0
∂_3	1	0	0	0	1
∂_4	0	1	0	1	0

6.3. Probabilistic linguistic evaluation information of second phase

In the second phase, based on the optimal two-sided matching scheme $\wp^* = \{\partial_1 \leftrightarrow \{\delta_2, \delta_3\}, \partial_2 \leftrightarrow \{\delta_1, \delta_3\}, \partial_3 \leftrightarrow \{\delta_1, \delta_5\}, \partial_4 \leftrightarrow \{\delta_2, \delta_4\}\}$ between enterprises and headhunting companies of the first phase, headhunting companies, acting as the intermediaries, recommend candidates for enterprises under position h_1, as shown in Table 7.

Remark 2. Considering the limitation of space, preference information in other positions is omitted, this paper only shows the matching decision-making process under position h_1 in second phase, and the matching decision-making under other positions is similar to that under position h_1, which is also omitted.

Furthermore, the considered linguistic term set is S={s_0: extremely poor; s_1: very poor; s_2: poor; s_3: middle-lower; s_4: middle; s_5: middle-upper; s_6: good; s_7: very good; s_8: extremely good}. Assuming that $U_i^{h_1} = \left(u_{i1}^{h_1}, \ldots, u_{in_{j_i}}^{h_1}\right)$ $(i = 1, \ldots, 4)$ are probabilistic linguistic evaluation vectors and $E_{\partial}^{h_1} = \left(e_1^{h_1}, \ldots, e_4^{h_1}\right)$ is the probabilistic linguistic expected vector of enterprises under position h_1; $Z_{f_{j_i}}^{h_1} = \left(z_{1f_{j_i}}^{h_1}, \ldots, z_{mf_{j_i}}^{h_1}\right)$ $(f_{j_i} = 1, \ldots, n_{j_i})$ are probabilistic linguistic evaluation vectors and $E_A^{h_1} = \left(e_1^{h_1}, \ldots, e_{12}^{h_1}\right)$

Table 7
Candidates recommended by headhunting companies to enterprises under position h_1.

Position	Enterprise	Candidates under each position
h_1	∂_1	$A_{h_1}^{\delta_2\leftrightarrow\partial_1} = \{A_5, A_6\}, A_{h_1}^{\delta_3\leftrightarrow\partial_1} = \{A_7, A_8\}$
	∂_2	$A_{h_1}^{\delta_1\leftrightarrow\partial_2} = \{A_1, A_2, A_3, A_4\}, A_{h_1}^{\delta_3\leftrightarrow\partial_2} = \{A_7, A_8\}$
	∂_3	$A_{h_1}^{\delta_1\leftrightarrow\partial_3} = \{A_1, A_2, A_3, A_4\}, A_{h_1}^{\delta_5\leftrightarrow\partial_3} = \{A_{11}, A_{12}\}$
	∂_4	$A_{h_1}^{\delta_2\leftrightarrow\partial_4} = \{A_5, A_6\}, A_{h_1}^{\delta_4\leftrightarrow\partial_4} = \{A_9, A_{10}\}$

is the probabilistic linguistic expected vector of candidates under position h_1; $V_{j_i}^{h_1} = \left(v_{j_i1}^{h_1}, \ldots, v_{j_in_{j_i}}^{h_1}\right)$ $(j_i = 1, \ldots, m_i)$ are probabilistic linguistic evaluation vectors of headhunting companies under position h_1, as shown in Tables 8-10.

6.4. Solution process of second phase

Step 6.1: According to probabilistic linguistic evaluation vectors $U_i^{h_1} = \left(u_{i1}^{h_1}, \ldots, u_{in_{j_i}}^{h_1}\right)$ $(i = 1, \ldots, 4)$ and $V_{j_i}^{h_1} = \left(v_{j_i1}^{h_1}, \ldots, v_{j_in_{j_i}}^{h_1}\right)$ $(j_i = 1, \ldots, m_i)$, and probabilistic linguistic expected vector $E_{\partial}^{h_1} = \left(e_1^{h_1}, \ldots, e_m^{h_1}\right)$, satisfaction vectors $W_i^{h_1} = \left(\alpha_{i1}^{h_1}, \ldots, \alpha_{in_{j_i}}^{h_1}\right)$ $(i = 1, \ldots, 4)$ of enterprises to candidates under position h_1 are constructed by Algorithm 3, as shown in Table 11.

Step 6.2: According to probabilistic linguistic evaluation vectors $Z_{f_{j_i}}^{h_1} = \left(z_{1f_{j_i}}^{h_1}, \ldots, z_{mf_{j_i}}^{h_1}\right)$ $(f_{j_i} = 1, \ldots, 12)$ and probabilistic linguistic expected vectors $E_A^{h_1} = \left(e_1^{h_1}, \ldots, e_{n_{j_i}}^{h_1}\right)$, satisfaction vectors $W_i^{h_1} = \left(\beta_{i1}^{h_1}, \ldots, \beta_{in_{j_i}}^{h_1}\right)$ $(f_{j_i} = 1, \ldots, 12)$ of candidates to enterprises under position h_1 are constructed by Algorithm 4, as shown in Table 12.

Step 6.3: According to two-sided matching vectors $X_{if_{j_i}}^{h_1} = \left(x_{i1}^{h_1}, \ldots, x_{in_{j_i}}^{h_1}\right)$ $(i = 1, \ldots, 4)$ and satisfaction vectors $W_i^{h_1} = \left(\alpha_{i1}^{h_1}, \ldots, \alpha_{in_{j_i}}^{h_1}\right)$ $(i = 1, \ldots, 4)$ and $W_{f_{j_i}}^{h_1} = \left(\beta_{1f_{j_i}}^{h_1}, \ldots, \beta_{mf_{j_i}}^{h_1}\right)$ $(f_{j_i} = 1, \ldots, 12)$, a one-to-many two-sided matching model (M−3) between enterprises and candidates under position h_1 is constructed.

Step 6.4: By the combination satisfaction matching method, model (M−3) can be transformed into a single-objective model (M−4). Then, assume that $\widehat{\omega}_1 = \widehat{\omega}_2 = 0.5$, optimal one-to-many matching vectors $X_{if_{j_i}}^{*h_1} = \left(x_{i1}^{*h_1}, \ldots, x_{in_{j_i}}^{*h_1}\right)$ $(i = 1, \ldots, 4)$ between the enterprise and candidates under position h_1 is obtained, as shown in Table 13. It can be seen from Table 13 that the optimal one-to-many matching scheme between enterprises and candidates is $\wp_1^* = \{\partial_1 \leftrightarrow \{A_1, A_8\}, \partial_2 \leftrightarrow \{A_4, A_7\}, \partial_3 \leftrightarrow \{A_{11}, A_{12}\}, \partial_4 \leftrightarrow \{A_9\}\}$.

Step 6.5: Satisfaction vectors $W_i^{h_1} = \left(\alpha_{i1}^{h_1}, \ldots, \alpha_{in_{j_i}}^{h_1}\right)$ $(i = 1, \ldots, 4)$ and $W_{f_{j_i}}^{h_1} = \left(\beta_{1f_{j_i}}^{h_1}, \ldots, \beta_{mf_{j_i}}^{h_1}\right)$ $(f_{j_i} = 1, \ldots, 12)$ are converted into extended satisfaction vectors $W_c^{h_1} = \left(\alpha_{c1}^{h_1}, \ldots, \alpha_{cn_{j_i}}^{h_1}\right)$ $(c = 1, \ldots, 7)$ and $W_{f_{j_i}}^{h_1} = \left(\beta_{1f_{j_i}}^{h_1}, \ldots,\right.$

Table 8
Evaluation matrix and excepted vector of enterprises to candidates under position h_1.

	δ_1				δ_2		
	A_1	A_2	A_3	A_4	A_5	A_6	
∂_1					$\{s_0(0.2), s_1(0.2), s_3(0.4)\}$	$\{s_0(0.3), s_3(0.4), s_4(0.3)\}$	
∂_2	$\{s_1(0.3), s_4(0.7)\}$	$\{s_3(0.6), s_4(0.4)\}$	$\{s_2(0.4), s_3(0.6)\}$	$\{s_0(0.2), s_3(0.3), s_4(0.5)\}$			
∂_3	$\{s_2(0.5), s_3(0.5)\}$	$\{s_1(0.2), s_2(0.5), s_3(0.3)\}$	$\{s_1(0.4), s_2(0.4), s_3(0.2)\}$	$\{s_0(0.2), s_1(0.4), s_3(0.4)\}$			
∂_4					$\{s_2(0.6), s_3(0.4)\}$	$\{s_2(0.2), s_3(0.6), s_4(0.2)\}$	
	δ_3		δ_4		δ_5		$e_i^{h_1}$
	A_7	A_8	A_9	A_{10}	A_{11}	A_{12}	
∂_1	$\{s_1(0.3), s_3(0.7)\}$	$\{s_0(0.2), s_3(0.3), s_4(0.5)\}$					$\{s_1(0.2), s_3(0.3), s_4(0.5)\}$
∂_2	$\{s_2(0.3), s_3(0.4), s_4(0.3)\}$	$\{s_2(0.6), s_3(0.4)\}$					$\{s_1(0.3), s_3(0.7)\}$
∂_3					$\{s_0(0.2), s_3(0.3), s_4(0.5)\}$	$\{s_2(0.3), s_3(0.4), s_4(0.3)\}$	$\{s_2(0.4), s_4(0.6)\}$
∂_4			$\{s_2(0.2), s_3(0.8)\}$	$\{s_2(0.6), s_3(0.4)\}$			$\{s_0(0.2), s_1(0.2), s_3(0.6)\}$

Table 9
Evaluation vectors of headhunting companies to candidates under position h_1.

	A_1	A_2	A_3	A_4	A_5	A_6
δ_1	$\{s_2(0.3), s_4(0.7)\}$	$\{s_3(0.5), s_4(0.5)\}$	$\{s_1(0.3), s_3(0.4), s_4(0.3)\}$	$\{s_2(0.6), s_3(0.4)\}$		
δ_2					$\{s_2(0.6), s_4(0.4)\}$	$\{s_1(0.2), s_2(0.5), s_3(0.3)\}$
δ_3						
δ_4						
δ_5						
	A_7	A_8	A_9	A_{10}	A_{11}	A_{12}
δ_1						
δ_2						
δ_3	$\{s_1(0.2), s_3(0.5), s_4(0.3)\}$	$\{s_1(0.2), s_3(0.8)\}$				
δ_4			$\{s_1(0.6), s_2(0.4)\}$	$\{s_1(0.6), s_3(0.4)\}$		
δ_5					$\{s_2(0.2), s_3(0.5), s_4(0.3)\}$	$\{s_0(0.3), s_3(0.4), s_4(0.3)\}$

Table 10
Evaluation matrix and excepted vector of candidates to enterprises under position h_1.

	δ_1				δ_2	
	A_1	A_2	A_3	A_4	A_5	A_6
∂_1					$\{s_1(0.2), s_3(0.8)\}$	$\{s_2(0.3), s_3(0.7)\}$
∂_2	$\{s_1(0.2), s_2(0.5), s_3(0.3)\}$	$\{s_1(0.3), s_3(0.7)\}$	$\{s_2(0.5), s_3(0.5)\}$	$\{s_2(0.2), s_3(0.5), s_4(0.3)\}$		
∂_3	$\{s_0(0.3), s_3(0.4), s_4(0.3)\}$	$\{s_1(0.2), s_2(0.5), s_3(0.3)\}$	$\{s_2(0.4), s_3(0.6)\}$	$\{s_0(0.3), s_3(0.4), s_4(0.3)\}$		
∂_4					$\{s_0(0.2), s_3(0.3), s_4(0.5)\}$	$\{s_1(0.2), s_2(0.5), s_3(0.3)\}$
$e_{f_{j_i}}^{h_1}$	$\{s_2(0.3), s_3(0.4), s_4(0.3)\}$	$\{s_2(0.4), s_3(0.6)\}$	$\{s_1(0.2), s_3(0.8)\}$	$\{s_1(0.3), s_3(0.7)\}$	$\{s_2(0.2), s_3(0.8)\}$	$\{s_1(0.4), s_3(0.6)\}$
	δ_3		δ_4		δ_5	
	A_7	A_8	A_9	A_{10}	A_{11}	A_{12}
∂_1	$\{s_1(0.3), s_3(0.7)\}$	$\{s_3(0.5), s_4(0.5)\}$				
∂_2	$\{s_2(0.3), s_3(0.4), s_4(0.3)\}$	$\{s_2(0.4), s_3(0.6)\}$				
∂_3					$\{s_0(0.2), s_3(0.3), s_4(0.5)\}$	$\{s_2(0.3), s_3(0.4), s_4(0.3)\}$
∂_4			$\{s_2(0.3), s_3(0.4), s_4(0.3)\}$	$\{s_2(0.2), s_3(0.8)\}$		
$e_{f_{j_i}}^{h_1}$	$\{s_2(0.2), s_3(0.8)\}$	$\{s_1(0.6), s_3(0.4)\}$	$\{s_0(0.2), s_3(0.3), s_4(0.5)\}$	$\{s_1(0.3), s_3(0.7)\}$	$\{s_1(0.3), s_3(0.3), s_4(0.4)\}$	$\{s_1(0.4), s_2(0.3), s_4(0.3)\}$

Table 11
Satisfactions of enterprises to candidates under position h_1.

	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	A_{11}	A_{12}
∂_1					0.2060	0.0836	0.1982	0.8561				
∂_2	0.8279	0.9109	0.4500	1.0000			0.7691	0.7916				
∂_3	0.1062	0.1596	0.3494	0.1138							0.6871	0.8063
∂_4					0.6176	0.8940			0.9642	0.8561		

Table 12
Satisfactions of candidates to enterprises under position h_1.

	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	A_{11}	A_{12}
∂_1					0.4649	0.6260	0.4331	1.0000				
∂_2	0.0382	0.3439	0.6131	0.8493			0.5392	0.6989				
∂_3	0.2602	0.5392	0.5999	0.8169							0.4331	0.6989
∂_4					0.8223	0.6752			0.6260	0.5863		

Table 13
Optimal matching scheme of enterprises and candidates under position h_1.

	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	A_{11}	A_{12}
∂_1	0	0	0	0	1	0	0	1	0	0	0	0
∂_2	0	0	0	1	0	0	1	0	0	0	0	0
∂_3	0	0	0	0	0	0	0	0	0	0	1	1
∂_4	0	0	0	0	0	0	0	0	1	0	0	0

Table 14
Optimal stable matching scheme of enterprises and candidates under position h_1.

	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	A_{11}	A_{12}
∂_1	0	0	0	0	1	0	0	1	0	0	0	0
∂_2	0	1	0	1	0	0	0	0	0	0	0	0
∂_3	0	0	0	0	0	0	0	0	0	0	1	1
∂_4	0	0	0	0	0	0	0	0	1	0	0	0

$\beta^{h_1}_{df_{j_i}}\Big)$ ($f_{j_i} = 1,\ldots,12$) of enterprises and candidates by converting $\partial = \{\partial_1, \ldots, \partial_m\}$ into the virtual subject set $\widehat{\partial} = \{\widehat{\partial}_1, \ldots, \widehat{\partial}_d\}$ of enterprises.

Step 6.6: According to two-sided matching vectors $X^{h_1}_{cf_{j_i}} = \left(x^{h_1}_{c1}, \ldots, x^{h_1}_{cn_{j_i}}\right)$ ($c = 1,\ldots,7$) and extended satisfaction vectors $W^{h_1}_c = \left(\alpha^{h_1}_{c1}, \ldots, \alpha^{h_1}_{cn_{j_i}}\right)$ ($c = 1,\ldots,7$) and $W^{h_1}_{f_{j_i}} = \left(\beta^{h_1}_{1f_{j_i}}, \ldots, \beta^{h_1}_{df_{j_i}}\right)$ ($f_{j_i} = 1,\ldots,12$) under position h_1, a one-to-one stable two-sided matching model (M−5) between enterprises and candidates under position h_1 is constructed.

Step 6.7: By the combination satisfaction matching method, model (M−5) is transformed into a single-objective model (M−6), and optimal stable one-to-many matching vectors $X^{**h_1}_{if_{j_i}} = \left(x^{**h_1}_{i1}, \ldots, x^{**h_1}_{in_{j_i}}\right)$ ($i = 1, \ldots, 4$) between enterprises and candidates under position h_1 are obtained, as shown in Table 14.

Table 15
Optimal matching schemes considering psychological tendency coefficients of enterprises under position h_1.

Method	Optimal matching schemes
Method based on model (M−4) and $\rho = 2$	$\{\partial_1 \leftrightarrow (A_5, A_8), \partial_2 \leftrightarrow (A_4, A_7), \partial_3 \leftrightarrow (A_{11}, A_{12}), \partial_4 \leftrightarrow (A_9)\}$
Method based on model (M−6) and $\rho = 2$	$\{\partial_1 \leftrightarrow (A_7, A_8), \partial_2 \leftrightarrow (A_2, A_4), \partial_3 \leftrightarrow (A_{11}, A_{12}), \partial_4 \leftrightarrow (A_9)\}$
Method based on model (M−4) and $\rho = 0.5$	$\{\partial_1 \leftrightarrow (A_1, A_8), \partial_2 \leftrightarrow (A_4, A_7), \partial_3 \leftrightarrow (A_{11}, A_{12}), \partial_4 \leftrightarrow (A_9)\}$
Method based on model (M−6) and $\rho = 0.5$	$\{\partial_1 \leftrightarrow (A_1, A_8), \partial_2 \leftrightarrow (A_2, A_4), \partial_3 \leftrightarrow (A_{11}, A_{12}), \partial_4 \leftrightarrow (A_9)\}$
Method based on model (M−4) and $\rho = 0.8$	$\{\partial_1 \leftrightarrow (A_6, A_8), \partial_2 \leftrightarrow (A_3, A_4), \partial_3 \leftrightarrow (A_{11}, A_{12}), \partial_4 \leftrightarrow (A_5)\}$
Method based on model (M−6) and $\rho = 0.8$	$\{\partial_1 \leftrightarrow (A_5, A_8), \partial_2 \leftrightarrow (A_2, A_4), \partial_3 \leftrightarrow (A_{11}, A_{12}), \partial_4 \leftrightarrow (A_6)\}$

It can be seen from Table 21 that the optimal one-to-many matching scheme between enterprises and candidates is $\wp^{**}_1 = \{\partial_1 \leftrightarrow \{A_1, A_8\}, \partial_2 \leftrightarrow \{A_2, A_4\}, \partial_3 \leftrightarrow \{A_{11}, A_{12}\}, \partial_4 \leftrightarrow \{A_9\}\}$. This case demonstrates that decision-making for multi-position two-phase person-position matching with intermediary participation under the probabilistic linguistic environment is proposed and implemented, which further extends the significance of the PLTS theory and person-position matching.

7. Discussion

In this section, the above example is further discussed to reveal the effectiveness, accuracy and advantages of the proposed method. First, the result analysis of different psychological tendency coefficients on the optimal person-position matching schemes is carried out. Second, the sensitivity analysis is proceeded on different weights of objective functions, and comparative analysis with other methods is presented. Third, advantages of the proposed method are expounded. Finally, theoretical and practical contributions of this paper are exhibited.

7.1. Analysis of different psychological tendency coefficients

To verify the feasibility and effectiveness of the proposed decision-making method, according to different psychological tendency coefficients of enterprises, this paper compares the different person-position matching schemes solved by the matching model (M−4) and the stable matching model (M−6) under position h_1, as shown in Table 15 and Figs. 4 and 5.

According to Table 15 and Figs. 4 and 5, when the psychological tendency coefficient ρ changes, satisfactions of enterprises changes

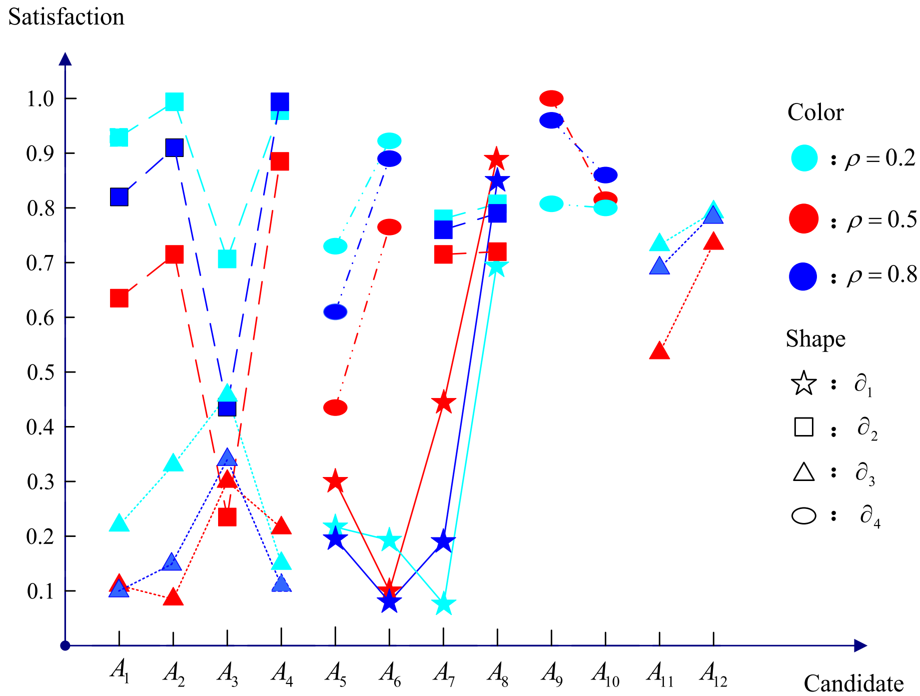

Fig. 4. Satisfactions of enterprises to candidates under different psychological tendency coefficients.

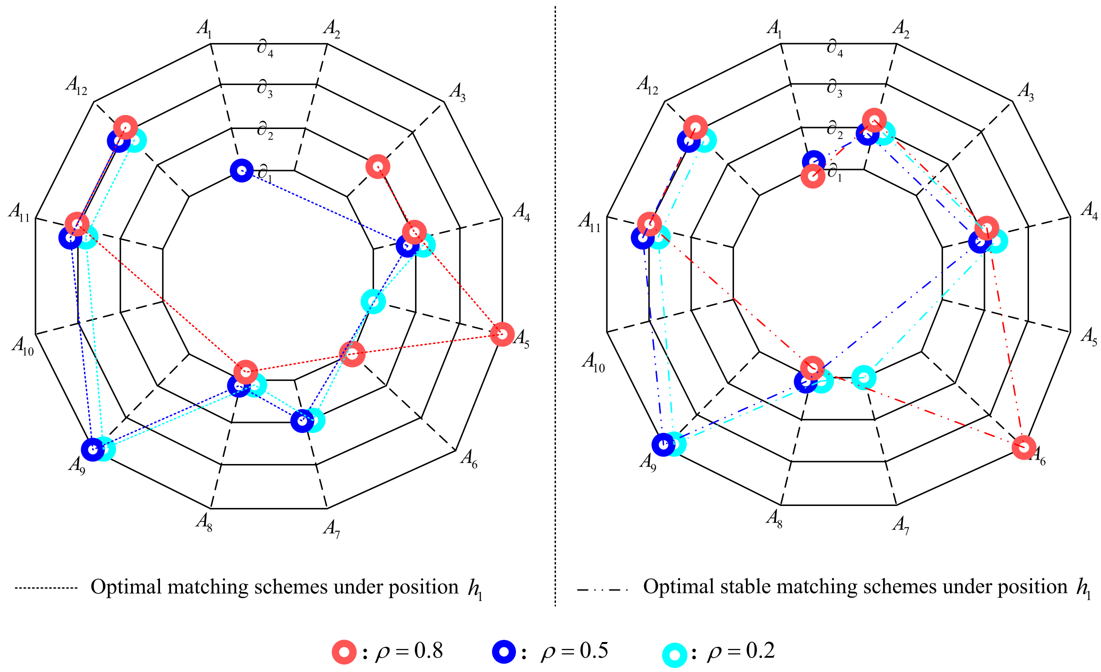

Fig. 5. Comparison of optimal matching schemes under different psychological tendency coefficients.

obviously and the optimal matching scheme is also quite different. This shows that the psychological tendency coefficient influence satisfactions of enterprises to candidates to a certain extent, and reflect on different optimal matching schemes. At the same time, it shows the necessity of considering stable constraints in the process of person-position matching.

7.2. Sensitivity analysis on different weights of objective functions

Considering that optimal matching schemes obtained by solving model (M−4) and (M−6) may be different with different weights of objective functions, to demonstrate the effectiveness of the proposed person-position matching decision-making method, the method of Liu and You (2017) and Jia et al. (2022) are extended to compare with the proposed method of this paper. The optimal matching schemes of different methods are shown in Table 16.

It can be seen from Table 16 that there are some differences between the optimal matching scheme and the optimal stable matching scheme obtained by using different weights of objective functions. Thus, the weights of objective functions should be fully considered to solve the

Table 16
Comparison of optimal matching schemes of different methods.

Method	Optimal matching scheme
Method based on model (M−4) ($\widehat{\omega}_1 = 0.2, \widehat{\omega}_2 = 0.8$)	$\{\partial_1 \leftrightarrow (A_5, A_8), \partial_2 \leftrightarrow (A_4, A_7), \partial_3 \leftrightarrow (A_{11}, A_{12}), \partial_4 \leftrightarrow (A_9)\}$
Method based on model (M−4) ($\widehat{\omega}_1 = 0.5, \widehat{\omega}_2 = 0.5$)	$\{\partial_1 \leftrightarrow (A_1, A_8), \partial_2 \leftrightarrow (A_4, A_7), \partial_3 \leftrightarrow (A_{11}, A_{12}), \partial_4 \leftrightarrow (A_9)\}$
Method based on model (M−4) ($\widehat{\omega}_1 = 0.8, \widehat{\omega}_2 = 0.2$)	$\{\partial_1 \leftrightarrow (A_6, A_8), \partial_2 \leftrightarrow (A_4, A_7), \partial_3 \leftrightarrow (A_{11}, A_{12}), \partial_4 \leftrightarrow (A_9)\}$
Method based on model (M−6) ($w_1 = 0.2, w_2 = 0.8$)	$\{\partial_1 \leftrightarrow (A_5, A_8), \partial_2 \leftrightarrow (A_2, A_4), \partial_3 \leftrightarrow (A_{11}, A_{12}), \partial_4 \leftrightarrow (A_9)\}$
Method based on model (M−6) ($w_1 = 0.5, w_2 = 0.5$)	$\{\partial_1 \leftrightarrow (A_1, A_8), \partial_2 \leftrightarrow (A_2, A_4), \partial_3 \leftrightarrow (A_{11}, A_{12}), \partial_4 \leftrightarrow (A_9)\}$
Method based on model (M−6) ($w_1 = 0.8, w_2 = 0.2$)	$\{\partial_1 \leftrightarrow (A_5, A_8), \partial_2 \leftrightarrow (A_2, A_4), \partial_3 \leftrightarrow (A_{11}, A_{12}), \partial_4 \leftrightarrow (A_9)\}$
Method based on the idea of Liu and You (2017) ($\widehat{\omega}_1 = 0.2, \widehat{\omega}_2 = 0.8$)	$\{\partial_1 \leftrightarrow (A_6, A_8), \partial_2 \leftrightarrow (A_4, A_7), \partial_3 \leftrightarrow (A_3, A_{12}), \partial_4 \leftrightarrow (A_5)\}$
Method based on the idea of Liu and You (2017) ($\widehat{\omega}_1 = 0.5, \widehat{\omega}_2 = 0.5$)	$\{\partial_1 \leftrightarrow (A_1, A_8), \partial_2 \leftrightarrow (A_4, A_7), \partial_3 \leftrightarrow (A_{11}, A_{12}), \partial_4 \leftrightarrow (A_9)\}$
Method based on the idea of Liu and You (2017) ($\widehat{\omega}_1 = 0.8, \widehat{\omega}_2 = 0.2$)	$\{\partial_1 \leftrightarrow (A_5, A_8), \partial_2 \leftrightarrow (A_2, A_4), \partial_3 \leftrightarrow (A_{11}, A_{12}), \partial_4 \leftrightarrow (A_9)\}$
Method based on the idea of Jia et al. (2022) ($\widehat{\omega}_1 = 0.2, \widehat{\omega}_2 = 0.8$)	$\{\partial_1 \leftrightarrow (A_5, A_8), \partial_2 \leftrightarrow (A_2, A_4), \partial_3 \leftrightarrow (A_{11}, A_{12}), \partial_4 \leftrightarrow (A_9)\}$
Method based on the idea of Jia et al. (2022) ($\widehat{\omega}_1 = 0.5, \widehat{\omega}_2 = 0.5$)	$\{\partial_1 \leftrightarrow (A_5, A_8), \partial_2 \leftrightarrow (A_4, A_7), \partial_3 \leftrightarrow (A_{11}, A_{12}), \partial_4 \leftrightarrow (A_9)\}$
Method based on the idea of Jia et al. (2022) ($\widehat{\omega}_1 = 0.8, \widehat{\omega}_2 = 0.2$)	$\{\partial_1 \leftrightarrow (A_5, A_8), \partial_2 \leftrightarrow (A_4, A_7), \partial_3 \leftrightarrow (A_{11}, A_{12}), \partial_4 \leftrightarrow (A_9)\}$

two-sided matching model. In addition, the method of Liu et al. [40] cannot be directly used to solve the problem in this paper and does not consider stable matching conditions to construct the matching model, which may lead to inaccurate matching schemes. In reference [41], Jia et al. only considers the profit and loss of subject expectations, but does not consider the profit and loss of intermediary evaluations, and solution methods of models are not the same, which lead to different matching schemes compared with this paper. Therefore, the method proposed in this paper is more feasible and effective for solving multi-position two-phase person-position matching decision-making problems, the matching schemes under multiple positions obtained by using the method proposed in this paper are relatively more reference. Next, advantages of the proposed method compared with other methods will be discussed.

7.3. Advantages of the proposed method

By virtue of the above comparative analysis, the main advantages of the proposed method are highlighted as follows:

(1) Under the probabilistic linguistic environment, the preferences of subjects are fully considered, which can better express the fuzziness and uncertainty of preference information and hesitation of tripartite subjects. Thus, the proposed method allows tripartite subjects to express their intuitive feelings flexibly.
(2) In the proposed method, the satisfaction calculation for enterprises is determined considering subject expectations and intermediary evaluations. However, many previous methods only solve the problem of person-position matching from the dimensions of enterprises and candidates, such as the methods proposed in Liu et al. [40] and Jia et al. [41]. Thus, the proposed method can better consider the gains and losses of enterprises and candidates on the intermediary dimension.
(3) In the proposed person-position matching decision-making, a many-to-many two-sided matching model of enterprises and headhunting companies in first phase, one-to-many two-sided matching models and one-to-one stable matching models of enterprises and candidates under multiple positions in second phase are structured. Whereas, most of the existing literature only established a model for the person-position matching under a certain position, and cannot solve person-position matching problems under the condition of multiple positions. Therefore, these designed models in this paper are more suitable for apply to the multi-position person-position matching decision-making and can help to improve the stability of matching schemes and maximize satisfactions of tripartite subjects.

7.4. Theoretical and practical contributions of this paper

The main theoretical and practical contributions of this paper are reflected in the following. Theoretical contributions are displayed in the following aspects: (1) A decision-making method for two-phase person-position matching with intermediary participation is proposed, which considers the gains and losses of subject expectations and intermediary evaluations. (2) A multi-position person-position matching decision-making method is developed, which is more conducive to solve the realistic person-position matching problem in multi-position situation of human resource allocation. (3) The theory of PLTSs is applied to the person-position matching decision-making process, which will better describe the uncertainty of linguistic preferences of two-sided subjects. (4) A satisfaction calculation method considering double reference points of subject expectations and intermediary evaluations based on PLTSs is designed. (5) A many-to-many two-sided matching model of enterprises and headhunting companies is established. (6) One-to-many two-sided matching models and one-to-one stable matching models of enterprises and candidates under multi positions are constructed.

Practical contributions are reflected in the following aspects: (1) A new two-phase person-position matching decision-making method is used to solve the multi-position person-position matching problem composed of enterprises, headhunting companies and candidates, which helps enterprises to quickly recruit required candidates, improve the efficiency of human resource allocation, and enhance actual benefits of enterprises. (2) The proposed method considers subject expectations and intermediary evaluations, and the gains and losses of enterprises, headhunting companies and candidates. It helps to reduce the potential conflicts among the enterprises, headhunting companies and candidates in the matching process and improve their satisfactions.

8. Conclusion

Considering the participation of intermediaries and the situation of multiple positions in person-position matching, a novel decision-making method is designed to solve this complex person-position matching problem. Specifically speaking, this paper proposed a multi-position two-phase person-position matching decision-making method. In real life, intermediaries often participate in the process of person-position matching under multiple positions. Hence, it is of theoretical and practical significance to study the person-position matching with intermediary participation from a two-phase perspective.

Different from the traditional person-position matching, this paper incorporated intermediaries into the person-position matching process and proposed a two-phase decision-making method. In the first phase, satisfactions of enterprises and headhunting companies are calculated based on prospect theory. On this basis, a many-to-many two-sided matching model is established, and the optimal many-to-many matching scheme between enterprises and headhunting companies is obtained by solving the model. In the second phase, considering subject expectations and intermediary evaluations, satisfactions of enterprises and candidates under multiple position are calculated. Then, to maximize satisfactions of enterprises and candidates, one-to-many matching models and one-to-one stable matching models for enterprises and candidates under multiple positions are developed; and optimal matching schemes under multiple positions are obtained by solving the models.

Compared with the existing methods, the main innovations of this paper are given as follows: (1) A novel decision-making method is proposed to solve the two-phase person-position matching problem with intermediary participation. (2) A multi-position person-position matching decision-making method is developed, which is more in line with the realistic multi-position situation of human resource allocation. (3) The theory of PLTSs is applied to person-position matching, which can flexibly reflect the complexity of subjects' linguistic preferences. (4) A satisfaction calculation considering the double reference points based on PLTSs is developed. (5) A many-to-many two-sided matching model of enterprises and headhunting companies, one-to-many two-sided matching models and one-to-one stable matching models of enterprises and candidates under multiple positions are structured.

Limitations of this paper are shown as follows: (1) The person-position matching decision-making is preliminarily discussed based on PLTSs in this paper. However, other complex types of linguistic preference information and the related theories are not taken into account. (2) Theories of group decision-making and multi-attribute two-sided matching are not considered in this paper. Thus, it is difficult to solve more complex person-position problems. (3) The calculation for satisfactions of candidates considering intermediary evaluations has not been studied.

For future research, the following potential directions are pointed out: (1) The person-position matching decision-making method with intermediary participation in other complex linguistic environment needs further research. (2) The decision-making problems in complex situations such as group decision-making and multi-attribute two-sided matching need further study. (3) In person-position matching decision-making, the calculation method for satisfactions of candidates considering intermediary evaluations needs further exploration. (4) It is necessary to strengthen the analysis of complexity of person-position matching and the dynamic changes in the process, and to establish a dynamic employment market ecosystem model.

CRediT authorship contribution statement

Qi Yue: Conceptualization, Methodology, Formal analysis, Software, Writing – review & editing. **Shijie Huang:** Conceptualization, Methodology, Formal analysis, Software, Writing – review & editing. **Bin Hu:** Conceptualization, Software, Writing – review & editing. **Yuan Tao:** Conceptualization, Software, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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