Feedback Systems - Astrom

Solution to Exercises

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1 Introduction

1.1 Eye motion: N/A

1.2 Identify 5 Feedback Systems

- **Air Conditioning**: sensor is the thermometer, actuator is the AC pumping cold air to the environment, and the control law specifies the amount and temperature of the cold air being pumped as a function of the difference between the desired temperature and the actual temperature of the environment. The disturbance could be the amount of humans in the environment, the temperature on the outside etc.

- **Gas Heater**: sensor is the human feeling the heat, actuator is the human turning the knob of the gas heater, and the control law is to turn it clockwise if the environment feels too cold, and counter-clockwise if it feels too hot. The amount to be turned is eyeballed by the human and learned from experience. The disturbance could be the outside temperature, which dictates how much heat the environment is losing via conduction through the walls.

- **Cruise Control**: sensor is the car speedometer, actuator is the power provided to the car engine, and the control law is to provide more power if the velocity is below the requested value, and less power if it's above. Disturbances could be uphill/downhill segments in the road.

- **Human Body Temperature**: sensor is hypothalamus sensing body temperature, actuator are the sweat glands and the control law is producing sweat to cool of the skin depending on how hot it is. The disturbance is the environment temperature, which directly affects our temperature.

- **Inverted Pendulum**: sensor is angle of the rod, actuator is the cart motor, and the control law is to move to the director of the rod angle to 'counteract' it. Disturbances could be wind or people poking consciously the rod.

1.3 Balance Systems

The control system comprises of two inputs: (1) our desire to stand upright, and (2) our perceived position, which is a feeling of falling down whenever it is too different from being upright. The system has one output, which is how much and in which direction to move the muscles of the foot that is touching the ground, as to counteract the direction towards which we are 'falling down'.

1.4 Cruise Control on MATLAB: N/A

1.5 Integral Action

The goal is to show that if the system reaches a steady state with a PI controller, than the error $e(t) = r(t) - y(t)$ must be equal to zero. One assumption we'll use is that the system is 'well-behaved', i.e. that $|u(t)|$ cannot grow unbounded as t increases. One counter example would be, for instance, a system that is not affected at all by $u(t)$, i.e. $y(t) = f(u(t)) = y_0$. Think of the control of such system as turning a knob that is not connected to the system. These strange systems will not be considered in this proof. The idea of the proof is that if $\lim_{t\to\infty} e(t) = e_0 \neq 0$, then the integrative part of the control law will make $|u(t)| \xrightarrow{t\to\infty} \infty$.

Let $r(t) = r_0$. If the system is steady state, then $\lim y(t) = y_0 < \infty$ and $\lim e(t) = r_0 - y_0 = e_0$. If the control law is a PI system, then:

$$u(t) = k_P e(t) + k_I \int_0^t e(\tau) d\tau$$

$$|\lim u(t)| = k_P |e_0| + k_I |\int_0^\infty e(\tau) d\tau|$$

Thus,

$$|\lim u(t)| < \infty \implies \lim |\int_0^\infty e(\tau) d\tau| < \infty \implies \lim e(t) = e_0 = 0$$

Therefore, if the well-behaved system uses a PI controller and reaches a steady state, than the error must be zero. Otherwise, $|u(t)|$ would grow unbounded.

1.6 Reading a Random Paper

The paper we chose was Design of the PID temperature controller for an alkaline electrolysis system with time delays, where (a) the system comprises an alkaline electrolysis system, (b) the sensor measures the temperature of the stack, (c) the actuator controls the flow rate of cooling water and (d) the computational element is not mentioned, but it could comprise an A/D converter to have both the stack temperature T_{stack} and the desired temperature T_{ref} in a digital form, calculate u and then have an D/A converter to send a voltage signal to a servomotor that will open or close the valve.