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UNICAP – Ciência da Computação – 2º período
Disciplina: Elementos da integralização computacional
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Lista #6 – integração por partes

Integrais indefinidas

① $\int \underbrace{x^2}_{u} \underbrace{e^x}_{dv} dx \rightarrow \begin{cases} u = x^2 \\ du = 2x \\ v = e^x \\ dv = e^x \end{cases} \rightarrow x^2 \cdot e^x - \int e^x 2x\, dx = x^2 e^x - 2\int \underbrace{x}_{u} \underbrace{e^x}_{dv} dx$ $\quad \begin{cases} u = x \\ du = 1 \\ v = e^x \\ dv = e^x \end{cases}$

$= x^2 e^x - 2\cdot\left(x e^x - \int e^x \cdot 1\right) = x^2 e^x - 2\cdot\left(x e^x - e^x\right) + C$

$\boxed{= x^2 e^x - 2x e^x - 2e^x + C}$

② $\int \arcsen x\, dx \rightarrow \begin{cases} u = \arcsen x \\ du = 1/\sqrt{1-x^2} \\ v = x \\ dv = 1 \end{cases} \rightarrow \arcsen x \cdot x - \int x \cdot \frac{1}{\sqrt{1-x^2}} dx$

$= x \arcsen x - \int \frac{x}{\sqrt{1-x^2}} dx$ $\quad \begin{cases} u = 1-x^2 \\ \frac{du}{dx} = -2x \rightarrow -\frac{du}{2} = x\, dx \end{cases}$

$\rightarrow = \int u^{-1/2} \cdot \frac{-du}{2} = -\frac{1}{2} \cdot \frac{u^{1/2}}{1/2} = -\frac{1}{2} \cdot u^{1/2} \cdot \frac{2}{1} = -u^{1/2} = -\sqrt{1-x^2}$

$\boxed{= x \arcsen x + \sqrt{1-x^2} + C}$

③ $\int \ln x^2 dx \rightarrow \begin{cases} u = \ln x^2 = 2\ln x \\ du = 2\cdot\frac{1}{x} = \frac{2}{x} \\ v = x \\ dv = 1 \end{cases} \rightarrow 2\ln x \cdot x - \int \cancel{x} \cdot \frac{2}{\cancel{x}} = 2x\ln x - \int 2 = 2x\ln x - 2x + C$

$\boxed{= 2x(\ln x - 1) + C}$

④ $\int \operatorname{sen}(\ln x) dx \begin{cases} u = \operatorname{sen}(\ln x) \\ du = \cos(\ln x)\cdot\frac{1}{x} = \frac{\cos(\ln x)}{x} \\ v = x \\ dv = 1 \end{cases} \rightarrow \operatorname{sen}(\ln x)\cdot x - \int \cancel{x} \cdot \frac{\cos(\ln x)}{\cancel{x}} dx$

$\begin{cases} u = \cos(\ln x) \\ du = -\frac{\operatorname{sen}(\ln x)}{x} \\ v = x \\ dv = 1 \end{cases} \rightarrow \cos(\ln x)\cdot x - \int \cancel{x} \cdot \frac{-\operatorname{sen}(\ln x)}{\cancel{x}} dx$

$= x\cos(\ln x) + \int \operatorname{sen}(\ln x) dx = I$

$= x \operatorname{sen}(\ln x) - \int \cos(\ln x) dx$

$\therefore x \operatorname{sen}(\ln x) - \left(x\cos(\ln x) + \int \operatorname{sen}(\ln x) dx\right) = I \rightarrow I = x\operatorname{sen}(\ln x) - x\cos(\ln x) - I$

$\therefore 2I = x\operatorname{sen}(\ln x) - x\cos(\ln x) \rightarrow I = \boxed{\frac{1}{2}\left(x\operatorname{sen}(\ln x) - x\cos(\ln x)\right) + C}$

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(5) $\int x \arctan x \, dx \begin{cases} u = \arctan x \\ du = \frac{1}{1+x^2} dx \\ v = x^2/2 \\ dv = x\,dx \end{cases} \rightarrow \arctan x \cdot \frac{x^2}{2} - \int \frac{x^2}{2} \cdot \frac{1}{1+x^2} dx$

$= \frac{x^2 \arctan x}{2} - \frac{1}{2} \int x^2 \cdot \frac{1}{1+x^2} dx \rightarrow \frac{x^2}{1+x^2} = \frac{1+x^2-1}{1+x^2} = \frac{\cancel{1+x^2}}{\cancel{1+x^2}} - \frac{1}{1+x^2} = 1 - \frac{1}{1+x^2}$

$= \frac{x^2 \arctan x}{2} - \frac{1}{2} \int \left(1 - \frac{1}{1+x^2}\right) dx = \frac{x^2 \arctan x}{2} - \frac{1}{2} \left(\int 1 dx - \int \frac{1}{1+x^2} dx \right)$ x arctan x

$= \frac{x^2 \arctan x}{2} - \frac{1}{2}(x - \arctan x) + C = \frac{x^2 \arctan x}{2} - \frac{x}{2} + \frac{\arctan x}{2} + C$

$\boxed{= \frac{1}{2}(x^2 \arctan x - x + \arctan x) + C}$

integrais definidas

(1) $\int_0^1 x e^x dx \begin{cases} u = x \\ du = 1dx \\ v = e^x \\ dv = e^x dx \end{cases} \rightarrow x e^x \Big|_0^1 - \int_0^1 e^x dx = x e^x - e^x \Big]_0^1 = (1 \cdot e^1 - e^1) - (0 \cdot e^0 - e^0)$

$= 0 - (-1) = \boxed{1}$

(2) $\int_1^e \ln x \, dx \begin{cases} u = \ln x \\ du = 1/x \, dx \\ v = x \\ dv = 1dx \end{cases} \rightarrow \ln x \cdot x \Big|_1^e - \int_1^e \cancel{x} \cdot \frac{1}{\cancel{x}} dx = x \ln x \Big|_1^e - \int_1^e 1 dx$

$= x \ln x - x \Big]_1^e = (e \ln e - e) - (1 \cdot \ln 1 - 1) = 0 - (-1) = \boxed{1}$

(3) $\int_0^{\pi/2} x \cos x \, dx \begin{cases} u = x \\ du = 1dx \\ v = \operatorname{sen} x \\ dv = \cos x \, dx \end{cases} \rightarrow x \cdot \operatorname{sen} x \Big|_0^{\pi/2} - \int_0^{\pi/2} \operatorname{sen} x \, dx = x \operatorname{sen} x \Big|_0^{\pi/2} - (-\cos x) \Big|_0^{\pi/2}$

$= x \operatorname{sen} x + \cos x \Big]_0^{\pi/2} = (\pi/2 \cdot \operatorname{sen} \pi/2 + \cos \pi/2) - (0 \cdot \operatorname{sen} 0 + \cos 0)$

$\boxed{= \frac{\pi}{2} - 1}$

(4) $\int_0^1 x^3 e^x dx \begin{cases} u = x^3 \\ du = 3x^2 dx \\ v = e^x \\ dv = e^x dx \end{cases} \rightarrow x^3 e^x \Big|_0^1 - \int_0^1 e^x 3x^2 dx = x^3 e^x - 3 \int e^x x^2 dx \begin{cases} u = x^2 \\ du = 2x\,dx \\ v = e^x \\ dv = e^x dx \end{cases}$

$\rightarrow x^2 e^x - \int e^x 2x \, dx$

$= x^2 e^x - 2 \int x e^x dx \begin{cases} u = x \\ du = 1dx \\ v = e^x \\ dv = e^x dx \end{cases}$

$\rightarrow x e^x - \int e^x dx$

$= x e^x - e^x$

$\therefore x^3 e^x - 3\left[x^2 e^x - 2(x e^x - e^x)\right] = x^3 e^x - 3(x^2 e^x - 2x e^x + 2e^x) \Big|_0^1$

$= x^3 e^x - 3x^2 e^x + 6x e^x - 6e^x \Big|_0^1 =$

$= (1^3 e^1 - 3 \cdot 1^2 e^1 + 6 \cdot 1 \cdot e^1 - 6e^1) - (0^3 e^0 - 3 \cdot 0^2 e^0 + 6 \cdot 0 e^0 - 6e^0)$

$= e - 3e + \cancel{6e} - \cancel{6e} + 6 = \boxed{-2e + 6}$

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$$\text{⑤ } \int_0^1 \underbrace{e^x \operatorname{sen} x \, dx}_{=I} \quad \begin{cases} u = \operatorname{sen} x \\ du = \cos x \, dx \\ v = e^x \\ dv = e^x dx \end{cases} \rightarrow \operatorname{sen} x \cdot e^x - \int e^x \cos x \, dx \quad \begin{cases} u = \cos x \\ du = -\operatorname{sen} x \\ v = e^x \\ dv = e^x dx \end{cases}$$

$$\rightarrow \cos x \, e^x - \int e^x \cdot -\operatorname{sen} x$$

$$= \cos x \, e^x + \underbrace{\int e^x \operatorname{sen} x}_{=I}$$

$$\therefore I = e^x \operatorname{sen} x - (e^x \cos x + I) \Big|_0^1$$

$$\hookrightarrow I = e^x \operatorname{sen} x - e^x \cos x - I \Big|_0^1 \rightarrow 2I = e^x \operatorname{sen} x - e^x \cos x \Big|_0^1$$

$$\therefore I = \frac{1}{2}\left(e^x \operatorname{sen} x - e^x \cos x\right)\Big|_0^1 = \left[\frac{1}{2}\left(e^1 \operatorname{sen} 1 - e^1 \cos 1\right)\right] - \left[\frac{1}{2}\left(e^0 \cancel{\operatorname{sen} 0} - \cancel{e^0} \cancel{\cos 0}\right)\right]$$

$$= \frac{1}{2}\left(e \cdot \operatorname{sen} 1 - e \cos 1\right) - \frac{1}{2}(-1) = \frac{1}{2}\left(e \operatorname{sen} 1 - e \cos 1\right) + \frac{1}{2}$$

$$= \frac{1}{2}\left(e \operatorname{sen} 1 - e \cos 1 + 1\right)$$