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UNICAP - Ciência da Computação - 2º período
Disciplina: Elementos da integralização computacional
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Lista #4 - integrais definidas (ignorar #14 e #18)

① $\int_0^2 3x^2\,dx = 3\int_0^2 x^2\,dx = \cancel{3}\,\frac{x^3}{\cancel{3}}\Big]_0^2 = (2^3 - 0^3) = 8$ //

② $\int_1^4 (2x+1)\,dx = \int_1^4 2x\,dx + \int_1^4 x^0\,dx = 2\int_1^4 x\,dx + \int_1^4 x^0\,dx$

$= \frac{\cancel{2}x^2}{\cancel{2}} + x\Big]_1^4 = \Big[(4^2+4) - (1^2+1)\Big] = [(16+4)-(2)] = 18$ //

③ $\int_0^1 x^3\,dx = \frac{x^4}{4}\Big]_0^1 = \Big[\frac{1^4}{4} - \cancel{\frac{0^4}{4}}\Big] = \frac{1}{4}$ //

④ $\int_0^\pi \operatorname{sen} x\,dx = -\cos x\Big]_0^\pi = [(-\cos\pi) - (-\cos 0)] = -\cos\pi + \underset{\hookrightarrow 1}{\cos 0}$

$\cos\pi = -1 \therefore |-\cos\pi = 1|$

$= 1 + 1 = 2$ //

⑤ $\int_0^{\pi/2} \cos x\,dx = \operatorname{sen} x\Big]_0^{\pi/2} = \operatorname{sen}\pi/2 - \operatorname{sen}0 = 1 - 0 = 1$ //

⑥ $\int_1^e \frac{1}{x}\,dx = \ln x\Big]_1^e = [(\overset{1}{\cancel{\ln e}}) - (\overset{0}{\cancel{\ln 1}})] = 1 - 0 = 1$ //

⑦ $\int_0^2 e^{2x}\,dx$; $u = 2x \to \frac{du}{dx} = 2 \to \frac{du}{2} = dx$

$= \int_0^2 e^u \frac{du}{2} = \frac{1}{2}\int_0^2 e^u\,du = \frac{1}{2}\cdot e^u\Big]_0^2 = \frac{1}{2}e^{2x}\Big]_0^2$

$= \frac{e^{2x}}{2}\Big]_0^2 = \Big[\Big(\frac{e^{2\cdot 2}}{2}\Big) - \Big(\frac{e^0}{2}\Big)\Big] = \frac{e^4}{2} - \frac{1}{2} = \frac{e^4-1}{2}$ //

⑧ $\int_0^{\pi/4} \tan x\,dx = \int_0^{\pi/4} \frac{\operatorname{sen} x}{\cos x}\,dx$; $|u = \cos x|$, $du = -\operatorname{sen} x$, $|-du = \operatorname{sen} x|$; $\int_0^{\pi/4} \frac{-du}{u} = -\ln|\cos x|\Big]_0^{\pi/4}$

$\int \frac{u'}{u} = \ln|u|$

$= [(-\ln|\cos \pi/4|) - (-\ln|\cos 0|)] = [(-\ln 1/\sqrt{2}) + (\overset{0}{\cancel{\ln 1}})] = -(-\ln\sqrt{2}) = \ln\sqrt{2}$

$\hookrightarrow = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

$\ln\frac{1}{a} = \overset{0}{\cancel{\ln 1}} - \ln a = -\ln a$

$= \ln a^{1/2} = \boxed{\frac{1}{2}\ln 2}$

→

$\frac{d}{dx}\arctan x = \frac{1}{x^2+1}$

$\tan 45^\circ = 1 \rightarrow \arctan 1 = 45^\circ = \pi/4$

(9) $\int_0^1 \frac{1}{1+x^2}\,dx = \arctan x \Big]_0^1 = \arctan 1 - \arctan 0 = \boxed{\frac{\pi}{4}}$

$\arctan 0 = x$

$\tan x = 0$

$\frac{\sin x}{\cos x} = 0 \quad \sin x = 0 \rightarrow x = 0 \; (\pi)$

↳ não pode ser 0.

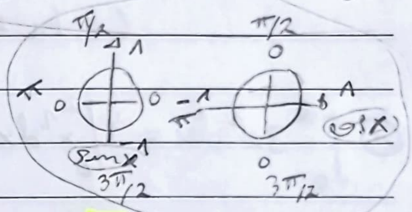
se $\frac{\sin \pi}{\cos \pi} = 0$ ↳ $= 0 =$ indet.

$\therefore \arctan 0 = 0$

(10) $\int_0^1 \sqrt{x}\,dx = \int_0^1 x^{1/2}\,dx$

$= \frac{x^{3/2}}{3/2}\Big]_0^1 = x^{3/2}\cdot\frac{2}{3}\Big]_0^1 = \frac{2}{3}\sqrt{x^3}\Big]_0^1 = \left[\left(\frac{2}{3}\sqrt{1^3}\right) - \left(\cancel{\frac{2}{3}\sqrt{0^3}}\right)\right]$

$\boxed{= \frac{2}{3}}$



(11) $\int_{-1}^1 x^5\,dx = \frac{x^6}{6}\Big]_{-1}^1 \left[\left(\frac{1^6}{6}\right) - \left(\frac{(-1)^6}{6}\right)\right] = \frac{1}{6} - \frac{1}{6} = 0$

(12) $\int_{-2}^2 x^4\,dx = \frac{x^5}{5}\Big]_{-2}^2 = \left[\left(\frac{2^5}{5}\right) - \left(\frac{(-2)^5}{5}\right)\right] = \frac{32}{5} + \frac{32}{5} = \boxed{\frac{64}{5}}$

(13) $\int_0^\pi \cos(2x)\,dx = \int_0^\pi \cos u\,dx = \int_0^\pi \cos u \frac{du}{2} = \frac{1}{2}\int_0^\pi \cos u\,du$

$u = 2x$

$du = u'\cdot dx \rightarrow \frac{du}{dx} = u' \rightarrow \frac{du}{dx} = 2 \rightarrow dx = \frac{du}{2}$

$= \frac{1}{2}\sin x\Big]_0^\pi = \left(\frac{1}{2}\cdot\sin\pi\right) - \left(\frac{1}{2}\sin 0\right) = \boxed{0}$

(14) $\int_0^1 \frac{2x}{x^2+1}\,dx = 2\int_0^1 \frac{x\,dx}{u} = 2\int_0^1 \frac{\frac{du}{2}}{u} = \cancel{2}\int_0^1 \frac{du}{\cancel{2}}\cdot\frac{1}{u}$

$u = x^2+1$

$\frac{du}{dx} = 2x \quad \frac{du}{2} = x\,dx$

$= \int_0^1 u^{-1}\,du = \ln|u|\Big]_0^1 = \ln|x^2+1|\Big]_0^1$

$= (\ln(1^2+1)) - (\ln(0^2+1)) = \ln 2 - \cancel{\ln 1} = \boxed{\ln 2}$

(15) $\int_0^{\ln 2} e^x\,dx = e^x\Big]_0^{\ln 2} = e^{\ln 2} - \cancel{e^0} = 2 - 1 = \boxed{1}$

$e^{\ln a} = a$

(16) $\int_0^1 (3x^2+4)\,dx = \int_0^1 3x^2\,dx + 4\int_0^1 x^0\,dx = \cancel{3}\cdot\frac{x^3}{\cancel{3}} + 4\cdot x\Big]_0^1 = x^3+4x\Big]_0^1$

$= (1^3+4\cdot 1) - (\cancel{0^3+4\cdot 0}) = 5$

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(17) $\int_1^2 \frac{1}{x^2}dx = \int_1^2 x^{-2}dx = \frac{x^{-1}}{-1}\Big]_1^2 = -\frac{1}{x}\Big]_1^2 = \left(-\frac{1}{2}\right) - \left(\frac{-1}{1}\right)$

$= \boxed{\frac{1}{2}}$

$= \frac{180}{6} = 30°$ $\quad \frac{d}{dx}\,\text{sen}^2x = \text{sen}\,2x = 2\,\text{sen}\,x\cos x \;\therefore\; \int 2\,\text{sen}\,x\cos x = \text{sen}^2x$

$2\int \text{sen}\,x\cos x = \text{sen}^2 x$

$\boxed{\frac{\text{sen}^2x}{2} = \int \text{sen}\,x\cos x}$

(18) $\int_0^{\pi/6} \text{sen}\,x\cos x\,dx = \frac{\text{sen}^2x}{2}\Big]_0^{\pi/6}$

$= \left(\frac{\text{sen}^2\,\pi/6}{2}\right) - \left(\cancel{\frac{\text{sen}^2 0}{2}}\right) = \frac{(1/2)^2}{2} = \frac{1}{4}\cdot\frac{1}{2} = \boxed{\frac{1}{8}}$

(19) $\int_0^1 \frac{x+1}{(x+1)^2}dx \rightarrow u = x+1$

$\frac{du}{dx} = 1 \rightarrow du = dx$

$\therefore = \int_0^1 \frac{u}{u^2}\,du = \int_0^1 u^{-1}\,du$

$= \ln u\Big]_0^1 = \ln(x+1)\Big]_0^1 = \ln 2 - \cancel{\ln 1} = \boxed{\ln 2}$

(20) $\int_0^2 (4x+1)\,dx = \int_0^2 4x\,dx + \int_0^2 x^0\,dx = \cancel{4}^{\,2}\cdot\frac{x^2}{\cancel{2}} + x\Big]_0^2$

$= 2x^2 + x\Big]_0^2 = (2\cdot 2^2 + 2) - \cancel{(2\cdot 0^2 + 0)} = \boxed{10}$

spiral