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28/10/25

UNICAP - Ciência da Computação - 2º período
Disciplina: Elementos da integralização computacional
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Lista #10 - integrais duplas

① Calcule a integral dupla da função $f(x,y) = x+y$ sobre o retângulo $0 \leq x \leq 1$; $0 \leq y \leq 2$:

$$\boxed{\iint_R (x+y)\,dA} \rightarrow \int_0^2\int_0^1 x\,dx\,dy + \int_0^2\int_0^1 y\,dx\,dy = \int_0^2 \underbrace{\frac{x^2}{2}\Big|_0^1}_{1/2} dy + y\int_0^2 \underbrace{x\Big|_0^1}_{1} dy$$

$$= \frac{1}{2}\int_0^2 dy + \int_0^2 y\,dy = \frac{1}{2}\cdot \underbrace{y\Big|_0^2}_{2} + \underbrace{\frac{y^2}{2}\Big|_0^2}_{2} = 1+2 = \boxed{3}$$

② Determine o valor da integral dupla da função $f(x,y) = xy$ sobre o quadrado $0 \leq x \leq 2$; $0 \leq y \leq 2$:

$$\boxed{\iint_R xy\,dA} \rightarrow \int_0^2\int_0^2 xy\,dx\,dy = y\int_0^2 \frac{x^2}{2}\Big|_0^2 dy = y\int_0^2 2\,dy = 2\int_0^2 y\,dy$$

$$= \cancel{2}\cdot\frac{y^2}{\cancel{2}}\Big|_0^2 = \boxed{4}$$

③ Calcule a integral dupla da função $f(x,y) = x^2+y^2$ sobre a região retangular $0 \leq x \leq 1$; $0 \leq y \leq 1$:

$$\boxed{\iint_R (x^2+y^2)\,dA} \rightarrow \int_0^1\int_0^1 x^2\,dx\,dy + \int_0^1\int_0^1 y^2\,dx\,dy = \int_0^1 \underbrace{\frac{x^3}{3}\Big|_0^1}_{1/3} dy + y^2\int_0^1 \underbrace{x\Big|_0^1}_{1} dy$$

$$= \frac{1}{3}\int_0^1 dy + \int_0^1 y^2\,dy = \frac{1}{3}\cdot \underbrace{y\Big|_0^1}_{1} + \underbrace{\frac{y^3}{3}\Big|_0^1}_{1/3} = \frac{1}{3} + \frac{1}{3} = \boxed{\frac{2}{3}}$$

④ Encontre o valor da integral dupla da função $f(x,y) = e^{x+y}$ sobre o retângulo $0 \leq x \leq 1$; $0 \leq y \leq 1$:

$$\boxed{\iint_R e^{x+y}\,dA} \rightarrow \int_0^1\int_0^1 e^x\cdot e^y\,dx\,dy = e^y\int_0^1 \underbrace{e^x\Big|_0^1}_{e-1} dy = (e-1)\int_0^1 e^y\,dy$$

$$= (e-1)\cdot e^y\Big|_0^1 = (e-1)(e-1) = \boxed{(e-1)^2}$$

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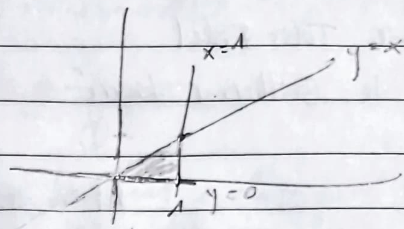
⑤ Calcule a integral dupla que representa a área da região limitada pelas retas $y=x$; $y=0$ e $x=1$:

$$\boxed{\iint_R 1\,dA} \to \int_0^1\int_0^x dy\,dx = \int_0^1 \overbrace{y\Big|_0^x}^{x} dx$$

$$= \int_0^1 x\,dx = \frac{x^2}{2}\Big|_0^1 = \boxed{\frac{1}{2}}$$




$x:[0,1]$
$y:[0,x]$

⑥ Calcule o volume sob a superfície $z=4-x-y$ e acima do retângulo definido por $0\leq x\leq 2$; $0\leq y\leq 2$:

$$\boxed{\iint_R (4-x-y)\,dA} = \int_0^2\int_0^2 (4-x-y)\,dx\,dy = \int_0^2\int_0^2 4\,dx\,dy - \int_0^2\int_0^2 x\,dx\,dy - \int_0^2\int_0^2 y\,dx\,dy$$

$$= 4\int_0^2 \overbrace{x\Big|_0^2}^{2} dy - \int_0^2 \overbrace{\frac{x^2}{2}\Big|_0^2}^{2} dy - y\int_0^2 \overbrace{x\Big|_0^2}^{2} dy = 4\cdot 2\cdot \overbrace{y\Big|_0^2}^{2} - 2\cdot \overbrace{y\Big|_0^2}^{2} - \cancel{2}\cdot \overbrace{\frac{y^2}{\cancel{2}}\Big|_0^2}^{4}$$

$$= 16 - 4 - 4 = \boxed{8}$$