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UNICAP - Ciência da Computação - 2º período
Disciplina - Elementos da Integralização Computacional
Docente - Tulio Fidel
Discente - Guilherme Larré

Lista #7 - integrais definidas c/ dicas

① $\int_0^1 x^2dx$ → dica: use $\int x^n dx = \frac{x^{n+1}}{n+1}$.

↳ $= \frac{x^3}{3}\Big|_0^1 = \frac{1^3}{3} - \frac{0^3}{3} = \boxed{\frac{1}{3}}$

② $\int_1^e \frac{1}{x}dx$ → dica: a int. de $\frac{1}{x}$ é $\ln x$.

↳ $= \ln x\Big|_1^e = \ln e - \cancel{\ln 1}^{0} = \ln e = \boxed{1}$

③ $\int_0^2 (3x+1)dx$ → Resolva separando os termos.

↳ $= \int_0^2 3x\,dx + \int_0^2 1\,dx = 3\int_0^2 x\,dx + \int_0^2 dx = \frac{3x^2}{2} + x\Big|_0^2 = \left(\frac{3\cdot 2^2}{2} + 2\right) - \left(\cancel{\frac{3\cdot 0^2}{2} + 0}\right)$

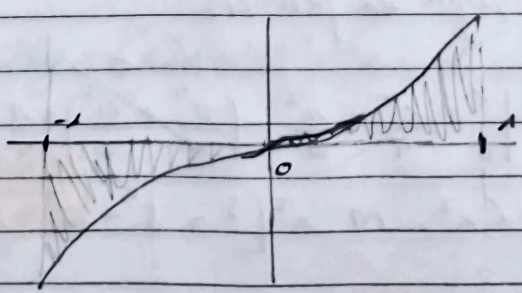
$= \frac{12}{2} + 2 = \boxed{8}$

④ $\int_0^1 e^x dx$ → A integral de e^x é e^x.

↳ $= e^x\Big|_0^1 = e^1 - \cancel{e^0}^{1} = \boxed{e-1}$

⑤ $\int_{-1}^1 x^3 dx$ → Função ímpar em intervalo simétrico.

$= 0$ →



→ áreas simétricas em rel. à origem é sempre 0 (integral norm., não área geom.), pois se cancelam!

$= \frac{x^4}{4}\Big|_{-1}^1 = \frac{1^4}{4} - \frac{(-1)^4}{4} = \frac{1}{4} - \frac{1}{4} = \boxed{0}$!

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⑥ $\int_0^4 \sqrt{x}\,dx$ → Reescreva como $x^{1/2}$

$= \int_0^4 x^{1/2}dx = \frac{x^{3/2}}{3/2} = \frac{2}{3} x^{3/2}\Big|_0^4 = \frac{2}{3} \cdot 4^{3/2} - \cancel{\frac{2}{3} \cdot 0^{3/2}} = \frac{2 \cdot \sqrt{4^3}}{3} = \frac{2\sqrt{64}}{3} = \frac{2 \cdot 8}{3} = \boxed{\frac{16}{3}}$

⑦ $\int_0^1 (2x+5)\,dx$ → Divida a integral em 2 partes

$= \int_0^1 2x\,dx + \int_0^1 5\,dx = 2\int_0^1 x\,dx + 5\int_0^1 dx = \cancel{2} \cdot \frac{x^2}{\cancel{2}} + 5x = x^2 + 5x \Big|_0^1$

$= 1^2 + 5 \cdot 1 - \cancel{(0^2 + 5 \cdot 0)} = 1 + 5 = \boxed{6}$

use a soma de integrais

⑧ $\int_0^3 (x^2+2)\,dx = \int_0^3 x^2 dx + \int_0^3 2\,dx = \int_0^3 x^2 dx + 2\int_0^3 dx = \frac{x^3}{3} + 2x \Big|_0^3$

$= \frac{3^{\cancel{3}2}}{\cancel{3}} + 2 \cdot 3 = 9 + 6 = \boxed{15}$

⑨ $\int_1^4 (x+1)\,dx$ → A integral de x é $\frac{x^2}{2}$.

$= \int_1^4 x\,dx + \int_1^4 1\,dx = \frac{x^2}{2} + x \Big|_1^4 = \frac{4^2}{2} + 4 - \left(\frac{1^2}{2} + 1\right) = \frac{16}{2} + 4 - \left(\frac{1}{2} + 1\right)$

$= 8 + 4 - \frac{1}{2} - 1 = 11 - \frac{1}{2} = \frac{22-1}{2} = \boxed{\frac{21}{2}}$

⑩ $\int_0^2 (4x^2 - 2x)\,dx$ → Calcule termo a termo

$= \int_0^2 4x^2 dx - \int_0^2 2x\,dx = 4\int_0^2 x^2 dx - 2\int_0^2 x\,dx = 4 \cdot \frac{x^3}{3} - \frac{\cancel{2}x^2}{\cancel{2}}\Big|_0^2 = \frac{4}{3}x^3 - x^2 \Big|_0^2$

$= \frac{4}{3} \cdot 2^3 - 2^2 - \cancel{\left(\frac{4}{3} \cdot 0^3 - 0^2\right)} = \frac{4 \cdot 8}{3} - 4 = \frac{32}{3} - 4 = \frac{32-12}{3} = \boxed{\frac{20}{3}}$

⑪ $\int_0^1 (x^3 + x)\,dx$ → Aplique a regra da potência

$= \int_0^1 x^3 dx + \int_0^1 x\,dx = \frac{x^4}{4} + \frac{x^2}{2} \Big|_0^1 = \frac{1^4}{4} + \frac{1^2}{2} - \cancel{\left(\frac{0^4}{4} + \frac{0^2}{2}\right)} = \frac{1}{4} + \frac{1}{2} = \frac{1+2}{4} = \boxed{\frac{3}{4}}$

⑫ $\int_2^5 2x\,dx$ → A integral de $2x$ é x^2. $\leadsto$ $x^2 \Big|_2^5 = 5^2 - 2^2 = 25 - 4 = \boxed{21}$

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→

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(13) $\int_0^{\ln 2} e^x dx$ → Avalie e^x nos limites.

$\Leftarrow = e^x \Big|_0^{\ln 2} = e^{\ln 2} - \cancel{e^0}^{1} = 2-1 = \boxed{1}$

$e^{\ln a} = a$

(14) $\int_1^3 \frac{1}{x^2} dx$ → Reescreva como x^{-2}

$\Leftarrow = \int_1^3 x^{-2} dx = \frac{x^{-1}}{-1}\Big|_1^3 = -x^{-1}\Big|_1^3 = \frac{-1}{x}\Big|_1^3 = -\frac{1}{3} - \left(-\frac{1}{1}\right) = -\frac{1}{3} + 1 = \frac{-1+3}{3}$

$= \boxed{\frac{2}{3}}$

(15) $\int_0^1 (5-4x)dx$ → integral de uma função linear

$= \int_0^1 5dx - \int_0^1 4x dx = 5x - \frac{4x^2}{2}\Big|_0^1 = 5x - 2x^2\Big|_0^1 = 5.1 - 2.1^2 - \cancel{(5.0 - 2.0^2)}^{0} = 5-2 = \boxed{3}$

(16) $\int_0^2 (x^2+3x)dx$ → Calcule cada parte separadamente

$\Leftarrow \int_0^2 x^2 dx + \int_0^2 3x dx = \int_0^2 x^2 dx + 3\int_0^2 x dx = \frac{x^3}{3} + \frac{3x^2}{2}\Big|_0^2 = \frac{2^3}{3} + \frac{3.2^2}{2} - \cancel{\left(\frac{0^3}{3} + \frac{3.0}{2}\right)}^{0}$

$= \frac{8}{3} + 6 = \frac{8+18}{3} = \boxed{\frac{26}{3}}$

(17) $\int_1^2 (x^3 - x)dx$ → Lembre-se da integral de x^3

$= \int_1^2 x^3 dx - \int_1^2 x dx = \frac{x^4}{4} - \frac{x^2}{2}\Big|_1^2 = \frac{2^4}{4} - \frac{2^2}{2} - \left(\frac{1^4}{4} - \frac{1^2}{2}\right) = \frac{16}{4} - 2 - \left(\frac{1}{4} - \frac{1}{2}\right)$

$= 4 - 2 - \left(\frac{1-2}{4}\right) = 2 - \left(\frac{-1}{4}\right) = 2 + \frac{1}{4} = \frac{8+1}{4} = \boxed{\frac{9}{4}}$

(18) $\int_0^1 (x^4 + 2x^2)dx$ → Use a soma de polinômios

$= \int_0^1 x^4 dx + \int_0^1 2x^2 dx = \int_0^1 x^4 dx + 2\int_0^1 x^2 dx = \frac{x^5}{5} + 2.\frac{x^3}{3}\Big|_0^1 = \frac{1^5}{5} + 2.\frac{1^3}{3} - \cancel{\left(\frac{0^5}{5} + \frac{2.0}{3}\right)}^{0}$

$= \frac{1}{5} + \frac{2}{3} = \frac{3+10}{15} = \boxed{\frac{13}{15}}$

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(19) $\int_0^2 \frac{x}{2}\,dx$ → Coloque $\frac{1}{2}$ em evidência.

$= \frac{1}{2}\int_0^2 x\,dx = \frac{1}{2}\cdot\frac{x^2}{2}\Big|_0^2 = \frac{x^2}{4}\Big|_0^2 = \frac{\cancel{2^2}}{\cancel{4}} - \frac{\cancel{0^2}}{\cancel{4}} = \boxed{1}$

(20) $\int_0^\infty e^{-x}\,dx$ → integral imprópria, use limites.

$u = -x \rightarrow \frac{du}{dx} = -1 \rightarrow -du = dx$

$= \int_0^\infty e^u \cdot -du = -\int_0^\infty e^u\,du = -e^u\Big|_0^\infty = -e^{-x}\Big|_0^\infty = -e^{-\infty} - (-e^{-0}) = \lim_{x\to+\infty} -e^{-x} + \cancel{e^0}^{\,1}$

$= \lim_{x\to+\infty}\left(-\frac{1}{e^x}\right) + 1 = \boxed{1}$

$\frac{1}{n^\infty} = 0^+$ e $-\frac{1}{n^\infty} = 0^-$

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