S T Q Q S S D

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UNICAP - Ciência da Computação - 2º período
Disciplina: Elementos da integralização Computacional
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Lista II S - integrais por substituição

① $\int_0^1 2x(x^2+1)^3\,dx = 2\int_0^1 x\underbrace{(x^2+1)}^3 dx = \cancel{2}\int_0^1 u^3 \frac{du}{\cancel{2}} = \int_0^1 u^3 du$

$u = x^2+1$

$\frac{du}{dx} = 2x \rightarrow \frac{du}{2} = x\,dx$

$= \frac{u^4}{4}\Big]_0^1 = \frac{(x^2+1)^4}{4}\Big]_0^1 = \left(\frac{(1^2+1)^4}{4}\right) - \left(\frac{(0^2+1)^4}{4}\right) = \left(\frac{2^4}{4}\right) - \left(\frac{1^4}{4}\right)$

$= 4 - \frac{1}{4} = \frac{16}{4} - \frac{1}{4} = \frac{15}{4}$

② $\int_0^{\pi/4} \underbrace{\tan x}\sec^2 x\,dx = \int_0^{\pi/4} u\,du = \frac{u^2}{2}\Big]_0^{\pi/4} = \frac{\tan^2 x}{2}\Big]_0^{\pi/4}$

$u = \tan x$

$\frac{du}{dx} = \sec^2 x \rightarrow du = \sec^2 x\,dx$

$= \left(\frac{\tan^2 \overset{45°}{\pi/4}}{2}\right) - \left(\frac{\tan^2 0}{2}\right) = \frac{1^2}{2} - \frac{0}{2} = \frac{1}{2}$

$\frac{\sin^2 0}{\cos^2 0} = \frac{0^2}{1} = 0$

	sen	cos	tan
30	$1/2$	$\sqrt{3}/2$	$\sqrt{3}/3$
45	$\sqrt{2}/2$	$\sqrt{2}/2$	1
60	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$

③ $\int_1^e \frac{\ln x}{x}\,dx = \int_1^e u\,du = \frac{u^2}{2}\Big]_1^e = \frac{(\ln x)^2}{2}\Big]_1^e$

$u = \ln x$

$\frac{du}{dx} = \frac{1}{x} \quad du = \frac{1}{x}dx$

$= \left(\frac{(\ln e)^2}{2}\right) - \left(\frac{(\ln 1)^2}{2}\right) = \frac{1^2}{2} - 0 = \frac{1}{2}$

④ $\int_0^{\pi/2} \sin^3 x \cos x\,dx = \int_0^{\pi/2} u^3 du = \frac{u^4}{4}\Big]_0^{\pi/2} = \frac{\sin^4 x}{4}\Big]_0^{\pi/2} = \left(\frac{1^4}{4}\right) - \left(\frac{0^4}{4}\right)$

$u = \sin x$

$\frac{du}{dx} = \cos x \rightarrow du = \cos x\,dx$

$= \frac{1}{4}$

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$u = x^2$ $\qquad \frac{du}{dx} = 2x \to \frac{du}{2} = x\,dx$

(5) $\int_0^1 x \cdot e^{x^2} dx = \int_0^1 e^u \frac{du}{2} = \frac{1}{2}\int_0^1 e^u du = \frac{1}{2} \cdot e^u \Big]_0^1 = \frac{1}{2} e^{x^2} \Big]_0^1$

$= \left(\frac{1}{2} e^{1^2}\right) - \left(\frac{1}{2} e^{0^2}\right) = \frac{e}{2} - \frac{1}{2} = \boxed{\frac{e-1}{2}}$

(6) $\int_0^1 \frac{1}{\sqrt{1-x^2}} dx$ $\qquad \sin^2 y + \cos^2 y = 1 \to \underbrace{1 - \sin^2 y}_{1-x^2} = \cos^2 y$

$x = \sin y \to \frac{dx}{dy} = \cos y \to dx = \cos y\, dy$

↳ reescrevendo os limites: $\begin{cases} x = 0 \to y = 0 \\ x = 1 \to y = \pi/2 \end{cases}$ $\therefore \int_0^1 \frac{1}{\sqrt{1-x^2}} dx = \int_0^{\pi/2} \frac{1}{\sqrt{1-\sin^2 y}} \cos y\, dy$

$= \int_0^{\pi/2} \frac{1}{\sqrt{\cos^2 y}} \cos y\, dy = \int_0^{\pi/2} \frac{1}{|\cos y|} \cos y\, dy = \int_0^{\pi/2} \frac{1}{\cancel{\cos y}} \cancel{\cos y}\, dy$

$|\cos y| = \cos y$ (em $0 \leq y \leq \pi/2$) (é positivo)

$= \int_0^{\pi/2} 1\, dy = y \Big]_0^{\pi/2} = \frac{\pi}{2} - 0 = \boxed{\frac{\pi}{2}}$

(7) $\int_{-1}^1 x^3 \underbrace{\sqrt{1+x^4}}_{u = 1+x^4} dx = \int_{-1}^1 \sqrt{u} \frac{du}{4} = \frac{1}{4}\int_{-1}^1 u^{1/2} du = \frac{1}{4} \cdot \frac{u^{3/2}}{\frac{3}{2}} \Big]_{-1}^1 = \frac{1}{4} u^{3/2} \cdot \frac{2}{3} \Big]_{-1}^1$

$\frac{du}{dx} = 4x^3 \to \frac{du}{4} = x^3 dx$

$= \frac{\cancel{2} u^{3/2}}{6\cancel{/2}} \Big]_{-1}^1 = \frac{\sqrt{u^3}}{6} \Big]_{-1}^1 = \frac{\sqrt{(1+x^4)^3}}{6} \Big]_{-1}^1 = \left(\frac{\sqrt{(1+1^4)^3}}{6}\right) - \left(\frac{\sqrt{(1+(-1)^4)^3}}{6}\right)$

$= \frac{\sqrt{2^3}}{6} - \frac{\sqrt{2^3}}{6} = \boxed{0}$ $\qquad$ OU: $\frac{\sqrt{u^3}}{6} \Big]_?^? \to \begin{cases} x = 1;\ u = 2 \\ x = -1;\ u = 2 \end{cases} \to \frac{\sqrt{u^3}}{6} \Big]_2^2 = \boxed{0}$

pois $\frac{\sqrt{2^3}}{6} - \frac{\sqrt{2^3}}{6} = 0$ ← ñ há variação.

(8) $\int_0^{\pi/2} \sin 2x\, dx = \int_0^{\pi/2} \sin u \frac{du}{2} = \frac{1}{2}\int_0^{\pi/2} \sin u\, du = \frac{1}{2} \cdot (-\cos u) \Big]_0^{\pi/2}$

$u = 2x$

$\frac{du}{dx} = 2 \to \frac{du}{2} = dx$ $\qquad = -\frac{1}{2} \cos u \Big]_0^{\pi/2} = -\frac{1}{2} \cos 2x \Big]_0^{\pi/2} = \left(-\frac{1}{2} \cos \pi\right) - \left(-\frac{1}{2} \cos 0\right)$

$= -\frac{1}{2} \cdot -1 + \frac{1}{2} \cdot 1 = \frac{1}{2} + \frac{1}{2} = \boxed{1}$

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$\sin^2 y + \cos^2 y = 1$

$1 - \sin^2 y = \cos^2 y$

(9) $\int_0^1 (1-x^2)^{1/2}\,dx = \int_0^1 \sqrt{1-x^2}\,dx$

$x = \sin y \to \frac{dx}{dy} = \cos y \to dx = \cos y\,dy$

$\therefore \int_0^{\pi/2} \sqrt{1-\sin^2 y}\,\cos y\,dy$

$= \int_0^{\pi/2} \sqrt{\cos^2 y}\,\cos y\,dy$ $\to = |\cos y| = \cos y$ no intervalo $0 \leq y \leq \pi/2$

e $\begin{cases} x = 1\ ;\ y = \pi/2 \\ x = 0\ ;\ y = 0 \to \text{ou } y = \pi\ ? \end{cases}$

$\to \cos\pi = -1$; π não interessa, precisa ser >0 por causa da raiz.

$= \int_0^{\pi/2} \cos y \cos y\,dy = \int_0^{\pi/2} \cos^2 y\,dy \to \left\{\cos^2 y = \frac{1+\cos 2y}{2}\right.$

$u = 2y$; $\frac{du}{dy} = 2 \to dy = \frac{du}{2}$

$\therefore = \int_0^{\pi/2} \frac{1+\cos 2y}{2}\,dy = \frac{1}{2}\int_0^{\pi/2} (1+\cos 2y)\,dy = \frac{1}{2}\int_0^{\pi/2} 1\,dy + \frac{1}{2}\int_0^{\pi/2} \cos 2y\,dy$

$= \frac{1}{2}\int_0^{\pi/2} 1\,dy + \frac{1}{2}\int_0^{\pi/2} \cos u\,\frac{du}{2} = \frac{1}{2}\int_0^{\pi/2} 1\,dy + \frac{1}{4}\int_0^{\pi/2} \cos u\,du$

$= \frac{1}{2}y + \frac{1}{4}\sin 2y\Big]_0^{\pi/2} = \left(\frac{1}{2}\cdot\frac{\pi}{2} + \cancel{\frac{1}{4}\sin\pi}\right) - \left(\cancel{\frac{1}{2}\cdot 0} + \cancel{\frac{1}{4}\sin 0}\right)$

$= \frac{\pi}{4} - 0 = \boxed{\frac{\pi}{4}}$

(10) $\int_1^e \frac{1}{x} e^{\ln x}\,dx$ $\to u = \ln x$; $\frac{du}{dx} = \frac{1}{x}$ $\to du = \frac{1}{x}\,dx$ $\therefore = \int_?^? e^u\,du = e^u\Big]_?^?$

$\begin{cases} x = e\ ;\ u = 1 \\ x = 1\ ;\ u = 0 \end{cases}$ $\therefore = e^u\Big]_0^1 = e^1 - e^0 = \boxed{e-1}$

ou: $e^{\ln x}\Big]_1^e = e^{\ln e} - e^{\ln 1} = e - e^0 = \boxed{e-1}$

$e^{\ln a} = a$

(11) $\int_0^1 3x^2 e^{x^3}\,dx$, $u = x^3 \to \frac{du}{dx} = 3x^2$; $du = 3x^2\,dx$ e $\begin{cases} x = 1\ ;\ u = 1 \\ x = 0\ ;\ u = 0 \end{cases}$ $\therefore \int_0^1 e^u\,du = e^u\Big]_0^1$

$= e^1 - e^0 = \boxed{e-1}$

(12) $\int_0^{\pi/4} \sec^2 x\,dx$ $\left(u = \tan x \to \frac{du}{dx} = \sec^2 x\right.$ $\to du = \sec^2 x\,dx$ e $\begin{cases} x = \pi/4\ ;\ u = 1 \\ x = 0\ ;\ u = 0 \end{cases}$

$\therefore \int_0^1 du = u\Big]_0^1 = 1 - 0 = \boxed{1}$