S T Q Q S S D

07/10/25

UNICAP - Ciência da Computação - 2º período
Disciplina: Elementos da integralização Computacional
Docente: Tulio Fidel
Discente: Guilherme Harré

Lista #7 - Área entre curvas

$y = 2 - x^2$ e $y = -x$

(1) $\underbrace{f(x) = 2 - x^2 \text{ e } g(x) = -x}_{a<b}$

$$A = \int_a^b [f(x) - g(x)]\,dx$$

↳ a e b = pontos de intersecção

$\therefore f(x) = g(x) \to 2 - x^2 = -x \to x^2 - x - 2 = 0 \to \underbrace{(x-2)}_{x'=2}\underbrace{(x+1)}_{x''=-1} = 0$

b a

$\therefore \int_{-1}^{2} [2 - x^2 - (-x)]\,dx$

$= \int_{-1}^{2} (2 - x^2 + x)\,dx = \int_{-1}^{2} 2\,dx - \int_{-1}^{2} x^2\,dx + \int_{-1}^{2} x\,dx = 2x - \frac{x^3}{3} + \frac{x^2}{2}\Big|_{-1}^{2}$

$= 2.2 - \frac{2^3}{3} + \frac{2^2}{2} - \left(2.-1 - \frac{(-1)^3}{3} + \frac{(-1)^2}{2}\right) = 4 - \frac{8}{3} + \frac{\cancel{4}^{2}}{\cancel{2}} - \left(-2 + \frac{1}{3} + \frac{1}{2}\right)$

$= 4 - \frac{8}{3} + 2 + 2 - \frac{1}{3} - \frac{1}{2} = \frac{24 - 16 + 12 + 12 - 2 - 3}{6} = \frac{48 - 21}{6} = \frac{27}{6} = \boxed{\frac{9}{2}}$ u.a.

(2) $y = 0$; $\underbrace{x' = y^2 \text{ e } x'' = y + 2}_{\text{ponto de intersecção} = b}$ $\to y^2 = y + 2 \to y^2 - y - 2 = 0 \to (y-2)(y+1)$

a $y' = 2$ e $y'' = -1$

b   pq a área é só $y \geq 0$. (no gráfico)

$\therefore \int_0^2 [x'' - x']\,dy$

Como integramos em rel. a y, pegamos a função + à direita − a mais à esquerda!

$= \int_0^2 (y + 2 - y^2)\,dy = \int_0^2 y\,dy + 2\int_0^2 dy - \int_0^2 y^2\,dy = \frac{y^2}{2} + 2y - \frac{y^3}{3}\Big|_0^2$

$= \frac{2^{\cancel{2}}}{\cancel{2}} + 2.2 - \frac{2^3}{3} - \left(\frac{0^2}{2} + 2.0 - \frac{0^3}{3}\right) = 2 + 4 - \frac{8}{3} - 0 = 6 - \frac{8}{3} = \frac{18 - 8}{3} = \boxed{\frac{10}{3}}$

③ $y=0$ e $y=x\sqrt{4-x^2}$; $[-2,2]$ → são áreas simétricas. pode-se calcular de $[0,2]$ e multiplicar por 2.

$$\therefore A_2=\int_0^2 (x\sqrt{4-x^2}-0)\,dx=\int_0^2 x\underbrace{\sqrt{4-x^2}}_{u=4-x^2}dx=\int_0^2 u^{1/2}\frac{du}{-2}=-\frac{1}{2}\cdot\frac{u^{3/2}}{3/2}\Big|_0^2$$

$\frac{du}{dx}=-2x$

$-\frac{du}{2}=dx$

$$=-\frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{3}\cdot u^{3/2}\Big|_0^2=-\frac{1}{3}(4-x^2)^{3/2}\Big|_0^2=-\frac{1}{3}\cdot(\cancel{4-2^2})^{3/2}-\left(-\frac{1}{3}(4-0^2)^{3/2}\right)$$

$$=-\frac{1}{3}\cdot 0-\left(-\frac{1}{3}\sqrt{4^3}\right)=\frac{1}{3}\cdot 4\sqrt{4}=\boxed{\frac{8}{3}}\to 2\cdot\frac{8}{3}=\boxed{\frac{16}{3}}\text{ u.a.}$$

④ $y=1$ e $y=\cos^2 x$, $[0,\pi]$ → $\int_0^\pi (1-\cos^2x)\,dx=\int_0^\pi \sin^2 x\,dx$ ($u=\sin x$, $\frac{du}{dx}=\cos x$)

$\sin^2x+\cos^2x=1$

$\sin^2x=1-\cos^2x$

$\frac{1-\cos 2x}{2}$

$$=\int_0^\pi \frac{1-\cos 2x}{2}dx=\frac{1}{2}\int_0^\pi (1-\cos 2x)\,dx$$

$$=\frac{1}{2}\left[\int_0^\pi 1\,dx-\int_0^\pi \cos 2x\,dx\right]=\frac{1}{2}\left(x-\frac{1}{2}\sin 2x\right)\Big|_0^\pi=\frac{1}{2}\left(\pi-\frac{1}{2}\cancel{\sin 2\pi}\right)-\frac{1}{2}\left(\cancel{0-\frac{1}{2}\sin 0}\right)$$

$u=2x$, $\frac{du}{dx}=2$, $\frac{du}{2}=dx$

$$=\frac{1}{2}(\pi-0)=\boxed{\frac{\pi}{2}}\text{ u.a.}$$

⑤ $y=2x^2$ e $y=x^4-2x^2$; $[?,?]$ pontos de interseção serão a e b. (-2, 2)

↳ são simétricas em rel. à origem, então calculamos a área p/ $[0,2]$ e multiplicamos por 2.

$$\begin{cases}2x^2=x^4-2x^2\\ x^4-4x^2=0 \quad :x^2\\ x^2-4=0\\ x^2=4 \quad x=\pm\sqrt{4}=\pm 2\end{cases}$$

$$\therefore A_2=\int_0^2 [2x^2-(x^4-2x^2)]\,dx=\int_0^2(2x^2-x^4+2x^2)\,dx=\int_0^2(4x^2-x^4)\,dx$$

$$=4\int_0^2 x^2dx-\int_0^2 x^4dx=4\cdot\frac{x^3}{3}-\frac{x^5}{5}\Big|_0^2=4\cdot\frac{2^3}{3}-\frac{2^5}{5}-\cancel{\left(\frac{4\cdot 0^3}{3}-\frac{0^5}{5}\right)}$$

$32 \times 5 = 160$; $160-96=64$

$$=4\cdot\frac{8}{3}-\frac{32}{5}=\frac{32}{3}-\frac{32}{5}=\frac{160-96}{15}=\frac{64}{15}=A_2$$

$$\therefore A_T=2A_2=2\cdot\frac{64}{15}=\boxed{\frac{128}{15}}\text{ u.a.}$$

spiral

S T Q Q S S D   ___/___/___

(6) $x = 12y^2 - 12y^3$ e $x = 2y^2 - 2y$; $[0,1]$

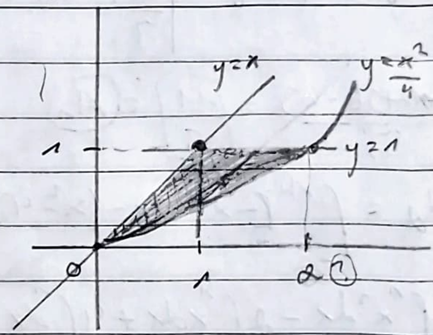
↳ $\int_0^1 [12y^2-12y^3-(2y^2-2y)]\,dy = \int_0^1 (12y^2-12y^3-2y^2+2y)\,dy$

$= \int_0^1(-12y^3+10y^2+2y)\,dy = -12\int_0^1 y^3\,dy + 10\int_0^1 y^2\,dy + 2\int_0^1 y\,dy$

$= -\cancel{12}^{3}\,\frac{y^4}{\cancel{4}} + 10\frac{y^3}{3} + \cancel{2}\frac{y^2}{\cancel{2}}\Big|_0^1 = -3y^4+\frac{10}{3}y^3+y^2\Big|_0^1$

$= -3\cdot1^4+\frac{10}{3}\cdot1^3+1^2-\left(\cancel{-3\cdot0^4+\frac{10}{3}\cdot0^3+0^2}\right) = -3+\frac{10}{3}+1 = \frac{-9+10+3}{3} = \boxed{\frac{4}{3}}$ u.a.

(7) $y=x$, $y=1$ e $y=\frac{x^2}{4}$



↳ inters. entre $y=1$ e $y=\frac{x^2}{4}$ é 2?

↳ $1 = \frac{x^2}{4} \rightarrow x^2=4 \rightarrow x=\pm 2$
só +2.

↳ 2 seções:

(I) $y=x$ e $y=\frac{x^2}{4}$ p/ $[0,1]$ e (II) $y=1$ e $y=\frac{x^2}{4}$ p/ $[1,2]$.

(I) $\int_0^1 \left(x-\frac{x^2}{4}\right)dx = \int_0^1 x\,dx - \frac14\int_0^1 x^2\,dx = \frac{x^2}{2}-\frac14\cdot\frac{x^3}{3}\Big|_0^1 = \frac{x^2}{2}-\frac{x^3}{12}\Big|_0^1$

$=\frac{6x^2-x^3}{12}\Big|_0^1 = \frac{6\cdot1^2-1^3}{12} - \left(\cancel{\frac{6\cdot0^2-0^3}{12}}\right)^0 = \boxed{\frac{5}{12}} = A_1$

(II) $\int_1^2\left(1-\frac{x^2}{4}\right)dx = \int_1^2 1\,dx - \frac14\int_1^2 x^2\,dx = x-\frac14\frac{x^3}{3}\Big|_1^2 = x-\frac{x^3}{12}\Big|_1^2$

$=\frac{12x-x^3}{12}\Big|_1^2 = \frac{12\cdot2-2^3}{12}-\left(\frac{12\cdot1-1^3}{12}\right) = \frac{24-8}{12}-\frac{11}{12}=\frac{16-11}{12}=\boxed{\frac{5}{12}}=A_2$

∴ $A_T = A_1+A_2 = \boxed{\frac{10}{12}} = \boxed{\frac{5}{6}\text{ u.a}}$

(8) $y=x^2-4$, $x=-3$ e $y=-x^2-2x$

↳ seções:

intersec. entre curvas.

(I) $y=x^2-4$ e $y=-x^2-2x$ p/ $[-3, ?]$ $\qquad x'=1 \qquad x''=-2$

↳ $x^2-4=-x^2-2x \rightarrow 2x^2+2x-4=0 \;\div 2 \rightarrow x^2+x-2=0 \rightarrow (x-1)(x+2)=0$

∴ p/ a figura, $[-3,-2]$ é a área de interesse.

→

__/__/__ S T Q Q S S D

II $y = -x^2 - 2x$ e $y = x^2 - 4$ p/ $[-2, 1]$ — intersec. entre curvas ($x>0$)

↳ Calculando.

I $A_1 = \int_{-3}^{-2} [x^2 - 4 - (-x^2 - 2x)]\,dx = \int_{-3}^{-2} (x^2 - 4 + x^2 + 2x)\,dx = \int_{-3}^{-2} (2x^2 + 2x - 4)\,dx$

$= 2\int_{-3}^{-2} x^2\,dx + 2\int_{-3}^{-2} x\,dx - 4\int_{-3}^{-2} dx = 2.\frac{x^3}{3} + \cancel{2}\frac{x^2}{\cancel{2}} - 4x \Big|_{-3}^{-2} = \frac{2}{3}x^3 + x^2 - 4x \Big|_{-3}^{-2}$

$= \frac{2}{3}(-2)^3 + (-2)^2 - 4.(-2) - \left(\frac{2}{3}(-3)^3 + (-3)^2 - 4(-3)\right)$

$= \frac{2}{3}.-8 + 4 + 8 - \left(\frac{2}{3}.-27 + 9 + 12\right) = -\frac{16}{3} + 12 - \left(-\frac{54}{3} + 21\right) = -\frac{16}{3} + 12 + \frac{54}{3} - 21$

$= \frac{54 - 16 + 36 - 63}{3} = \boxed{\frac{11}{3}} = A_1$

II $A_2 = \int_{-2}^{1} [-x^2 - 2x - (x^2 - 4)]\,dx = \int_{-2}^{1} (-x^2 - 2x - x^2 + 4)\,dx = \int_{-2}^{1} (-2x^2 - 2x + 4)\,dx$

$= -2\int_{-2}^{1} x^2\,dx - 2\int_{-2}^{1} x\,dx + 4\int_{-2}^{1} dx = -2\frac{x^3}{3} - \cancel{2}\frac{x^2}{\cancel{2}} + 4x \Big|_{-2}^{1} = -\frac{2}{3}x^3 - x^2 + 4x \Big|_{-2}^{1}$

$= -\frac{2}{3}.1^3 - 1^2 + 4 - \left(-\frac{2}{3}.(-2)^3 - (-2)^2 + 4(-2)\right) = -\frac{2}{3} - 1 + 4 - \left(\frac{16}{3} - 4 - 8\right)$

$= -\frac{2}{3} + 3 - \frac{16}{3} + 12 = \frac{-2 + 9 - 16 + 36}{3} \overset{20}{} = \boxed{\frac{27}{3}} = A_2$

∴ $A_T = A_1 + A_2 = \frac{11}{3} + \frac{27}{3} = \boxed{\frac{38}{3}}$ u.a.

9) $y = 4 - x^2$, $x = -2$, $x = 3$, $y = -x + 2$ p/ $[-2, 3]$

cosseções: I $y' = -x + 2$, $y'' = 4 - x^2$; $[-2, ?]$ (intrs.)

$4 - x^2 = -x + 2$
$x^2 - x - 2 = 0$
$(x+1)(x-2) = 0$
$\begin{cases} x' = -1 \\ x'' = 2 \end{cases}$ ∴ $[-2, -1]$ //

II $y' = 4 - x^2$, $y'' = -x + 2$; $[-1, 2]$ //

III $y' = -x + 2$, $y'' = 4 - x^2$; $[2, 3]$ //

↳ I $A_1 = \int_{-2}^{-1} [-x + 2 - (4 - x^2)]\,dx = \int_{-2}^{-1} (-x + 2 - 4 + x^2)\,dx = \int_{-2}^{-1} (x^2 - x - 2)\,dx$

$= \int_{-2}^{-1} x^2\,dx - \int_{-2}^{-1} x\,dx - 2\int_{-2}^{-1} dx = \frac{x^3}{3} - \frac{x^2}{2} - 2x \Big|_{-2}^{-1} = \frac{(-1)^3}{3} - \frac{(-1)^2}{2} - 2(-1) - \left(\frac{(-2)^3}{3} - \frac{(-2)^2}{2} - 2(-2)\right)$

→

$= -\frac{1}{3}-\frac{1}{2}+2-\left(-\frac{8}{3}-\frac{4}{2}+4\right) = -\frac{1}{3}-\frac{1}{2}+\cancel{2}+\frac{8}{3}+\cancel{2}-\cancel{4} = \frac{7}{3}-\frac{1}{2} = \frac{14-3}{6} = \boxed{\frac{11}{6}} = A_1$

↳ (II) $A_2 = \int_{-1}^{2}[4-x^2-(-x+2)]\,dx = \int_{-1}^{2}(4-x^2+x-2)\,dx = \int_{-1}^{2}(-x^2+x+2)\,dx$

$= -\int_{-1}^{2}x^2dx + \int_{-1}^{2}x\,dx + 2\int_{-1}^{2}dx = -\frac{x^3}{3}+\frac{x^2}{2}+2x\Big|_{-1}^{2} = -\frac{2^3}{3}+\frac{2^2}{2}+2\cdot 2-\left(-\frac{(-1)^3}{3}+\frac{(-1)^2}{2}+2\cdot(-1)\right)$

$= -\frac{8}{3}+\frac{4}{2}+4-\left(\frac{1}{3}+\frac{1}{2}-2\right) = -\frac{8}{3}+\frac{4}{2}+4-\frac{1}{3}-\frac{1}{2}+2 = \frac{-16+36-2-3+12}{6}$

$48 - 21 = 27$

$= \frac{27}{6} = \boxed{\frac{9}{2}} = A_2$

↳ (III) $\int_{2}^{3}[-x+2-(4-x^2)]\,dx = \int_{2}^{3}(-x+2-4+x^2)\,dx = \int_{2}^{3}(x^2-x-2)\,dx$

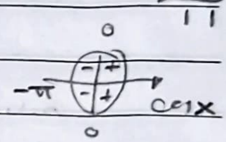
$= \int_{2}^{3}x^2dx - \int_{2}^{3}x\,dx - 2\int_{2}^{3}dx = \frac{x^3}{3}-\frac{x^2}{2}-2x\Big|_{2}^{3} = \frac{3^3}{3}-\frac{3^2}{2}-2\cdot 3-\left(\frac{2^3}{3}-\frac{2^2}{2}-2\cdot 2\right)$

$27 + 16 = 43$

$= 9-\frac{9}{2}-6-\frac{8}{3}+2+4 = 9-\frac{9}{2}-\frac{8}{3} = \frac{54-27-16}{6} = \boxed{\frac{11}{6}} = A_3$

$54 - 43 = 11$

$\therefore A_T = A_1+A_2+A_3 = \frac{11}{6}+\frac{9}{2}+\frac{11}{6} = \frac{22}{6}+\frac{9}{2} = \frac{22+27}{6} = \boxed{\frac{49}{6}}$



(10) $y=0$ e $y = 3\,\text{sen}\,x\sqrt{1+\cos x}$; $[-\pi, 0] \to |0 - 3\,\text{sen}\,x\sqrt{1+\cos x}|$ pq $y=0$ está acima

↳ $\int_{-\pi}^{0} -3\,\text{sen}\,x\sqrt{1+\cos x} = -3\int_{-\pi}^{0}\text{sen}\,x\sqrt{1+\cos x}\,dx \to -3\int_{-\pi}^{0}v^{1/2}\cdot -dv = 3\frac{v^{3/2}}{3/2}\Big|_{-\pi}^{0}$

$v = 1+\cos x$

$\frac{dv}{dx} = -\text{sen}\,x$ ∴ $-dv = \text{sen}\,x\,dx$

$= \cancel{3}\cdot\frac{2}{\cancel{3}}\sqrt{v^3}\Big|_{-\pi}^{0} = 2\sqrt{v^3}\Big|_{0}^{2} = 2\sqrt{2^3} - \cancel{2\cdot\sqrt{0^3}} = 2\cdot 2\sqrt{2} = \boxed{4\sqrt{2}}$ u.a.

$v = 1+\cos(-\pi)$
$v = 1-1 = 0$
$v = 1+\cos 0 = 1+1$
$v = 2$

(11) $x = 1-y^4$, $x = y^3-y$; $[-1,1] \to \int_{-1}^{1}[1-y^4-(y^3-y)]\,dy = \int_{-1}^{1}(1-y^4-y^3+y)\,dy$

$= \int_{-1}^{1}(-y^4-y^3+y+1)\,dy = -\int_{-1}^{1}y^4dy - \int_{-1}^{1}y^3dy + \int_{-1}^{1}y\,dy + \int_{-1}^{1}dy = -\frac{y^5}{5}-\frac{y^4}{4}+\frac{y^2}{2}+y\Big|_{-1}^{1}$

$= -\frac{1^5}{5}-\frac{1^4}{4}+\frac{1^2}{2}+1-\left(-\frac{(-1)^5}{5}-\frac{(-1)^4}{4}+\frac{(-1)^2}{2}-1\right) = -\frac{1}{5}-\frac{1}{4}+\frac{1}{2}+1-\left(\frac{1}{5}-\frac{1}{4}+\frac{1}{2}-1\right)$

$= -\frac{1}{5}-\cancel{\frac{1}{4}}+\cancel{\frac{1}{2}}+1-\frac{1}{5}+\cancel{\frac{1}{4}}-\cancel{\frac{1}{2}}+1 = -\frac{2}{5}+2 = \frac{-2+10}{5} = \boxed{\frac{8}{5}\text{ u.a.}}$

→

___/___/___ S T Q Q S S D

12) $y=-1,\ y=1,\ x=y^2-2$ e $x=e^y,\ [-1,1]$

$$A=\int_{-1}^{1}\left[e^y-(y^2-2)\right]dy=\int_{-1}^{1}(e^y-y^2+2)\,dy=\int_{-1}^{1}e^y\,dy-\int_{-1}^{1}y^2\,dy+2\int_{-1}^{1}dy$$

$$=e^y-\frac{y^3}{3}+2y\Big|_{-1}^{1}=e-\frac{1}{3}+2-\left(e^{-1}-\frac{(-1)^3}{3}+2(-1)\right)=e-\frac{1}{3}+2-\left(e^{-1}+\frac{1}{3}-2\right)$$

$$=e-\frac{1}{3}+2-e^{-1}-\frac{1}{3}+2=e-\frac{2}{3}-e^{-1}+4=\boxed{e-e^{-1}+\frac{10}{3}}\text{ u.a.}$$