UNICAP - Ciência da Computação - 2º período

Disciplina: Elementos da Integralização Computacional

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Lista #2 - integrais

$$\int u^p \, du = \frac{u^{p+1}}{p+1} + C \quad (p \neq 1)$$

① $\int k\,dx \rightarrow kx + C$

② $\int x^n\,dx \rightarrow \frac{x^{n+1}}{n+1} + C \quad (n \neq 1)$

③ $\int \frac{1}{x}\,dx = \int x^{-1}\,dx \rightarrow \ln(x) + C$

④ $\int e^x\,dx \rightarrow e^x + C$

⑤ $\int a^x\,dx \rightarrow \frac{a^x}{\ln(a)} + C \quad (a > 0 \text{ e } a \neq 1)$

↳ e.g. $\int (8e^x + 2\cdot 5^x)\,dx = \int 8e^x\,dx + \int 2\cdot 5^x\,dx$

$= 8\underbrace{\int e^x dx}_{e^x + C_1} + 2\int 5^x\,dx = 8e^x + C_1 + 2\frac{5^x}{\ln 5} + C_2$

$= 8e^x + 2\left(\frac{5^x}{\ln 5}\right) + C$ //

$d/dx(\sin x) = \cos x$

$d/dx(\cos x) = -\sin x$

⑥ $\int \operatorname{sen} x\,dx \rightarrow -\cos x + C$

⑦ $\int \cos x\,dx \rightarrow \operatorname{sen} x + C$

⑧ $\int \sec^2 x\,dx \rightarrow \tan x + C$

⑨ $\int \csc^2 x\,dx \rightarrow -\cot x + C$

⑩ $\int \sec x \tan x\,dx \rightarrow \sec x + C$

⑪ $\int \csc x \cot x\,dx \rightarrow -\csc x + C$

⑫ $\int \frac{1}{\sqrt{1-x^2}}\,dx \rightarrow \arcsin x + C$

$d/dx(\tan x) = \sec^2 x$

$d/dx(\csc x) = -\csc x \cot x$

$d/dx(\sec x) = \sec x \tan x$

$d/dx(\cot x) = -\csc^2 x$

↳ calculando $dy/dx(\arcsin x)$:

se $y = \arcsin x \rightarrow \boxed{\sin y = x} \xrightarrow{\text{derivando ambos os lados}} \frac{d \sin y}{dx} = \frac{dx}{dx} \xrightarrow{\text{regra da cadeia}} \cos y \cdot \frac{dy}{dx} = 1$ ↗ isolando

↳ $\frac{dy}{dx} = \frac{1}{\cos y} \rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{1-\sin^2 y}} \rightarrow \boxed{\frac{d(\arcsin x)}{dx} = \frac{1}{\sqrt{1-x^2}}}$!

$y = \arcsin x$

$\sin y = x$

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$\sin^2 x + \cos^2 y = 1$

$\sqrt{\cos^2 y} = \sqrt{1 - \sin^2 x}$

$\boxed{\cos y = \sqrt{1-\sin^2 x}}$

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(13) $\int \frac{1}{1+x^2}\,dx \to \arctan x + c$

(14) $\int \frac{1}{\sqrt{x^2-1}}\,dx$ $\quad \boxed{\sec^2 y - 1 = \tan^2 y} \to$ se $\boxed{x = \sec y} \to x^2 - 1 = \sec^2 y - 1$

$\hookrightarrow \int \frac{1}{\sqrt{\sec^2 y - 1}}\,dx = \int \frac{1}{\sqrt{\tan^2 y}}\,dx \quad \frac{dx}{dy} = \frac{d\sec y}{dy} = \sec y \tan y \to \boxed{d\sec y = \sec y \tan y\,dy}$

$\hookrightarrow \int \frac{1}{\cancel{\tan y}} \cdot \sec y \cancel{\tan y}\,dy = \int \sec y\,dy = \int \sec y \cdot \underbrace{\frac{\sec y + \tan y}{\sec y + \tan y}}_{= \frac{1}{1} = 1} dy$

$= \int \frac{\sec^2 y + \tan y}{\sec y + \tan y}\,dy$

$\quad \lfloor u = \sec y + \tan y \quad \frac{du}{dy} = \boxed{\sec y \tan y + \sec^2 y}$

$\quad \Rightarrow \boxed{du = \sec y \tan y + \sec^2 y\,dy}$

$\therefore \boxed{\int \frac{du}{u}} = \ln u + c$

$\hookrightarrow$ pq $\frac{du}{dx} \ln u = \boxed{\frac{1}{u} \cdot u'}$ $\qquad \therefore = \int \frac{(\sec y + \tan y)'}{\sec y + \tan y}\,dy$

$\therefore \int \sec y\,dy = \ln(\sec y + \tan y) + c$