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UNICAP - Ciência da Computação - 2º período

Disciplina: Elementos da integralização computacional

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Lista #8 - Comprimento do arco

$$\int_a^b \sqrt{1+[f'(x)]^2}\,dx$$

1) a) $y = 5x - 2$; $-2 \leq 2$

$\frac{dy}{dx} = 5 \therefore \int_{-2}^{2} \sqrt{1+5^2}\,dx = \int_{-2}^{2} \sqrt{26}\,dx$

$= \sqrt{26}\int_{-2}^{2} dx = \sqrt{26}\,x \Big|_{-2}^{2} = 2\sqrt{26} - (-2\sqrt{26}) = \boxed{4\sqrt{26}\ u.c.}$

b) $y = x^{2/3} - 1$; $1 \leq x \leq 2$

$\frac{dy}{dx} = \frac{2}{3}x^{-1/3} \therefore \int_1^2 \sqrt{1+\left(\frac{2}{3}x^{-1/3}\right)^2}\,dx = \int_1^2 \left[1+\left(\frac{2}{3}\cdot\frac{1}{x^{1/3}}\right)^2\right]^{1/2} dx$

$= \int_1^2 \left(1+\frac{4}{9}\cdot\frac{1}{x^{2/3}}\right)^{1/2} dx = \int_1^2 \left(1+\frac{4}{9x^{2/3}}\right)^{1/2} dx = \int_1^2 \left(\frac{9x^{2/3}+4}{9x^{2/3}}\right)^{1/2} dx$

$= \int_1^2 \frac{\sqrt{9x^{2/3}+4}}{3x^{1/3}}\,dx = \frac{1}{3}\int_1^2 \frac{\sqrt{9x^{2/3}+4}}{x^{1/3}}\,dx = \frac{1}{3}\int_1^2 x^{-1/3}\sqrt{\underbrace{9x^{2/3}+4}_{u}}\,dx$

$\frac{du}{dx} = 9\cdot\frac{2}{3}\cdot x^{-1/3}$

$\frac{du}{dx} = \frac{\cancel{18}^{6}}{\cancel{3}} x^{-1/3}$

$\frac{1}{6}du = x^{-1/3}dx$

$= \frac{1}{3}\int_1^2 u^{1/2}\frac{1}{6}du = \frac{1}{18}\int_1^2 u^{1/2}du = \frac{1}{18}\frac{u^{3/2}}{3/2}\Big|_1^2$

$= \frac{1}{\cancel{18}_{9}}\cdot\frac{2}{3}\,u^{3/2}\Big|_1^2 = \frac{1}{27}(9x^{2/3}+4)^{3/2}\Big|_1^2$

$= \frac{1}{27}(9\cdot 2^{2/3}+4)^{3/2} - \frac{1}{27}(9\cdot 1^{2/3}+4)^{3/2} = \frac{1}{27}(9\cdot 2^{2/3}+4)^{3/2} - \frac{1}{27}(13)^{3/2}$

$\boxed{= \frac{1}{27}\left[(9\cdot 2^{2/3}+4)^{3/2} - (13\sqrt{13})\right] u.c.}$

c) $y = \frac{1}{3}(2+x^2)^{3/2}$; $0 \leq x \leq 3$. $\to \int_0^3 \sqrt{1+[x(2+x^2)^{1/2}]^2}\,dx$

$\frac{dy}{dx} = 0\cdot\cancel{(2+x^2)^{3/2}} + \frac{1}{\cancel{3}}\cdot\frac{\cancel{3}}{\cancel{2}}(2+x^2)^{1/2}\cdot\cancel{2}x$

$\frac{dy}{dx} = x(2+x^2)^{1/2}$

$= \int_0^3 \sqrt{1+x^2(2+x^2)}\,dx$

$(a+b)^2 = a^2+2ab+b^2$

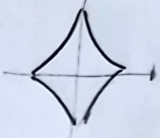
$= \int_0^3 \sqrt{1+2x^2+x^4}\,dx$; $x^4+2x^2+1^2 = (x^2+1)^2$

$= \int_0^3 \sqrt{(x^2+1)^2}\,dx = \int_0^3 (x^2+1)\,dx$ // $\int_0^3 x^2dx + \int_0^3 dx$

$= \frac{x^3}{3} + x\Big|_0^3$

$= \frac{3^{\cancel{3}\,2}}{\cancel{3}} + 3 - \left(\frac{0^3}{3}+0\right) = 9+3+0 = \boxed{12\ u.c.}$

spiral

__/__/__ → é um "astroid". $[0, 2\pi]$  S T Q Q S

d) $x^{2/3} + y^{2/3} = 2^{2/3}$

$x = a\cos^3 t$ e $y = a\,\mathrm{sen}^3 t$ entre $t \in [0, 2\pi]$ $\therefore$ $\boxed{x = 2\cos^3 t}$ e $\boxed{y = 2\,\mathrm{sen}^3 t}$

$a = 2$

↳ Fórmula p/ curva paramétrica:

$$\boxed{L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt}$$

$\frac{dx}{dt} = 2 \cdot 3\cos^2 t \cdot -\mathrm{sen}\, t$

$\frac{dy}{dt} = 2 \cdot 3 \cdot \mathrm{sen}^2 t \cdot \cos t$

$$\therefore L = \int_0^{2\pi} \sqrt{(-6\cos^2 t\,\mathrm{sen}\,t)^2 + (6\,\mathrm{sen}^2 t \cos t)^2}\,dt = \int_0^{2\pi} \sqrt{36\cos^4 t\,\mathrm{sen}^2 t + 36\,\mathrm{sen}^4 t \cos^2 t}\,dt$$

$$= \int_0^{2\pi} \sqrt{36\cos^2 t\,\mathrm{sen}^2 t\,(\cos^2 t + \mathrm{sen}^2 t)}\,dt = \int_0^{2\pi} [6\cos t\,\mathrm{sen}\,t\,\underbrace{(\cos^2 t + \mathrm{sen}^2 t)}_{1}{}^{1/2}]\,dt$$

$1 \to 1^{1/2} = 1$

$$= \int_0^{2\pi} (6\cos t\,\mathrm{sen}\,t)\,dt \to \int_0^{2\pi} (\cancel{6}^{3} \cdot \tfrac{1}{\cancel{2}}\,\mathrm{sen}\,2t)\,dt = 3\int_0^{2\pi} \mathrm{sen}\,2t\,dt$$

$\cos x\,\mathrm{sen}\,x = \frac{1}{2}\mathrm{sen}\,2x$

$$= 3 \cdot \int_0^{2\pi} \mathrm{sen}\,u\,\frac{du}{2}$$

$\left(\frac{du}{dt} = 2 \to \frac{du}{2} = dt\right.$

$$= 3 \cdot \frac{1}{2} \cdot \int_0^{2\pi} \mathrm{sen}\,u\,du \to \frac{3}{2} \cdot -\cos 2t \Big|_0^{2\pi}$$

↳ Se fizer de 0 a 2π, dará 0, pois a função é simétrica. Calculemos de 0 a $\frac{\pi}{2}$ e multiplicamos por 4:

$$-\frac{3}{2}\cos 2t \Big|_0^{\pi/2} = -\frac{3}{2}\cos\left(\cancel{2}\cdot\frac{\pi}{\cancel{2}}\right) - \left(-\frac{3}{2}\cos(2 \cdot 0)\right) = +\frac{3}{2} + \frac{3}{2} = \frac{6}{2} = \boxed{3}$$

$\cos\pi = -1$

↳ $[0, \pi/2] = 3 \times 4 = \boxed{12\ \text{u.c.}}$ p/ $[0, 2\pi]$.

e) $y = \ln x$; $\sqrt{3} \leq x \leq \sqrt{8}$

$\left(\frac{dy}{dx} = \frac{1}{x}\right.$

$$\therefore \int_{\sqrt{3}}^{\sqrt{8}} \sqrt{1 + (x^{-1})^2}\,dx = \int_{\sqrt{3}}^{\sqrt{8}} \sqrt{1 + x^{-2}}\,dx = \int_{\sqrt{3}}^{\sqrt{8}} \sqrt{1 + \frac{1}{x^2}}\,dx$$

$$= \int_{\sqrt{3}}^{\sqrt{8}} \sqrt{\frac{x^2+1}{x^2}}\,dx = \int_{\sqrt{3}}^{\sqrt{8}} \frac{\sqrt{x^2+1}}{x}\,dx$$ → subst. trigonométrica: $\boxed{x = \tan t}$

p/ integrandos:
- $\sqrt{a^2 - x^2} \to x = a\,\mathrm{sen}\,\theta$
- $\sqrt{a^2 + x^2} \to x = a\tan\theta$
- $\sqrt{x^2 - a^2} \to x = a\sec\theta$

$\therefore$ $\boxed{x = \tan t}$ e, derivamos ambos os lados p/ encontrar dx:

$$\frac{dx}{dt} = \frac{d(\tan t)}{dt} ; \boxed{dx = \sec^2 t\,dt}$$

↳ $\sqrt{x^2+1} \to \sqrt{\tan^2 t + 1} = \sqrt{\sec^2 t} = \sec t$ (p/ o intervalo positivo desejado)

$$\therefore \int_{\sqrt{3}}^{\sqrt{8}} \frac{\sec t}{\tan t} \cdot \sec^2 t\,dt = \int_{\sqrt{3}}^{\sqrt{8}} \frac{\sec^3 t}{\tan t}\,dt$$

	30	45	60
sen	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$
cos	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$
tan	$\sqrt{3}/3$	1	$\sqrt{3}$

→

$$\int_{\sqrt3}^{\sqrt8} \frac{\frac{1}{\cos^3 t}}{\frac{\sin t}{\cos t}}\,dt = \int_{\sqrt3}^{\sqrt8} \frac{1}{\cos^{3} t}\cdot\frac{\cos t}{\sin t}\,dt = \int_{\sqrt3}^{\sqrt8} \frac{1}{\cos^2 t\sin t}\,dt\left(\frac{\sin t}{\sin t}\right) = \int_{\sqrt3}^{\sqrt8} \frac{\sin t}{\cos^2 t\sin^2 t}\,dt$$

$$= \int_{\sqrt3}^{\sqrt8} \frac{\sin t}{\cos^2 t(1-\cos^2 t)}\,dt = \int_{\sqrt3}^{\sqrt8} \frac{-du}{u^2(1-u^2)} = -\int_{\sqrt3}^{\sqrt8} \frac{1}{u^2(1-u^2)}\,du \qquad \frac{1}{u^2}\cdot\frac{1}{1-u^2} = \frac{1}{u^2(1-u^2)}$$

$u = \cos t \quad \frac{du}{dt} = -\sin t$

$-du = \sin t\,dt$

$$= \frac{1-u^2+u^2}{u^2(1-u^2)} = \frac{1}{u^2} + \frac{1}{1-u^2} \to \frac{1/2}{1-u} + \frac{1/2}{1+u}$$

$$\therefore -\int_{\sqrt3}^{\sqrt8} \frac{1}{u^2(1-u^2)}\,du = -\int_{\sqrt3}^{\sqrt8}\left(\frac{1}{u^2}+\frac{1/2}{1-u}+\frac{1/2}{1+u}\right)du$$

$$= -\left(\int_{\sqrt3}^{\sqrt8}\frac{1}{u^2}\,du + \int_{\sqrt3}^{\sqrt8}\left(\frac12\cdot\frac{1}{1-u}\right)du + \int_{\sqrt3}^{\sqrt8}\left(\frac12\cdot\frac{1}{1+u}\right)du\right)$$

$$= -\left(\int_{\sqrt3}^{\sqrt8} u^{-2}\,du + \frac12\int_{\sqrt3}^{\sqrt8}(1-u)^{-1}\,du + \frac12\int_{\sqrt3}^{\sqrt8}(1+u)^{-1}\,du\right)$$

$$= -\left(\frac{u^{-1}}{-1} + \frac12\ln|1-u| + \frac12\ln|1+u|\right)\Big|_{\sqrt3}^{\sqrt8}$$

$$= \frac{1}{u} - \frac12\ln(1-u) - \frac12\ln(1+u)\Big|_{\sqrt3}^{\sqrt8} \qquad \begin{cases} u = \cos t \\ x = \tan t \end{cases} \quad \begin{cases} \cos t = \frac{1}{\sqrt{1+x^2}} \\ \sin t = \frac{x}{\sqrt{1+x^2}} \end{cases}$$

$\llcorner \sqrt{1+x^2}$

$$-\frac12\ln\left(\frac{1-u}{1+u}\right) = \frac12\ln\left(\frac{1+u}{1-u}\right) \qquad \frac{1+\frac{1}{\sqrt{1+x^2}}}{1-\frac{1}{\sqrt{1+x^2}}} \to \frac{P(x)}{Q(x)}$$

$$\hookrightarrow P(x) = \left(1+\frac{1}{\sqrt{1+x^2}}\right)\cdot\sqrt{1+x^2}$$

$$= \sqrt{1+x^2}+1 \quad e \quad Q(x) = \left(1-\frac{1}{\sqrt{1+x^2}}\right)\sqrt{1+x^2} = \sqrt{1+x^2}-1$$

$$\therefore \sqrt{1+x^2} - \frac12\ln\left(\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1}\right)\Big|_{\sqrt3}^{\sqrt8} = \sqrt{1+8} - \frac12\ln\left(\frac{\sqrt{1+8}+1}{\sqrt{1+8}-1}\right) - \left(\sqrt{1+3} - \frac12\ln\left(\frac{\sqrt{1+3}+1}{\sqrt{1+3}-1}\right)\right)$$

$$= \sqrt9 - \frac12\ln\left(\frac{\sqrt9+1}{\sqrt9-1}\right) - \left(\sqrt4 - \frac12\ln\left(\frac{\sqrt4+1}{\sqrt4-1}\right)\right) = 3 - \frac12\ln\left(\frac42\right) - \left(2 - \frac12\ln\left(\frac31\right)\right)$$

$$= 3 - \frac12\ln 2 - 2 + \frac12\ln 3 = 1 + \frac12\left(-\ln 2 + \ln 3\right) = \boxed{1+\frac12\ln\left(\frac32\right)} \text{ v.c.}$$

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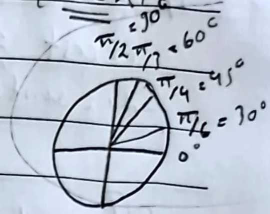
f) $y = 1 - \ln(\sin x)$, $\pi/6 \le x \le \pi/4$

$\frac{dy}{dx} = -\frac{1}{\sin x}\cdot \cos x = -\cot x \to \int_{\pi/6}^{\pi/4}\sqrt{1+(-\cot x)^2}\,dx$

$= \int_{\pi/6}^{\pi/4}\sqrt{\underbrace{1+\cot^2 x}_{\csc^2 x}}\,dx$ → pois: $\frac{\sin^2 x + \cos^2 x}{\sin^2 x} = \frac{1}{\sin^2 x} \to 1+\cot^2 x = \csc^2 x$



$= \int_{\pi/6}^{\pi/4}\sqrt{\csc^2 x}\,dx = \int_{\pi/6}^{\pi/4}\csc x\,dx \to \ln|\csc x - \cot x|\Big|_{\pi/6}^{\pi/4}$

(positivo p/ intervalo desejado)

	30	45	60
sen	1/2	√2/2	√3/2
cos	√3/2	√2/2	1/2
tan	√3/3	1	√3

$= \ln\left(\csc\frac{\pi}{4} - \cot\frac{\pi}{4}\right) - \ln\left(\csc\frac{\pi}{6} - \cot\frac{\pi}{6}\right)$

• $\frac{1}{\sin\frac{\pi}{4}} = \frac{1}{\sqrt{2}/2} = \frac{2}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$

$\frac{\cos\frac{\pi}{4}}{\sin\frac{\pi}{4}} = \frac{\sqrt{2}/2}{\sqrt{2}/2} = \frac{\sqrt{2}}{2}\cdot\frac{2}{\sqrt{2}} = \frac{2\sqrt{2}}{2\sqrt{2}} = 1$

• $\csc\frac{\pi}{6} = \frac{1}{\sin\frac{\pi}{6}} = \frac{1}{1/2} = 2$

• $\cot\pi/6 = \frac{\cos\pi/6}{\sin\pi/6} = \frac{\sqrt{3}/2}{1/2} = \frac{\sqrt{3}}{\cancel{2}}\cdot\frac{\cancel{2}}{1} = \sqrt{3}$

∴ $\ln(\sqrt{2}-1) - \ln(2-\sqrt{3}) = \boxed{\ln\left(\frac{\sqrt{2}-1}{2-\sqrt{3}}\right)\ u.c}$

g) $y = \sqrt{x^3}$ de $(0,0)$ a $(4,8)$

$\frac{dy}{dx} = \frac{3}{2}\cdot x^{1/2} \to \int_0^4\sqrt{1+\left(\frac{3}{2}x^{1/2}\right)^2}\,dx = \int_0^4\sqrt{1+\frac{9}{4}x}\,dx = \int_0^4\sqrt{\frac{4+9x}{4}}\,dx$

$= \int_0^4\frac{\sqrt{4+9x}}{2}\,dx = \frac{1}{2}\int_0^4\sqrt{\underbrace{4+9x}_{u=4+9x}}\,dx$ — $\frac{du}{dx} = 9 \to \frac{du}{9} = dx$

$= \frac{1}{2}\int_0^4 u^{1/2}\frac{du}{9} = \frac{1}{18}\frac{u^{3/2}}{3/2}\Big|_0^4 = \frac{1}{9\cancel{18}}\cdot\frac{\cancel{2}}{3}\cdot u^{3/2}\Big|_0^4 = \frac{1}{27}\cdot u^{3/2}\Big|_{(0)}^{(4)} \to \frac{1}{27}\cdot u^{3/2}\Big|_4^{40}$

$= \frac{1}{27}\cdot\sqrt{40^3} - \frac{1}{27}\cdot\sqrt{4^3} = \frac{1}{27}\cdot 80\sqrt{10} - \frac{1}{27}\cdot 4\sqrt{4} = \frac{80\sqrt{10}}{27} - \frac{8}{27}$

$(40^{1/2})^3 = (\sqrt{40})^3 = (\sqrt{4\cdot 10})^3$

$= (2\sqrt{10})^3 = 8\cdot\sqrt{10^3} = 8\cdot 10\sqrt{10} = 80\sqrt{10}$

$\boxed{= \frac{8}{27}(10\sqrt{10}-1)\ u.c.}$

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h) $y = 4\sqrt{x^3} + 2$ de $(0,2)$ a $(1,6)$ $\quad (x^{3/2})$

$$\frac{dy}{dx} = 4\cdot\frac{3}{2}\cdot x^{1/2} = 6x^{1/2} \therefore \int_0^1 \sqrt{1+(6x^{1/2})^2}\,dx = \int_0^1 \sqrt{\underbrace{1+36x}}\,dx = \int_0^1 u^{1/2}\frac{du}{36}$$

$u = 36x+1$

$\frac{du}{dx} = 36 \to \frac{du}{36} = dx$

$$= \frac{1}{36}\cdot\frac{u^{3/2}}{3/2}\Big|_1^{37} = \frac{1}{18\cdot 36}\cdot\frac{2}{3}\cdot u^{3/2}\Big|_1^{37} = \frac{1}{54}\cdot u^{3/2}\Big|_1^{37}$$

$$= \frac{1}{54}\cdot 37^{3/2} - \frac{1}{54}\cdot 1^{3/2} = \frac{37\sqrt{37}}{54} - \frac{1}{54} = \boxed{\frac{37\sqrt{37}-1}{54}\ u.c.}$$

i) $y = 6(x^{2/3} - 1)$, de $(1,0)$ a $(2\sqrt{2}, 6)$

$\boxed{y = 6x^{2/3} - 6}$

$$\frac{dy}{dx} = \cancel{6}\cdot\frac{2}{\cancel{3}}\cdot x^{-1/3} = 4x^{-1/3} \therefore L = \int_1^{2\sqrt{2}} \sqrt{1+(4x^{-1/3})^2}\,dx = \int_1^{2\sqrt{2}} \sqrt{1+16x^{-2/3}}\,dx = \int_1^{2\sqrt{2}} \sqrt{1+\frac{16}{x^{2/3}}}\,dx$$

$$= \int_1^{2\sqrt{2}} \sqrt{\frac{x^{2/3}+16}{x^{2/3}}}\,dx = \int_1^{2\sqrt{2}} \frac{\sqrt{x^{2/3}+16}}{x^{1/3}}\,dx = \int_1^{2\sqrt{2}} \frac{\sqrt{u^2+16}\cdot 3u^2\,du}{u} = \int_1^{2\sqrt{2}} \sqrt{u^2+16}\cdot 3u\,du$$

$(x^{2/3})^{1/2} = x^{1/3}$ $\quad u = x^{1/3} \therefore x = u^3 \to \frac{dx}{du} = \frac{d}{du}u^3 \to dx = 3u^2\,du$

$$= 3\int_1^{2\sqrt{2}} u\sqrt{\underbrace{u^2+16}}\,du = 3\int t^{1/2}\cdot\frac{dt}{2} = \frac{3}{2}\int_a^b t^{1/2}\,dt = \frac{\cancel{3}}{\cancel{2}}\cdot\frac{t^{3/2}}{\cancel{3/2}}\Big|_a^b = t^{3/2}\Big|_{17}^{18}$$

$t = u^2+16$

$\frac{dt}{du} = 2u$

$\frac{dt}{2} = u\,du$

$\begin{cases} u = x^{1/3} \\ t = u^2+16 = x^{2/3}+16 \end{cases} \to \begin{cases} a = 1^{2/3}+16 = 17 \\ b = (2\sqrt{2})^{2/3}+16 = \sqrt[3]{(2\sqrt{2})^2}+16 \end{cases}$

$= \sqrt[3]{4\cdot 2}+16 = 2+16 = 18$

$\therefore 18^{3/2} - 17^{3/2} = 18\sqrt{18} - 17\sqrt{17}$

$(18^{1/2})^3 = (\sqrt{18})^3$

$= (\sqrt{9\cdot 2})^3 = (3\sqrt{2})^3 = 27\cdot(2^{1/2})^3 = 27\sqrt{2^3} = 27\cdot 2\sqrt{2} = 54\sqrt{2}$

$\boxed{\therefore L = 54\sqrt{2} - 17\sqrt{17}\ u.c.}$

j) $(y-1)^2 = (x+4)^3$ de $(-3,2)$ a $(0,9)$ $\therefore \int_{-3}^0 \sqrt{1+\left(\frac{3}{2}(x+4)^{1/2}\right)^2}\,dx = L$

$4^{3/2} = \sqrt{4^3} = 4\sqrt{4} = 8$

$y - 1 = \pm(x+4)^{3/2}$

$y = (x+4)^{3/2} + 1 \to \frac{dy}{dx} = \frac{3}{2}\cdot(x+4)^{1/2}\cdot 1 + 0 = \frac{3}{2}(x+4)^{1/2}$

positivo no intervalo desejado.

$$L = \int_{-3}^0 \sqrt{1+\frac{9}{4}(x+4)}\,dx = \int_{-3}^0 \sqrt{\frac{4+9x+36}{4}}\,dx = \int_{-3}^0 \sqrt{\frac{9x+40}{4}}\,dx = \int_{-3}^0 \frac{\sqrt{9x+40}}{2}\,dx$$

$$= \frac{1}{2}\int_{-3}^0 \sqrt{\underbrace{9x+40}}\,dx \to \frac{1}{2}\int_{-3}^0 u^{1/2}\frac{du}{9} = \frac{1}{18}\cdot\frac{u^{3/2}}{3/2}\Big|_{-3}^0 = \frac{1}{9\cdot 18}\cdot\frac{2}{3}\cdot u^{3/2}\Big|_{-3}^0 = \frac{1}{27}u^{3/2}\Big|_{-3}^0 \to$$

$u = 9x+40$

$\frac{du}{dx} = 9 \to dx = \frac{du}{9}$

___/___/___ $u = 9x + 40$
intervalos em $x = [-3, 0]$
interv. em $u = [13, 40]$

$$= \frac{1}{27} \cdot u^{3/2} \Big|_{13}^{40} = \frac{1}{27} \cdot \sqrt{40^3} - \frac{1}{27} \cdot \sqrt{13^3} = \boxed{\frac{80\sqrt{10} - 13\sqrt{13}}{27} \text{ u.c.}}$$

$$40^{3/2} = (40^{1/2})^3$$
$$= (\sqrt{40})^3 = (\sqrt{10 \cdot 4})^3 = (2\sqrt{10})^3$$
$$= 8 \cdot 10\sqrt{10} = 80\sqrt{10}$$

k) $x^2 = y^3$, de $(0,0)$ a $(8,4)$ $\therefore L = \int_0^4 \sqrt{1 + \left(\frac{3}{2} y^{1/2}\right)^2} dy$

$x = \pm y^{3/2} = y^{3/2}$ → p/ intervalo desejado. → $\frac{dx}{dy} = \frac{3}{2} \cdot y^{1/2}$

$$= \int_0^4 \sqrt{1 + \frac{9}{4} y}\, dy = \int_0^4 \sqrt{\frac{4 + 9y}{4}}\, dy = \int_0^4 \frac{\sqrt{4 + 9y}}{2} dy = \frac{1}{2} \int_0^4 \underbrace{\sqrt{4 + 9y}}_{u = 4 + 9y}\, dy = \frac{1}{2} \int u^{1/2} \cdot \frac{du}{9}$$

$$\frac{du}{dy} = 9 \quad \frac{du}{9} = dy$$

$$= \frac{1}{18} \cdot \frac{u^{3/2}}{3/2} \Big|_a^b = \frac{1}{\cancel{9}} \cdot \frac{\cancel{2}}{3} \cdot u^{3/2} \Big|_a^b = \frac{1}{27} u^{3/2} \Big|_a^b \quad \begin{cases} u = 9y + 4 \therefore \boxed{a = 4} \text{ e} \\ \boxed{b = 40} \end{cases}$$

$$\frac{1}{27} u^{3/2} \Big|_4^{40} = \frac{1}{27} \sqrt{40^3} - \frac{1}{27} \cdot \sqrt{4^3} = \frac{80\sqrt{10}}{27} - \frac{8}{27} = \boxed{\frac{8}{27} (10\sqrt{10} - 1) \text{ u.c.}}$$

$$40^{3/2} = (\sqrt{40})^3 = (\sqrt{10 \cdot 4})^3$$
$$= (2\sqrt{10})^3 = 8 \cdot 10\sqrt{10} = 80\sqrt{10}$$