Lista - 4 Calculo 3 - Guilber Leal

Questão 34

Vamos encontrar possíveis valores de $I = \int_C \frac{(x+y)\,dx + (y-x)\,dy}{x^2+y^2}$, onde C é uma curva fechada qualquer que não passa pela origem, temos:

$$I = \int_C \frac{x+y}{x^2+y^2}\,dx + \frac{y-x}{x^2+y^2}\,dy$$

$$I = \iint_R \left[\frac{\partial}{\partial x}\left(\frac{y-x}{x^2+y^2}\right) - \frac{\partial}{\partial y}\left(\frac{x+y}{x^2+y^2}\right) \right] dx\,dy$$

$$\Rightarrow \iint_R \left[\frac{-(x^2+y^2) - (y-x)\cdot(2x)}{(x^2+y^2)^2} - \left[\frac{x^2+y^2 - (x+y)(2y)}{(x^2+y^2)^2}\right] \right] dx\,dy$$

$$\Rightarrow \iint_R \left[\frac{-(x^2+y^2) - (x^2+y^2) - 2xy + 2x^2 + 2xy + 2y^2}{(x^2+y^2)^2} \right] dx\,dy$$

$$\Rightarrow \iint_R \left[\frac{(-2x^2+2x^2) + (-2y^2+2y^2) + (-2xy+2xy)}{(x^2+y^2)^2} \right] dx\,dy$$

$$\Rightarrow \iint_R (0)\,dx\,dy$$

$$\Rightarrow 0 \iint_R dx\,dy$$

$$\Rightarrow 0_{//}$$

Questão 13

Vamos verificar o teorema de Green para $\vec{F}(x,y) = (4x-2y,\ 2x+6y)$, onde D é a região interior à elipse $\frac{x^2}{4}+y^2=1$. temos:

$$\oint_C (4x-2y)\,dy - (2x+6y)\,dx = \iint_D \frac{\partial}{\partial x}(4x-2y) + \frac{\partial}{\partial y}(2x+6y)\,dA$$

$= \iint_D (4+6)\,dx\,dy$
$= 10\iint_D dx\,dy$
$= 10$ Area (Elipse)
$= 10(\pi(2)(1))$
$= 20\pi$,,

Questão 36

Vamos calcular $\int_C \vec{F}.d\vec{r}$, onde $\vec{F}(x,y) = (y^3+1)i + (3xy^2+1)j$ e C é a semicircunferência $(x-1)^2+y^2=1$, com $y \geq 0$, começando de $(0,0)$ a $(2,0)$. temos:

$$\int_C \vec{F}.d\vec{r} = \iint_R \left[\frac{\partial}{\partial x}(3xy^2+1) - \frac{\partial}{\partial y}(y^3+1)\right] dx\,dy$$

$\Rightarrow \iint_R (3y^2-3y^2)\,dx\,dy$

$\Rightarrow \iint_R (0)\,dx\,dy$

$\Rightarrow 0$,,

Lista 5

Questão 6

$$A(S) = \iint_S 1\, dS$$

$$A(S) = \frac{14\pi}{3}$$

$$\iint_S 1\, dS = \frac{14\pi}{3}$$

Estamos lidando com uma paraboloide, então

$$x = r\cos\theta$$
$$y = r\sin\theta$$

Como $x^2 + y^2 = r^2$

$$2z = x^2 + y^2 \rightarrow 2z = r^2 \rightarrow z = \frac{r^2}{2}$$

$$\vec{\sigma}(r,\theta) = \left(r\cos\theta, r\sin\theta, \frac{r^2}{2}\right)$$

$0 \leq \theta \leq 2\pi$

$$z = \frac{r^2}{2}$$

$z = 0$

$r = 0$

$r = \sqrt{2k}$

$$dS = |N|\, dr\, d\theta$$

$$\vec{N} = \frac{\sigma(r,\theta)}{\partial r} \times \frac{\sigma(r,\theta)}{\partial \theta}$$

$$N = \frac{\partial r(u,v)}{\partial u} \times \frac{\partial r(u,v)}{\partial v}$$

$\vec{N} = -r^2 \sin\theta \vec{j} + r\cos^2\theta \vec{k} - (r^2 \cos\theta \vec{i} - r\sin\theta \vec{k})$

$\vec{N} = -r^2 \cos\theta \vec{i} - r^2 \sin\theta \vec{j} + r\cos^2\theta \vec{k} + r\sin^2\theta \vec{k}$

$\vec{N} = -r^2 \cos\theta \vec{i} - r^2 \sin\theta \vec{j} + r(\cos^2\theta + \sin^2\theta) \vec{k}$

$\vec{N} = -r^2 \cos\theta \vec{i} - r^2 \sin\theta \vec{j} + r\vec{k}$

$\vec{N} = (-r^2 \cos\theta, -r^2 \sin\theta, r)$

$|\vec{N}| = \sqrt{(-r^2\cos\theta)^2 + (-r^2 \sin\theta)^2 + (r)^2}$

$|\vec{N}| = \sqrt{r^4 \cos^2\theta + r^4 \sin^2\theta + r^2}$

$|\vec{N}| = \sqrt{r^4(\cos^2\theta + \sin\theta) + r^2}$

$|\vec{N}| = \sqrt{r^4 + r^2}$

$|\vec{N}| = \sqrt{r^2(1+r^2)}$

$|\vec{N}| = r\sqrt{1+r^2}$

$dS = r\sqrt{(1+r^2)}\, dr\, d\theta$

$14\pi = \iint_S 1\, dS$

$$\iint_S 1\, dS = \int_0^{2\pi}\int_0^{\sqrt{2k}} r\sqrt{(1+r^2)}\, dr\, d\theta$$

$$\int_0^{2\pi}\int_0^{\sqrt{2k}} r\sqrt{(1+r^2)}\, dr\, d\theta$$

$$\int_0^{2\pi}\int_0^{\sqrt{2k}} r\sqrt{(1+r^2)}\, dr\, d\theta = \int_0^{2\pi}\int_1^{1+2k} \sqrt{t}\, \frac{dt}{2}\, d\theta$$

$$\int_0^{2\pi} \frac{1}{2}\left[\left(\frac{2}{3} t^{\frac{3}{2}}\right)\right]_1^{1+2k} d\theta = \int_0^{2\pi} \frac{1}{2}\left[\left(\frac{2}{3} t\sqrt{t}\right)\right]_1^{1+2k} d\theta$$

$$\int_0^{2\pi} \frac{1}{2}\left(\frac{2}{3}(1+2k)\sqrt{1+2k} - \frac{2}{3}\right) d\theta$$

$$\int_0^{2\pi} \frac{1}{3}(1+2k)\sqrt{1+2k} - \frac{1}{3}) d\theta$$

$$\left(\frac{(1+2k)\sqrt{1+2k} - 1}{3}\right)\int_0^{2\pi} d\theta$$

$$\frac{\sqrt{(1+2k)^3}-1}{3} \cdot 2\pi$$

$$\frac{14}{3}\pi = \frac{(\sqrt{(1+2k)^3}-1}{3} \cdot 2\pi$$

$$\sqrt{(1+2k)^3}-1 = 7$$

$$\sqrt{(1+2k)^3} = 8$$

$$(1+2k)^3 = 64$$

$$1+2k = \sqrt{64}$$

$$1+2k = 4$$

$$k = \frac{3}{2}$$ //

Questão 7 -

$$\text{Area} = \iint_S ||\vec{N}||\, dS$$

$z = 1 - x^2$

$x^2 = y^2$

$y = |x|$

$$\vec{N} = \left(-\frac{\partial F}{\partial x}, -\frac{\partial F}{\partial y}, 1\right)$$

$$\frac{\partial F}{\partial x} = -2x$$

$$\frac{\partial F}{\partial y} = 0$$

$$\vec{N} = (2x, 0, 1)$$

$$||\vec{N}|| = \sqrt{(2x)^2 + 0^2 + 1^2} = \sqrt{4x^2+1}$$

$$\iint_S ||\vec{N}||\, dS = \int_{-1}^{0}\int_{0}^{-x} \sqrt{4x^2+1}\, dy\, dx + \int_{0}^{1}\int_{0}^{x} \sqrt{4x^2+1}\, dy\, dx$$

$$\int_0^1 x\sqrt{4x^2+1}\, dx - \int_{-1}^{0} x\sqrt{4x^2+1}\, dx$$

$$4x^2+1 = u$$

$$8x\, dx = du$$

$$x\, dx = \frac{du}{8}$$

$$\frac{1}{8}\int_1^5 \sqrt{u}\,du - \frac{1}{8}\int_5^1 \sqrt{u}\,du$$

$$\frac{1}{8}\int_1^5 \sqrt{u}\,du + \frac{1}{8}\int_1^5 \sqrt{u}\,du$$

$$\frac{1}{4}\int_1^5 \sqrt{u}\,du = \frac{1}{4}\left[\frac{2}{3}u^{\frac{3}{2}}\right]_1^5$$

$$\frac{1}{6}(5\sqrt{5}-1)$$

$$\frac{\pi}{6}(5\sqrt{5}-1)_{//}$$

Questão 16

$y(t) = t$ $\qquad \varphi(\theta, t) = (\ln t, t\cos\theta, t\,\mathrm{sen}\,\theta)$

$x(t) = \ln t$

$$m = \iint_S d_{ox}\,dS$$

$$d_{ox} = \sqrt{y^2+z^2}\,dS$$

$$m = \iint_S k\sqrt{y^2+z^2}\,dS$$

$$m = k\iint_S \sqrt{y^2+z^2}\,dS$$

$N(t,\theta) = (t\cos^2\theta)\,i + (0)\,j + (-\mathrm{sen}\,\theta)\,k - (0)\,k - (\cos\theta)\,j - (-t\,\mathrm{sen}^2\theta)\,i.$

$$m = k\iint_S \sqrt{y^2+z^2}\,dS$$

$$m = k\int_0^{2\pi}\int_{\sqrt{3}}^{\sqrt{8}} \sqrt{(t\cos\theta)^2+(t\,\mathrm{sen}\,\theta)^2}\,\sqrt{t^2+1}\,dt\,d\theta$$

$$m = k\int_0^{2\pi}\int_{\sqrt{3}}^{\sqrt{8}} \sqrt{t^2}\,\sqrt{t^2+1}\,dt\,d\theta$$

$$m = k\int_0^{2\pi}\int_{\sqrt{3}}^{\sqrt{8}} t\sqrt{t^2+1}\,dt\,d\theta$$

$$\int t\sqrt{t^2+1}\,dt$$

$du = 2t\,dt$

$t\,dt = \frac{du}{2}$

$$\int \frac{\sqrt{u}\,du}{2} = \int \frac{u^{\frac{1}{2}}}{2} = \frac{u^{\frac{1}{2}+1}}{2(\frac{1}{2}+1)} = \frac{u^{\frac{3}{2}}}{2(\frac{3}{2})} = \frac{u^{\frac{3}{2}}}{3}.$$

$$\frac{u^{\frac{3}{2}}}{3} = \frac{(t^2+1)^{\frac{3}{2}}}{3}$$

$$M = k\int_0^{2\pi}\int_{\sqrt{3}}^{\sqrt{8}} t\sqrt{t^2+1}\,dt\,d\theta = \int_0^{2\pi}\left[\frac{(t^2+1)^{\frac{3}{2}}}{3}\right]_{t=\sqrt{3}}^{\sqrt{8}} d\theta$$

$$M = k\int_0^{2\pi}\left[\frac{((\sqrt{8})^2+1)^{\frac{3}{2}}}{3} - \frac{((\sqrt{3})^2+1)^{\frac{3}{2}}}{3}\right] d\theta$$

$$M = k\int_0^{2\pi}\left[\left(\frac{(8+1)^{\frac{3}{2}}}{3} - \frac{(3+1)^{\frac{3}{2}}}{3}\right)\right] d\theta$$

$$M = k\int_0^{2\pi}\left[\left(\frac{9^{\frac{3}{2}}}{3}\right) - \left(\frac{4^{\frac{3}{2}}}{3}\right)\right] d\theta$$

$$M = k\int_0^{2\pi}\left[\frac{27}{3} - \frac{8}{3}\right] d\theta$$

$$M = k\int_0^{2\pi}\left[\frac{19}{3}\right] d\theta$$

$$M = k\left[\frac{19}{3}\theta\right]_{\theta=0}^{2\pi}$$

$$M = k\left[\frac{19}{3}(2\pi - 0)\right]$$

$$M = k\,\frac{19}{3}\,2\pi = \frac{38\,k\,\pi}{3}$$

Lista 6

Questão 2

$$\iint_S \vec{F} \cdot d\vec{S}$$

$$\iint_S \vec{F} \cdot d\vec{S} = \iiint_W div(\vec{F}) dV$$

$$\iiint_W div(\vec{F}) dV = \iiint_W 9\, dV = 9 \iiint_W dV$$

$$9 \iiint_W dV = 9\left(\frac{4}{3} \pi 5^3\right) = 12\pi \cdot 125 = 1500\pi$$

$$\iint_S \vec{F} \cdot d\vec{S} = 1500\pi_{//}$$

Questão 4

$$\iint_S \vec{F} \cdot \vec{n}\, dS$$

$$\iint_S \vec{F} \cdot \vec{n}\, d\vec{S} = \iiint_W div(\vec{F})\, dV$$

$$\iiint_W div(\vec{F})\, dV = \iiint_W 3(x^2+y^2+z^2)\, dV$$

$$\iiint_W 3(x^2+y^2+z^2)\, dV$$

$$\iiint_W 3(x^2+y^2+z^2)\, dV = \int_0^{2\pi} \int_0^1 \int_0^{\pi} 3\rho^2 \cdot \cancel{\rho} d\varphi d\rho d\theta$$

$$\int_0^{2\pi} \int_0^1 \int_0^{\pi} 3\rho^2 \cdot \rho^2 \sin\varphi\, d\varphi\, d\rho\, d\theta =$$

$$3\int_0^{2\pi} \int_0^1 \rho^4 \left(\int_0^{\pi} \sin\varphi\, d\varphi\right) d\rho\, d\theta = 3\int_0^{2\pi} \int_0^1 \rho^4 (-\cos\pi - (-\cos 0)$$

$d\rho d\theta$.

$$3 \cdot 2 \int_0^{2\pi} \int_0^1 \rho^4\, d\rho\, d\theta = 6 \int_0^{2\pi} \left(\int_0^1 \rho^4\, d\rho\right) d\theta$$

$$6\int_0^{2\pi}\left[\frac{p^5}{5}\right]_0^1 d\theta$$

$$\frac{6}{5}\int_0^{2\pi} d\theta = \frac{12}{5}\pi$$

$$\iint_S \vec{F}\cdot\vec{n}\, dS = \frac{12}{5}\pi$$

Questão 17

$$\iint_S \frac{\partial F}{\partial \vec{n}}\, dS = \iiint_W \nabla^2 F\, dx\, dy\, dz$$

$$\iint_S \vec{F}\cdot\vec{n}\, dS = \iiint_W \operatorname{div}(\vec{F})\, dx\, dy\, dz$$

$$\iint_S \frac{\partial F}{\partial \vec{n}}\, dS = \iint_S \nabla F\cdot\vec{n}\, dS$$

$$\iint_S \nabla F\cdot\vec{n}\, dS = \iint_S \vec{F}\cdot\vec{n}\, dS$$

$$\iiint_W \operatorname{div}(\vec{F})\, dx\, dy\, dz$$

$$\operatorname{div}\vec{F} = \nabla\cdot\vec{F}$$

$$\nabla\cdot\nabla F = \nabla^2 F$$

$$\iint_S \vec{F}\cdot\vec{n}\, dS = \iiint_W \operatorname{div}(\vec{F})\, dx\, dy\, dz$$

$$\iint_S \frac{\partial F}{\partial \vec{n}}\, dS = \iiint_W \nabla^2 F\, dx\, dy\, dz$$