Prova - Calculo 3 - Guilber Leal

$X = [-1, 1]$

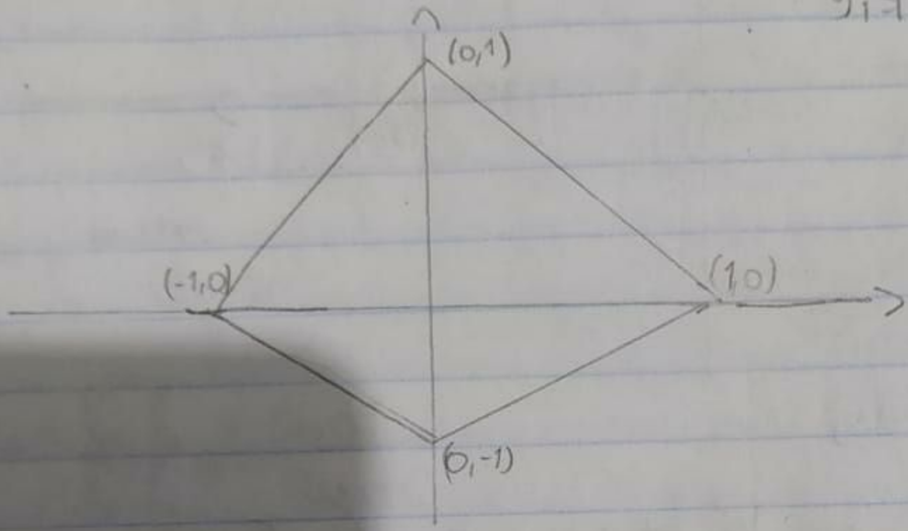
$y_1 = |x| - 1$

$y_2 = -|x| + 1$

$y_1 = |x| - 1, y_2 = -|x| + 1$

(1)

Graficamente



$$\int_{-1}^{1} \int_{|x|-1}^{-|x|+1} dy\, dx$$

$$\int_{|x|-1}^{-|x|+1} dy \Rightarrow \left[y \right]_{-1+|x|}^{1-|x|} \Rightarrow 1 - |x| - (-1 + |x|) = -2.|x| + 2$$

$$\Rightarrow \int_{-1}^{1} (-2.|x| + 2)\, dx \Rightarrow \int_{-1}^{0} -2(-x) + 2\, dx + \int_{0}^{1} -2x + 2\, dx$$

$$\Rightarrow 2\left[\frac{x^2}{2} \right]_{-1}^{0} + \left[2x \right]_{-1}^{0} - 2\left[\frac{x^2}{2} \right]_{0}^{1} + \left[2x \right]_{0}^{1}$$

$$\Rightarrow 0 - 1 + 2 - 1 - 0 + 2 - 0 \Rightarrow 2_{//}$$

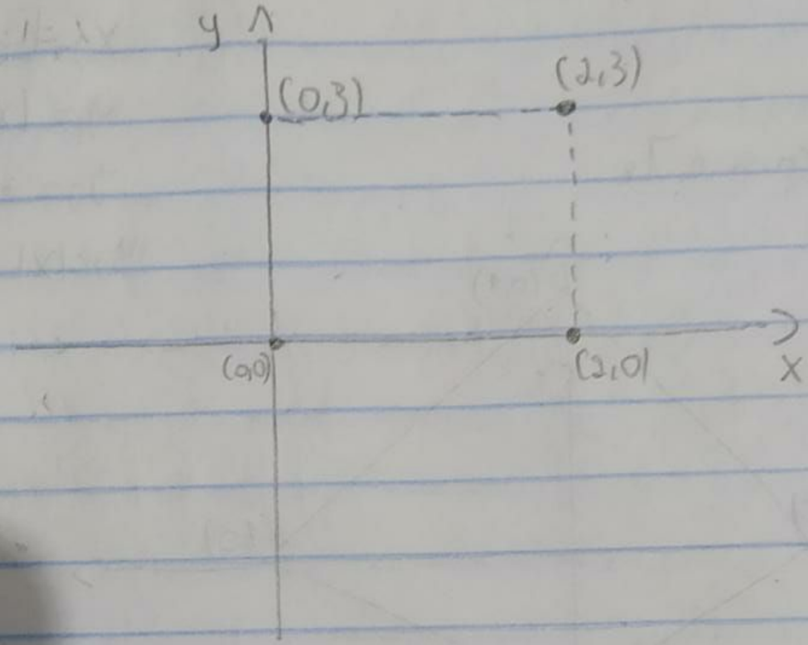
R: $2_{//}$

②

$0 = 4 - x^2$

$x^2 = 4$

$x = \pm 2$



$x = [0,2]$

$y = [0,3]$

$$\int_0^3 \int_0^2 (4 - x^2)\,dx\,dy$$

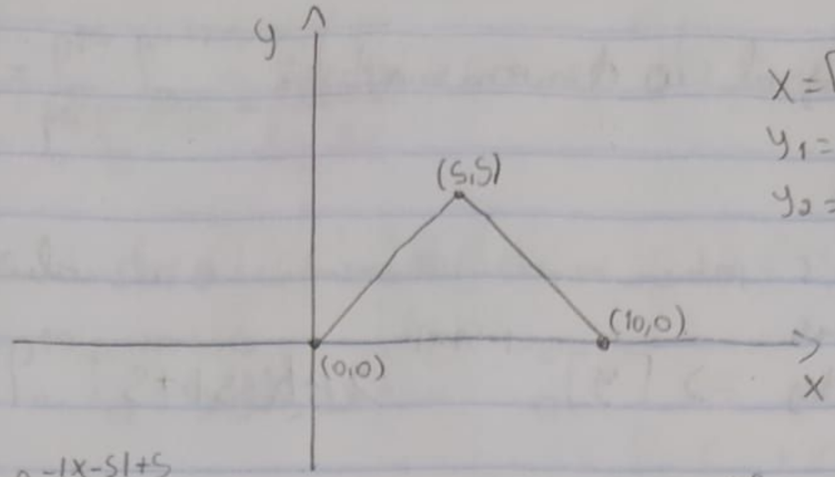
$$\int_0^2 4 - x^2\,dx \Rightarrow \int_0^2 4\,dx - \int_0^2 x^2\,dx \Rightarrow [4x]_0^2 - \left[\frac{x^3}{3}\right]_0^2$$

$$\Rightarrow 8 - \frac{8}{3} \Rightarrow \frac{16}{3}$$

$$\int_0^3 \frac{16}{3}\,dy \Rightarrow \left[\frac{16}{3}y\right]_0^3 \Rightarrow \left(\frac{16}{3}.3 - \frac{16}{3}.0\right) \Rightarrow 16,,$$

R: 16,,

③



$x = [0,10]$

$y_1 = 0$

$y_2 = -|x-5|+5$

$$\int_0^{10}\int_0^{-|x-5|+5} dy\,dx$$

$$x_c = \frac{\iint_D x\,\sigma(x,y)\,dS}{\iint_D \sigma(x,y)\,dS}$$

$$x_c = \frac{\int_0^{10}\int_0^{-|x-5|+5} x\,dy\,dx}{\int_0^{10}\int_0^{-|x-5|+5} dy\,dx}$$

$$\int_0^{-|x-5|+5} x\,dy \Rightarrow [xy]_0^{5-|-5+x|} \Rightarrow x(-|x-5|+5)$$

$$\int_0^{5} x(-(-x+5)+5)\,dx + \int_0^{10} x(-(x-5)+5)\,dx$$

$$\int_0^{5} x(-(-x+5)+5\,dx \Rightarrow \int_0^5 x^2\,dx \Rightarrow \left[\frac{x^3}{3}\right]_0^5 \Rightarrow \frac{125}{3},$$

$$\int_5^{10} x(-(x-5)+5)\,dx \Rightarrow \int_5^{10} x(-x+10)\,dx \Rightarrow \int_5^{10} -x^2+10x\,dx$$

$$\Rightarrow -\int_5^{10} x^2\,dx + \int_5^{10} 10x\,dx \Rightarrow -\left[\frac{x^3}{3}\right]_5^{10} + 10\left[\frac{x^2}{2}\right]_5^{10}$$

$$\Rightarrow -\frac{875}{3} + 375 \Rightarrow \frac{250}{3},$$

$$\int_0^{5} x(-(-x+5)+5)\,dx + \int_5^{10} x(-(x-5)+5)\,dx = \frac{125}{3} + \frac{250}{3} =$$

$$\Rightarrow 125,$$

Calculando a integral do denominador:

$$\int_0^{10}\int_0^{-|x-5|+5} dy\,dx$$

interna: $\int_0^{-|x-5|+5} dy \Rightarrow [y]_0^{5-|-5+x|} \Rightarrow -|x-5|+5$

$$\int_0^{10} -|x-5|+5\,dx$$

Eliminando os absolutos

$$\int_0^{5} -(-x+5)+5\,dx + \int_0^{10} -(x-5)+5\,dx$$

Primeiro: $\int_0^{5} -(-x+5)+5\,dx \Rightarrow \int_0^{5} x\,dx \Rightarrow \left[\frac{x^2}{2}\right]_0^5 \Rightarrow \frac{25}{2},$

Segundo: $\int_5^{10} -(x-5)+5\,dx \Rightarrow \int_5^{10} -x+10\,dx \Rightarrow -\int_5^{10} x\,dx + \int_5^{10} 10\,dx$

$$\Rightarrow \left[\frac{x^2}{2}\right]_5^{10} + \left[10x\right]_5^{10} \Rightarrow -\frac{75}{2}+5 \Rightarrow \frac{25}{2},$$

Então:

$$\int_0^{5} -(-x+5)+5\,dx + \int_5^{10} -(x-5)+5\,dx = \frac{25}{2}+\frac{25}{2} = 25$$

$$x_c = \frac{\int_0^{10}\int_0^{-|x-5|+5} x\,dy\,dx}{\int_0^{10}\int_0^{-|x-5|+5} dy\,dx} \Rightarrow \frac{125}{25} = 5$$

$$y_c = \frac{\int_0^{10}\int_0^{-|x-5|+5} y \, dy\, dx}{\int_0^{10}\int_0^{-|x-5|+5} dy\, dx}$$

Calculando o numerador, denominador = 25.

$$\int_0^{10}\int_0^{-|x-5|+5} y \, dy\, dx$$

interna: $\int_0^{-|x-5|+5} y\, dy \Rightarrow \left[\frac{y^2}{2}\right]_0^{-|x-5|+5} \Rightarrow \frac{(-|x-5|+5)^2}{2}$

Voltando: $\int_0^{10}\left(\frac{(-|x-5|+5)^2}{2}\right) dx$

Eliminando os absolutos

$$\int_0^5 \frac{(-(-x+5)+5)^2}{2}dx + \int_5^{10}\frac{(-(x-5)+5)^2}{2}dx$$

Primeiro: $\int_0^5 (-(-x+5)+5)^2\, dx \Rightarrow \frac{1}{2}\cdot\int_0^5 (-(-x+5)+5)^2 dx$

$$\Rightarrow \frac{1}{2}\cdot\int_0^5 x^2\, dx \Rightarrow \frac{1}{2}\left[\frac{x^3}{3}\right]_0^5 \Rightarrow \frac{1}{2}\cdot\frac{125}{3} \Rightarrow \frac{125}{6}$$

Segundo: $\int_5^{10}\frac{(-(x-5)+5)^2}{2}dx \Rightarrow \frac{1}{2}\cdot\int_5^{10}(-(x-5)+5)^2 dx$

$$\Rightarrow \frac{1}{2}\cdot\int_5^{10}(-x+10)^2 dx \Rightarrow \frac{1}{2}\cdot\int_5 x^2 - 20x + 100\, dx$$

$$\Rightarrow \frac{1}{2}\cdot\left(\int_5^{10} x^2 dx - \int_5^{10} 20x\, dx + \int_0^{10} 100\, dx\right)$$

data / /

$$\Rightarrow \frac{1}{2}\left(\left[\frac{x^3}{3}\right]_5^{10} - 20.\left[\frac{x^2}{2}\right]_5^{10} + \left[100x\right]_5^{10}\right)$$

$$\Rightarrow \frac{1}{2}\left[\left(\frac{10^3}{3} - \frac{5^3}{3}\right) - 20.\left(\frac{10^2}{2} - \frac{5^2}{2}\right) + \left(100.10 - 100.5\right)\right]$$

$$\Rightarrow \frac{1}{2}\left(\frac{875}{3} - 750 + 500\right) \Rightarrow \frac{125}{6},$$

Então:

$$\int_0^5 \frac{(-(-x+5)+5)^2}{2}\,dx + \int_5^{10} \frac{(-(x-5)+5)^2}{2}\,dx = \frac{125}{6} + \frac{125}{6}$$

$$\Rightarrow \frac{125}{3},$$

Então:

$$y_c = \frac{\int_0^{10}\int_0^{-|x-5|+5} y\,dy\,dx}{\int_0^{10}\int_0^{-|x-5|+5} dy\,dx} = \frac{\frac{125}{3}}{25} = \frac{5}{3}$$

R: $(x_c, y_c) = \left(5, \frac{5}{3}\right)$

4 a)

$x^2 = y^2 = r^2$ $\quad r = [\sqrt{1}, \sqrt{4}] = [1,2]$

$dxdy = r\,dr\,d\theta$ $\quad \theta = [0, \frac{\pi}{2}]$

$$\int_0^{\frac{\pi}{2}} \int_1^2 \frac{\log(r^2)}{r^2} r\,dr\,d\theta \qquad , \log(r^2) = \ln(r^2).$$

$$\int_0^{\frac{\pi}{2}} \int_1^2 \frac{\ln(r^2)}{r^2} r\,dr\,d\theta$$

Interno:

$$\int_1^2 \frac{\ln(r^2)}{r^2} r\,dr \Rightarrow \int_1^2 \frac{\ln(r^2)}{r}\,dr$$

$u = r^2$

$du = 2r\,dr$ $\quad \Rightarrow \int \frac{\ln(u)}{2r^2}\,du \Rightarrow \int \frac{\ln(u)}{2.u}\,du \Rightarrow \int_1^4 \frac{\ln(u)}{u}\,du$

$u(1) = 1^2 = 1$

$u(2) = 2^2 = 4$

$$\Rightarrow \frac{1}{2} \int_1^4 \frac{\ln(u)}{u}\,du$$

$v = \ln(u)$

$dv = \frac{1}{u}$ $\quad \Rightarrow \frac{1}{2}\left[\frac{v^2}{2}\right]_0^{2\ln(2)} \Rightarrow \ln^2(2)$

$V(1) = \ln 1 = 0$

$V(4) = \ln 4 = \ln 2^2 = 2\ln 2$

Voltando:

$$\int_0^{\frac{\pi}{2}} \ln^2(2)\,d\theta \Rightarrow [\ln^2(2)\,\theta]_0^{\frac{\pi}{2}} \Rightarrow \frac{\pi}{2}\ln^2(2)_{\,//}$$

R: $\frac{\pi}{2}\ln^2(2)$

b)

$x = [0,1]$ $\int_0^1 \int_0^1 \frac{x}{1+xy}\, dx\, dy$

$y = [0,1]$

Irei inverter a ordem de integração:

$$\int_0^1 \int_0^1 \frac{x}{1+xy}\, dy\, dx$$

Interno:

$$\int_0^1 \frac{x}{1+xy}\, dy$$

$u = 1+xy$

$du = x\,dy$

$u(1) = 1+x(1) = 1+x$ $\Rightarrow x.\int_1^{1+x} \frac{1}{xu}\, du \Rightarrow x.\frac{1}{x}.\int_1^{1+x} \frac{1}{u}\, du$

$u(0) = 1+x(0) = 1$

$$\Rightarrow [\ln(u)]_1^{1+x} \Rightarrow \ln(x+1)$$

Voltando:

$$\int_0^1 \ln(x+1)\, dx$$

$u = x+1$

$du = dx$ $\Rightarrow \int_1^2 \ln u\, du$

$u(0) = 0+1 = 1$

$u(1) = 1+1 = 2$

$u = \ln(u)$ $\int uv' \Rightarrow uv - \int u'v \Rightarrow [\ln(u)u - \int \frac{1}{u}.u\, du]_1^2$

$v' = 1$ $\Rightarrow [u\ln(u) - \int \frac{1}{2} u\, du]_1^2 \Rightarrow [u\ln(u) - u]_1^2$

$u' = \frac{1}{u}$

$v = u$ $\Rightarrow 2\ln(2) - 2 - (-1) \Rightarrow 2\ln(2) - 1$ //

R: $2\ln(2) - 1$ //

(5)

$x = 0$

$z = 0$

$x + z = 2$

$z = 1 - y^2$

$x + 1 - y^2 = 2$

$x = 2 + y^2 - 1$

$0 = y^2 - 1$

$y = \pm 1$

$$\int_{-1}^{1}\int_{0}^{1-y^2}\int_{0}^{2-z} y - 1 \, dx\, dz\, dy$$

$y = [-1, 1]$

$x = [0, 2 - z]$

$z = [0, 1 - y^2]$

Mais Interna:

$$\int_{0}^{2-z} y - 1 \, dx \Rightarrow \left[(-1+y)\,x\right]_{0}^{2-z}$$

$$\Rightarrow (-z+2).(y-1)$$

Intermediario:

$$\int_{0}^{1-y^2} (-z+2).(y-1)\, dz$$

$$\Rightarrow (-1+y).\int_{0}^{1-y^2} 2 - z \, dz$$

$$\Rightarrow (-1+y)\left(\int_{0}^{1-y^2} 2\, dz - \int_{0}^{1-y^2} z \, dz\right)$$

$$\Rightarrow (-1+y)\left(\left[2z\right]_{0}^{1-y^2} - \left[\frac{z^2}{2}\right]_{0}^{1-y^2}\right)$$

$$\Rightarrow (-1+y)\left(2(-y^2+1) - \frac{(-y^2+1)^2}{2}\right)$$

$$\Rightarrow \int_{-1}^{1} (-1+y)\left(2(-y^2+1) - \frac{(-y^2+1)^2}{2}\right) dy$$

$$\Rightarrow \int_{-1}^{1} \left(\frac{y^4}{2} + y^2 + \frac{1}{2} - \frac{y^5}{5} - y^3 + \frac{3y}{2} - 2 \, dy\right.$$

$$\Rightarrow \int_{-1}^{1} \frac{y^4}{2}\, dy + \int_{-1}^{1} y^2\, dy + \int_{-1}^{1} \frac{1}{2}\, dy - \int_{-1}^{1} \frac{y^5}{2}\, dy - \int_{-1}^{1} y^3\, dy + \int_{-1}^{1} \frac{3y}{2}\, dy - \int_{-1}^{1} 2$$

$$\Rightarrow \frac{1}{2}\left[\frac{y^5}{5}\right]_{-1}^{1} + \left[\frac{y^3}{3}\right]_{-1}^{1} + \left[\frac{1}{2}y\right]_{-1}^{1} - 0 - 0 + 0 - \left(2y\right]_{-1}^{1}$$

$$\Rightarrow \frac{1}{5} + \frac{2}{3} + 1 - 0 - 0 + 0 - 4$$

$$\Rightarrow -\frac{32}{15} //$$

R: $-\frac{32}{15}$ //