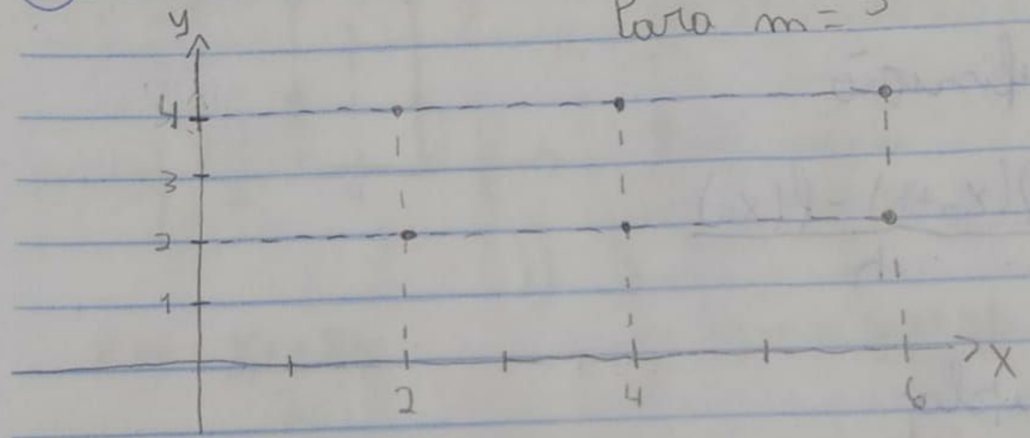
Lista 1 - Cálculo 3

1

$m = 2$

Para $m = 3$



$\Delta A = \Delta x . \Delta y$

$\Delta A = 4$

$\Delta x = \dfrac{x_i - x_{i-1}}{m}$ $\Delta y = \dfrac{y_j - y_{j-1}}{n}$

$\Delta x = \dfrac{6 - 0}{3}$ $\Delta y = \dfrac{4 - 0}{2}$

$\Delta x = 2$ $\Delta y = 2$

$$V = \sum_{i=1}^{3} \sum_{i=1}^{2} f(x_{ij}, y_{ij}) \Delta A$$

$f(2,2).4 + f(2,4).4 + f(4,2).4 + f(4,4).4 + f(6,2).4 +$
$f(6,4).4$

② $\int_1^3\int_0^1 (1+4xy)\,dx\,dy$ ⓐ

$\int_1^3 \left(\int_0^1 1\,dx \int_0^1 4xy\,dx\right) dy$

$\int_1^3 (1.1-1.0) + \left(\frac{4.1^2.y}{2} - 4.0.y\right)$

$\int_1^3 1+2y\,dy$

$[y]_1^3 + \left[\frac{2y^2}{2}\right]_1^3$

$[3-1] + \left[\frac{\cancel{2}.3^2}{\cancel{2}} - \frac{\cancel{2}.1^2}{\cancel{2}}\right]$

$2+9-1$

10,,

ⓑ $\int_2^4\int_{-1}^1 (x^2+y^2)\,dy\,dx$

$\int_2^4\left[\int_{-1}^1 x^2\,dy + \int_{-1}^1 y^2\,dy\right] dx$

$\int_2^4 \left([x^2.y]_{-1}^1 + \left[\frac{y^3}{3}\right]_{-1}^1\right) dx$

$\int_2^4 \left((x^2.1 - x^2.-1) + \frac{1^3}{3} - \left(-\frac{1^3}{3}\right)\right)$

$\int_2^4 2x^2 + \frac{2}{3}\,dx$

$\left[\frac{2x^3}{3}\right]_2^4 + \left[\frac{2}{3}x\right]_2^4$

$\left[\frac{2.4^3}{3} - \frac{2.2^3}{3}\right] + \left[\frac{2}{3}.4 - \frac{2}{3}.2\right]$

$\frac{128}{3} - \frac{16}{3} + \frac{8}{3} - \frac{4}{3}$

$\frac{116}{3}$,,

c) $\int_0^{\pi/2} \int_0^{\frac{\pi}{2}} \operatorname{sen} x \cos y \, dy \, dx$

$\int_0^{\frac{\pi}{2}} ([\operatorname{sen} x] dy + [\cos y] dy) \, dx$

$\int_0^{\frac{\pi}{2}} \operatorname{sen} x . (\operatorname{sen} \frac{\pi}{2} - \operatorname{sen} 0) \, dx$

$\int_0^{\frac{\pi}{2}} \operatorname{sen} x . 1 \, dx$

$[-\cos \frac{\pi}{2} - (-\cos 0)]$

$\underline{1}$

d) $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \int_{-1}^{5} \cos y \, dx \, dy$

$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \int_{-1}^{5} \cos . y \, dx \, dy$

$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos . y \, [5 - (-1)] \, dx$

$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos . y . 6 \, dx$

$6 . [\operatorname{sen} \frac{\pi}{2} - \operatorname{sen} \frac{\pi}{6}]$

$6 . [1 - \frac{1}{2}]$

$6 . \frac{1}{2}$

3

E) $\int_0^2 \int_0^1 (2x+y)^{8}\, dx\, dy$

$u = 2x+y$

$\frac{du}{2} = dx$

$\int_0^2 \int_0^1 u^8 \, dx \, \frac{du}{2}\, dy$

$\int_0^2 \left(\frac{1}{2}\left[\frac{u^9}{9}\right]_0^1\right) dy$

$\int_0^2 \left(\frac{1}{18}\left[(2+y)^9 - (y)^9\right]\right) dy$

$\frac{1}{18}\left(\int_0^2 (2+y)^9 - \int_0^2 (y)^9\right) dy$

Para Δy

$u = 2+y$

$du = 1\, dy$

$\frac{1}{18}\left(\int_0^2 u^9\, du - \int_0^2 (y)^9\, dy\right)$

$\frac{1}{18}\left(\int_0^2 \left[\frac{u^{10}}{10}\right]_0^2 - \left[\frac{y^{10}}{10}\right]_0^2\right)$

$\frac{1}{18}\cdot\frac{1}{10}\left[(2+y)^{10}\right]_0^2 - \left[y^{10}\right]_0^2$

$\frac{1}{180}\left(\left[4^{10} - 2^{10} - 2^{10}\right]\right)$

$\frac{4^{10} - 2 \cdot 2^{10}}{180} \Rightarrow \frac{4^{10} - 2^{11}}{180}$

F) $\int_0^1 \int_1^2 \frac{x e^x}{y}\, dy\, dx$

$\int_0^1 \int_1^2 \frac{x e^x}{y}\, dy\, dx$

$\int_0^1 x e^x \cdot \int_1^2 \left[\frac{1}{y}\right] dx$

$\int_0^1 x e^x \cdot \ln(2)\, dx$

$\int_0^1 x e^x \cdot \ln 2\, dx$

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T
E

$\int u\, dv = u.v - \int v\, du$

$\int x e^x\, dx$

$u = x \qquad dv = e^x\, dx$

$du = 1 dx \qquad v = e^x$

$\ln 2 (x . e^x - \int e^x\, dx)$

$\ln 2 . (1 . e^1 - 0 . e^0) - [e^x]_0^1$

$\ln 2 . e^1 - [e^1 - e^0]$

$\ln 2 . (e^1 - e^1 + 1)$

$\ln 2 . (0+1) = \ln 2$,,

g) $\int_1^4 \int_1^2 \left(\frac{x}{y} + \frac{y}{x}\right) dy\, dx$

$\int_1^4 \left(\int_1^2 \frac{x}{y} + \int_1^2 \frac{y}{x}\right) dy\, dx$

$\int_1^4 \left(x \int_1^2 \frac{1}{y} + \frac{1}{x} \int_1^2 y\right) dy\, dx$

$\int_1^4 \left(x \left[\ln(y)\right]_1^2 + \frac{1}{x}\left[\frac{y^2}{2}\right]_1^2\right) dx$

$\int_1^4 \left(x \ln(2) + \frac{3}{2x}\right) dx$

$\left(\int_1^4 x \ln(2) + \int_1^4 \frac{3}{2x}\right) dx$

$\ln 2 . \frac{1}{2}\left[16 - 1\right] + \frac{3}{2}\left[\ln(4) - \ln(1)\right]$

$\ln(2) . \frac{15}{2} + \frac{3}{2} \ln 4$

$\frac{15}{2} \ln(2) + 3 \ln(2) \Rightarrow \frac{21}{2} \ln(2)$,,

H) $\int_0^1 \int_0^3 e^{x+3y}\, dx\, dy$

$u = x + 3y$

$du = 1\, dx$

Para Δx

$\int_0^1 \int_0^3 e^u\, du\, dy$

$\int_0^1 \left(\left[e^u\right]_0^3\right) dy$

$\int_0^1 \left(\left[e^{x+3y}\right]_0^3\right) dy$

$\int_0^1 \left(e^{3+3y} - e^{3y}\right) dy$

$\int_0^1 \left(e^{3+3y}\right) dy - \int_0^1 e^{3y}\, dy$

Para $\Delta y'$

$u1 = 3 + 3y$

$\frac{du}{3} = dy$

Para $\Delta y''$

$u2 = 3y$

$\frac{du}{3} = dy$

$$\int_0^1 e^{u_1} \frac{du}{3} - \int_0^1 e^{u_2} \frac{du}{3}$$

$$\left([e^{u_1}]_0^1 - [e^{u_2}]_0^1 \right) \frac{1}{3}$$

$$\left([e^{3+3y}]_0^1 - [e^{3y}]_0^1 \right) \frac{1}{3}$$

$$\left[(e^6 - e^3) - (e^3 - e^0) \right] \frac{1}{3}$$

$$\frac{e^6 - 2e^3 + 1}{3}$$

i) $\int_0^1 \int_0^1 (u-v)^5 \, du \, dv$

$x = u - v$

$dx = 1 du$

Para Δu

$$\int_0^1 \int_0^1 x^5 \, dx \, dv$$

$$\int_0^1 \left[\frac{x^6}{6} \right]_0^1 dv$$

$$\int_0^1 \left(\frac{1}{6} \left[(u-v)^6 \right]_0^1 \right) dv$$

$$\int_0^1 \frac{1}{6} \, (1-v)^6 - (0-v)^6 \, dv$$

$$\frac{1}{6} \int_0^1 \left((1-v)^6 + \int v^6 \right) dv$$

$x = 1 - v$

$dx = -1 dv$

$-dx = dv$

Para Δv

$$\frac{1}{6} \left(\int_0^1 -x^6 \, dx + \int_0^1 v^6 \, dv \right)$$

$$\frac{1}{6} \left(-1 \left[\frac{x^7}{7} \right]_0^1 + \left[\frac{v^7}{7} \right]_0^1 \right)$$

$$\frac{1}{42} \left(-1 \left[(1-1)^7 - (1-0)^7 \right] + \left[1^7 - 0^7 \right] \right)$$

$$\frac{1}{42} \left(-1 - 1 + 1 \right)$$

$$\frac{2}{42} \Rightarrow \frac{1}{21} \,,,$$

j) $\int_0^1 \int_0^1 xy\sqrt{x^2y^2}\, dy\, dx$

$\int_0^1 \int_0^1 xy\sqrt{x^2y^2} = \int_0^1 \int_0^1 x^2y^2$

onde: $xy\sqrt{x^2y^2} \to xy\sqrt{x^2}\sqrt{y^2} \to xy\,xy \to x^2y^2$

$\int_0^1 x^2\left(\left[\frac{y^3}{3}\right]_0^1\right) dx$

$\int_0^1 x^2\left(\frac{1}{3}\left[1^3 - 0^3\right]\right) dx$

$\int_0^1 \frac{x^2}{3} \cdot 1\, dx \Rightarrow \frac{1}{3}\left(\left[\frac{x^3}{3}\right]_0^1\right)$

$\frac{1}{3}\left(\frac{1}{3}\left[1^3 - 0^3\right]\right)$

$\frac{1}{9} \cdot (1-0) \Rightarrow \frac{1}{9}$,,

k) $\int_0^2 \int_0^{\pi} r \sin^2(\theta)\, d\theta\, dr$

$\int_0^2 r\left(\int_0^{\pi} \frac{1-\cos(2\cdot\theta)}{2}\, d\theta\right) dr$

$\int_0^2 r\left(\frac{1}{2}\int_0^{\pi} 1-\cos(2\theta)\, d\theta)\, d\theta\right) dr$

$\int_0^2 r\left(\frac{1}{2}\left[\int_0^{\pi} 1\, d\theta - \int_0^{\pi} \cos(2\cdot\theta)\, d\theta\right]\right) dr$

Para $\Delta\theta$

$u = 2\theta$

$\frac{du}{2} = d\theta$

$\int_0^2 \frac{r}{2}\left(\int_0^{\pi} 1\, d\theta - \int_0^{\pi} \cos(u)\,\frac{dx}{2}\right) dr$

$\int_0^2 \frac{\pi}{2}\left(\left[\theta\right]_0^{\pi} - \frac{1}{2}\left[\operatorname{sen}(u)\right]_0^{\pi}\right) dr$

Substituindo u

$$\int_0^2 \frac{r}{2}\left(\left[\pi - 0\right] - \frac{1}{2}\left[\operatorname{sen}(2.\theta)\right]_0^{\pi}\right) dr$$

$$\int_0^2 \frac{r}{2}\left(\pi - \frac{1}{2}\left[\operatorname{sen}(2\pi) - \operatorname{sen}(2-0)\right]\right) dr$$

sendo $\operatorname{sen}(\pi = 0$, então $\operatorname{sen}(2\pi) = 0$

$$\int_0^2 \frac{r\pi}{2}\, dr$$

$$\left(\frac{\pi}{2}\left[\frac{r}{2}\right]_0^2\right)$$

$$\frac{\pi}{2}\cdot\left(\frac{1}{2}\left[2^2 - 0^2\right]\right)$$

$$\frac{\pi}{2}\cdot 2 \Rightarrow \pi_{,,}$$

l) $\int_0^1 \int_0^1 \sqrt{s+t}\; ds\, dt$

$u = s+t$

$du = 1\, ds$

Para ds

$$\int_0^1 \int_0^1 \sqrt{u}\; du\, dt$$

$$\int_0^1 \int_0^1 u^{\frac{1}{2}}\; du\, dt$$

$$\int_0^1 \left(\left[\frac{u^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^1\right) dt$$

$$\int_0^1 \left(\frac{2}{3}\left[(1+t)^{\frac{3}{2}} - (s+t)^{\frac{3}{2}}\right]\right) dt$$

Lista 2

① a) $\int_1^e \int_0^{\ln x} f(x,y)\, dy\, dx$

$x = 1$
$x = e$
$y = 0$
$y = \ln x$

$1 \leq x \leq e$
$0 \leq y \leq \ln x$

$y = \ln x = \ln e$
$y = 1$
$0 \leq y \leq 1$
$e^y \leq x \leq e$

$\int_0^1 \int_{e^y}^e f(x,y)\, dx\, dy$

$x = -\sqrt{1-y^2}$
$x = -1\sqrt{y}$
$y = x^2 - 2x + 1$

$\int_{-1}^0 \int_0^{g(x)} f(x,y)\, dx\, dy$

$\int_{-1}^0 \int_0^{g(x)} f(x,y)\, dy\, dx + \int_0^1 \int_0^{h(x)} f(x,y)\, dy\, dx$,,

b)

$x = -\sqrt{1-y^2} \rightarrow y = \sqrt{1-x^2}$
$x = 1 - \sqrt{y} \rightarrow y = x^2 - 2x + 1$

$\int_{-1}^0 \int_0^{\sqrt{1-x^2}} f(x,y)\, dy\, dx + \int_0^1 \int_0^{x^2-2x+1} f(x,y)\, dy\, dx$,,

c)

$\int_{-1}^2 \int_{y^2-4}^{y-2} f(x,y)\, dx\, dy$

$y = -1$
$y = 2$
$x = y^2 - 4$
$x = y - 2$

$-1 \leq y \leq 2$
$y^2 - 4 \leq x \leq y - 2$

$x = y - 2 \Rightarrow y - 2 = y^2 - 4$
$x = y^2 - 4 \Rightarrow y^2 - y - 2 = 0$

$y_1 = -1$
$y_2 = 2$

Substitui, $x = y - 2$
$y_1 = -1 \rightarrow x_1 = -3$
$y_2 = 2 \rightarrow x_2 = 0$

$-4 \leq x \leq -3$

$X = y^2 - 4 \rightarrow y^2 = x + 4$

$y = \pm\sqrt{x+4}$

$-\sqrt{x+4} \leq y \leq +\sqrt{x+4}$

$y_1 = -1 \rightarrow x_1 = -3$

$y_2 = 2 \rightarrow x_2 = 0 \qquad -3 \leq x \leq 0$

$x+2 \leq y \leq +\sqrt{x+4}$

$-4 \leq x \leq -3$

$-\sqrt{x+4} \leq y \leq +\sqrt{x+4}$

$$\int_{-4}^{-3} \int_{-\sqrt{x+4}}^{\sqrt{x+4}} f(x,y)\, dx\, dy$$

$-3 \leq x \leq 0$

$x+2 \leq y \leq +\sqrt{x+4}$

$$\int_{-3}^{-0} \int_{x+2}^{\sqrt{x+4}} f(x,y)\, dy\, dx$$

$$\int_{-4}^{-3} \int_{-\sqrt{x+4}}^{\sqrt{x+4}} f(x,y)\, dy\, dx + \int_{-3}^{-0} \int_{x+2}^{\sqrt{x+4}} f(x,y)\, dy\, dx$$

2 a)

$x = y^3$

$x + y = 2$

$y = 0$

$y = 2 - x$

$y = \sqrt[3]{x}$

$$\begin{cases} x + y = 2 \\ x = y^3 \end{cases}$$

$y^3 + y = 2$

$0 \leq y \leq 1$

$y^3 \leq x \leq 2 - y$

$A = \iint_R 1 \, dA$

$0 \leq y \leq 1$
$y^3 \leq x \leq 2 - y$

$\int_0^1 \int_{y^3}^{2-y} 1 \, dx \, dy$

$\int_0^1 \int_{y^3}^{2-y} 1 \, dx \, dy = \int_0^1 \left(\int_{y^3}^{2-y} 1 \, dx \right) dy = \int_0^1 [x]_{y^3}^{2-y} \, dy$

$\int_0^1 (2-y) - (y^3) \, dy = \int_0^1 2 - y - y^3 \, dy = \left[2y - \frac{y^2}{2} - \frac{y^4}{4} \right]_0^1$

$\left(2 - \frac{1}{2} - \frac{1}{4} \right) - (0 - 0 - 0) = \frac{5}{4}$ //

D)
$y = x$
$y = 4x$
$xy = 36$

$Area = \iint_D dA$

$4x = \frac{36}{x} \rightarrow x^2 = 9$

$x = \pm 3 \rightarrow x = 3$

$\iint_{D_1} dA = \int_0^3 \int_x^{4x} dy \, dx$

$\int_0^3 \int_x^{4x} dy \, dx = \int_0^3 3x \, dx = \left[\frac{3x^2}{2} \right]_0^3 = \frac{27}{2}$

$x = \frac{36}{x} \rightarrow x^2 = 36$

$x = \pm 6 \rightarrow x = 6$

$$\iint_{D_2} dA = \int_3^6 \int_x^{\frac{36}{x}} dy\, dx$$

$$\int_3^6 \frac{36}{x} dx - \int_3^6 x\, dx = 36\ln[x]_3^6 - \left[\frac{x^2}{2}\right]_3^6$$

$$36(\ln 6 - \ln 3) - \frac{36-9}{2} = 36\ln\frac{6}{3} - \frac{27}{2} = 36\ln 2 - \frac{27}{2}$$

$$\frac{27}{2} + 36\ln 2 - \frac{27}{2} = 36\ln 2_{//}$$

c)

$y^2 = -x$ $\qquad$ $x - y = 4 \rightarrow y = x - 4$

$x - y = 4$ $\qquad$ $x = -y^2$

$y = -1$ $\qquad$ $-1 \leq y \leq 2$

$y = 2$ $\qquad$ $-y^2 \leq x \leq 4 + y$

$$\int_{-1}^{2} \int_{-y^2}^{4+y} 1\, dx\, dy$$

$$\int_{-1}^{2} \int_{-y^2}^{4+y} 1\, dx\, dy$$

$$\int_{-1}^{2} \left(\int_{-y^2}^{4+y} 1\, dx\, dy \right) = \int_{-1}^{2} [x]_{-y^2}^{4+y} dy$$

$$\int_{-1}^{2} (4+y) - (-y^2)\, dy = \int_{-1}^{2} 4 + y + y^2\, dy$$

$$\left[4y + \frac{y^2}{2} + \frac{y^3}{3}\right]_{-1}^{2} = \left(8 + \frac{4}{2} + \frac{8}{3}\right) - \left(-4 + \frac{1}{2} - \frac{1}{3}\right)$$

$$12 + \frac{3}{2} + \frac{9}{3} = \frac{33}{2}_{//}$$

3 a)

$$\iint_D (x-y)\,dx\,dy$$

$y = -2x + 4$

$y = -2x + 7$

$y = x - 2$

$y = x + 1$

$y = x + 1 = -2x + 7$

$3x = 6$

$x = 2$

$$\iint_D (x-y)\,dx\,dy = \int_1^2 \int_{-2x+4}^{x+1} (x-y)\,dy\,dx + \int_2^3 \int_{x-2}^{-2x+7} (x-y)\,dy\,dx$$

b)

$$\int_1^4 \int_{\frac{\ln y}{2}}^{\ln 2} \frac{1}{e^x+1}\,dx\,dy$$

$\ln y = 2x \to e^{2x} = y$

$1 \leq y \leq 4$

$\frac{\ln y}{2} \leq x \leq \ln 2$

$y = 1$

$y = 4$

$x = \ln 2$

$x = \frac{\ln y}{2}$

$0 \leq x \leq \ln 2$

$1 \leq y \leq e^{2x}$

$0 \leq x \leq \ln 2$

$1 \leq y \leq e^{2x}$

$$\int_0^{\ln 2} \int_1^{e^{2x}} \frac{1}{e^x+1}\,dy\,dx$$

$$\int_0^{\ln 2} \int_1^{e^{2x}} \frac{1}{e^x+1}\,dy\,dx$$

$$\int_0^{\ln 2} \left(\int_1^{e^{2y}} \frac{1}{e^x+1}\,dy \right) dx \Rightarrow \int_0^{\ln 2} \frac{1}{e^{x+1}} \left(\int_1^{e^{2y}} dy \right) dx$$

$$\int_0^{\ln 2} \frac{1}{e^x+1} \left[y \right]_1^{e^{2y}} dx$$

$$(e^{2x}-1) = ((e^x)^2-1) \Rightarrow (e^x+1).(e^x-1)$$

substituindo

$$\int_0^{\ln 2} \frac{1}{e^x+1}(e^{2x}-1)\,dx \Rightarrow \int_0^{\ln 2} \frac{1}{e^x+1}.(e^x+1).(e^x-1)\,dx$$

$$\int_0^{\ln 2}(e^x-1)\,dx \Rightarrow \left[e^x-x\right]_0^{\ln 2} \Rightarrow (e^{\ln 2}-\ln 2)-(e^0-0)$$

$$(2-\ln 2)-(1) \Rightarrow 1-\ln 2 \,//$$

c) $\int_0^1 \int_{\arcsin y}^{\frac{\pi}{2}} \cos x\sqrt{1+\cos^2 x}\,dx\,dy$

$\arcsin y \leq x \leq \frac{\pi}{2}$

$0 \leq y \leq 1$

$y = \operatorname{sen} x$

$0 \leq y \leq \operatorname{sen} x$

$0 \leq x \leq \frac{\pi}{2}$

$$\int_0^{\frac{\pi}{2}} \int_0^{\operatorname{sen} x} \cos x\sqrt{1+\cos^2 x}\,dy\,dx$$

$$\int_0^{\frac{\pi}{2}} \left(\cos x\sqrt{1+\cos^2 x}\right)\left[y\right]_0^{\operatorname{sen} x} dx$$

$$\int_0^{\frac{\pi}{2}} \left(\operatorname{sen} x \cdot \cos x\sqrt{1+\cos^2 x}\right) dx$$

$u = 1+\cos^2 x$

$du = 2\cos x(-\operatorname{sen} x)\,dx$

$-\frac{du}{2} = \cos x \operatorname{sen} x\,dx$

$$\int_0^{\frac{\pi}{2}} \left(\underbrace{\operatorname{sen} x \cos x}_{-\frac{du}{2}}\sqrt{\underbrace{1+\cos^2 x}_{u}}\right) dx$$

$$\int_2^1 \sqrt{u}\,.\left(-\frac{du}{2}\right) \Rightarrow -\frac{1}{2}\int_2^1 \sqrt{u}\,du$$

$x = 0 \rightarrow u = 1+\cos^2 0 = 2$

$$-\frac{1}{2}\left[\frac{2}{3}u^{\frac{3}{2}}\right]_2^1 \Rightarrow -\frac{1}{3}(1-2\sqrt{2})$$

$x = \frac{\pi}{2} \rightarrow u = 1+\cos^2 \frac{\pi}{2} = 1$

$$\frac{(2\sqrt{2}-1)}{3} \,//$$

Lista 3

1

$x+\frac{y}{2}+0=4$

$x+\frac{y}{2}=4$

$\frac{y}{2}=4-x$

$y=8-2x$

$0\leq x\leq 4$

$0\leq y\leq 8-2x$

$$\int_0^4\int_0^{8-2x}\int_0^{4-x-\frac{y}{2}} x\,dz\,dy\,dx$$

$$\int_0^4\int_0^{8-2x}\left(\int_0^{4-x-\frac{y}{2}} x\,dz\right)dy\,dx$$

$$\int_0^4\int_0^{8-2x} x\left[z\right]_0^{4-x-\frac{y}{2}}dy\,dx$$

$$\int_0^4\left(\int_0^{8-2x} 4x-x^2-\frac{xy}{2}\,dy\right)dx$$

$$\int_0^4\left[4xy-x^2y-\frac{x}{2}\left(\frac{y^2}{2}\right)\right]_0^{8-2x}dx$$

$$\int_0^4(16x-8x^2+x^3)\,dx$$

$$\left[\frac{16x^2}{2}-\frac{8x^3}{3}+\frac{x^4}{4}\right]_0^4$$

$$\left(\frac{16.4^2}{2}-\frac{8.4^3}{3}+\frac{4^4}{4}\right)$$

$$\frac{64}{3}_{//}$$

2

$0\leq z\leq y$

$0\leq y\leq 2$

$0\leq x\leq 4-y^2$

$$\int_0^2\int_0^{4-y^2}\int_0^{y} dz\,dy\,dx$$

$$\int_0^2\int_0^{4-y^2}\left(\int_0^{y} dz\right)dx\,dy$$

$$\int_0^2\int_0^{4-y^2}\left[z\right]_0^{y}dx\,dy$$

$$\int_0^2 \int_0^{4-y^2} y \, dx\, dy$$

$$\int_0^2 y \left(\int_0^{4-y^2} dx \right) dy$$

$$\int_0^2 y [x]_0^{4-y^2} dy \Rightarrow \int_0^2 y(4-y^2)\, dy \Rightarrow \int_0^2 (4y - y^3)\, dy$$

$$\Rightarrow \left[\frac{4y^2}{2} - \frac{y^4}{4} \right]_0^2 \Rightarrow \left(\frac{4.4}{2} - \frac{2^4}{4} \right) \Rightarrow 4_{//}$$

3

$0 \leq z \leq x^2 + y^2$

$y = 2 - x$

$0 \leq x \leq 2$

$0 \leq y \leq 2 - x$

$$\int_0^2 \int_0^{2-x} \int_0^{x^2+y^2} dz\, dy\, dx$$

$$\int_0^2 \int_0^{2-x} [z]_0^{x^2+y^2} dy\, dx$$

$$\int_0^2 \int_0^{2-x} (x^2+y^2)\, dy\, dx$$

$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$$\int_0^2 \left(\int_0^{2-x} (x^2+y^2)\, dy \right) dx$$

$$\int_0^2 \left[x^2 y + \frac{y^3}{3} \right]_0^{2-x} dx$$

$$\int_0^2 \left(2x^2 - x^3 + \frac{(2-x)^3}{3} \right) dx$$

$$\int_0^2 \left(2x^2 - x^3 + \frac{2^3 - 3.(2)^2 x + 3.(2) x^2 - x^3}{3} \right) dx$$

$$\int_0^2 \left(2x^2 - x^3 + \frac{8}{3} - 4x + 2x^2 - \frac{x^3}{3} \right) dx$$

$$\int_0^2 \left(\frac{8}{3} - 4x + 4x^2 - \frac{4x^3}{3} \right) dx$$

$\frac{8}{3}{}_{//}$

$$\left[\frac{8}{3}x - \frac{4x^2}{2} + \frac{4x^3}{3} - \frac{4}{3}.\frac{x^4}{4} \right]_0^2 \Rightarrow \left(\frac{8}{3}.2 - \frac{4.2^2}{2} + \frac{4.2^3}{3} - \frac{2^4}{3} \right) =$$