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## Vectors

### Parametric representation of lines

A line (**L**) in an  $n$  dimensional space can be represented using a vector equation, such as the one below:

$$\mathbf{L} = \{k \cdot \vec{v}; k \in \mathbb{R}\}$$

### Linear combination

Given a set of vectors (**V**) and a set of scalars (**C**), a linear combination can be represented as below:

$$\mathbf{V} = \{\vec{v}_1, \vec{v}_2, \vec{v}_3, \dots\}$$

$$\mathbf{C} = \{c_1, c_2, c_3, \dots\}$$

$$(c_1 \cdot \vec{v}_1 + c_2 \cdot \vec{v}_2 + c_3 \cdot \vec{v}_3 + \dots) \text{ Is a linear combination of } \mathbf{C} \text{ \& } \mathbf{V}$$

### Span

The span of a set of vectors ( $\text{Span}(\vec{v}_1, \vec{v}_2, \vec{v}_3, \dots)$ ) is the set of all possible linear combinations between that set of vectors and the set of real numbers;

## Linear independence

A set of vectors is said to be linearly independent if at least one vector in that set can't be represented as a linear combination of other vectors in that set;

Only if there is no solution for  $(c_1 \cdot \vec{v}_1 + c_2 \cdot \vec{v}_2 + c_3 \cdot \vec{v}_3 + \dots = 0)$  other than when  $\forall c_i = 0$  that the set of vectors is linearly independent;

## Subspace of a vector set

A subset  $\mathbf{U}$  is said to be a subspace of a superset  $\mathbf{V}$  when  $\mathbf{U}$  follows all the conditions below:

1.  $\vec{0} \in U$
2.  $\mathbf{U}$  contains all the linear combinations of vectors in itself (Is closed in vector addition and scalar multiplication)

**Note:**  $\text{Span}(\mathbf{V})$  are always valid subspaces;

## Vector transformations

Vector transformations are the association between vectors from a  $\mathbb{D}$  to a vector of a  $\mathbb{CD}$  (In other words, functions operating on vectors)

$$T : \mathbb{R}^n \rightarrow \mathbb{R}^m; T(\vec{v}_1) = \vec{v}_2$$

## Linear transformations

A transformation is said to be linear if it follows both the **additive property** and the **homogeneous property**

Additive property

$$T(\vec{a} + \vec{b}) = T(\vec{a}) + T(\vec{b})$$

Homogeneous property

$$T(c \cdot \vec{a}) = c \cdot T(\vec{a}); \forall c \in \mathbb{R}$$

Linear transformations have some properties that are listed below:

$$T \text{ Is a linear transformation} \implies T(\vec{0}) = \vec{0}$$

$T$  Is a linear transformation  $\implies T$  can be represented by a transformation matrix  $A$

**Note:**  $A \iff A \cdot \vec{v} = T(\vec{v})$

## Null space

The null space is the set of vectors that when transformed, equal the null vector;

$$N(T) = \{\vec{v}; T(\vec{v}) = \vec{0}\}$$

## Eigenvalues and eigenvectors

$$\vec{a} \text{ is an eigenvector of } A \iff A \cdot \vec{a} = \lambda \cdot \vec{a}$$

$$\lambda \text{ is an eigenvalue of } A \iff (A \cdot \vec{v}), \vec{v} \in \text{span}(\vec{v})$$

## Determining the matrix of eigenvalues of a matrix

### Using determinants

$$\det(A - \lambda \cdot I) = 0$$
$$\Lambda = \begin{bmatrix} \lambda_1 & 0 & 0 & \dots \\ 0 & \lambda_2 & 0 & \dots \\ 0 & 0 & \lambda_3 & \dots \\ \dots & \dots & \dots & \dots \end{bmatrix}$$

### Using the matrix of eigenvectors

$$\Lambda = S^{-1} \cdot A \cdot S$$

## Determining the eigenvectors using the eigenspace

$$E_{\lambda_n} = N(A - \lambda_n \cdot I)$$

Determining the eigenspaces for all  $\lambda$  values, you will find all the eigenvectors for the matrix  $A$ . You can create a matrix of eigenvectors as noted below:

$$S = [B_{\lambda_1} | B_{\lambda_2} | B_{\lambda_3} | \dots]$$

## Determining the transformation matrix using eigenvalues and eigenvectors

$$A = S \cdot \Lambda \cdot S^{-1}$$