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Types of linear systems

- Undetermined: Fewer equations than unknowns (Either has none or an infinite number of solutions);
- Determined: Same number of equations as unknowns (Has only one solution)
- Overdetermined: More equations that unknowns (Has no solution, but one can find the best approximate solution using linear regression)

Representing linear systems with matrices

A system of linear equations can be summarized as the following:

$$A \cdot X = b$$

1. The matrix of the coefficients;

$$A = \begin{bmatrix} a_1 & b_1 & c_1 & ... & z_1 \\ a_2 & b_2 & c_2 & ... & z_2 \\ a_3 & b_3 & c_3 & ... & z_3 \\ ... & ... & ... & ... & ... \\ a_n & b_n & c_n & ... & z_n \end{bmatrix}$$

2. The matrix of the unknowns;

$$X = \begin{bmatrix} x \\ y \\ z \\ ... \\ w \end{bmatrix}$$

3. The matrix of the constants;

$$b = \begin{bmatrix} K \\ L \\ M \\ ... \\ J \end{bmatrix}$$

Solving linear systems

Inversion method

$$X = A^{-1} \cdot b$$

Gaussian elimination

To solve the linear system using gaussian elimination, one must follow the steps listed below:

1. Create augmented matrix $(A|b)$;

$$(A|b) = \left[\begin{array}{ccc|c} 3 & -2 & 1 & 2 \\ 4 & 3 & -2 & 4 \\ 5 & -3 & 3 & 8 \end{array}\right]$$

2. Find the equivalent linear system with the row echelon form;

$$\left[\begin{array}{ccc|c} 3 & -2 & 1 & 2 \\ 4 & 3 & -2 & 4 \\ 5 & -3 & 3 & 8 \end{array}\right] \sim \left[\begin{array}{ccc|c} 3 & -2 & 1 & 2 \\ 0 & 17 & -10 & 4 \\ 0 & 0 & 78 & 234 \end{array}\right]$$

3. Solve using the resulting system using the substitution method;

$$\begin{array}{rrrcl} 3x & -2y & 1z & = & 2 \\ & 17y & -10z & = & 4 \\ & & 78z & = & 234 \end{array} \implies \{(x, y, z)\} = \{(1, 2, 3)\}$$