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Finding approximate root of a function using numerical values

Using the intermediate value theorem (listed below), there are numerical methods to find the root between an interval (a, b);

$$\text{Intermediate value theorem:}$$
$$f(a) > 0 \;\;\&\;\; f(b) < 0 \implies \exists\, x_k\, ;\, a < x_k < b \;\;\&\;\; f(x) \text{ is continuous}$$

Using the Newton method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}\, ;\, f'(x) = \frac{d}{dx} f(x)$$

Numerical integration (Quadrature)

By using numerical techniques, you can approximate the computation of integral values;

Definite integrals using the Newton-Cotes method

$$\int_a^b f(x) \approx (b-a) \cdot \sum_{i=0}^{n} f(S_i) \cdot w_i$$

$$S_i = \left\{ \frac{(b-a) \cdot (i+1)}{n}, \ldots \right\}$$

n	Factor	$\frac{w_i}{\text{Factor}}$				
1	$\frac{1}{2}$	1	1			
2	$\frac{1}{6}$	1	4	1		
3	$\frac{1}{8}$	1	3	3	1	
4	$\frac{1}{90}$	7	32	12	32	7

Ex.:

Solve the definite integral below using the Newton-Cotes method of order 4

$$\int_1^5 \sqrt[3]{x}$$

Resolution:

$$S_i = \left\{ \frac{4 \cdot 1}{4}, \frac{4 \cdot 2}{4}, \frac{4 \cdot 3}{4}, \frac{4 \cdot 4}{4}, \frac{4 \cdot 5}{4} \right\} = \{1, 2, 3, 4, 5\}$$

$$\int_1^5 \sqrt[3]{x} \approx (5-1) \sum_{i=0}^{4} \sqrt[3]{S_i} \cdot w_i = 4 \cdot \left(\left(\sqrt[3]{1} \cdot \frac{7}{90} \right) + \left(\sqrt[3]{2} \cdot \frac{32}{90} \right) + \left(\sqrt[3]{3} \cdot \frac{12}{90} \right) + \left(\sqrt[3]{4} \cdot \frac{32}{90} \right) + \left(\sqrt[3]{5} \cdot \frac{7}{90} \right) \right) \approx 5.66$$