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Vectors

Parametric representation of lines

A line ($\mathbf{L}$) in an n dimensional space can be represented using a vector equation, such as the one below:

$$\mathbf{L} = \{k \cdot \vec{v};\ k \in \mathbb{R}\}$$

Linear combination

Given a set of vectors ($\mathbf{V}$) and a set of scalars ($\mathbf{C}$), a linear combination can be represented as below:

$$\mathbf{V} = \{\vec{v_1}, \vec{v_2}, \vec{v_3}, ...\}$$
$$\mathbf{C} = \{c_1, c_2, c_3, ...\}$$
$$(c_1 \cdot \vec{v_1} + c_2 \cdot \vec{v_2} + c_3 \cdot \vec{v_3} + ...) \text{ Is a linear combination of } \mathbf{C} \ \& \ \mathbf{V}$$

Span

The span of a set of vectors ($\operatorname{Span}(\vec{v_1}, \vec{v_2}, \vec{v_3}, ...)$) is the set of all possible linear combinations between that set of vectors and the set of real numbers;

Linear independence

A set of vectors is said to be linearly independent if at least one vector in that set can't be represented as a linear combination of other vectors in that set;

Only if there is no solution for $(c_1 \cdot \vec{v_1} + c_2 \cdot \vec{v_2} + c_3 \cdot \vec{v_3} + ... = 0)$ other than when $\forall c_i = 0$ that the set of vectors is linearly independent;

Subspace of a vector set

A subset **U** is said to be a subspace of a superset **V** when **U** follows all the conditions below:

1. $\vec{0} \in U$
2. **U** contains all the linear combinations of vectors in itself (Is closed in vector addition and scalar multiplication)

Note: Span(**V**) are always valid subspaces;

Vector transformations

Vector transformations are the association between vectors from a $\mathbb{D}$ to a vector of a $\mathbb{CD}$ (In other words, functions operating on vectors)

$$T : \mathbb{R}^n \to \mathbb{R}^m;\, T(\vec{v_1}) = \vec{v_2}$$

Linear transformations

A transformation is said to be linear if it follows both the **additive property** and the **homogeneous property**

$$\text{Additive property}$$
$$T\left(\vec{a} + \vec{b}\right) = T\left(\vec{a}\right) + T\left(\vec{b}\right)$$
$$\text{Homogeneous property}$$
$$T\left(c \cdot \vec{a}\right) = c \cdot T\left(\vec{a}\right);\, \forall c \in \mathbb{R}$$

Linear transformations have some properties that are listed below:

$$T \text{ Is a linear transformation } \implies T\left(\vec{0}\right) = \vec{0}$$

$$T \text{ Is a linear transformation } \implies T \text{ can be represented by a transformation matrix } A$$

Note: $A \iff A \cdot \vec{v} = T\left(\vec{v}\right)$

Null space

The null space is the set of vectors that when transformed, equal the null vector;

$$N(T) = \left\{\vec{v};\, T\left(\vec{v}\right) = \vec{0}\right\}$$

Eigenvalues and eigenvectors

$$\vec{a} \text{ is an eigenvector of } A \iff A \cdot \vec{a} = \lambda \cdot \vec{a}$$

$$\lambda \text{ is an eigenvalue of } A \iff (A \cdot \vec{v}), \vec{v} \in \operatorname{span}(\vec{v})$$

Determining the matrix of eigenvalues of a matrix

Using determinants

$$\det(A - \lambda \cdot I) = 0$$

$$\Lambda = \begin{bmatrix} \lambda_1 & 0 & 0 & ... \\ 0 & \lambda_2 & 0 & ... \\ 0 & 0 & \lambda_3 & ... \\ ... & ... & ... & ... \end{bmatrix}$$

Using the matrix of eigenvectors

$$\Lambda = S^{-1} \cdot A \cdot S$$

Determining the eigenvectors using the eigenspace

$$E_{\lambda_n} = N(A - \lambda_n \cdot I)$$

Determining the eigenspaces for all λ values, you will find all the eigenvectors for the matrix A. You can create a matrix of eigenvectors as noted below:

$$S = [E_{\lambda_1}|E_{\lambda_2}|E_{\lambda_3}|...]$$

Determining the transformation matrix using eigenvalues and eigenvectors

$$A = S \cdot \Lambda \cdot S^{-1}$$