Introduction to quantum information

June 2025

Exam Instructions and Evaluation Criteria

Nature of the Project

This project is an **open problem**, meaning that there is no predefined solution. Instead, students are provided with guidelines and suggestions for exploration. However, students are encouraged and expected to go beyond the suggestions and bring their own insight, creativity, and analysis into the topic.

Important: Projects are **individual**. Collaboration or group work is not permitted.

Grading Breakdown

The final grade will be composed of two components:

1. **Absolute Grade (80% of final score)**
   This component is assigned individually to each student based on what was achieved in the submitted work. It is graded from 0 to 100%, independently of the rest of the class.
2. **Relative Grade (20% of final score)**
   This component is based on how the submitted project compares with those of other students. All projects will be ranked, with the best receiving 100% for this component. The rest of the submissions will be graded proportionally according to their relative ranking.
3. **Excellence Bonus (up to +20%)**
   An additional bonus may be awarded to projects that demonstrate **exceptionally high quality** or extend significantly **beyond the scope of the course**. This includes original contributions, innovative simulations, or outstanding analytical or simulational insights.

Submission Requirements

Each student must submit the following two components:

1. **A Jupyter Notebook** containing:
   - Full description of the work and results
   - All code used for the simulations and analysis
   - The notebook must be executable on **Google Colab**
2. **A PDF printout** of the notebook. This acts as a backup and is required in case execution issues occur with the Colab environment.

Remarks

- Projects that demonstrate originality, clarity of thought, and deeper investigation into the topic are highly valued.

- Cloning or direct copying of code or text from online sources or peers will result in disqualification.
- Proper documentation, clear visualizations, and insightful discussion of results are essential for full credit.

1. Theoretical Introduction: Quantum Model of a Ring of Spins

A one-dimensional ring of spin-1/2 particles with nearest-neighbor interactions is a canonical model in quantum many-body physics. The most common Hamiltonian describing such a system is the transverse-field Ising model (TFIM):

$$H = J \sum_{i=1}^{N} Z_i Z_{i+1} + h \sum_{i=1}^{N} X_i \qquad [0.1]$$

where Z_i and X_i are Pauli operators acting on site i, J is the coupling constant, h is the transverse field strength, and periodic boundary conditions imply $Z_{N+1} = Z_1$.

This model captures rich physics including quantum phase transitions, frustration (particularly when N is odd and $J > 0$), and entanglement dynamics. For $N \leq 10$, numerical simulations using quantum circuits become feasible and allow the study of state evolution, magnetization, and correlation functions.

Historically, these systems have been investigated using exact diagonalization, tensor networks, and quantum Monte Carlo. In recent years, they have also been explored using digital quantum computers via circuit-based simulations.

Relevant references:

- Nielsen & Chuang, *Quantum Computation and Quantum Information*
- Ladd et al., Nature 464, 45 (2010)
- Qiskit Textbook
- Kandala et al., Nature 549, 242 (2017)

Qiskit enables simulation by constructing quantum circuits to approximate the evolution operator e^{-iHt} using Trotter-Suzuki decomposition. Operators such as `RZZGate`, `RXGate`, and circuit composition tools allow one to model the Hamiltonian dynamics effectively.

Problem 1: Theoretical Derivation of a Trotterized Model

Objective: Derive the first-order Trotterized form of the time evolution operator for a ring of spins.

Brief Description: The Trotter-Suzuki formula allows approximating the exponential of a sum of non-commuting operators as a product of exponentials of individual terms:

$$e^{-i(H_Z+H_X)t} \approx \left(e^{-iH_Z\Delta t}e^{-iH_X\Delta t}\right)^r, \quad \Delta t = \frac{t}{r} \qquad [0.2]$$

Questions:

1. Write the decomposition of the Hamiltonian into commuting parts: $H_Z = J\sum Z_i Z_{i+1}$ and $H_X = h\sum X_i$.
2. Derive the form of the time evolution operator using first-order Trotterization.
3. For each term $e^{-iJZ_iZ_{i+1}\Delta t}$, express the equivalent quantum circuit using RZZ gates.
4. For each term $e^{-ihX_i\Delta t}$, express the equivalent circuit using RX gates.
5. Sketch the quantum circuit for one Trotter step on $N = 4$ spins.

Problem 2: Implementation in Qiskit

Objective: Construct and simulate a Trotterized quantum circuit for the ring model.

Tasks:

1. Implement a function to construct one Trotter step for a system of N spins.
2. Build a quantum circuit with r repeated Trotter steps.
3. Simulate the circuit using `Aer.get_backend('statevector_simulator')`.
4. Plot the probabilities of the computational basis states at the final time.

Problem 3: Simulations

Perform the following simulation:

1. Compare the time evolution for different values of the transverse field h (e.g., $h = 0.5, 1.0, 2.0$) and coupling strength (e.g., $J = \pm 0.5, \pm 1.0, \pm 2.0$).
2. Fix h and vary N from 3 to 10. Observe how the distribution changes.
3. Initialize in the all-zero state and track how population spreads over time.
4. Evaluate how the initial state influences the outputs.
5. Introduce a dissipative model of noise which allows the system to loose energy in order to evaluate the minimum (local or global) energy configuration of the system. My suggestion is to couple the target system with another which has a well defined energy minima, and apply the dissipative noise to the ancilla system.
6. Use odd N and observe frustration effects for $J > 0$.

Expected results:

- For small h, the system stays close to the classical spin configurations (low superposition).
- For large h, population spreads more due to strong transverse mixing.
- Frustration in odd N will show non-uniform distributions even in ground-state oriented circuits.

Problem 4: Analysis and Interpretation

Questions:

1. Analyze how the number of Trotter steps r affects the accuracy of the simulation.
2. Discuss how the choice of h and J changes the observable probabilities.
3. What do the most probable basis states suggest about the spin alignment?
4. Compute the expectation value of $\sum X_i$ and comment on the quantum magnetization.
5. What are the limitations of this simulation with regard to circuit depth and classical comparison?
6. How would noise affect your results and analysis?
7. Try to run you model and analyse your results in a real quantum computer.