Question 2-4

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Question 2.4: Focus distance and depth of field: Figure out how the focus distance and depth of field indicators on a lens are determined.

2. Compute the depth of field (minimum and maximum focus distances) for a given focus setting z_o as a function of the circle of confusion diameter c (make it a fraction of the sensor width), the focal length f , and the f-stop number N (which relates to the aperture diameter d). Does this explain the usual depth of field markings on a lens that bracket the in-focus marker, as in Figure 2.20a? We have initially a focused image with distance from the lens to image plane given by z_i, then we change the focus so we have a circle of confusion c. The new z_i' is given by $z_i + \Delta z_i$.

We have then:

$$z_i = \frac{f}{1 - f/z_o} \; z_i' = \frac{f}{1 - f/z_o'}$$

First, from the biggest distance we have the following triangle similarity:

$$\frac{c}{z_i - z_i'} = \frac{d}{z_i'} \; c = d\left(\frac{z_i}{z_i'} - 1\right) \; z_o' = \frac{dfz_o}{df + c(f - z_o)}$$

By similar arguments we find that the minimum z_o is:

$$z_o'' = \frac{dfz_o}{df + c(z_o - f)}$$

The depth of field is the difference between them $DOF = z_o' - z_o''$, then we have:

$$DOF = \frac{dfz_o}{df + c(f - z_o)} - \frac{dfz_o}{df + c(z_o - f)}$$

$$DOF = \frac{2dfz_o c(z_o - f)}{d^2f^2 + c^2(z_o - f)^2}, \text{ using that } d = f/N \text{ we have}$$

$$DOF = \frac{2Nf^2 z_o c(z_o - f)}{f^4 + c^2N^2(z_o - f)^2}$$