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Points: 50 **Due Date: April 25, 2019**

1 a**)** Perform unit and limit checks on the analytical solution for the steady state temperature distribution within the fruit derived in the class. For the limit check consider the temperature of the fruit at the outer edge (i.e., radius, r = a), when thermal conductivity, k approaches zero (i.e., fruit is an insulator). Is the result meaningful? Note that the limit check should be done analytically (i.e., not using plots). **8pts.**

b) Using MATLAB[1] evaluate temperature distribution within the fruit during harvest and during cold storage. Plot the temperature as a function of radius of the fruit. Use subplots (one per window) for this purpose. Use following data: **12 pts**.

Ambient Temperature at Harvest: 26.7 °C
Density of orange: 998 kg/m^3
Specific heat of orange: 3900 J/kg/°C
Storage temperature = 3 °C
Radius of the fruit = 0.036 m
Heat production due to respiration = 4.71 + 3.55 * Temperature (°C), J/(s-m^3)
Thermal conductivity of fruit = 0.47 W/m/°C
Convective heat transfer coefficient at the fruit surface = 6 W/m^2/°C

c) Determine the amount of heat that needs to be removed from the orange to keep it at its steady state temperature when the cold storage is held at 3°C. **5 pts.**

d) Determine the amount of heat that needs to be removed to cool the fruit from its ambient condition to cold storage condition (Do the integration analytically – i.e., do not do the numerical integration in MATLAB)? **12 pts**

e) Extend these results to say 600 bins of fruits each containing 500 kg of fruits. How could an engineer use this model to design a cold storage facility (- consider initial cooling and keeping the storage temperature at steady state)? **8 pts.**

[1] Include MATLAB program with your submission.

f) What are some of the limitations of the generalizations you are proposing to do in (2e) **5 pts.**

2. **Extra Credit:** Using the attached derivation of heat transfer equation in cylindrical coordinates as an example, show that the form of this equation in spherical system becomes:

$$\rho C \frac{\partial T}{\partial t} = \frac{1}{r^2}\left[\frac{\partial}{\partial r}\left\{k_r r^2 \frac{\partial T}{\partial r}\right\} + \frac{1}{\sin^2 \phi}\frac{\partial}{\partial \theta} k_\theta \frac{\partial T}{\partial \theta} + \frac{1}{\sin(\phi)}\frac{\partial}{\partial \phi}\left\{k_\phi \sin(\phi)\frac{\partial T}{\partial \phi}\right\}\right] \pm A$$

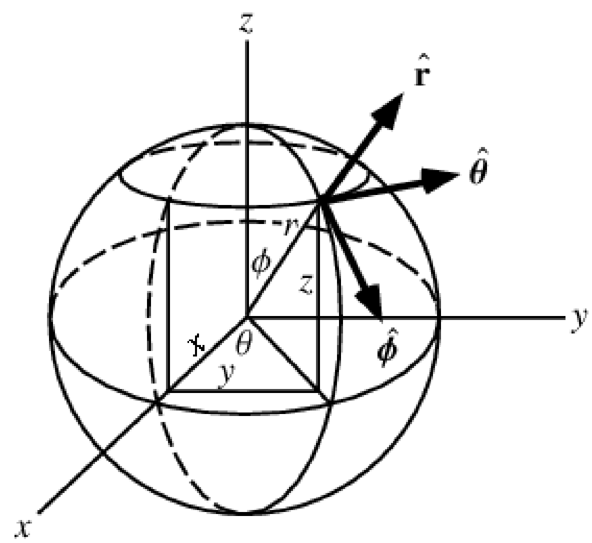
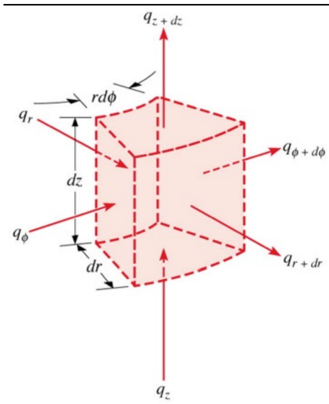


Figure 1. Spherical coordinate system (r, θ, ϕ)

Where T is the temperature, t is time, k_r, k_θ, k_ϕ are the orthotropic components of the heat transfer coefficient in spherical coordinate system, A is the generation/dissipation rate term per unit volume, and r, θ, and ϕ are spherical coordinates as shown in Figure 1. **5 points**

Heat Equation for Cylindrical Coordinate System - EBS 270 Student work (2013 W)



$$Differential\ volume:\ \ dV = rd\theta drdz \qquad (1)$$

$Differential\ area\ \ (r\ direction):$ $dA_r = rd\theta drdz$ (2)

(2)$Differential\ area\ (\theta\ direction):$ $dA_{r\theta} = drdz$ (3)

$Differential\ area\ (\phi\ direction):$ $dA_z = rd\theta dr$ (4)

Note: The area in the radial direction is not constant since it the arc length ($s = r\theta$) increases with r:

- $dA_r = rd\theta dz$
- $dA_{r+dr} = (r + dr)d\theta dz$

But, distribution gives,

$dA_{r+dr} = rd\theta dz + drd\theta dz$

and as dV gets very small, the second term becomes negligible and the two areas converge:

$dA_{r+dr} = \ rd\theta dz = \ dA_r$

1. Energy Balance on the differential volume:

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = \dot{E}_{stored} \qquad (5)$$

where

$$\dot{E}_{gen} = AdV \qquad (6)$$

$$\dot{E}_{store} = \rho C_p \frac{\partial T}{\partial t} dV \qquad (7)$$

2. Find $\dot{E}_{in} - \dot{E}_{out}$

$$\dot{E}_{in} - \dot{E}_{out} = (q_r - q_{r+dr}) + (q_{r\theta} - q_{\theta+rd\theta}) + (q_z - q_{z+dz}) \qquad (8)$$

Using the Taylor expansion, $f(x) = f(a) + f'(a)(x - a)$ we can define

$$q_{r+dr} = q_r + \frac{\partial q_r}{\partial r}(r + dr - r) = q_r + \frac{\partial q_r}{\partial r} dr \qquad (9)$$

$$q_{r\theta+rd\theta} = q_{r\theta} + \frac{\partial q_{r\theta}}{r\partial\theta}(r\theta + rd\theta - r\theta) = q_\theta + \frac{\partial q_{r\theta}}{\partial\theta} d\theta \qquad (10)$$

$$q_{z+dz} = q_z + \frac{\partial q_z}{\partial z}(z + dz - z) = q_z + \frac{\partial q_z}{\partial z} dz \qquad (11)$$

The right side of equation 8 can now be written,

$$\left(q_r - q_r - \frac{\partial q_r}{\partial r}dr\right) + \left(q_\theta - q_\theta - \frac{\partial q_{r\theta}}{\partial \theta}d\theta\right) + \left(q_z - q_z - \frac{\partial q_z}{\partial z}dz\right) \tag{12}$$

and simplified to,

$$-\frac{\partial q_r}{\partial r}dr - \frac{\partial q_{r\theta}}{\partial \theta}d\theta - \frac{\partial q_z}{\partial z}dz \tag{13}$$

Fourier's law says,

$$q_r = -k\frac{\partial T}{\partial r}dA_r; \quad q_{r\theta} = -k\frac{\partial T}{r\partial \theta}dA_{r\theta}; \quad q_z = -k\frac{\partial T}{\partial z}dA_z \tag{14}$$

Subbing into (13) gives,

$$\frac{\partial}{\partial r}\left(k\frac{\partial T}{\partial r}rd\theta dz\right)dr + \frac{\partial}{\partial \theta}\left(k\frac{\partial T}{r\,\partial \theta}drdz\right)d\theta + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}rd\theta dr\right)dz \tag{15}$$

Factoring out all factorable terms yields

$$\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right)d\theta drdz + \frac{\partial}{\partial \theta}\left(k\frac{\partial T}{r\,\partial \theta}\right)d\theta drdz + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)rd\theta drdz \tag{16}$$

Using the definition of dV (1) we can write,

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right)dV + \frac{1}{r}\frac{\partial}{\partial \theta}\left(k\frac{\partial T}{r\,\partial \theta}\right)dV + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)dV \tag{17}$$

Factoring dV, we can rewrite (8),

$$\dot{E}_{in} - \dot{E}_{out} = \left[\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \theta}\left(k\frac{\partial T}{\partial \theta}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)\right]dV \tag{18}$$

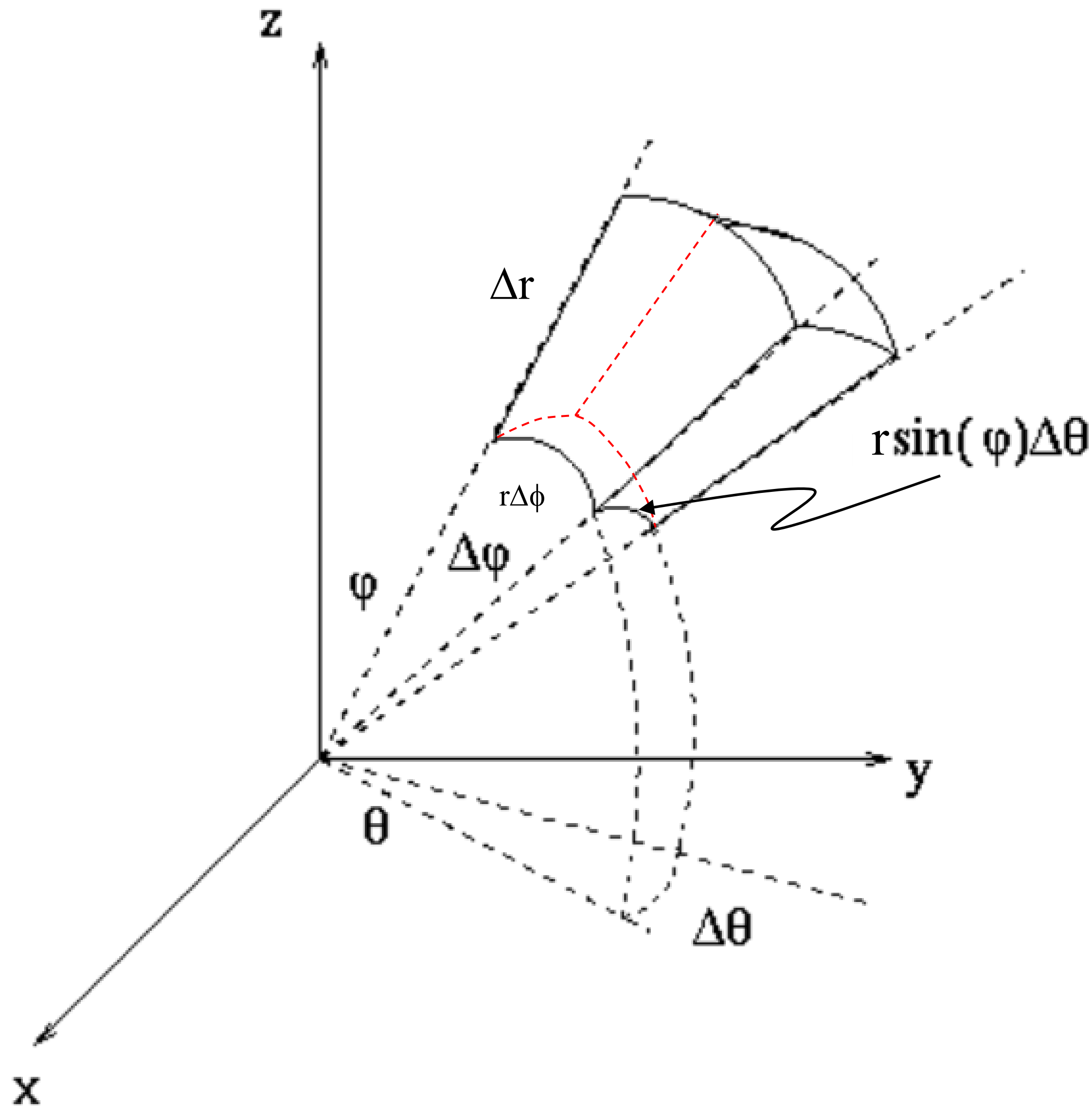
3. Rewrite energy balance

Subbing (6), (7), and (18) into (5) gives the energy balance

$$\left[\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \theta}\left(k\frac{\partial T}{\partial \theta}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)\right]dV + AdV = \rho C_p\frac{\partial T}{\partial t}dV \tag{19}$$

Finally cancelling dV from both sides,

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \theta}\left(k\frac{\partial T}{\partial \theta}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + A = \rho C_p\frac{\partial T}{\partial t} \tag{20}$$


z
Δr
rsin(φ)Δθ
rΔϕ
Δφ
φ
y
θ
Δθ
x