

Biological and Agricultural Engineering Department

Modeling and Analysis of Physical and Biological Processes: EBS 270

Homework No. 3

Points: 120

Due Date: May 24, 2019

- 15 1) Solve for the steady state temperature distribution within the orange using FDM. Derive matrix equations first and then write your own MATLAB code to do this problem. Verify the FDM solution against the steady state solution obtained in HW#1. Use the following data:

Ambient Temperature at Harvest: 26.7 °C

Storage temperature, 3 °C

Radius of the fruit = 0.036 m

Heat production due to respiration = $4.71 + 3.55 * \text{Temperature (°C)}$, J/(s·m³)

Fruit and rind density = 998 kg/m³

Sp. heat of fruit and rind = 3900 J/kg/°C

Thermal conductivity of fruit = 0.47 W/m/°C

Convective heat transfer coef. at the rind surface = 6 W/m²/°C

- 15 2a) Solve the transient orange cooling problem using “pdepe” routine in MATLAB. Assume that the fruit was harvested at 26.7 °C. Note that the distribution of temperature at t=0 needs to be determined using “pdepe” itself¹). Hint: You need to think about how to input IC for orange cooling problem. You can start with ambient (harvest) condition (IC) and go for a long time with convective heat loss corresponding to ambient temperature (this creates IC for cold storage condition), then change convective BC to represent cold storage condition. Compare the steady state solutions obtained using theoretical, FD in #1, and pdepe by plotting all of them on the same figure.

- 10 2b) Repeat Prob#2a without making the assumption that heat of respiration is a linear function of temperature (i.e., Use Arrhenius equation for heat of respiration, i.e, $Q=Q^* e^{-\frac{E}{RT}}$, where $Q^*= 11.6*10^{12}$ J/m³, $E/R=7569$ °K. How much does the steady state temperature distribution differ from the one obtained in #2a? Is the linear heat source assumption reasonable?

¹ You could use your analytical solution for this purpose. However, when you go to problem #2b, you will need numerically obtained IC.

- 20 3a) Derive the following forward and backward difference second order approximation to the first derivative at node ℓ analytically:

Forward Difference:
$$\left. \frac{\partial \phi}{\partial x} \right|_{x_\ell} = \frac{-3\phi_\ell + 4\phi_{\ell+1} - \phi_{\ell+2}}{2\Delta x} + O(\Delta x^3) \quad (1a)$$

Backward Difference:
$$\left. \frac{\partial \phi}{\partial x} \right|_{x_\ell} = \frac{\phi_{\ell-2} - 4\phi_{\ell-1} + 3\phi_\ell}{2\Delta x} + O(\Delta x^3) \quad (1b)$$

- 10 3b) Repeat Prob #2b assuming that there is material discontinuity between rind and the flesh. How much does the steady state temperature distribution differ from the one obtained in #2b? Is the assumption of homogeneity between rind and flesh (i.e., same thermal conductivity for flesh and rind) reasonable? Use following data for the rind:

Thickness of the rind is 0.003 m;

Thermal conductivity of the rind is 0.23 W/m²/°C

- 5 3c) Determine the amount of heat that should be removed after the fruit reaches steady state for the case given in 3b above. How does this amount compare with the analytically estimated amount in HW#1?

Hint: Heat that needs to be removed during cold storage under steady condition can be determined by knowing the temperature of the last node at steady state and evaluating the convective heat loss for transient problems.

- 15 3d) Estimate the amount of heat that needs to be removed for cooling the orange from the harvest condition to cold storage condition. How does this amount differ from the one obtained in HW#1.

Hint: Heat removed during cooling can be determined by knowing the temperature of the last node on the rind with time and evaluating the convective heat loss for the transient problem.

- 30 4) Solve the bioreactor problem (Prob#3, HW#2) using pdepe. Refer to Example #2 in the “pdepe” document. You can use “help” feature of MATLAB to get this information.

Remember that the growth of *A. niger* follows the following growth equation:

$$\frac{dX}{dt} = \mu_{\max} X \left[1 - \frac{X}{X_{\max}} \right] \quad (3a)$$

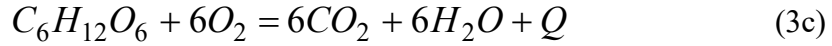
$$\frac{d[m_{CO_2}]}{dt} = \left[a \frac{dX}{dt} + bX \right] \quad (3b)$$

where,

$$\mu_{\max} = \frac{Ae^{\left(\frac{-E_{a1}}{RT_{abs}}\right)}}{1 + Be^{\frac{-E_{a2}}{RT_{abs}}}}, \text{ Maximum growth rate}^2$$

(Saucedo-Costaneda et al³, 1990).
 X = Concentration of the microorganism
 X_{\max} = Maximum biomass concentration that depends on temperature, expressed as fraction of dry matter
 T_{abs} = Absolute temperature
 $A = 2.694 \cdot 10^{11}$ (1/h)
 $B = 1.3 \cdot 10^{47}$
 $E_{a1} = 70,225$ J/(g-mole)
 $E_{a2} = 283,356$ J/(g-mole)
 $R = 8.31447$ J/(mol-°K)
 $a = 0.37$ g-CO₂/cc-S
 $b = 0.41 \cdot 10^{-5}$ g-CO₂/(cc-S h)
 m_{CO_2} = mass of CO₂ per unit volume of substrate, S⁴.

Moreover, respiration which leads to CO₂ generation is given by:



where $Q = 674$ Kcal/mole of glucose.

Use coupled pdes (both X and T) and solve the nonlinear problem.

$$\frac{\partial X}{\partial t} = \mu_{\max} X \left[1 - \frac{X}{X_{\max}} \right] + \frac{1}{r} \frac{\partial}{\partial r} \left(k_1 r \frac{\partial X}{\partial r} \right) \quad (4)$$

with $X = 8.74 \cdot 10^{-4}$ g/g-DM. when $t=0$ and $\frac{\partial X}{\partial r} = 0$ at both $r=0$ and $r=6$ cm.

$$\rho C_p \frac{\partial T}{\partial t} = Q + \frac{1}{r} \frac{\partial}{\partial r} \left(k_2 r \frac{\partial T}{\partial r} \right) \quad (5)$$

with $T = 25^\circ\text{C}$ at $t=0$ and BCs are given by:

² Note that (E/RT) has no units. As long as E , R , and T are in consistent units, you do not need to convert them to some other units.

³ Saucedo-Castaneda, G., M. Gutierrez-Rojas, G. Bacquet, M. Raimbault, and Viniegra-Gonzalez. 1990. Heat transfer simulation in solid substrate fermentation. *Biotechnology and Bioengineering*. 35:802-808.

⁴ This is given in terms of unit volume of substrate after appropriately accounting for the density of substrate.

$$\frac{\partial T}{\partial r} = 0 \quad \text{at } r=0$$

and

$$-k_2 \frac{\partial T}{\partial r} = h(T - T_\infty) \quad \text{at } r=3 \text{ cm}$$

Note that $f_1 = k_1 \frac{\partial X}{\partial r} = 0$ in equation (5)!

Note that X_{\max} should be a fraction (not percent) when you use in pdepe. Saucedo-Castaneda et. al.(1990) give following expression for X_{\max} :

$$X_{\max} = (-127.08 + 7.95T - 0.016T^2 - 4.03 \cdot 10^{-3}T^3 + 4.73 \cdot 10^{-5}T^4)/100 \quad (6)$$

For $30^\circ\text{C} < T < 45^\circ\text{C}$

Use following data:

$$\begin{aligned} k_2 &= 18 \text{ cal}/(\text{cm } ^\circ\text{C h}) \\ h &= 10.14 \text{ cal}/(\text{cm}^2 \text{ } ^\circ\text{C h}) \\ \rho &= 0.7 \text{ gm}/\text{cm}^3 \\ C_p &= 4.19 \text{ cal}/(\text{g } ^\circ\text{C}) \end{aligned}$$

If you need additional data please refer Saucedo-Costaneda et al(1990).

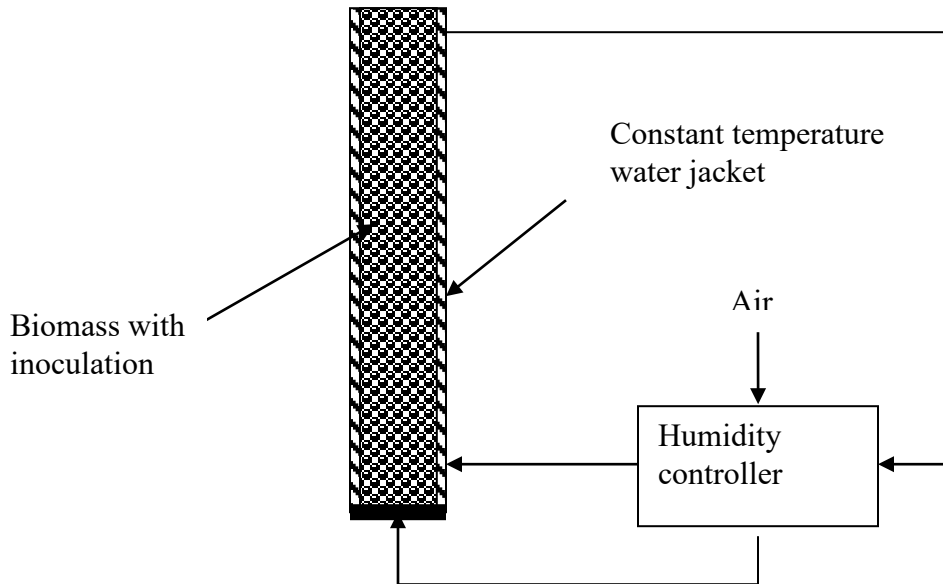


Figure 1. A flexiglass, cylindrical bioreactor of radius 6 cm and height 35 cm with a water jacket that is maintained at 35°C .