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Points: 60 Due Date: June 6, 2019[1]

Note: Include mesh, 3-D plot, 2-D plot for last time step, and relevant 1-D plot to show the key points (In problem #2 show surface temperature, in problem #3 show concentration of oil at selected times – 1year, 2,year, 5year etc., and in problem #4 temperature distribution in the radial direction at key times). ***For Prob#1, you need to write your own MATLAB code. For problems 2, 3 and 4 use COMSOL.***

20 1) Determine steady state temperature distribution during storage within the homogeneous and isotropic spherical orange using FEM. Write down the matrix equation for a typical element as well as the first and the last elements. Develop your own MATLAB code to solve this problem. Compare your results with the analytical and FDM solutions. Use the following data:

Ambient Temperature at Harvest: 26.7 °C
Storage temperature, 3 °C
Radius of the orange = 0.036 m
Heat production due to respiration = 4.71 + 3.55 * Temperature (°C), J/(s-m^3)
Fruit and rind density = 998 kg/m^3
Sp. heat of fruit and rind = 3900 J/kg/˚C
Thermal conductivity of fruit = 0.47 W/m/˚C
Convective heat transfer coef. at the rind surface = 6 W/m^2/˚C

Hint: Use equation #5 with associated BC (equations 6a and b) to solve this problem. Your solution will be in terms of u. You will have to solve it for $T=u/r + T_{\infty}$. Refer to the class handout for the variable definitions.

10 2a) Use FEM to solve the orange cooling problem, if orange is treated as an ellipsoid of revolution with semi-major axis equal to 0.045 m and semi-minor axis equal to 0.035 m. Assume that the rind material has a thermal conductivity value of 0.23 W/m/ºC. Compare your results with FDM solution obtained in HW#3. Use the relevant data from prob#1.

b) Estimate the amount of heat that needs to be removed per orange after it reaches steady state[2]? Does this agree with the theoretical and Finite Difference solution?

15 3) Evaluate the peanut oil concentration in peanut butter cup with time, if peanut butter cup is assumed axisymmetric in geometry and has a height of 18 mm with a realistic distribution of peanut butter in the middle (perhaps a hemisphere of radius 12 mm with the origin of the sphere 3 mm

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[1] This homework will be accepted without penalty until June 13, 2018.

[2] For simplicity, you can use the average surface temperature at steady state and the surface area of the ellipsoid to perform appropriate computation. You can better estimation if you plot total heat flux from each surface element at steady state. If you did this estimation at each time step, you can determine the amount of heat to be removed as it cools.

below the center of the top surface)[3]. Assume that the top of the cup has a radius of 15 mm and the bottom of the cup has a radius of 10 mm[4]. Assume also that the initial concentration of peanut oil in peanut butter layer is 18%. Assume that the diffusion coefficient is $5.5*10^{-15}$ m^2/s. Solve the axisymmetric problem completely (i.e., r and z dependency). Do the results of FEM agree with the analytical results? If not, what changes did you find and reasons for this change?

15 4) Solve the bioreactor problem (Prob#3, HW#2/3) using COMSOL.
Remember that the growth of A. niger follows the following growth equation[5]:

$$\frac{dX}{dt} = \mu_{\max} X \left[1 - \frac{X}{X_{\max}}\right] \tag{3a}$$

$$\frac{d[m_{CO2}]}{dt} = \left[a\frac{dX}{dt} + bX\right] \tag{3b}$$

where,
$\mu_{max} = 9.2 \bullet 10^{-5}$ s^{-1}, Maximum growth rate (Saucedo-Costaneda et al[6]., 1990).
X = Concentration of the microorganism, expressed as fraction
X_{max}= 0.28, Maximum biomass concentration, expressed as fraction
a = 0.53 kg-CO_2/kg-S
b = $1.61 \bullet 10^{-9}$ kg-CO_2/(s-kg-S)
m_{CO2} = mass of CO_2 per unit volume of substrate.

Initial concentration of the biomass, X_0 is $8.74*10^{-6}$ (kg of biomass/kg of substrate). Moreover, respiration which leads to CO_2 generation is given by:

$$Q = 7.5 \bullet 10^9 [0.53\frac{dX}{dt} + 1.61 \bullet 10^{-9} X] \text{ W/(m}^3\text{)} \tag{3c}$$

Use following data:

Thermal conductivity, k = 2.1 W/(m-°C)
Convective heat transfer coefficient, h = 117.85 W/(m^2 °C)
Density, ρ = 700 kg/m^3
Specific heat, C_p = 17531 J/(kg -°C)

Note: I have converted all numbers into SI units. You may use the explicit solution for equation (3a) for X, i.e.,

$$X = \frac{X_{\max} \bullet C_0 e^{\mu_{\max} t}}{1 + C_0 e^{\mu_{\max} t}} \tag{3d}$$

where $C_0 = \dfrac{X_0}{X_{\max} - X_0}$

[3] Note that you can just put in a quarter circle to represent peanut butter cup, by putting a 90 degree sector and rotating it by -90 degrees.
[4] Define the four corner points. Use Bezier polygon to connect them (i.e., define all four line segments by entering coordinates for each segment in a small table and then close it to get the trapezoidal shape.
[5] μ_{max} and X_{max} are expressed as constants corresponding to their maximum values for simplicity. Temperature dependency of these parameters can be built into the model relatively easily.
[6] Saucedo-Castaneda, G., M. Gutierrez-Rojas, G. Bacquet, M. Raimbault, and Viniegra-Gonzalez. 1990. Heat transfer simulation in solid substrate fermentation. Biotechnology and Bioengineering.35:802-808.

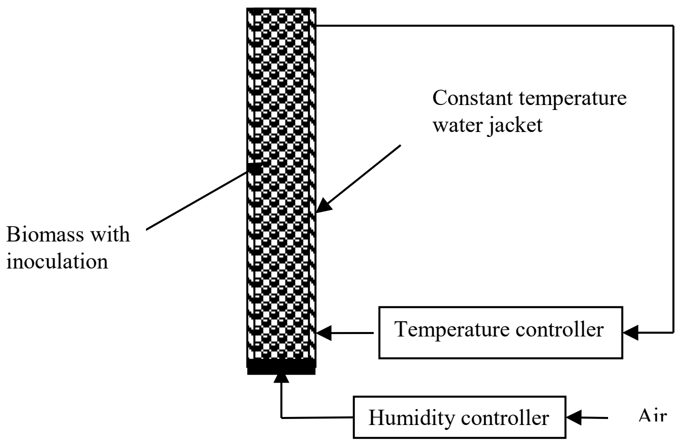

Figure 1. A flexiglass, cylindrical bioreactor of radius 0.06 m and height 0.35 m with a water jacket that is maintained at 35 ºC.