HOMEWORK ASSIGNMENT #3 – EBS270
Spring, 2019

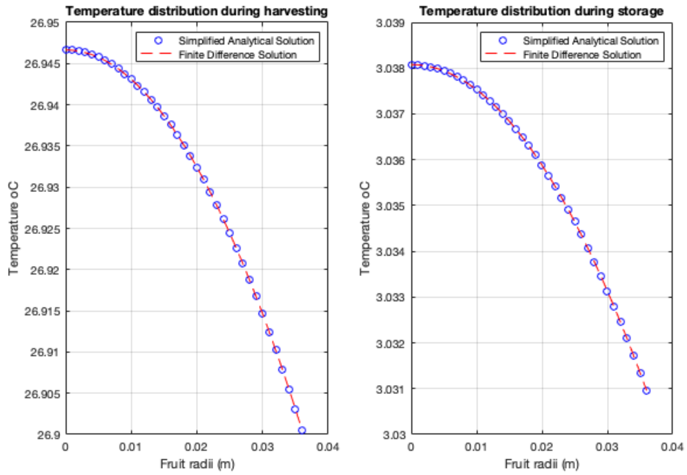
Student: Guilherme De Moura Araujo

MATLAB CODE

```
%% Biological and Agricultural Engineering Department
% Modeling and Analysis of Physical and Biological Processes: EBS 270
% Homework No. 3 - Due Date: May 28, 2019
% Student: Guilherme De Moura Araujo
%%
clear all; clc;

global T_Harvest T_Storage rho cp a a0 a1 k h w he

T_Harvest = 26.7; % Ambiente Temperature at Harvest, (oC)
T_Storage = 3; % Storage temperature (oC)
rho = 998; % Density of orange (km/m3)
cp = 3900; % Specific heat of orange (J/kg/oC)
a = 0.036; % Radius of fruit (m)
a0 = 4.71;
a1 = 3.55;
%A = a0 + a1*T; % Heat production due to respiration (J/(s-m3))
k = 0.47; % Thermal Conductivity of the fruit (W/m/oC)
h = 6; % Convective heat transfer coefficient at the fruit surface (W/m2/oC)
w = sqrt(a1/k);
he = h-k/a;
dr = 0.001; %Random value adopted by student

%% Analytical Solution from Homework #1
% Harvesting condiions
T_Inf = T_Harvest;
alpha = -(a0 + a1*T_Inf)/a1;
A = -(alpha*(k+he*a))/(he*sin(w*a)+k*w*cos(w*a));
j = 1;
for r=0:dr:a
   if r == 0
     T_H(j) = A*w-((a0+a1*T_Inf)/a1)+T_Inf;
   else
   u_H(j) = A*sin(w*r)+alpha*r;
   T_H(j) = u_H(j)/r+T_Inf;
   end
   j = j+1;
end
%-----------------------------------------------------------------------
% Storage conditions
T_Inf = T_Storage;
alpha = -(a0 + a1*T_Inf)/a1;
A = (-alpha*(k+he*a))/(he*sin(w*a)+k*w*cos(w*a));
j = 1;
for r=0:dr:a
   if r == 0
     T_S(j) = A*w-((a0+a1*T_Inf)/a1)+T_Inf;
```

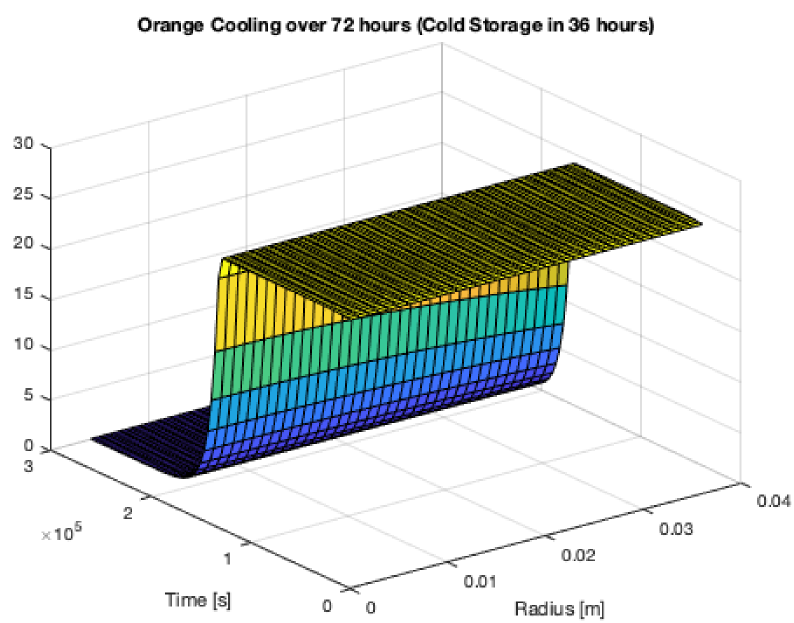
```matlab
    else
    u_S(j) = A*sin(w*r)+alpha*r;
    T_S(j) = u_S(j)/r+T_Inf;
    end
    j = j+1;
end

%% Finite Difference Solution

N = 37; % Number of nodes
dr = a/(N-1); % Delta r -> Same used for analytical solution
alpha = k/(dr*dr); % Constant
beta = a1-2*alpha;

AN = zeros(10,10);
for i=1:N
    for j=1:N
        if i>=2
            if i==j
                AN(i,j) = beta;
                AN(i,j-1) = (alpha-alpha/(i-1));
                if i<N
                AN(i,j+1) = (alpha+alpha/(i-1));
                end
            end
        end
    end
end

AN(1,1) = a1-6*alpha;
AN(1,2) = 6*alpha;
AN(N,N-1) = 2*alpha;
AN(N,N) = -2*(alpha+h/a+h/dr)+a1;

FNH = -a0*ones(N,1);
FNH(N)= -a0-2*h*(1/a+1/dr)*T_Harvest;

FNS = -a0*ones(N,1);
FNS(N)= -a0-2*h*(1/a+1/dr)*T_Storage;

TNH = inv(AN)*FNH;
TNS = inv(AN)*FNS;

j = 0:dr:a;
subplot(1,2,1)
plot(0:a/(length(T_H)-1):a,T_H,'bo');
hold on
plot(j,TNH,'r--');
legend('Simplified Analytical Solution','Finite Difference Solution')
title('Temperature distribution during harvesting');
xlabel('Fruit radii (m)');ylabel('Temperature oC');
grid
subplot(1,2,2)
plot(0:a/(length(T_H)-1):a,T_S,'bo');
```

```
hold on
plot(j,TNS,'r--');
legend('Simplified Analytical Solution','Finite Difference Solution');
title('Temperature distribution during storage');
xlabel('Fruit radii (m)');ylabel('Temperature oC');
grid
```

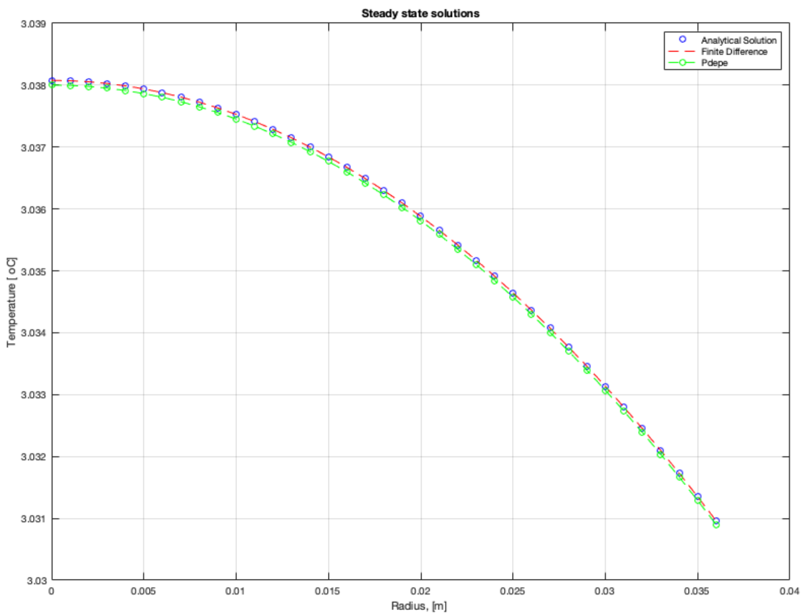



```
%% Problem 1 comments
% After plotting Simplified Analytical Solution vs. Finite Difference we can notice that both results
are in perfect accord with each other.

%% Alternative hand written solution (PLEASE DON'T RUN THIS SECTION IF YOU RAN THE
PREVIOUS SECTION)
N = 10; % Total number of Nodes (From T0 to T9)
dr = a/(N-1);
alpha = k/(dr*dr); % Constant
beta = a1-2*alpha;

A = [a1-6*alpha 6*alpha 0 0 0 0 0 0 0 0;
    0 beta 2*alpha 0 0 0 0 0 0 0;
    0 alpha/2 beta 3/2*alpha 0 0 0 0 0 0;
    0 0 2/3*alpha beta 4/3*alpha 0 0 0 0 0;
    0 0 0 3/4*alpha beta 5/4*alpha 0 0 0 0;
    0 0 0 0 4/5*alpha beta 6/5*alpha 0 0 0;
    0 0 0 0 0 5/6*alpha beta 7/6*alpha 0 0;
    0 0 0 0 0 0 6/7*alpha beta 8/7*alpha 0;
    0 0 0 0 0 0 0 7/8*alpha beta 9/8*alpha;
    0 0 0 0 0 0 0 0 2*alpha -2*(alpha+h/a+h/dr)+a1];

FH = [-a0;-a0;-a0;-a0;-a0;-a0;-a0;-a0;-a0;-a0-2*h*(1/a+1/dr)*T_Harvest];
TH = inv(A)*FH;

FS = [-a0;-a0;-a0;-a0;-a0;-a0;-a0;-a0;-a0;-a0-2*h*(1/a+1/dr)*T_Storage];
TS = inv(A)*FS;

j = 0:a/(N-1):a;
subplot(1,2,1)
```

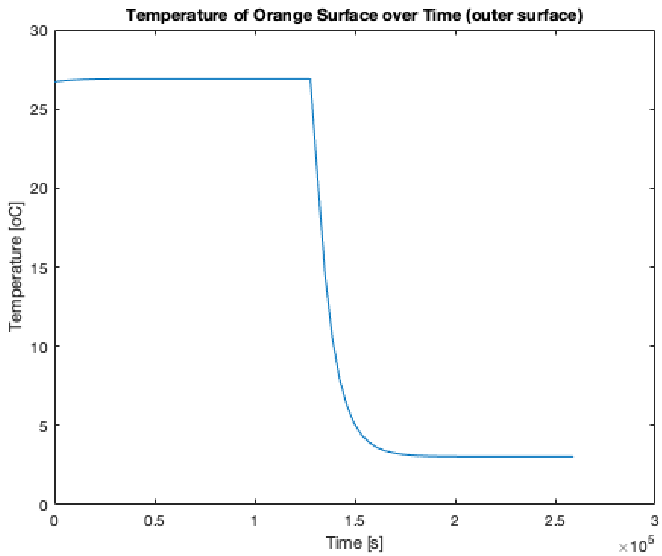
```
plot(0:a/(length(T_H)-1):a,T_H,'bo');
hold on
plot(j,TH,'r--');
legend('Simplified Analytical Solution','Finite Difference Solution')
title('Temperature distribution during harvesting');
xlabel('Fruit radii (m)');ylabel('Temperature oC');
grid
subplot(1,2,2)
plot(0:a/(length(T_H)-1):a,T_S,'bo');
hold on
plot(j,TS,'r--');
legend('Simplified Analytical Solution','Finite Difference Solution');
title('Temperature distribution during storage');
xlabel('Fruit radii (m)');ylabel('Temperature oC');
grid

%% Transient Orange Cooling using PDEPE

global Q ER
Q = 11.6e+12;
ER = 7569;
r = 0:dr:a;

m = 2;
t = 0:(259200/71):259200; %3600 s/h * 24/day * 3days = 259200 s
sol = pdepe(m, @pdefun, @pdeic, @pdebc, r, t);

% Surface plot
surf(r, t, sol)
title('Orange Cooling over 72 hours (Cold Storage in 36 hours)')
xlabel('Radius [m]')
ylabel('Time [s]')
```



```
%% Problem 2a

figure
plot(r, T_S, 'bo', r, TNS, 'r--', r, sol(end,:), 'g--o');
title('Steady state solutions')
xlabel('Radius, [m] ');
ylabel('Temperature [ oC]');
legend('Analytical Solution', 'Finite Difference', 'Pdepe');
grid;
```




```
% Comments:
% We can see that all three methods in accord with each other.
%% Temperature as a function of time for the last node (orange outer surface)

figure
plot(t, sol(:,end))
title('Temperature of Orange Surface over Time (outer surface)')
xlabel('Time [s]')
ylabel('Temperature [oC]')
```

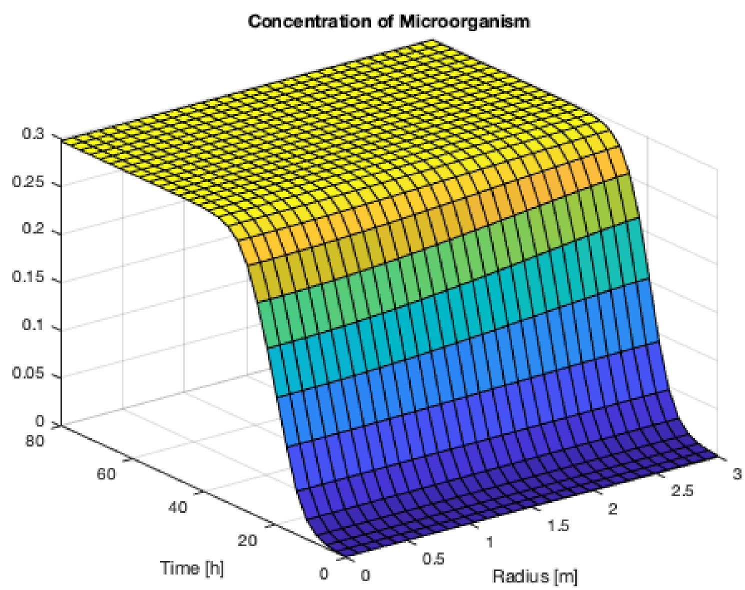


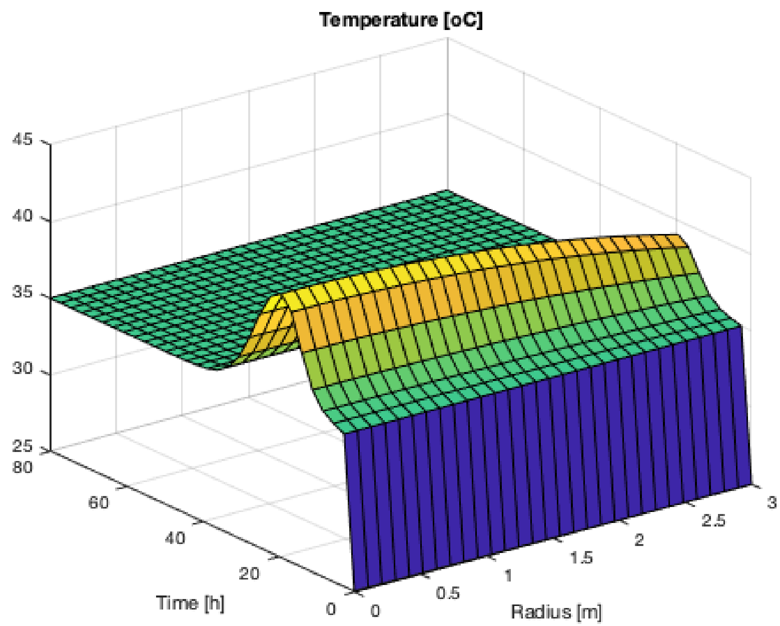

```
%% Problem 2b

T_Linear = sol(end,:);  % Comment out for 2b
T_Arrhenius = sol(end,:);
DiffT = T_Linear - T_Arrhenius;
avgT = mean(DiffT(:));

fprintf('Average difference temperature in the steady state: %.5f oC\n',avgT);

% Comments: after running pdepe two times, the average temperature
% difference between the linear assumption and the equation of Arrhenius
% was calculated as 0.0027 oC, which is slightly insignificant. Therefore
% we conclude that our previous assumption was good.

%% Problem 3b

T_homogenous = sol(end,:);  % Comment out for 3b
T_descontinuity = sol(end,:);
DiffT2 = T_homogenous - T_descontinuity;
avgT2 = mean(DiffT2(:));

fprintf('Average difference temperature in the steady state: %.5f oC\n',avgT2);

% Comments: after running pdepe two times, the average temperature
% difference between the homogenous assumption and the more complex model
% (material descontinuity flesh-rind) was calculated as -0.00111 oC, which
% is slightly insignificant. Therefore we conclude that our previous
% assumption (material is homogenous) was good.
```

```
%% Problem 3c

T = sol(end,end);
area = 4*pi*a^2;
Qremoved = h * area * (T-T_Storage);
fprintf('Heat required to be removed: Q = %.5f J/s\n',Qremoved);

% Comments: From homework 1 we calculated Q = 0.003 J/s; and now we found Q = 0.028 J/s.
Again, the difference is very small. Suggesting that a simpler model yields good results just as a
more complex model.

%% Problem 3d

V = 4/3*pi*a^3;
Q2 = rho * cp * V * (T_Harvest - T) % calculates E for 3d
fprintf('Heat required to be removed: Q = %.2f J/s\n',Q2);

% Comments: From homework 1 we calculated Q = 18,100 J/s; and now we found
% Q = 18,000 J/S. Again, the difference is very small. Suggesting that a
% simpler model yields good results just as a more complex model.


%% Functions Required for PDEPE
% pdepe
function [c,f,s] = pdefun(x,t,u,DuDx)
global a0 a1 cp h he Q ER rho k
c = rho * cp;
% Uncomment the if statement for parts 3b, 3c and 3d
% if x <= 0.033
%   k = 0.47;
% else
%   k = 0.23;
% end
f = k * DuDx;
s = a0 + a1 * u; % comment this out for 2b and after
% s = Q * exp(-ER/(u + 273)); % uncomment this out for 2b and after
end
% IC
function u0 = pdeic(x)
global T_Harvest
u0 = T_Harvest;
end
% BC
function [pl,ql,pr,qr] = pdebc(xl,ul,xr,ur,t)
global T_Harvest T_Storage h
pl = 0;
ql = 1;
if t < 129600
   pr = h * (ur - T_Harvest);
else
   pr = h * (ur - T_Storage);
end
qr = 1;
end
```

```
%% Problem 4
clear all; clc;
global A B Ea1 Ea2 R a b k1 k2 h rho cp Tinf
A = 2.694 * 10^11;
B = 1.3 * 10^47;
Ea1 = 70225;
Ea2 = 283356;
R = 8.31447;
a = 0.53;
b = 0.58*10^-5;
k1 = 0;
k2 = 18;
h = 10.14;
rho = 0.7;
cp = 4.19;
Tinf = 35;
m = 1;
x = linspace(0,3,30);
t = linspace(0,80,30);
sol = pdepe(m, @pdefun, @pdeic, @pdebc, x, t);
u1 = sol(:,:,1);
u2 = sol(:,:,2);

%% Plot Results
figure
surf(x, t, u1);
title('Concentration of Microorganism');
xlabel('Radius [m]');
ylabel('Time [h]');

figure
surf(x, t, u2);
title('Temperature [oC]');
xlabel('Radius [m]');
ylabel('Time [h]');

%% Comments:
% We can see that the microorganisms grow until a certain point and
then
% mantain its concentration constant, where the growth of new microbes
% offsets the death of old ones. Therefore, we conclude that upon a
certain
% point, the concentration of microorganisms stays constant as time
goes.
% It makes sense if we look at the system equation, in which we have a
term
% umax*X*[1-X/Xmax], suggesting that after X = Xmax there's no change
in
% the concentration.
% The temperature has somehow a similar behavior. As the original
% concentration of microorganisms goes up the heat generation also goes
up,
% which increases the temperature. As the concentration of
microorganisms
% stabilizes the temperature goes down and remain constant, mostly due
to
```

```
% the convective heat loss between the bioreactor surface and the water
% jacket (which acts as a cooling jacket everytime the bioreactor
% temperatures is above 35 oC). The steady state temperature is
maintained
% at 35 oC, which is the temperature of the water jacket.

%% Functions

% pdepe
function[c, f, s] = pdefun(x, t, u, DuDx)
global A B Ea1 Ea2 R a b k2 rho cp
c = [1; rho*cp];
f = [0; k2].*DuDx;
exp1 = exp(-Ea1/(R * (u(2) + 273)));
exp2 = exp(-Ea2/(R * (u(2) + 273)));
umax = A * exp1/(1 + B * exp2);
xmax = (-127.08 + 7.95*u(2) - 0.016*u(2)^2 - 4.03*10^(-3)*u(2)^3 +
4.73*10^(-5)*u(2)^4)/100;
DxDt = umax*u(1)*(1-u(1)/xmax);
mCO2 = 674000/(6*44)*(a*DxDt+b*u(1));
s = [DxDt; mCO2];
end
% IC
function u0 = pdeic(x)
u0 = [8.74*10^-4; 25];
end
% BC
function [pl, ql, pr, qr] = pdebc(xl, ul, xr, ur, t)
global h Tinf
pl = [0;0];
ql = [1;1];
pr = [0;h*(ur(2)-Tinf)];
qr = [1;1];
end
```




Temperature [oC]
45
40
35
30
25
80
60
40
20
0
Time [h]
0
0.5
1
1.5
2
2.5
3
Radius [m]