



## **A HEURISTIC APPROACH FOR THE ELECTRIC VEHICLE ROUTING PROBLEM**

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### **ABSTRACT**

Electric vehicles are gaining popularity as companies strive to employ more sustainable solutions. Due to their higher costs compared to traditional vehicles, applying operations research techniques becomes critical to ensure efficiency and cost competitiveness. This paper presents a mathematical model and a heuristic approach to solve the electrical vehicle routing problem for a last mile delivery scenario with capacity and battery restrictions. The heuristic approach employs a modified version of the Split algorithm that delivers the optimal breakdown into routes for a given sequence of customers. In order to improve the efficiency, a two-phase process is proposed. First, we remove the battery restrictions and get a good quality solution for the corresponding CVRP. Second, we use the solution for the CVRP as the initial solution for the EVRP using the modified Split algorithm with a local search heuristic.

**KEYWORDS.** Electric vehicles. Heuristics. Split algorithm.

**Paper topics:** MH – Metaheuristics, OC – Combinatorial Optimization

## 1. Introduction

A couple is visiting a shopping mall and enters a bookstore. The husband picks a book from a shelf, takes a look and tells his wife he will buy it. His wife gets her mobile phone, goes to the Internet and tells him not to buy because she can order it online at a lower price including freight. This real-life example illustrates how popular and attractive is online shopping.

Online shopping has reshaped how companies look at their logistics function. It is not just a matter of developing a product but also ensuring that the product can get to the consumer homes, and not any longer just to a store. It moves from the traditional truck full of merchandise being unloaded at a store to smaller vehicles that are more suited to the environment of city centers where they need to cope with congested streets and parking restrictions to reach consumer homes. Today small vans, motorcycles and even bicycles can be used to perform the last mile delivery in many urban areas. [FERGUNSON 2012] indicated that 25% of all products and 50% of all light products can migrate from traditional transportation vehicles to bicycles in urban areas. [DE DECKER 2012] pointed out that bicycles can be designed to carry weight even above 200kg. Considering that the last mile delivery may represent 28% of the total delivery cost, then optimization in this area can bring considerable gain opportunities [RANIERI et al. 2018].

Going a step further, these small vans, motorcycles and bicycles can be electric vehicles aiming to reduce CO<sub>2</sub> emissions which is a growing worldwide concern. [MASSINK et al. 2011] indicated that 23% of the global CO<sub>2</sub> emissions comes from the transportation sector. Therefore, electric vehicles have received more attention in the past years. Nevertheless, these electric vehicles bring a new challenge to the design of a logistics network which is the need for battery recharging during their delivery routes. This is the Electric Vehicle Routing Problem, known as EVRP.

## 2. Literature Review

The vehicle routing problem was introduced by [DANTZIG and RAMSER 1959] opening a door for extensive research. Several variations of the problem were formulated considering vehicle capacity restrictions (capacitated vehicle routing problem - CVRP), customers with service time windows (vehicle routing problem with time windows - VRPTW), different types of vehicles (heterogeneous vehicle routing problem – hVRP), more than one depot as the origin of the routes (multi-depot vehicle routing problem – MDVRP) and others. Recently, the electric vehicle routing problem was added to the variations of the original problem.

The first appearance of the EVRP model was in [ERDOGAN and MILLER-HOOKS 2012] even though they named it as the Green VRP. They considered using alternative fuel vehicles (AFVs) that would depart from a central depot to serve customers. However, these vehicles could eventually require a stop in alternative fuel stations (AFSs) in order to refill their tank, which for the EVRP can be understood as recharging the battery. They proposed a mixed-integer-linear programming (MILP) formulation for the problem as well as two heuristics to solve it: a modified Clarke and Wright savings algorithm (MCWS) and a density-based clustering algorithm (DBCA) that creates clusters and then runs the MCWS for each cluster of customers.

[ZHANG et al. 2018] describe two possible solutions to address the Green VRP. The first solution is based on a two-phase heuristic which solves a traveling salesman problem with the nodes from the VRP using the Nearest Neighbor Criteria (NNC) and then creates routes based on this TSP route. The second solution is an ant colony system (ACS) algorithm which also employs a local search scheme aiming to improve the ACS solutions. The ACS algorithm is able to deliver better quality solutions but takes more time than the two-phase heuristic algorithm. [PENG et al. 2019] present a memetic algorithm that employs an adaptative local search to explore neighborhood moves for both intra-route and inter-route moves.

[QIN et al. 2021] present a review of the literature on the electric vehicle routing problem (EVRP) describing its main variants and solving algorithms. A more recent study was presented by [CATALDO-DÍAZ et al. 2022] where they provide mathematical models considering time windows and comparing the scenario where batteries are fully recharged at stations versus a

scenario allowing partial recharging, and versus a scenario with partial recharging but also limiting battery discharge.

This paper makes use of the Split algorithm that was proposed by [PRINS 2004] where he used the concept of route-first, cluster-second to solve the CVRP. The Split algorithm starts with a chromosome as an initial solution containing a sequence of customers (route-first) without delimiters of routes and finishes by indicating the optimal breakdown of that sequence in routes (cluster-second). This algorithm has some important advantages. The fact that the chromosome does not contain route delimiters makes the implementation of the genetic algorithm easier by removing all the complexity of verifying the routes feasibility and fixing them during its execution. Additionally, as the Split algorithm will always deliver the optimal breakdown into routes for that sequence, then it works as a local search procedure aiming to intensify the solution embedded in the genetic algorithm. [CHANG and CHEN 2007] modified the Split algorithm to solve the problem with time windows but without end service time at the customers. [PRINS 2009] presented a modified version of the algorithm to address the problem using a mixed fleet with different types of vehicles. [SOUZA 2023] modified the Split algorithm to consider capacity restrictions, time windows (with both start and end service time at customers) and two different types of vehicles: bicycles and motorcycles. In the case of bicycles, a penalization scheme considering how the inclination angle for going from one customer to another could impact a biker was added to the formulation of the problem. Although the literature frequently associates the Split algorithm with genetic algorithms, its versatility and ease of use allow it to be used in other meta-heuristics. In this paper a solution using local search was used to demonstrate that.

[MONTROYA et al. 2016] also made use of the Split algorithm as part of a two-phase heuristic to solve the GVRP. In the first phase they use the Split algorithm employing a repair procedure to insert alternative fuel stations based on a constrained shortest path algorithm. In the second phase a set-partitioning formulation is employed.

### 3. Mathematical Model

The first mathematical model for the EVRP comes from [ERDOGAN and MILLER-HOOKS 2012] where the green vehicle routing problem is presented. There we can consider refueling the tank as an equivalent task as recharging the battery. [ZHANG et al. 2018] modified the model to also take customer demand and vehicle capacity into consideration. [QIN et al. 2021] present the model considering constraints for capacity and battery. The model presented below is based on the last version but without tracking the time between the vertices.

Set definitions:

$V$ : set of  $n$  customers  $\{c_1, c_2, \dots, c_n\}$ ;

$F$ : set of  $m$  recharging stations  $\{s_1, s_2, \dots, s_m\}$ ;

$F'$ : set of recharging stations with dummies to allow the same station to be used more than once if needed, example  $\{s_1, s_2, \dots, s_m, s'_1, s'_2, \dots, s'_m\}$ ;

$V'$ : set of customers and recharging stations, including dummies  $(V \cup F')$ ;

$V'_{n+1}$ : set of customers, recharging stations and including the depot as node  $n+1$  to be used for the end of routes;

$V'_0$ : set of customers, recharging stations and including the depot as node  $0$  to be used for the beginning of routes;

$F'_0$ : set of recharging stations and the depot as node  $0$  to be used for the beginning of routes.

Variable definitions:

$x_{ij}$ : binary decision variable that equals  $1$  if there is a route connecting customer  $i$  with customer  $j$  and  $0$  otherwise;

$u_i$ : auxiliary variable to accumulate the remaining cargo in the vehicle after serving customer  $i$ ;

$y_i$ : auxiliary variable to indicate the remaining battery when the vehicle arrives at customer  $i$ .

Parameter definitions:

$d_{ij}$ : distance between customers  $i$  and  $j$ ;

$q_i$ : demand for customer  $i$  (valid only for customers and assumed as zero for recharging stations).

Constant definitions:

$Q$ : battery capacity, which is considered as uniform for this particular model;

$C$ : vehicle cargo capacity, which is also uniform for this particular model;

$h$ : discharge rate, which is also a constant as this model considers linear discharging as a function of the distance travelled.

Mathematical formulation:

$$\min \sum_{\substack{i \in V'_0 \\ i \neq j}} \sum_{\substack{j \in V'_{n+1} \\ i \neq j}} d_{ij} x_{ij} \quad (1)$$

Subject to:

$$\sum_{\substack{j \in V'_{n+1} \\ i \neq j}} x_{ij} = 1 \quad \forall i \in V \quad (2)$$

$$\sum_{\substack{j \in V'_{n+1} \\ i \neq j}} x_{ij} \leq 1 \quad \forall i \in F' \quad (3)$$

$$\sum_{\substack{i \in V'_{n+1} \\ i \neq j}} x_{ji} - \sum_{\substack{i \in V'_0 \\ i \neq j}} x_{ij} = 0 \quad \forall j \in V' \quad (4)$$

$$0 \leq y_j \leq y_i - h \cdot d_{ij} x_{ij} + Q(1 - x_{ij}) \quad \forall j \in V'_{n+1}, i \in V, i \neq j \quad (5)$$

$$0 \leq y_j \leq Q - h \cdot d_{ij} x_{ij} \quad \forall i \in F'_0, j \in V'_{n+1}, i \neq j \quad (6)$$

$$0 \leq u_j \leq u_i - q_i x_{ij} + C(1 - x_{ij}) \quad \forall i \in V'_0, j \in V'_{n+1}, i \neq j \quad (7)$$

$$0 \leq u_0 \leq C \quad (8)$$

$$x_{ij} \in \{0,1\} \quad \forall i \in V'_0, j \in V'_{n+1} \quad i \neq j \quad (9)$$

The objective function (1) aims to minimize the total length of all routes. Constraint (2) has the purpose of ensuring that each customer has one, and only one, successor, which can be either another customer, or a recharging station, or the depot. Constraint (3) indicates that each recharging station, including the dummies used to allow more than one stop at the same station, has at most one successor. This constraint differs from the original CVRP which obliges all nodes to have one route passing through them. In the case of the EVRP this is not the case as some recharging stations may end up not being used at all. Constraint (4) is the flow conservation

equation that ensures that the number of vehicles going into a node will be equal to the number coming out of the node. For all customers it will mean one vehicle entering and one vehicle departing. For the recharging stations it will mean zero or one vehicle entering and the same zero or one vehicle departing. Of course, more than one vehicle can pass through the same recharging station but for the mathematical model dummy stations are used for that purpose. For the depot it will mean that the same number of vehicles that depart will also arrive. Constraints (5) and (6) model the battery discharge when going to a customer, the full recharge when at the recharging station and restricting it to not going below zero. Constraints (7) and (8) model the vehicle unloading the demand at each customer in the route. The auxiliary variable  $u$  is used to accumulate the cargo being carried by each vehicle. This cargo cannot exceed the vehicle capacity and decreases according to the demand from each customer in its route. Constraints (5) and (7) act as Big-M type constraints. The variable  $x_{ij}$  is binary per definition (9).

#### 4. Heuristic Implementation

Current modern solvers can execute the EVRP mathematical model and indicate whether an optimal solution is achieved or not. Nevertheless, as the number of customers increase more execution time is required and easily reaches a few hours. It limits the use of exact methods for combinatorial optimization to small problems. Therefore, we need to look for heuristic solutions which may be able to deliver a good quality solution, even though without being able to guarantee it is the optimal solution. For many business applications, it is better to get a good solution within some seconds or even a few minutes rather than some hours. If we think on a company that every morning needs to define and provide routes to drivers, then it is rather obvious that a good solution within seconds or a few minutes is preferred and that waiting a few hours for the optimal solution is unacceptable from the business point of view.

In this work we started designing a solution that employed the Split algorithm originally designed by [PRINS 2014]. In this algorithm the input is a sequence of customers, and the output is the optimal breakdown of the given sequence indicating where each route should start and end without changing the sequence of customers. The Split algorithm does not give the optimal solution of the VRP but gives the optimal breakdown into routes for a given sequence. Since this algorithm was published, many authors worked in modifying it and published their solutions incorporating the Split algorithm. [MONTROYA et al. 2016] modified and used the Split algorithm as part of a heuristic solution for the GVRP. This paper does something similar in the sense that it also incorporates the Split algorithm but differs in other aspects.

The Split algorithm calculates all feasible routes considering a given sequence of customers and then selects the shortest path from the beginning of the sequence to its end. The algorithm calculates the cumulative cost of adding customers in this sequence to a route and verifies whether feasibility constraints are violated or not. In the case of the EVRP described in the previous section, we deal not only with the cargo capacity constraints but also the battery constraints. The verification of cargo capacity constraints is already inside the Split algorithm as initially proposed. [MONTROYA et al. 2016] added the verification of the battery to the Split algorithm. This verification is a simple process using the cumulative distance from the route and the discharge rate to verify whether the battery has enough charge to support the route. In case it does not, then a repair procedure is called with the purpose of identifying which recharging stations need to be inserted in the route and where to be inserted. Different than [MONTROYA et al. 2016] this work creates a table including each edge in the route being analyzed and adds to the table the lowest difference in terms of distance for reaching each recharging station for that edge. Then, the table is sorted in descending order. In the end, we will have for each edge, the indication of the best recharging station to be inserted in that edge and what the difference in distance will be. With this information, we select the edge with the lowest impact in distance. The logic of this heuristic solution is to privilege insertions with the least impact in the distance. The algorithm still needs to



analyze the feasibility of the route. If a recharging station is inserted in the first edges of the route, it may be possible that the vehicle still runs out of battery before reaching the end.

During the design and implementation, we started with a random initial sequence and executed a tabu search algorithm that would use the Split algorithm for calculating the total distance with the charging stations inserted, and then would execute modifications in the sequence by exchanging the positions of two randomly selected customers with the aid of a tabu memory. This tabu search meta-heuristic worked but its performance was slow, and the solutions delivered did not achieve a good quality in a short timeframe. The slowness was due to the high number of calculations embedded in the modified Split algorithm where all recharging stations are analyzed to be inserted into each edge of the route. The quality is naturally impacted by the slowness of the algorithm as more iterations would be required in order to reach a better quality solution but the time required would be significant. However, it is rather intuitive that an optimal solution for the EVRP should be near to the optimal solution of the corresponding CVRP. Therefore, the approach was changed to start the problem by removing the battery capacity constraint and transforming the EVRP into a CVRP. Then, we used a tabu search algorithm for the CVRP based on [SOUZA 2023] in order to get an initial solution for the EVRP. The next step is running a heuristic implementation using the Split algorithm and the repair procedure. This two-step process resulted in a faster execution with better quality solutions. Of course, the slowness of the Split algorithm with all verifications for inserting a recharging station in the path still exists but less iterations are required due to the good initial solution obtained from the corresponding CVRP.

The Split algorithm with its modification to consider the battery restriction is described in Algorithm 1. A few new variables are introduced and the original denomination from [PRINS 2014] was maintained.  $S_j$  with  $j \in \{1, \dots, n\}$  is used to indicate each customer in the given sequence of customers, where  $S_j$  means the customer that occupies the  $j$ -position in the sequence that will be examined by the algorithm.  $c_{S_j, S_{j+1}}$  is used to indicate the cost of going from the customer that occupies the  $j$ -position in the sequence  $S$  to the customer that occupies the position  $j + 1$  in the same sequence, which here is considered as the distance between these two customers (in other words it would be equal to  $d$  from the mathematical model described before). The variables *cost* and *load* are used to carry the cumulative cost (considered as distance here) of the path and cumulative load being delivered by the vehicle in that route.  $V_j$  and  $P_j$  are auxiliary variables used by the algorithm, the first to calculate the best cumulative cost (here distance) for a sequence and the second to be used in the end to extract the points where the sequence should be broken in order to come up with the optimal division of routes.  $W$  is the vehicle capacity which was defined as  $C$  in the mathematical model.

Different than [MONTROYA et al. 2016] the algorithm here does not consider an end time limit. The function *charge\_test*( $i, j$ ) in Line 16 will verify whether the sequence of customers from index  $i$  to index  $j$  in the vector of customers, for the given sequence, can be performed or whether the vehicle will run out of battery before reaching the depot as the ending point. It returns *True* if the sequence is feasible or *False* if running out of battery. The function *charger*( $i, j$ ) in Line 22 will only be executed when getting a *False* result from the previous function. It returns the revised cost of the sequence being analyzed after the inclusion of charging points in the route. It identifies the first node of the given sequence where the vehicle runs out of battery before reaching it. Then, for each edge since the initial point (starting with the depot) until the node where the vehicle could not reach because of not enough battery it verifies what is the closest charging point. For this verification we calculate the additional distance for inserting each one of the charging points in the edge and select the one with the least incremental distance. After that, we verify if the vehicle can reach the end of the route in the depot, if not then we run these steps again with start point as the node that can now be reached instead of running out of battery before and the next node that cannot be reached. Again, for each edge in this path we verify the closest charging point to be inserted. The process continues until the vehicle can reach the depot as ending point with enough battery.

The modified Split algorithm above was embedded in a heuristic solution that executed a local search verification in the neighborhood of the initial solution. Considering that the initial solution is a good solution for the CVRP, we expect that after the insertion of the closest charging points the resulting solution be still good. The proposed heuristic, for each iteration, randomly selects a node in the sequence and verifies the improvement by changing its position with the first, the second or the third node in the sequence. For each verification the Split algorithm is executed to verify the cost of the new solution. The proposed approach worked well, as it can be noticed in the section about the results, for problems with size between 20 to 50 customers. Of course, as more customers are added and the size of the problem increased, then the algorithms may need to be revisited to better explore the neighborhood and become more effective. Other techniques, beyond the pure local search described here, may need to be applied in other to achieve a good execution performance. Anyhow, for the size of problems investigated here the algorithm delivered an acceptable performance.

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Algorithm 1: Split algorithm modified for considering the battery restriction

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1:   $V_0 := 0$ 
2:  for  $i := 1$  to  $n$  do
3:     $V_i := +\infty$ 
4:  endfor
5:   $P_0 := +\infty$ 
6:  for  $i := 1$  to  $n$  do
7:     $load := 0$ ;  $cost := 0$ ;  $j := i$ 
8:    repeat
9:       $load := load + q_{S_j}$ 
10:     if  $i = j$  then
11:        $cost := c_{0,S_j} + c_{S_j,0}$ 
12:     else
13:        $cost := cost - c_{S_{j-1},0} + c_{S_{j-1},S_j} + c_{S_j,0}$ 
14:     endif
15:     if ( $load \leq W$ ) then
16:       if  $charge\_test(i,j)=True$  then
17:         if  $V_{i-1} + cost < V_j$  then
18:            $V_j := V_{i-1} + cost$ 
19:            $P_j := i - 1$ 
20:         endif
21:       else do
22:          $cost := charger(i,j)$ 
23:         if  $V_{i-1} + cost < V_j$  then
24:            $V_j := V_{i-1} + cost$ 
25:            $P_j := i - 1$ 
26:         endif
27:       endif
28:        $j := j+1$ 
29:     endif
30:   until ( $j > n$ ) or ( $load > W$ )
31: endfor
  
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The procedure to insert the recharging stations in the path is described in Procedure 1. Although the idea may sound intuitive to look for the insertion with the least impact in distance, its implementation involves some complexity. Overall, the algorithm needs to identify in which point of the route the vehicle will run out of battery and then identify for each of the route edges prior to this point which one is the best suited for deviating to a recharging station. Of course, the algorithm still needs to ensure that the vehicle doesn't run out of battery before arriving at the recharging station as well. Then, after being recharged the algorithm needs to verify whether the vehicle can get to the end of the route with enough battery charge or will need to stop at another recharging station again. The implementation accepts that the same recharging station can handle more than one vehicle simultaneously. More complex implementations can define a limit for simultaneous vehicles being charged but would require tracking the arrival time of the vehicle at the recharging stations and the recharging time would need to be considered.

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#### Procedure 1: Procedure for inserting recharging stations

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STEP 1: Get sequence of customers making a path to be analyzed and add the depot to the beginning and the end of the sequence

STEP 2: Calculate the battery status for arriving at each node of the path

STEP 3: Identify the first node with a negative battery status and name it as POS2 while POS1 is the first node

STEP 4: For each edge in the path between POS1 and POS2 identify the insertion of the recharging station with the least incremental distance

The charging station to be inserted must comply with three restrictions: (1) the battery status for reaching the recharging station coming from the previous node must be positive; (2) the battery status for reaching the next node coming from the recharging stations must be positive; and (3) the same recharging station cannot be added to the route following itself

STEP 5: A table is created with the index of the node that precedes the inserted recharging station, the identification of the recharging station, the incremental distance by adding the recharging station after that node

STEP 6: Using the table, the recharging station with the least incremental distance is inserted in the route

STEP 7: The node after the recharging station is selected as the new POS1

STEP 8: Repeat STEP 2, procedure ends if there is no negative value

STEP 9: Repeat STEP 3 but maintain POS1 as defined in STEP 7

STEP 10: Repeat STEP 4, 5, 6 and 7 but if getting to the last node of the path without any feasible insertion in STEP 4 (meaning empty table in STEP 5) then returns to STEP 3 but using the origin of the path again as POS1

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The procedure above is executed when called by Algorithm 1 but returning only the revised cost of the path after recharging stations are inserted to create a feasible solution for the EVRP. There is no need to return the positions where the recharging stations were added, and which stations were added at this moment. For the execution of Algorithm 1, during all iterations we care only with the cost and the respective sequence of customers. The sequence of customers with the lowest cost is stored. Once all iterations are over, Algorithm 1 is executed once more to provide the optimal breakdown for the stored sequence with the lowest cost and this time the procedure returns which recharging stations and in which positions they were inserted to provide the full information for each route. Vector  $P$ , used by Algorithm 1, is used to extract the break points in the sequence to indicate where each route starts and ends. The extraction of the information from vector  $P$  is indicated in [PRINS 2014].

## 5. Computational Results

The mathematical model and the heuristic approach were both implemented in Python 3.10 (adding Numpy, Pandas and Matplotlib) and ran in a PC using the Core i7 processor, 1,3GHz clock, 8GB of RAM memory and 256GB SSD. The mathematical model was implemented using the PuLP framework and proved to be capable of delivering optimal solutions for randomly generated small instances. The two-phase heuristic approach was also implemented in Python in the same platform. Figures 1 and 2 below show the results comparing the corresponding CVRP to the complete EVRP for one of the instances that were tested. In the case instance E-n22-k4, which contains 21 customers plus the central depot with index 0, a total of 8 recharging stations possible to be used, and for which the best solution found contains 4 vehicles. It can be seen that if there is an adequate availability of recharging stations and also properly located, then the disturbance of the corresponding CVRP is minimal. The red dot represents the depot, the green dots represent the customers and the blue dots represent the recharging stations. The instances (data and indication of best solution found) used for testing the proposed heuristic approach can be obtained from the following address <https://mavrovouniotis.github.io/EVRPcompetition2020/>.

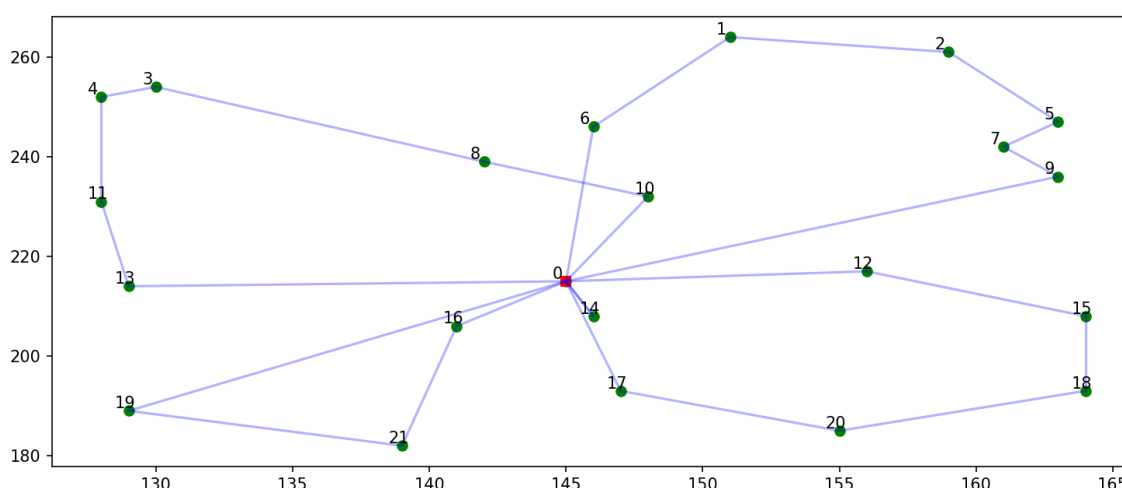


Figure 1: Corresponding CVRP for the E-n22-k4 instance

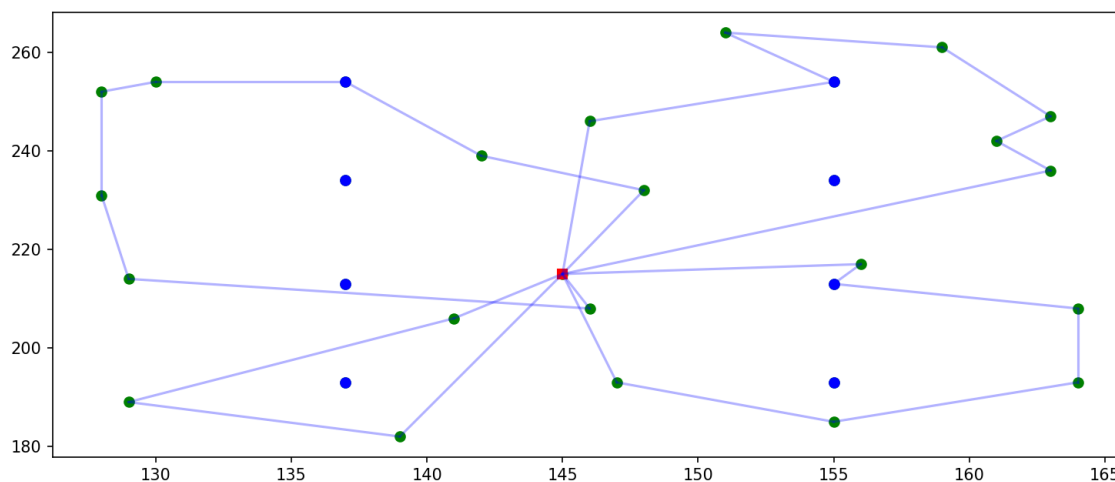


Figure 2: EVRP for the E-n22-k4 instance

Analyzing Figure 1 and Figure 2, we can make a few interesting observations. As seen in Figure 1 we have the result from the tabu search for the corresponding CVRP (EVRP from E-n22-k4 removing the battery restriction) and we can identify 5 routes:

Route 1: 0-13-11-4-3-8-10-0

Route 2: 0-6-1-2-5-7-9-0

Route 3: 0-12-15-18-20-17-0

Route 4: 0-16-21-19-0

Route 5: 0-14-0

The solution above became the initial solution of the second phase of the algorithm where the battery restriction is added back to the problem and the Split algorithm associated with a local search heuristic will look for a good solution complying with all restrictions. Figure 2 shows the final result from the two-phase process and we can verify that the solution from the corresponding CVRP and the complete EVRP are close to each other. A few observations:

Route 5 and route 1 merged.

Route 4 was modified by changing the position of neighbor customers in the same route.

Route 1, route 2 and route 3 cannot be completed without recharging the vehicle battery.

Selection of the route deviation for going to a recharging station is made in order to cause minimum impact in the route while complying with the restrictions.

Different than the CVRP, the routes are not bi-directional anymore as they can comply with the battery restriction in one direction but not in the other.

Of course, as we are considering a heuristic approach, there is no guarantee of optimality. However, a visual inspection indicates a good quality of the solution. A total of 5 different instances were tested and similar conclusions were observed. The execution yielded results close to the upper bounds provided in the reference demonstrating a good performance for the proposed two-phase heuristic approach. The algorithm proposed here was executed a minimum of 25 times for each instance but 10 times for the instance E-n51-k5 (where execution time is longer due to the higher number of customers involved), and the best solution found for each instance is indicated. Usually, the best solution was reached in almost all executions that were also limited to 20 iterations and the convergence to these results was obtained fast. Table 1 summarizes the results showing that the achieved results are close the best solution found (BSF).

Table 1: Results for selected instances

Instance	BSF	Solution found	Deviation	Comments
E-n22-k4	384.67	394.00	2.43%	4 vehicles
E-n23-k3	571.94	572.56	0.11%	3 vehicles
E-n30-k3	509.47	513.76	0.84%	4 vehicles
E-n33-k4	840.14	848.83	1.03%	4 vehicles
E-n51-k5	529.90	576.76	8.84%	6 vehicles

## 6. Conclusion and future research

The results demonstrate that using the problem relaxation to first solve the corresponding CVRP to get an initial solution and then performing a heuristic approach based on local search and using the Split algorithm is a fast method for delivering good quality solutions for the EVRP for the size of problems analyzed here. Future research may incorporate additional complexity to the model to consider the recharging time and time restrictions to the problem. As the recharging time is still one of the challenges faced by electric vehicles and battery manufacturers, it is also intuitive that the solution for the EVRP may move away from the corresponding CVRP as a considerable stop at a recharging station may impact the feasibility of still meeting time windows for the next customers in the route. Needless to say, reducing the battery recharging time is one of the main areas of attention for development in this business. Another practical scenario to be studied is limiting the number of vehicles that can be served by the same recharging station simultaneously. In order to do so, it will be needed to track the time when each vehicle arrives at the recharging station and consider the recharging time taking into consideration it would also depend on the battery level in the moment of the vehicle arrival. The implementation presented in this paper, although addressing a basic version of the EVRP formulation, offers good practice and a good foundation for building additional complexity on top of it.

Sustainability will continue to be a major concern for many companies and should increase its importance in the logistics sector. Therefore, applying operations research techniques for optimizing resources may help companies to offset their investments and additional cost by increased efficiency in their operations. Because of that, the EVRP offers good insights and solutions for the industry.

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