

FUNDAÇÃO GETÚLIO VARGAS

MODELAGEM MATEMÁTICA

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# Programming a cryptocurrency in Agda

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# 1 Introduction

## 1.1 Cryptocurrencies

In 1983, David Chaum created ecash (Panurach, 1996), an anonymous cryptographic electronic money. This cryptocurrency uses RSA blind signatures (Chaum, 1983) to spend transactions. Later, in 1989, David Chaum founded an electronic money corporation called DigiCash Inc. It was declared bankrupt in 1998.

Adam Back developed a proof-of-work (PoW) scheme for spam control, Hashcash (Back et al., 2002). To send an email, the hash of the content of this email plus a nonce has to have a numerically value smaller than a defined target. So, to create a valid email, the sender (miner) has to spend a considerable CPU resource on it. Because, hash functions produce practically random values, so the miner has to guess a lot of nonce values before finding some nonce that makes the hash of the email less than the target value. This idea is the same that is used in Bitcoin proof of work, because each block has a nonce guessed by the miner and the hash of the block has to be less than the target value.

Wei Dai proposed b-money (Dai, 1998) for the first proposal for distributed digital scarcity. And Hal Finney created Bit Gold, a reusable proof of work for hashcash for its algorithm of proof of work.

In 31 October 2008, Satoshi Nakamoto registered the website “bitcoin.org” and put a link for his paper (Nakamoto et al., 2008) in a cryptography mailing list. In January 2009, Nakamoto released the bitcoin software as open-source code. The identity of Satoshi Nakamoto is still unknown. Since that time, the total market of Bitcoin came to 330 billions dollars in 17 of December of 2018 and its value has a historic record of 20 thousands dollars.

Other cryptocurrencies like Ethereum (Wood et al., 2014), Monero (Noether, 2015) and ZCash (Hopwood, Bowie, Hornby, & Wilcox, 2016) were created after Bitcoin, but Bitcoin is still the cryptocurrency with the biggest market value.

Ethereum is a cryptocurrency that uses account model instead of UTXO used in Bitcoin for its transaction data structure. It uses Solidity as its programming language for smart contracts, it looks like JavaScript, so it is easier to program in it than in stack machine programming language of Bitcoin. Ethereum is now changing from proof of work (used in Bitcoin) to proof of stake.

Monero and ZCash are both cryptocurrencies that focus on fungibility, privacy and decentralization. Monero uses obfuscated public ledger, so anyone can send transactions, but nobody can tell the source, amount or destination. Zcash uses the

concept of zero-knowledge proof called zk-SNARKs, which guarantee privacy for its users.

## 1.2 Bitcoin

The bitcoin was made to be a peer to peer electronic cash. It was made in one way that users can save and verify transactions without the need of a trust party. Because of that no authority or government can block the bitcoin.

Transactions in bitcoins are one array of input of previous transactions and one array of outputs. The mining transaction does not need to have an input. For each input of the transaction, it is necessary a signature signed with a private key to prove the ownership of the bitcoins.

Transactions are grouped in a block. Each block has in its header the timestamp that the block was created, the hash of the block, the previous hash and a nonce. A nonce is an arbitrary value that the miner has to choose to make the hash of the block respect some specific characteristics.

Each block has a limit size of 1 MB. Because of that, bitcoin has a blockchain (one array of blocks). Each block should be created in an average of 10 minutes. This time was chosen because in 10 minutes, it is enough to propagate the block in all the world. To make it possible, there is a concept called proof of work in bitcoin. So the miner has to choose a random value as nonce that makes the hash of the block less than a certain value. This value is chosen in a way that each block should be generated in 10 minutes in average. If the value is too low, miners will take more time to find a nonce that makes the hash block less than the target. If it is too high, it will be easier to find a nonce and they will find it faster.

When two blocks are mined in nearly the same time, there are two valid blockchains. It is because the last block in the both blockchains are valid but different. Because of this problem, in Bitcoin protocol, the largest chain is always the right chain. While two valid chains have the same size, it is not possible to know which chain is the right. This situation is called fork and when it happens, it is necessary to wait to see in which chain the new block will be.

In Bitcoin, there is a possibility of 51% attack. It happens when some miner, with more power than all network, secretly mines blocks. So if the main network has 50 blocks, the miner could produce hidden blocks from 46 to 55 and he would have 10 hidden blocks from the network. When he shows their hidden blocks, his chain becomes the valid chain, because it is bigger. So all transactions from previous blockchain from 46 to 50 blocks become invalid. Because of that, when someone makes a big

transaction in the blockchain, it is a good idea to wait more time. So it is becoming harder and harder to make a 51Bitcoin has the highest market value nowadays, so attacking the bitcoin network is very expensive. Nowadays, this kind of attack is more common in new altcoins.

### 1.3 Agda Introduction

Agda is a dependently typed functional language developed by Norell at Chalmers University of Technology as his PhD Thesis. The current version of Agda is Agda 2.

### 1.4 Martin-Löf type theory

Agda is also a proof assistance based on intensional Martin-Löf type theory.

### 1.5 Types

In Martin-Löf type theory, there are 3 finites types and 5 types constructors. The 0 type contain 0 terms, it is called empty type and it is written `bot`.

The 1 type is the type with just 1 canonical term and it represents existence. It is called unit type and it is written `top`.

The 2 type contains 2 canonical terms. It represents a choice between two values.

The Boolean Type is defined using the Trivial type and the Either type

If statement is defined using booleans

### 1.6 Types Constructors

The sum-types contain an ordered pair. The second type can depend on the first type. It has the same meaning of exist.

```
data  $\sum$  (A : Set) (B : A  $\rightarrow$  Set) : Set where
   $\langle \_, \_ \rangle$  : (x : A)  $\rightarrow$  B x  $\rightarrow$   $\sum$  A B
```

```
 $\sum$ -elim :  $\forall$  {A : Set} {B : A  $\rightarrow$  Set} {C : Set}
 $\rightarrow$  ( $\forall$  x  $\rightarrow$  B x  $\rightarrow$  C)
```

$$\begin{array}{c}
 \rightarrow \sum A B \\
 \hline
 \rightarrow C \\
 \sum\text{-elim } f \langle x, y \rangle = f x y
 \end{array}$$

The pi-types contain functions. So given an input type, it will return an output type. It has the same meaning of a function

In Inductive types, it is a self-referential type. Naturals numbers are examples of that

```

data ℕ : Set where
  zero : ℕ
  suc : ℕ → ℕ

```

Other data structs like linked list of natural numbers, trees, graphs are too. Proofs in inductive types are made by induction.

```

ℕ-elim : (target : ℕ) (motive : (ℕ → Set)) (base : motive zero)
  (step : (n : ℕ) → motive n → motive (suc n) ) → motive target
ℕ-elim zero motive base step = base
ℕ-elim (suc target) motive base step = step target (ℕ-elim target motive base step)

```

Universe types are created to allow proofs written in all types. For example, the type of Nat is U0.

It looks like CoQ, but does not have tactics. Agda is a total language, so it is guaranteed that the code always terminal and coverage all inputs. Agda needs it to be a consistent language.

Agda has inductive data types that are similar to algebraic data types in non-dependently typed programming language. The definition of Peano numbers in Agda:

```

data ℕ : Set where
  zero : ℕ
  suc : ℕ → ℕ

```

Definitions in Agda are done using induction. For example, the sum of two numbers in Agda:

```

_+_ : ℕ → ℕ → ℕ
zero +_ m = m
suc n +_ m = suc (n + m)

```

In Agda, because of dependent types, it is possible to make some restrictions in types that is not possible in other language. For example, get the first element of a vector. For it, it is necessary to specify in the type that the vector should have at size greater or equal than one.

```
head : {A : Set} {n : ℕ} (vec : Vector A (suc n)) → A
head (x :: vec) = x
```

Another good example is that in sum of two matrices, they should have the same dimensions.

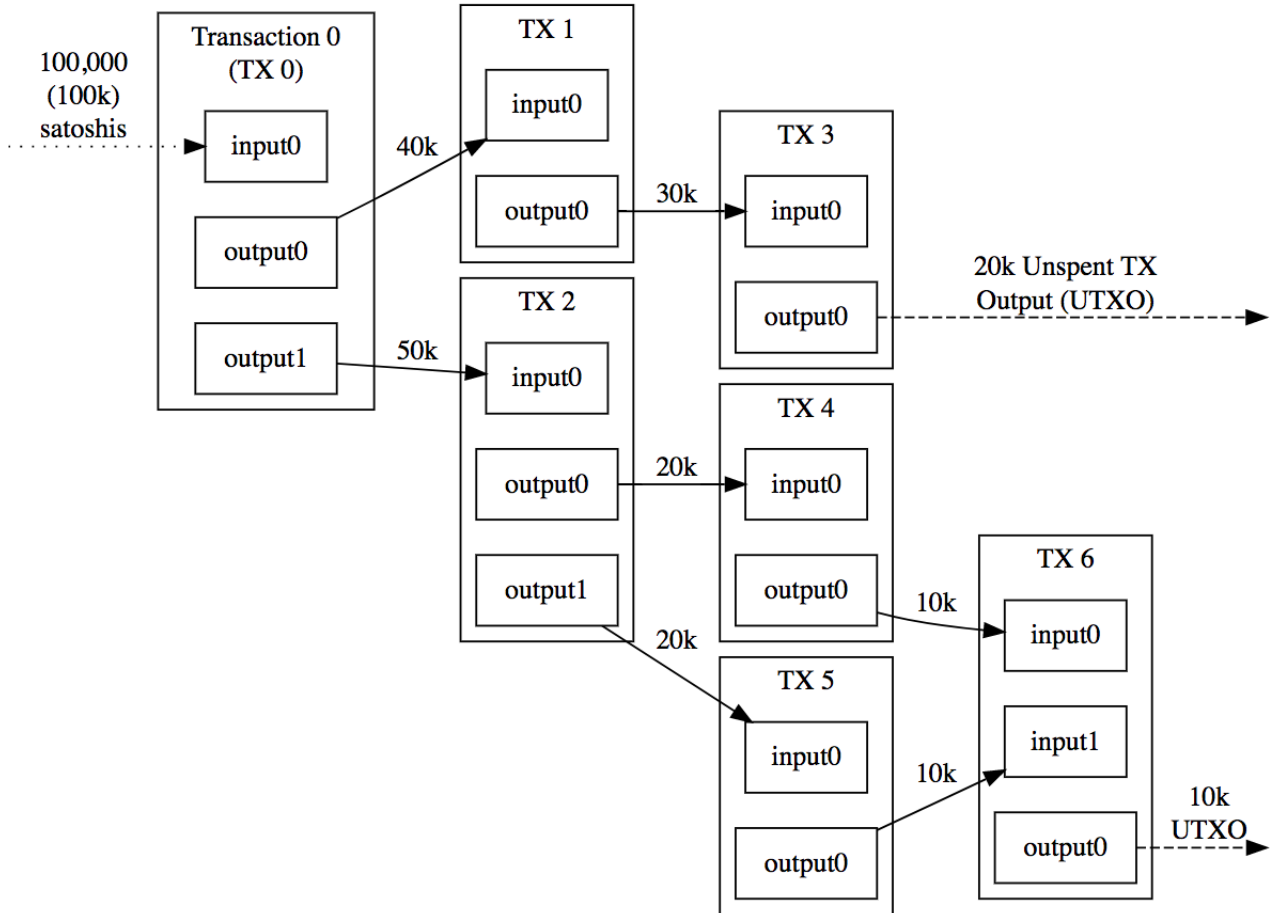
```
_+m_ : {m n : ℕ} (P Q : Matrix ℕ m n) → Matrix ℕ m n
[] +m [] = []
(vx :: P) +m (vy :: Q) = (vx +v vy) :: (P +m Q)
```

## 1.7 UTXO Bitcoin

There are two kinds of data structures to modeling accounts records and savings states. The UTXO model used in Bitcoin and the account model used in Ethereum.



In account model, it is saved the address and the balance of each address. For example, the data struct will look like this  $[(0xabc01, 1.01), (0xabc02, 2.02)]$ . So the address  $0xabc01$  has 1.01a of balance and the address  $0xabc02$  has 2.02 of balance. In this way, it is possible to easily know how much of balance each address has, but it is not possible to know how they got in this state.



Triple-Entry Bookkeeping (Transaction-To-Transaction Payments) As Used By Bitcoin

In UTXO model, each transaction is saved in the transaction tree. Every transaction is composed of multiples inputs and multiples outputs. But all inputs have to never been spent before.

Because of that, in UTXO model, it is easy to make a new transaction from previous one, but it is harder to know how much each one has. The wallet that calculate how much balance each address has.



In account model, there could be one kind of vulnerability that is less probabable to happen in UTXO model. Because there is an undesirable intermediary state that there is some address without balance while another has not already received his money.

For example:

bobBalance -= 1

Intermediary State

aliceBalance += 1

In account model, it is straight foward to know how much balance each address has. In UTXO model, this calculation is made offchain. It can be a good thing, because each user has more privacy.

## 1.8 TXTree in Agda

## 2 Methods

## 3 Conclusion

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