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Resumo sobre "The dynamics of a Family of One-Dimensional Maps"

For given a and b $(0<a<1, 0\leqslant b\leqslant 1)$. Let f be a function from [0,1] to [0,1], algebrically defined as:

$$f(x)=\left\{\begin{matrix} b+\frac{(1-b)}{a}x & if\ 0\leqslant x<a \\ \frac{(1-x)}{(1-a)} & if\ a\leqslant x\leqslant 1 \end{matrix}\right\}$$

1. Fixed Points

Definition: A fixed point x is a point for which f(x) = x.

From this definition it is easy to see that if b = 0, it follows that f(x) = 0 for each x such that $0\leqslant x<a$, therefore that f(0) = 0, which means that 0 is a fixed point.

And as for the second part of f (attention: b does not pay any influence on this part). Let x be such that $a\leqslant x<1$ and f(x) = x:

$$f(x)=x \;\Rightarrow\; \frac{(1-x)}{(1-a)}=x \;\Rightarrow\; 1-x=x-ax \;\Rightarrow\; 2x-ax=1 \;\Rightarrow\; x=\frac{1}{(2-a)}$$

Therefore it is possible that f contains one or two fixed points.

From now on we shall ignore the first fixed point presented although it really exists, the second fixed point (which is in [a,1]) shall be called c, therefore:

$$c\in[a,1], c=\frac{1}{(2-a)}\Rightarrow f(c)=c$$

2. Slope on point c.

We are interested on the slope at the point c:

$$|slope(c)|=\frac{1}{(1-a)}>1$$

Therefore, for any interval I such tha $(I\subset[a,1])\Rightarrow|f|(I)>|I|$.