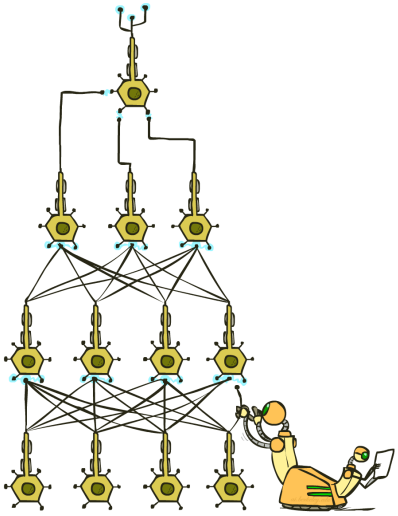

CS 188: Artificial Intelligence

Deep Learning II

Instructors: Pieter Abbeel & Anca Dragan --- University of California, Berkeley

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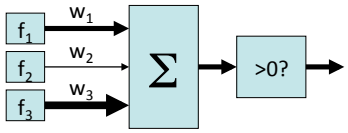
- Demo-site:
  - http://playground.tensorflow.org/

- Recap / softmax multi-class perceptron
- Multi-layer network
- Gradient descent/momentum

Perceptron



- Objective: Classification Accuracy

$$l^{\text{acc}}(w) = \frac{1}{m}\sum_{i=1}^{m}\left(\text{sign}(w^\top f(x^{(i)})) == y^{(i)}\right)$$

  - Issue: many plateaus → how to measure incremental progress?

Soft-Max

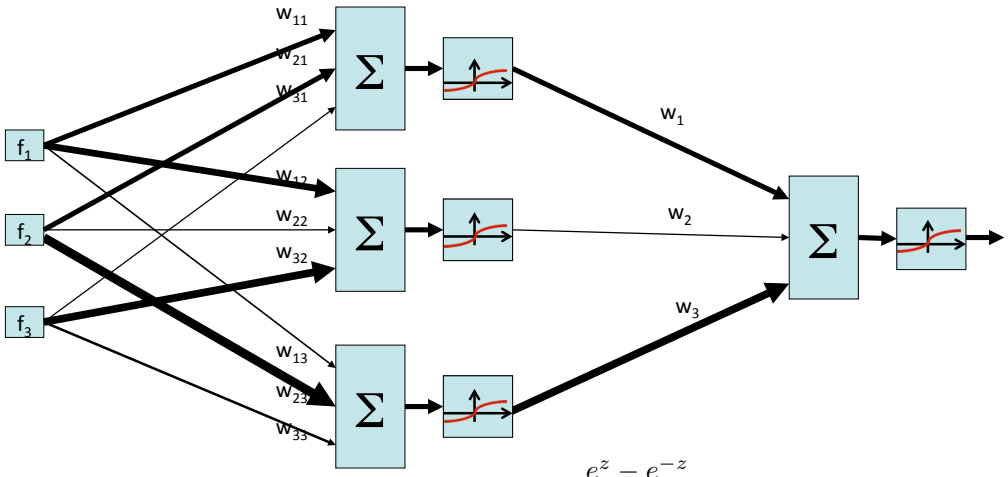
- Score for y=1: $w^\top f(x)$ Score for y=-1: $-w^\top f(x)$
- Probability of label:

$$p(y=1|f(x);w) = \frac{e^{w^\top f(x^{(i)})}}{e^{w^\top f(x)} + e^{-w^\top f(x)}}$$

$$p(y=-1|f(x);w) = \frac{e^{-w^\top f(x)}}{e^{w^\top f(x)} + e^{-w^\top f(x)}}$$

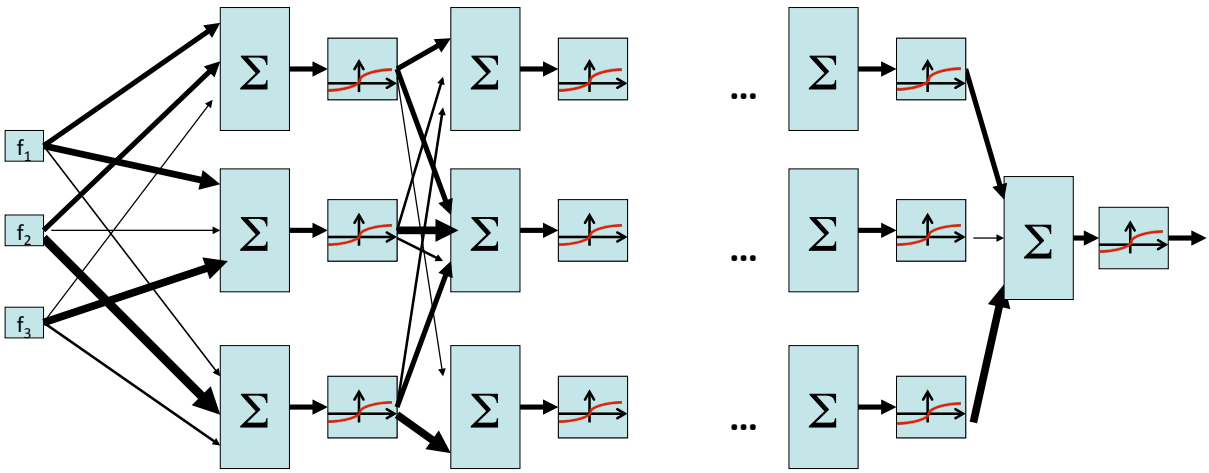
- Objective: $l(w) = \prod_{i=1}^{m} p(y=y^{(i)}|f(x^{(i)});w)$
- Log: $ll(w) = \sum_{i=1}^{m} \log p(y=y^{(i)}|f(x^{(i)});w)$

Two-Layer Neural Network



$$z \rightarrow \tanh(z) = \frac{e^z - e^{-z}}{e^z + e^{-z}}$$

N-Layer Neural Network



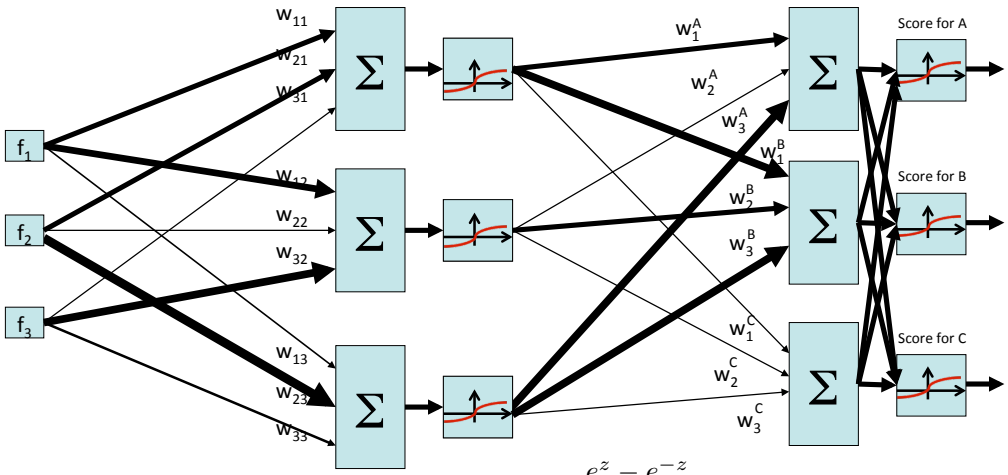
Multi-class Softmax

- 3-class softmax – classes A, B, C
  - 3 weight vectors: w_A, w_B, w_C
- Probability of label A: (similar for B, C)

$$p(y = A|f(x); w) = \frac{e^{w_A^\top f(x)}}{e^{w_A^\top f(x)} + e^{w_B^\top f(x)} + e^{w_C^\top f(x)}}$$

- Objective: $l(w) = \prod_{i=1}^{m} p(y = y^{(i)}|f(x^{(i)}; w)$
- Log: $ll(w) = \sum_{i=1}^{m} \log p(y = y^{(i)}|f(x^{(i)}; w)$

Multi-class Two-Layer Neural Network



$$z \to \tanh(z) = \frac{e^z - e^{-z}}{e^z + e^{-z}}$$

Steepest Descent

- Idea:
  - Start somewhere
  - Repeat: Take a step in the steepest descent direction

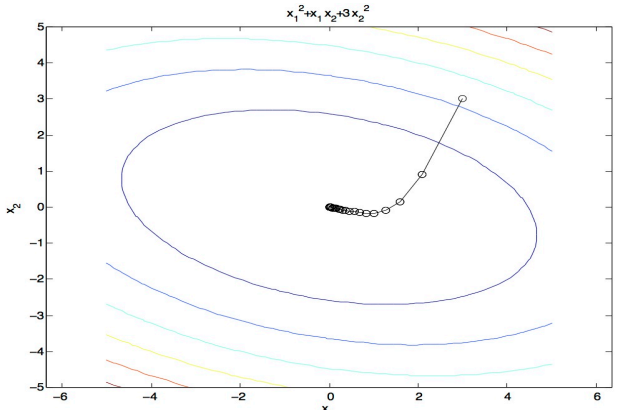


Figure source: Mathworks

What is the Steepest Descent Direction?

$$\min_{\Delta:\Delta_1^2+\Delta_2^2\leq\epsilon} g(w+\Delta)$$

- First-Order Taylor Expansion: $g(w+\Delta) \approx g(w) + \frac{\partial g}{\partial w_1}\Delta_1 + \frac{\partial g}{\partial w_2}\Delta_2$
- Steepest Descent Direction: $\min_{\Delta:\Delta_1^2+\Delta_2^2\leq\epsilon} \frac{\partial g}{\partial w_1}\Delta_1 + \frac{\partial g}{\partial w_2}\Delta_2$
- Recall: $\min_{a:\|a\|\leq\epsilon} a^\top b \quad \rightarrow \quad a = -b\frac{\epsilon}{\|b\|}$
- Hence, solution: $-\nabla g \frac{\epsilon}{\|\nabla g\|}$ $\qquad \nabla g = \begin{bmatrix} \frac{\partial g}{\partial w_1} \\ \frac{\partial g}{\partial w_2} \end{bmatrix}$

Generally, Steepest Direction

- Steepest Direction = direction of the gradient

$$\nabla g = \begin{bmatrix} \frac{\partial g}{\partial w_1} \\ \frac{\partial g}{\partial w_2} \\ \cdots \\ \frac{\partial g}{\partial w_n} \end{bmatrix}$$

Optimization Procedure 2: Momentum

- Gradient Descent

- Init: w
- For i = 1, 2, …

$$w \leftarrow w - \alpha * \nabla g(w)$$

- Momentum

- Init: w
- For i = 1, 2, …

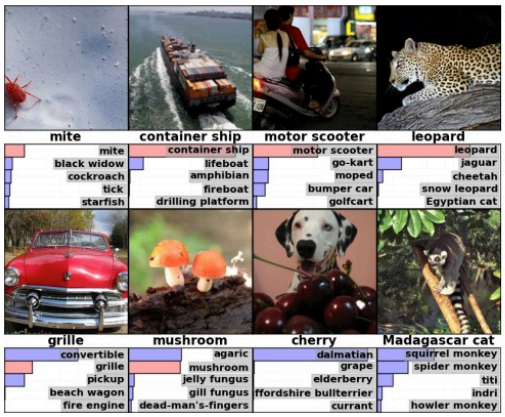
$$v \leftarrow \mu * v - \alpha * \nabla g(w)$$

$$w \leftarrow w + v$$

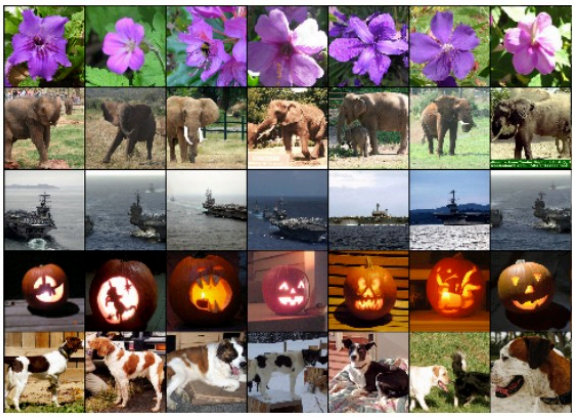
- Physical interpretation as ball rolling down the loss function + friction (mu coefficient).
- mu = usually ~0.5, 0.9, or 0.99 (Sometimes annealed over time, e.g. from 0.5 -> 0.99)

ConvNets are everywhere

Classification



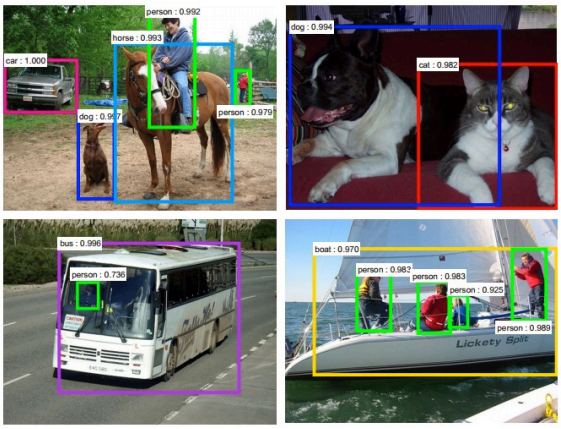
Retrieval



[Krizhevsky 2012]

ConvNets are everywhere

Detection

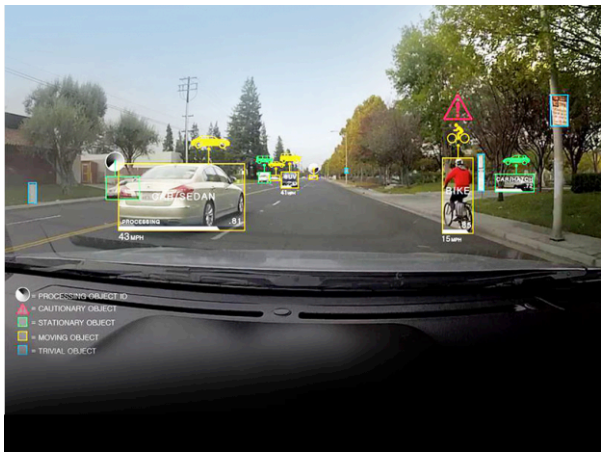


[Faster R-CNN: Ren, He, Girshick, Sun 2015]

Segmentation

[Farabet et al., 2012]

ConvNets are everywhere

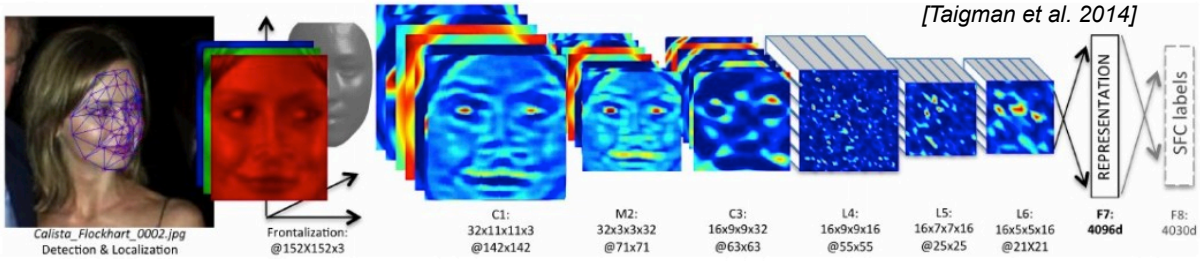


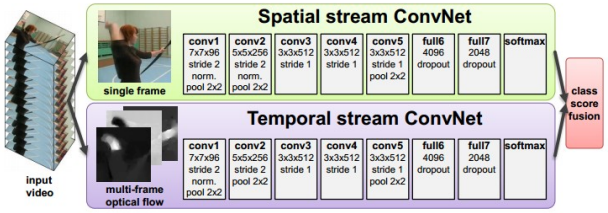
self-driving cars

NVIDIA Tegra X1

ConvNets are everywhere

[Taigman et al. 2014]





[Simonyan et al. 2014]

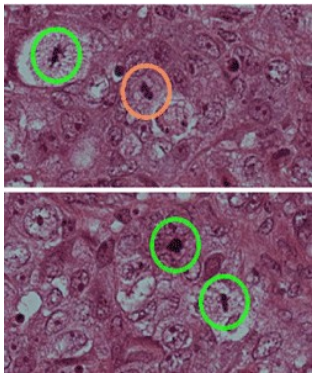
[Goodfellow 2014]

ConvNets are everywhere

[Toshev, Szegedy 2014]

[Mnih 2013]

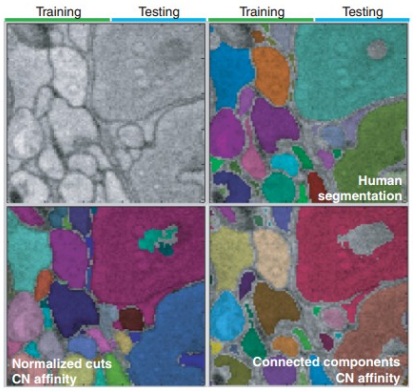
ConvNets are everywhere



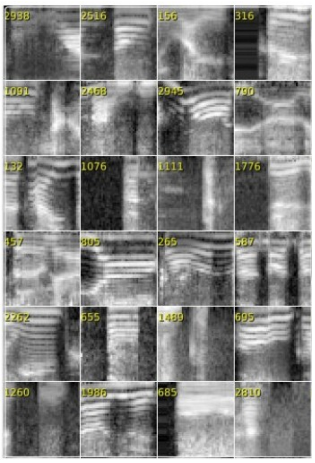
[Ciresan et al. 2013]

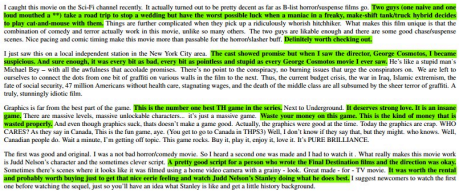
[Sermanet et al. 2011]
[Ciresan et al.]

ConvNets are everywhere



[Turaga et al., 2010]





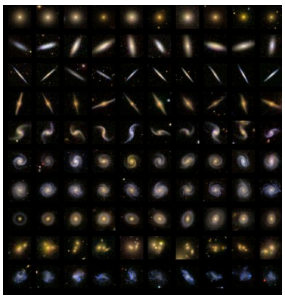
I caught this movie on the Sci-Fi channel recently. It actually turned out to be pretty decent as far as B-list horror/suspense films go. Two guys (one naive and one loud mouthed a **) take a road trip to stop a wedding but have the worst possible luck when a manic in a freaky, make-shift tank/truck hybrid decides to play cat-and-mouse with them. Things are further complicated when they pick up a ridiculously whorish hitchhiker. What makes this film unique is that the combination of comedy and terror actually work in this movie, unlike so many others. The two guys are likable enough and there are some good chase/suspense scenes. Nice pacing and comic timing make this movie more than passable for the horror/slasher buff. Definitely worth checking out.

I just saw this on a local independent station in the New York City area. The cast showed promise but when I saw the director, George Cosmotos, I became suspicious. And sure enough, it was every bit as bad, every bit as pointless and stupid as every George Cosmotos movie I ever saw. He's like a stupid man's Michael Bey -- with all the awfulness that accolade promises. There's no point to the conspiracy, no burning issues that urge the conspirators on. We are left to ourselves to connect the dots from one bit of graffiti on various walls in the film to the next. Thus, the current budget crisis, the war in Iraq, Islamic extremism, the fate of social security, 47 million Americans without health care, stagnating wages, and the death of the middle class are all subsumed by the sheer terror of graffiti. A truly, stunningly idiotic film.

Graphics is far from the best part of the game. This is the number one best TH game in the series. Next to Underground. It deserves strong love. It is an insane game. There are massive levels, massive unlockable characters... it's just a massive game. Waste your money on this game. This is the kind of money that is wasted properly. And even though graphics suck, thats doesn't make a game good. Actually, the graphics were good at the time. Today the graphics are crap. WHO CARES? As they say in Canada, This is the fun game, aye. (You get to go to Canada in THPS3) Well, I don't know if they say that, but they might. who knows. Well, Canadian people do. Wait a minute, I'm getting off topic. This game rocks. Buy it, play it, enjoy it, love it. It's PURE BRILLIANCE.

The first was good and original. I was a not bad horror/comedy movie. So I heard a second one was made and I had to watch it . What really makes this movie work is Judd Nelson's character and the sometimes clever script. A pretty good script for a person who wrote the Final Destination films and the direction was okay). Sometimes there's scenes where it looks like it was filmed using a home video camera with a grainy - look. Great made - for - TV movie. It was worth the rental and probably worth buying just to get that nice eerie feeling and watch Judd Nelson's Stanley doing what he does best. I suggest newcomers to watch the first one before watching the sequel, just so you'll have an idea what Stanley is like and get a little history background.

[Denil et al. 2014]



ConvNets are everywhere

Whale recognition, Kaggle Challenge

Mnih and Hinton, 2010

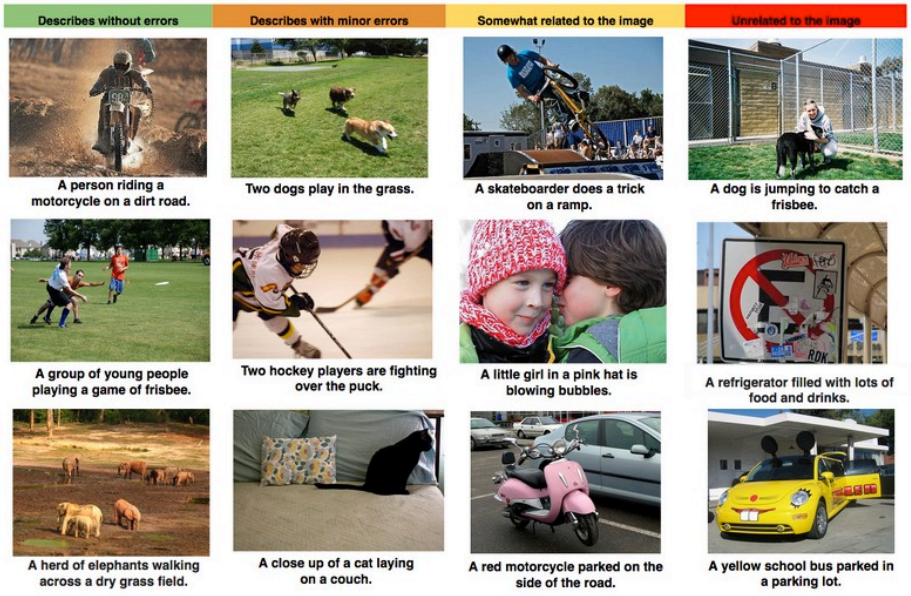


Image Captioning

[Vinyals et al., 2015]

reddit.com/r/deepdream

Remaining Pieces

- Optimizing machine learning objectives:
  - Stochastic Descent
  - Mini-batches
- Improving generalization
  - Drop-out
- Activation functions
- Initialization and batch normalization
- Computing the gradient $\nabla g(w)$
  - Backprop
  - Gradient checking

Mini-batches and Stochastic Gradient Descent

- Typical objective:

$$ll(w) = \frac{1}{m}\sum_{i=1}^{m} \log p(y = y^{(i)}|f(x^{(i)}); w)$$

= average log-likelihood of label given input

$$\approx \frac{1}{k}\sum_{i=1}^{k} \log p(y = y^{(i)}|f(x^{(i)}); w)$$

= estimate based on mini-batch 1…k

- Mini-batch gradient descent: compute gradient on mini-batch (+ cycle over mini-batches: 1..k, k+1…2k, ... ; make sure to randomize permutation of data!)
- Stochastic gradient descent: k = 1

Remaining Pieces

- Optimizing machine learning objectives:
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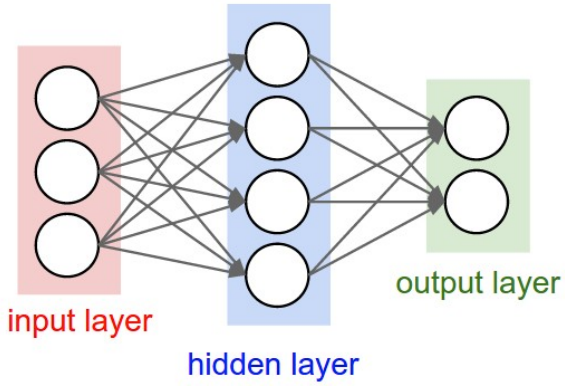
- Q: what happens when W=0 init is used?



- First idea: **Small random numbers**
(gaussian with zero mean and 1e-2 standard deviation)

```
W = 0.01* np.random.randn(D,H)
```


- First idea: **Small random numbers**
(gaussian with zero mean and 1e-2 standard deviation)

```
W = 0.01* np.random.randn(D,H)
```

Works ~okay for small networks, but can lead to non-homogeneous distributions of activations across the layers of a network.

Lets look at some activation statistics

E.g. 10-layer net with 500 neurons on each layer, using tanh non-linearities, and initializing as described in last slide.

```
# assume some unit gaussian 10-D input data
D = np.random.randn(1000, 500)
hidden_layer_sizes = [500]*10
nonlinearities = ['tanh']*len(hidden_layer_sizes)
```

```
act = {'relu':lambda x:np.maximum(0,x), 'tanh':lambda x:np.tanh(x)}
Hs = {}
for i in xrange(len(hidden_layer_sizes)):
    X = D if i == 0 else Hs[i-1] # input at this layer
    fan_in = X.shape[1]
    fan_out = hidden_layer_sizes[i]
    W = np.random.randn(fan_in, fan_out) * 0.01 # layer initialization

    H = np.dot(X, W) # matrix multiply
    H = act[nonlinearities[i]](H) # nonlinearity
    Hs[i] = H # cache result on this layer
```

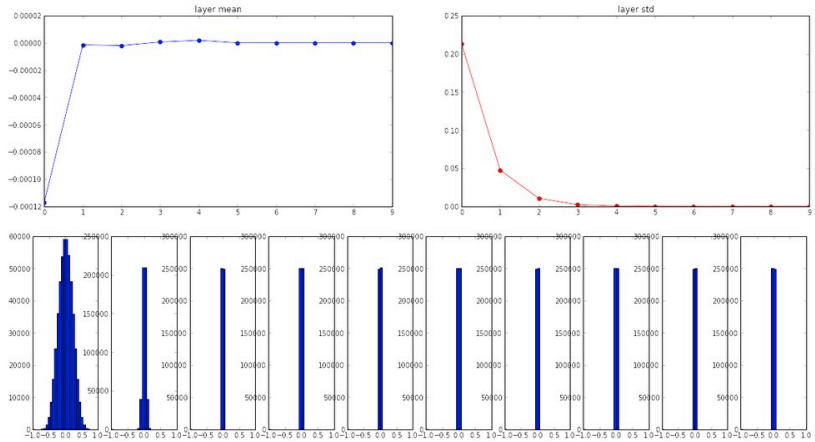
```
# look at distributions at each layer
print 'input layer had mean %f and std %f' % (np.mean(D), np.std(D))
layer_means = [np.mean(H) for i,H in Hs.iteritems()]
layer_stds = [np.std(H) for i,H in Hs.iteritems()]
for i,H in Hs.iteritems():
    print 'hidden layer %d had mean %f and std %f' % (i+1, layer_means[i], layer_stds[i])

# plot the means and standard deviations
plt.figure()
plt.subplot(121)
plt.plot(Hs.keys(), layer_means, 'ob-')
plt.title('layer mean')
plt.subplot(122)
plt.plot(Hs.keys(), layer_stds, 'or-')
plt.title('layer std')

# plot the raw distributions
plt.figure()
for i,H in Hs.iteritems():
    plt.subplot(1,len(Hs),i+1)
    plt.hist(H.ravel(), 30, range=(-1,1))
```



```
input layer had mean 0.000927 and std 0.998388
hidden layer 1 had mean -0.000117 and std 0.213081
hidden layer 2 had mean -0.000001 and std 0.047551
hidden layer 3 had mean -0.000002 and std 0.010630
hidden layer 4 had mean 0.000001 and std 0.002378
hidden layer 5 had mean 0.000002 and std 0.000532
hidden layer 6 had mean -0.000000 and std 0.000119
hidden layer 7 had mean 0.000000 and std 0.000026
hidden layer 8 had mean -0.000000 and std 0.000006
hidden layer 9 had mean 0.000000 and std 0.000001
hidden layer 10 had mean -0.000000 and std 0.000000
```



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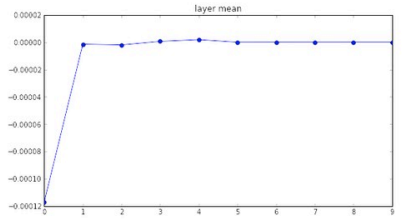
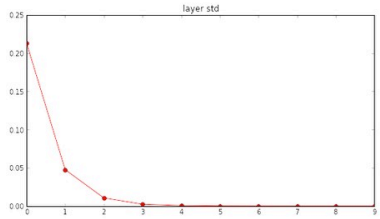
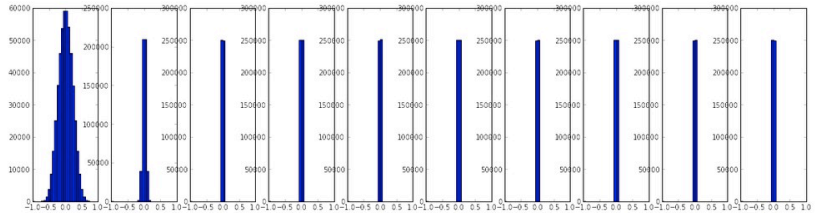
hidden layer 8 had mean -0.000000 and std 0.000006
hidden layer 9 had mean 0.000000 and std 0.000001
hidden layer 10 had mean -0.000000 and std 0.000000
```







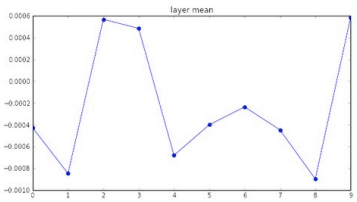
All activations become zero!

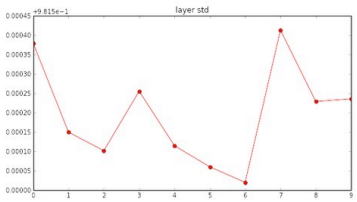
Q: What do the gradients look like?

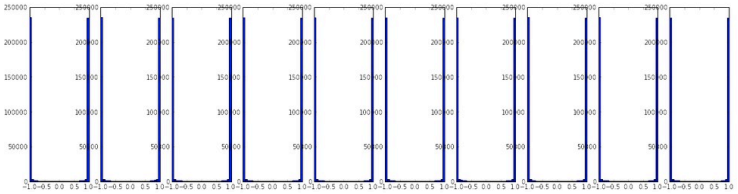

```
W = np.random.randn(fan_in, fan_out) * 1.0 # layer initialization
```

```
input layer had mean 0.001800 and std 1.001311
hidden layer 1 had mean -0.000430 and std 0.981879
hidden layer 2 had mean -0.000849 and std 0.981649
hidden layer 3 had mean 0.000566 and std 0.981601
hidden layer 4 had mean 0.000483 and std 0.981755
hidden layer 5 had mean -0.000682 and std 0.981614
hidden layer 6 had mean -0.000401 and std 0.981560
hidden layer 7 had mean -0.000237 and std 0.981520
hidden layer 8 had mean -0.000448 and std 0.981913
hidden layer 9 had mean -0.000899 and std 0.981728
hidden layer 10 had mean 0.000584 and std 0.981736
```

*1.0 instead of *0.01





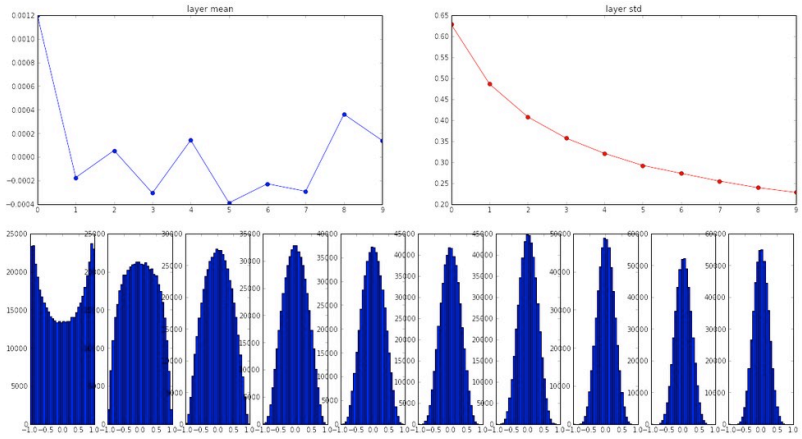


Almost all neurons completely saturated, either -1 and 1. Gradients will be all zero.

```
input layer had mean 0.001800 and std 1.001311
hidden layer 1 had mean 0.001198 and std 0.627953
hidden layer 2 had mean -0.000175 and std 0.486051
hidden layer 3 had mean 0.000055 and std 0.407723
hidden layer 4 had mean -0.000306 and std 0.357108
hidden layer 5 had mean 0.000142 and std 0.320917
hidden layer 6 had mean -0.000389 and std 0.292116
hidden layer 7 had mean -0.000228 and std 0.273387
hidden layer 8 had mean -0.000291 and std 0.254935
hidden layer 9 had mean 0.000361 and std 0.239266
hidden layer 10 had mean 0.000139 and std 0.228008
```

```
W = np.random.randn(fan_in, fan_out) / np.sqrt(fan_in) # layer initialization
```

"Xavier initialization"
[Glorot et al., 2010]

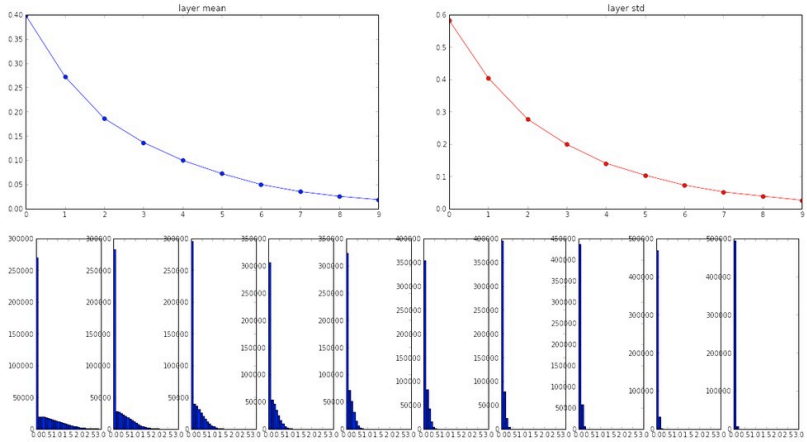


Reasonable initialization.
(Mathematical derivation assumes linear activations)


```
input layer had mean 0.000501 and std 0.999444
hidden layer 1 had mean 0.398623 and std 0.582273
hidden layer 2 had mean 0.272352 and std 0.403795
hidden layer 3 had mean 0.186076 and std 0.276912
hidden layer 4 had mean 0.136442 and std 0.198685
hidden layer 5 had mean 0.099568 and std 0.140299
hidden layer 6 had mean 0.072234 and std 0.103280
hidden layer 7 had mean 0.049775 and std 0.072748
hidden layer 8 had mean 0.035138 and std 0.051572
hidden layer 9 had mean 0.025404 and std 0.038583
hidden layer 10 had mean 0.018408 and std 0.026076
```

```
W = np.random.randn(fan_in, fan_out) / np.sqrt(fan_in) # layer initialization
```

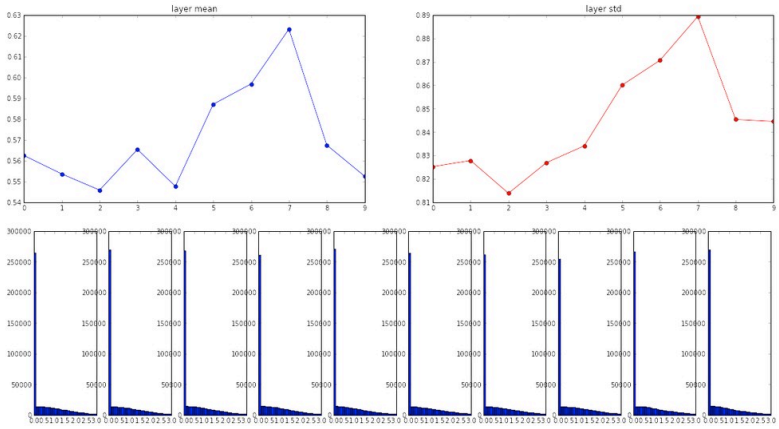
but when using the ReLU nonlinearity it breaks.



```
input layer had mean 0.000501 and std 0.999444
hidden layer 1 had mean 0.562488 and std 0.825232
hidden layer 2 had mean 0.553614 and std 0.827835
hidden layer 3 had mean 0.545867 and std 0.813855
hidden layer 4 had mean 0.565396 and std 0.826902
hidden layer 5 had mean 0.547678 and std 0.834092
hidden layer 6 had mean 0.587103 and std 0.860035
hidden layer 7 had mean 0.596867 and std 0.870610
hidden layer 8 had mean 0.623214 and std 0.889348
hidden layer 9 had mean 0.567498 and std 0.845357
hidden layer 10 had mean 0.552531 and std 0.844523
```

```
W = np.random.randn(fan_in, fan_out) / np.sqrt(fan_in/2) # layer initialization
```

He et al., 2015
(note additional /2)

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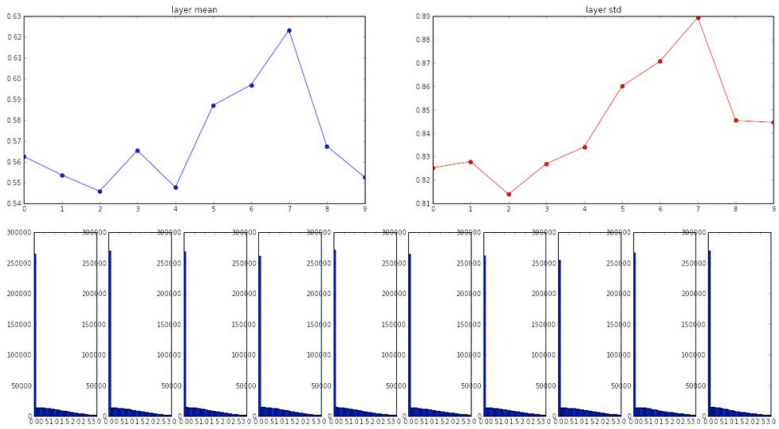

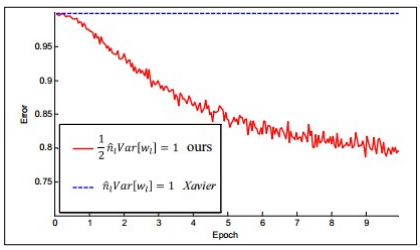

Proper initialization is an active area of research…

Understanding the difficulty of training deep feedforward neural networks by Glorot and Bengio, 2010

Exact solutions to the nonlinear dynamics of learning in deep linear neural networks by Saxe et al, 2013

Random walk initialization for training very deep feedforward networks by Sussillo and Abbott, 2014

Delving deep into rectifiers: Surpassing human-level performance on ImageNet classification by He et al., 2015

Data-dependent Initializations of Convolutional Neural Networks by Krähenbühl et al., 2015

All you need is a good init, Mishkin and Matas, 2015
…

Batch Normalization

[Ioffe and Szegedy, 2015]

"you want unit gaussian activations? just make them so."

consider a batch of activations at some layer.
To make each dimension unit gaussian, apply:

$$\widehat{x}^{(k)} = \frac{x^{(k)} - \mathrm{E}[x^{(k)}]}{\sqrt{\mathrm{Var}[x^{(k)}]}}$$

this is a vanilla differentiable function...

Batch Normalization

[Ioffe and Szegedy, 2015]

"you want unit gaussian activations?
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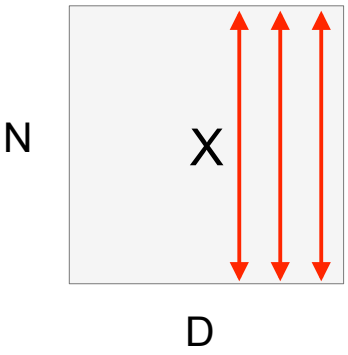


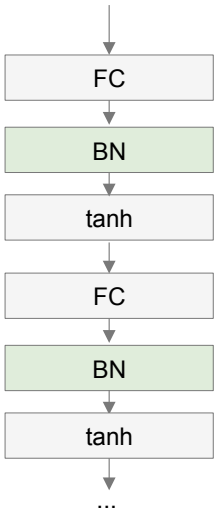
1. compute the empirical mean and variance independently for each dimension.

2. Normalize

$$\hat{x}^{(k)} = \frac{x^{(k)} - \mathrm{E}[x^{(k)}]}{\sqrt{\mathrm{Var}[x^{(k)}]}}$$

Batch Normalization

[Ioffe and Szegedy, 2015]



Usually inserted after Fully Connected / (or Convolutional, as we'll see soon) layers, and before nonlinearity.

Problem: do we necessarily want a unit gaussian input to a tanh layer?

$$\hat{x}^{(k)} = \frac{x^{(k)} - \mathrm{E}[x^{(k)}]}{\sqrt{\mathrm{Var}[x^{(k)}]}}$$

Batch Normalization

[Ioffe and Szegedy, 2015]

Normalize:

$$\widehat{x}^{(k)} = \frac{x^{(k)} - \mathrm{E}[x^{(k)}]}{\sqrt{\mathrm{Var}[x^{(k)}]}}$$

And then allow the network to squash the range if it wants to:

$$y^{(k)} = \gamma^{(k)}\widehat{x}^{(k)} + \beta^{(k)}$$

Note, the network can learn:

$$\gamma^{(k)} = \sqrt{\mathrm{Var}[x^{(k)}]}$$

$$\beta^{(k)} = \mathrm{E}[x^{(k)}]$$

to recover the identity mapping.

Batch Normalization

[Ioffe and Szegedy, 2015]

Input: Values of x over a mini-batch: $\mathcal{B} = \{x_{1...m}\}$;
Parameters to be learned: γ, β

Output: $\{y_i = \mathrm{BN}_{\gamma,\beta}(x_i)\}$

$$\mu_{\mathcal{B}} \leftarrow \frac{1}{m}\sum_{i=1}^{m} x_i \qquad \text{// mini-batch mean}$$

$$\sigma^2_{\mathcal{B}} \leftarrow \frac{1}{m}\sum_{i=1}^{m}(x_i - \mu_{\mathcal{B}})^2 \qquad \text{// mini-batch variance}$$

$$\widehat{x}_i \leftarrow \frac{x_i - \mu_{\mathcal{B}}}{\sqrt{\sigma^2_{\mathcal{B}} + \epsilon}} \qquad \text{// normalize}$$

$$y_i \leftarrow \gamma\widehat{x}_i + \beta \equiv \mathrm{BN}_{\gamma,\beta}(x_i) \qquad \text{// scale and shift}$$

- Improves gradient flow through the network
- Allows higher learning rates
- Reduces the strong dependence on initialization
- Acts as a form of regularization in a funny way, and slightly reduces the need for dropout, maybe

Batch Normalization

[Ioffe and Szegedy, 2015]

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$$y_i \leftarrow \gamma\widehat{x}_i + \beta \equiv \text{BN}_{\gamma,\beta}(x_i) \qquad \text{// scale and shift}$$

Note: at test time BatchNorm layer functions differently:

The mean/std are not computed based on the batch. Instead, a single fixed empirical mean of activations during training is used.

(e.g. can be estimated during training with running averages)

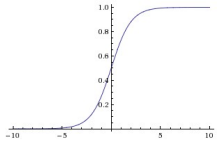
Remaining Pieces

- Optimizing machine learning objectives:
  - Stochastic Descent
  - Mini-batches
- Improving generalization
  - Drop-out
- Activation functions
- Initialization and batch normalization
- Computing the gradient $\nabla g(w)$
  - Gradient checking
  - Backprop

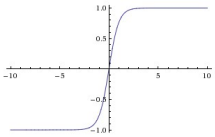
Activation Functions

Sigmoid

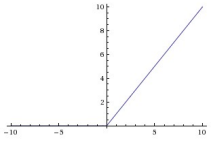
$$\sigma(x) = 1/(1+e^{-x})$$



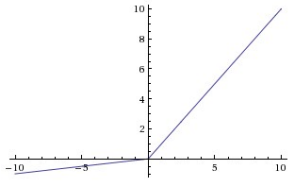
tanh tanh(x)



ReLU max(0,x)



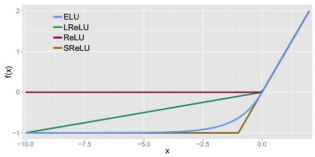
Leaky ReLU
max(0.1x, x)



Maxout $\max(w_1^T x + b_1, w_2^T x + b_2)$

ELU

$$f(x) = \begin{cases} x & \text{if } x > 0 \\ \alpha\,(\exp(x) - 1) & \text{if } x \le 0 \end{cases}$$



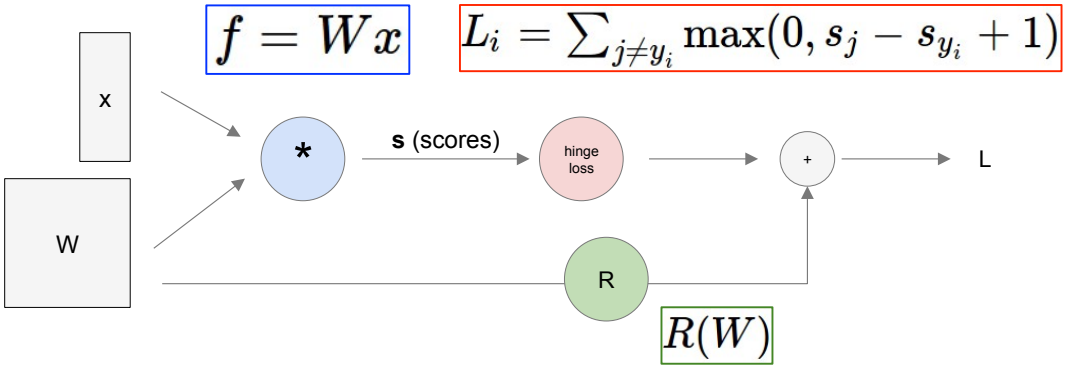
Gradient Descent

$$\frac{df(x)}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

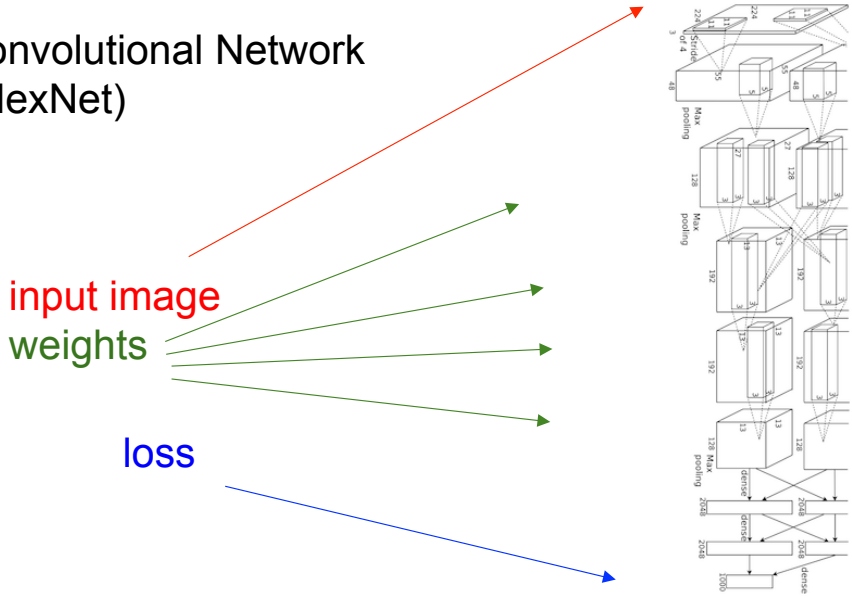
Numerical gradient: slow :(, approximate :(, easy to write :)
Analytic gradient: fast :), exact :), error-prone :(

In practice: Derive analytic gradient, check your implementation with numerical gradient

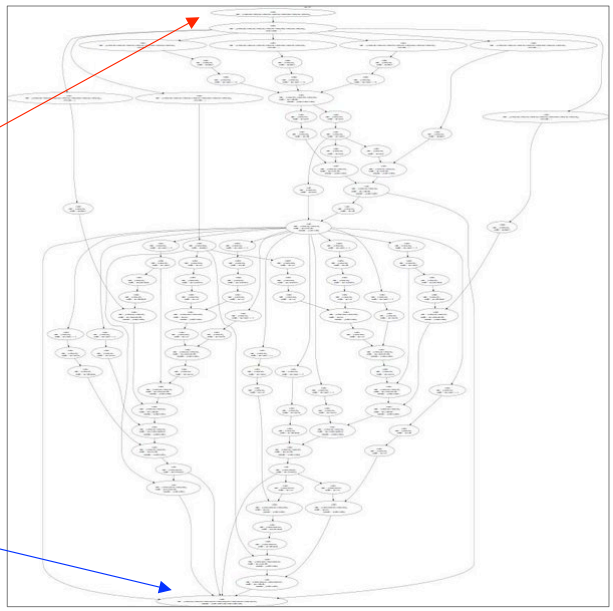
Computational Graph



Convolutional Network
(AlexNet)



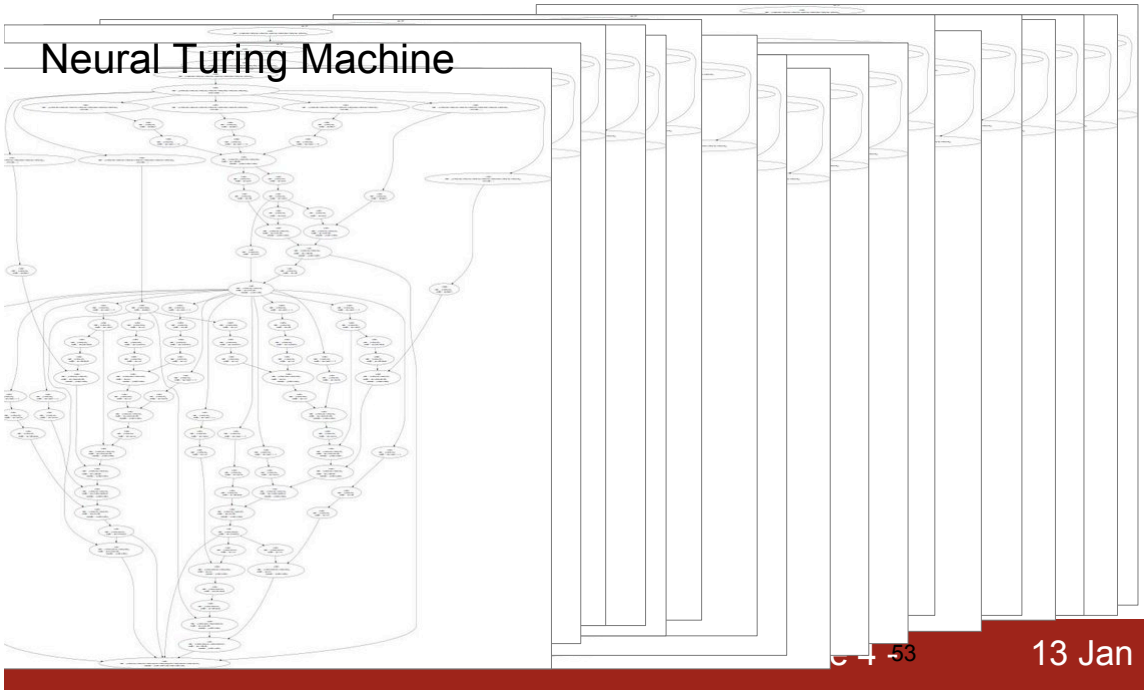
Neural Turing Machine



input tape

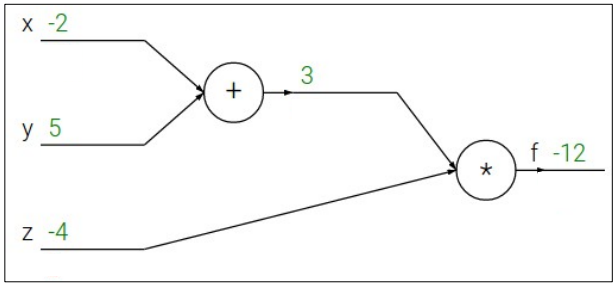
loss

Neural Turing Machine



$$f(x, y, z) = (x + y)z$$

e.g. x = -2, y = 5, z = -4



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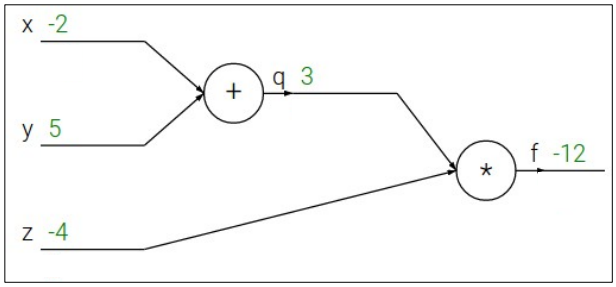
Want: $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$



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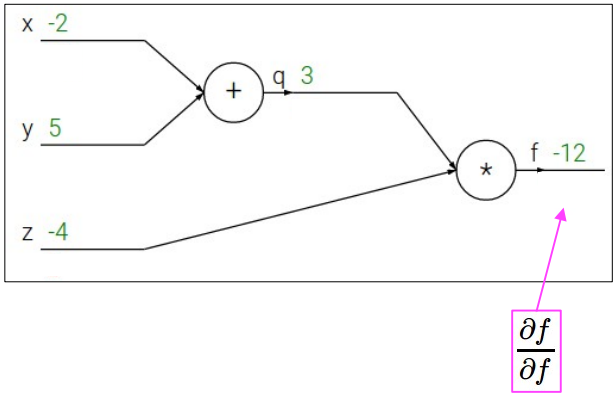

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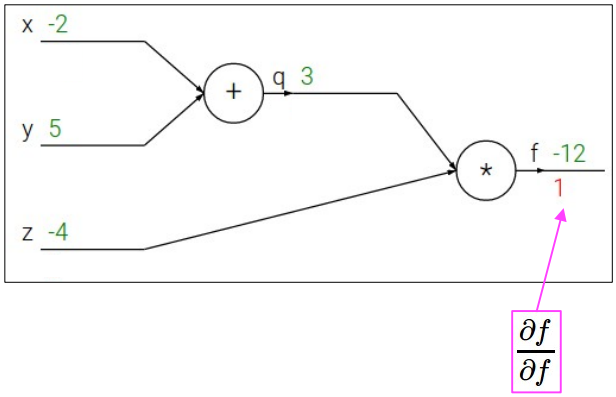

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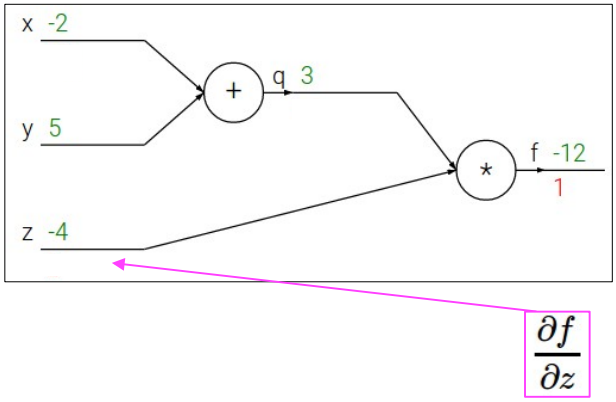

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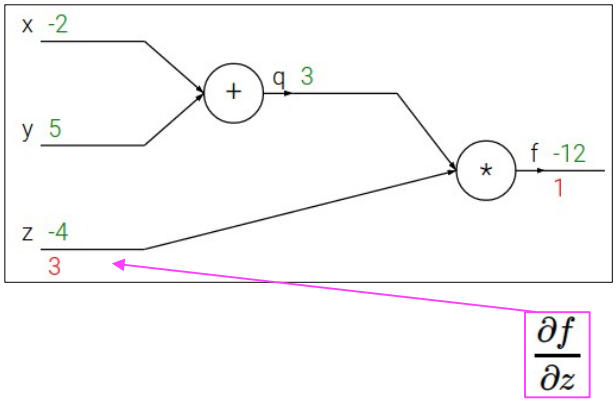

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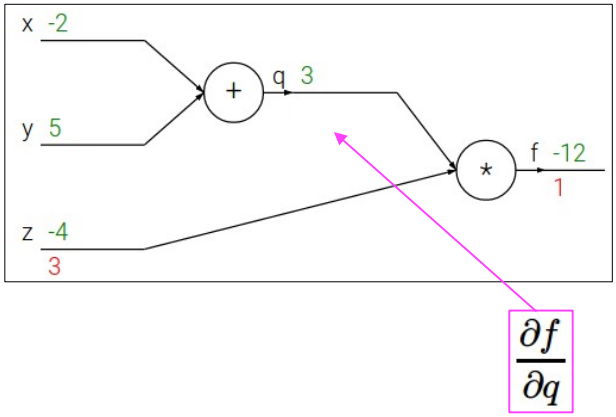

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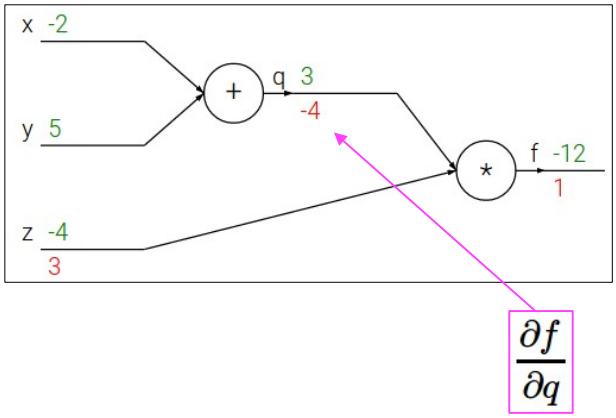

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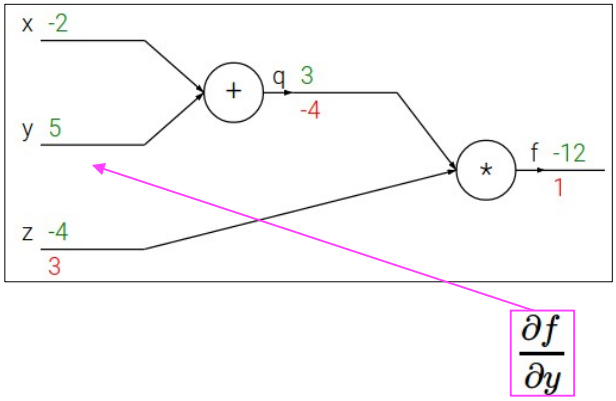

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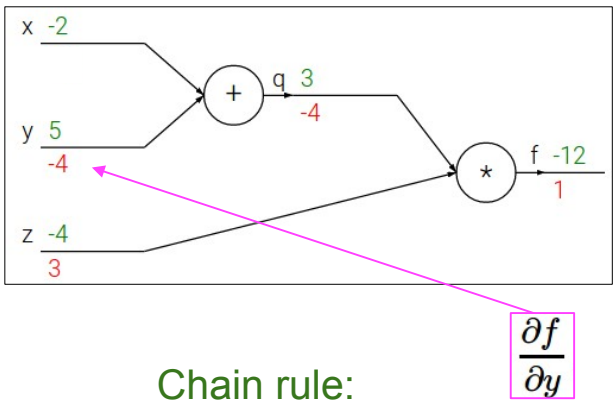

Chain rule:

$$\frac{\partial f}{\partial y} = \frac{\partial f}{\partial q}\frac{\partial q}{\partial y}$$

$$f(x, y, z) = (x + y)z$$

e.g. x = -2, y = 5, z = -4

$$q = x + y \qquad \frac{\partial q}{\partial x} = 1, \frac{\partial q}{\partial y} = 1$$

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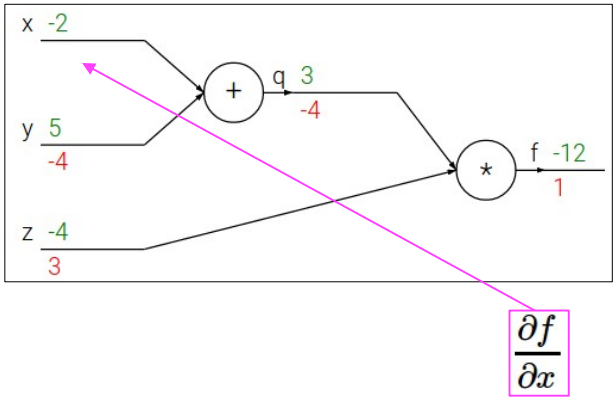
Want: $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$



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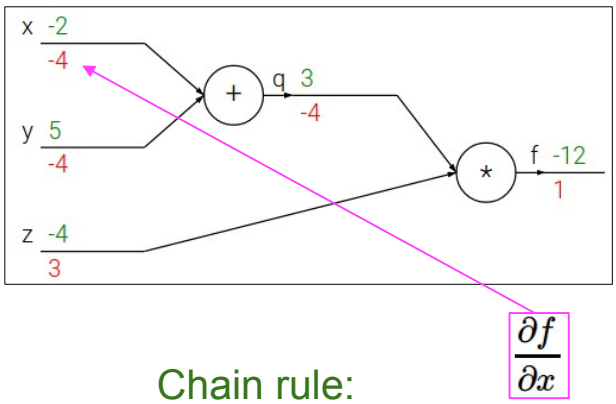
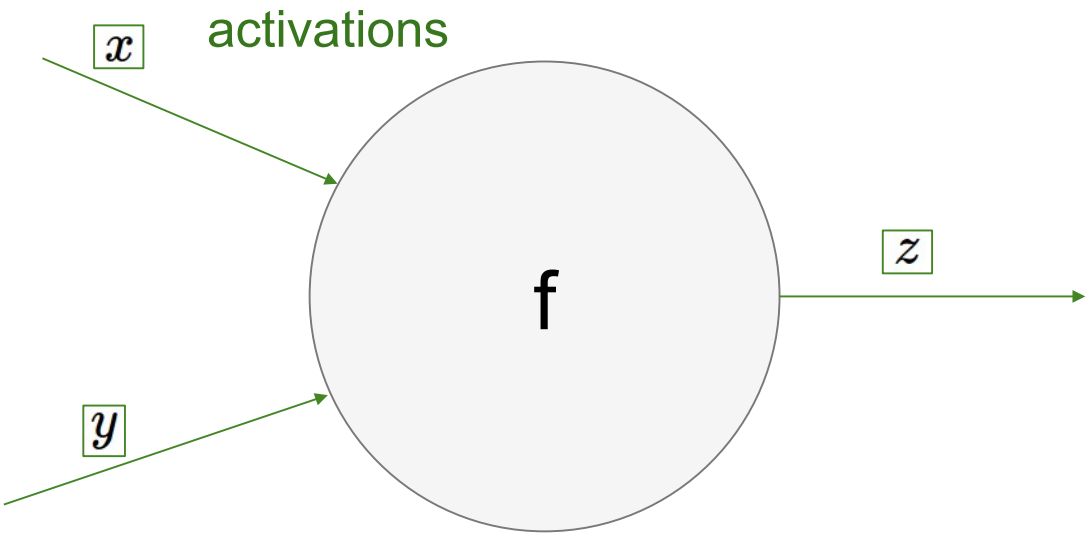
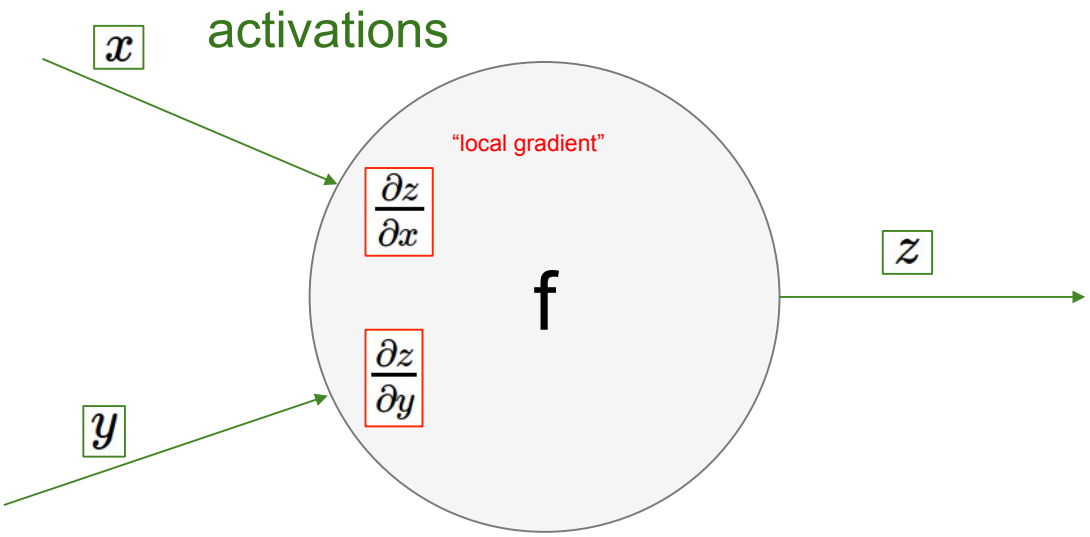
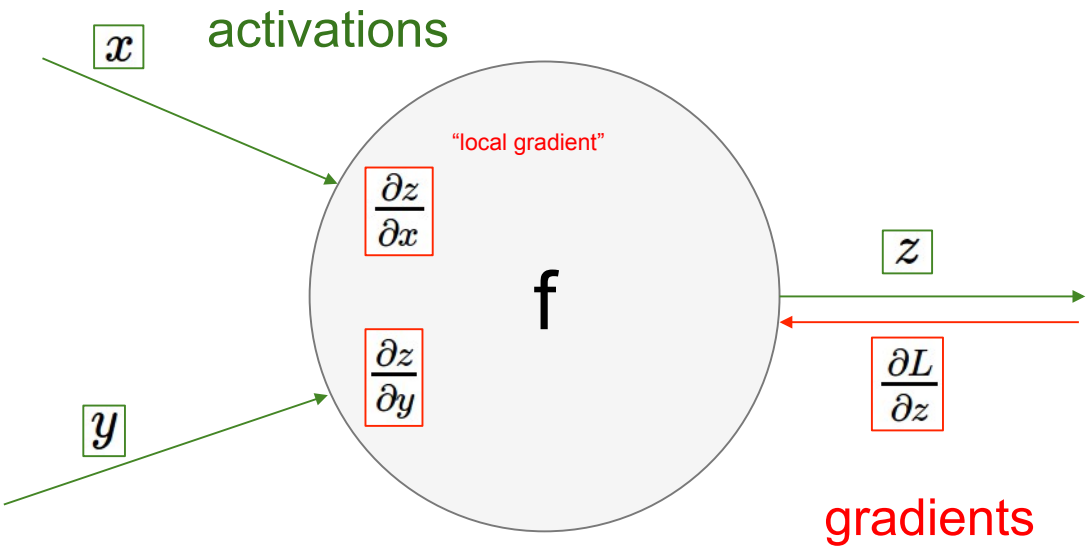


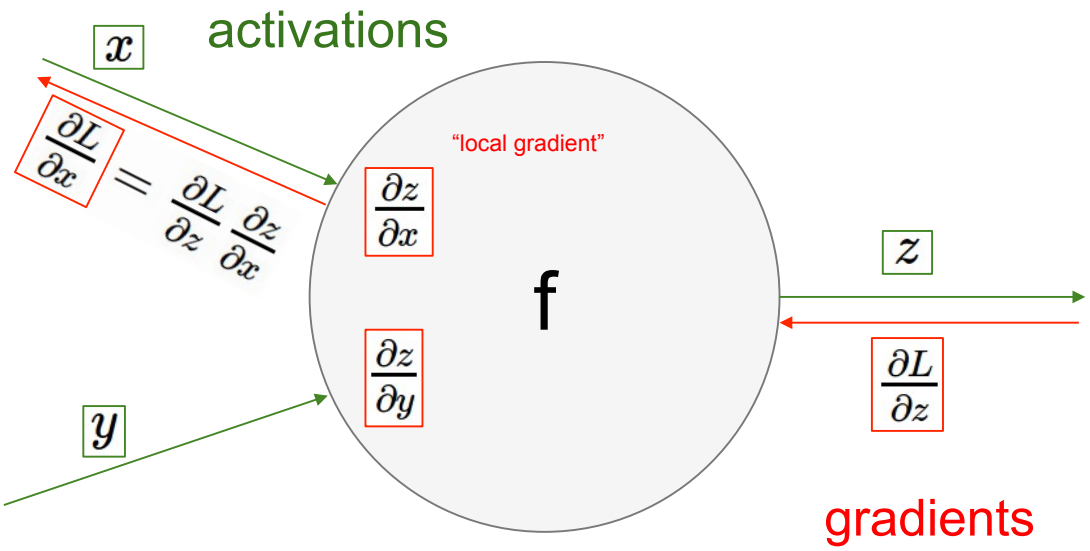
Chain rule:

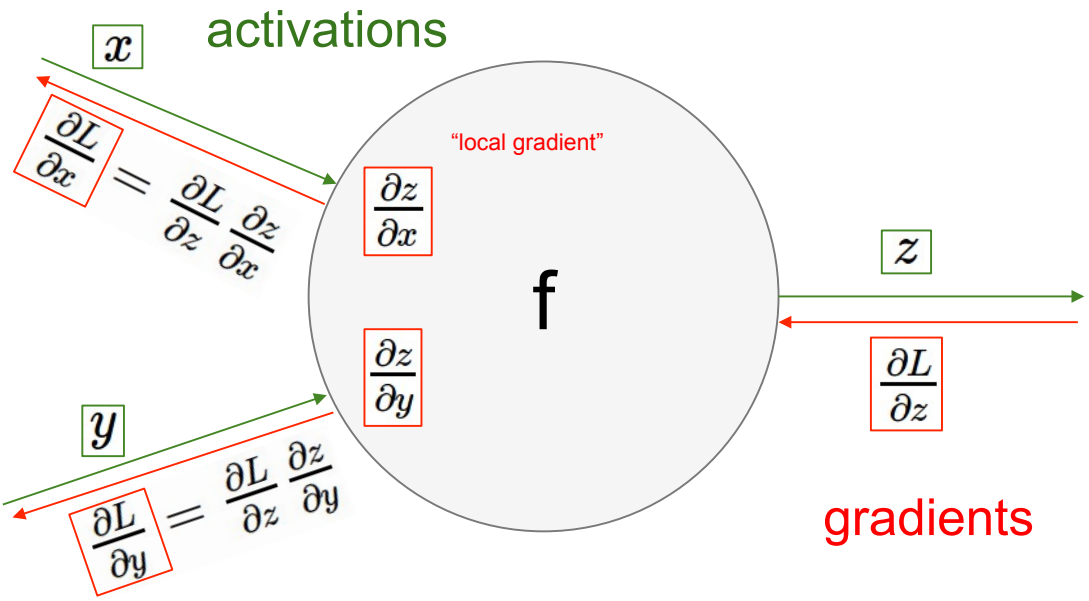
$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial q}\frac{\partial q}{\partial x}$$

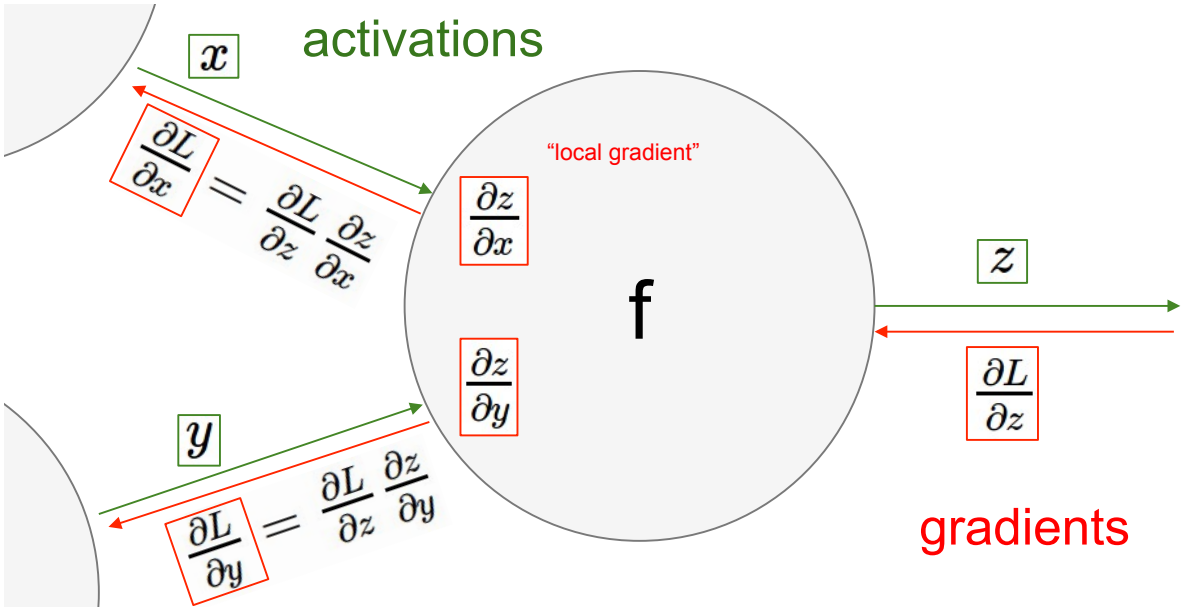

activations
x
y
f
z


activations
x
y
"local gradient"
$\frac{\partial z}{\partial x}$
$\frac{\partial z}{\partial y}$
f
z

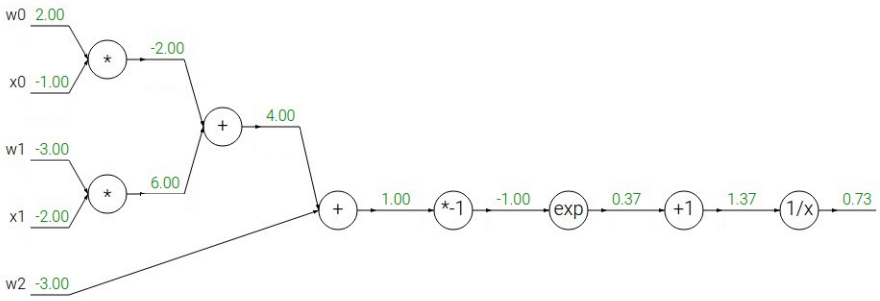

x
activations
"local gradient"
$\frac{\partial z}{\partial x}$
f
z
$\frac{\partial z}{\partial y}$
$\frac{\partial L}{\partial z}$
y
gradients


x
activations
$\frac{\partial L}{\partial x} = \frac{\partial L}{\partial z}\frac{\partial z}{\partial x}$
"local gradient"
$\frac{\partial z}{\partial x}$
f
z
$\frac{\partial z}{\partial y}$
$\frac{\partial L}{\partial z}$
y
gradients


activations
x
$\frac{\partial L}{\partial x} = \frac{\partial L}{\partial z}\frac{\partial z}{\partial x}$
"local gradient"
$\frac{\partial z}{\partial x}$
f
z
$\frac{\partial z}{\partial y}$
$\frac{\partial L}{\partial z}$
y
$\frac{\partial L}{\partial y} = \frac{\partial L}{\partial z}\frac{\partial z}{\partial y}$
gradients


activations
x
$\frac{\partial L}{\partial x} = \frac{\partial L}{\partial z}\frac{\partial z}{\partial x}$
"local gradient"
$\frac{\partial z}{\partial x}$
f
z
$\frac{\partial z}{\partial y}$
$\frac{\partial L}{\partial z}$
y
$\frac{\partial L}{\partial y} = \frac{\partial L}{\partial z}\frac{\partial z}{\partial y}$
gradients

Another example: $$f(w,x) = \frac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$



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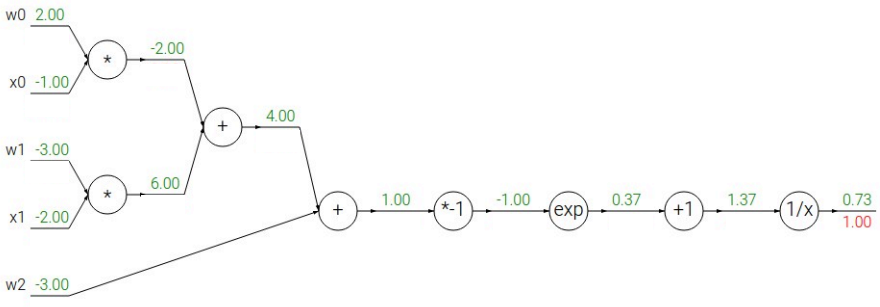
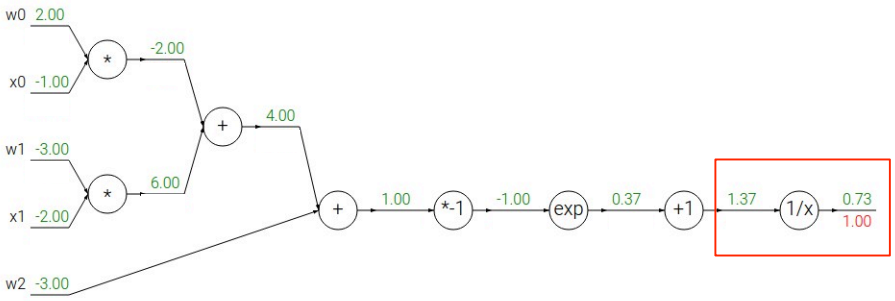


$$f(x) = e^x \rightarrow \frac{df}{dx} = e^x \qquad f(x) = \frac{1}{x} \rightarrow \frac{df}{dx} = -1/x^2$$

$$f_a(x) = ax \rightarrow \frac{df}{dx} = a \qquad f_c(x) = c + x \rightarrow \frac{df}{dx} = 1$$

Another example: $f(w,x) = \frac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$



$f(x) = e^x \rightarrow \frac{df}{dx} = e^x$

$f_a(x) = ax \rightarrow \frac{df}{dx} = a$

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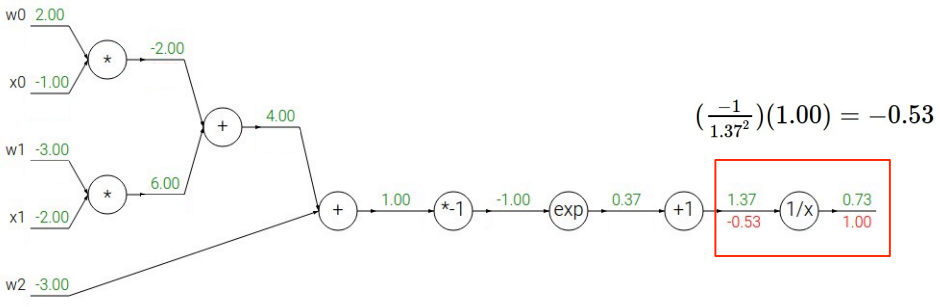

$(\frac{-1}{1.37^2})(1.00) = -0.53$

$f(x) = e^x \rightarrow \frac{df}{dx} = e^x$

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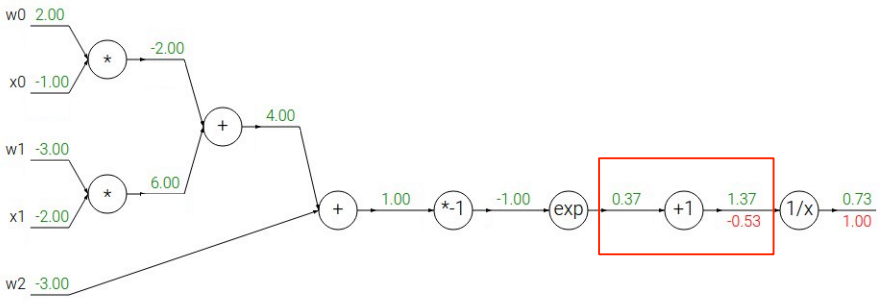

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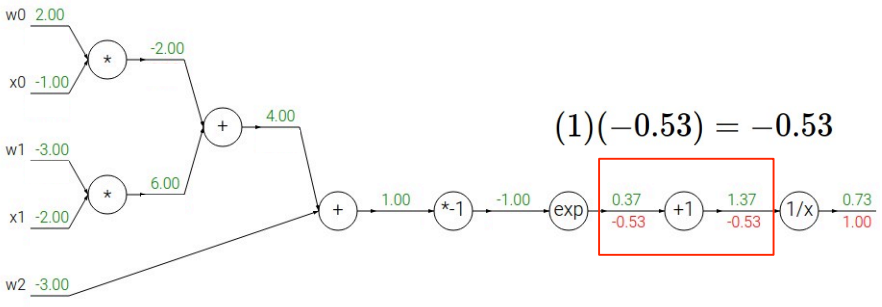

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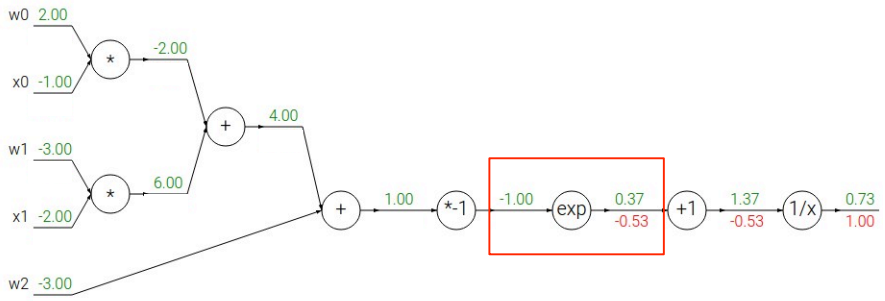

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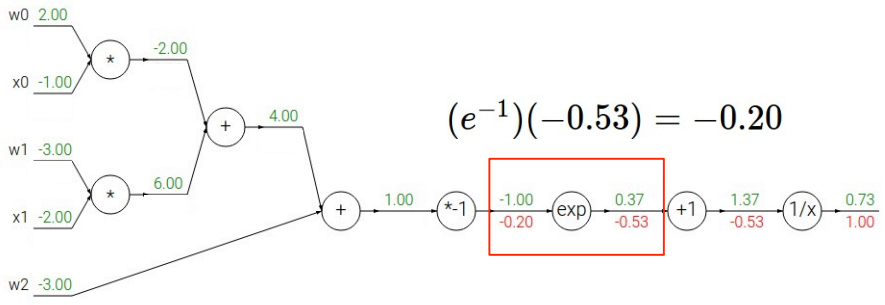

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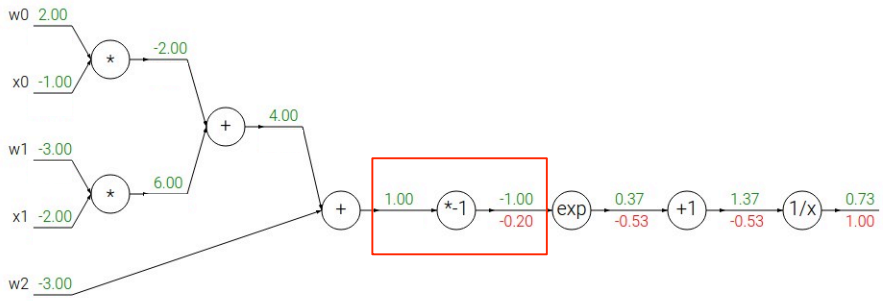

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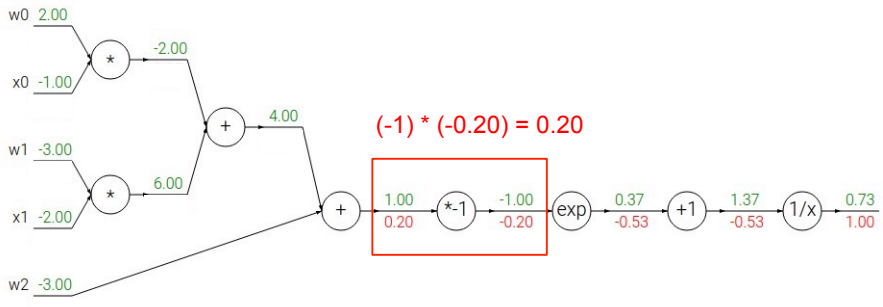

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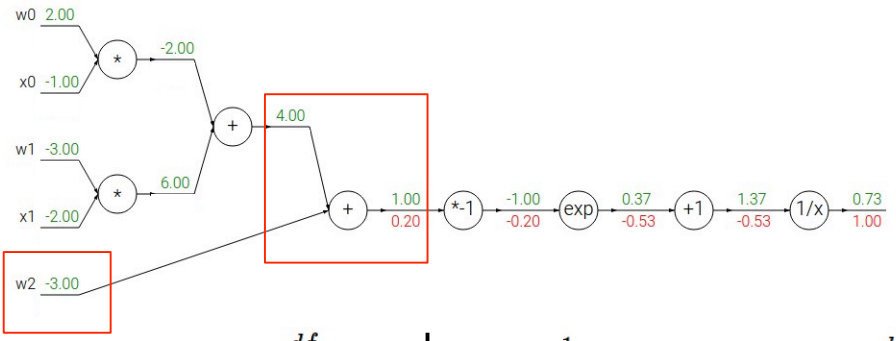

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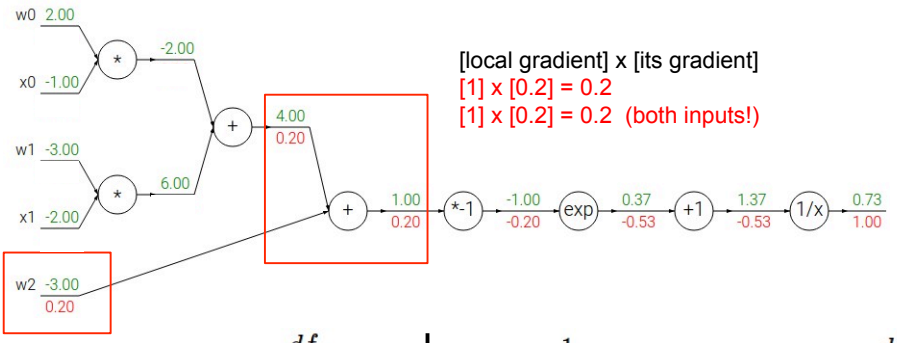

[local gradient] x [its gradient]
[1] x [0.2] = 0.2
[1] x [0.2] = 0.2 (both inputs!)

$f(x) = e^x \rightarrow \frac{df}{dx} = e^x$

$f_a(x) = ax \rightarrow \frac{df}{dx} = a$

$f(x) = \frac{1}{x} \rightarrow \frac{df}{dx} = -1/x^2$

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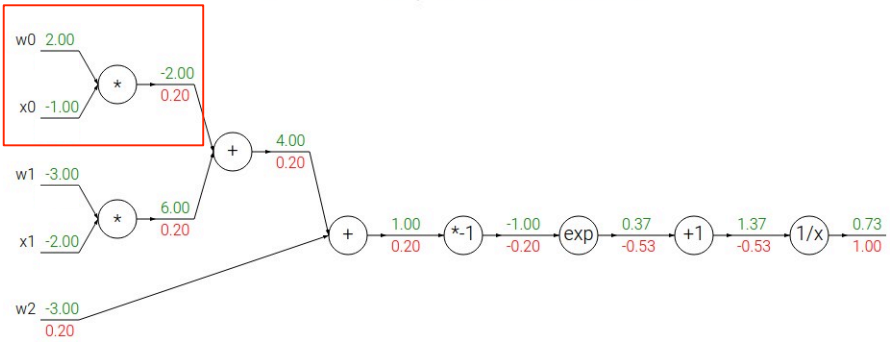

$f(x) = e^x \rightarrow \frac{df}{dx} = e^x$

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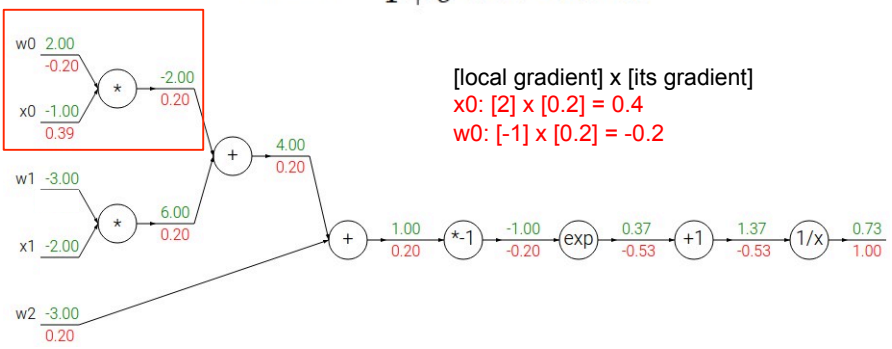


[local gradient] x [its gradient]
x0: [2] x [0.2] = 0.4
w0: [-1] x [0.2] = -0.2

$f(x) = e^x \rightarrow \frac{df}{dx} = e^x$

$f_a(x) = ax \rightarrow \frac{df}{dx} = a$

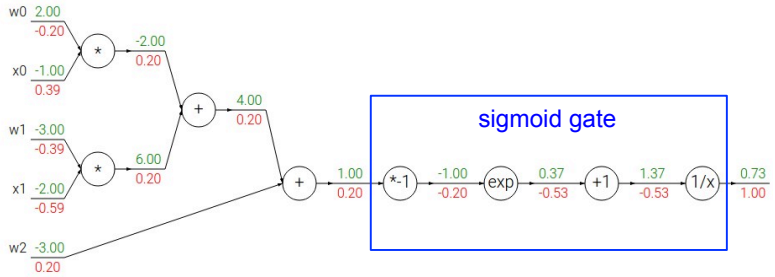
$f(x) = \frac{1}{x} \rightarrow \frac{df}{dx} = -1/x^2$

$f_c(x) = c + x \rightarrow \frac{df}{dx} = 1$

$$f(w,x) = \frac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$

$$\sigma(x) = \frac{1}{1+e^{-x}}$$ sigmoid function

$$\frac{d\sigma(x)}{dx} = \frac{e^{-x}}{(1+e^{-x})^2} = \left(\frac{1+e^{-x}-1}{1+e^{-x}}\right)\left(\frac{1}{1+e^{-x}}\right) = (1-\sigma(x))\,\sigma(x)$$



$$f(w,x) = \frac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$

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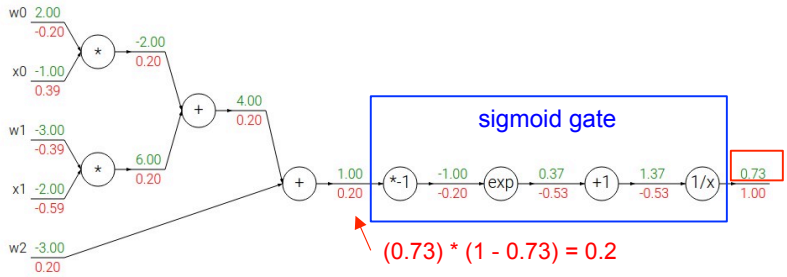
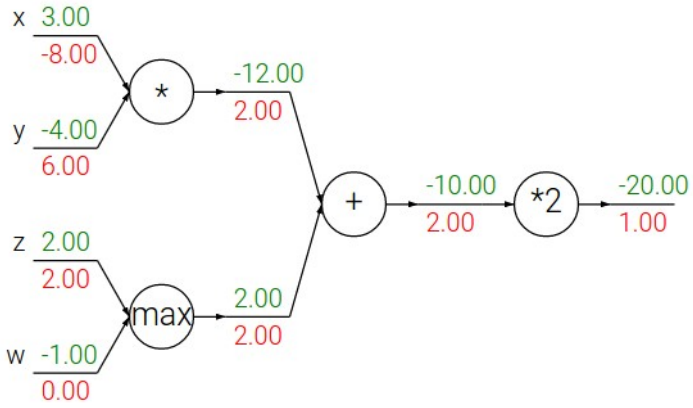
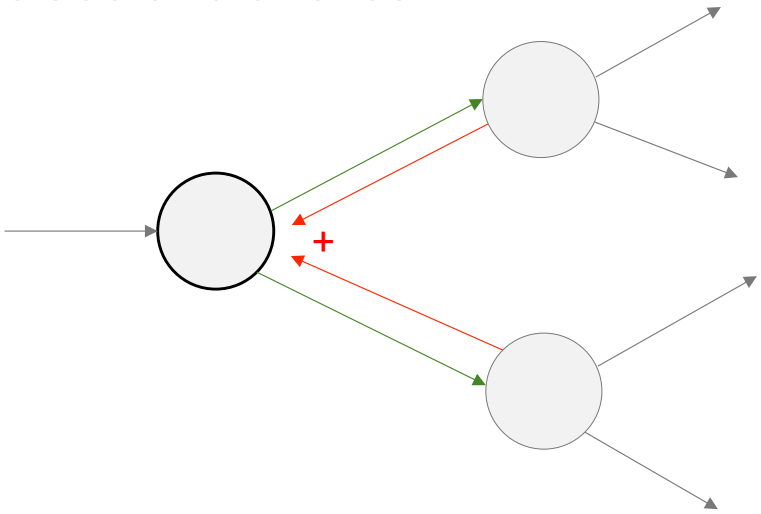


Patterns in backward flow

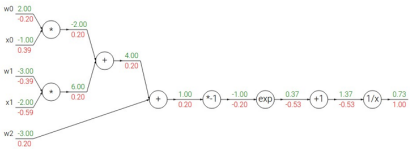
add gate: gradient distributor
max gate: gradient router
mul gate: gradient…?



Gradients add at branches



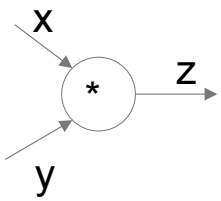
Implementation: forward/backward API



Graph (or Net) object. *(Rough psuedo code)*

```
class ComputationalGraph(object):
    #...
    def forward(inputs):
        # 1. [pass inputs to input gates...]
        # 2. forward the computational graph:
        for gate in self.graph.nodes_topologically_sorted():
            gate.forward()
        return loss # the final gate in the graph outputs the loss
    def backward():
        for gate in reversed(self.graph.nodes_topologically_sorted()):
            gate.backward() # little piece of backprop (chain rule applied)
        return inputs_gradients
```


Implementation: forward/backward API



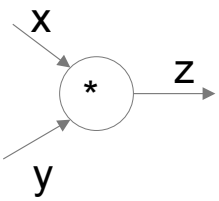
(x,y,z are scalars)

```
class MultiplyGate(object):
    def forward(x,y):
        z = x*y
        return z
    def backward(dz):
        # dx = ... #todo
        # dy = ... #todo
        return [dx, dy]
```

$\frac{\partial L}{\partial z}$

$\frac{\partial L}{\partial x}$

Implementation: forward/backward API




```
class MultiplyGate(object):
    def forward(x,y):
        z = x*y
        self.x = x # must keep these around!
        self.y = y
        return z
    def backward(dz):
        dx = self.y * dz # [dz/dx * dL/dz]
        dy = self.x * dz # [dz/dy * dL/dz]
        return [dx, dy]
```

(x,y,z are scalars)

That's it for ML

Up next: Advanced Applications!