# **Optimization Benchmarking for A2 algorithm**

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## **ABSTRACT**

The aim of COCO Platform is to benchmark continuous algorithms. Here we deal with single objective functions, and we benchmark two implementations of A2 aglorithm (coded with python and C languages) dealing with the evolution strategies issues with 2 other performing algorithms BIPOP-C and BFGS.

# Keywords

Benchmarking, Black-box optimization, A2

# 1. INTRODUCTION

Inspired by biology and Darwin's, evolutions strategies ES are stochastic optimization algorithms designed for continuous search space. Unlike the gradient based algorithms which are local search algorithms, ES are global optimization algorithms and perform very well on difficult problems such as badly scaled, non-continuously differentiable or even not completely defined functions (Blackbox). [2, 12] ES are heuristic population-based search algorithm that incorporate random variation and selection. In each iteration called generation, ES algorithm generates offsprings from  $\mu$  parents. The offsprings are generated by adding a mutation vector to the parents. The mutation vectors are Gaussian distributed with mean equal to zero and standard deviation  $\sigma$  called step size. Then environmental selection reduces the population to its original size. [2, 12] The important features of ES are:

- Unbiasedness: the mutation is based on normally distributed vectors so the information injected at each generation is unbiased.
- Self-adaptation:strategy parameter control, step size adaptation.

Environmental selection: survival of the fittest, unlike the genetic algorithms where the individuals are randomly selected, the selection process in ES is deterministic.

Various step size adaptation concepts have been imagined since the creation of ES algorithms and perform well but when the scaling of the parameters to be optimized is not known, the idea of individual step sizes has to be implemented. Schwefel and Rechenberg have introduced the idea of mutative step size control. This concept is based on the idea that both objective parameters (solution, individual) and strategy parameters (here step size) undergo mutation and selection. One shortcoming of the adaptation of individual step sizes is that it is impossible for small population. This is due to the fact that the size of the parameter variation is not taken into account. For example, the object parameter can undergo a large variation even if the step size variation is small because the randomly generated vector (the one added to mutate the offspring) is large. Another reason is that the step size variation between the offsprings of a generation is the same as the one from between generations. This makes the individual step size irrelevant [10]. This problem has been addressed by introducing the derandomized mutative step-size control. The algorithm studied A2 is a good example of the derandomization.

## 2. ALGORITHM PRESENTATION

In this part, we use the same notations than in [8] and we make reference to some equations of this paper.

A2 is a  $(\mu$ ,  $\lambda)$  evolutionary strategy. The algorithm takes place in three different steps : the mutation of object variables, the adaptation of strategy parameters and the selection[8].

#### 2.1 Mutation

The mutation is done from a parent randomly chosen  $(E_k)$  to produce a new individual  $(N_k)$ . The equation of the mutation is:

$$x_{i}^{N^{k}} = x_{i}^{E_{\xi}} + \delta_{i}^{E_{k}}.z_{i}^{k} + \delta_{r}^{E_{k}}.z_{r}^{k}.r_{i}^{E_{\xi}}$$
 (1)

This mutation can be decomposed in 2 parts. The individual mutation (independent for each vector's component) and the global mutation in one direction (r).

We make this mutation on  $\mu$  parents for each generation.

GECCO'13, July 6-10, 2013, Amsterdam, The Netherlands. ACM ISBN TBA. DOI: 10.1145/1235 This mutations needs to be adapted, which means that the mutations which were good in the past have a higher probability to be chosen.

# 2.2 Mutation adaptation

There are two types of adaptation: individual step size adaptation and direction adaptation.

Individual step size adaptation enables different variances of the mutation on each axis and consequently to have good results on ill-conditioned functions. However, the mutation is dependant of the coordinate system[8].

$$\delta_i^N = \delta_i^E . exp(\beta(\|s^N\| - \chi_n)) . exp(\beta_{ind}(|s_i^N| - \chi_1))$$
 (2)

The direction adaptation is done by accumulation which makes it possible to keep the information of previous mutations

$$r' = (1 - c_r) \cdot \delta_r^{E_{\xi}} \cdot r^{E_{\xi}} + c_r \cdot (x^{N_k} - x^{E_{\xi}})$$
 (3)

#### Derandomization.

These two adaptations are derandomized which reduces the stochastic noise of the procedure.

# $\mathbf{1}^{st}$ derandomization

It is done by adding  $\beta$  and  $\beta_{ind}$  in the formula of  $\delta$ . If they are inferior to one, it reduces the evolution of the parameters without changing the strength of the mutation[8, 7].

# $2^{nd}$ derandomization

In the formulas, it is represented by :  $||s^N|| - \chi_n$  and  $|s_i^N| - \chi_1$ 

If s is greater than his expectation and that the mutation is efficient (it means that the son beget is present in the next generation) then,  $\delta$  needs to be increased[8, 7].

S represents the accumulation of the mutations. It is better to use accumulation than only the last  $z_i$  because our adaptation will be based on all the previous mutations. Accumulation indicates if a sequence of mutations is efficient (the mutations must go in the same direction).[8, 7]

#### 2.3 Selection

The selection is the step in which the algorithm selects the  $\mu$  best individuals from the offspring to produce the new generation.

# 3. ALGORITHM IMPLEMENTATION

The algorithm was implemented in Python. First, a class Individu was created which contains as attributes the object and strategy parameters :

• X : numpy vector of the object variables

• Delta: numpy vector of individual step sizes

• DeltaR : step-size for direction

• R : numpy direction vector

• S : accumulation of the realized vectors z

• Sr: weighted sum of DeltaR

. It also contains the method mutation which from a parent creates a child.

The second class Population is constituted of an array of individuals. It contains the following attributes :

• F: the function to be optimized

• N : problem dimensionality

• Lan: number of parents

• Mu : number of children

• Individus: python list containing the population

The method nextgen creates the next generation.

## 4. RESULTS

Results from experiments according to [9] and [3] on the benchmark functions given in [1, 6] are presented in Figures 1, 2 and 3 and in Tables 1 and 2. The experiments were performed with COCO [5], version 1.0.1, the plots were produced with version 1.0.4.

The average runtime (aRT), used in the figures and tables, depends on a given target function value,  $f_t = f_{\text{opt}} + \Delta f$ , and is computed over all relevant trials as the number of function evaluations executed during each trial while the best function value did not reach  $f_t$ , summed over all trials and divided by the number of trials that actually reached  $f_t$  [4, 11]. Statistical significance is tested with the rank-sum test for a given target  $\Delta f_t$  using, for each trial, either the number of needed function evaluations to reach  $\Delta f_t$  (inverted and multiplied by -1), or, if the target was not reached, the best  $\Delta f$ -value achieved, measured only up to the smallest number of overall function evaluations for any unsuccessful trial under consideration.

## Figure 1:

Globally, we observe that the arT in number of f-evaluations is better for the BFGS algorithm on all the functions. Alsoit has the best statistical result compared to all other algorithms. Zoubab and A2 algorithm are competitive regarding the arT.Zoubab does better for Sphere and Ellipsoid Separable algorithms but for the rest they similar results.

#### Figure 2:

Our algorithm perfoms rather correctly compared to the others. The BIPOP-C alogorithm is definitely the best one. So if we compare to Zoubab, we have better results, and compared to BFGS, it has better results for weakly structured multi-modalfunctions but for the rest we are either better or equal.

#### Figure 3:

For the 20-D the trends are differents. A2 and carepediem Algorithms are globally performing equally. Regarding BFGS it does better for weakly structured multi-modal functions (weakly) and we have better resluts for multimodal functions.

## 5. CONCLUSION

In this project, we implemented A2 evolution strategy using python programming language, and compared its performance with the other algorithms (CMA-ES, BFGSâĂę) and the implementation of the same algorithm with C language by team Zoubab. The results figures illustrate the fact that the average running time of our algorithm is lesser than the other teamâĂŹs algorithm (Zoubab) in most of the cases. Even if it is quite performant, our algorithm is still not as effective the CMA-ES, which was predictable. We will analyse our results thoroughly and present our final conclusion during the oral presentation of our work

## 6. REFERENCES

- S. Finck, N. Hansen, R. Ros, and A. Auger. Real-parameter black-box optimization benchmarking 2009: Presentation of the noiseless functions. Technical Report 2009/20, Research Center PPE, 2009. Updated February 2010.
- [2] N. Hansen, D. V. Arnold, and A. Auger. Evolution strategies. In Springer Handbook of Computational Intelligence, pages 871–898. Springer, 2015.
- [3] N. Hansen, A. Auger, D. Brockhoff, D. Tušar, and T. Tušar. COCO: Performance assessment. ArXiv e-prints, arXiv:1605.03560, 2016.
- [4] N. Hansen, A. Auger, S. Finck, and R. Ros. Real-parameter black-box optimization benchmarking 2012: Experimental setup. Technical report, INRIA, 2012.
- [5] N. Hansen, A. Auger, O. Mersmann, T. Tušar, and D. Brockhoff. COCO: A platform for comparing continuous optimizers in a black-box setting. ArXiv e-prints, arXiv:1603.08785, 2016.
- [6] N. Hansen, S. Finck, R. Ros, and A. Auger. Real-parameter black-box optimization benchmarking 2009: Noiseless functions definitions. Technical Report RR-6829, INRIA, 2009. Updated February 2010.
- [7] N. Hansen and A. Ostermeier. Completely derandomized self-adaptation in evolution strategies. *Evolutionary computation*, 9(2):159–195, 2001.
- [8] N. Hansen, A. Ostermeier, and A. Gawelczyk. On the adaptation of arbitrary normal mutation distributions in evolution strategies: The generating set adaptation. In ICGA, pages 57–64, 1995.
- [9] N. Hansen, T. Tušar, O. Mersmann, A. Auger, and D. Brockhoff. COCO: The experimental procedure. ArXiv e-prints, arXiv:1603.08776, 2016.
- [10] A. Ostermeier, A. Gawelczyk, and N. Hansen. A derandomized approach to self-adaptation of evolution strategies. *Evolutionary Computation*, 2(4):369–380, 1994.
- [11] K. Price. Differential evolution vs. the functions of the second ICEO. In *Proceedings of the IEEE* International Congress on Evolutionary Computation, pages 153–157, 1997.
- [12] R. Salomon. Evolutionary algorithms and gradient search: similarities and differences. *IEEE Transactions* on Evolutionary Computation, 2(2):45–55, 1998.

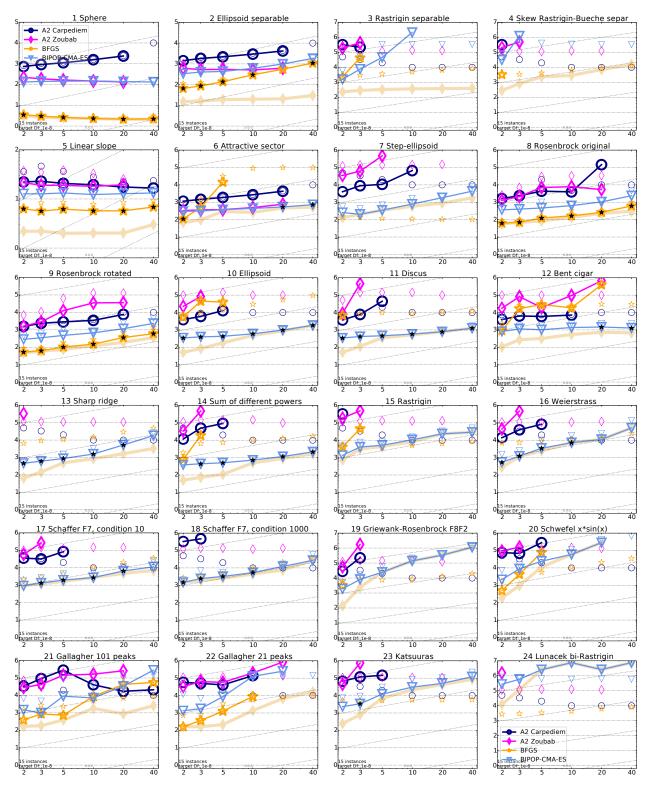


Figure 1: Average running time (aRT in number of f-evaluations as  $\log_{10}$  value), divided by dimension for target function value  $10^{-8}$  versus dimension. Slanted grid lines indicate quadratic scaling with the dimension. Different symbols correspond to different algorithms given in the legend of  $f_1$  and  $f_{24}$ . Light symbols give the maximum number of function evaluations from the longest trial divided by dimension. Black stars indicate a statistically better result compared to all other algorithms with p < 0.01 and Bonferroni correction number of dimensions (six). Legend:  $\circ$ : A2 Carpediem,  $\diamond$ : A2 Zoubab,  $\star$ : BFGS,  $\nabla$ : BIPOP-CMA-ES

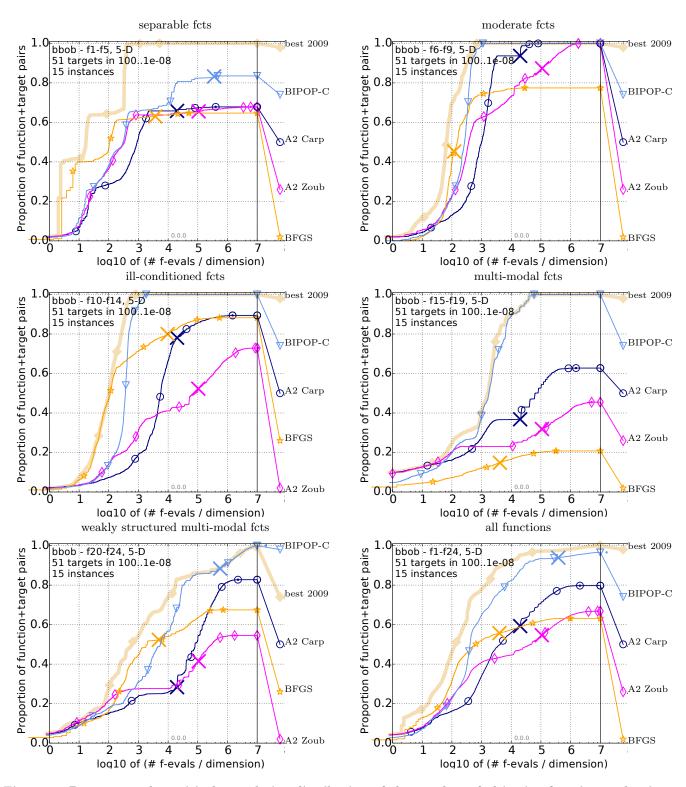


Figure 2: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 51 targets with target precision in  $10^{[-8..2]}$  for all functions and subgroups in 5-D. The "best 2009" line corresponds to the best aRT observed during BBOB 2009 for each selected target.

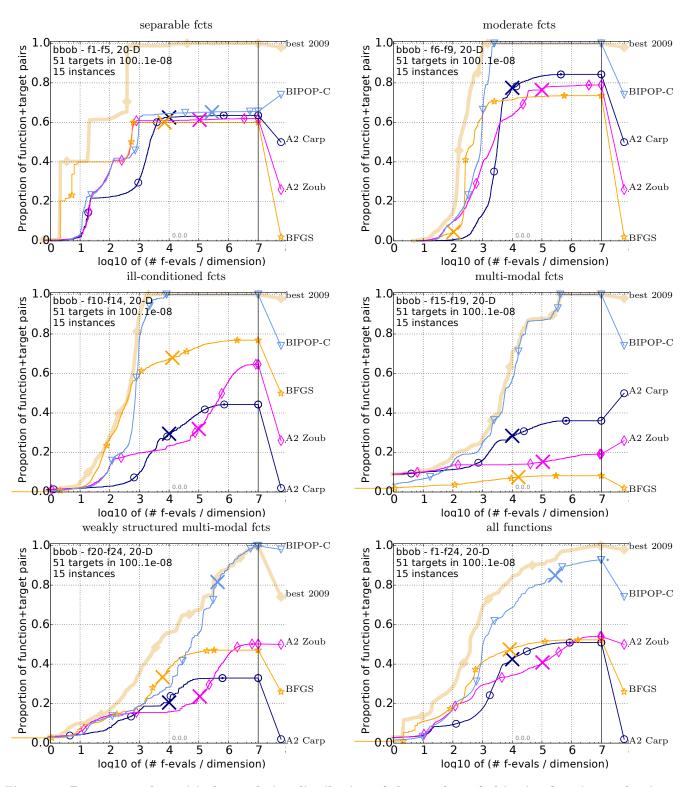


Figure 3: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 51 targets with target precision in  $10^{[-8..2]}$  for all functions and subgroups in 20-D. The "best 2009" line corresponds to the best aRT observed during BBOB 2009 for each selected target.

$\Delta f_{ m opt}$	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ		1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f1	11	12	12	12	12	12	12	15/15	f13	132 15(2)	195	250	319 5)4408(774	1310	1752	2255 ∞ 1e5	15/15 0/15
A2 Carp A2 Zoub		11(6) 7.9(3)	50(7) 15(4)	107(26) 22(5)	167(26) 30(5)	283(24) 45(4)	391(32) 60(9)	$\frac{15}{15}$			613(159)				∞ 61) ∞	∞ 1e5 ∞ 5e5	0/15
BFGS	1.2(0)		4 1.1(0)*4		4 1.1(0)*	4 1.1(0)*	4 1.1(0)*4		BFGS		4 1(0.1)*		*4 <b>1</b> (0.0		1) 136(93		0/15
BIPOP-C			15(3)	21(2)	27(4)	40(3)		15/15	BIPOP-C		5.4(3)	5.9(2)	5.4(0.			0.3) <b>1.4</b> (0.2	
$\Delta f_{ m opt}$	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ	$\Delta f_{ m opt}$	1e1	1e0	1e-1	le-2 1	e-3 1	e-5	1e-7	#succ
f2	83	87	88	89	90	92	94	15/15	f14	10	41	58	90	139	251	476	15/15
A2 Carp A2 Zoub	$25(3) \\ 15(5)$	32(3) $19(7)$	42(6) $21(4)$	53(4)	65(3)	86(4)	103(4) 28(9)	15/15	A2 Carp	1.1(1)						297(386)	3/15
BFGS	3.8(3)	4 <b>5.6</b> (2)	*4 <b>6.2</b> (2)	22(8) *4 <b>6.5</b> (1)*	23(5) *4 <b>6.6</b> (1)**	26(7) 4 <b>6.9</b> (2)*4		15/15 15/15	A2 Zoub BFGS		1.7(1)			5.0(1) 58 1.3(0.4)*4		∞ 6e5	0/15 0/15
BIPOP-C	13(4)	16(3)	18(1)	19(2)	20(2)	21(3)	22(2)	15/15	BIPOP-C		2.8(1)		. ,	4.6(1)	5.4(1)	4.5(0.3)	
$\Delta f_{ m opt}$	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ		11.1(0.5)	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f3	716	1622	1637	1642	1646	1650	1654	15/15	f15	511	9310				20769	21359	14/15
A2 Carp	25(1)	404(38		∞	∞	∞	∞ 1e5	0/15	A2 Carp	36(99)	∞	∞	∞	∞	∞	∞ 1e5	0/15
A2 Zoub BFGS	107(71)	) 4781(63 ∞	90)∞	∞	∞	∞ ∞	$\infty$ 5e5 $\infty$ 2e4	0/15 0/15	A2 Zoub				∞	∞	∞	∞ 5e5	0/15
BIPOP-C		.9) <b>16</b> (11			1) <b>139</b> (563			14/15	BFGS BIPOP-C	87(137) 1.6(0.		∞ □ *4 • •/•	∞ 7) 1.0(0.	∞ 6) <b>1.2</b> (0.5	∞ \ 1.0(0.5)	∞ 2e4	0/15 $15/15$
$\Delta f_{ m opt}$	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ		1	,	1e-1	,	,	) 1.2(0.5) 1e-5	1.2(0.5)	. /
f4	809	163	33 168	8 1758	8 1817	1886	1903	15/15	$\frac{\Delta f_{\text{opt}}}{\mathbf{f} 16}$	1e1 120	1e0 612	2662	1e-2 10163	1e-3 10449	1e-5 11644	1e-7 12095	#succ 15/15
A2 Carp	34(0.8)		∞	∞	∞	∞	∞ 1e5	0/15	A2 Carp	3.8(5)				39(31)	35(47)	34(27)	3/15
A2 Zoub BFGS	170(206 169(147		∞ ∞	∞	∞	∞	∞ 5e5 ∞ 2e4	0/15 0/15	A2 Zoub	1.6(2)	237(357	658(587			) ∞	$\infty$ 5e5	0/15
BIPOP-C			∞	∞	∞	∞	∞ 2e6	0/15	BFGS	153(102)			∞ .	∞	∞ +2 .	∞ 4e4	0/15
$\Delta f_{ m opt}$	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ	BIPOP-C		` '	, ,		.7)*2 <b>1.3</b> (2)			15/15
f5	10	10	10	10	10	10	10	15/15	$\Delta f_{ m opt}$	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
A2 Carp	7.7(4)	10(4)	10(4)	10(5)	10(6)	10(5)	10(4)	15/15	f17 A2 Carp	5.2 1.6(2)	215 1.7(0.	899 9) 2.6(0.3	2861 3) 4.1(0.2	3669 ) 6.0(7)	6351 33(20)	7934 52(107)	15/15 3/15
A2 Zoub	7.3(5)	9.5(3)	9.5(4)	9.5(4)	9.5(5)	9.5(4)	9.5(5) *3 <b>3.1</b> (1)*3	15/15	A2 Zoub	1.9(2)			90(148)	347(591)	∞	∞ 5e5	0/15
BIPOP-C		6.5(2)	6.6(2)	6.6(2)	6.6(2)	6.6(2)		15/15	BFGS	120(203			∞	∞		∞ 2e4	0/15
	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ	BIPOP-C	3.4(3)	1(0.2)	<b>1</b> (2)	<b>1</b> (1)	1(0.6)	*2 <b>1</b> (0.6)*	3 <b>1.2</b> (0.4)	15/15
	114	214	281	404	580	1038	1332	15/15		1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
A2 Carp			7.4(2)	7.9(0.8)		6.0(0.3)	6.3(0.3)	15/15	f18 A2 Carp	103	378 4.0(1)	3968 4.2(0.1	8451 ) 14(15)	9280 44(40)	10905 ∞	12469 ∞ 1e5	15/15
A2 Zoub BFGS	1.7(0.8) 3.0(2)	1.6(0.3 3.3(1)	) 1.8(0.6 3.4(2)	3.0(1.0)		1.2(0.3) 2.0(0.8)	1.3(0.2) 7.8(7)	$\frac{15}{15}$			77(193)	78(123)	298(208)		∞	∞ 1e5 ∞ 5e5	$0/15 \ 0/15$
BIPOP-C					1.7(0.2)			15/15	BFGS	57(96)	∞` ´	∞ ` ´	∞ `´	∞	_ ∞	∞ 2e4	0/15
$\Delta f_{ m opt}$	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ	BIPOP-C	<b>1</b> (0.8)	<b>3.4</b> (3)	<b>1</b> (1)	1(0.4)	, ,	*2 <b>1.2</b> (0.6	)* <b>4.3</b> (0.7)	45/15
			1171	1451	1572	1572	1597	15/15	$\Delta f_{ m opt}$	1e1	1e0	1e-1	1e-	2 1e-3	1e-5	1e-7	#succ
f7	24	324															
A2 Carp	2.9(3)	2.0(2)	7.6(0.3)	19(35)	25(32)	25(32)	25(47)	10/15	f19	1 (0)	1			0e5 1.2ε	5 1.2e5	1.2e5	15/15
	2.9(3)	2.0(2)	7.6(0.3)	19(35) 542(874)	25(32)	25(32)	25(47) )1471(1975)	10/15 3/15	f19 A2 Carp	1(0)	1(0	) 5960	42 1. (8774) ∞ ∞				$\frac{15/15}{0/15}$
A2 Carp A2 Zoub	2.9(3) 3.6(9) 5 ∞	2.0(2)	7.6(0.3) 658(1149)	19(35) 542(874)	25(32) 1112(1394)	25(32)	25(47)	10/15	f19 A2 Carp A2 Zoub BFGS	1(0) 1(0) 1655(12	1(0 1(0 40) 2.2e4(	) 5960 ) ∞ 4e4) 1780	(8774) ∞ ∞ (2389) ∞	0e5 1.2e ∞ ∞ ∞	e5 1.2e5 ∞ ∞ ∞	1.2e5 ∞ 1e5 ∞ 5e5 ∞ 3e4	15/15 0/15 0/15 0/15 0/15
A2 Carp A2 Zoub BFGS BIPOP-C	2.9(3) 3.6(9) 5 ∞	2.0(2) 577(1002) ∞	7.6(0.3) 658(1149) ∞	19(35) 542(874) 1 ∞	$25(32)$ $1112(1394)$ $\infty$	$25(32)$ $1112(1652)$ $\infty$	$25(47)$ $)1471(1975)$ $\infty 600$	10/15 3/15 0/15	f19 A2 Carp A2 Zoub	1(0) 1(0) 1655(12	1(0 1(0 40) 2.2e4(	) 5960 ) ∞ 4e4) 1780	$(8774) \propto \\ \infty \\ (2389) \propto \\ (161) 1(0)$	0e5 1.2e ∞ ∞ ∞	e5 1.2e5 ∞ ∞ ∞	$1.2e5$ $\infty 1e5$ $\infty 5e5$ $\infty 3e4$ $1(0.7)$	15/15 $0/15$ $0/15$ $0/15$ $0/15$ $15/15$
A2 Carp A2 Zoub BFGS BIPOP-0 $\Delta f_{ m opt}$ f8	2.9(3) 3.6(9) 5 \$\infty\$ (5.0(4)] 1e1 73	$ \begin{array}{c} 2.0(2) \\ 577(1002) \\ \infty \\ 1.5(1) \\ 1e0 \\ \hline 273 \end{array} $	7.6(0.3) 658(1149) ∞ 1(1) 1e-1	$19(35)$ $542(874)$ $\infty$ $1(0.2)$ $1e-2$ $372$	$ \begin{array}{c} 25(32) \\ 1112(1394) \\ \infty \\ 1(0.7) \\ \underline{1e-3} \\ 391 \end{array} $	25(32) 1112(1652) ∞ 1(0.9) 1e-5 410	$ \begin{array}{c} 25(47) \\ 1471(1975) \\                                    $	10/15 3/15 0/15 15/15 #succ 15/15	$f19$ A2 Carp A2 Zoub BFGS BIPOP-C $\Delta f_{ m opt}$	1(0) 1(0) 1655(12- 20(18)	1(0 1(0 40) 2.2e4( ) 2801(1 1e0	) 5960 ) ∞ 4e4) 1780 434) <b>161</b> 1e-1	$(8774) \infty$ $\infty$ $(2389) \infty$ $(161) 1(0)$ $1e-2$	0e5 1.2e	5 1.2e5	1.2e5 ∞ 1e5 ∞ 5e5 ∞ 3e4 1(0.7) 1e-7	15/15 0/15 0/15 0/15 0/15 15/15
A2 Carp A2 Zoub BFGS BIPOP-0 $\Delta f_{ m opt}$ f8 A2 Carp	2.9(3) 3.6(9) \$\infty\$ (25.0(4) 1e1 73 7.8(2)	$2.0(2)$ $577(1002)$ $\infty$ $1.5(1)$ $1e0$ $273$ $35(2)$	7.6(0.3) 658(1149) $\infty$ 1(1) 1e-1 336 36(2)	$ \begin{array}{c} 19(35) \\ 542(874) \\ \infty \\ 1(0.2) \\ 1e-2 \\ \hline 372 \\ 37(68) \end{array} $	$ \begin{array}{c} 25(32) \\ 11112(1394) \\ \infty \\ 1(0.7) \\ 1e-3 \\ \hline 391 \\ 37(3) \end{array} $	$ \begin{array}{c} 25(32) \\ 11112(1652) \\ \infty \\ 1(0.9) \\ 1e-5 \\ \hline 410 \\ 42(123) \end{array} $	$ \begin{array}{c} 25(47) \\ )1471(1975) \\                                    $	10/15 3/15 0/15 15/15 #succ 15/15 14/15	$f19$ A2 Carp A2 Zoub BFGS BIPOP-C $\Delta f_{ m opt}$ $f20$	1(0) 1(0) 1655(12- 20(18) 1e1 16	1(0 1(0 40) 2.2e4( ) 2801(1 1e0 851	$\begin{array}{cccc} ) & 5960 \\ ) & \infty \\ 4e4) & 1780 \\ 434) & \textbf{161} \\ \hline & & & \\ \end{array}$	$(8774) \infty$ $\infty$ $(2389) \infty$ $(161) 1(0)$ $\frac{1e-2}{51362}$	0e5 1.2e $\infty$ $\infty$ $\infty$ 1.9) 1(0.9 $\frac{1e-3}{54470}$	1.2e5	$1.2e5$ $\infty 1e5$ $\infty 5e5$ $\infty 3e4$ $1(0.7)$ $1e-7$	15/15 0/15 0/15 0/15 15/15 #succ 14/15
A2 Carp A2 Zoub BFGS BIPOP-O $\Delta f_{ m opt}$ f8 A2 Carp A2 Zoub	2.9(3) 3.6(9) 5.0(4) 1e1 73 7.8(2) 4.0(2)	$2.0(2)$ $577(1002)$ $\infty$ $1.5(1)$ $1e0$ $273$ $35(2)$ $54(86)$	7.6(0.3) 658(1149) ∞ 1(1) 1e-1 336 36(2) 50(12)	$ \begin{array}{c} 19(35) \\ 542(874) \\                                    $	$ \begin{array}{c} 25(32) \\ 1112(1394) \\ \infty \\ 1(0.7) \\ 1e-3 \\ \hline 391 \\ 37(3) \\ 55(69) \end{array} $	$ \begin{array}{c} 25(32) \\ 1112(1652) \\ \infty \\ 1(0.9) \\ 1e-5 \\ 410 \\ 42(123) \\ 64(37) \end{array} $	$ 25(47) )1471(1975) $ $ \approx 600 $ $ 1(0.9) $ $ 1e-7 $ $ 422 $ $ 48(6) $ $ 76(48) $	10/15 3/15 0/15 15/15 #succ 15/15 14/15 15/15	$f19$ A2 Carp A2 Zoub BFGS BIPOP-C $\Delta f_{ m opt}$ $f20$	1(0) 1(0) 1655(12- 20(18) 1e1 16 2.4(2)	1(0 1(0 40) 2.2e4( ) 2801(1 1e0	) 5960 ) ∞ 4e4) 1780 434) <b>161</b> 1e-1	$(8774) \infty$ $\infty$ $(2389) \infty$ $(161) 1(0)$ $1e-2$	0e5 1.2e	5 1.2e5	1.2e5 ∞ 1e5 ∞ 5e5 ∞ 3e4 1(0.7) 1e-7	15/15 0/15 0/15 0/15 0/15 15/15
A2 Carp A2 Zoub BFGS BIPOP-O $\frac{\Delta f_{\mathrm{opt}}}{\mathrm{f8}}$ A2 Carp A2 Zoub BFGS	2.9(3) 3.6(9) 0 5.0(4) 1e1 73 7.8(2) 4.0(2) 2.1(1)	$2.0(2)$ $577(1002)$ $\infty$ $1.5(1)$ $1e0$ $273$ $35(2)$ $54(86)$ $1.8(3)$ *	7.6(0.3) 658(1149)  1(1) 1e-1 336 36(2) 50(12) 2 1.6(2)	$ \begin{array}{c} 19(35) \\ 542(874) \\                                    $	$ \begin{array}{c} 25(32) \\ 1112(1394) \\ \infty \\ 1(0.7) \\ 1e-3 \\ \hline 37(3) \\ 55(69) \\ ^{3} 1.5(0.4) \end{array} $	$ \begin{array}{c} 25(32) \\ 1112(1652) \\                                    $	$25(47)$ $1471(1975)$ $\approx 600$ $1(0.9)$ $1e-7$ $422$ $48(6)$ $76(48)$ $*^3$ $1.5(0.2)^7$	10/15 3/15 0/15 15/15 #succ 15/15 14/15 15/15	$f19$ A2 Carp A2 Zoub BFGS BIPOP-C $\Delta f_{ m opt}$ $f20$ A2 Carp A2 Zoub BFGS	1(0) 1(0) 1655(12- 20(18) 1e1 16 2.4(2) 1.7(1) 1.8(0.9)	$\begin{array}{c} 1(0) \\ 1(0) \\ 4(0) \\ 2.2e4() \\ 2801(1) \\ \hline 1e0 \\ \hline 851 \\ 45(30) \\ 835(790) \\ 2.5(2) \\ \end{array}$	$\begin{array}{cccc} ) & 5960 \\ ) & \infty \\ 4e4) & 1780 \\ 434) & 161 \\ \hline & & & \\ \hline & & & \\ \hline & & & \\ & & & \\ \hline & & & \\ & &$	$(8774) \propto \infty$ $(2389) \propto 0$ $(161) 1(0)$ $1e-2$ $51362$ $27(33) \propto 0$ $7.6(10)$	0e5 1.2e	5 1.2e5	$ \begin{array}{c} 1.2e5 \\                                    $	15/15 0/15 0/15 0/15 15/15 #succ 14/15 1/15 0/15 1/15
A2 Carp A2 Zoub BFGS BIPOP-C $\Delta f_{\mathrm{opt}}$ f8 A2 Carp A2 Zoub BFGS BIPOP-C	2.9(3) 3.6(9) 0 5.0(4) 1e1 73 7.8(2) 4.0(2) 2.1(1)	$2.0(2)$ $577(1002)$ $\infty$ $1.5(1)$ $1e0$ $273$ $35(2)$ $54(86)$	7.6(0.3) 658(1149) ∞ 1(1) 1e-1 336 36(2) 50(12)	$ \begin{array}{c} 19(35) \\ 542(874) \\                                    $	$ \begin{array}{c} 25(32) \\ 1112(1394) \\ \infty \\ 1(0.7) \\ 1e-3 \\ \hline 391 \\ 37(3) \\ 55(69) \end{array} $	$ \begin{array}{c} 25(32) \\ 1112(1652) \\ \infty \\ 1(0.9) \\ 1e-5 \\ 410 \\ 42(123) \\ 64(37) \end{array} $	$25(47)$ $1471(1975)$ $\approx 600$ $1(0.9)$ $1e-7$ $422$ $48(6)$ $76(48)$ $*^3$ $1.5(0.2)^7$	10/15 3/15 0/15 15/15 #succ 15/15 14/15 15/15	$\begin{array}{c} \textbf{f19} \\ \textbf{A2 Carp} \\ \textbf{A2 Zoub} \\ \textbf{BFGS} \\ \textbf{BIPOP-C} \\ \hline \textbf{f20} \\ \textbf{A2 Carp} \\ \textbf{A2 Carp} \\ \textbf{A2 Zoub} \\ \textbf{BFGS} \\ \textbf{BIPOP-C} \end{array}$	$ \begin{vmatrix} 1(0) \\ 1(0) \\ 1655(12 \\ 20(18) \end{vmatrix} $ $ \begin{vmatrix} 1e1 \\ 16 \\ 2.4(2) \\ 1.7(1) \\ 1.8(0.9) \\ 3.3(2) \end{vmatrix} $	1(0 1(0 40) 2.2e4( ) 2801(1 1e0 851 45(30) 835(790) 2.5(2) 8.2(9)	) 5960 ) ∞ 4e4) 1780 434) 161 1e-1 38111 37(37) ∞ 10(10) 2.8(3)	$(8774) \propto \infty$ $(2389) \propto 0$ $(161) 1(0)$ $1e-2$ $51362$ $27(33) \sim 0$ $7.6(10)$ $2.2(1)$	$\begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0.9 \\ 0$	$\begin{array}{cccc} & 1.2e5 \\ & \infty \\ & \infty \\ & \infty \\ & ) & 1(0.7) \\ \hline & 1e-5 \\ \hline & 54861 \\ 26(26) \\ & \infty \\ & & 7.1(5) \\ 2.2(1) \end{array}$	$\begin{array}{c} 1.2e5\\ \infty \ 1e5\\ \infty \ 5e5\\ \infty \ 5e5\\ \infty \ 3e4\\ 1(0.7)\\ 1e-7\\ 55313\\ 25(27)\\ \infty \ 5e5\\ 7.1(8)\\ 2.2(1)\\ \end{array}$	15/15 0/15 0/15 0/15 15/15 #succ 14/15 1/15 0/15 1/15 15/15
A2 Carp A2 Zoub BFGS BIPOP-C $\Delta f_{\mathrm{opt}}$ f8 A2 Carp A2 Zoub BFGS BIPOP-C $\Delta f_{\mathrm{opt}}$ f9	2.9(3) 3.6(9) 5.0(4)  1e1 73 7.8(2) 4.0(2) 2.1(1) 3.2(2)  1e1 35	2.0(2) 577(1002) 1.5(1) 1e0 273 35(2) 54(86) 1.8(3)* 3.7(3) 1e0 127	7.6(0.3) 658(1149)  1(1) 1e-1 336 36(2) 50(12) 2 1.6(2), 4.5(0.6 1e-1 214	$ \begin{array}{c} 19(35) \\ 542(874) \\ \infty \\ 1(0.2) \\ 1e-2 \\ \hline 37(68) \\ 52(122) \\ ^2 \\ 1.5(1)^* \\ ) \\ 4.7(1) \end{array} $	$\begin{array}{c} 25(32) \\ 1112(1394) \\ \infty \\ 1(0.7) \\ 1e-3 \\ \hline 391 \\ 37(3) \\ 55(69) \\ {}^{(3)} 3 \\ 1.5(0.4) \\ 4.8(2) \\ 1e-3 \\ \hline 300 \\ \end{array}$	25(32) 1112(1652)  1(0.9) 1e-5 410 42(123) 64(37) *31.5(0.4) 5.1(4) 1e-5 335	$\begin{array}{c} 25(47) \\ 25(47) \\ )1471(1975) \\ \approx 600 \\ 1(0.9) \\ 1e-7 \\ \hline 422 \\ 48(6) \\ 76(48) \\ *^{3}1.5(0.2) \\ 5.4(3) \\ 1e-7 \\ \hline 369 \\ \end{array}$	10/15 3/15 0/15 15/15 #succ 15/15 14/15 15/15 15/15 15/15 #succ 15/15 #succ 15/15	$\begin{array}{c} \textbf{f19} \\ \text{A2 Carp} \\ \text{A2 Zoub} \\ \text{BFGS} \\ \text{BIPOP-C} \\ \hline \textbf{f20} \\ \text{A2 Carp} \\ \text{A2 Carp} \\ \text{A2 Zoub} \\ \text{BFGS} \\ \text{BIPOP-C} \\ \hline \Delta f_{\text{opt}} \\ \end{array}$	1(0) 1(0) 1655(12- 20(18) 1e1 16 2.4(2) 1.7(1) 1.8(0.9) 3.3(2) 1e1	$\begin{array}{c} 1(0) \\ 1(0) \\ 1(0) \\ 2($	$\begin{array}{cccc} ) & 5960 \\ ) & \infty \\ 4e4) & 1780 \\ 434) & 161 \\ \hline 1e-1 \\ \hline 38111 \\ 37(37) \\ \infty \\ 10(10) \\ 2.8(3) \\ 1e-1 \\ \end{array}$	$\begin{array}{c} (8774) & \infty \\ & \infty \\ (2389) & \infty \\ (161) & 1(0) \\ \hline 1e-2 \\ \hline 51362 \\ 27(33) \\ \infty \\ \hline 7.6(10) \\ 2.2(1) \\ 1e-2 \\ \end{array}$	$\begin{array}{c} 0e5 & 1.2e \\ \infty & \infty \\ \infty & \infty \\ 0.9) & 1(0.9) \\ \hline 1e-3 & \\ \hline 54470 & 26(60) \\ \infty & \\ 7.2(3) & \\ 2.1(0.9) & \\ 1e-3 & \end{array}$	$\begin{array}{cccc} & 1.2e5 \\ & \infty \\ & \infty \\ & \infty \\ & ) & 1(0.7) \\ \hline & 1e-5 \\ \hline & 54861 \\ 26(26) \\ & \infty \\ & 7.1(5) \\ 2.2(1) \\ 1e-5 \end{array}$	$\begin{array}{c} 1.2e5\\ \infty \ 1e5\\ \infty \ 5e5\\ \infty \ 5e5\\ \infty \ 3e4\\ 1(0.7)\\ 1e-7\\ \hline 55313\\ 25(27)\\ \infty \ 5e5\\ 7.1(8)\\ 2.2(1)\\ 1e-7\\ \end{array}$	15/15 0/15 0/15 0/15 15/15 #succ 14/15 1/15 0/15 1/15 1/15 1/15 #succ
A2 Carp A2 Zoub BFGS BIPOP-C $\Delta f_{\mathrm{opt}}$ f8 A2 Carp A2 Zoub BFGS BIPOP-C $\Delta f_{\mathrm{opt}}$ f9 A2 Carp	2.9(3) 3.6(9) 5 5.0(4)  1e1 73 7.8(2) 4.0(2) 2.1(1) 3.2(2)  1e1 35 18(9)	2.0(2) $5.77(1002)$ $0.0(2)$	7.6(0.3) 658(1149)  1(1) 1e-1 336 36(2) 50(12) 2 1.6(2) 4.5(0.6 1e-1 214 22(2)	$\begin{array}{c} 19(35) \\ 542(874) \\ 1542(874) \\ \infty \\ 1(0.2) \\ 1e-2 \\ 372 \\ 37(68) \\ 52(122) \\ ^{-2} \\ 21.5(1)^* \\ 0 \\ 4.7(1) \\ 1e-2 \\ 263 \\ 25(6) \end{array}$	$\begin{array}{c} 25(32) \\ 1112(1394) \\ \infty \\ \hline \\ 1(0.7) \\ 1e-3 \\ \hline 391 \\ 37(3) \\ 55(69) \\ r^{3} \\ 1.5(0.4) \\ 4.8(2) \\ 1e-3 \\ \hline 300 \\ 26(6) \\ \end{array}$	25(32) 1112(1652 1(0.9) 1e-5 410 42(123) 64(37) )*31.5(0.4) 5.1(4) 1e-5 335 31(7)	$\begin{array}{c} 25(47) \\ 25(47) \\ 1471(1975) \\ \approx 600 \\ 1(0.9) \\ 1e-7 \\ 422 \\ 48(6) \\ 76(48) \\ *^3 1.5(0.2) \\ 5.4(3) \\ 1e-7 \\ \hline 369 \\ 34(6) \end{array}$	10/15 ) 3/15 0/15 15/15  #succ 15/15 14/15 15/15 15/15  #succ 15/15  #succ 15/15	$\begin{array}{c} \textbf{f19} \\ \textbf{A2 Carp} \\ \textbf{A2 Zoub} \\ \textbf{BFGS} \\ \textbf{BIPOP-C} \\ \hline & \textbf{42 Carp} \\ \textbf{A2 Carp} \\ \textbf{A2 Carp} \\ \textbf{A2 Carp} \\ \textbf{A2 Zoub} \\ \textbf{BFGS} \\ \textbf{BIPOP-C} \\ \hline & \textbf{42 Dopper Carps} \\ \hline & \textbf{43 Dopper Carps} \\ \hline & \textbf{43 Dopper Carps} \\ \hline & \textbf{44 Dopper Carps} \\ \hline \\ \hline & \textbf{44 Dopper Carps} \\ \hline \\ \hline & \textbf{44 Dopper Carps} \\ \hline \\ $	1(0) 1(0) 1655(12: 20(18] 1e1 16 2.4(2) 1.7(1) 1.8(0.9) 3.3(2) 1e1 41	$\begin{array}{c} 1(0)\\ 1(0)\\ 4(0) \ 2.2e4()\\ 2801(1)\\ 1e0\\ \hline 851\\ 45(30)\\ 835(790)\\ 2.5(2)\\ 8.2(9)\\ 1e0\\ \hline 1157\\ \end{array}$	$\begin{array}{c} ) & 5960 \\ ) & \infty \\ 4e4) & 1780 \\ 434) & 161 \\ \hline & 38111 \\ 37(37) \\ \infty \\ 10(10) \\ 2.8(3) \\ 1e-1 \\ \hline & 1674 \\ \end{array}$	$\begin{array}{c} (8774) & \infty \\ & \infty \\ (2389) & \infty \\ (161) & 1(0) \\ \hline 1e-2 & \\ \hline 51362 \\ 27(33) & \infty \\ & \\ \hline 7.6(10) & \\ 2.2(1) \\ \hline 1e-2 & \\ \hline 1692 & \\ \end{array}$	0e5	1.2e5 0 1(0.7) 1e-5 54861 26(26) 7.1(5) 2.2(1) 1e-5 1729	$\begin{array}{c} 1.2e5 \\ \infty \ 1e5 \\ \infty \ 5e5 \\ \infty \ 5e5 \\ \infty \ 5e5 \\ \infty \ 3e4 \\ 1(0.7) \\ 1e-7 \\ 55313 \\ 25(27) \\ \infty \ 5e5 \\ 7.1(8) \\ 2.2(1) \\ 1e-7 \\ 1757 \end{array}$	15/15 0/15 0/15 0/15 15/15 #succ 14/15 1/15 1/15 1/15 1/15 1/15 1/15 1/15 1/15
A2 Carp A2 Zoub BFGS BIPOP-C $\Delta f_{\mathrm{opt}}$ f8 A2 Carp A2 Zoub BFGS BIPOP-C $\Delta f_{\mathrm{opt}}$ f9 A2 Carp A2 Zoub	$\begin{array}{c} \textbf{2.9(3)} \\ 3.6(9) \\ & \infty \\ \hline & \infty \\ 1e1 \\ \hline & 73 \\ 7.8(2) \\ 4.0(2) \\ \textbf{2.1(1)} \\ 3.2(2) \\ 1e1 \\ \hline & 35 \\ 18(9) \\ 4.5(2) \\ \end{array}$	2.0(2) 577(1002)  1.5(1) 1e0 273 35(2) 54(86) 1.8(3)* 3.7(3) 1e0 127 18(8) 243(646)	7.6(0.3) $658(1149)$ $0.00$ $1(1)$ $1e-1$ $0.00$	$\begin{array}{c} 19(35) \\ 542(874) \\ 10.2) \\ \hline 1(0.2) \\ \hline 1e-2 \\ \hline 37(68) \\ 52(122) \\ 2 \\ 1.5(1)^* \\ ) \\ 4.7(1) \\ 1e-2 \\ \hline 263 \\ 25(6) \\ 133(115) \end{array}$	$\begin{array}{c} 25(32) \\ 1112(1394) \\ \infty \\ 1(0.7) \\ 1e-3 \\ \hline 391 \\ 37(3) \\ 55(69) \\ ^{13} 1.5(0.4 \\ 4.8(2) \\ 1e-3 \\ \hline 300 \\ 26(6) \\ 128(284) \end{array}$	25(32) 1112(1652) 21(0.9) 1e-5 410 42(123) 64(37) *31.5(0.4) 5.1(4) 1e-5 335 31(7) 140(93)	$\begin{array}{c} 25(47) \\ 25(47) \\ 1471(1975) \\ \approx 600 \\ 1(0.9) \\ 1e-7 \\ \hline 422 \\ 48(6) \\ 76(48) \\ *31.5(0.2) \\ 5.4(3) \\ 1e-7 \\ \hline 369 \\ 34(6) \\ 156(416) \\ \end{array}$	10/15 3/15 0/15 15/15 #succ 15/15 14/15 15/15 15/15 #succ 15/15 15/15 15/15 15/15 15/15	$\begin{array}{c} \textbf{f19} \\ \text{A2 Carp} \\ \text{A2 Zoub} \\ \text{BFGS} \\ \text{BIPOP-C} \\ \hline \textbf{f20} \\ \text{A2 Carp} \\ \text{A2 Carp} \\ \text{A2 Zoub} \\ \text{BFGS} \\ \text{BIPOP-C} \\ \hline \textbf{521} \\ \text{A2 Carp} \\ \text{A2 Carp} \end{array}$	1(0) 1(0) 1655(12: 20(18) 1e1 16 2.4(2) 1.7(1) 1.8(0.9) 3.3(2) 1e1 41 2.0(2)	$\begin{array}{c} 1(0) \\ 1(0) \\ 1(0) \\ 2($	$\begin{array}{cccc} ) & 5960 \\ ) & \infty \\ 4e4) & 1780 \\ 434) & 161 \\ \hline 1e-1 \\ \hline 38111 \\ 37(37) \\ \infty \\ 10(10) \\ 2.8(3) \\ 1e-1 \\ \end{array}$	$\begin{array}{c} (8774) & \infty \\ & \infty \\ (2389) & \infty \\ (161) & 1(0) \\ \hline 1e-2 \\ 51362 \\ 27(33) & \infty \\ & 7.6(10) \\ 2.2(1) \\ \hline 1e-2 \\ \hline 1692 \\ 828(931) \\ \end{array}$	$\begin{array}{c} 0e5 & 1.2e \\ \infty & \infty \\ \infty & \infty \\ \infty & \infty \\ 0.9) & 1(0.9 \\ 1e-3 & 54470 \\ 26(60) & \infty \\ \infty & 7.2(3) \\ 2.1(0.9) & 1e-3 \\ 1705 & 822(836) \\ 422(495) & 1.2e \\ \end{array}$	$\begin{array}{cccc} & 1.2e5 \\ & \infty \\ & \infty \\ & \infty \\ & ) & 1(0.7) \\ \hline & 1e-5 \\ \hline & 54861 \\ 26(26) \\ & \infty \\ & 7.1(5) \\ 2.2(1) \\ 1e-5 \end{array}$	$\begin{array}{c} 1.2e5\\ \infty \ 1e5\\ \infty \ 5e5\\ \infty \ 5e5\\ \infty \ 3e4\\ 1(0.7)\\ 1e-7\\ \hline 55313\\ 25(27)\\ \infty \ 5e5\\ 7.1(8)\\ 2.2(1)\\ 1e-7\\ \end{array}$	15/15 0/15 0/15 0/15 15/15 #succ 14/15 1/15 0/15 1/15 1/15 1/15 #succ
A2 Carp A2 Zoub BFGS BIPOP-C $\frac{\Delta f_{\mathrm{opt}}}{f8}$ A2 Carp A2 Zoub BFGS BIPOP-C $\frac{\Delta f_{\mathrm{opt}}}{f9}$ A2 Carp A2 Carp BFGS BIPOP-C $\frac{\Delta f_{\mathrm{opt}}}{f9}$ A2 Carp BFGS	$\begin{array}{c} \textbf{2.9(3)} \\ 3.6(9) \\ \infty \\ 5.0(4) \\ \hline \\ \textbf{1e1} \\ \hline \\ 73 \\ 4.0(2) \\ \textbf{2.1(1)} \\ 3.2(2) \\ \hline \\ \textbf{1e1} \\ \hline \\ 35 \\ 18(9) \\ 4.5(2) \\ \textbf{3.6(3)} \\ \end{array}$	2.0(2) 577(1002) 1.5(1) 1e0 273 35(2) 54(86) 1.8(3) 3.7(3) 1e0 127 18(8) 243(646) 3.0(0.	7.6(0.3) $658(1149)$ $0.56(0.3)$ $1(1)$ $1e-1$ $1e-1$ $16e-1$ $16e-$	$\begin{array}{c} 19(35) \\ 542(874) \\ 10(2) \\ \hline \\ 1(0.2) \\ \hline \\ 1e-2 \\ \hline \\ 37(68) \\ 52(122) \\ 2 \\ 1.5(1)^* \\ ) \\ 4.7(1) \\ 1e-2 \\ \hline \\ 256 \\ 133(115) \\ *3 \\ 1.8(6). \end{array}$	$\begin{array}{c} 25(32) \\ 1112(1394) \\ \infty \\ 1(0.7) \\ 1e-3 \\ 37(3) \\ 55(69) \\ 3 \\ 1.5(0.4) \\ 4.8(2) \\ 1e-3 \\ \hline 300 \\ 26(6) \\ 128(284) \\ 7)^{*3} 1.6(0.5) \\ \end{array}$	25(32) 1112(1652 ~ 1(0.9) 1e-5 410 42(123) 64(37) *31.5(0.4) 5.1(4) 1e-5 335 31(7) 140(93) 5) *31.5(0.8)	$25(47)$ $25(47)$ $1471(1975)$ $\infty 600$ $1(0.9)$ $1e-7$ $422$ $48(6)$ $76(48)$ $*3.5(0.2)$ $5.4(3)$ $1e-7$ $369$ $34(6)$ $156(416)$ $8)*4.4(0.6)$	10/15 3/15 0/15 15/15 #succ 15/15 14/15 15/15 15/15 15/15 #succ 15/15 15/15 15/15 15/15 15/15 15/15	19 A2 Carp A2 Zoub BFGS BIPOP-C Δfopt f20 A2 Carp A2 Zoub BFGS BIPOP-C Δfopt f21 A2 Carp A2 Zoub BFGS	1(0) 1(0) 1(0) 1655(12- 20(18) 1e1 16 2.4(2) 1.7(1) 1.8(0.9) 3.3(2) 1e1 41 2.0(2) 93(0.7) 3.8(4)	$\begin{array}{c} 1 \\ 1 \\ 0 \\ 10 \\ 0 \\ 20 \\ 10 \\ 10 \\ 10 $	$\begin{array}{cccc} ) & 5960 \\ ) & \infty \\ 4e4) & 1780 \\ 434) & 161 \\ \hline & 1e-1 \\ 38111 \\ 37(37) \\ \infty \\ 10(10) \\ 2.8(3) \\ 1e-1 \\ \hline & 1674 \\ 836(926) \\ 430(445) \\ 1.9(3) \end{array}$	$\begin{array}{c} (8774) & \infty \\ \infty \\ (2389) & \infty \\ (161) & 1(0) \\ \hline 1e-2 \\ 51362 \\ 27(33) \\ \infty \\ 7.6(10) \\ 2.2(1) \\ \hline 1e-2 \\ \hline 1692 \\ 828(931) \\ 425(544) \\ 1.9(3) \end{array}$	$\begin{array}{c} 0e5 & 1.2e \\ \infty & \infty \\ \infty & \infty \\ \infty & \infty \\ 0.9) & 1(0.9) \\ 1e-3 & \\ 54470 & \\ 26(60) & \infty \\ 0.7.2(3) & \\ 2.1(0.9) & \\ 1e-3 & \\ 1705 & 822(836) \\ 422(495) & \\ 1.9(2) & \\ \end{array}$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{c} 1.2e5\\ \infty \ le5\\ 10.7)\\ 1e-7\\ 55313\\ 25(27)\\ \infty \ le5\\ 5e5\\ 7.1(8)\\ 2.2(1)\\ 1e-7\\ 1757\\ 798(512)\\ 410(534)\\ 2.0(2)\\ \end{array}$	15/15 0/15 0/15 0/15 15/15 #succ 14/15 1/15 1/15 15/15 #succ 14/15 1/15 1/15 1/15 1/15 1/15 1/15
A2 Carp A2 Zoub BFGS BIPOP-C $\frac{\Delta f_{\rm opt}}{\rm f8}$ A2 Carp A2 Zoub BFGS BIPOP-C $\frac{\Delta f_{\rm opt}}{\rm f9}$ A2 Carp A2 Zoub BFGS BIPOP-C BIPOP-C	$\begin{array}{c} \textbf{2.9(3)} \\ 3.6(9) \\ \infty \\ 5.0(4) \\ \hline \\ \textbf{1e1} \\ \hline \\ 73 \\ 4.0(2) \\ \textbf{2.1(1)} \\ 3.2(2) \\ \hline \\ \textbf{1e1} \\ \hline \\ 35 \\ 18(9) \\ 4.5(2) \\ \textbf{3.6(3)} \\ \end{array}$	$2.0(2)$ $2.0(2)$ $577(1002)$ $\infty$ $1.5(1)$ $1e0$ $273$ $35(2)$ $54(86)$ $1.8(3)^*$ $3.7(3)$ $1e0$ $127$ $18(8)$ $243(646)$ $3.0(0.8.7(4))$	7.6(0.3) $658(1149)$ $0.56(0.3)$ $1(1)$ $1e-1$ $1e-1$ $16e-1$ $16e-$	$\begin{array}{c} 19(35) \\ 542(874) \\ 10(2) \\ \hline \\ 1(0.2) \\ \hline \\ 1e-2 \\ \hline \\ 37(68) \\ 52(122) \\ 2 \\ 1.5(1)^* \\ ) \\ 4.7(1) \\ 1e-2 \\ \hline \\ 256 \\ 133(115) \\ *3 \\ 1.8(6). \end{array}$	$\begin{array}{c} 25(32) \\ 1112(1394) \\ \infty \\ \hline \\ 1(0.7) \\ 1e-3 \\ \hline \\ 37(3) \\ 55(69) \\ r^3 \\ 1.5(0.4) \\ 4.8(2) \\ 1e-3 \\ \hline \\ 300 \\ 26(6) \\ 128(284) \\ 7)^* ^3 1.6(0.4) \\ 6.4(4) \\ \end{array}$	25(32) 1112(1652) 21(0.9) 1e-5 410 42(123) 64(37) *31.5(0.4) 5.1(4) 1e-5 335 31(7) 140(93)	$25(47)$ $25(47)$ $1471(1975)$ $\infty 600$ $1(0.9)$ $1e-7$ $422$ $48(6)$ $76(48)$ $*3.5(0.2)$ $5.4(3)$ $1e-7$ $369$ $34(6)$ $156(416)$ $8)*4.4(0.6)$	10/15 3/15 0/15 15/15 #succ 15/15 14/15 15/15 15/15 15/15 15/15 15/15 15/15 15/15 15/15 15/15	19 A2 Carp A2 Zoub BFGS BIPOP-C Δfopt f20 A2 Carp A2 Zoub BFGS BIPOP-C Δfopt f21 A2 Carp A2 Zoub BFGS BIPOP-C Δfopt β190 β190 β190 β190 β190 β190 β190 β190	1(0) 1(0) 1655(12: 20(18) 1e1 16 2.4(2) 1.7(1) 1.8(0.9) 3.3(2) 1e1 41 2.0(2) 93(0.7) 3.8(4) 2.3(2)	$\begin{array}{c} 1(0)\\ 1(0)\\ 4(0) & 2.2e4(\\ ) & 2801(1)\\ \hline 1e0\\ \hline 851\\ 45(30)\\ 835(790)\\ 2.5(2)\\ 8.2(9)\\ \hline 1e0\\ \hline 1157\\ 562(821)\\ 217(192)\\ 1.4(2)\\ 14(6)\\ \end{array}$	$\begin{array}{c} )  5960 \\ )  \infty \\ 4e4)  1780 \\ 434)  161 \\ \hline 1e-1 \\ 38111 \\ 37(37) \\ \infty \\ 10(10) \\ 2.8(3) \\ 1e-1 \\ \hline 1674 \\ 836(926) \\ 430(445) \\ 1.9(3) \\ 24(74) \\ \end{array}$	$\begin{array}{c} (8774) & \infty \\ \infty \\ (2389) & \infty \\ (161) & 1(0) \\ \hline 1e-2 \\ 51362 \\ 27(33) \\ \infty \\ 7.6(10) \\ 2.2(1) \\ 1e-2 \\ 1692 \\ 828(931) \\ 425(544) \\ 1.9(3) \\ 25(119) \\ \end{array}$	$\begin{array}{c} 0e5 & 1.2e \\ \infty & \infty \\ \infty & \infty \\ \infty & \infty \\ 0.9) & 1(0.9 \\ 1e-3 \\ \hline 54470 \\ 26(60) & \infty \\ 7.2(3) \\ 2.1(0.9) \\ 1e-3 \\ \hline 1705 \\ 822(836) \\ 422(495) \\ 1.9(2) \\ 25(21) \\ \end{array}$	$\begin{array}{cccc} & 1.2e5 \\ & \infty \\ & \infty \\ & \infty \\ & & \infty \\ & & & \times \\ & \times \\ & & \times \\ & & \times \\ & \times $	$\begin{array}{c} 1.2e5\\ \infty \ le5\\ \infty \ le4\\ 1(0.7)\\ 1e-7\\ 55313\\ 25(27)\\ \infty \ le6\\ 7.1(8)\\ 2.2(1)\\ 1e-7\\ 1757\\ 798(512)\\ 410(534)\\ 2.0(2)\\ 25(20)\\ \end{array}$	15/15 0/15 0/15 0/15 15/15 #succ 14/15 1/15 1/15 15/15 #succ 14/15 1/15 1/15 1/15 1/15 1/15 1/15 1/15 1/15
A2 Carp A2 Zoub BFGS BIPOP-C $\frac{\Delta f_{\mathrm{opt}}}{f8}$ A2 Carp A2 Zoub BFGS BIPOP-C $\frac{\Delta f_{\mathrm{opt}}}{f9}$ A2 Carp A2 Carp BFGS BIPOP-C $\frac{\Delta f_{\mathrm{opt}}}{f9}$ A2 Carp BFGS	2.9(3) 3.6(9) 3.6(9) 1.10	2.0(2) 577(1002) 1.5(1) 1e0 273 35(2) 54(86) 1.8(3) 3.7(3) 1e0 127 18(8) 243(646) 3.0(0.	$7.6(0.3)$ $658(1149)$ $\infty$ $1(1)$ $1e-1$ $336$ $36(2)$ $50(12)$ $^{\prime 2}$ $1.6(2)^{\prime 4}$ $4.5(0.6)$ $1e-1$ $214$ $22(2)$ $1.50(75)$ $7)^{*2}2.0(1)$ $7.2(2)$	$\begin{array}{c} 19(35) \\ 542(874) & 1 \\ 542(874) & 1 \\ \infty \\ \hline & 1(0.2) \\ 1e-2 \\ \hline & 372 \\ 37(68) \\ 52(122) \\ ^2 & 1.5(1)^* \\ ) & 4.7(1) \\ 1e-2 \\ \hline & 263 \\ 25(6) \\ 33(115) \\ *3 & 1.8(0.6.7(5)) \end{array}$	$\begin{array}{c} 25(32) \\ 1112(1394) \\ \infty \\ 1(0.7) \\ 1e-3 \\ 37(3) \\ 55(69) \\ 3 \\ 1.5(0.4) \\ 4.8(2) \\ 1e-3 \\ \hline 300 \\ 26(6) \\ 128(284) \\ 7)^{*3} 1.6(0.5) \\ \end{array}$	$25(32)$ $1112(1652)$ $1(0.9)$ $1e-5$ $410$ $42(123)$ $64(37)$ $*3_1.5(0.4)$ $5.1(4)$ $1e-5$ $335$ $31(7)$ $140(93)$ $5)*3_1.5(0.8)$ $6.3(4)$	25(47) 1471(1975) 2600 1(0.9) 1e-7 422 48(6) 76(48) *31.5(0.2) 5.4(3) 1e-7 369 34(6) 156(416) 6.2(5) 1e-7 880	10/15 3/15 0/15 15/15 #succ 15/15 14/15 15/15 15/15 15/15 #succ 15/15 15/15 15/15 15/15 15/15 15/15	$f19$ A2 Carp A2 Zoub BFGS BIPOP-C $\frac{\Delta f_{\rm opt}}{f20}$ A2 Carp A2 Zoub BFGS BIPOP-C $\frac{\Delta f_{\rm opt}}{f21}$ A2 Carp A2 Zoub BFGS BIPOP-C $\frac{\Delta f_{\rm opt}}{f21}$ A2 Carp A2 Zoub BFGS BIPOP-C $\Delta f_{\rm opt}$	1(0) 1(0) 1655(12. 20(18 1e1 16 2.4(2) 1.7(1) 1.8(0.9) 3.3(2) 1e1 2.0(2) 93(0.7) 3.8(4) 2.3(2) 1e1	$\begin{array}{c} 1(0\\ 1(0)\\ 4(0) \ 2.2e4(\\ ) \ 2801(1\\ 1e0\\ \hline 851\\ 45(30)\\ 835(790)\\ 2.5(2)\\ 8.2(9)\\ 1e0\\ \hline 1157\\ 562(821)\\ 217(192)\\ 1.4(2)\\ 14(6)\\ 1e0\\ \end{array}$	$\begin{array}{c} )  5960 \\ )  \infty \\ 4e4)  1780 \\ 434)  161 \\ \hline 1e-1 \\ 38111 \\ 37(37) \\ \infty \\ 10(10) \\ 2.8(3) \\ 1e-1 \\ \hline 1674 \\ 836(926) \\ 430(445) \\ 1.9(3) \\ 24(74) \\ 1e-1 \end{array}$	$\begin{array}{c} (8774) & \infty \\ (2389) & \infty \\ (161) & 1(0) \\ \hline 1e-2 & \\ 51362 \\ 27(33) & \infty \\ & \\ & \\ 7.6(10) \\ 2.2(1) \\ \hline 1e-2 & \\ 1692 \\ 828(931) \\ 425(544) \\ 1.9(3) \\ 25(119) \\ 1e-2 & \\ \end{array}$	$\begin{array}{c} 0e5 & 1.2e \\ \infty \\ \infty \\ \infty \\ \infty \\ 0.9) & 1(0.9 \\ 1e-3 \\ \hline 54470 \\ 26(60) \\ \infty \\ 7.2(3) \\ 2.1(0.9) \\ 1e-3 \\ \hline 1705 \\ 822(836) \\ 422(495) \\ 1.9(2) \\ 25(21) \\ 1e-3 \\ \end{array}$	$\begin{array}{cccc} & & & & & & \\ & & & & & \\ & & & & \\ & & & & \\ & & & $	$\begin{array}{c} 1.2e5\\ \infty \ le5\\ 10.7\\ 1e-7\\ 1757\\ 18\\ 2.2(1)\\ 1e-7\\ 1757\\ 798(512)\\ 410(534)\\ 2.0(2)\\ 25(20)\\ 1e-7\\ \end{array}$	15/15 0/15 0/15 15/15 15/15 #succ 14/15 1/15 1/15 15/15 #succ 14/15 11/15 15/15 1/
A2 Carp A2 Zoub BFGS BIPOP-C $\frac{\Delta f_{\rm opt}}{\rm f8}$ A2 Carp A2 Zoub BFGS BIPOP-C $\frac{\Delta f_{\rm opt}}{\rm f9}$ A2 Carp A2 Zoub BFGS BIPOP-C $\frac{\Delta f_{\rm opt}}{\rm f10}$ A2 Carp A2 Carp A2 Zoub BFGS BIPOP-C $\frac{\Delta f_{\rm opt}}{\rm f10}$ A2 Carp A2 Carp	2.9(3) 3.6(9) 5.0(4) le1 73 7.8(2) 4.0(2) 2.1(1) 3.2(2) le1 35 18(9) 4.5(2) 3.6(3) 5.8(1) le1 3.4(3) 4.5(2) 4.0(2) 4.5(2)	$\begin{array}{c} 2.0(2) \\ 577(1002) \\ \infty \\ 1.5(1) \\ 1e0 \\ 273 \\ 35(2) \\ 54(86) \\ 1.8(3)^* \\ 3.7(3) \\ 1e0 \\ 127 \\ 18(8) \\ 243(646) \\ 3.0(0.) \\ 8.7(4) \\ 1e0 \\ \hline 500 \\ 42(18) \end{array}$	$\begin{array}{c} 7.6(0.3) \\ 658(1149) \\ \infty \\ 1(1) \\ 1e^{-1} \\ 336 \\ 36(2) \\ 50(12) \\ 2 \\ 1.6(2)^4 \\ 4.5(0.6 \\ 1e^{-1} \\ 214 \\ 22(2) \\ 150(75) \\ 7)^{\star 2} 2.0(1) \\ 7.2(2) \\ 1e^{-1} \\ 574 \\ 43(14) \end{array}$	$\begin{array}{l} 19(35) \\ 542(874) \ \ 1\\ 542(874) \ \ 1\\  \\ \infty \\ \hline & 1(0.2) \\ 1e-2 \\ \hline & 372 \\ 37(68) \\ 52(122) \\ 2 \ \ 1.5(1)^3 \\ ) \ \ 4.7(1) \\ 1e-2 \\ \hline & 263 \\ 25(6) \\ 133(115) \\ ^{*3} \ \ 1.8(0.6.7(5) \\ 1e-2 \\ \hline & 607 \\ 49(16) \end{array}$	$\begin{array}{c} 25(32) \\ 1112(1394) \\ & \\ & \\ \hline \times \\ 1(0.7) \\ 1e-3 \\ \hline 3911 \\ 37(3) \\ 37(3) \\ 37(3) \\ 37(3) \\ 31.5(0.4) \\ 4.8(2) \\ 1e-3 \\ \hline 300 \\ 26(6) \\ 128(284) \\ 7)^{\star} 3_{1}.6(0.4) \\ 6.4(4) \\ 1e-3 \\ \hline 626 \\ 56(15) \\ \end{array}$	$\begin{array}{c} 25(32) \\ 25(32) \\ 1112(1652) \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ $	$\begin{array}{c} 25(47) \\ 25(47) \\ 1)1471(1975) \\ \infty 600 \\ 1(0.9) \\ 1e-7 \\ 422 \\ 48(6) \\ 76(48) \\ *31.5(0.2)' \\ 5.4(3) \\ 1e-7 \\ 34(6) \\ 156(416) \\ 8)^{*4}1.4(0.6] \\ 6.2(5) \\ 1e-7 \\ 880 \\ 68(20) \end{array}$	10/15 3/15 0/15 15/15 #succ 15/15 14/15 15/15 15/15 15/15 15/15 15/15 15/15 15/15 15/15 15/15 15/15 15/15 15/15	19 A2 Carp A2 Zoub BFGS BIPOP-C Δfopt f20 A2 Carp A2 Zoub BFGS BIPOP-C Δfopt f21 A2 Carp A2 Zoub BFGS BIPOP-C Δfopt A2 Carp A2 Coub BFGS BIPOP-C Δfopt f21 A2 Carp A2 Toub	1(0) 1(0) 1655(12.2) 20(18) 1e1 16 2.4(2) 1.7(1) 1.8(0.9) 3.3(2) 1e1 41 2.0(2) 93(0.7) 3.8(4) 2.3(2) 1e1 1e1	$\begin{array}{c} 1(0)\\ 1(0)\\ 1(0)\\ 2(0)\\$	$\begin{array}{c} )  5960 \\ \infty \\ 4e4)  1780 \\ 434)  161 \\ \hline 1e-1 \\ \hline 38111 \\ 37(37) \\ \infty \\ \hline 10(10) \\ 2.8(3) \\ \hline 1e-1 \\ \hline 1674 \\ 836(926) \\ 1.9(3) \\ 24(74) \\ \hline 1e-1 \\ \hline 938 \\ \end{array}$	$\begin{array}{c} (8774) & \infty \\ (2389) & \infty \\ (161) & 1(0) \\ \hline 1e-2 \\ \hline 51362 \\ 27(33) \\ \infty \\ \hline 7.6(10) \\ 2.2(1) \\ \hline 1e-2 \\ 1692 \\ 828(931) \\ 425(544) \\ 1.9(3) \\ 25(119) \\ 1e-2 \\ \hline 980 \end{array}$	$\begin{array}{c} 0e5 & 1.2e\\ & & \\$	$\begin{array}{c} & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ &$	$\begin{array}{c} 1.2e5\\ \infty \ leb\\ \infty \ feb\\ 0.7)\\ 1e-7\\ 55313\\ 25(27)\\ \infty \ feb\\ 0.7.1(8)\\ 2.2(1)\\ 1e-7\\ 1757\\ 798(512)\\ 410(534)\\ 2.0(2)\\ 25(20)\\ 1e-7\\ 1068\\ \end{array}$	15/15 0/15 0/15 0/15 15/15 #succ 14/15 1/
A2 Carp A2 Zoub BFGS BIPOP-C $\frac{f_{\text{opt}}}{f8}$ A2 Carp A2 Zoub BFGS BIPOP-C $\frac{\Delta f_{\text{opt}}}{f9}$ A2 Carp A2 Zoub BFGS BIPOP-C $\frac{\Delta f_{\text{opt}}}{f10}$ A2 Carp A2 Zoub	2.9(3) 3.6(9) 73 7.8(2) 4.0(2) 2.1(1) 3.2(2) 18(9) 4.5(2) 3.6(3) 5.8(1) 1et 349 42(18) 241(907)	$2.0(2)$ $577(1002)$ $\infty$ $1.5(1)$ $1e0$ $273$ $35(2)$ $54(86)$ $1.8(3)^*$ $3.7(3)$ $1e0$ $127$ $18(8)$ $243(646)$ $3.0(0)$ $8.7(4)$ $1e0$ $500$ $42(88)$ $679(88)$	$\begin{array}{c} 7.6(0.3) \\ 658(149) \\ \infty \\ 1(1) \\ 1e-1 \\ 336 \\ 36(2) \\ 50(12) \\ ^2 1.6(2)^4 \\ 4.5(0.6 \\ 1e-1 \\ 22(2) \\ 1.50(75) \\ 7)^{\star 2} 2.0(1) \\ 7.2(2) \\ 1e-1 \\ 574 \\ 43(14) \\ 1446(119) \end{array}$	$\begin{array}{c} 19(35) \\ 542(874) \ : \\ \hline 10(2) \\ \hline 10($	$\begin{array}{c} 25(32) \\ 1112(1394) \\ \hline & 1(0.7) \\ 1e-3 \\ \hline & 391 \\ 37(3) \\ 55(69) \\ ^3 \\ 1.5(0.4) \\ 4.8(2) \\ 1e-3 \\ \hline & 300 \\ 26(6) \\ 128(284) \\ 7)^* ^3 1.6(0.4) \\ 6.4(4) \\ 1e-3 \\ \hline & 626 \\ 56(15) \\ 35960(1e4) \end{array}$	25(32) 1112(1652) 1(0.9) 1e-5 410 42(123) 64(37) *3.15(0.4) 5.1(4) 1e-5 335 31(7) 140(93) 35) *3.5(0.4) 6.3(4) 1e-5 829 56(14) 4513(762)	25(47) 25(47) 1471(1975) 600 1(0.9) 1e-7 422 48(6) 76(48) *3.1.5(0.2)* 5.4(3) 1e-7 369 34(6) 156(416) 8)* <sup>4</sup> 1.4(0.6) 6.2(5) 1e-7 880 68(20) 39469(7777	10/15 3/15 3/15 15/15	719 A2 Carp A2 Zoub BFGS BIPOP-C Δfopt 720 A2 Carp A2 Carp A2 Zoub BFGS BIPOP-C Δfopt 721 A2 Carp A2 Zoub BFGS BIPOP-C Δfopt 721 A2 Carp A2 Zoub A2 Zoub A2 Zoub	1(0) 1(0) 1655(12: 20(18) 1e1 16 2.4(2) 1.7(1) 1.8(0.9) 3.3(2) 1e1 41 2.0(2) 93(0.7) 3.8(4) 2.3(2) 1e1 71 218(705) 170(228)	1(0 1(0 40) 2.2e4(2) 2801(1 1e0 851 45(30) 835(790) 2.5(2) 8.2(9) 1e0 1157 562(821) 217(192) 1.4(2) 1e0 386) 386) 386)	$\begin{array}{c} )  5960 \\ )  \infty \\ 4e4)  1780 \\ 434)  161 \\ 1e-1 \\ 38111 \\ 37(37) \\ \infty \\ 10(10) \\ 2.8(3) \\ 1e-1 \\ 1674 \\ 836(926) \\ 1.9(3) \\ 24(74) \\ 1e-1 \\ 938 \\ 214(240) \\ 304(431) \end{array}$	$\begin{array}{ll} (8774) & \infty \\ (2389) & \infty \\ (161) & 1(0) \\ \hline 1e-2 & \\ 51362 \\ 27(33) & \infty \\ \hline & 7.6(10) \\ 2.2(1) \\ 1e-2 & \\ \hline 1692 \\ 828(931) \\ 425(544) \\ 1.9(3) \\ 25(119) \\ 1e-2 & \\ \hline 980 \\ 206(230) \\ 292(95) & \\ \end{array}$	0e5 1.2e  ∞  ∞  0e7  10e3  54470  26(60)  ∞  7.2(3)  2.1(0.9)  1e-3  1705  822(836)  422(495)  1.9(2)  1e-3  1008  200(174)  284(371)	$\begin{array}{c} 5 & 1.2e5 \\ & \infty \\ & 10.7 \\ & 1e-5 \\ & 54861 \\ 26(26) \\ & \infty \\ & 7.1(5) \\ & 2.2(1) \\ & 1e-5 \\ & 1729 \\ 811(795) \\ & 416(663) \\ & 1.9(3) \\ & 2.9(3) \\ & 1.9(3) \\ & 1.9(3) \\ & 1.9(3) \\ & 1.9(3) \\ & 2.7(3)(6) \\ & 1.9(3) \\ & 2.7(3)(6) \\ & 1.9(3) \\ & 2.7(3)(6) \\ & 1.9(3) \\ & 2.7(3)(6) \\ & 1.9(3) \\ & 2.7(3)(6) \\ & 1.9(3) \\ & 1$	$\begin{array}{c} 1.2e5\\ \infty \ leb\\ \infty \ feb\\ 10.7)\\ 1e-7\\ \hline 55313\\ 25(27)\\ \infty \ feb\\ 7.1(8)\\ 2.2(1)\\ 1e-7\\ \hline 1757\\ 798(512)\\ 410(532)\\ 2.0(2)\\ 2.5(20)\\ 1e-7\\ \hline 1068\\ 19(234)\\ 269(734)\\ \end{array}$	15/15 0/15 0/15 0/15 15/15 #succ 14/15 1/15 15/15 #succ 14/15 15/15 8/15 15/15 8/15 15/15 #succ 14/15 15/15 15/15 15/15 15/15 15/15
A2 Carp A2 Zoub BFGS BIPOP-C	2.9(3) 3.6(9) 5.0(4) le1 73 7.8(2) 4.0(2) 2.1(1) 3.2(2) le1 35 18(9) 4.5(2) 3.6(3) 5.8(1) le1 349 42(18) 241(907) 1(0.3)	$\begin{array}{c} 2.0(2) \\ 577(1002) \\ \infty \\ 1.5(1) \\ 1e0 \\ 273 \\ 35(2) \\ 54(86) \\ 1.8(3)^4 \\ 3.7(3) \\ 1e0 \\ 127 \\ 18(8) \\ 243(646) \\ 3.0(0) \\ 8.7(4) \\ 1e0 \\ 500 \\ 42(18) \\ 0679(850) \\ ^*4 \\ 1(0.3) \end{array}$	$\begin{array}{c} 7.6(0.3) \\ 658(1149) \\ \infty \\ 1(1) \\ 1e-1 \\ 336 \\ 36(2) \\ 50(12) \\ ^2 1.6(2)^4 \\ 4.5(0.6 \\ 1e-1 \\ 22(2) \\ 1) 150(75) \\ 7)^{*2} 2.0(1) \\ 7.2(2) \\ 1e-1 \\ 574 \\ 43(14) \\ 11446(119) \\ ^*4 1(0.2) \end{array}$	$\begin{array}{l} 19(35) \\ 542(874) \ \ 1\\ 542(874) \ \ 1\\ \hline \\ 10.2) \\ 1 \\ 10.2) \\ 1 \\ 10.2) \\ 37(68) \\ 52(122) \\ 2\\ 37(68) \\ 52(122) \\ 2\\ 1.5(1)^4 \\ 1.47(1) \\ 1 \\ 10.2 \\ 263 \\ 25(6) \\ 133(115) \\ *3\\ 1.8(0.6.7(5)) \\ 10.2 \\ 607 \\ 49(16) \\ 98(14(623)^{*4} \ \ 1(0.2)^{*4} \\ 1(0.2) \\ \end{array}$	$\begin{array}{c} 25(32) \\ 1112(1394) \\ \hline \times \\ 1(0.7) \\ 1e-3 \\ \hline \times \\ 391 \\ 37(3) \\ 55(69) \\ 3 \\ 1.5(0.4 \\ 4.8(2) \\ 1e-3 \\ \hline 300 \\ 26(6) \\ 128(284) \\ 7)^*3_{1.6}(0.1 \\ 6.4(4) \\ 1e-3 \\ \hline 626 \\ 56(15) \\ 339960(1e4) \\ *^4 \\ 1(0.4) \\ \end{array}$	25(32) 1112(1652) 1(0.9) 1e-5 410 42(123) 64(37) )*3.1.5(0.4) 5.1(4) 1e-5 337 31(7) 1440(93) 5)*31.5(0.4) 6.3(4) 1e-5 829 56(14) )4513(762) *4 1.1(0.6)	25(47) 1471(1975) 25(47) 1471(1975) 2600 1(0.9) 1e-7 422 48(6) 76(48) *31.5(0.2) 5.4(3) 1e-7 369 34(6) 6.2(5) 1e-7 880 68(20) 39469(7777 2.2)*23(39)	10/15 3/15 10/15 15/15	$\begin{array}{c} {\bf f.16p} \\ {\bf f.16p} \\ {\bf A2 \ Zoub} \\ {\bf BFGS} \\ {\bf BIFOP-C} \\ {\bf \Delta f_{opt}} \\ {\bf f20} \\ {\bf A2 \ Zoub} \\ {\bf BFGS} \\ {\bf BIPOP-C} \\ {\bf \Delta f_{opt}} \\ {\bf f21} \\ {\bf A2 \ Carp} \\ {\bf BFGS} \\ {\bf A2 \ Carp} \\ {\bf A2 \ Carp} \\ {\bf BFGS} \\ {\bf BFGS} \\ {\bf BFGS} \\ {\bf A2 \ Carp} \\ {\bf A2 \ Carp} \\ {\bf A2 \ Carp} \\ {\bf BFGS} \\ {\bf A2 \ Carp} \\ {\bf A3 \ Carp} \\ {\bf A3 \ Carp} \\ {\bf A4 \ Carp} \\ {\bf A5 \ $	1(0) 1(0) 1655(12-20(18) 1e1 16 2.4(2) 1.7(1) 1.8(0.9) 3.3(2) 1e1 2.0(2) 93(0.7) 3.8(4) 2.3(2) 1e1 71 218(705) 170(228) 3.1(3)	1(0 1(0) 2.2e4(4) 2801(1) 2801(1) 160 851 45(30) 8835(790) 2.55(2) 8.2(9) 1e0 1157 562(821) 217(192) 1.4(2) 14(6) 160 (386) 389(259) 425(545) 2.9(2) 2.9(2)	$\begin{array}{c} )  5960 \\ )  \infty \\ 4e4)  1780 \\ 434)  161 \\ \hline \\ 1e-1 \\ \hline \\ 38111 \\ 37(37) \\ \infty \\ 10(10) \\ 2.8(3) \\ 1e-1 \\ \hline \\ 1674 \\ 836(926) \\ 430(445) \\ 1.9(3) \\ 24(74) \\ 1e-1 \\ \hline \\ 938 \\ 214(240) \\ 304(431) \\ 2.1(2) \\ 2.1(2) \end{array}$	$\begin{array}{lll} (8774) & \infty & \\ (2389) & \infty \\ (161) & 1(0) \\ \hline & 16 & 2 \\ \hline & 51362 \\ 27(33) & \infty \\ \hline & 7.6(10) \\ 2.2(1) \\ \hline & 1e-2 \\ \hline & 1692 \\ 828(931) \\ 425(544) \\ 1.9(3) \\ 25(119) \\ \hline & 1e-2 \\ \hline & 980 \\ 206(230) \\ 292(95) \\ 2.1(2) \end{array}$	$\begin{array}{c} 0e5 & 1.2e\\ & \infty\\ & \infty\\ & \infty\\ & \infty\\ & \infty\\ & \infty\\ & 0.9) & 1(0.9\\ 16.9\\ & 524470\\ 26(60)\\ & \infty\\ & 7.2(3)\\ 2.1(0.9)\\ 1e-3\\ & 1705\\ 822(836)\\ 422(495)\\ 1.9(2)\\ 25(21)\\ 1e-3\\ & 1008\\ 200(174)\\ 284(371)\\ 2.0(2)\\ 2.0(2)\\ 2.0(2)\\ \end{array}$	5 1.2e5	$\begin{array}{c} 1.2e5\\ \infty\ leb \\ \infty\ feb\\ \infty\ feb\\ \infty\ feb\\ \infty\ feb\\ 0.5e5\\ \infty\ feb\\ 0.5e5\\ 0.2e10.7\\ 1e-7\\ 1757\\ 798(512)\\ 410(534)\\ 2.0(2)\\ 25(20)\\ 1e-7\\ 1068\\ 192(234)\\ 269(734)\\ 2.6(4)\\ \end{array}$	15/15 0/15 0/15 0/15 15/15  #succ 14/15 1
A2 Carp A2 Zoub BFGS BIPOP-C $\frac{\Delta f_{\rm opt}}{f8}$ A2 Carp A2 Zoub BFGS BIPOP-C $\frac{\Delta f_{\rm opt}}{f9}$ A2 Carp A2 Zoub BFGS BIPOP-C $\frac{\Delta f_{\rm opt}}{f10}$ A2 Carp A2 Zoub BFGS BIPOP-C SIDDER A2 Soub BFGS BIPOP-C	2.9(3) 3.6(9) 3 	$\begin{array}{c} 2.0(2) \\ 577(1002) \\ \infty \\ 1.5(1) \\ 1e0 \\ 273 \\ 35(2) \\ 54(86) \\ 1.8(3)^{*} \\ 3.7(3) \\ 1e0 \\ 127 \\ 18(8) \\ 243(646) \\ 8.7(4) \\ 1e0 \\ 42(18) \\ 0.500 \\ 44(18) \\ 0.500 \\ 44(18) \\ 0.500 \\ 44(18) \\ 0.500 \\ 64(18) \\ 0.500 \\ 64(18) \\ 0.500 $	$\begin{array}{c} 7.6(0.3) \\ 658(1149) \\ \infty \\ 1(1) \\ 1e^{-1} \\ 336 \\ 36(2) \\ 50(12) \\ 2 \\ 1.6(2)^4 \\ 4.5(0.6) \\ 1e^{-1} \\ 214 \\ 22(2) \\ 1.50(75) \\ 7)^{*2} 2.0(1) \\ 7.2(2) \\ 1e^{-1} \\ 43(14) \\ 144(119) \\ 4^4 \\ 1(0.2) \\ 5) \\ 2.7(0) \end{array}$	$\begin{array}{l} 19(35) \\ 542(874) \text{ i} \\ 542(874) \text{ i} \\ \hline \\ 1(0.2) \\ 1e-2 \\ 37(68) \\ 52(122) \\ 2 \text{ 1.5}(1)^3 \\ 0 \text{ 4.7}(1) \\ 1e-2 \\ 263 \\ 25(6) \\ 33(115) \\ *3 \text{ 1.8}(0) \\ 6.7(5) \\ 1e-2 \\ \hline \\ 607 \\ 49(16) \\ 99\$414(623) \\ *4 \text{ 1}(0.2) \\ 4) \text{ 2.7}(0.2) \\ \end{array}$	$\begin{array}{c} 25(32) \\ 1112(1394) \\ \hline \times \\ 1(0.7) \\ 1e-3 \\ \hline 391 \\ 37(3) \\ 55(69) \\ 3 \\ 1.5(0.4 \\ 4.8(2) \\ 1e-3 \\ \hline 300 \\ 26(6) \\ 0.128(284) \\ 7)^*3_1.6(0.4 \\ (1e-3 \\ 626 \\ 56(15) \\ 3\$960(1e4 \\ 4.4) \\ 1e-3 \\ \hline 626 \\ 56(15) \\ 3\$960(1e4 \\ 4.3) \\ 2.8(0.4) \\ 3) \\ 2.8(0.4) \\ 3) \\ 2.8(0.4) \\ 3) \\ 3) \\ 3) \\ 4 \\ 4 \\ 4 \\ 4 \\ 4 \\ 3) \\ 3)$	25(32) 1112(1652) 1(0.9) 1e-5 410 42(123) 64(37) *3.5(0.4) 5.1(4) 1e-5 335 31(7) 140(93) 5)*3.5(0.4) 1e-5 829 56(14) 56(14) 33.3(0.3) 56(14) 10.3(0.3)	25(47) 1471(1975) 2600 1(0.9) 1e-7 422 48(6) 76(48) *31.5(0.2) 369 34(6) 156(416) 8) *41.4(0.6 6.2(5) 1e-7 880 68(20) 39469(7777 2.2)*23(39) 2.4(0.2)	10/15 3/15 10/15 15/15 15/15 14/15 15/15 15/15 15/15 15/15 15/15 15/15 15/15 15/15 15/15 15/15 15/15 15/15 15/15 15/15 15/15 15/15 15/15 15/15	$\begin{array}{c} f16\\ \hline f16\\ \hline f16\\ \hline p1\\ \hline f20\\ \hline f20\\ \hline f20\\ \hline f20\\ \hline f20\\ \hline A2 \ Carp \\ f20\\ \hline A2 \ Carp \\ \hline f21\\ \hline A2 \ Carp \\ \hline f21\\ \hline A2 \ Carp \\ \hline f21\\ \hline f21\\ \hline f22\\ \hline A2 \ Carp \\ \hline f22\\ \hline A2 \ Carp \\ \hline f22\\ \hline f22\\ \hline G0pt \\ \hline f22\\ \hline G0pt \\ \hline f23\\ \hline G0pt \\ \hline f22\\ \hline G0pt \\ \hline f23\\ \hline G0pt \\ \hline f22\\ \hline G0pt \\ \hline G0pt \\ \hline f22\\ \hline G0pt \\ G0pt \\ \hline G0pt \\ G0pt$	1(0) 1(0) 1655(12: 20(18) 1e1 16 2.4(2) 1.7(1) 1.8(0.9) 3.3(2) 1e1 2.0(2) 93(0.7) 3.8(4) 2.3(2) 1e1 71 218(705) 170(228) 3.1(3) 6.9(18)	$\begin{array}{c} 1,0\\ 1,0\\ 1,0\\ 1,0\\ 1,0\\ 1,0\\ 1,0\\ 1,0\\$	$\begin{array}{c} )  5960 \\ )  \infty \\ 4e4)  1780 \\ 434)  161 \\ \hline 1e-1 \\ 38111 \\ 37(37) \\ \infty \\ 10(10) \\ 2.8(3) \\ \hline 1e-1 \\ 1674 \\ 836(926) \\ 430(445) \\ 1.9(3) \\ 24(74) \\ 1e-1 \\ \hline 938 \\ 214(240) \\ 304(431) \\ 2.1(22) \\ 45(105) \end{array}$	$\begin{array}{ll} (8774) & \infty \\ (2389) & \infty \\ (161) & 1(0) \\ \hline 1e^2 \\ 51362 \\ 27(33) \\ \infty \\ 7.6(10) \\ 2.2(1) \\ \hline 1e^2 \\ \hline 1692 \\ 828(931) \\ 425(544) \\ 1.9(3) \\ 25(119) \\ \hline 1e^2 \\ \hline 980 \\ 206(230) \\ 292(95) \\ 2.1(2) \\ 43(69) \end{array}$	$\begin{array}{c} 0e5 & 1.2e\\ & \infty\\ & \infty\\ & \infty\\ & \infty\\ & \infty\\ & \infty\\ & 0.9\\ & 10.9\\ & 1.0\\ & 2.0\\ & 2.6\\ & 600\\ & 2.2\\ & 2.1\\ & 10.9\\ & 1.0\\ & 1.0\\ & 2.1\\ & 1.0\\ & 1.0\\ & 1.0\\ & 2.0\\ & 1.0\\ & 1.0\\ & 2.0\\ & 1.0\\ & 1.0\\ & 2.0\\ & 1.0\\ & 2.0\\ & 1.0\\ & 2.0\\ & 1.0\\ & 2.0\\ & 1.0\\ & 2.0\\ & 1.0\\ & 2.0\\ & 1.0\\ & 2.0\\ & 1.0\\ & 2.0\\ & 1.0\\ $	$\begin{array}{c} 5 & 1.2e5 \\ & \infty \\ & 10.7 \\ 1e-5 \\ \hline & 1729 \\ & 12.2(1) \\ 1e-5 \\ & 1729 \\ & 1811(795) \\ & 16663) \\ & 1.9(3) \\ & 25(36) \\ & 1e-5 \\ \hline & 1040 \\ & 195(120) \\ & 275(376) \\ & 2.0(2) \\ & 41(84) \end{array}$	$\begin{array}{c} 1.2e5 \\ \approx 165 \\ \approx 5e5 \\ \approx 5e5 \\ \approx 5e4 \\ 1(0.7) \\ 1e-7 \\ 55313 \\ 25(27) \\ \approx 5e5 \\ 7.1(8) \\ 2.2(1) \\ 1e-7 \\ 1757 \\ 798(512) \\ 410(534) \\ 2.0(2) \\ 25(20) \\ 1e-7 \\ 1068 \\ 192(234) \\ 269(734) \\ 2(4) \\ 2(4) \\ 2(6) \\ 40(93) \\ \end{array}$	15/15 0/15 0/15 0/15 15/15 15/15 15/15 1/
A2 Carp A2 Zoub BFGS BIPOP-C	2.9(3) 3.6(9) 5.0(4) le1 73 7.8(2) 4.0(2) 2.1(1) 3.2(2) le1 35 18(9) 4.5(2) 3.6(3) 5.8(1) le1 349 42(18) 241(907) 1(0.3)	$\begin{array}{c} 2.0(2) \\ 2.0(2) \\ 577(1002) \\ \infty \\ 1.5(1) \\ 1e0 \\ 273 \\ 35(2) \\ 54(86) \\ 1.8(3)^* \\ 3.7(3) \\ 1e0 \\ 127 \\ 18(8) \\ 243(646) \\ 3.0(0. \\ 8.7(4) \\ 1e0 \\ 500 \\ 42(18) \\ 679(850 \\ ^{\star 4} 1(0.3) \\ 6) 2.9(0. \\ 1e0 \\ \end{array}$	$\begin{array}{c} 7.6(0.3) \\ 7.6(0.3) \\ 658(1149) \\ \infty \\ 11) \\ 1e-1 \\ 336 \\ 36(12) \\ 50(12) \\ 2 \\ 1.6(12) \\ 4.5(0.6) \\ 1e-1 \\ \hline \\ 214 \\ 22(2) \\ 150(75) \\ 7.2(2) \\ 1e-1 \\ \hline \\ 574 \\ 43(14) \\ 1446(119) \\ 44 \\ 1(0.2) \\ 5) \\ 2.7(2) \\ 1e-1 \\ \hline \\ 1.5(2) \\ 1.5(2) \\ 1.5(3) \\ $	$\begin{array}{c} 19(35) \\ 542(874) & 1 \\ \hline & (0) \\ \hline \\ & (0) \\ \hline & (0) \\ \hline \\ \\ & (0) \\ \hline \\ \\ & (0) \\ \hline \\ \\ \hline \\ & (0) \\ \hline \\ \\ & (0) \\ \hline \\ \\ \\ & (0) \\ \hline \\ \\ \\ \\ \\ \hline \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ $	$\begin{array}{c} 25(32) \\ 25(32) \\ 1(12)(1394) \\ \infty \\ 1(0.7) \\ 1e-3 \\ 391 \\ 37(3) \\ 55(69) \\ 3 \\ 1.5(0.4) \\ 4.8(2) \\ 1e-3 \\ 300 \\ 22(68) \\ 128(284) \\ 7)^{*3}_{1.6}(0.6) \\ 6.4(4) \\ 1e-3 \\ 626 \\ 56(15) \\ 3\$960(1e4) \\ *^{4} \\ 1(0.4)_{3} \\ 2.8(0.1e-3) \\ 1e-3 \\ 1.5(1.2e-3) \\ 1.5(1.2e$	$\begin{array}{c} 25(32) \\ 1112(1652) \\ \hline 1(0.9) \\ 1e-5 \\ \hline 410 \\ 42(123) \\ 64(37) \\ \star^3 1.5(0.4) \\ 5.1(4) \\ 1e-5 \\ 31(7) \\ 140(93) \\ 5)^{\star} \frac{3}{1.5}(0.6) \\ 6.3(4) \\ 1e-5 \\ \hline 829 \\ 56(14) \\ 4513(762) \\ \star^4 \\ 1.1(0.3) \\ 3) \\ 2.3(0.116) \\ 1.1(16) \\$	25(47) 25(47) 27(17) 27(17) 27(17) 28(6) 28(6) 28(6) 28(6) 28(6) 28(18)	10/15 3/15 0/15 15/15 14/15 15/15/15 1		1(0) 1(0) 1655(12-2) 20(18) 1e1 16 2.4(2) 1.7(1) 1.8(0.9) 3.3(2) 1e1 41 2.0(2) 93(0.7) 3.8(4) 2.3(2) 1e1 71 218(705) 170(228) 3.1(3) 6.9(18)	1(0 1(0 1(0 1(0) 2.2e4(2) 2801(1 1e0 851 45(30) 835(790) 1e0 1157 562(821) 217(192) 1.4(2) 14(6) 1e0 386 386 386 399 425(545) 2.9(2) 10 10 10 10 10 10 10 10 10 10	$\begin{array}{c} )  5960 \\ )  \infty \\ 4e4)  1780 \\ 434)  161 \\ 1e-1 \\ 38111 \\ 37(37) \\ \infty \\ 10(10) \\ 2.8(3) \\ 1e-1 \\ 1674 \\ 836(926) \\ 1.9(3) \\ 24(74) \\ 1e-1 \\ 938 \\ 214(240) \\ 304(431) \\ 2.1(2) \\ 45(105) \\ 1e-1 \end{array}$	$\begin{array}{ll} (8774) & \infty \\ (2389) & \infty \\ (161) & 1(0) \\ \hline 1e^2 \\ 51362 \\ 27(33) \\ \infty \\ 7.6(10) \\ 2.2(1) \\ \hline 1e^2 \\ 1692 \\ 828(931) \\ 425(544) \\ 1.9(3) \\ 25(119) \\ 1e^2 \\ \hline 980 \\ 206(230) \\ 292(95) \\ 2.1(2) \\ 43(69) \\ 1e-2 \\ \hline \end{array}$	$\begin{array}{c} 0e5 & 1.2e \\ & \infty \\ & 0.9 \\ & 1.0 \\ & 2.26(60) \\ & \infty \\ & 0.226(60) \\ & \infty \\ & 0.226(60) \\ & \infty \\ & 0.226(20) \\ & 0.232(20)$	$\begin{array}{c} 5 & 1.2e5 \\ & \infty \\ & 0 \\ & \infty \\ & 0 \\$	$\begin{array}{c} 1.2e5\\ \infty \ leb\\ \infty \ feb\\ 0.7\\ 10.7\\ 11e-7\\ 55313\\ 25(27)\\ 2.2(1)\\ 1e-7\\ 1757\\ 7757\\ 1757\\ 2.0(2)\\ $	15/15 0/15 0/15 0/15 15/15 #succ 14/15 1/
$ \begin{array}{c} {\rm A2\ Carp} \\ {\rm A2\ Zoub} \\ {\rm BFGS} \\ {\rm BIPOP-C} \\ \hline                                  $	2.9(3) 3.6(9) 1 0 0 15.0(4) 1 1e1 73 7.8(2) 4.0(2) 2.1(1) 23.2(2) 1 1e1 85 1.5(3) 3.5(3) 2.5 8.6(3) 2.5	$\begin{array}{c} 2.0(2) \\ 577(1002) \\ \infty \\ 1.5(1) \\ 1e0 \\ 273 \\ 35(2) \\ 54(86) \\ 1.8(3)^{*} \\ 3.7(3) \\ 1e0 \\ 127 \\ 18(8) \\ 243(646) \\ 8.7(4) \\ 1e0 \\ 42(18) \\ 679(850 \\ *4 10.3) \\ 61 \\ 2.9(0) \\ 1e0 \\ 200 \\ 143(6, 0) \\ 143($	$\begin{array}{c} 7.6(0.3) \\ 658(1149) \\ \infty \\ 1(1) \\ 1e^{-1} \\ 336 \\ 36(2) \\ 50(12) \\ 2 \\ 1.6(2)^4 \\ 4.5(0.6) \\ 1e^{-1} \\ 214 \\ 22(2) \\ 150(75) \\ 7)^{\star} 2_{2}.0(1) \\ 7.2(2) \\ 1e^{-1} \\ \hline 574 \\ 43(14) \\ 1446(119) \\ 444(14) \\ 164(119) \\ 65) \\ 2.7(0.2) \\ 5 \\ 2.7(2) \\ 3) \\ 54(31) \\ 54(32) \\ 3) \\ 54(33) \\ 5$	$\begin{array}{c} 19(35) \\ 542(874) \\ \vdots \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0$	$\begin{array}{c} 25(32) \\ 1112(1394) \\ \hline \times \\ 1(0.7) \\ 1e-3 \\ \hline 391 \\ 37(3) \\ 55(69) \\ 3 \\ 1.5(0.4 \\ 4.8(2) \\ 1e-3 \\ \hline 300 \\ 26(6) \\ 0.128(284) \\ 7)^*3_1.6(0.4 \\ (1e-3 \\ 626 \\ 56(15) \\ 3\$960(1e4 \\ 4.4) \\ 1e-3 \\ \hline 626 \\ 56(15) \\ 3\$960(1e4 \\ 4.3) \\ 2.8(0.4) \\ 3) \\ 2.8(0.4) \\ 3) \\ 2.8(0.4) \\ 3) \\ 3) \\ 3) \\ 4 \\ 4 \\ 4 \\ 4 \\ 4 \\ 3) \\ 3)$	25(32) 1112(1652) 1(0.9) 1e-5 410 42(123) 64(37) *3.5(0.4) 5.1(4) 1e-5 335 31(7) 140(93) 5)*3.5(0.4) 1e-5 829 56(14) 56(14) 33.3(0.3) 56(14) 10.3(0.3)	25(47) )1471(1975) \$\tilde{6}000\$ \$1(0.9)\$ 1e-7 422 48(6) 76(48) *31.5(0.2) 5.4(3) 1e-7 369 34(6) 156(416) 8) *41.4(0.6 6.2(5) 1e-7 880 68(20) 39469(7777 20) 21 22,4(0.2) 16-7 1673 112(142)	10/15 3/15 0/15 15/15 14/15 15/15/15 1	$\begin{array}{c} f16\\ \hline f16\\ \hline f16\\ \hline p1\\ \hline f20\\ \hline f20\\ \hline f20\\ \hline f20\\ \hline f20\\ \hline A2 \ Carp \\ f20\\ \hline A2 \ Carp \\ \hline f21\\ \hline A2 \ Carp \\ \hline f21\\ \hline A2 \ Carp \\ \hline f21\\ \hline f21\\ \hline f22\\ \hline A2 \ Carp \\ \hline f22\\ \hline A2 \ Carp \\ \hline f22\\ \hline f22\\ \hline G0pt \\ \hline f22\\ \hline G0pt \\ \hline f23\\ \hline G0pt \\ \hline f22\\ \hline G0pt \\ \hline f23\\ \hline G0pt \\ \hline f22\\ \hline G0pt \\ \hline G0pt \\ \hline f22\\ \hline G0pt \\ G0pt \\ \hline G0pt \\ G0pt$	1(0) 1(0) 1655(12: 20(18) 1e1 16 2.4(2) 1.7(1) 1.8(0.9) 3.3(2) 1e1 2.0(2) 93(0.7) 3.8(4) 2.3(2) 1e1 71 218(705) 170(228) 3.1(3) 6.9(18)	$\begin{array}{c} 1,0\\ 1,0\\ 1,0\\ 1,0\\ 1,0\\ 1,0\\ 1,0\\ 1,0\\$	$\begin{array}{c} )  5960 \\ )  \infty \\ 4e4)  1780 \\ 434)  161 \\ \hline 1e-1 \\ 38111 \\ 37(37) \\ \infty \\ 10(10) \\ 2.8(3) \\ \hline 1e-1 \\ 1674 \\ 836(926) \\ 430(445) \\ 1.9(3) \\ 24(74) \\ 1e-1 \\ \hline 938 \\ 214(240) \\ 304(431) \\ 2.1(22) \\ 45(105) \end{array}$	$\begin{array}{ll} (8774) & \infty \\ (2389) & \infty \\ (161) & 1(0) \\ \hline 1e^2 \\ 51362 \\ 27(33) \\ \infty \\ 7.6(10) \\ 2.2(1) \\ \hline 1e^2 \\ \hline 1692 \\ 828(931) \\ 425(544) \\ 1.9(3) \\ 25(119) \\ \hline 1e^2 \\ \hline 980 \\ 206(230) \\ 292(95) \\ 2.1(2) \\ 43(69) \end{array}$	$\begin{array}{c} 0e5 & 1.2e\\ & \infty\\ & \infty\\ & \infty\\ & \infty\\ & \infty\\ & \infty\\ & 0.9\\ & 10.9\\ & 1.0\\ & 2.0\\ & 2.6\\ & 600\\ & 2.2\\ & 2.1\\ & 10.9\\ & 1.0\\ & 1.0\\ & 2.1\\ & 1.0\\ & 1.0\\ & 1.0\\ & 2.0\\ & 1.0\\ & 1.0\\ & 2.0\\ & 1.0\\ & 1.0\\ & 2.0\\ & 1.0\\ & 2.0\\ & 1.0\\ & 2.0\\ & 1.0\\ & 2.0\\ & 1.0\\ & 2.0\\ & 1.0\\ & 2.0\\ & 1.0\\ & 2.0\\ & 1.0\\ & 2.0\\ & 1.0\\ $	$\begin{array}{c} 5 & 1.2e5 \\ & \infty \\ & 10.7 \\ 1e-5 \\ \hline & 1729 \\ & 12.2(1) \\ 1e-5 \\ & 1729 \\ & 1811(795) \\ & 16663) \\ & 1.9(3) \\ & 25(36) \\ & 1e-5 \\ \hline & 1040 \\ & 195(120) \\ & 275(376) \\ & 2.0(2) \\ & 41(84) \end{array}$	$\begin{array}{c} 1.2e5 \\ \approx 165 \\ \approx 5e5 \\ \approx 5e5 \\ \approx 5e4 \\ 1(0.7) \\ 1e-7 \\ 55313 \\ 25(27) \\ \approx 5e5 \\ 7.1(8) \\ 2.2(1) \\ 1e-7 \\ 1757 \\ 798(512) \\ 410(534) \\ 2.0(2) \\ 25(20) \\ 1e-7 \\ 1068 \\ 192(234) \\ 269(734) \\ 2(4) \\ 2(4) \\ 2(6) \\ 40(93) \\ \end{array}$	15/15 0/15 0/15 0/15 15/15 15/15 15/15 1/
A2 Carp A2 Zoub BFGS BIPOP-C  f8 A2 Carp A2 Zoub BFGS BIPOP-C	2.9(3) 3.6(9) 1 	$\begin{array}{c} 2.0(2) \\ 2.0(2) \\ 577(1002) \\ \infty \\ 1.5(1) \\ 1e0 \\ 273 \\ 35(2) \\ 54(86) \\ 1.8(3)^4 \\ 3.7(3) \\ 1e0 \\ 127 \\ 18(8) \\ 243(646; 3.0(0.8,7(4) \\ 1e0 \\ 500 \\ 42(18) \\ 0679(850) \\ *^4 1(0.3) \\ 6) 2.9(0.16,10) \\ 1e0 \\ 100 \\ 17(1.8e4) \\ 17(1.8e4) \\ 17(1.8e4) \\ 18(1.8e4) \\ 18($	$\begin{array}{c} 7.6(0.3) \\ 7.6(0.3) \\ 658(1149) \\ \infty \\ 1(1) \\ 1e-1 \\ 336 \\ 36(2) \\ 50(12) \\ 2 \\ 1.6(2)^4 \\ 4.5(0.6) \\ 1e-1 \\ 22(2) \\ 1.5(0.6) \\ 1.50(75) \\ 7.2(2) \\ 1e-1 \\ 574 \\ 43(14) \\ 1446(119) \\ 1446(119) \\ 1446(119) \\ 142(2) \\ 1.5(2) $	$\begin{array}{c} 19(35) \\ 542(874) \text{ i} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ $	$\begin{array}{c} 25(32) \\ 1112(1394) \\ \hline 1(0.7) \\ 1e-3 \\ \hline 391 \\ 37(3) \\ 55(69) \\ ^{3} 1.5(0.4) \\ 4.8(2) \\ 1e-3 \\ \hline 300 \\ 26(6) \\ 128(284) \\ 7)^{*3} 1.6(0.4) \\ 6.4(4) \\ 1e-3 \\ \hline 626 \\ 56(15) \\ 35960(1e4) \\ ^{4} 4 \\ 1(0.4) \\ .3) \\ 2.8(0.1) \\ 1e-3 \\ \hline 1177 \\ 60(69) \\ \infty \end{array}$	$\begin{array}{c} 25(32) \\ 1112(1652) \\ \hline 1(0.9) \\ 1e-5 \\ \hline 410 \\ 42(123) \\ 64(37) \\ \star^3 1.5(0.4) \\ 5.1(4) \\ 1e-5 \\ 335 \\ 31(7) \\ 140(93) \\ 5)^{\star 3} 1.5(0.4) \\ 6.3(4) \\ 1e-5 \\ \hline 829 \\ 56(14) \\ 4513(762) \\ \star^4 \\ 1.1(0.3) \\ 2.3(0.166) \\ 1e-5 \\ \hline 1467 \\ 89(55) \\ \infty \end{array}$	$\begin{array}{c} 25(47) \\ 25(47) \\ 25(47) \\ 27(47) \\ 27(47) \\ 28(6) \\ 29(48) \\ 39(48) \\ 31.5(0.2) \\ 369 \\ 34(6) \\ 369 \\ 34(6) \\ 369 \\ 34(6) \\ 369 \\ 34(6) \\ 369 \\ 34(6) \\ 369 \\ 34(6) \\ 369 \\ 34(6) \\ 369 \\ 34(6) \\ 369 \\ 34(6) \\ 369 \\ 34(6) \\ 369 \\ 34(6) \\ 369 \\ 34(6) \\ 369 \\ 340 \\ 369 \\ 394 \\ $	10/15 3/15 3/15 10/15 18/15 18/15 14/15 15/1	f19	1(0) 1(0) 1655(12-2) 20(18) 1e1 16 2.4(2) 1.7(1) 1.8(0.9) 3.3(2) 1e1 41 2.0(2) 93(0.7) 3.8(4) 2.3(2) 1e1 71 218(702 3.1(3) 6.9(18) 1.7(2) 1.7(2) 1.7(2) 1.7(2) 1.7(2) 1.7(2) 1.7(2) 1.7(2) 1.7(2) 1.7(2) 1.7(2) 1.7(2) 1.7(2) 1.7(2) 1.7(2) 1.7(2)	$\begin{array}{c} 1(0)\\ 1(0)\\ 2(2)\\ 1(0)\\ 2(2)\\$	$\begin{array}{c} )  5960 \\ )  \infty \\ 4e4)  1780 \\ 434)  161 \\ \hline 1e-1 \\ \hline 81111 \\ 37(37) \\ \infty \\ 10(10) \\ 2.8(3) \\ \hline 1e-1 \\ 1674 \\ 836(926) \\ 430(445) \\ 1.9(3) \\ 24(74) \\ \hline 1e-1 \\ 938 \\ 214(240) \\ 304(431) \\ 2.1(2) \\ 45(105) \\ \hline 1e-1 \\ 14249 \end{array}$	$\begin{array}{l} (8774) & \infty \\ (2389) & \infty \\ (161) & 1(0) \\ \hline 1e-2 \\ 51362 \\ 27(33) \\ \hline \begin{array}{c} 7.6(10) \\ 2.2(1) \\ \hline 1e-2 \\ \hline 1692 \\ 828(931) \\ 425(544) \\ 1.9(3) \\ 25(119) \\ \hline 1e-2 \\ \hline 980 \\ 206(230) \\ 292(95) \\ 2.1(2) \\ 43(69) \\ \hline 1e-2 \\ \hline 27890 \\ 12(12) \\ \hline \end{array}$	$\begin{array}{c} 0e5 & 1.2e\\ & & \\$	$\begin{array}{c} 1.2e5 \\ & \times \\ & \times$	$\begin{array}{c} 1.2e5\\ \infty \ leb\\ \infty \ leb\\ \infty \ feb\\ 0.71\\ 1(0.7)\\ 1e-7\\ 55313\\ 25(27)\\ \infty \ feb\\ 7.1(8)\\ 1e-7\\ 1757\\ 798(512)\\ 410(534)\\ 2.0(2)\\ 25(20)\\ 1e-7\\ 1068\\ 192(234)\\ 2.6(4)\\ 40(93)\\ 1e-7\\ 34256\\ 21(45)\\ \infty \ feb\\ 5e5\\ \end{array}$	15/15 0/15 0/15 15/15 15/15 1/1
A2 Carp A2 Zoub BFGS BIPOP-C	2.9(3) 3.6(9) 5.0(4) 1e1 73 7.8(2) 4.0(2) 2.1(1) 3.2(2) 1e1 35 18(9) 4.5(2) 3.6(3) 5.8(1) 1e1 349 42(18) 241(907 1(0.3) 3.5(0. 1e1 143 138(61 3236(35 1(0.2)	$\begin{array}{c} 2.0(2) \\ 2.0(2) \\ 577(1002) \\ \infty \\ 1.5(1) \\ 1e0 \\ 273 \\ 35(2) \\ 54(86) \\ 1.8(3)^4 \\ 3.7(3) \\ 1e0 \\ 127 \\ 18(8) \\ 243(646) \\ 3.0(0) \\ 8.7(4) \\ 1e0 \\ 500 \\ 42(18) \\ 0.679(850) \\ ^{1} 44(0.3) \\ 6) 2.9(0. \\ 143(6.12) \\ 177) 1.864 \\ 2)^{1} 4.160 \\ 177) 1.864 \\ 2)^{1} 4.160 \\ 200 \\ 3.060 \\ 2.0$	$\begin{array}{c} 7.6(0.3) \\ 658(1149) \\ \infty \\ 1(1) \\ 1e-1 \\ 336 \\ 36(2) \\ 50(12) \\ 2 \\ 1.6(2)^4 \\ 4.5(0.6 \\ 1e-1 \\ 22(2) \\ 1.50(7) \\ 7)^{*2} 2.5(1) \\ 7.2(2) \\ 1e-1 \\ 574 \\ 43(14) \\ 1446(119) \\ 4^4 \\ 1(0.2) \\ 5) \\ 2.7(0. \\ 1e-1 \\ 2 \\ 763 \\ 3) \\ 54(31) \\ 4.1(0.2) \\ 1$	$\begin{array}{c} 19(35) \\ 542(874) \ \ : \\ \hline 10(.2) \\ 1e-2 \\ \hline 37(68) \\ 52(122) \ \ : \\ 21.5(1)^4 \\ 1e-2 \\ \hline 263 \\ 25(6) \\ 133(115) \\ \star^3 1.8(0.6.7(5) \\ 1e-2 \\ \hline 607 \\ 49(16) 99141(623) \\ \star^4 1(0.2 \\ 4) \ \ 2.7(0.16) \\ 1e-2 \\ \hline : \ \ 977 \\ \vdots \\ 62(25) \\ \odot .8)^*1.9(2.16) \end{array}$	$\begin{array}{c} 25(32) \\ 1112(1394) \\ \hline \hline 1(0.7) \\ 1e-3 \\ \hline 391 \\ 37(3) \\ 55(69) \\ ^3 \\ 1.5(0.4) \\ 4.8(2) \\ 1e-3 \\ \hline 300 \\ 26(6) \\ 128(284) \\ 7)^* \\ ^3 \\ 1.6(0.3) \\ 6.4(4) \\ 1e-3 \\ \hline 626 \\ 56(15) \\ 35960(1e4) \\ ^4 \\ 1(0.4) \\ .3) \\ 2.8(0. \\ 1e-3 \\ \hline 1177 \\ 60(69) \\ \hline \\ $	25(32) 1112(1652) 1(0.9) 1e-5 410 42(123) 64(37) 1,31,5(0.4) 5.1(4) 1e-5 335 31(7) 140(93) 5)*31.5(0.4) 1e-5 829 56(14) 24513(762) 44 1.10(0.3) 2.3(0.1e-5 1467 89(55) 99(272)	$\begin{array}{c} 25(47) \\ 25(47) \\ 25(47) \\ 27(47) \\ 27(47) \\ 27(47) \\ 27(48$	10/15 3/15 0/15 15/15 #succ 15/15 14/15 15/15 15/15 15/15 15/15 15/15 15/15 15/15 15/15 15/15 15/15 15/15 15/15 15/15 15/15 15/15 15/15 15/15 15/15 0/15 0/15 0/15 0/15	$\begin{array}{c} f19\\ \hline f.16\\ \hline f.16\\ \hline p.16\\ \hline f.20\\ \hline A2\\ \hline Couple & Coupl$	1(0) 1(0) 1655(12-2) 20(18 1e1 16 2.4(2) 1.7(1) 1.8(0.9) 3.3(2) 1e1 41 2.0(2) 93(0.7) 3.8(4) 2.3(2) 1e1 71 218(705 170(228 3.1(3) 6.9(18 1e1 3.0 2.3(2) 1.7(	$\begin{array}{c} 1,0\\ 1,0\\ 1,0\\ 1,0\\ 1,0\\ 1,0\\ 1,0\\ 2.54\\ 2.54\\ 2.5(2)\\ 2.5(2)\\ 2.5(2)\\ 2.5(2)\\ 1.62\\ 2.17(192)\\ 1.4(2)\\ 1.4(2)\\ 1.4(2)\\ 1.4(3)\\ 389(259)\\ 425(545)\\ 2.9(25)\\ 2.9(25)\\ 1.9(117)\\ 95(60)\\ 31(35)\\$	$\begin{array}{c} )  5960 \\ )  \infty \\ 4e4)  1780 \\ 434)  161 \\ 1e-1 \\ 38111 \\ 37(37) \\ \infty \\ 10(10) \\ 2.8(3) \\ 1e-1 \\ 1674 \\ 836(926) \\ 430(445) \\ 1.9(3) \\ 24(74) \\ 1e-1 \\ 938 \\ 214(240) \\ 304(431) \\ 2.1(22) \\ 45(105) \\ 1e-1 \\ 14249 \\ 19(15) \\ 165(121) \\ \infty \end{array}$	$\begin{array}{lll} (8774) & \infty & \\ (2389) & \infty & \\ (161) & 1(0) \\ \hline & 1e^{-2} & \\ 51362 & 27(33) \\ \infty & \\ 7.6(10) & 2.2(1) \\ \hline & 1e^{-2} & \\ 1692 & 828(931) \\ 425(544) & 1.9(3) \\ 25(119) & \\ 1e^{-2} & \\ 980 & \\ 206(230) & \\ 21(2) & \\ 27890 & \\ 1e^{-2} & \\ 27890 & \\ 12(12) & \\ \infty & \\ \end{array}$	$\begin{array}{c} 0e5 & 1.2e\\ & \infty\\ & \infty\\ & \infty\\ & \infty\\ & \infty\\ & \infty\\ & 0.9) & 1(0.9\\ 16.93\\ & 54470\\ 26(60)\\ & \infty\\ & 7.2(3)\\ 2.1(0.9)\\ & 1e-3\\ & 1705\\ 822(836)\\ 822(836)\\ 822(836)\\ 422(495)\\ 1.9(2)\\ 25(21)\\ 1e-3\\ & 1008\\ 200(174)\\ 284(371)\\ 284(371)\\ 242(47)\\ 1e-3\\ & 31654\\ 15(18)\\ & \infty\\ & \infty\\ \end{array}$	$\begin{array}{c} 5  \mathbf{1.2e5} \\ & \infty \\ & 1(0.7) \\ \mathbf{1e-5} \\ & 54861 \\ 26(26) \\ & \sim \\ & 7.1(5) \\ 1729 \\ & 811(795) \\ 416(663) \\ 1.96 \\ 1.96 \\ 215(36) \\ \mathbf{1e-5} \\ & 1040 \\ 195(120) \\ 275(376) \\ 2.0(2) \\ 41(84) \\ \mathbf{1e-5} \\ & 33030 \\ 22(15) \\ & \infty \\ & \infty \\ \end{array}$	$\begin{array}{c} 1.2e5\\ \infty\ lefs\\ \infty\ fess\\ \infty\ fess\\ \infty\ fess\\ \infty\ fess\\ \infty\ fess\\ 1(0.7)\\ 1e-7\\ 55313\\ 25(27)\\ 2.2(1)\\ 1e-7\\ 1757\\ 798(512)\\ 410(534)\\ 2.0(2)\\ 25(20)\\ 1e-7\\ 1068\\ 192(234)\\ 269(734)\\ 269(734)\\ 2(145)\\ \infty\ fess\\ 5ess\\ \infty\ fess\\ 24(45)\\ \infty\ fess\\ \infty\ fess\\ 2ess\\ 2ess\\$	15/15 0/15 0/15 0/15 15/15 #succ 14/15 1/
A2 Carp A2 Zoub BFGS BIPOP-C	2.9(3) 3.6(9) 5.0(4) 1e1 73 7.8(2) 4.0(2) 2.1(1) 3.2(2) 1e1 35 18(9) 4.5(2) 3.6(3) 5.8(1) 1e1 241(907 1(0.3) 3.5(0.1) 1e1 143 138(61 3236(35 1(0.2) 1.8(2) 8.4(3)	$\begin{array}{c} 2.0(2) \\ 577(1002) \\ \infty \\ 1.5(1) \\ 1e0 \\ 273 \\ 35(2) \\ 54(86) \\ 1.8(3)^{*} \\ 3.7(3) \\ 1e0 \\ 127 \\ 18(8) \\ 243(646) \\ 3.0(0. \\ 8.7(4) \\ 1e0 \\ 500 \\ 42(18) \\ 0.679(850) \\ ^{*4} 1(0.3) \\ 6) 2.9(0. \\ 1e0 \\ 200 \\ 1.8e4 \\ 2)^{*4} 1(0.3) \\ 3) \\ 7.2e4 \\ 1.8e4 \\ 2)^{*4} 1(0.3) \\ 3) \\ 7.2e4 \\ 3) \\ 7.2e4 \\ 4.0e4 \\ 1.8e4 \\ 2.8e4 \\ 3) \\ 7.2e4 \\ 3) \\ 7.2e4 \\ 4.0e4 \\ 4.0e4 \\ 3) \\ 7.2e4 \\ 4.0e4 \\ 4.0e$	$\begin{array}{c} 7.6(0.3) \\ 658(1149) \\ \infty \\ 1(1) \\ 1e-1 \\ 336 \\ 36(2) \\ 50(12) \\ 2 \\ 1.6(2)^4 \\ 4.5(0.6) \\ 1e-1 \\ 22(2) \\ 1.50(75) \\ 7)^{*2} 2.0(1) \\ 7.2(2) \\ 1e-1 \\ 574 \\ 43(14) \\ 1446(119) \\ *^{4} 1(0.2) \\ 5) \\ 2.7(0.1) \\ 1e-1 \\ 2 \\ 763 \\ 3) \\ 54(31) \\ 1.1)^{*4} 1.1(1.1) \\ 1.1)^{*4} 1.1(1.1) \\ 1.1)^{*4} 1.1(1.1) \\ 1.1)^{*4} 1.1(1.1) \\ 1.1)^{*4} 1.1(1.1) \\ 1.1)^{*4} 1.1(1.1) \\ 1.1)^{*4} 1.1(1.1) \\ 1.1(2) \\ 1.2(2) $	$\begin{array}{c} 19(35) \\ 542(874) \text{ : } \\ 542(874) \text{ : } \\ 600 \\ \hline 1(0.2) \\ 1e-2 \\ 37(68) \\ 52(122) \\ 2 \\ 1.5(1)^4 \\ ) \\ 4.7(1) \\ 1e-2 \\ 263 \\ 25(6) \\ 133(115)^5 \\ \star^3 \\ 1.8(0. \\ 6.7(5) \\ 1e-2 \\ 607 \\ 49(16) \\ 98141(623) \\ \star^4 \\ 1(0.2 \\ 4) \\ 2.7(0. \\ 1e-2 \\ 4) \\ 2.7(0. \\ 1e-2 \\ 5) \\ 62(25) \\ 0.8)^*1.9(2. \\ 0.3) \\ 1.8(0. \\ 1.8(0. \\ 0.3) \\ 1$	$\begin{array}{c} 25(32) \\ 1112(1394) \\ \hline \times \\ 1(0.7) \\ 1e-3 \\ \hline 391 \\ 37(3) \\ 55(69) \\ 3 \\ 1.5(0.4 \\ 4.8(2) \\ 1e-3 \\ \hline 300 \\ 26(68) \\ 1.28(284) \\ (6.4(4) \\ 1e-3 \\ \hline 626 \\ 56(15) \\ 35960(1e4) \\ *^4 \\ 1(0.4) \\ 3) \\ 2.8(0. \\ 1e-3 \\ \hline 1177 \\ 60(69) \\ \hline \\ \infty \\ 0 \\ 8.2(4) \\ .2) \\ 1.6(0.2. \\ 1$	25(32) 1112(1652) 1(0.9) 1e-5 410 42(123) 64(37) 1,31,5(0.4) 5.1(4) 1e-5 337 31(7) 140(93) 15)*31,5(0.8) 6.3(4) 1e-5 829 56(14) 3015(3762) 44 1.1(0.3) 1e-5 8(14) 16-5 1467 89(55) 89(55) 20 199(272) 21* 1.4(0.1	$\begin{array}{c} 25(47) \\ 25(47) \\ 1471(1975) \\ 267 $	10/15 3/15 0/15 15/15 14/15 15/15 14/15 15/15/15 1	$\begin{array}{c} f19 \\ \hline f19 \\ A2 \ Carp \\ A2 \ Zoub \\ BFGS \\ BIPOP-C \\ \hline 20 \\ A2 \ Carp \\ A2 \ Carp \\ A2 \ Carp \\ A2 \ Carp \\ A2 \ Zoub \\ BFGS \\ BIPOP-C \\ \hline 21 \\ A2 \ Carp \\ A2 \ Zoub \\ BFGS \\ BIPOP-C \\ \hline \Delta f_{opt} \\ f21 \\ A2 \ Carp \\ A2 \ Zoub \\ BFGS \\ BIPOP-C \\ \hline 22 \\ A2 \ Carp \\ A2 \ Zoub \\ BFGS \\ BIPOP-C \\ \hline \Delta f_{opt} \\ f23 \\ A2 \ Zoub \\ BIPOP-C \\ \hline BIPOP-C \\ BIPOP-C \\ \hline BIPOP-C \\$	1(0) 1(0) 1655(12-2) 20(18) 1e1 16 2.4(2) 1.7(1) 1.8(0.9) 3.3(2) 1e1 41 2.0(2) 3.8(4) 2.3(2) 1e1 71 218(705) 170(228) 3.1(3) 6.9(15) 1.7(1) 1.7(1) 1.7(1)	$\begin{array}{c} 1,0\\ 1,0\\ 1,0\\ 1,0\\ 1,0\\ 1,0\\ 1,0\\ 1,0\\$	$\begin{array}{c} )  5960 \\ )  \infty \\ 4e4)  1780 \\ 434)  161 \\ 1e-1 \\ 38111 \\ 37(37) \\ \infty \\ 10(10) \\ 2.8(3) \\ 1e-1 \\ 1674 \\ 836(926) \\ 430(445) \\ 1.9(3) \\ 24(74) \\ 1e-1 \\ 938 \\ 214(240) \\ 304(431) \\ 2.1(2) \\ 45(105) \\ 1e-1 \\ 14249 \\ 19(15) \\ 165(121) \\ \infty \\ 3.7(5) \end{array}$	$\begin{array}{lll} (8774) & \infty & \\ (2389) & \infty & \\ (161) & 1(0) \\ \hline 1e^2 & \\ 51362 & \\ 27(33) & \infty & \\ & 7.6(10) & \\ 2.2(1) & \\ 1e^2 & \\ 1692 & \\ 828(931) & \\ 1.9(3) & \\ 2206(230) & \\ 11.9(3) & \\ 206(230) & \\ 2.06(230) & \\ 2.1(2) & \\ & 43(69) & \\ 1e^2 & \\ & 27890 & \\ 12(12) & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & $	$\begin{array}{c} 0e5 & 1.2e \\ & \infty \\ & 0.9 \\ & 1.0 \\ & 2.0 \\ & 2.26 \\ & 0.0$	$\begin{array}{c} 5  \mathbf{1.2e5} \\ & \infty \\ & 1(0.7) \\ \mathbf{1e-5} \\ & 54861 \\ 26(26) \\ & \infty \\ & 7.1(5) \\ 2.2(1.5) \\ & 179 \\ 2.1(1.5) \\ & 2.1(1.5) \\ & 2.1(1.5) \\ & 2.1(1.5) \\ & 2.1(1.5) \\ & 2.1(1.5) \\ & 2.1(1.5) \\ & 2.1(1.5) \\ & \infty \\ & 1.8(1.5) \\ & 2.1(1.5) \\ &$	$\begin{array}{c} 1.2e5\\ \approx 1te5\\ \approx 1te5\\ \approx 5e5\\ \approx 5e5\\ \approx 5e4\\ 1(0.7)\\ 1e-7\\ 55313\\ 25(27)\\ 1e-7\\ 175\\ 788(512)\\ 410(534)\\ 2.0(2)\\ 25(20)\\ 1e-7\\ 1068\\ 192(234)\\ 269(734)\\ 2.6(4)\\ 40(93)\\ 1e-7\\ 34256\\ 21(455)\\ \approx 5e5\\ \approx 2e4\\ 1.8(2)\\ \end{array}$	15/15 0/15 0/15 0/15 15/15 15/15 1/1
A2 Carp A2 Zoub BFGS BIPOP-C $\Delta f_{\rm opt}$ fil A2 Zoub BFGS BIPOP-C $\Delta f_{\rm opt}$ fil A2 Zoub BFGS BIPOP-C $\Delta f_{\rm opt}$ fil A2 Carp A2 Zoub BFGS BIPOP-C $\Delta f_{\rm opt}$ fil A2 Carp A2 Zoub BFGS BIPOP-C $\Delta f_{\rm opt}$ fil A2 Carp A2 Zoub BFGS BIPOP-C $\Delta f_{\rm opt}$ fil A2 Carp A2 Zoub BFGS BIPOP-C $\Delta f_{\rm opt}$ fil A2 Carp A2 Zoub BFGS BIPOP-C $\Delta f_{\rm opt}$	2.9(3) 3.6(9) 3.6(9) 5.0(4) 1e1 73 7.8(2) 4.0(2) 2.1(1) 3.2(2) 1e1 35 18(9) 4.5(2) 3.6(3) 2.5.8(1) 1e1 349 42(18) 241(907 1(0.3) 3.5(0. 1e1 143 138(61) 138(61) 12(0.2) 2.6(3.6(3) 2.7(1.0.3) 3.8(3.6(3) 3.8(3.6(3) 3.8(3.6(3) 3.8(3.6(3) 3.8(3.6(3) 3.8(3.6(3) 3.8(3.6(3) 3.8(3.6(3) 3.8(3.6(3) 3.8(4) 3.8(4.6(3) 3.8(4) 3.8(4.6(3) 3.8(4) 3.8(4) 3.8(4) 3.8(4) 3.8(4) 3.8(4) 3.8(4) 3.8(4) 3.8(4) 3.8(4) 3.8(4) 3.8(4	$\begin{array}{c} 2.0(2) \\ 2.0(2) \\ 5777(1002) \\ \infty \\ 1.5(1) \\ 1e0 \\ 273 \\ 35(2) \\ 1.8(3)^3 \\ 3.7(3) \\ 1e0 \\ 127 \\ 18(8) \\ 3.0(0.8,7(4)) \\ 18(9.8,7(4)) \\ 160 \\ 500 \\ 42(18) \\ 679(850) \\ ^{\star 4} 1(0.3) \\ 6) 2.9(0.1) \\ 1e0 \\ 100 \\ 11.8(4) \\ 100 \\ 10$	$\begin{array}{c} 7.6(0.3) \\ 7.6(0.3) \\ 658(1149) \\ \infty \\ 1(1) \\ 1e-1 \\ 336 \\ 36(2) \\ 50(12) \\ 2 \\ 1.6(2)^4 \\ 4.5(0.6) \\ 1e-1 \\ \hline \\ 214 \\ 22(2) \\ 150(75) \\ 7.2(2) \\ 1e-1 \\ \hline \\ 574 \\ 43(14) \\ 1446(119) \\ 44 \\ 1(0.2) \\ 5) \\ 2.7(2) \\ 1e-1 \\ 2 \\ 762 \\ 3) \\ 54(31) \\ 1(2) \\ 2 \\ 2 \\ 2 \\ 2(1) \\ 1e-1 \\ 2 \\ 3) \\ 1.1((2) \\ 2 \\ 2 \\ 2(1) \\ 1e-1 \\ 2 \\ 3) \\ 1.1((2) \\ 2 \\ 2(2) \\ 1e-1 \\ 2 \\ 3) \\ 1.1((2) \\ 2 \\ 2(2) \\ 1e-1 \\ 2 \\ 1.1((2) \\ 2(2) \\ 1e-1 \\ 1.1((2) \\ 1e-1 \\ 1.1((2) \\ 1e-1 \\ 1e-$	$\begin{array}{c} 19(35) \\ 542(874) & 1 \\ (0.2) \\ \hline 10-2 \\ 37(8) \\ 52(122) \\ 2 & 1.5(1)^* \\ ) & 4.7(1) \\ 1e-2 \\ \hline 263 \\ 25(6) \\ 133(115) \\ *3 & 1.8(0 \\ 6.7(5) \\ 1e-2 \\ \hline 607 \\ 49(16) \\ 98141(623) \\ *4 & 1(0.2) \\ 49 & (10.2) \\ 1e-2 \\ \vdots & 977 \\ 0 & 62(25) \\ \infty \\ \infty \\ 0.8) *1.9(2) \\ 0.3) & 1.8(0 \\ 1e-2 \\ \hline \end{array}$	$\begin{array}{c} 25(32) \\ 25(32) \\ 1(12)(1394) \\ \infty \\ 1(0.7) \\ 1e-3 \\ 391 \\ 37(3) \\ 37(3) \\ 15(69) \\ 3 \\ 1.5(0.4) \\ 4.8(2) \\ 1e-3 \\ 300 \\ 28(68) \\ 128(284) \\ 7)^{*3}_{1.5}(6(0.6) \\ 6.4(4) \\ 1e-3 \\ 626 \\ 56(15) \\ 35960(1e4) \\ *^{4} \\ 1(0.4) \\ .3) \\ 2.8(0.1e-3) \\ 1177 \\ 60(69) \\ \infty \\ .2) \\ 1.6(0.2) \\ 1e-3 \\ 11e-3 \\ .2) \\ 1.6(0.2) \\ 1e-3 \\ .2) \\ 1.6(0.2) \\ 1e-3 \\ .2) \\ 1.6(0.$	$\begin{array}{c} 25(32) \\ 125(32) \\ 1112(1652) \\ \hline & 1(0.9) \\ 1e-5 \\ \hline & 410 \\ 42(123) \\ 64(37) \\ 1e-5 \\ \hline & 315 \\ 31(7) \\ 140(93) \\ 5)^{*}\frac{4}{3}, 5(0.4) \\ 1e-5 \\ \hline & 829 \\ 56(14) \\ 4513(762) \\ *^{4} \\ 1.1(0.3) \\ 2.3(0.166) \\ 11e-5 \\ \hline & 1467 \\ 89(55) \\ \hline & 99(272) \\ *^{1} \\ 1.4(0.166) \\ \hline & 1.4(0.166) $	$\begin{array}{c} 25(47) \\ 25(47) \\ 25(47) \\ 27(47) \\ 28(69) \\ 28(69) \\ 38(69) \\ 28(69) \\ 38(69) \\ 38(69) \\ 38(69) \\ 38(69) \\ 38(69) \\ 38(69) \\ 39(69) \\ 48(29) \\ 39(469) \\ 48(29) \\ 39(469) \\ 48(29) \\ 39(469) \\ 48(29) \\ 39(469) \\ 48(29) \\ 29(29) \\ 2$	10/15 3/15 3/15 15/15 #succ 15/15 14/15 15/15 15/15 15/15 15/15 15/15 15/15 15/15 15/15 15/15 15/15 15/15 15/15 15/15 15/15 15/15 15/15 0/15 15/15 0/15 15/15/15 15/15 15/15 15/15 15/15 15/15 15/15 15/15 15/15 15/	$\begin{array}{c} \textbf{f.19} \\ \textbf{A2 Carp} \\ \textbf{A2 Carp} \\ \textbf{A2 Zoub} \\ \textbf{BFGS} \\ \textbf{BIPOP-C} \\ \textbf{A2 Carp} \\ \textbf{BFGS} \\ \textbf{BIPOP-C} \\ \textbf{D1 Carp} \\ \textbf{C2 Carp} \\ \textbf{C3 Carp} \\ \textbf{C4 Carp} \\ \textbf{C4 Carp} \\ \textbf{C4 Carp} \\ \textbf{C5 Carp} \\ \textbf{C6 Carp} \\ \textbf{C6 Carp} \\ \textbf{C7 Carp} \\ C7 Ca$	1(0) 1(0) 1655(12-2) 20(18) 1e1 16 2.4(2) 1.7(1) 1.8(0.9) 3.3(2) 1e1 2.0(2) 93(0.7) 3.8(4) 2.3(2) 1e1 71 218(705) 3.6(9) 15 16 170(228) 3.1(3) 6.9(18) 1e1 1.7(1) 1e1 1.7(1)	1(0) 1(0) 1(0) 1(0) 1(0) 1(0) 1(0) 1(0)	$\begin{array}{c} )  5960 \\ )  \infty \\ 4e4)  1780 \\ 494)  161 \\ 1e-1 \\ 38111 \\ 37(37) \\ \infty \\ 10(10) \\ 2.8(3) \\ 1e-1 \\ 1674 \\ 836(926) \\ 1.9(3) \\ 24(74) \\ 1e-1 \\ \hline 938 \\ 214(240) \\ 304(431) \\ 2.1(2) \\ 45(105) \\ 1e-1 \\ \hline 14249 \\ 19(15) \\ 165(121) \\ \infty \\ 3.7(5) \\ 1e-1 \end{array}$	$\begin{array}{lll} (8774) & \infty & \\ (2389) & \infty & \\ (2389) & \infty & \\ (161) & 1(0) & \\ \hline & & \\ 162 & \\ \hline & & \\ 51362 & \\ 27(33) & \infty & \\ \hline & & \\ 7.6(10) & & \\ 2.2(1) & \\ 1e-2 & \\ \hline & & \\ 1692 & \\ 828(931) & \\ 425(544) & \\ 1.9(3) & \\ 25(119) & \\ \hline & & \\ 1980 & \\ 206(230) & \\ 292(95) & \\ 2.1(2) & \\ 43(69) & \\ \hline & & \\ 1e-2 & \\ \hline & & \\ 27890 & \\ 12(12) & \infty & \\ \hline & & \\ & & $	$\begin{array}{c} 0e5 & 1.2e\\ & & \\$	$\begin{array}{c} 1.2e5 \\ & \times \\ & \times$	$\begin{array}{c} 1.2e5 \\ \infty \ leb \\ \infty \ leb \\ \infty \ feb \\ \infty \ feb \\ \infty \ feb \\ \infty \ feb \\ 10.7) \\ 1e-7 \\ 55313 \\ 25(27) \\ \infty \ feb \\ 7.1(8) \\ 2.2(21) \\ 1e-7 \\ 1757 \\ 798(512) \\ 410(534) \\ 2.0(2) \\ 25(20) \\ 1e-7 \\ 1068 \\ 192(234) \\ 2.6(4) \\ 40(93) \\ 1e-7 \\ 34256 \\ 21(45) \\ \infty \ feb \\ 2e4 \\ 1.8(7) \\ $	15/15 0/15 0/15 0/15 15/15 15/15 1/1
A2 Carp A2 Zoub BFGS BIPOP-C	2.9(3) 3.6(9) 5.0(4) 1e1 73 7.8(2) 4.0(2) 2.1(1) 3.2(2) 1e1 35 18(9) 4.5(2) 3.6(3) 5.8(1) 1e1 349 42(18) 241(907 1(0.3) 3.5(0.1) 1e1 143 138(61 3236(35) 1(0.2) 8.4(3)	$\begin{array}{c} 2.0(2) \\ 2.0(2) \\ 577(1002) \\ \infty \\ 1.5(1) \\ 1e0 \\ 273 \\ 35(2) \\ 54(86) \\ 1.8(3)^4 \\ 3.7(3) \\ 1e0 \\ 127 \\ 18(8) \\ 243(646) \\ 3.0(0. \\ 8.7(4) \\ 1e0 \\ \hline 500 \\ 42(18) \\ 0679(850) \\ ^{*4}4 \ 1(0.3) \\ 6) \ 2.9(0. \\ 1e0 \\ 10 \\ 200 \\ 17) \ 1.8e4 \\ 2)^{*4} \ 1(0.3) \\ 6) \ 7.2(1.8e4) \\ 1.8e4 \\ 1.9e4 \\ 1.9$	$\begin{array}{c} 7.6(0.3) \\ 7.6(0.3) \\ 658(1149) \\ \infty \\ 1(1) \\ 1e-1 \\ 336 \\ 36(2) \\ 50(12) \\ 2 \\ 1.6(2)^4 \\ 4.5(0.6) \\ 1e-1 \\ 22(2) \\ 1.5(0.6) \\ 1.50(75) \\ 7)^{\star 2} 2.0(1) \\ 7.2(2) \\ 1e-1 \\ 574 \\ 43(14) \\ 1446(119) \\ 1446(119) \\ 1446(119) \\ 12 \\ 763 \\ 3) \\ 54(31) \\ (24) \\ \infty \\ 1)^{\star 4} \\ 1.1((2) \\ 2.2((110) \\ 1.0) \\ 1.0) \\ 1.0 \\$	$\begin{array}{c} 19(35) \\ 542(874) & 1 \\ 542(874) & 1 \\ \hline & (0.2) \\ \hline & (0.2$	$\begin{array}{c} 25(32) \\ 1112(1394) \\ \hline 1(0.7) \\ 1e-3 \\ \hline 391 \\ 37(3) \\ 55(69) \\ \cdot ^3 \\ 1.5(0.4) \\ 4.8(2) \\ 1e-3 \\ \hline 300 \\ 26(6) \\ 128(284) \\ 7)^{*3} \\ 1.6(0.4) \\ 6.4(4) \\ 1e-3 \\ \hline 626 \\ 56(15) \\ 35960(1e4) \\ *^4 \\ 1(0.4) \\ .3) \\ 2.8(0.1) \\ 1e-3 \\ \hline 1177 \\ 60(69) \\ \infty \\ ) \\ 8.2(4) \\ .2) \\ 1.6(0.2) \\ 1.6(0.2) \\ 461 \\ \hline \end{array}$	25(32) 1112(1652) 1(0.9) 1e-5 410 42(123) 64(37) *3.1.5(0.4) 5.1(4) 1e-5 335 31(7) 140(93) 5)*3.1.5(0.6) 6.3(4) 1e-5 829 56(14) 4513(762) *4 1.1(0.3) 2.3(0.1e-5 1467 89(55) 29(272) 1-1-5 1303	$\begin{array}{c} 25(47) \\ 25(47) \\ 25(47) \\ 27(47) \\ 27(47) \\ 27(47) \\ 27(48$	10/15 3/15 3/15 10/15 15/15 14/15 15/15	$\begin{array}{c} f.14\\ \hline f.12\\ \hline f.12\\ \hline A.2 \ Zoub\\ BFGS\\ BIPOP-C\\ \hline f.20\\ \hline f.20\\ \hline f.20\\ \hline f.20\\ \hline A.2 \ Zoub\\ BFGS\\ BIPOP-C\\ \hline \Delta f_{opt}\\ f.21\\ \hline A.2 \ Carp\\ F.23\\ \hline A.2 \ Carp\\ A.2 \ Carp\\ A.2 \ Carp\\ f.23\\ \hline A.2 \ Carp\\ A.2 \ Carp\ A.2 \ $	1(0) 1(0) 1655(12-2) 20(18) 1e1 16 2.4(2) 1.7(1) 1.8(0.9) 3.3(2) 1e1 41 2.0(2) 3.8(4) 2.3(2) 1e1 71 218(705) 170(228) 3.1(3) 6.9(15) 1.7(1) 1.7(1) 1.7(1)	$\begin{array}{c} 1,0\\ 1,0\\ 1,0\\ 1,0\\ 1,0\\ 1,0\\ 1,0\\ 1,0\\$	$\begin{array}{c} )  5960 \\ )  \infty \\ 4e4)  1780 \\ 434)  161 \\ 1e-1 \\ 38111 \\ 37(37) \\ \infty \\ 10(10) \\ 2.8(3) \\ 1e-1 \\ 1674 \\ 836(926) \\ 430(445) \\ 1.9(3) \\ 24(74) \\ 1e-1 \\ 938 \\ 214(240) \\ 304(431) \\ 2.1(2) \\ 45(105) \\ 1e-1 \\ 14249 \\ 19(15) \\ 165(121) \\ \infty \\ 3.7(5) \end{array}$	$\begin{array}{lll} (8774) & \infty & \\ (2389) & \infty & \\ (161) & 1(0) \\ \hline 1e^2 & \\ 51362 & \\ 27(33) & \infty & \\ & 7.6(10) & \\ 2.2(1) & \\ 1e^2 & \\ 1692 & \\ 828(931) & \\ 1.9(3) & \\ 2206(230) & \\ 11.9(3) & \\ 206(230) & \\ 2.06(230) & \\ 2.1(2) & \\ & 43(69) & \\ 1e^2 & \\ & 27890 & \\ 12(12) & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & $	$\begin{array}{c} 0e5 & 1.2e \\ & \infty \\ & 0.9 \\ & 1.0 \\ & 2.0 \\ & 2.26 \\ & 0.0$	$\begin{array}{c} 5  \mathbf{1.2e5} \\ & \infty \\ & 1(0.7) \\ \mathbf{1e-5} \\ & 54861 \\ 26(26) \\ & \infty \\ & 7.1(5) \\ 2.2(1.5) \\ & 179 \\ 2.1(1.5) \\ & 2.1(1.5) \\ & 2.1(1.5) \\ & 2.1(1.5) \\ & 2.1(1.5) \\ & 2.1(1.5) \\ & 2.1(1.5) \\ & 2.1(1.5) \\ & \infty \\ & 1.8(1.5) \\ & 2.1(1.5) \\ &$	$\begin{array}{c} 1.2e5\\ \approx 1te5\\ \approx 1te5\\ \approx 5e5\\ \approx 5e5\\ \approx 5e4\\ 1(0.7)\\ 1e-7\\ 55313\\ 25(27)\\ 1e-7\\ 175\\ 788(512)\\ 410(534)\\ 2.0(2)\\ 25(20)\\ 1e-7\\ 1068\\ 192(234)\\ 269(734)\\ 2.6(4)\\ 40(93)\\ 1e-7\\ 34256\\ 21(455)\\ \approx 5e5\\ \approx 2e4\\ 1.8(2)\\ \end{array}$	15/15 0/15 0/15 0/15 15/15 15/15 1/1
A2 Carp A2 Zoub BFGS BIPOP-C $\Delta f_{\rm opt}$ fil A2 Zoub BFGS BIPOP-C $\Delta f_{\rm opt}$ fil A2 Zoub BFGS BIPOP-C $\Delta f_{\rm opt}$ fil A2 Carp A2 Zoub BFGS BIPOP-C $\Delta f_{\rm opt}$ fil A2 Carp A2 Zoub BFGS BIPOP-C $\Delta f_{\rm opt}$ fil A2 Carp A2 Zoub BFGS BIPOP-C $\Delta f_{\rm opt}$ fil A2 Carp A2 Zoub BFGS BIPOP-C $\Delta f_{\rm opt}$ fil A2 Carp A2 Zoub BFGS BIPOP-C $\Delta f_{\rm opt}$	2.9(3) 3.6(9) 3.6(9) 5.0(4) 1e1 73 7.8(2) 4.0(2) 2.1(1) 3.2(2) 1e1 35 18(9) 5.8(1) 1e1 349 42(18) 241(907 1(0.3) 3.5(0.1) 1e1 143 138(61) 3236(35) 1(0.5) 8.4(3) 161 161 161 173 186(173) 186(173) 186(173) 186(173) 186(173)	$2.0(2)$ $2.0(2)$ $577(1002)$ $\infty$ $1.5(1)$ $1e0$ $273$ $35(2)$ $54(86)$ $1.8(3)^*$ $3.7(3)$ $1e0$ $127$ $18(8)$ $243(646;$ $3.0(0.$ $8.7(4)$ $1e0$ $500$ $44(810)$ $60$ $10$ $10$ $10$ $10$ $10$ $10$ $10$ $1$	$\begin{array}{c} 7.6(0.3) \\ 7.6(0.3) \\ 658(1149) \\ \infty \\ 1(1) \\ 1e-1 \\ 336 \\ 36(2) \\ 50(12) \\ 2 \\ 1.6(2)^4 \\ 4.5(0.6 \\ 1e-1 \\ 22(2) \\ 1.50(75) \\ 7.2(2) \\ 1e-1 \\ 574 \\ 43(14) \\ 1446(119) \\ 1446(119) \\ 1446(119) \\ 1446(119) \\ 1446(119) \\ 12 \\ 7.2(2) \\ 1.2(2) \\ 1.3(2) \\$	$\begin{array}{c} 19(35) \\ 542(874) \ \ 1\\                                $	$\begin{array}{c} 25(32) \\ 1112(1394) \\ \hline \\ 1(0.7) \\ 1e-3 \\ \hline \\ 391 \\ 37(3) \\ 55(69) \\ 3 \\ 1.5(0.4) \\ 4.8(2) \\ 1e-3 \\ \hline \\ 300 \\ 26(6) \\ 128(284) \\ 7)^{*3} 1.6(0.4) \\ 6.4(4) \\ 1e-3 \\ \hline \\ 626 \\ 56(15) \\ 33960(1e4)^{*4} \\ 1(0.4) \\ 3) \\ 2.8(0. \\ 1e-3 \\ \hline \\ 1177 \\ 60(69) \\ \infty \\ 0) \\ 8.2(4) \\ .2) \\ 1.6(0.2) \\ 1$	25(32) 1112(1652) 1(0.9) 1e-5 410 42(123) 64(37) *3.1.5(0.4) 5.1(4) 1e-5 335 31(7) 140(93) 5)*3.1.5(0.8) 6.3(4) 1e-5 829 56(14) 4513(762) *4 1.1(0 3) 2.3(0.16-5) 199(272) 2)*1.4(0.1 1e-5 1303) 9) 73(98)	25(47) $25(47)$ $25(47)$ $25(47)$ $25(47)$ $25(48)$ $2$	10/15 3/15 3/15 15/15 #succ 15/15 14/15 15/15 15/15 15/15 15/15 15/15 15/15 15/15 15/15 15/15 15/15 15/15 15/15 15/15 15/15 15/15 15/15 0/15 15/15 0/15 15/15/15 15/15 15/15 15/15 15/15 15/15 15/15 15/15 15/15 15/	A2 Carp A2	1(0) 1(0) 1(0) 1655(12-2) 20(18 1e1 16 2.4(2) 1.7(1) 1.8(0.9) 3.3(2) 1e1 41 2.0(2) 93(0.7) 3.8(4) 2.3(2) 1218(705) 170(228 3.1(3) 6.9(18 114) 1.7(2) 1e1 1.7(2) 1e1 1.7(2) 1e1 1.7(2) 1e1 1.7(2) 1e1 1.7(2) 1e1 1.7(2) 1e1 1.7(2) 1e1 1.7(2) 1.8(705)	$\begin{array}{c} 1(0\\ 1(0)\\ 1(0)\\ 2(0)\\ $	$\begin{array}{c} )  5960 \\ )  \infty \\ 4e4)  1780 \\ 434)  161 \\ \hline 1e-1 \\ 38111 \\ 37(37) \\ \infty \\ 10(10) \\ 2.8(3) \\ \hline 1e-1 \\ 1674 \\ 836(926) \\ 430(445) \\ 1.9(3) \\ 24(74) \\ 1e-1 \\ \hline 938 \\ 214(240) \\ 304(431) \\ 2.1(2) \\ 45(105) \\ \hline 1e-1 \\ 14249 \\ 19(15) \\ 165(121) \\ \infty \\ 3.7(5) \\ \hline 1e-1 \\ 6.4e6 \\ \infty \\ \infty \end{array}$	$\begin{array}{lll} (8774) & \infty & \\ (2389) & \infty \\ (161) & 1(0) \\ \hline & 1e^{-2} \\ \hline & 51362 \\ 27(33) & \infty \\ \hline & 7.6(10) \\ 2.2(1) \\ \hline & 1e^{-2} \\ \hline & 1692 \\ 828(931) \\ 425(544) \\ 1.9(3) \\ 425(5149) \\ 1e^{-2} \\ \hline & 980 \\ 206(230) \\ 292(95) \\ 2.1(22) \\ 43(69) \\ \hline & 1e^{-2} \\ \hline & 27890 \\ 12(12) & \infty \\ \hline & 2.1(2) \\ \hline & 2 \\ \hline & 9.666 \\ \hline & \infty \\ \end{array}$	$\begin{array}{c} 0e5 & 1.2e\\ & \infty\\ & \infty\\ & \infty\\ & \infty\\ & \infty\\ & 0.9) & 1(0.9\\ & 1(0.9\\ & 1(0.9)\\ & 2.4(10.9)\\ & 2.1(10.9)\\ & 1e-3\\ & 1705\\ & 822(836)\\ & 422(495)\\ & 1.9(2)\\ & 25(21)\\ & 1e-3\\ & 1008\\ & 200(174)\\ & 284(371)\\ & 284(371)\\ & 1e-3\\ & 31654\\ & 15(18)\\ & \infty\\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ &$	$\begin{array}{c} 1.2e5 \\ & \sim \\ & 10.729 \\ & \sim \\ & 1.22(1) \\ & 1e-5 \\ & 1729 \\ & 11.92(1) \\ & 1e-5 \\ & 1729 \\ & 11.92(1) \\ & $	$\begin{array}{c} 1.2e5\\ \infty \ let5\\ \infty \ let5\\ \infty \ fee5\\ \infty \ fee5\\ \infty \ fee5\\ \infty \ fee4\\ 1 (0.7)\\ 1e-7\\ 55313\\ 2.5(27)\\ \infty \ fee5\\ 7.1(8)\\ 2.2(1)\\ 1e-7\\ 1757\\ 798(512)\\ 410(534)\\ 2.0(2)\\ 2.5(20)\\ 1e-7\\ 1068\\ 192(234)\\ 2.6(4)\\ 40(93)\\ 1e-7\\ 34256\\ 2.1(45)\\ \infty \ fee5\\ \infty \ fe$	15/15 0/15 0/15 0/15 15/15 1/15
A2 Carp A2 Zoub BFGS BIPOP-C  A fopt f8 A2 Carp A2 Zoub BFGS BIPOP-C  A fopt f9 A2 Carp	2.9(3) 3.6(9) 3.6(9) 5.0(4) 1e1 73 7.8(2) 4.0(2) 2.1(1) 3.2(2) 1e1 35 18(9) 4.5(2) 3.6(3) 5.8(1) 1e1 349 42(18) 241(907 1(0.3) 3.5(0. 1e1 143 138(61 3236(35 1(0.2) 5.8.4(1) 1e1 108 73(37) 165(593 1.1(1)	$\begin{array}{c} 2.0(2) \\ 2.0(2) \\ 577(1002) \\ \infty \\ 1.5(1) \\ 1e0 \\ 273 \\ 35(2) \\ 54(86) \\ 1.8(3)^4 \\ 3.7(3) \\ 1e0 \\ 127 \\ 18(8) \\ 243(646) \\ 3.0(0. \\ 8.7(4) \\ 1e0 \\ \hline \\ 500 \\ 42(18) \\ 0.679(850) \\ *^4 \ 1(0.3) \\ 6) \ 2.9(0. \\ 143(6) \\ 177) \ 1.8e4 \\ 2)^4 \ 1(0.6) \\ 39 \ 7.2(16) \\ 1e0 \\ 268 \\ 39(25) \\ 9.10(5) \\ *^4 \ 1(0.5) \\ *^6 \ 1(0.5) \\ *^6 \$	$\begin{array}{c} 7.6(0.3) \\ 658(1149) \\ \infty \\ 1(1) \\ 1e-1 \\ 336 \\ 36(2) \\ 50(12) \\ 2 \\ 1.6(2)^4 \\ 4.5(0.6 \\ 1e-1 \\ 22(2) \\ 1.65(2)^4 \\ 4.5(0.6 \\ 1e-1 \\ 22(2) \\ 1.50(75) \\ 7)^{\star 2} 2.0(1) \\ 7.2(2) \\ 1e-1 \\ 574 \\ 43(14) \\ 1446(119) \\ 1446(119) \\ 1446(119) \\ 12 \\ 763 \\ 3) \\ 54(31) \\ (2e4) \\ \infty \\ 1e-1 \\ 2 \\ 763 \\ 3) \\ 54(31) \\ (2e4) \\ \infty \\ 11) \\ 4 \\ 1.1((2e) \\ 1e-1 \\ 371 \\ 1) \\ 35(15) \\ 154(51) \\ 35(15) \\ 154(51) \\ 35(15) \\ 154(51) \\ 35(15) \\ 154(51) \\ 35(15$	$\begin{array}{c} 19(35) \\ 542(874) & \text{i} \\ 60(2) \\ 16-2 \\ 37(68) \\ 52(122) \\ 2 & 1.5(1)^4 \\ 16-2 \\ 263 \\ 25(6) \\ 133(115) \\ \star^3 & 1.8(0.6, 7(5) \\ 16-2 \\ 607 \\ 49(16) \\ 99(14)(623) \\ \star^4 & 1(0.2) \\ 40(2, 10) \\ 16-2 \\ 99(13) \\ 18(0.2) \\ 19(2, 10) \\ 10(2, 10$	$\begin{array}{c} 25(32) \\ 1112(1394) \\ \hline \times \\ 1(0.7) \\ 1e-3 \\ \hline \times \\ 391 \\ 37(3) \\ 55(69) \\ 3 \\ 1.5(0.4) \\ 4.8(2) \\ 1e-3 \\ \hline 300 \\ 26(6) \\ 128(284) \\ 7)^{*3} \\ 1.6(0.1) \\ 6.4(4) \\ 1e-3 \\ \hline 626 \\ 56(15) \\ 339960(1e4) \\ ^{*4} \\ 1(0.4) \\ .3) \\ 2.8(0.1) \\ 1e-3 \\ \hline \times \\ 461 \\ 1) \\ 166(42) \\ 4^{*4} \\ 1(0.1) \\ 10.1$	25(32) 1112(1652) 1(0.9) 1e-5 410 42(123) 64(37) 1,*3-1.5(0.4) 5.1(4) 1e-5 335 31(7) 140(93) 5)*3-1.5(0.3 6.3(4) 1e-5 829 56(14) 4513(762) *4 1.1(0.3) 2.3(0.1e-5 1467 89(55) \$9(272) 2)* 1.4(0.1 1e-5 1303) 1 8(3) 9) 73(8)	$\begin{array}{c} 25(47) \\ 25(47) \\ 25(47) \\ 27(47) \\ 27(47) \\ 27(47) \\ 3600 \\ 1(0.9) \\ 1e-7 \\ 422 \\ 48(6) \\ 76(48) \\ *31.5(0.2) \\ 5.4(3) \\ 1e-7 \\ 369 \\ 34(6) \\ 156(416) \\ 880 \\ 6.2(5) \\ 1e-7 \\ 880 \\ 68(20) \\ 20 \\ 2.4(0.5) \\ 1e-7 \\ 1673 \\ 112(142) \\ \infty \\ 5e5 \\ \infty \\ 4e4 \\ 1)^{4} \\ 1.3(0.1) \\ 1e-7 \\ 1494 \\ 19(2) \\ 64(85) \\ 19(92) \end{array}$	10/15 3/15 3/15 10/15 15/15 14/15 15/15	$\begin{array}{c} \textbf{f}_{19} \\ \textbf{A2} & \textbf{Carp} \\ \textbf{A2} & \textbf{Zoub} \\ \textbf{BFGS} \\ \textbf{BIPOP-C} \\ \textbf{\Delta}f_{\text{opt}} \\ \textbf{f}_{\textbf{20}} \\ \textbf{A2} & \textbf{Carp} \\ \textbf{A2} & \textbf{Carp} \\ \textbf{A2} & \textbf{Carp} \\ \textbf{A2} & \textbf{Carp} \\ \textbf{A2} & \textbf{Zoub} \\ \textbf{BFGS} \\ \textbf{BIPOP-C} \\ \textbf{\Delta}f_{\text{opt}} \\ \textbf{f}_{\textbf{22}} \\ \textbf{A2} & \textbf{Zoub} \\ \textbf{BFGS} \\ \textbf{BIPOP-C} \\ \textbf{\Delta}f_{\text{opt}} \\ \textbf{f}_{\textbf{23}} \\ \textbf{A2} & \textbf{Zoub} \\ \textbf{BFGS} \\ \textbf{BIPOP-C} \\ \textbf{\Delta}f_{\text{opt}} \\ \textbf{f}_{\textbf{24}} \\ \textbf{A2} & \textbf{Zoub} \\ \textbf{BFGS} \\ \textbf{BIPOP-C} \\ \textbf{\Delta}f_{\text{opt}} \\ \textbf{f}_{\textbf{24}} \\ \textbf{A2} & \textbf{Zoub} \\ \textbf{BFGS} \\ \textbf{BIPOP-C} \\ \textbf{D3} & \textbf{D4} \\ \textbf{D4} & \textbf{D5} \\ \textbf{D5} \\$	1(0) 1(0) 1655(12-2) 20(18) 1e1 16 2.4(2) 1.7(1) 1.8(0.9) 3.3(2) 1e1 41 2.0(2) 3.8(4) 2.3(2) 1e1 71 218(705) 170(228) 3.1(3) 6.9(15) 1.7(2) 1.7(2) 1.7(2) 1.7(2) 1.7(1) 1.6(226) 6.9(16) 1.7(1) 1.6(226) 6.9(16) 1.7(1) 1.6(226) 6.9(16) 1.7(1)	$\begin{array}{c} 1(0\\ 1(0)\\ 1(0)\\ 2(2) 2(4)\\ 2(2)\\ 2(3)\\ 2$	$\begin{array}{c} )  5960 \\ )  \infty \\ 4e4)  1780 \\ 434)  161 \\ 1e-1 \\ 38111 \\ 37(37) \\ \infty \\ 10(10) \\ 2.8(3) \\ 1e-1 \\ 1674 \\ 836(926) \\ 430(445) \\ 1.9(3) \\ 24(74) \\ 1e-1 \\ 938 \\ 214(240) \\ 304(431) \\ 2.1(2) \\ 45(105) \\ 1e-1 \\ 14249 \\ 19(15) \\ 165(121) \\ \infty \\ 3.7(5) \\ 1e-1 \\ 6.4e6 \\ \infty \\ \infty \\ \end{array}$	$\begin{array}{lll} (8774) & \infty & \\ (2389) & \infty & \\ (161) & 1(0) \\ \hline & 1e^2 & \\ \hline & 51362 \\ 27(33) & \infty & \\ & 7.6(10) \\ \textbf{2.2}(1) \\ \hline & 1e^2 & \\ \hline & 1692 \\ 828(931) \\ 425(544) & 1.9(3) \\ 226(230) & 292(95) \\ \textbf{2.1}(2) & \\ 43(69) & \\ \hline & 1e^2 & \\ \hline & 27890 \\ 12(12) & \infty & \\ \hline & \textbf{2.1}(2) \\ \hline & \infty & \\ \hline & \textbf{2.1}(2) \\ \hline & 9.666 & \\ & \infty & \\ & \infty & \\ \hline \end{array}$	0e5 1.2e  ∞  ∞  0e7  10e3  10e3  54470  26(60)  ∞  7.2(3)  2.1(0.9)  1e-3  1705  822(836)  422(495)  1.9(2)  1e-3  1008  200(174)  224(47)  1e-3  31654  15(18)  ∞  1.8(1)  1e-3  9.666  ∞  ∞  ∞	$\begin{array}{c} 1.2e5 \\ \infty \\ $	$\begin{array}{c} 1.2e5\\ \infty \ leb \\ 1.2e5\\ \infty \ leb \\ 5.6e5\\ \infty \ 5.6e5\\ \infty \ 5.6e5\\ \infty \ 5.6e5\\ \infty \ 5.6e5\\ 0.1(0.7)\\ 1e-7\\ 175\\ 798(512)\\ 410(534)\\ 2.0(2)\\ 2.5(20)\\ 1e-7\\ 1068\\ 192(234)\\ 2.6(4)\\ 40(93)\\ 1e-7\\ 1.3e7\\ \infty \ 5.e5\\ 0.2e4\\ 1.8(2)\\ 1e-7\\ 1.3e7\\ \infty \ 5.e5\\ \infty \ 5.e$	15/15 0/15 0/15 0/15 15/15 15/15 1/1
A2 Carp A2 Zoub BFGS BIPOP-C	2.9(3) 3.6(9) 3.6(9) 5.0(4) 1e1 73 7.8(2) 4.0(2) 2.1(1) 3.2(2) 1e1 35 18(9) 4.5(2) 3.6(3) 5.8(1) 1e1 349 42(18) 241(907 1(0.3) 3.5(0. 1e1 143 138(61 3236(35 1(0.2) 5.8.4(1) 1e1 108 73(37) 165(593 1.1(1)	$2.0(2)$ $2.0(2)$ $577(1002)$ $\infty$ $1.5(1)$ $1e0$ $273$ $35(2)$ $54(86)$ $1.8(3)^*$ $3.7(3)$ $1e0$ $127$ $18(8)$ $243(646;$ $3.0(0.$ $8.7(4)$ $1e0$ $500$ $44(810)$ $60$ $10$ $10$ $10$ $10$ $10$ $10$ $10$ $1$	$\begin{array}{c} 7.6(0.3) \\ 658(1149) \\ \infty \\ 1(1) \\ 1e-1 \\ 336 \\ 36(2) \\ 50(12) \\ 2 \\ 1.6(2)^4 \\ 4.5(0.6 \\ 1e-1 \\ 22(2) \\ 1.6(2)^4 \\ 4.5(0.6 \\ 1e-1 \\ 214 \\ 22(2) \\ 1.5(0.6) \\ 1.50(75) \\ 7)^{\star 2} 2.0(1) \\ 7.2(2) \\ 1e-1 \\ 574 \\ 43(14) \\ 1446(119) \\ 1446(119) \\ 2 \\ 763 \\ 3) \\ 54(31) \\ (2e4) \\ \infty \\ 1.1)^{\star 4} \\ 1.1((2) \\ 2.2((2e4) \\ 1e-1 \\ 371 \\ 35(15) \\ 154(51) \\ 35(15) \\ 154(51) \\ 35(15) \\ 15(51) \\ 154(51) \\ 37(15) \\ 154(51) \\ 37(15) \\ $	$\begin{array}{c} 19(35) \\ 542(874) & \text{i} \\ 60(2) \\ 16-2 \\ 37(68) \\ 52(122) \\ 2 & 1.5(1)^4 \\ 16-2 \\ 263 \\ 25(6) \\ 133(115) \\ \star^3 & 1.8(0.6, 7(5) \\ 16-2 \\ 607 \\ 49(16) \\ 99(14)(623) \\ \star^4 & 1(0.2) \\ 40(2, 10) \\ 16-2 \\ 99(13) \\ 18(0.2) \\ 19(2, 10) \\ 10(2, 10$	$\begin{array}{c} 25(32) \\ 1112(1394) \\ \hline \times \\ 1(0.7) \\ 1e-3 \\ \hline \times \\ 391 \\ 37(3) \\ 55(69) \\ 3 \\ 1.5(0.4 \\ 4.8(2) \\ 1e-3 \\ \hline \\ 300 \\ 26(6) \\ 128(284) \\ 7)^{*3} \\ 1.6(0.3 \\ 6.4(4) \\ 1e-3 \\ \hline \\ 626 \\ 56(15) \\ 35960(1e4) \\ *^{4} \\ 1(0.4) \\ .3) \\ 2.8(0. \\ 1e-3 \\ \hline \\ 1177 \\ 60(69) \\ \infty \\ ) \\ 8.2(4) \\ .2) \\ 1.6(0.2 \\ 1e-3 \\ \hline \\ 4611 \\ 1) \\ 166(42) \\ 4)^{*4} \\ 1(0.1) \\ 10.1 \\ 10.2 \\ 1$	25(32) 1112(1652) 1(0.9) 1e-5 410 42(123) 64(37) 1,*3-1.5(0.4) 5.1(4) 1e-5 335 31(7) 140(93) 5)*3-1.5(0.3 6.3(4) 1e-5 829 56(14) 4513(762) *4 1.1(0.3) 2.3(0.1e-5 1467 89(55) \$9(272) 2)* 1.4(0.1 1e-5 1303) 1 8(3) 9) 73(8)	$\begin{array}{c} 25(47) \\ 25(47) \\ 25(47) \\ 27(47) \\ 27(47) \\ 27(47) \\ 3600 \\ 1(0.9) \\ 1e-7 \\ 422 \\ 48(6) \\ 76(48) \\ *31.5(0.2) \\ 5.4(3) \\ 1e-7 \\ 369 \\ 34(6) \\ 156(416) \\ 880 \\ 6.2(5) \\ 1e-7 \\ 880 \\ 68(20) \\ 20 \\ 2.4(0.5) \\ 1e-7 \\ 1673 \\ 112(142) \\ \infty \\ 5e5 \\ \infty \\ 4e4 \\ 1)^{4} \\ 1.3(0.1) \\ 1e-7 \\ 1494 \\ 19(2) \\ 64(85) \\ 19(92) \end{array}$	10/15 3/15 3/15 10/15 18/15 18/15 14/15 15/15	A2 Carp A2	1(0) 1(0) 1655(12-2) 20(18) 1e1 16 2.4(2) 1.7(1) 1.8(0.9) 3.3(2) 1e1 41 2.0(2) 3.8(4) 2.3(2) 1e1 71 218(705) 170(228) 3.1(3) 6.9(15) 1.7(2) 1.7(2) 1.7(2) 1.7(2) 1.7(1) 1.6(226) 6.9(16) 1.7(1) 1.6(226) 6.9(16) 1.7(1) 1.6(226) 6.9(16) 1.7(1)	$\begin{array}{c} 1(0\\ 1(0)\\ 1(0)\\ 2(0)\\ $	$\begin{array}{c} )  5960 \\ )  \infty \\ 4e4)  1780 \\ 434)  161 \\ \hline 1e-1 \\ 38111 \\ 37(37) \\ \infty \\ 10(10) \\ 2.8(3) \\ \hline 1e-1 \\ 1674 \\ 836(926) \\ 430(445) \\ 1.9(3) \\ 24(74) \\ 1e-1 \\ \hline 938 \\ 214(240) \\ 304(431) \\ 2.1(2) \\ 45(105) \\ \hline 1e-1 \\ 14249 \\ 19(15) \\ 165(121) \\ \infty \\ 3.7(5) \\ \hline 1e-1 \\ 6.4e6 \\ \infty \\ \end{array}$	$\begin{array}{lll} (8774) & \infty & \\ (2389) & \infty \\ (161) & 1(0) \\ \hline & 1e^{-2} \\ \hline & 51362 \\ 27(33) & \infty \\ \hline & 7.6(10) \\ 2.2(1) \\ \hline & 1e^{-2} \\ \hline & 1692 \\ 828(931) \\ 425(544) \\ 1.9(3) \\ 425(5149) \\ 1e^{-2} \\ \hline & 980 \\ 206(230) \\ 292(95) \\ 2.1(22) \\ 43(69) \\ \hline & 1e^{-2} \\ \hline & 27890 \\ 12(12) & \infty \\ \hline & 2.1(2) \\ \hline & 2 \\ \hline & 9.666 \\ \hline & \infty \\ \end{array}$	$\begin{array}{c} 0e5 & 1.2e\\ & \infty\\ & \infty\\ & \infty\\ & \infty\\ & \infty\\ & 0.9) & 1(0.9\\ & 1(0.9\\ & 1(0.9)\\ & 2.4(10.9)\\ & 2.1(10.9)\\ & 1e-3\\ & 1705\\ & 822(836)\\ & 422(495)\\ & 1.9(2)\\ & 25(21)\\ & 1e-3\\ & 1008\\ & 200(174)\\ & 284(371)\\ & 284(371)\\ & 1e-3\\ & 31654\\ & 15(18)\\ & \infty\\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ &$	$\begin{array}{c} 1.2e5 \\ & \sim \\ & 10.729 \\ & \sim \\ & 1.22(1) \\ & 1e-5 \\ & 1729 \\ & 11.92(1) \\ & 1e-5 \\ & 1729 \\ & 11.92(1) \\ & $	$\begin{array}{c} 1.2e5\\ \infty \ let5\\ \infty \ let5\\ \infty \ fee5\\ \infty \ fee5\\ \infty \ fee5\\ \infty \ fee4\\ 1 (0.7)\\ 1e-7\\ 55313\\ 2.5(27)\\ \infty \ fee5\\ 7.1(8)\\ 2.2(1)\\ 1e-7\\ 1757\\ 798(512)\\ 410(534)\\ 2.0(2)\\ 2.5(20)\\ 1e-7\\ 1068\\ 192(234)\\ 2.6(4)\\ 40(93)\\ 1e-7\\ 34256\\ 2.1(45)\\ \infty \ fee5\\ \infty \ fe$	15/15 0/15 0/15 0/15 15/15 1/15

Table 1: Average running time (aRT in number of function evaluations) divided by the respective best aRT measured during BBOB-2009 in dimension 5. The aRT and in braces, as dispersion measure, the half difference between 10 and 90%-tile of bootstrapped run lengths appear for each algorithm and target, the corresponding best aRT in the first row. The different target  $\Delta f$ -values are shown in the top row. #succ is the number of trials that reached the (final) target  $f_{\rm opt}+10^{-8}$ . The median number of conducted function evaluations is additionally given in *italics*, if the target in the last column was never reached. Entries, succeeded by a star, are statistically significantly better (according to the rank-sum test) when compared to all other algorithms of the table, with p = 0.05 or  $p = 10^{-k}$  when the number k following the star is larger than 1, with Bonferroni correction of 110. A  $\downarrow$  indicates the same tested against the best algorithm of BBOB-2009. Best results are printed in bold.

_70bt	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ	$\frac{\Delta f_{\mathrm{opt}}}{\mathbf{f13}}$	1e1 652	1e0 2021	1e-1 2751	1e-2 3507	1e-3	1e-5 9 2445	1e-7 5 30201	#succ
	43 $159(16)$	43 273(16)	43 393(22)	43 $507(23)$	43 $746(19)$	43 $979(23)$	$\frac{15/15}{15/15}$	A2 Carp	33(0.4)	39(26)	72(37)	89(79)	76(6	7) ∞	∞ 2e5	0/15
A2 Zoub 5.6(0.8) BFGS 1(0)*4	12(1) 1(0)*4	18(2) 1(0)*4	25(2) 1(0)*4	31(3) 1(0)*4	43(3) 1(0)*4	56(2) 1(0)*4	15/15 15/15	A2 Zoub BFGS	131(488) 1.7(0.3	248(415) 3)* <sup>3</sup> <b>1</b> (0.0)	938(1292 (0.0)	!) 2517(217 *2 <b>1</b> (0.1			$\infty 2e6$ $\infty 5e5$	0/15 0/15
BIPOP-C 7.9(1)	14(2)	20(2)	26(3)	33(3)	45(3)	57(4)	15/15	BIPOP-C		2.7(5)	5.1(6)	6.2(4		(0.7) <b>2.3</b> $(2)$		15/15
$ \begin{array}{c c} \Delta f_{\text{opt}} & \text{1e1} \\ \hline \mathbf{f2} & 385 \end{array} $	1e0 386	1e-1 387	1e-2 388	1e-3 390	1e-5 391	1e-7 393	#succ 15/15	$\frac{\Delta f_{\mathrm{opt}}}{\mathbf{f} 14}$	1e1 75	1e0	1e-1 304	1e-2 451	1e-3	1e-5	1e-7	#succ 15/15
A2 Carp 56(5)	74(6)	93(7)	112(6)	127(7)	162(8)	197(10)	15/15	A2 Carp	5.4(3)	22(4)	39(2)	48(4)	41(2)	∞	∞ 2e5	0/15
A2 Zoub 21(2) BFGS <b>20</b> (4)	25(4) 24(5)	26(4) 26(4)	27(4) 27(4)	28(3) 27(4)	29(6) 28(3)	31(5) 28(2)	$\frac{15/15}{15/15}$	A2 Zoub BFGS	2.3(2) 2.7(1)	2.4(0.6) 1.8(0.6)	3.3(0.6) 2.0(0.8)*	3.8(0.3) 31.8(0.4)*			∞ 2e6 1 * 4 2e5	0/15 0/15
BIPOP-C 35(7)	40(3)	44(2)	45(3) 1e-2	47(3)	48(2)	50(2) 1e-7	15/15	BIPOP-C		2.9(0.5)			4.1(0.4)		1.2(0.1)*	
	1e0 7626	1e-1 7635	7637	1e-3 7643	1e-5 7646	7651	#succ 15/15	$\Delta f_{ m opt}$	1e1 30378	1e0 1.5e5	1e-1 3.1e5	1e-2 3.2e5	1e-3 3.2e5	1e-5 4.5e5	1e-7	#succ
A2 Carp ∞ A2 Zoub ∞	∞ ∞	∞ ∞	∞ ∞	∞ ∞	∞ ∞	$\infty$ 2e5 $\infty$ 2e6	0/15 0/15	f15 A2 Carp	∞	∞	∞	∞	∞	∞	4.6e5 ∞ 2e5	$\frac{15/15}{0/15}$
BFGS ∞	∞	∞	∞	∞	∞	$\infty$ 1e5	0/15	A2 Zoub BFGS	∞ ∞	∞	∞	∞	∞ ∞	∞	∞ 2e6 ∞ 1e5	0/15 0/15
BIPOP-C $12(6)^{*4}$ $\Delta f_{opt}$   1e1	∞ 1e0	∞ 1e-1	∞ 1e-2	∞ 1e-3	∞ 1e-5	∞ 6e6 1e-7	0/15 #succ	BIPOP-C	1(0.4)*4	<b>2.0</b> (1.0)	1.4(0.4)	1.4(0.5)	1.4(0.5)	1(0.4)	1(0.3)	15/15
$ \begin{array}{c c} \Delta f_{\text{opt}} & \text{1e1} \\ \hline \mathbf{f4} & 4722 \end{array} $	7628	7666	7686	7700	7758	1.4e5	9/15	$\frac{\Delta f_{\text{opt}}}{\mathbf{f} 16}$	1e1 1384	1e0 27265	1e-1 77015	1e-2 1.4e5	1e-3 1.9e5	1e-5 2.0e5	1e-7 2.2e5	#succ
A2 Carp ∞ A2 Zoub ∞	∞ ∞	∞ ∞	∞ ∞	∞ ∞	∞ ∞	$\infty 2e5$ $\infty 2e6$	$0/15 \\ 0/15$	A2 Carp	20(2)	14(9)	<i>∞</i>	 ∞	∞	2.0e5 ∞	$\infty 2e5$	0/15
BFGS ∞	∞	∞	∞	∞	∞	$\infty$ 2e5	0/15	A2 Zoub BFGS	246(399) ∞	∞	∞	∞ ∞	∞ ∞	∞	$\infty$ 2e6 $\infty$ 3e5	0/15 0/15
BIPOP-C $\infty$ $\Delta f_{opt}$   1e1	∞ 1e0	∞ 1e-1	∞ 1e-2	∞ 1e-3	∞ 1e-5	∞ 6e6 1e-7	0/15 #succ	BIPOP-C	1.7(0.5	5) <b>1.0</b> (0.	6)* <b>1.2</b> (1.0	)*4(0.6)*	4 <b>1</b> (0.9)*4	1 <b>1</b> (0.5)*4		15/15
$\frac{\Delta f_{ m opt}}{{ m f5}}$ 1e1	41	41	41	41	41	41	15/15	$\Delta f_{ m opt}$	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
A2 Carp 7.4(2) A2 Zoub 8.2(2)	8.5(2) 9.4(2)	8.6(3) 10(2)	8.6(2) 10(2)	8.6(2) 10(2)	8.6(2) 10(2)	8.6(2) 10(3)	$\frac{15}{15}$	f17 A2 Carp	63 2.4(3)	1030 10(2)	4005 7.0(1)	12242 5.0(4)	30677 9.1(7)	56288 ∞	80472 $\infty 2e5$	$\frac{15/15}{0/15}$
BFGS 2.4(0.4)*	4.7(0.3)*	42.8(0.3)*	42.8(0.3)*	42.8(0.1)	$^{4}$ <b>2.8</b> $(0.8)$	$^{*4}$ <b>2.8</b> $(0.5)$	45/15	A2 Zoub BFGS	1.5(1) 359(591)	3.2e4(3 ∞	ie4)∞	∞ ∞	∞ ∞	∞	$\infty 2e6$ $\infty 4e5$	0/15 0/15
BIPOP-C 5.1(0.8) $\Delta f_{\text{opt}}$   1e1	6.2(0.9) 1e0	6.3(1) 1e-1	6.3(1) 1e-2	6.3(1) 1e-3	6.3(1) 1e-5	6.3(1) 1e-7	15/15  #succ	BIPOP-C			*4 <b>1</b> (0.8)*			*3.3(0.8)		15/15
	2343	3413	4255	5220	6728	8409	15/15	$\Delta f_{ m opt}$	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
A2 Carp 10(0.7) A2 Zoub 1.4(0.2)	8.9(0.3) 1.3(0.4)		8.8(0.4) $1.3(0.3)$	8.8(0.5) $1.3(0.4)$	9.2(0.4) $1.5(0.4)$	9.3(0.2) $1.6(0.3)$	$\frac{15/15}{15/15}$	f18 A2 Carp	621 7.4(2)	3972 5.8(0.9)	19561 4.0(3)	28555 32(42)	67569 44(54)	1.3e5 ∞	1.5e5 ∞ 2e5	$\frac{15/15}{0/15}$
BFGS 3.6(2)	3.5(1)	3.4(0.9)	3.5(0.8)	3.5(1.0)	3.6(0.7)	45(38)	0/15	A2 Zoub BFGS	622(1925 ∞	) ∞ ∞	∞ ∞	∞ ∞	∞ ∞	∞ ∞	$\infty$ 2e6 $\infty$ 4e5	0/15 0/15
! ` ′	1.3(0.2) 1e0	1.2(0.2) 1e-1	1.1(0.2) 1e-2	1.1(0.1) 1e-3	1.2(0.1)* 1e-5	<sup>3</sup> 1.2(0.1)* 1e-7	#succ	BIPOP-C		1)*2.4(2)*				* <b>1.7</b> (0.5)		15/15
f7 1351	4274					16969	15/15	$\Delta f_{ m opt}$	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
A2 Carp 6.9(1) A2 Zoub 1166(133		150(14	12) ∞	∞	∞ ∞	$\infty$ 2e5 $\infty$ 2e6	0/15 0/15	f19 A2 Carp	1 1(0)	1(0)	3.4e5 ∞	4.7e6 ∞	6.2e6 ∞	6.7e6 ∞	6.7e6 ∞ 2e5	$\frac{15/15}{0/15}$
BFGS ∞	∞	∞	∞ ∞	∞	∞	≈ 2100	0/15	A2 Zoub BFGS	1(0) 1.2e6(1e	1(0) 6) ∞	∞ ∞	∞ ∞	∞ ∞	∞ ∞	$\infty$ 2e6 $\infty$ 2e5	$0/15 \\ 0/15$
BIPOP-C $1(0.9)$ $\Delta f_{\text{opt}}$ 1e1	1e0	1e-1	1) <b>2.2</b> (0	1e-3	2) 2.2(0.2 1e-5	) <b>2.1</b> (0.2) 1e-7	15/15  #succ	BIPOP-C		2.4e4(3	Be4 <b>].2</b> (1)	1(0.3)	1(0.3)	1(0.2)	1(0.2)	15/15
f8 2039	3871	4040	4148	4219	4371	4484	15/15	$\frac{\Delta f_{\text{opt}}}{\mathbf{f} 20}$	1e1 82	1e0 46150	1e-1 3.1e6	1e-2 5.5e6	1e-3 5.5e6	1e-5 5.6e6	1e-7 5.6e6	#succ 14/15
A2 Carp 15(1) A2 Zoub 5.4(2)	12(0.3) $17(2)$	12(7) $18(40)$	14(0.7) $18(3)$	16(0.5) 19(38)	20(6) 21(2)	42(33) 23(3)	1/15 $15/15$	A2 Carp	28(11)	2.0(2)	∞	∞	∞	∞	$\infty$ 2e5	0/15
BFGS 1.8(0.4)	*21.2(0.1)	*4 <b>1.2</b> (0.1	$^{*4}_{1.2}(0.1)$	$*4_{1.2(0.2)}$	$*4_{1.2(0.2)}$	*41.2(0.2)	15/15	A2 Zoub	3.7(0.8) 2.1(0.4)	∞ 1*35.8(4)	∞ ∞	∞	∞	∞	$\infty$ 2e6 $\infty$ 4e5	0/15 0/15
BIPOP-C $4.0(1)$ $\Delta f_{\text{opt}} = 1e1$	4.0(0.7) 1e0	4.3(0.3) 1e-1		, ,	4.6(1.0) 1e-5	4.6(0.5) 1e-7	#succ	BIPOP-C	4.3(0.9)	9.2(2)	<b>1</b> (0.0)	<b>1</b> (0.9)	<b>1</b> (0.3)	<b>1</b> (0.5)	1(0.3)	14/15
f9   1716	3102	3277	3379	3455	3594	3727	15/15	$\frac{\Delta f_{\text{opt}}}{\text{f21}}$	1e1 561	1e0 6541	1e-1 14103	1e-2 14318	1e-3	1e-5 15567	1e-7	#succ 15/15
A2 Carp 19(2) A2 Zoub 10(6)	22(16) 59(168)	23(9) 66(153)		26(22) $99(174)$ 1	27(15) 138(60)	31(15) 174(217)	$\frac{13}{15}$	A2 Carp	43(3)	49(88)	23(34)	23(12)	23(36)	22(26)	19(28)	6/15
BFGS <b>2.2</b> (0.4) BIPOP-C 4.7(1)	*4.2(1)**	4 <b>2.1</b> (0.9) 6.0(3)	*4.1(1)*4	2.0(0.9)* 6.1(3)	42.0(1)*4	1.9(1)*4 6.1(0.9)		A2 Zoub BFGS	457(1020 1.9(3)	)388(701) <b>5.5</b> (6)	368(393) 4.6(5)	362(651) 4.6(2)	354(441) 4.5(5)	333(517) 4.3(3)	295(405) <b>7.3</b> (9)	5/15 2/15
$\Delta f_{\mathrm{opt}}$ 1e1	5.7(5) 1e0	6.0(3) 1e-1	6.1(3) 1e-2	6.1(3) 1e-3	6.1(4) 1e-5	1e-7	#succ	BIPOP-C	!	5) 55(25)	48(23)	47(58)	46(93)	43(86)	39(54)	13/15
f10 7413	8661	10735	13641	14920	17073	17476	15/15	$\frac{\Delta f_{\text{opt}}}{\mathbf{f22}}$	1e1 467	1e0 5580	1e-1 23491	1e-2 24163	1e-3 24948	1e-5 26847	1e-7 1.3e5	#succ 12/15
A2 Carp 403(465) A2 Zoub 900(1015		∞ )1341(251	∞ 1)1060(20	∞ 20)973(637	∞ 7) 1746(17	$\infty 2e5$ $(28) \infty 2e6$	0/15 0/15	A2 Carp		151(188)	∞	∞	∞	∞	$\infty$ 2e5	0/15
		1(0.3)		0.6) 1.1(0			0/15	A2 Zoub BFGS	2.5(1)	1.8(4)	8.1(11)	7.9(4)	677(694) <b>7.7</b> (14)	10(6)	125(221) 14(11)	$\frac{2}{15}$ $0/15$
	1.8(0.0	1.6(0. 1e-1	1) 1.3(0 1e-2	.1) 1.2(0 1e-3	.0) <b>1.1</b> (	0.0) <b>1.1</b> (0.0	#succ	BIPOP-C	6.8(14)	13(21) 1e0	215(276) 1e-1	209(326) 1e-2	202(240) 1e-3	188(273) 1e-5	37(35) 1e-7	5/15 #succ
f11 1002	2228	6278	8586	9762	12285	14831	15/15	$\frac{\Delta f_{\text{opt}}}{\mathbf{f23}}$	3.2	1614	67457	3.7e5	4.9e5	8.1e5	8.4e5	#succ 15/15
A2 Carp ∞ A2 Zoub ∞	∞ ∞	∞ ∞	∞ ∞	∞	∞	$\infty$ 2e5 $\infty$ 2e6	0/15	A2 Carp	2.0(3) 1.9(1)	∞ 312(449)	∞	∞ ∞	∞ ∞	∞ ∞	∞ 2e5 ∞ 2e6	$0/15 \\ 0/15$
BFGS 1(0.7)*4	1 <b>1</b> (0.8)*4	<b>1.3</b> (0.5)	*2.6(3)	147(89)	∞	$\infty$ 2e5	0/15	BFGS	47(16)	304(261)	∞	∞	∞	∞	∞ 1e5	0/15
BIPOP-C 10(0.5)	` ′	1.9(0.0)	` ,	,	, , ,	*4. <b>o</b> (0.0)*	'	$\Delta f_{ m opt}$	4.3(4)  1e1	32(24) 1e0	1(0.9) 1e-1	1.7(0.7) 1e-2	2.0(1) 1e-3	1.2(0.8) 1e-5	1.2(0.7) 1e-7	15/15 #succ
$\frac{\Delta f_{ m opt}}{{ m f12}}  \frac{1{ m e1}}{1042}$	1e0 1 1938	1e-1 2740	1e-2 3156	1e-3 4140	1e-5 12407	1e-7 13827	#succ 15/15	f24	1.3e6	7.5e6	5.2e7	5.2e7	5.2e7	5.2e7	5.2e7	3/15
A2 Carp 34(10)	32(11)	32(14)	45(19)	125(233)	233(197		0/15	A2 Carp A2 Zoub	∞ ∞	∞ ∞	∞ ∞	∞ ∞	∞ ∞	∞ ∞	$\infty$ 2e5 $\infty$ 2e6	$0/15 \\ 0/15$
BFGS 1.6(2)*5			) <b>1.7</b> (0.5		*2 1.8(3)	45(68)	1/15	BFGS BIPOP-C	∞ 21(1)	∞ 1(0.9)	∞ 1(0.8)	∞ 1(2)	∞ 1(1)	∞ 1(0.6)	∞ 1e5 1(1)	$0/15 \\ 3/15$
BIPOP-C 3.0(0.2)	4.0(4)	4.5(4)	4.9(3)	4.5(2)	1.9(0.	9) <b>2.0</b> (0.7	15/15	211 01 -0	-(±)	1(0.0)	1(0.0)	-(2)	-(1)	1(0.0)	-(1)	P/ 10

Table 2: Average running time (aRT in number of function evaluations) divided by the respective best aRT measured during BBOB-2009 in dimension 20. The aRT and in braces, as dispersion measure, the half difference between 10 and 90%-tile of bootstrapped run lengths appear for each algorithm and target, the corresponding best aRT in the first row. The different target  $\Delta f$ -values are shown in the top row. #succ is the number of trials that reached the (final) target  $f_{\rm opt} + 10^{-8}$ . The median number of conducted function evaluations is additionally given in *italics*, if the target in the last column was never reached. Entries, succeeded by a star, are statistically significantly better (according to the rank-sum test) when compared to all other algorithms of the table, with p = 0.05 or  $p = 10^{-k}$  when the number k following the star is larger than 1, with Bonferroni correction of 110. A  $\downarrow$  indicates the same tested against the best algorithm of BBOB-2009. Best results are printed in bold.