FLEXIBILITY VERSUS SECURITY IN AGENCY CONTRACTS WITH MORAL HAZARD

Lorenzo Bozzoli* and Guillaume Pommey†

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Abstract

We extend the principal-agent model of moral hazard with limited liability by introducing uncertainty and private information concerning the principal's reservation payoff. We characterize the optimal design of the principal's exit rights from the incentive contract, highlighting a trade-off between securing the agent's investments and preserving the principal's flexibility to abandon trade when opportunity costs are high. We argue that our framework provides an efficiency rationale for employment protection laws that impose a minimal liquidation fee for contractual terminations, and we discuss how this study could offer a micro-foundation for limited commitment that does not rely on contractual incompleteness or bounded rationality.

1 Introduction

When contracting over a project for a long period of time, it is common for one party to encounter better outside opportunities before the formal end of the relationship. For example, in 2016 Disney gave Netflix exclusive rights to stream all Disney's new releases and promote their content on their platform. Three years later, foreseeing a better future on its own, Disney was ready to launch their in-house streaming services, Disney+, and unilaterally decided to remove its content from Netflix by the end of 2019.[1]

While early termination clauses are useful to seize attractive outside opportunities, they also evidently weaken the strength of the initial agreement by subjecting parties to the risk of exerting relation-specific effort without reward. Therefore, any such agreement faces a trade-off between providing some flexibility to exit but also guaranteeing enough security for parties to be properly incentivized to invest in the relationship.

*Tor Vergata University of Rome.
†Tor Vergata University of Rome.
[1]Source: https://www.imdb.com/news/ni61387953/.

This trade-off has long been recognized but its causes and its analysis are usually left implicit and modelled in reduced-form. It appears most notably in the literature on limited commitment and the holdup problem which typically assumes that due to the complexity of the environment or the bounded rationality of the parties, it is too costly to write a contract that incorporates all possible contingencies.[2] The demand for flexibility is often modeled as an exogenous constraint on the set of available contracts rather than an equilibrium outcome.[3]

In this paper, we explicitly address this trade-off without relying on prior assumption on limited commitment nor contract incompleteness. We assume that a principal can hire an agent to work on a *project* whose success depends on the unobservable level of effort exerted by the agent. This problem gives rise to the standard principal-agent moral hazard setting in which we also assume bilateral risk-neutrality and limited liability of the agent. We further assume that the principal has an *outside option* that she discovers in the course of the relationship with the agent. More precisely, the outside option value is privately revealed to the principal after the agent has exerted effort but before the project outcome is realized. We assume that the principal possesses full commitment power and can design contracts that condition the decision to carry on the project on the realized value of the outside option, as well as the final payments on both the project outcome and the outside option's value.

We show that three types of contracts can emerge at equilibrium: *lock-in* contracts that force the principal to always complete the project; *partially-secured* contracts that allow the principal to cancel the project upon paying a penalty fee to the agent; *at-will* contracts that enable the principal to withdraw without consequences. We characterize the principal's optimal choice among these three types which is ultimately determined by three key parameters: the intensity of the agency costs, the asymptotic hazard rate of the outside option's distribution, and the expected output when the agent exerts high effort.

Importantly, we also find that the principal optimally chooses to distort the allocative efficiency of the termination policy. In our framework, ex ante allocative efficiency is such that the project should be terminated if and only if its expected value (gross of the *sunk* agency costs) is lower than the realized value of the outside option. We show instead that the equilibrium termination policy depends only on the direct comparison between the agency costs and the statistical properties of the outside option. Substantially, the costlier it is to incentivize effort-taking – either because of the magnitude of the investment or the noisiness of output as a signal of effort– the more secure is the agent's contract with respect to project completion. A lower asymptotic hazard rate of the reservation payoff's distribution also pushes in favor of a more secure contract. The gross expected value of the project, however, is only relevant in determining the amount of the termination fee

[2]See for example (Laffont and Tirole, 1988, p. 1154): *"contracts are costly to write and contingencies are often hard to foresee, which gives rise to the allocation of discretion to some members of the organization"*. Also, concerning the foundation of the hold-up problem as an issue of incomplete contracting, Tirole (1986, p. 239) writes: *"traditional explanation of incomplete contracts relies on the existence of transaction costs. First, some contingencies may not be foreseeable. Second, it may be expensive and time consuming to write the (high number of) foreseeable ones into a contract. Third, some contingencies may be private information (as is the case here). A complete contract must then specify an "incentive-compatible" mechanism of information transmission, which makes it particularly expensive"*.

[3]See, for example, (Brzustowski et al., 2023, p. 1338): *"In other words, contracting parties may refrain from signing long-term, binding contracts due to potential unforeseen or noncontractible contingencies even if such contracts were feasible [...] Although we do not model these contingencies, our assumption that the seller cannot commit not to switch off a deployed contract embodies the idea that she prefers a contract allowing for discretion in the future"*.

and the type of the contract (partially-secure or at-will). This highlights that the termination fee serves the purpose of disciplining the principal future self's exit behavior; in particular, it allows to enhance trade security through the reduction of the frequency with which she abandons the project, at the cost of some allocative distortions.[4]

This distortive effect and the role of the termination fee stand in apparent contrast with the seminal works of the law and economics literature. Shavell (1980), for instance, shows that introducing an appropriate cancellation fee induces contractual termination whenever it is Pareto-efficient, while still preserving the incentives for efficient investments.[5] This may wrongly suggest that a monopolistic principal should simply maximize gains from trade by picking the ex ante efficient fee, and then appropriate the whole surplus. Our results demonstrate however that this insight does not hold in the moral hazard problem with limited liability and asymmetric information on the principal's side. In that setting, the agent's effort lacks a selfish component, and the reward for effort-taking is fully embedded in the state-contingent payments, with the implication that the termination fee cannot play a direct role in restoring the agent's incentives when trade lacks security. As a result, more intricate dynamics come into play, which make the use of termination fees non-trivial.

We want to stress that our results crucially stem from our key assumption on the ex ante uncertainty about the principal's outside option and its private nature at the interim stage. Despite her ability to write the initial contract contingent on any future realized value of the outside option, the fact that the principal privately observes this value creates an incomplete information problem at the interim stage. In other words, when writing the initial contract, the principal must take into account the incentive constraint that ensures truthful reporting by her future self. This additional asymmetric information friction is enough to give rise to contractual forms typically associated with limited commitment or contract incompleteness features, without requiring additional institutional or behavioral assumptions. We believe that the existence of *at-will* contracts offers a micro-foundation for contractual forms typically associated with limited commitment. Additionally, we show that partially-secured contracts endogenously exhibit a form of *apparent* contract incompleteness, as the liquidation fee does not depend on the value of the outside option.

Finally, we provide a normative assessment of the optimal contractual policy from the point of view of an utilitarian social planner. Our analysis shows that the equilibrium outcome generally fails to maximize social surplus because of the time-inconsistency of the principal's preferred level of trade security before and after the effort choice is irreversibly taken by the agent. We demonstrate in particular that, whenever an at-will contract is optimal for the principal at equilibrium, it is not socially optimal. Therefore, in our context, contracts featuring at-will termination are always linked to the presence of allocative distortions brought by the principal's monopolistic position. We argue that this creates an efficiency rationale for a social planner to impose a minimum compensation fee, strengthening the security of trade when decentralized contracting fails to induce an efficient level, even when the planner is not biased in favor of the agent or inequality-averse. This observation aligns, for example, with the existence of employment protection laws in various countries that

[4]These distortions are similar to the virtual costs in Rochet and Stole (2002) model of random participation constraints, and are likewise captured by the inverse hazard rate of the distribution of the principal's outside payoff.

[5]The fact that such fees are sometimes unobserved or not enforced in practice, or lead to an ex ante inefficient investment level, is typically attributed to the presence of renegotiation frictions (Rogerson, 1984; Tirole, 1986).

prohibit at-will termination policies in labor contracts.

Related literature. The seminal work by Shavell (1980) provides the standard framework for the study of optimal damage compensations for contractual breach, comparing the impact of different policies on ex ante investments. Rogerson (1984) extends the analysis to the case of ex post renegotiation. Both models address a different framework than ours, and provide a normative analysis, while ours lets the principal design the penalty in a monopolistic setting.[6] Tirole (1986) also critiques the effectiveness of termination fees, in a model of procurement with renegotiation, by arguing that their adverse effects on the parties' bargaining power may actually discourage investments.

On a more related note to ours, Levitt and Snyder (1997) tackle the topic of contractual termination within the textbook moral hazard setting; in particular, they assume that the agent privately observes whether trade is profitable for the principal after exerting effort. They show that the principal may optimally provide the agent with the power to rescind the contract as a way to extract this private information.[7] Along the same lines, information transmission within organizations has also been studied in multiple applied contexts: Laux (2008), Inderst and Mueller (2010), Garrett and Pavan (2012) and Varas (2018) assess CEO turnover in companies' boards; Inderst and Ottaviani (2009) study the problem of information transmission in sales; Green and Taylor (2016) assess non-cooperative self-reporting of progress in multi-stage principal-agent projects; Simester and Zhang (2010) study the optimal policy for the termination of unprofitable product lines in firms; Heider and Inderst (2012) and Bruche and Llobet (2014) analyze the optimal information transmission within financial organizations. Finally, Halac and Yared (2020) also analyze a trade-off between flexibility and commitment in an unrelated environment to ours, in which the principal delegates a task to a biased agent, showing that the optimal policy with costly verification may include a *threshold* structure with an escape clause on the agent's set of possible delegated actions.

Concerning the intuition that exit design is a form of *endogenous* commitment, the interpretation of limited commitment as the principal's power to arbitrarily cancel long-term contracts has been explicitly proposed by Brzustowski et al. (2023) in a recent contribution, within the Coase conjecture environment. However, a similar interpretation was already suggested informally by Tirole (1986, p. 240): *"The breach of contract argument rests on the impossibility of enforcing the initial contract [...] These features put ("individual rationality") constraints on contracts that can be enforced"*.[8] In this contribution, we aim to provide a micro-foundation of this idea without imposing exogenous constraints on contracting and rationality.[9]

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[6]Also, in their analyses, damage compensation directly improves the provision of incentives, whereas it fails to do so in ours.

[7]Differently from ours, their model concentrates all sources of private information on the side of the agent and is thus not suitable to explore commitment issues. We compare our framework to theirs in the discussion section, showing that, if the principal's outside option is asymmetrically observed by the agent, the principal has no way to extract and effectively use such information.

[8]A similar point is made by Laffont and Tirole (1988, p. 1154): *"We are thus assuming that the regulator cannot commit himself not to use in the second period the information conveyed by the firm's first-period performance. This assumption, which, we believe, is reasonable in a wide range of applications, certainly deserves comment. The simplest way to motivate it is the changing principal framework. For instance, the current administration cannot bind future one"*.

[9]Asymmetric information explanations for incomplete contracting have already been proposed in the past, notably

Structure. In section 2, we introduce the environment, the timing of the payoff-relevant actions and the information structure. We present the socially optimal policy and show that it can be achieved whenever the outside option is publicly observable. In section 3 we solve for the principal's equilibrium contract and characterize the three possible contractual forms (lock-in, partially-secured and at-will contracts) according to the environment's characteristics. We provide an application with a uniformly distributed outside option in section 4 and show that it yields peculiar results. In section 5 we argue that a benevolent planner can improve the equilibrium outcome by imposing a mandatory minimum level of the liquidation fee; Finally, section 6 provides a discussion on our assumption and on the generality of our approach. All the proofs can be found in the Appendix.

2 Model

2.1 Fundamentals

A principal owns a *project* whose outcome $\omega \in \{g,b\}$ depends on the unobservable effort of an agent. The effort decision $e \in \{H,L\}$ affects the distribution of the project's outcome $p_e := \mathbb{P}(\omega = g)$. We assume that $g > b \geq 0$, $p_H > p_L > 0$, and define $\Delta p := p_H - p_L$. The agent incurs a non-monetary cost $D(e)$, where we assume $D(H) = d > 0$ and $D(L) = 0$. Both players are risk-neutral and the agent is subject to limited liability. We also let

$$y_e := p_e g + (1 - p_e) b$$

denote the expected value of the project's output conditional on the agent's effort decision. From $g > b \geq 0$ and $p_H > p_L > 0$ it follows that $y_H > y_L > 0$. .

The principal has an *outside option* whose value V is unknown at the beginning of the project. It is common knowledge that V is drawn from an absolutely continuous cumulative distribution function F with support on $\mathscr{V} := [V_a, V_b]$. We assume that $V_a \in \mathbb{R}_+$, $V_b \in \mathbb{R} \cup \infty$ and $V_0 := \mathbb{E}_F V < \infty$, and we define $f := F'$, assuming that f is continuously differentiable. The outside option's value is privately revealed to the principal only after the agent has exerted effort, but before the project's outcome ω realizes. Furthermore, we assume that $V_b > y_H$ and $y_L > V_a$. Importantly, the reservation payoff is only observed by the principal after the agent has already sunk his effort decision. As the analysis will clarify, this timing creates a tension between the need to provide long-term incentives at the outset of a relationship and the desire to retain flexibility to adapt to its later developments.

We call a *contract with exit rights* $\mathscr{C}$ the collection of a *project contract* $(t_g, t_b) \in \mathbb{R}^2_+$, and a *liquidation fee* $z \in \mathbb{R}_+ \cup +\infty$. By proposing a contract with exit rights at the ex ante stage, the principal retains the option to abandon trade after observing the realized value of V, while compensating the agent with a liquidation fee.[10] Let $\rho \in \{0,1\}$ denote the principal's interim decision to carry on with the project. When $\rho = 1$, the project's outcome ω accrues to the principal and the agent receives the outcome-contingent transfer t_ω. Instead, when $\rho = 0$, the value of the

by Spier (1992) who proposes that complete contracts may be seen as negative signals about contingencies on which one party is privately informed. In this contribution, we propose another explanation which does not rely on information transmission.

[10]For convenience, we use the notation $z = \infty$ to indicate that the principal fully commits to trade.

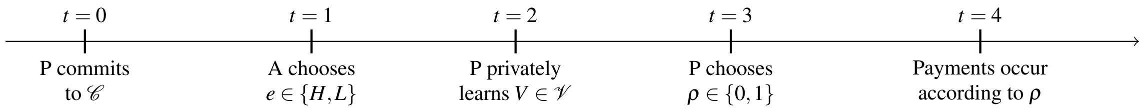


Figure 1: Timing of the game.

outside option V accrues to the principal and the agent receives the liquidation fee z. The timing of the game is given in Figure 1.

Remarkably, when committing to a contract $\mathscr{C}$, the principal accounts for the incentives of her interim self to exploit her private knowledge of the reservation payoff in ways that may not be ex ante efficient. Consequently, the optimal design of the exit clause – which may permit at-will termination, as is typical in limited commitment environments – stems from the principal's need to address this standard problem of screening with a random participation constraint, under full commitment and complete contracting assumptions.

For a given contract with exit rights $\mathscr{C}$ and effort level $e \in \{H,L\}$, the ex ante expected value of the project for the principal is given by

$$\pi(e,t_g,t_b) := y_e - p_e t_g - (1-p_e)t_b,$$

where $y_e = p_e g + (1-p_e)b$ denotes the expected project's output. The ex ante expected value of the project for the agent is given by

$$R(e,t_g,t_b) = p_e t_g + (1-p_e)t_b - D(e).$$

For convenience, we also define $\hat{V}(\phi) = F^{-1}(\phi)$ the ϕ^{th} quantile of F and $\bar{V}(\phi) = \mathbb{E}_F[V|V > V(\phi)]$ the expectation of F conditional on the fact that V lies in the upper $(1-\phi)^{th}$ tail of the distribution, that is, the *tail value-at-risk* of F at the level of confidence ϕ.[11] The inverse hazard rate of F is denoted by $I(V) = (1-F(V))/f(V)$. We assume throughout our analysis that F verifies the monotone hazard rate property.

Threshold trade policies. We first establish a preliminary result about the generality of contracts with exit rights $\mathscr{C} = \{(t_g,t_b),z\}$ with respect to richer mechanisms.

Lemma 1 *Any incentive-compatible and deterministic allocation rule* $(\rho,t_g,t_b,z) : V \to \{0,1\} \times [0,\infty)^3$ *can be implemented by the principal offering a contract with exit righs* $\mathscr{C} = \{(t_g,t_b),z\}$*. Furthermore, for any such* $\mathscr{C}$*, there exists a threshold* $\hat{V} \in \mathscr{V}$ *such that* $\rho(V) = \mathbb{I}\{V \leq \hat{V}\}$*.*

The first statement of Lemma 1 allows us to restrict attention, without loss of generality, to the simple class of contracts $\mathscr{C} = \{(t_g,t_b),z\}$. Payments in this class are independent from the value of the principal's outside option, not because of incomplete contracting, but, as an endogenous characteristics arising from the configuration of the principal's private information.[12]

[11]This equates the *expected loss* of the principal in terms of opportunity costs by forgoing the outside option, *conditional* on being in the worst $100(1-\phi)\%$ of the cases.

[12]We do not address stochastic mechanisms in this work, and leave the topic for further research.

The second statement of the Lemma shows that the decision to implement the project is fully characterized by a threshold value of the outside option. On this basis, we find it convenient to cast the principal's problem in a different way. Instead of considering a choice of contract with exit rights $\mathscr{C} = \{(t_g, t_b), z\}$ together with a trading rule $\rho = \mathbb{I}(\hat{V})$ to be followed in the interim stage, we let the principal directly announce an ex ante probability of trade $\phi \in [0,1]$ together with a contract $\mathscr{C} = \{(t_g, t_b), z\}$.[13] From now on, we will refer to $\phi \in [0,1]$ as the *frequency*, or the *security*, of trade.

Given a contract $\mathscr{C}$, an effort level e and a frequency of trade $\phi \in [0,1]$, the principal's ex ante expected utility rewrites as

$$v(e,\phi,\mathscr{C}) = \phi[y_e - p_e t_g - (1-p_e)t_b] + (1-\phi)(\bar{V}(\phi) - z),$$

while the agent's ex ante expected utility now writes as

$$u(e,\phi,\mathscr{C}) = \phi[p_e t_g + (1-p_e)t_b] + (1-\phi)z - D(e).$$

We call the collection $(e,\phi,\mathscr{C})$ a *trade policy*. A trade policy is said to be *consistent* if the frequency of trade ϕ is compatible with the principal's sequentially rational decisions to continue or terminate the project, after observing the realized values of $V \in \mathscr{V}$. In other words, if the principal announces to trade with frequency ϕ, consistency imposes that the probability that the value of continuing the project exceeds that of terminating it is exactly equal to ϕ. Formally, $\mathbb{P}(y_e - p_e t_g - (1-p_e)t_b \geq V - z) = \phi$ which can be equivalently rewritten as

$$y_e - p_e t_g - (1-p_e)t_b = \hat{V}(\phi) - z. \qquad (ICC_\phi)$$

The *consistency* constraint (ICC_ϕ) describes how, under the configuration of private information assumed in this environment, the principal's own incentives limit her allocative possibilities, even in the presence of full commitment and complete contracts.

In particular, if the value of a project falls shorts of $\hat{V}(\phi)$, the ϕ^{th} quantile of F, the trade frequency ϕ can be made consistent by either reducing the agent's payments, thereby making trade more attractive, or increasing the liquidation fee, thereby penalizing the outside option. The opposite reasoning applies if the project's value exceeds $\hat{V}(\phi)$. Notably, since a trade policy $(e,\phi,\mathscr{C})$ can be interpreted as a direct mechanism with a threshold trading rule, (ICC_ϕ) represents the constraint faced by the principal to ensure truthful revelation of her future reservation payoff.

Finally, to facilitate the interpretation of our results, we define the following typologies of contracts with exit rights according to the value of the liquidation fee $z \in [0,+\infty]$.

- **Lock-in contracts:** $z = +\infty$**.** The principal fully commits to the completion of the project, no interim exit allowed.
- **Partially secured contracts:** $z \in (0,\infty)$**.** The principal can unilaterally decide to terminate the project before its end but must compensate the agent with z.

[13] It is straightforward that, under the trading rule $\rho = \mathbb{I}(\hat{V})$, the probability of trade is $\phi = F(\hat{V})$.

- **At-will contracts:** $z = 0$. The principal can freely withdraw the original offer without having to compensate the agent.

2.2 Benchmarks

Before proceeding to the analysis of our main framework, we establish a couple of results to serve as benchmark cases.

Project value. Our first benchmark consists in establishing the value of the project for the principal in the absence of an outside option. Hence, assume for now that the principal can either proceed with the project or obtain a zero outside option. Our framework collapses to a standard moral hazard problem with limited liability (see Laffont and Martimort, 2009, ch.4).

In this environment, if the principal wants to induce effort level $e = H$ or $e = L$, transfers must satisfy the respective incentive constraints:

$$\Delta p(t_g - t_b) \geq d, \tag{ICC_H}$$

$$\Delta p(t_g - t_b) \leq d. \tag{ICC_L}$$

In both cases, $e \in \{L, H\}$, the principal must also satisfy the agent's participation constraint:

$$p_e t_g + (1 - p_e) t_b - D(e) \geq 0, \tag{IRC_e}$$

and limited liability constraints for each $\omega \in \{g, b\}$:

$$t_\omega \geq 0. \tag{LL_ω}$$

We call Π^e the value of the project to the principal when inducing effort e. Formally, Π^e is the value of the principal's maximization problem:

$$\Pi^e := \max_{t_g, t_b} \quad \pi(e, t_g, t_b) = y_e - p_e t_g - (1 - p_e) t_b,$$

subject to (ICC_e), (IRC_e), and LL_ω for $\omega \in \{g, b\}$.

The detailed analysis of this problem is standard and therefore omitted. The simplest case is that of $e = L$ which allows the principal to fully appropriate the gross value of the project $\Pi^L = y_L$ and leaves no rent to the agent. Instead, when the principal wants to induce effort $e = H$, moral hazard and limited liability impose an agency cost so that $\Pi^H = y_H - \frac{p_H}{\Delta p} d$. For future reference, we explicitly define the agency cost in the case $e = H$ as:

$$C^H := \frac{p_H}{\Delta p} d. \tag{1}$$

Furthermore, the agent enjoys a strictly positive limited liability rent $R^H = C^H - d > 0$.

Anticipating the characterization of the optimal contract with exit rights when $e = H$, it would be tempting to think that the principal's decision to exit the project will depend on the comparison of the net project value Π^H and the outside option value V. Instead, we will show that the exit decision relies in a nontrivial way of the interplay between the gross value of the project y_H and the agency cost C^H.

For the rest of the paper, we assume that the project value is higher under $e = H$ than under $e = L$, that is, we assume that $\Pi^H > \Pi^L$.

Social optimum. As a benchmark, we characterize the trade policies that maximizes the expected social surplus $v(\cdot) + u(\cdot) = \phi y_e + (1-\phi)\bar{V}(\phi) - D(e)$ when only the moral hazard problem is present.

We restrict the analysis to the nontrivial case in which it is socially optimal to induce effort $e = H$. For that matter we make the following assumption.

Assumption 1 *Assume the following holds:*

$$\int_{y_L}^{y_H} F(V)dV \geq C^H.$$

Proposition 1 *The set of the optimal trade policies coincides with those such that $e = H$ and trade happens with the frequency $\phi_H^* = F(y_H)$.*

More specifically, the proof of Proposition 1 shows that, conditional on the agent's effort level, the project should be implemented if and only if its gross value y_e exceeds the value of the outside option V. This is formally equivalent to a frequency of trade $\phi_e^* = F(y_e)$. Note also that, in the socially efficient decision to trade or not, the agent's cost of effort is ignored, as it is already sunk at the time in which trade takes place.

The fact that the effort decision $e = H$ is ex ante efficient is instead guaranteed by Assumption 1. To elaborate on this, note that, after integrating by parts, one obtains

$$\int_{y_L}^{y_H} F(V)dV = \phi_H^* y_H - \phi_L^* y_L - (\phi_H^* - \phi_L^*)\mathbb{E}[V|y_L < V < y_H],$$

with the implication that $e = H$ is efficient if

$$\phi_H^* y_H - \phi_L^* y_L > C^H + (\phi_H^* - \phi_L^*)\mathbb{E}[V|y_L < V < y_H]. \tag{2}$$

In the expression (2), the LHS coincides with the (expected) output difference between the efficient trade policies associated to the two effort levels $e \in \{L, H\}$.[14] In order for $e = H$ to be optimal, this difference must offset the sum of two terms. The first term C^H captures the severity of the agency costs needed to induce $e = H$. The second term $(\phi_H^* - \phi_L^*)\mathbb{E}[V|y_L < V < y_H]$ is the expected opportunity cost of the trades that only happen under the policy inducing high effort: hence, the larger is this term, the less advantageous it is to induce $e = H$ with respect to $e = L$.

Observable outside option. Remarkably, a socially optimal policy naturally emerges at equilibrium provided that the outside option is not private information of the principal.[15]

[14] In fact, if the effort level e is induced together with a socially efficient trading policy, the expected output is y_e, and trade happens with probability ϕ_e^*

[15] Because of our complete contracting assumption, we do not distinguish the cases in which V is public and verifiable, as any conditioning on V could be at least partially implemented by means of a *forcing* contract which extracts the shared information from the parties (Laffont and Martimort, 2009, ch. 6).

Proposition 2 *Assume that the value of the outside option $V \in \mathscr{V}$ is observed by both the principal and the agent at the interim stage. Then, the optimal contract offered by the principal is socially efficient.*

In fact, the principal's optimal trading policy in this scenario leads to the maximization of the joint surplus from trade, achieved by committing *ex ante* to the socially efficient trading rule $\rho = \mathbb{I}(V \leq y_H)$, and extracts as much as possible from it, by pushing the agent to its lowest level of expected utility that is incentive-compatible once the limited liability and obedience constraints are taken into account.

More in the detail, as shown in the proof of Proposition 2, any contract inducing the efficient effort decision $e = H$ together with the trade frequency $\phi \in (0,1]$ is subject to the following incentive-compatibility constraint.

$$\phi[p_H t_g + (1-p_H)t_b] + (1-\phi)z - d \geq \phi[p_L t_g + (1-p_L)t_b] + (1-\phi)z \qquad (ICC_H)$$

which can be rewritten as

$$t_g \geq t_b + \frac{C^H}{p_H \phi} \qquad (ICC_H)$$

Hence, whenever trade does not happen with probability one, the optimal project contract is distorted to compensate the adverse effects on the agent's incentives for effort-taking, due to the associated risk of not implementing the project. The optimal contract looks as follows:

$$t_g^* = \frac{C^H}{p_H \phi_H^*}; \quad t_b^* = 0; \quad z^* = 0.$$

Namely, the lower the security of trade, the higher the transfer that the agent must obtain, in the good state of the world, to guarantee that high effort is incentive-compatible. On the contrary the liquidation fee serves no purpose in providing incentives and is optimally set to zero. The agent's rent is

$$\phi_H^* \frac{C^H}{\phi_H^*} - d = C^H - d = R^H$$

and thus coincides with the limited liability rent obtained in the benchmark case without a principal's outside option.

Furthermore, the value for the principal of the optimal incentive-compatible project inducing trade with frequency ϕ and effort $e = H$ is

$$\Pi^{IC}(\phi) := y_H - \frac{C^H}{\phi}.$$

Once can note that $\Pi^{IC}(1) = \Pi^H$ and $\lim_{\phi \to 0} \Pi^{IC}(\phi) = -\infty$, where $\phi = 0$ is impossible to implement together with $e = H$ since, if trade is shut down, the agent never receives any returns from effort-taking. Furthermore, $\Pi^{IC}(\phi)$ has first derivative

$$\kappa(\phi) := \frac{\partial \Pi^{IC}(\phi)}{\partial \phi} = \frac{C^H}{\phi^2} > 0,$$

meaning that enhancing the frequency of trade improves the project's value. The effect is channeled by the mitigating influence that trade security has on the cost of providing incentives to the agent. Also notably, $\lim_{\phi \to 0} \kappa(\phi) = \infty$ and $\kappa(1) = C^H$.

Finally, the principal's expected value from the optimal policy when the reservation payoff is observable coincides with

$$v_H^* = \phi^* \Pi^{IC}(\phi_H^*) + (1 - \phi_H^*)\bar{V}(\phi_H^*) = \phi_H^* y_H + (1 - \phi_H^*)\bar{V}(\phi_H^*) - C^H.$$

The following result ensues.

Lemma 2 *Under the Assumption* 1*, it is optimal for the principal to induce* $e = H$ *in the public information benchmark.*

The proof of Lemma 2 shows that the best policy inducing $e = L$ yields

$$v_L^* = \phi_L^* y_L + (1 - \phi_L^*)\bar{V}(\phi_L^*)$$

to the principal, that is, the principal is able to extract the whole surplus from trade. Then, the fact that inducing $e = H$ dominates $e = L$ is a straightforward consequence of Assumption 1. Furthermore, the proof shows that $v_L^* > V_0$, meaning that the principal's prefers to induce any effort level than abstaining from trade and consuming her outside option with probability one.

3 Equilibrium trade policies

We now turn to the main analysis. We characterize the equilibrium trade policies when the value of the outside option is *privately* revealed to the principal *after* the exertion of effort by the agent and *before* the realization of the output as presented in Figure 1.

When the principal's reservation payoff is unobservable, the principal's impossibility to commit ex ante to an interim participation rule that conditions on the value of the outside option generates a dynamic inconsistency problem. That is, for certain realizations of V, the principal may prefer to opt out of trade after observing her outside option, even though doing so would be inefficient from an ex ante perspective.

In fact, once V has been realized, which happens after the agent's effort is sunk, the indirect effect of the trading rule $\rho(V)$ on the agent's incentives for effort provision is no longer relevant for the principal. At that point, the principal's decision to trade or not is based solely on the direct comparison between the profits from trade $\pi(e, t_g, t_b)$ and the outside option V, while, since the effort decision has already been irreversibly exerted by the agent, the principal has no interest anymore in the security of trade. Consequently, the ex ante efficient trading rule $\rho = \mathbb{I}(V \leq y_H)$ is sequentially inefficient whenever a value

$$V \in \left(y_H - \frac{C^H}{\phi_H^*}, y_H \right)$$

of the outside option is realized.[16]

In this section, we provide a characterization of the ex ante optimal policies in the presence of the consistency constraint (ICC_ϕ) described in section 2, which totally captures the requirements imposed by the principal's sequential rationality.

We show that their optimal design is governed by a trade-off between the security and the flexibility of trade, and, in general, fails to implement an ex ante efficient trading rule.

More in detail, the design of the optimal trading rule inducing $e = H$ in the private information case is fully governed by three parameters: the agent's productivity y_H, the costs of agency C^H, and the statistical parameter

$$C^* := \lim_{\phi \to 1} I(\hat{V}(\phi))$$

which describes the behavior of the reservation payoff's distribution at its upper tail, by taking the limit of the inverse hazard rate at the right boundary of $\mathscr{V}$. The following result provides a summary of our main findings, for the case in which $I(V)$ is increasing which, as our analysis will point out, yields a well-behaved optimization program for the principal.

Theorem 1 *Assume that $I(V)$ is non-decreasing and recall $C^* \equiv \lim_{\phi \to 1} I(\hat{V}(\phi))$. There exists a continuous and strictly increasing function $\bar{y} : (0, C^*) \to \mathbb{R}$, such that $\lim_{C^H \to 0} \bar{y}(C^H) = V_a$, $\lim_{C^H \to C^*} \bar{y}(C^H) = \infty$, and:*

- *A **lock-in** contract is optimal if and only if $C^H \geq C^*$.*
- *An **at-will** contract is optimal if and only if $C^H \in (0, C^*)$ and $y_H \geq \bar{y}(C^H)$.*
- *A **partially-secured** contract is optimal if and only if $C^H \in (0, C^*)$ and $y_H < \bar{y}(C^H)$.*

Theorem 1 provides a simple mapping of every parametrization (C^H, C^*, y_H) of the environment to the three classes of *lock-in*, *partially secured* and *at-will* contracts highlighted in section 2. Figure 2 provides a visual representation of the result.

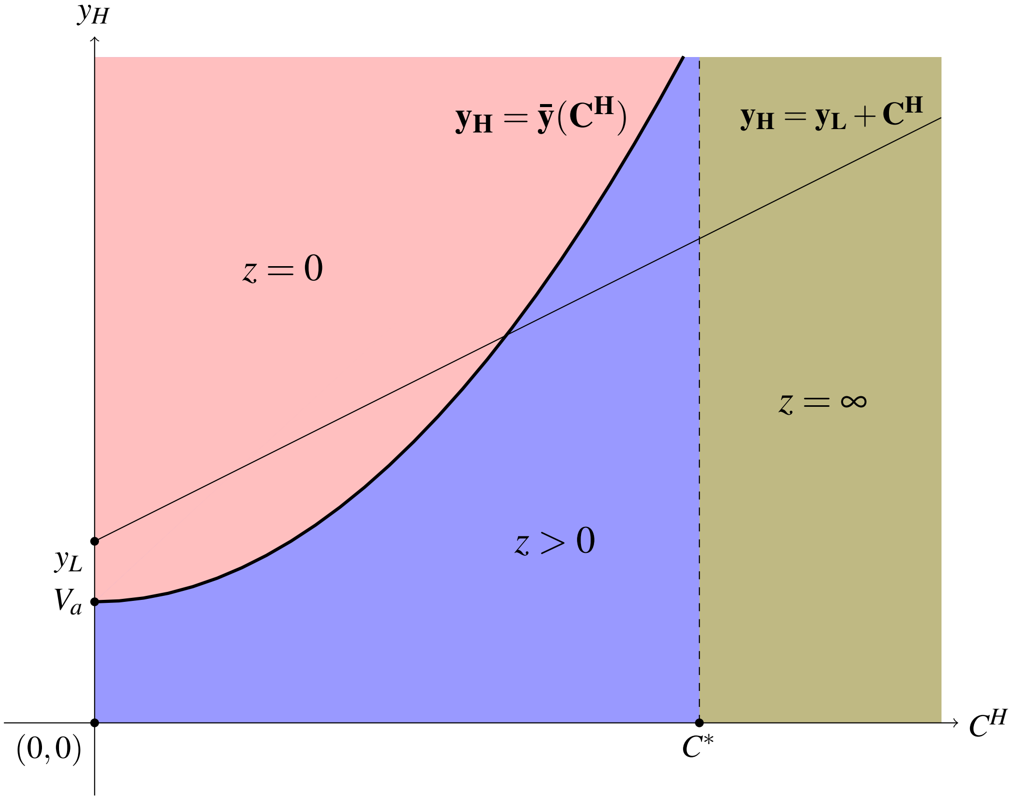
The black curve represents the $\bar{y}(C^H)$ function, which is strictly increasing as $C^H < C^*$ and explodes to infinity as $C^H \to C^*$; the increasing line $y_H = y_L + C^H$ separates the points of the parameter space according to the associated second-best effort decision in the fully deterministic benchmark. In particular, $e = H$ is optimal when the parameters lie in the region north-west of the line.

In the region in yellow where $C^H \geq C^*$, the agency costs are intense to the point that a lock-in contract is optimal, regardless of the agent's productivity. It should be noted that $C^* = \infty$ is possible, meaning that, for some distributions of V, a lock-in contract may never be optimal. When $C^* < C^H$, the principal instead retains some flexibility: in particularly, if (C^H, y_H) lie within the region in red, it is optimal for the principal to offer an at-will contract. Finally, when (C^H, y_H) lies in the blue region, the principal offers a contract that partially compensates the agent in case of unilateral termination.

The shape of the function $\bar{y}$ reflects the existence of a flexibility-security trade-off in the principal's optimal design of a trade policy. Specifically, as our analysis will clarify, the principal,

[16] For any such V, the economic profit from trade $\Pi^{IC}(\phi_H^*) - V$ is in fact negative.

Figure 2: Optimal policies in the (C^H, y_H) space when $I(V)$ is non-decreasing.



in designing the optimal liquidation fee z, must balance retaining flexibility in collecting the outside option, against reducing her profits from trade due to the adverse incentive of trade *insecurity* on incentives. From this viewpoint, Figure 2 leads to two considerations.

First, at parity of productivity, it is optimal to provide the agent with more secure contracts when agency costs are higher. In particular, this means either that the cost of effort is high, or that output is an imprecise signal of effort, as both conditions cause $C^H = \frac{p_H}{\Delta p} d$ to be large.

Second, at parity of agency costs, the more productive is the agent, the *less* secure is the optimal contract. This finding, which may seem counterintuitive, is explained by the fact that the role of a high liquidation fee is to discipline the principal's participation behavior, adjusting the consistency constraint (ICC_ϕ) in case the outside option is attractive enough to induce the principal to abandon trade *too often*. Therefore, if the agent is already very productive, the project's value secures itself and there is less need to introduce (distortionary) liquidation fees to bulk up incentive-provision.

For the more complicated case in which $I(V)$ is decreasing, alternative considerations can be made. In particular, in section 4, the case in which V is extracted from a uniform distribution is addressed, and the associated intricacies are highlighted.

In the rest of this section, the reasoning that leads to the formulation of Theorem 1 is exhibited.

3.1 High effort

To find the principal's optimal policy among those inducing $e = H$, one must solve the following program.

$$
\begin{aligned}
\max_{(\phi,\mathscr{C})\in[0,1]\times[0,\infty)^3} \quad & v(H,\phi,\mathscr{C}) = \phi[y_H - p_H t_g - (1-p_H)t_b] + (1-\phi)(\bar{V}(\phi) - z) && (3)\\
\text{s.t.:} \quad & \phi[p_H t_g + (1-p_H)t_b] + (1-\phi)z - d \geq 0 && (IRC_H)\\
& t_g \geq t_b + \frac{d}{\phi \Delta p} && (ICC_H)\\
& t_\omega \geq 0 \ \ \forall \omega \in \{g,b\} && (LL_\omega)\\
& z \geq 0 && (LL_z)\\
& y_H - p_H t_g - (1-p_H)t_b = \hat{V}(\phi) - z && (ICC_\phi)
\end{aligned}
$$

Note that (IRC_H) can be disregarded without loss of generality as it is automatically implied by (ICC_H, LL_z, LL_b). In the rest of this section, we solve first a relaxed version of (3), in which the limited liability constraint (LL_z) on the liquidation fee is ignored; as we shall observe, the relaxed problem alone conveys much of the economic insight. Then, we show that (LL_z) may turn out to be occasionally binding at the optimum, providing a necessary and sufficient condition for that to happen when $I(V)$ is non-decreasing, and discuss the economic interpretation of this result. Finally, the optimal policy inducing low effort from the agent is discussed and compared with that inducing high effort.

Relaxed program. Denote (ϕ^r, c^r) the policy that solves the relaxed version of program (3), in which (LL_z) is discarded. To find such policy, it is convenient to substitute out the (ICC_ϕ) by setting

$$z = \hat{V}(\phi) - y_H - p_H t_g - (1-p_H)t_b \tag{4}$$

within the objective function, thus obtaining

$$v^r(\phi, t_g, t_b) := y_H - p_H t_g - (1-p_H)t_b + (1-\phi)(\bar{V}(\phi) - \hat{V}(\phi)).$$

Hence, $t_b = 0$ and $t_g = \frac{C^H}{p_H \phi}$ follow from the optimally binding constraints (LL_b) and (ICC_H). These are the same transfers already characterized in the full observability case. Therefore, in the relaxed program, the principal's expected profit in the event that trade happens coincides with the upper bound $\Pi^{IC}(\phi)$ of the incentive-compatible profits associated to (H,ϕ), as characterized in the previous section.

Thus, by substituting $\Pi^{IC}(\phi)$ in (4), the optimal liquidation fee to induce the arbitrary frequency of trade $\phi \in (0,1]$ is identified by the function

$$z^r(\phi) := \Pi^{IC}(\phi) - \hat{V}(\phi).$$

As a consequence, the optimal contract to implement a trade frequency $\phi \in (0,1]$ is[17]

[17] Note that $\phi = 0$ is never incentive-feasible in association with $e = H$, since the RHS of the (ICC_H) in program (3)

$$c^r(\phi) = \left(\frac{C^H}{p_H \phi}, 0, z^r(\phi)\right).$$

At this point, we are only left to characterize the optimal frequency of trade, which, after substituting $c^r(\phi)$ in the objective function, is found at the solution of the following unconstrained optimization program.

$$\max_{\phi \in (0,1]} \quad v^r(\phi) = \Pi^{IC}(\phi) + (1-\phi)[\bar{V}(\phi) - \hat{V}(\phi)].$$

To proceed with the analysis, recall now that $I(V)$ is defined as the inverse hazard rate of F: the following result, which conveys the main economic insights of this section, can be provided on this basis.

Proposition 3 *If $\phi^r \in (0,1)$, then $I(\hat{V}(\phi^r)) = \kappa(\phi^r)$.*

Proposition 3 exhibites a necessary condition that holds at any *interior* optimum in the relaxed program. The condition says that the marginal effect of the optimal trade frequency on trade profitability must be equal to the inverse hazard rate of F.

To provide an economic interpretation of Proposition 3, it must be stressed that the problem of choosing an optimal ϕ, faced by the principal at the ex ante stage, can be regarded as deciding which *utility-types $V \in \mathscr{V}$* of her future self should take part in trade, and which ones should opt out. The condition in the statement captures the two countervailing, marginal effects of varying the share ϕ of utility-types that participate.

On one side, by rising ϕ, the principal improves trade profitability by loosening the (ICC_H) constraint, as captured by the term $\kappa(\phi)$; on the other hand, in order to make a larger trade frequency consistent, the principal needs to induce more *well off* utility-types of her interim self to participate, thereby increasing the liquidation fee $z^r(\phi)$ required to meet the (ICC_ϕ) constraint .

Hence, the inverse hazard rate $I(\hat{V}(\phi))$ can be interpreted as the marginal *virtual cost* for the ex ante principal of attracting her own future type $\hat{V}(\phi)$ to take part in trade, and $\kappa(\phi)$ as the associated marginal benefit. The proof of Lemma 3 shows that, once accounting for all the first and second-order effects, these end up being the only two factors that matter in identifying the optimal frequency of trade.

To get some insights on how the result obtains, let us compare the analysis under incomplete information to the case in which V is observable, meaning that the principal is not subject to the consistency constraint (ICC_ϕ). If V is publicly observable, the principal maximizes

$$v_H^*(\phi) = \phi \Pi^{IC}(\phi) + (1-\phi)\bar{V}(\phi)$$

with derivative

$$\frac{\partial v_H^*(\phi)}{\partial \phi} = \phi \kappa(\phi) + \Pi^{IC}(\phi) - \hat{V}(\phi),$$

while, under private information, the principal maximizes

$$v^r(\phi) = \phi \Pi^{IC}(\phi) + (1-\phi)[\bar{V}(\phi) - \hat{V}(\phi) + \Pi^{IC}(\phi)] = v_H^*(\phi) - (1-\phi)z^r(\phi)$$

explodes to infinity as $\phi \to 0$.

with derivative
$$\frac{\partial v^r(\phi)}{\partial \phi} = \frac{\partial v_H^*(\phi)}{\partial \phi} + z^r(\phi) - (1-\phi)\frac{\partial z^r(\phi)}{\partial \phi}.$$
Hence, in the (relaxed) program under private information, one has to take into account, in addition to the factors already considered in the public information benchmark, the effect of ϕ on the associated liquidation transfer $z^r(\phi)$ that makes the trade frequency ϕ consistent. In particular, this effect can be decomposed as follows:
$$\frac{\partial z^r(\phi)}{\partial \phi} = \frac{1}{f(\hat{V}(\phi))} - \kappa(\phi). \tag{5}$$

Then, the optimality condition $I(\hat{V}(\phi)) = \kappa(\phi)$ arises from substituting equation (5) within the expression for $\frac{\partial v^r(\phi)}{\partial \phi}$.

For the well-behaved case in which $I(V)$ is non-decreasing, which ensures in particular that $v^r(\phi)$ is concave, it is possible to provide a sufficient and necessary condition for the optimality of a lock-in contract.

Proposition 4 *If $I(V)$ is non-decreasing, there exists $C^* \in \mathbb{R} \cup \infty$ such that $\phi^r \in (0,1)$ if $C^H \in (0, C^*)$, and $\phi^r = 1$ otherwise.*

To interpret Proposition 4, recall that C^H can be thought as an index of how *severe* the agency problem is in this environment; the result thus states that locking in to a contract is only optimal for the principal when the agency problem is more *intense* than some threshold level C^H which, as the proof shows, is related to the shape of the distribution F and can be loosely associated to the weight of the distribution's tail. Hence, intuitively, the principal optimally proposes a lock-in contract in the relaxed problem when the agency problem is *severe* with respect to the weight given to very high opportunity costs. This, quite surprisingly, leaves other important elements out of the picture: in particular, the sheer profitability of trade as captured by the expected output y_H is irrelevant in determining whether the principal would prefer to lock in trade.

Furthermore, note that the statement of Proposition 4 allows $C^* = \infty$, meaning that, since C^H is necessarily finite, there may be some distributions of the reservation payoff under which a lock-in contract is never optimal, no matter the intensity of the agency problem. The following corollary to Proposition 4 helps qualifying this point.

Corollary 1 *C^* is finite only if $\lim_{\hat{V} \to \infty} \mathbb{E}[V - \hat{V} | V \geq \hat{V}] < \infty$.*

Corollary 1, which exploits Lauermann and Wolinsky (2016, p. 272), provides a necessary condition on F that, if violated, implies that a lock-in contract is *never* optimal for the principal; that is, the distribution of the outside option must be asymptotically *memoryless* as V goes to infinity. Intuitively, when the condition is not respected, extreme values from the tail of the distribution have a heavy weight, exhacerbating the cost of foregoing the outside option.

To provide other economic insights, however, we must look at the full program in which the (LL_z) is considered. Furthermore, one must stress that Proposition 4 fails when $I(V)$ is decreasing; to cover this case, we discuss in the section 4 the scenario in which V is uniformly distributed.

Full program. It can be the case that, for the optimal ϕ^r in the relaxed program, (LL_z) is violated, meaning that $z^r(\phi^r) < 0$, that is,

$$\Pi^{IC}(\phi^r) > \hat{V}(\phi^r).$$

This happens when the relaxed optimal contract $c^r(\phi^r)$ is *too profitable* to make the trade frequency ϕ^r consistent. In this occurrence, the consistency of ϕ^r would be (efficiently) restored in the relaxed program by having the agent to pay a cash transfer to the principal in case the latter unilaterally terminates trade; this, however, is ruled out in the full program by the limited liability constraint (LL_z).

Before analyzing in detail how the presence of the (LL_z) constraint impacts the optimal policy, let us provide a preliminary lemma.

Lemma 3 *There exists $\underline{\phi} \in [0, F(\Pi^H))$ such that $z^r(\phi) \geq 0$ for each $\phi \in [\underline{\phi}, 1]$, with strict inequality for each $\phi \in (\underline{\phi}, 1]$; furthermore, $z^r(\underline{\phi}) = 0$ if $\underline{\phi} \in (0, F(\Pi^H))$.*

Lemma 3 identifies a closed interval of trade probabilities $\phi \in [\underline{\phi}, 1] \subseteq [0, 1]$, including $\phi = 1$, such that the contract $c^r(\phi)$, that is optimal conditionally on ϕ in the relaxed program, does not violate (LL_z). This interval, since $\Pi^H < V_b$ by assumption and thus $\underline{\phi} < F(\Pi^H) < 1$, is always non-empty.

On this basis we can provide the following result.

Proposition 5 *The optimal policy (H, ϕ^*, c^*) inducing $e = H$ can only assume one of the following three forms:*

i. $\phi^* = 1$ *and* $z^* \in [V_b - \Pi^H, \infty]$.

ii. $\phi^* \in [\underline{\phi}, 1]$ *and* $z^* = z^r(\phi^*) > 0$.

iii. $\phi^* = \underline{\phi}$ *and* $z^* = 0$.

Furthermore, $t_b^* = 0$ *and* $t_g^* = \frac{C^H}{p_H \phi^*}$.

Note that case *i.* is a *lock-in* contract that can be interpreted as full commitment on the initial offer; case *ii.* is a *partially secured* contract, where the principal imposes a liquidation fee on her future self in case of unilateral termination; case *iii.* is a contract with at-will termination, which can be seen as endogenous no-commitment on the initial offer.

The most important finding from Proposition 5 is that the efficient frequency of trade must lie within the closed interval $[\underline{\phi}, 1]$. In particular, since for any $\phi \in [\underline{\phi}, 1]$ the relaxed optimal contract $c^r(\phi)$ respects (LL_z), this means that the whole program (3) including all the limited liability constraints can be reduced to the following problem of unconstrained optimization:

$$\max_{\phi \in [\underline{\phi}, 1]} v^r(\phi) = \Pi^{IC}(\phi) + (1 - \phi)(\bar{V}(\phi) - z^r(\phi)).$$

Notably, Proposition 5 does *not* hinge on whether $I(V)$ is non-decreasing. Therefore, $v^r(\phi)$ need not be concave and may have multiple local optima.[18] This implies that, when the relaxed

[18]The case of uniformly distribution reservation payoff is a notable example, as highlighted in section 4.

optimal frequency ϕ^r does not respect (LL_z), the latter constrain may not bind at the optimum, and, in principle, the optimal frequency of trade lies everywhere in the unit interval.

Because of the possible non-concavity of $v^r(\phi)$, Proposition 5 must exploit non-trivial arguments to show that $\phi \in [\underline{\phi}, 1]$. The complete proof is provided in the Appendix; it is instructive to review in this section some economic insights that can be drawn from it.

In particular, assume for the rest of this section that $z^r(\phi^r) < 0$ and thus the relaxed solution does not straightforwardly coincide with the full solution of (3). As shown in Lemma 3, this has the implication that $\underline{\phi} \in (0, F(\Phi^H))$.

In this scenario, let us provide a graphical comparison of the ex-post payoff conditional on each realized $V \in \mathscr{V}$ under the optimal policies $c^*(\phi)$ that induce three different trade frequencies $\phi < \underline{\phi}$, $\phi = \underline{\phi}$ and $\phi > \underline{\phi}$, together with the effort decision $e = H$.

Figure 3 illustrates the case in which $\phi = \underline{\phi}$. The contract $c^r(\underline{\phi})$ pinned down by $\underline{\phi}$ in the relaxed program is such that $z^r(\underline{\phi}) = 0$ by construction of $\underline{\phi}$, and yields the principal $\Pi^{IC}(\underline{\phi})$ when trade succeeds. The red locus highlights the principal's ex-post payoff for each realized quantile of $V = \hat{V}(\phi)$, under this policy.[19] The straight black line represents the quantile function $\hat{V}(\phi)$.

Figure 3: Policy with $\phi = \underline{\phi}$.

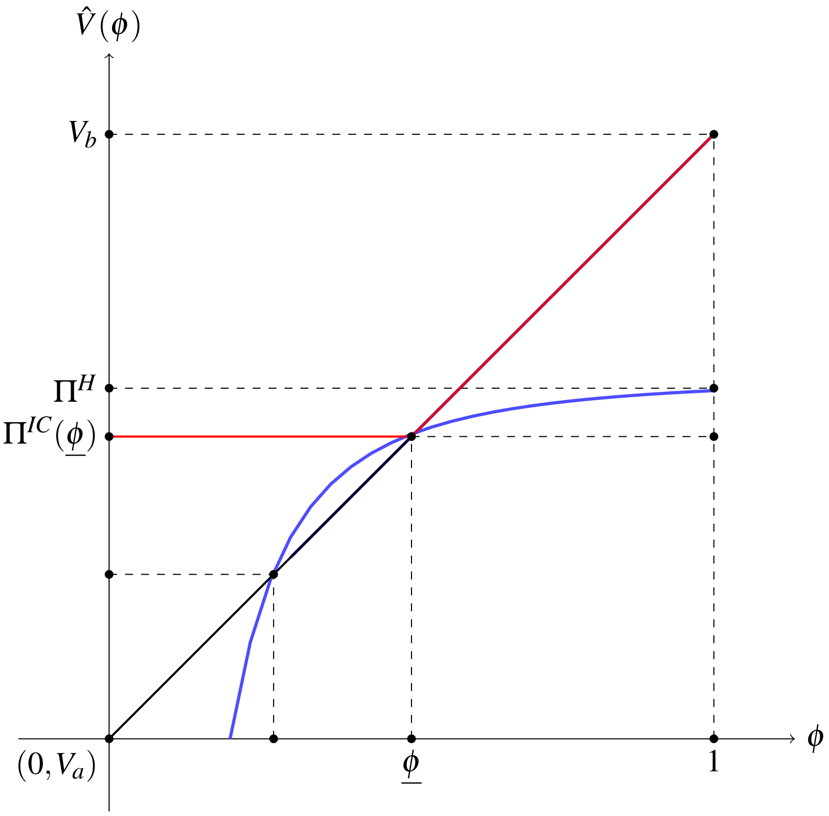


Figure 4, instead, illustrates the optimal policy associated to a frequency of trade ϕ' such that $z^r(\phi') < 0$. In this case, the contract $c^r(\phi')$ pinned down in the relaxed program is unfeasible, as it features a negative transfer $z^r(\phi) < 0$; that is, trade is too profitable to make such a low frequency as ϕ' consistent. To solve the issue efficiently, as shown in the proof of Proposition 5, the

[19]Recall that $\Pi^{IC}(\phi)$ is increasing in ϕ since rising ϕ secures the agent's investment and relaxes the (ICC_H).

principal's profit from trade must be curbed below the incentive-compatible upper bound, by an amount $|z^r(\phi)| > 0$.[20] Under this penalization, the principal's profits from trade as a function of ϕ shift from the dotted blue curve to the thick blue curve.

On this basis, one can visualize that the principal is strictly worse off by efficiently targeting $\phi' < \underline{\phi}$ rather than $\underline{\phi}$. Indeed, whenever $V \in [V_a, \hat{V}(\phi')]$, which means that a level of the reservation payoff lying in the $\phi \in [0, \phi']$ quantile realizes, the principal trades under both policies but only undergoes the penalty $|z^r(\phi)|$ under ϕ'; if $V \in (\hat{V}(\phi'), \hat{V}(\underline{\phi})]$ is extracted, so that the realized quantile lies between ϕ' and $\underline{\phi}$, the principal only trades under the $\underline{\phi}$ policy and obtains a strictly higher value than any reservation payoff in this interval, and is thus strictly better off when $\phi = \underline{\phi}$ than $\phi = \phi'$; if $V \in (\hat{V}(\underline{\phi}), V_b]$ is extracted, that is, a realized quantile $\phi \in (\underline{\phi}, 1]$, the principal is indifferent.

However, the same reasoning does *not* apply to the trade frequencies such that $\phi'' > \underline{\phi}$.

In this case, since $z^r(\phi'') > 0$ by definition of $\underline{\phi}$, to ensure the trade policy's consistency, the principal must introduce a liquidation fee $z^r(\phi)$ which shifts her quantile function from the dotted to the thick line in Figure 5.

Now, this means that whenever $V \in (\hat{V}(\phi''), V_b]$ realizes, meaning that V coincides with the $\phi \in (\phi'', 1]$ quantile of the distribution, the principal is better off under the trade frequency $(\underline{\phi}, c^*(\underline{\phi})$ than $(\phi'', c^*(\phi''))$, since $\rho = 0$ in both cases, but, only to make the trade frequency ϕ'' consistent, she has to pay a strictly positive penalty. On the other hand, the principal is better off under $(\phi'', c^*(\phi''))$ than $(\underline{\phi}, c^*(\underline{\phi}))$ whenever $V \in [V_a, \hat{V}, \phi''))$, with V corresponding to the $\phi \in [0, \phi'')$ quantile of the distribution, since in this case trade always happens under the ϕ'' policy, yielding the principal $\Pi^{IC}(\phi'')$ which is strictly greater than what she obtains when $\phi = \underline{\phi}$.[21]

Therefore, whether the principal prefers the trade frequency $\underline{\phi}$ or ϕ'', depends in hindsight on the realized value of V: if a large reservation payoff realizes, $\underline{\phi}$ is better as it provides the principal with the flexibility to reap it; if a low reservation payoff realizes, meaning that trade takes place, ϕ'' is better as it improves profits by securing more the agent's investment.

Characterization of the optimal policies. It can be shown that the characterization in Theorem 1 can be fully pinned down from the behavior of the occasionally binding constraint (LL_z) in program (3).

To start illustrating that, let us state the following proposition, which provides a condition for (LL_z) to be active at the full program solution: in particular, it shows that the relaxed program of (3) respects (LL_z) at the solution, and is this without loss of generality, if and only if the agent has a high productivity level, as captured by the expected output y_H.

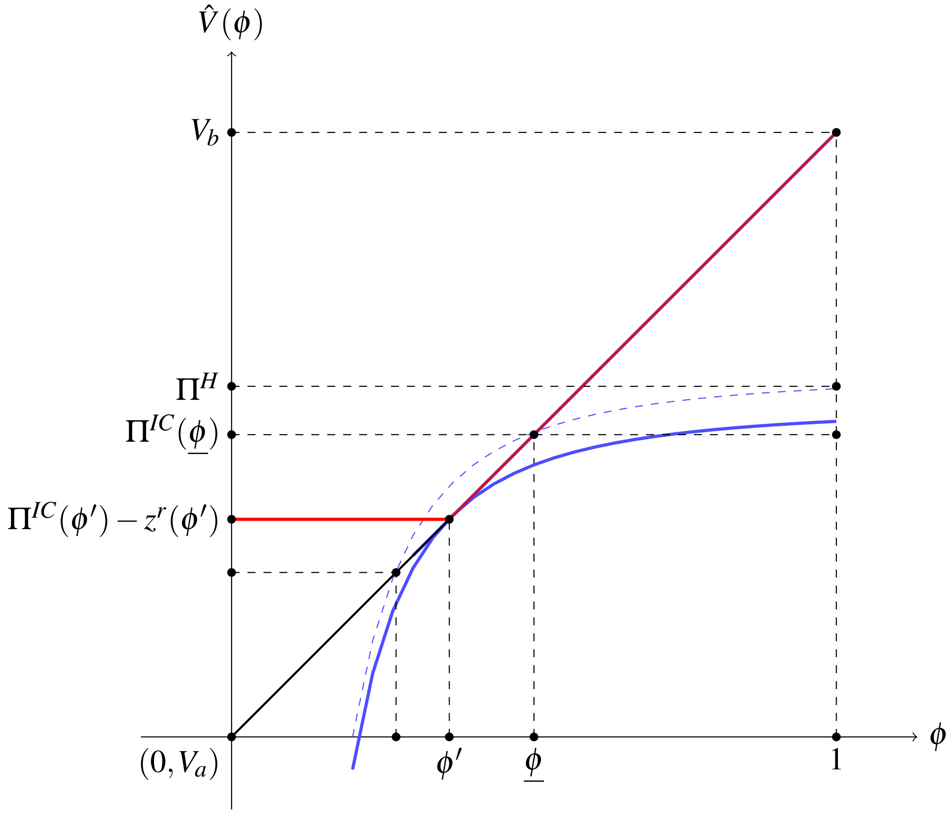
Proposition 6 *If $\phi^r < 1$, there exists y^* such that, fixing all the other parameters, $z^r(\phi^r) \leq 0$ if and only if $y_H \geq y^*$, and $z^r(\phi^r) < 0$ if and only if $y_H > y^*$. Furthermore, if $I(V)$ is increasing or constant, $(\phi^*, c^*) = (\underline{\phi}, c^r(\underline{\phi}))$ if and only if $y_H \geq y^*$.*

The proof is based on the fact that the optimal ϕ^r, which is found at the intersection of $I(V(\phi))$ and $\kappa(\phi)$, is not affected by the value of y_H, which enters none of them. Therefore, by changing

[20] This is done by paying an additional cash transfer $t = |z^r(\phi)|$ to the agent, for each realization of $\omega \in \{g, b\}$.

[21] Indeed, for each realized V such that $(\underline{\phi}, c^r(\underline{\phi}))$ prescribes trade, the principal gets $\Pi^{IC}(\underline{\phi}) < \Pi^{IC}(\phi'')$, where the inequality holds since $\Pi^{IC}(\phi)$ is strictly increasing; if V is such that $(\underline{\phi}, c^r(\underline{\phi}))$ prescribes no trade, the principal obtains a reservation payoffs $V < \Pi^{IC}(\phi'')$ by construction.

Figure 4: Policy with $\phi' < \underline{\phi}$ and $z^r(\phi') < 0$.



y_H of the expected output at parity of other conditions, one can shift $\Pi^{IC}(\phi)$ up or down without affecting ϕ^r, rendering (LL_z) violated or not at the relaxed optimum.

Hence, whenever $\phi^r \in (0,1)$, there exists a threshold level $y^* < V_b$ of the expected output such that, ceteris paribus, (LL_z) constrains the relaxed program if and only if $y_H > y^*$. In addition, Proposition 6 shows that, only when $I(V)$ is non-decreasing, $y_H > y^*$ also implies that (LL_z) is optimally binding and the optimal contract is at-will. This is not the case for $I(V)$ decreasing since, in that case, $v^r(\phi)$ may be not single-peaked and, although the relaxed maximum violates (LL_z), it may be that another local maximum of $v^r(\phi)$ such that (LL_z) is slack emerges as the global optimum in the full program.

Economically speaking, Proposition 6 shows that the productivity of the agent only affects the shape of the optimal contract via its effect on (LL_z) constraint.

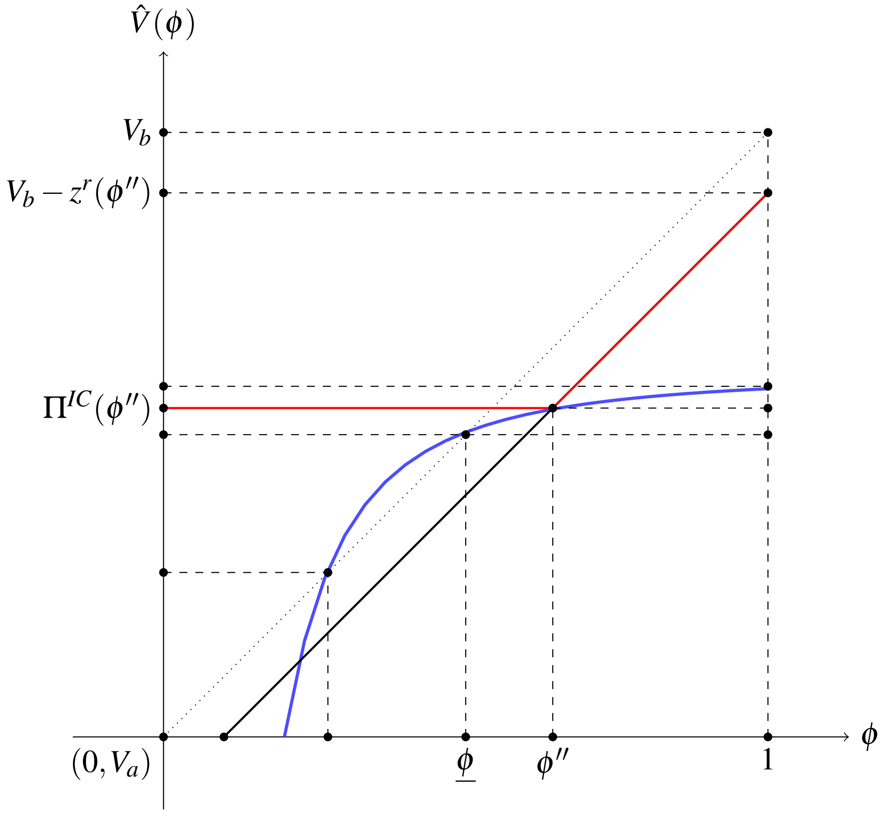
This highlights that the use of liquidation fees in the principal's monopolistic design is to *bulk up* the value of trade, when it is insufficient to guarantee a high trade frequency and protect the agent's investment.[22] Thus, if trade is itself very valuable, the principal has no need to introduce distortionary liquidation fees to prevent the disruption of the incentive scheme, as trade is secured *per se* due to its high value.

On the basis of Propositions 4 and 6,the results already stated in Theorem 1 hold.

Examples. *Pareto-distributed reservation payoff.* Assume the principal's reservation utility is

[22]The fact that $z_r^H(\phi) < 0$ when $y_H > y^*$, implies for example that monopolistic contracts for high-skilled workers would include termination fees on the worker's side, in case the worker had a collateral to post.

Figure 5: Policy with $\phi'' > \underline{\phi}$.



extracted from a Pareto distribution with shape parameter $\alpha \in (1, \infty)$, with unbounded support $[V_a, \infty)$ and CDF $F(V) = 1 - \left(\frac{V_a}{V}\right)^{\alpha}$.

The quantile function is $\hat{V}(\phi) = V_a(1-\phi)^{-\frac{1}{\alpha}}$, and the tail value-at-risk at the level of confidence ϕ is $\bar{V}(\phi) = \frac{\alpha}{\alpha-1} V_a (1-\phi)^{-\frac{1}{\alpha}}$, implying

$$\zeta(\phi) = (1-\phi)\bar{V}(\phi) = \frac{\alpha}{\alpha-1} V_a (1-\phi)^{\frac{\alpha-1}{\alpha}}.$$

Note also that, for a Pareto-distributed random variable V, the inverse hazard rate is $I(V) = \frac{V}{\alpha}$, and is thus strictly increasing. Hence,

$$I(\hat{V}(\phi)) = V_a \frac{(1-\phi)^{-\frac{1}{\alpha}}}{\alpha}$$

and therefore $I(\hat{V}(0)) = \frac{V_a}{\alpha}$ and $\lim_{\phi \to 1} I(\hat{V}(\phi)) = \infty$.

In this case, $C^* = \infty$, meaning that a lock-in contract cannot be optimal. Also, the optimal relaxed frequency of trade $\phi^r(C^H)$ as a function of C^H is pinned down by inverting

$$\hat{C}^H(\phi) = V_a \frac{\phi^2(1-\phi)^{-\frac{1}{\alpha}}}{\alpha}.$$

Then, the principal offers an at-will policy if

$$y_H \geq \bar{y}(\hat{C}^H) = \frac{\phi^r(C^H)+\alpha}{\alpha}(1-\phi^r(C^H))^{-\frac{1}{\alpha}}V_a \quad \forall C^H \in (0,\infty).$$

Exponentially distributed reservation payoff. Consider now the case in which F is exponential with parameter λ, meaning that $F(V) = 1-e^{-\lambda V}$ and $I(V) = \frac{1}{\lambda}$ is constant. Hence, $\hat{V}(\phi) = -\frac{log(1-\phi)}{\lambda}$.

In this case, $C^* = \lim_{\phi \to 1} I(V(\phi)) = \frac{1}{\lambda}$, meaning that, when $C^H \geq \frac{1}{\lambda}$, a lock-in contract is optimal in the relaxed program; in particular, the relaxed optimal frequency of trade is

$$\phi^r = \min\{\sqrt{\lambda C^H}, 1\}.$$

That is, the optimal frequency of trade is increasing in both the λ parameter of the exponential, and the intensity of the agency problem.

Thus,

$$\bar{y}(C^H) = \frac{\sqrt{\lambda C^H} - \log(1-\sqrt{\lambda C^H})}{\lambda} \quad \forall C^H \in \left(0, \frac{1}{\lambda}\right).$$

3.2 Low effort

Finally, it can also be noted that, once accounting for asymmetric information, the effort decision $e = L$ may dominate $e = H$ from the principal's point of view, even when $\Pi^L < \Pi^H$ and Assumption 1 holds, meaning that high effort is optimal in when the reservation payoff is either deterministic or publicly observable. In fact, the presence of private information does not affect the principal's optimal payoff when inducing $e = L$. The following lemma formalizes the point.

Lemma 4 *When V is her private information, the principal can induce $e = L$ and reach the same expected payoff v_L^* as when V is publicly observable.*

Since however, in general, private information constrains the principal when inducing $e = H$, the following remark can be advanced.

Remark 1 *When V is private information of the principal, it is less likely than inducing $e = H$ is optimal in the monopolistic problem, with respect to the case in which V is publicly observable.*

4 Uniformly distributed reservation payoff

Consider the scenario in which V is uniformly distributed over $[0, V_b]$ with V_b finite. In this case,

$$F(V) = \frac{V}{V_b}, \quad \hat{V}(\phi) = \phi V_b, \quad \bar{V}(\phi) = \frac{V_b + \hat{V}(\phi)}{2} = \frac{1+\phi}{2}V_b$$

meaning that

$$v^r(\phi) = y_H - \frac{C^H}{\phi} + (1-\phi)\left(\frac{1+\phi}{2}V_b - \phi V_b\right) = y_H - \frac{C^H}{\phi} + \frac{(1-\phi)^2}{2}V_b.$$

Hence the first derivative of the relaxed objective function,

$$\frac{\partial v^r(\phi)}{\partial \phi} = \frac{C^H}{\phi^2} - (1-\phi)V_b,$$

is weakly positive if

$$(1-\phi)\phi^2 V_b \leq C^H. \tag{6}$$

This implies that $\phi = 1$ is a local maximum. Studying the function $v^r(\phi)$ yields the result that another local maximum exists if and only if $V_b > \frac{27}{4}C^H$.

Denote the local optimum, which is obtained at the lower solution of

$$\phi^2(1-\phi)V_b - C^H = 0$$

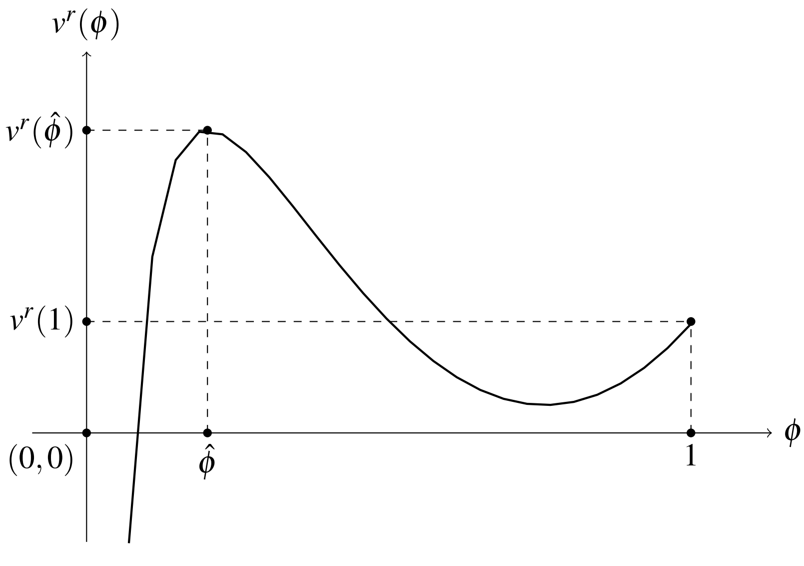
as $\hat{\phi} \in (0,1)$. Then, $\phi^r = \hat{\phi}$ if and only if

$$v^r(\hat{\phi}) = y_H - \frac{C^H}{\hat{\phi}} + \frac{(1-\hat{\phi})^2}{2}V_b < v^r(1) = y_H - C^H \implies \hat{\phi} > \frac{2C^H}{V_b}$$

and, otherwise, $\phi^r = 1$ is the relaxed optimum, meaning that a lock-in trade is optimal.

In the case in which $\phi^r = \hat{\phi}$, the objective function $v^r(\phi)$ looks as in Figure 6.

Figure 6: A double-peaked $v^r(\phi)$ such that $\phi^r = \hat{\phi}$ (uniform case).



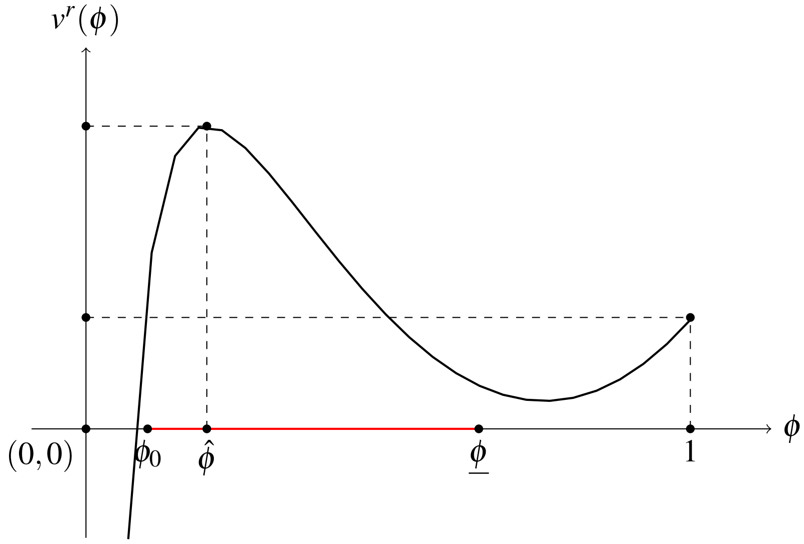
Then, it can be shown that the set of $\phi \in [0,1]$ such that (LL_z) is violated in the relaxed program is a (possibly empty) convex interval $[\phi_0, \underline{\phi}]$ identified by the inequality

$$\phi V_b > y_H - \frac{C^H}{\phi}.$$

This means that, in principle, if $c^r(\phi^r)$ violates (LL_z), such constraint may turn out not to be binding in the full optimum and the full optimal trade frequency may be $\phi^* = 1$ rather than $\phi^* = \underline{\phi}$.

Figure 7 stylizes the situation, where the red segment on the ϕ-axis higlights the trade frequencies such that $c^r(\phi)$ violates (LL_z). In this case, $v^r(\hat{\phi}) > v^r(1) > v^r(\underline{\phi})$, meaning that, a lock-in contract with $\phi^* = 1$ is fully optimal even if $\phi^r < 1$ in the relaxed program.

Figure 7: Non-binding (LL_z) in the constrained optimum (uniform case).



5 Public intervention

As already noted in Proposition 2, the monopolistic trading policy when V is publicly observable is such that $\phi = \phi_H^*$ and is thus efficient from the point of view of a utilitarian social planner.

However, this is no longer guaranteed in case V is unobservable. Note in fact that

$$\phi_H^* \in (\underline{\phi}, 1)$$

as ensured by the facts that $V_b > y_H > \Pi^H$ and $\underline{\phi} < F(\Pi^H)$ as seen in Lemma 3. Thus, the socially efficient trade frequency ϕ_H^* is just one of the possible values that ϕ can achieve at the monopolistic optimum, and there is no particular reason to believe that it emerges autonomously from decentralized contracting.

Notably, at-will contracts always result in the socially inefficient frequency of trade $\underline{\phi} < \phi_H^*$. Consequently, within the framework of this model, contracts typically linked to limited commitment that include at-will termination can be viewed as outcomes of monopolistic distortions, imposed by a principal with full commitment to maximize her own profits.

To correct this inefficiency, a utilitarian social planner would ideally constrain the parties by prescribing a mandatory level of the liquidation fee

$$z^r(\phi_H^*) = \hat{V}(\phi_H^*) - \Pi^{IC}(\phi_H^*) > 0.$$

Under the constraint that $z = z^r(\phi_H^*)$, it would be in the principal's interest to trade with the exact frequency ϕ_H^*, indirectly leading to the maximization of the utilitarian social welfare function.

Since, in a more realistic fashion, the planner may not know the actual value of the model's parameters, a policy imposing a *mandatory minimum level* $\bar{z}$ of the liquidation fee can be seen as an approximation of the ideal solution. The following lemma provide some insights in this respect.

Lemma 5 *Say that $I(V)$ is non-decreasing and $\phi^r \in (0,1)$. Then, the effect of a mandatory minimal liquidation fee $\bar{z}$ on social welfare is weakly positive when $\bar{z} \leq z^r(\phi_H^*)$ and ambiguous if $\bar{z} > z^r(\phi_H^*)$.*

Hence, if $I(V)$ is non-decreasing, by forbidding at-will contracts and imposing a prudently low minimum fee, a utilitarian benevolent planner can improve social welfare.

6 Discussion

Extensions and relationship with limited commitment. In this study, focusing on a textbook model of moral hazard, we have shown that a rational principal that has access to complete contracts may optimally retain an interim participation decision even if this negatively affects incentive-provision. We suggest that our model could serve as a baseline to study models with limited commitment, such as the holdup problem, by making the degree of the principal's commitment endogenous.

In this respect, it is noteworthy that, by adding the possibility for the principal to re-trade and propose a new contract when the original one is terminated, as in the holdup problem, other interesting dynamics could be generated, in which the principal rescinds the original contract even if she has a low reservation payoff, and replaces it with a predatory offer that seeks to extract all the surplus from the agent, appropriating his sunk costs.[23]

Another interesting extension involves considering other kinds information frictions, such as adverse selection in the *ratchet effect* model of Laffont and Tirole (1988), or to explicitly model competition among the agents, so that the principal's outside option would derive from her possible desire to switch from an agent to another in the interim stage.

Generality of the model. Our analysis refers to a stylized model of moral hazard; it is our conjecture that qualitatively similar results would extend to other models of agency, possibly entailing risk-aversion, multiple levels of effort and performance, or adverse selection on the agent's side as in mixed models in the style of Laffont and Martimort (2009, ch. 9), provided that, in the public information benchmark, the incentive-compatibility constraint of the agent is binding.

Different results obtain instead with (ICC_H) slack in the public information benchmark, meaning that the principal faces no agency costs to induce the high effort level $e = H$.

Consider for example the following alternative model. A risk-neutral agent is the producer of a good in capacity-constrained quantities $q \in [0,1]$. The agent's cost of production is $C(e,q) = \gamma_e q$ with $\gamma_H < \gamma_L + d$, and the value of trade is $S(e,q) = s_e q$ with $s_H > s_L$; a contract $c = (q,t,z) \in [0,1] \times [0,\infty)^2$ specifies a quantity-transfer pair in case $\rho = 1$ and a liquidation fee in case $\rho = 0$.

The highest incentive-compatible profit from trade when $e = H$, obtains from the maximization of $\pi(H,q,t) = s_H q - t$ subject to the (ICC_H) constraint $t - \gamma_H q - d \geq t - \gamma_L q$ and the (IRC_H)

[23] Such ex-post renegotiation possibilities are already studied in the law and economics literature on contractual remedies (see Rogerson, 1984).

constraint $t \geq \gamma_H q + d$. The optimum is $(q^H = 1, t^H = \gamma_H + d)$, with value $\Pi^H = s_H - \gamma_H - d$, where the former constraint is slack and the latter binds (Gul, 2001).

The full analysis of this model is deferred to the Appendix; the following takeaway point can be made under the condition that $F(s_H - \gamma_H) > \frac{d}{\gamma_L - \gamma_H}$, which ensures that the (ICC_H) is not optimally binding in the public information case.

Lemma 6 *Say $F(s_H - \gamma_H) > \frac{d}{\gamma_L - \gamma_H}$. Then, the principal obtains the same optimal payoff when V is either observable or unobservable, by offering $q = 1$, $t = \gamma_H + d$, $z = d$ and inducing the frequency of trade $\phi = \phi_H^* = F(s_H - \gamma_H)$.*

This indicates that the flexibility-security trade-off as we describe it is only relevant when the agent's incentives for effort provision are *tight* in the benchmark version of the model, in which the reservation payoff is publicly observed, which is due to the fact that (ICC_H) is non-responsive to z and thus it is impossible to compensate a decrease in trade security with an appropriate liquidation fee. Instead, when the only active constraint of the agent is the participation one, the liquidation fee can be straightforwardly adapted to secure that the agent's participation requirements are met even if trade fails with positive probability.

Agent's private information. Levitt and Snyder (1997) consider the same environment as ours, with the two differences that the agent has private information in the interim stage rather than the principal, and also, the agent privately observes the probability p_e that the state of the world $\omega = g$ realizes rather than the outside option V.

In particular, Levitt and Snyder (1997) aim to analyze the conflict of interest of an agent that is asked to report whether a project in which he has previously exerted effort has turned out to be unprofitable. They show that the principal can use the agent's private information to successfully terminate some loss-making projects, by committing to an appropriate design of a compensation scheme that also remunerates the agent in case trade is *not* carried out. Therefore, the optimal contracts characterized by Levitt and Snyder (1997) give the agent the power to unilaterally cancel trade, and prescribe an appropriate monetary repayment persuading the agent to (sometimes) terminate the relation when trade destroys value.

In our account of contractual termination, instead, it is the principal that may decide to retain the power to unilaterally stop trade, thus endogenously exposing her counter-party to a certain amount of contractual risk. By committing to a liquidation fee in the ex ante stage, the principal is able to discipline her future self in this respect; however, the fee does *not* serve the role of providing incentives to the agent, or compensating him of the possible losses.

Furthermore, it is noteworthy that the construction in Levitt and Snyder (1997) fully rests on the assumption that the agent's private information is correlated with the output's realization; if the agent were to observe some *market information* which does not concern directly the profitability of the trade, which is a key element in the interpretation of our model, the principal would have no way to extract such information from her and use it to affect her trading decisions. The following Lemma formalizes the point.

Lemma 7 *If V is privately observed by the agent rather than the principal, the latter cannot attain a higher payoff than the optimal profit Π^H of a lock-in contract.*

This means that, in order to effectively condition the execution of trade on *external* information, it is vital that such information is observed by the principal, as assumed in our model.

Appendix

Proof of Lemma 1. Suppose the principal can propose any direct and incentive-compatible deterministic mechanism, inducing an allocation rule $\mu : V \to \{0,1\} \times [0,\infty)^3$.

For an arbitrary such mechanism, let us denote $\mathscr{V}_0$ the subset of the principal's of $V \in \mathscr{V}$ reports that induce $\rho = 0$, and V_1 the subset inducing $\rho = 1$, where $\mathscr{V}_0 \cup \mathscr{V}_1 = \mathscr{V}$. Note first that $z(V) = z$ must be constant, for incentive-compatibility reasons, among all $V \in \mathscr{V}_0$, and similarly, for all $V \in \mathscr{V}_1$, trade must yield the same profit $\pi(e, t_H(V), t_b(V)) = \pi_1$ to the principal. Furthermore, it is incentive-compatible for the principal to truthfully report $V \in \mathscr{V}_1$, that is, a value of V inducing trade, if and only if $V - z \leq \pi_1$. As a consequence, any incentive-compatible mechanism can induce trade if and only if $V < \pi_1 + z$ realizes. That is, it enforces a threshold trading rule where $\rho = \mathbb{I}(V \leq \pi_1 + z)$. Notably, the same trading rule can be expressed as $\rho = \mathbb{I}(V \leq \hat{V}(\phi))$, where $\phi = F(\pi_1 + z)$. Thus, an incentive-compatible direct, deterministic mechanism is fully identified by the associated frequency of trade ϕ.

Denote $T_\omega = \mathbb{E}[t_\omega(V) | V \leq \hat{V}(\phi)]$. Then, relaxing the limited liability constraint at each state of the world $\omega \in \{g, b\}$ by taking it at the ex ante stage, thus reformulating it as $T_\omega \geq 0$, an upper bound on the principal's payoff can be found at the solution of the following program.

$$\max_{\mu} \quad \phi[y_H - p_H T_g - (1 - p_H) T_b] + (1 - \phi)(\bar{V}(\phi) - z) \tag{7}$$

$$\text{s.t.:} \quad T_g \geq T_b + \frac{d}{\phi \Delta p} \tag{ICC_H}$$

$$p_H T_g + (1 - p_H) T_b - d \geq 0 \tag{IRC_H}$$

$$T_\omega \geq 0 \;\; \forall \omega \in \{g, b\} \tag{LL_ω}$$

$$z \geq 0 \tag{LL_z}$$

$$y_H - p_H T_g - (1 - p_H) T_b = \hat{V}(\phi) - z \tag{ICC_ϕ}$$

In this program, there is no loss of generality in focusing on mechanisms such that $t_\omega(V) = T_\omega$ for each $V \in [V_a, V_b]$. However, any solution as such automatically respects the stricter ex-post limited liability constraint $t_\omega(V) \geq 0$ for each (ω, V), meaning that the relaxed program has the same value as the full program in which the ex-post limited liability constraints are considered. Thus, the principal can always achieve her optimal payoff by proposing a contract with exit rights $\mathscr{C} = \{(t_g, t_b); z\}$ in which the transfers and the penalty fee do not depend on V, rather than a more complicated direct mechanism. ■

Proof of Proposition 1. Denote W_e^* the optimal level of utilitarian social welfare conditional on the agent's effort decision. Note that the social welfare from an allocation μ is

$$w(\mu) = \int_{V_a}^{V_b} [\rho(V) y_e + (1 - \rho(V)) V f(V)] dV$$

meaning that, for a given level of $e \in \{L,H\}$ it is optimal to set $\rho(V) = 1$ if and only if $V \leq y_e$.

Now, define the function $\nu : [0,1] \to \mathbb{R}$ where

$$\nu(V) = F(V)V + (1-F(V))\bar{V}(F(V)).$$

Remarkably, the efficient level of social welfare conditional on $e \in \{L,H\}$, gross of the effort cost, is equal to $\nu(y_e)$. Therefore, $e = H$ is optimal if $W_H^* - W_L^* = \nu(y_H) - d - \nu(y_L) > 0$, or $\nu(y_H) - \nu(y_L) > d$. As guaranteed by Leibniz' rule, one can also note that

$$\frac{\partial \nu(V)}{\partial V} = Vf(V) + F(V) - f(V)\mathbb{E}[V'|V' > V] + (1-F(V))\frac{f(V)}{1-F(V)}\big(\mathbb{E}[V'|V' > V] - V\big) = F(V).$$

Thus,

$$\nu(y_H) - \nu(y_L) = \int_{y_L}^{y_H} F(V)dV > C^H = \frac{p_H d}{\Delta p} > d$$

where the former inequality is true by Assumption 1 and the latter since $p_H > \Delta p$. ■

Proof of Proposition 2. The principal's preferred policy inducing the effort level $e = H$ when V is observable, meaning that ρ can be explicitly designed by the principal to ex ante condition on $V \in \mathscr{V}$, is identified at the solution of the following program:

$$\begin{aligned}
\max_{(\phi,c)\in[0,1]\times[0,\infty)^2\times[0,\infty]} \quad & \phi[y_H - p_H t_g - (1-p_H)t_b] + (1-\phi)(\bar{V}(\phi) - z) && (8)\\
\text{s.t.:} \quad & \phi[p_H t_g + (1-p_H)t_b] + (1-\phi)z \geq 0 && (IRC_H)\\
& t_g \leq t_b + \frac{d}{\phi\Delta p} && (ICC_H)\\
& t_\omega \geq 0 \ \ \forall\omega \in \{g,b\} && (LL_\omega)\\
& z \geq 0 && (LL_z)
\end{aligned}$$

The (ICC_H), (LL_b) and (LL_z) constraints in (8) are optimally binding, and thus the optimal contract is such that

$$t_g = \frac{C^H}{p_H\phi}, \quad t_b = 0, \quad z = 0.$$

This means that the principal's profits from trade associated to an optimal incentive-compatible contract are increasing in ϕ, and equal to

$$\Pi^{IC}(\phi) = \pi(e, t_g^{IC}(\phi), t_b^{IC}(\phi)) = y_H - \frac{C^H}{\phi}.$$

To see how this affects the design of the principal's preferred policy, and in particular the optimal frequency of trade, let us substitute $\Pi^{IC}(\phi)$ in the objective function of (8). By doing so, one obtains the following expression:

$$\nu_H^*(\phi) = \phi y_H - C^H + (1-\phi)\bar{V}(\phi).$$

Thus, the principal's optimal trading rule is such that $\rho(V) = \mathbb{I}(V \leq y_H)$ or equivalently $\phi = \phi_H^*$, which is the same as in the utilitarian benchmark. Furthermore, it can be noted that the agent's ex ante expected payoff from the policy $\mathscr{C}_H^*$ characterized at the solution of (8) is

$$u(H,\phi_H^*,\mathscr{C}_H^*) = \phi_H^* p_H \frac{C^H}{p_H \phi_H^*} - d = R^H > 0,$$

meaning that the participation constraint (8) is also satisfied with slack.

Proof of Lemma 2. To induce the effort level $e = L$ the principal can propose a contract paying $t_g = t_b = z = 0$ to the agent. This yields y_L when trade happens. The principal's optimal participation rule is thus, for the same reason exhibited in the proof of Proposition 2 when $e = H$, to take part in trade only when $V \leq y_L$, which happens with frequency $\phi_L^* := F(y_L)$.

Now, if ν is as defined in the proof of Proposition 2, the principal's expected payoff from optimally inducing $e = L$ is

$$v_L^* = \phi_L^* y_L + (1-\phi_L^*)\bar{V}(\phi_L^*) = \nu(y_L) < \nu(y_H) - C^H$$

where the inequality is shown to hold in the proof of Proposition 1 when Assumption 1 is respected.

Furthermore, $V_0 = \nu(V_a)$, $v_L^* = \nu(y_L)$; also, ν is increasing with $y_L > V_a$, implying $v_L^* > V_0$. Since, then, $v_H^* > v_L^* > V_0$, the principal strictly prefers inducing $e = H$ than $e = L$ or abstaining from trade, as stated in the Lemma. ■

Proof of Proposition 3. The (relaxed) optimal frequency of trade ϕ^r must be such that

$$\phi^r \in \arg\max_{\phi\in(0,1]} \quad v^r(\phi) = y_H - \frac{C^H}{\phi} + (1-\phi)[\bar{V}(\phi) - \hat{V}(\phi)],$$

and, in particular, any interior solution must be a stationary point of the objective function $v^r(\phi)$. Note now that, as guaranteed by Leibniz' rule,

$$\frac{\partial \mathbb{E}_F[V|V>\hat{V}]}{\partial \hat{V}} = \frac{f(\hat{V})}{1-F(\hat{V})}\left(\mathbb{E}_F[V|V>\hat{V}] - \hat{V}\right)$$

and also that, since $\hat{V}(\phi) = F^{-1}(\phi)$,

$$\frac{\partial \hat{V}(\phi)}{\partial \phi} = \frac{1}{f(\hat{V}(\phi))}.$$

But, $\bar{V}(\phi) = \mathbb{E}_F[V|V>\hat{V}(\phi)]$ is the composition of the truncated expectation with the quantile function $\hat{V}(\phi)$, meaning that

$$\frac{\partial \bar{V}(\phi)}{\partial \phi} = \frac{\bar{V}(\phi) - \hat{V}(\phi)}{1-\phi},$$

using the fact that $F(\hat{V}(\phi)) = \phi$. Thus,

$$\frac{\partial (1-\phi)\bar{V}(\phi)}{\partial \phi} = (1-\phi)\frac{\bar{V}(\phi) - \hat{V}(\phi)}{1-\phi} - \bar{V}(\phi) = -\hat{V}(\phi).$$

Finally, this means that the derivative of the objective function with respect to ϕ is

$$\kappa(\phi) - \hat{V}(\phi) - \frac{1-\phi}{f(\hat{V}(\phi))} + \hat{V}(\phi) = \kappa(\phi) - I(\hat{V}(\phi))$$

which is null if and only if $\kappa(\phi) = I(\hat{V}(\phi))$. ∎

Proof of Proposition 4. Recall that the objective function of the principal is

$$v^r(\phi) = \Pi^{IC}(\phi) + (1-\phi)[\bar{V}(\phi) - \hat{V}(\phi)],$$

and that we look for an interior maximizer of $v^r(\phi)$.

To characterize it, start by listing some properties of the function $\kappa : (0,1] \to \mathbb{R}$. Namely, $\kappa(\phi)$ is defined and smooth for $\phi \in (0,1]$; it is such that $\lim\limits_{\phi \to 0} \kappa(\phi) = \infty$ and $\kappa(1) = C^H \in (0,\infty)$; also, it is strictly decreasing and concave in its domain.

Define now $C^* := \lim\limits_{\phi \to 1} I(\hat{V}(\phi))$ and note that it coincides with $\sup_{\phi \in (0,1)} I(\hat{V}(\phi))$ since $I(\hat{V}(\phi))$ is increasing or constant. We consider two mutually exclusive cases.

Suppose first that $C^* > C^H$.

Note that, since $f(V_a) > 0$ by assumption, $I(\hat{V}(0)) = \frac{1-F(V_a)}{f(V_a)} = \frac{1}{f(V_a)}$ is defined and finite; but then, since $\lim\limits_{\phi \to 0} \kappa(0) = \infty$, the continuity of both functions guarantees that there exists an interval $(0,\varepsilon)$ with $\varepsilon \in (0,1)$ such that $I(\hat{V}(\phi)) < \kappa(\phi)$ for any interior ϕ. Furthermore, by continuity of $I(\hat{V}(\phi))$ and the fact that

$$\lim_{\phi \to 1} I(\hat{V}(\phi)) = C^* > \kappa(1) = C^H$$

by construction, there also exists an open interval $(\delta,1)$, with $\delta \in (\varepsilon,1)$ such that $I(\hat{V}(\phi)) > \kappa(\phi)$ for each ϕ interior. Hence, the intermediate value theorem, taken between any two interior points of $(0,\varepsilon)$ and $(\delta,1)$, ensures that $I(\hat{V}(\phi^r)) = \kappa(\phi^r)$ for some $\phi^r \in (0,1)$. Therefore, ϕ^r is an interior stationary point of v^r.

To show that it is uniquely identified, and it is a global maximizer of $v^r(\phi)$ in its domain $(0,1]$, note that from the fact that $\frac{\partial \kappa(\phi)}{\partial \phi} < 0$ and $\frac{\partial I(\hat{V}(\phi))}{\partial \phi} \geq 0$ it follows that $\frac{\partial^2 v^r(\phi)}{\partial \phi^2} < 0$ which implies that the objective function $v^r(\phi)$ of the relaxed program is concave for $\phi \in (0,1]$.

Suppose instead that $C^H \geq C^*$. In this case, $I(\hat{V}(1)) < \kappa(1)$. Since $I(\hat{V}(\phi))$ is increasing and $\kappa(\phi)$ strictly decreasing, it follows that $I(\hat{V}(\phi)) < \kappa(\phi)$ for any $\phi \in (0,1)$. This means that $\frac{\partial v^r(\phi)}{\partial \phi}(\phi) > 0$ for all $\phi \in (0,1]$, so that $v^r(\phi)$ is strictly increasing in its entire domain and the optimal value ϕ^r coincides with its upper boundary $\phi^r = 1$. ∎

Proof of Corollary 1. Note that, since $I(V)$ is assumed to be non-decreasing, F has unbounded support: in fact, if on the contrary $V_b < \infty$, then $f(V_b) > 0$ and $F(V_b) = 1$, implying that $I(V_b) = \frac{1-F(V_b)}{f(V_b)} = 0 < I(V_a) = \frac{1-F(V_a)}{f(V_a)}$, and thus contradicting the assumption that $I(V)$ is non-decreasing. But then, Lauermann and Wolinsky (2016, p. 272) show that the condition in the statement is necessary to have that $\lim\limits_{V \to \infty} I(V) = \lim\limits_{\phi \to 1} I(\hat{V}(\phi)) = C^*$ exists finite. ∎

Proof of Lemma 3. Consider two mutually exclusive cases.

Suppose first that $z^r(\phi) > 0$ for each $\phi \in [0,1]$: in this case, define $\underline{\phi} = 0$ and note that it respects the statement of the lemma by construction.

Suppose then, that there exists $\phi' \in [0,1]$ such that $z^r(\phi') \leq 0$. In this case, let us define $\underline{\phi}$ as the supremum of the set of the zeros of the function $z^r(\phi) = \hat{V}(\phi) - \Pi^{IC}(\phi)$, over its domain $(0,1]$, and then show that $\underline{\phi}$ respects the statement of the Lemma.

It is convenient to begin by showing that, if $\underline{\phi}$ exists finite, it is such that $\underline{\phi} \in (0, F(\Pi^H))$. To see it, note that $\hat{V}(F(\Pi^H)) = \Pi^H$ by definition of $\hat{V}$. Then, since $\hat{V}(\phi)$ is increasing, it follows that, for each $\phi \in [F(\Pi^H), 1]$,

$$\hat{V}(\phi) > \hat{V}(F(\Pi^H)) = \Pi^H > \Pi^{IC}(\phi)$$

where the latter inequality holds since $\Pi^H > \Pi^{IC}(\phi)$ for each $\phi \in (0,1)$. Hence, $z^r(\phi) > 0$ for any $\phi \in [F(\Pi^H), 1]$.

To see that $\underline{\phi}$ exists finite, note that $z^r(\phi)$ is defined and continuous in $\phi \in (0,1)$ because both $\hat{V}(\phi)$ and $\Pi^{IC}(\phi)$ are defined and continuous in $\phi \in (0,1)$. Hence, due to the continuity of $z^r(\phi)$ and the fact that $z^r(F(\Pi^H)) > 0$, the intermediate value theorem applied over the interval $[\phi', F(\Pi^H)]$ guarantees that $z^r(\phi) = 0$ for at least one choice of $\phi \in [\phi', F(\Pi^H))$. Therefore, the set of the zeros of $z^r(\phi)$ is non-empty in $\phi \in (0,1]$ and its supremum exists finite.

Let us now verify that $\underline{\phi}$ respects the statement of the Lemma.

To see that $z^r(\phi) > 0$ for each $\phi \in (\underline{\phi}, 1]$, note that the existence of any $\phi \in (\underline{\phi}, 1]$ such that $z^r(\phi) = 0$ is in contradiction with the definition of $\underline{\phi}$; also, if there is $\phi \in (\underline{\phi}, 1]$ such $z^r(\phi) < 0$, the intermediate value theorem guarantees that z^r has a zero within the interval $(\phi, F(\Pi^H))$, contradicting the definition of $\underline{\phi}$.

Finally, to see that $z^r(\underline{\phi}) = 0$, note that, if $z^r(\underline{\phi}) \neq 0$, by continuity of z^r there exists $\varepsilon > 0$ such that $z^r(\phi) \neq 0$ for each $\phi \in (\underline{\phi} - \varepsilon, \underline{\phi})$: but this straightforwardly contradicts definition of $\underline{\phi}$ as the supremum of the set of the zeros of z^r. ■

Proof of Proposition 5. Start by noting that, if $z^r(\phi^r) \geq 0$, then the relaxed program has the same solution as the full program. In particular since $z^r(1) > 0$ as already shown in the proof of Lemma 3, $\phi^r = 1$ implies that $\phi^* = 1$: anytime a lock-in contract is optimal in the relaxed program, it is also optimal in the full one. In this case, any penalty $z^r \in [V_b - \Pi^H, \infty]$ can enforce the consistency of $\phi = 1$, and the optimal transfers are same as in the deterministic case, that are, $(t_g = \frac{C^H}{p_H \phi}, t_b = 0)$ yielding Π^H to the principal (case *i.* in the statement of the proposition).

Suppose instead that $\phi^r \in (0,1)$ and $z^r(\phi^r) > 0$: in this case, $\phi^r = \phi^*$ and $z^* = z^r(\phi^r) > 0$. Again, the optimal transfers yield the principal the upper bound of the incentive-compatible value of trade, $\Pi^{IC}(\phi^*)$, meaning that the solution belongs to case *ii.* in the statement of the proposition.

Suppose finally that $z^r(\phi^r) \leq 0$, which is mutually exclusive with respect to the cases already addressed. This has the implication, as seen in the proof of Lemma 3, that $\underline{\phi}$ is interior in the $(0, F(\Pi^H))$ interval.

One can then observe that $\phi^* \notin [0, \underline{\phi})$. To see that, one can verify that any such choice of ϕ is dominated by the policy $(\underline{\phi}, c^{IC}(\underline{\phi}))$.

To see it, note that the principal's optimal policy is found at the solution of the following program.

$$\max_{(\phi,c)\in[0,1]\times[0,\infty)^3} \quad \phi[y_H - p_H t_g - (1-p_H)t_b] + (1-\phi)(\bar{V}(\phi) - z) \tag{9}$$

$$\text{s.t.:} \quad \phi[p_H t_g + (1-p_H)t_b] + (1-\phi)z - d \geq 0 \tag{IRC_H}$$

$$t_g \geq t_b + \frac{C^H}{p_H \phi} \tag{ICC_H}$$

$$t_\omega \geq 0 \ \forall \omega \in \{g,b\} \tag{LL_ω}$$

$$z \geq 0 \tag{LL_z}$$

$$y_H - p_H t_g - (1-p_H)t_b = \hat{V}(\phi) - z \tag{ICC_ϕ}$$

where $t_g = \frac{C^H}{p_H \phi} + t_b$ is optimal since (ICC_H) is optimally binding, meaning that the profits from trade must be $\Pi^{IC}(\phi) - t_b$ at the optimum. Hence, the (ICC_ϕ) can be rewritten

$$z = \hat{V}(\phi) - \Pi^{IC}(\phi) + t_b.$$

By noting this, and also substituting out the (ICC_ϕ) within the objective and the (LL_z), the program becomes:

$$\max_{(\phi,t_b)\in[0,1]\times[0,\infty)} \quad \Pi^{IC}(\phi) - t_b + (1-\phi)(\bar{V}(\phi) - \hat{V}(\phi)) \tag{10}$$

$$\text{s.t.:} \quad t_b = \max\{0, \Pi^{IC}(\phi) - \hat{V}(\phi)\} \tag{LL}$$

Indeed, the binding, unified limited liability constraint (LL) emerges from the fact that the principal optimally selects the lowest feasible t_b, which corresponds to $t_b = 0$ when $z^r(\phi) \geq 0$ or $t_b = -z^r(\phi) = \Pi^{IC}(\phi) - \hat{V}(\phi)$ if $z^r(\phi) < 0$.

It is convenient to split the $[0,1]$ interval in two subsets, say Φ^+, containing all the ϕ such that $z^r(\phi) \geq 0$, and Φ^- containing those such that $z^r(\phi) \leq 0$. Note that, $\Phi^+ \cup \Phi^- = [0,1]$ and $\Phi^+ \cap \Phi^-$ contains all the $\phi \in [0,1]$ such that $z^r(\phi) = 0$.

Furthermore, Lemma 3 shows that $[\underline{\phi}, 1] \subset \Phi^-$, and that $\underline{\phi} = \max \Phi^+$.[24]

As a first step, let us show that $\phi^* \in \Phi^-$. Consider, indeed, the problem (10) in the restricted domain $\Phi^+ \times [0,\infty)$. Then, $t_b = \Pi^{IC}(\phi) - \hat{V}(\phi)$ is optimally binding and the objective can be rewritten as

$$\hat{V}(\phi) + (1-\phi)(\bar{V}(\phi) - \hat{V}(\phi)) = \phi\hat{V}(\phi) + (1-\phi)\bar{V}(\phi) = \nu(\hat{V}(\phi)).$$

The function ν has been already shown in the proof of Proposition 1 to be strictly increasing in $V \in \mathscr{V}$, and, since $\hat{V}$ is strictly increasing in $\phi \in [0,1]$, it holds that $\nu(\hat{V}(\phi))$ is increasing in $\phi \in \Phi^+$. Therefore, in the restricted domain Φ^+, the principal optimally selects the largest value of $\phi \in \Phi^+$ which, indeed, coincides with $\underline{\phi}$. But this is also contained in Φ^-, and thus, there is no loss of generality in focusing on Φ^- in order to find ϕ^*.

[24]To see why the latter statement holds, note that $z^r(\underline{\phi}) = 0$ implies that $\underline{\phi} \in \Phi^+$, and, for each $\phi \in (\underline{\phi}, 1]$, it holdsthat $z^r(\phi) > 0$ and thus $\phi \notin \Phi^+$, making ϕ the largest trade frequency belonging to Φ^+.

At this point, it only remains to check that $\phi^* \in [\underline{\phi}, 1]$. To see it, start by noting that, for each $\phi \in \Phi^-$, $t_b = 0$ is optimal. Hence, after substituting it, the value function of (10) looks as

$$\Pi^{IC}(\phi) + (1-\phi)(\bar{V}(\phi) - \hat{V}(\phi)) = \hat{V}(\phi) - z^r(\phi) + (1-\phi)(\bar{V}(\phi) - \hat{V}(\phi))$$

where we substitute $\Pi^{IC}(\phi) = \hat{V}(\phi) - z^r(\phi)$.

But then, at $\phi = \underline{\phi}$, the value of the objective function is $\hat{V}(\underline{\phi}) + (1-\underline{\phi})(\bar{V}(\underline{\phi}) - \hat{V}(\underline{\phi}))$ since $z^r(\underline{\phi}) = 0$, while, if $\phi < \underline{\phi}$, it is equal to $\hat{V}(\phi) - z^r(\phi) + (1-\phi)(\bar{V}(\phi) - \hat{V}(\phi))$. Since $z^r(\phi) \geq 0$ as implied by $\phi \in \Phi^+$, and since $\hat{V}(\phi) < \hat{V}(\underline{\phi})$ as $\phi < \underline{\phi}$, $\underline{\phi}$ dominates any such choice of ϕ.

Therefore, $\phi^* \in [\underline{\phi}, 1]$. Since then $z^r(\phi^*) \geq 0$, and the associated contract $c^r(\phi^*)$ is thus feasible in the relaxed program, this implies that, for any parametrization of the environment, the optimal policy must belong to either case *i.*, *ii.* or *iii.*, and can assume no other form. ■

Proof of Proposition 6. The part concerning the case $\phi^r = 1$ has already been established in the proof of Proposition 5.

Suppose instead that $\phi^r \in (0,1)$. Let us start from the first part of the statement, that is, $z^r(\phi^r) \leq 0$ if and only if $y_H \geq y^*$, with strict inequality if and only if $y_H > y^*$, for some y^* finite.

To proceed, note that $I(\hat{V}(\phi^r)) = \kappa(\phi^r)$ fully identifies ϕ^r as already seen in Lemma 3. But then, since $\kappa(\phi) = \frac{C^H}{\phi^2}$ and y_H does not enter the function $I(V)$, ϕ^r is independent from the value of y_H.

Note also that $z^r(\phi^r) \leq 0$ can be equivalently expressed as $y_H \geq y^* := \hat{V}(\phi^r) + \frac{p_H}{\phi^r \Delta p} d$. Since y^* does *not* depend on y_H through ϕ^r, $y_H \geq y^*$ is thus necessary and sufficient to have that $z^r(\phi^r) \leq 0$. The exact same argument can be pointed if one considers strict inequalities instead.

To move to the second part of the statement, let us assume that $I(V)$ is increasing or constant and $y_H \geq y^*$.

Let us prove, first, that $y_H \geq y^*$ implies $\phi^* = \underline{\phi}$.

Note that, by definition, $\underline{\phi}$ is such that there is no $\phi \in (\underline{\phi}, 1]$ such that $z^r(\phi) \leq 0$. Therefore, since $y_H \geq y^*$ implies that $z^r(\phi^r) \leq 0$, it must be that $\phi^r \in (0, \underline{\phi}]$.

Now, from Proposition 4, we know that $\phi^* \in [\underline{\phi}, 1]$ and thus $c^* = c^r(\phi^*)$, meaning that

$$\phi^* = \arg \max_{\phi \in [\underline{\phi}, 1]} v^r(\phi). \tag{11}$$

Also, as already argued in the proof of Proposition 4, the fact that $I(V)$ is non-decreasing implies that v^r is concave. Then, since $\phi^r \in (0, \underline{\phi}]$ is the global maximizer of $v^r(\phi)$, the latter is decreasing over the whole interval $[\underline{\phi}, 1]$. It follows that, since ϕ^* maximizes $v^r(\phi)$ over the interval $[\underline{\phi}, 1]$ in which it is decreasing, $\phi^* = \underline{\phi}$.

To show that the opposite implication holds, that is, $\phi^* = \underline{\phi}$ only if $y^H \geq y^*$, note simply that, if on the contrary $y_H < y^*$, $z^r(\phi^r) > 0$ as already proved. Then, $\phi^* = \phi^r$ since the relaxed solution is feasible in the fully constrained program. But then, since $z^r(\underline{\phi}) = 0$ as shown in Lemma 3, it follows that $\phi^r \neq \underline{\phi}$. ■

Proof of Theorem 1. In this proof, we break down the space of all possible pairs (y_H, C_H) according to the class of contracts that they induce in the principal's optimum: at-will ($z = 0$), partially secured ($z > 0$ finite) or fully secured ($z = \infty$).

It is convenient to separate two possible cases: $C_H \geq C^*$ and $C_H < C^*$.

If $C^H > C^* = \lim_{\phi \to 1} I(\hat{V}(\phi))$, as already argued in Propositions 4 and 5, the optimal frequency of trade is $\phi^* = 1$, which is also featured at the solution of the relaxed program. This means that the optimal contract offered by the principal is fully secured and exhibits $z = \infty$.

If $C^H < C^*$, Proposition 6 illustrates that the class of the optimal contract depends fully on whether y_H lies above or below a threshold level y^*, and in particular that $z = 0$ is optimal when $y_H \geq y^*$ and $z > 0$ finite otherwise. In the rest of the proof, we show that there exists a continuous, strictly increasing function $\bar{y} : (0, C^*) \to (0, \infty)$ which, for any value of $C^H \in (0, C^*)$, returns the associated threshold value $y^* = \bar{y}(C^H)$ of the agent's expected output.

To define $\bar{y}$, start by noting that, for a given C^H, the relaxed optimal frequency of trade $\phi^r \in (0, 1)$ is interior and pinned down by the necessary and sufficient condition

$$C^H = (\phi^r)^2 I(\hat{V}(\phi^r)). \tag{12}$$

We now argue that, for each that the parameter C^H can assume in the interval $(0, C^*)$, there exists one and only one value of ϕ^r possibly arising at the relaxed optimum. To see that, begin by noting that the RHS of (12) is defined and continuous in $\phi^r \in (0, 1)$, being the product of functions verifying these properties; also, it is strictly increasing in $\phi^r \in (0, 1)$, since its first derivative is positive and equal to

$$2(\phi^r) I(\hat{V}(\phi^r)) + (\phi^r)^2 I'(\hat{V}(\phi^r)) > 0.$$

Furthermore, since $I(\hat{V}(0)) = \frac{1}{f(V_a)}$ is defined and finite, the RHS of (12) is defined and equals zero when $\phi^r = 0$, and consequently, it approaches zero as $\phi^r \to 0$.

Therefore, the RHS of (12) is a strictly increasing function of $\phi^r \in (0, 1)$ ranging in the open interval $(0, C^*)$. But then, for each value of $C^H \in (0, C^*)$, there exists one and only one value of ϕ^r verifying (12), and thus, inducing the trade frequency ϕ^r at the relaxed optimum. Define $\hat{\phi} : (0, C^*) \to (0, 1)$ the function associating such a frequency of trade to each value of $C^H \in (0, C^*)$. This function, as the argument above guarantees, is defined, continuous and increasing in $(0, C^*)$.[25]

Consider now the threshold $y^* := \frac{C^H}{\phi^r} + \hat{V}(\phi^r)$ already identified in the proof of Proposition 6. By substituting $\phi^r = \hat{\phi}(C^H)$, one obtains that $y^* = \bar{y}(C^H) = \frac{C^H}{\hat{\phi}(C^H)} + \hat{V}(\hat{\phi}(C^H))$. Using the fact that $C^H = (\hat{\phi}(C^H))^2 I(\hat{V}(\hat{\phi}(C^H))$, this rewrites as

$$\bar{y}(C^H) = \phi^r(C^H) I(\hat{V}(\phi^r(C^H)) + \hat{V}(\hat{\phi}(C^H)).$$

Start by noting that $\bar{y}(C^H)$ is defined for each $C^H \in (0, C^*)$ as ensured by our analysis of (12), and is continuous in C^H being the composition of continuous functions.

Note also that

$$\frac{\partial \bar{y}(C^H)}{\partial C^H} = \left[I(\hat{V}(\hat{\phi}(C^H))) + \hat{\phi}(C^H) I'(\hat{V}(\hat{\phi}(C^H))) + \frac{1}{f(\hat{V}(\hat{\phi}(C^H)))} \right] \frac{\partial \hat{\phi}(C^H)}{\partial C^H} > 0.$$

Indeed, the term in the parenthesis is positive as we assume, in the statement of the Theorem, that

[25] In particular, it is continuous because it is the inverse of the continuous function defined by expressing C^H in terms of ϕ^r, as shown in the RHS of (12).

$I(V)$ is non-decreasing. Also, $\frac{\partial \hat{\phi}(C^H)}{\partial C^H} > 0$ since $\hat{\phi}$ is strictly increasing in C^H as already argued. Thus, $\bar{y}(C^H)$ is strictly increasing in $C^H \in (0, C^*)$.

Finally,

$$\lim_{C^H \to 0} \bar{y}(C^H) = 0 \cdot I(\hat{V}(0)) + \hat{V}(0) = V_a$$

and

$$\lim_{C^H \to C^*} \bar{y}(C^H) = 1 \cdot \lim_{\phi \to 1} I(\hat{V}(\phi)) + \hat{V}(1) = C^* + V_b = \infty$$

where the last equality holds because, as already noted in Corollary 1, $V_b = \infty$ if $I(V)$ is non-decreasing. ■

Proof of Lemma 4. Note first that the principal's problem when $V \in \mathscr{V}$ is private is more constrained than when it is public, and thus cannot have a higher value; hence, v_L^* is an upper bound on the value of the principal's payoff.

Note also that the policy $(\phi_L^*, \mathscr{C}_L^*)$ where $\mathscr{C}_L^* = (0,0,0)$ is incentive-compatible, as it respects constraints $(IRC_L, ICC_L, LL_g, LL_b, LL_z)$ as already verified and, also, it respects (ICC_ϕ) since $\hat{V}(\phi_L^*) = \Pi^L = y_L$ by definition of ϕ_L^*. Therefore, the principal attains the upper bound v_L^* and $(\phi_L^*, \mathscr{C}_L^*)$ is an optimal policy. ■

Proof of Lemma 5. Suppose that $\phi^r \in (0,1)$ and recall from equation 5 that

$$\frac{\partial z^r(\phi)}{\partial \phi}\Big|_{\phi=\phi^r} = \frac{1}{f(\hat{V}(\phi^r))} - \kappa(\phi^r).$$

Recall also the optimality condition $\kappa(\phi^r) = \frac{1-\phi^r}{f(\hat{V}(\phi^r))}$ from Proposition 3, and note that since, $\kappa(\phi^r)$ is strictly decreasing in ϕ, it follows that $\kappa(\phi) < \frac{1-\phi^r}{f(\hat{V}(\phi^r))}$ for each $\phi \in (\phi^r, 1]$. Furthermore, from the fact that $I(\hat{V}(\phi^r))$ is strictly increasing in ϕ^r,[26] it follows that $\frac{1}{f(\hat{V}(\phi))} > \frac{1}{f(\hat{V}(\phi^r))}$ for each $\phi \in (\phi^r, 1]$.

Hence, it must be that

$$\kappa(\phi^r) < \frac{1-\phi^r}{f(\hat{V}(\phi^r))} < \frac{1}{f(\hat{V}(\phi^r))} < \frac{1}{f(\hat{V}(\phi))}$$

for each $\phi \in [\phi^r, 1]$, meaning that $\frac{\partial z^r(\phi)}{\partial \phi} > 0$ in the $[\phi^r, 1]$ interval, and that $z^r(\phi)$ is strictly increasing for such values of ϕ.

Consider now program (3), but assume that, because of a minimal mandatory liquidation fee imposed by the government, (LL_z) is changed to $z \geq \bar{z}$.

To see what happens as $\bar{z}$ varies in the $(0, \infty)$ interval, let us start from the case in which $\bar{z} \in (0, z^r(\phi^r)]$.

[26] Note indeed that, in order to have that $I(\hat{V}(\phi))$ is increasing, we need that

$$\frac{\partial I(\hat{V}(\phi))}{\partial \phi} = (1-\phi)\frac{\partial \frac{1}{f(\hat{V}(\phi))}}{\partial \phi} - \frac{1}{f(\hat{V}(\phi))} > 0$$

which is only possible if $\frac{\partial \frac{1}{f(\hat{V}(\phi))}}{\partial \phi} > 0$.

In this occurrence, since $\bar{z} > 0$, it also holds that $z^r(\phi^r) > 0$, meaning that $\phi^* = \phi^r$ and $c^* = c^r(\phi^r)$ must hold in the *status quo* case that $\bar{z} = 0$. Furthermore, since $z^r(\phi^r) > \bar{z}$, the solution $(\phi^r, c^r(\phi^r))$ of the relaxed program disregarding (LL_z) is still feasible once the constraint $z \geq \bar{z}$ is accounted for, meaning that, $(\phi^r, c^r(\phi^r))$ is still optimal when a minimal liquidation fee lying in this interval is introduced by the government. Therefore, in this scenario, the government's intervention bears no effect.

Assume now that $\bar{z} \in (z^r(\phi^r), V_b - \Pi^H)$. In this case, the relaxed optimum $(\phi^r, c^r(\phi^r))$ is by construction unfeasible in the *new* program with $z \geq \bar{z}$. Define $\bar{\phi}$ to be the largest value of $\phi \in [0,1]$ such that $z^r(\phi) \leq \bar{z}$, and denote $\tilde{\phi}$ to be the largest value of $\phi \in [0, \bar{\phi})$ such that $z^r(\phi) \geq \bar{z}$. Note also that, since $z^r(1) = V_b - \Pi^H$, $\overline{\phi} < 1$ by construction.

But then, since $z^r(\phi^r) < \bar{z}$ and $z^r(1) > \bar{z}$, and since $z^r(\phi)$ is continuous and increasing in the $(\phi^r, 1)$ interval as already shown, it follows that $\bar{\phi}$ exists and that it is identified as the unique frequency of trade $\bar{\phi} \in (\phi^r, 1)$ such that $z^r(\bar{\phi}) = \bar{z}$, as guaranteed by the application of the intermediate value theorem on the $[\phi^r, 1]$ interval.

An immediate implication is that, since the reasoning above implies that $z^r(\phi) < \bar{z}$ for each $\phi \in [\phi^r, \bar{\phi})$, it must be that $\tilde{\phi} < \bar{\phi}$. Now, since $\lim_{\phi \to 0} z^r(\phi) = \infty$, it must also be that $\tilde{\phi} > 0$. Also, since $z^r(\phi^r) < \bar{z}$, the continuity of $z^r(\phi)$ implies that $\tilde{\phi}$ is such that $z^r(\tilde{\phi}) = \bar{z}$: if, by contradiction, $z^r(\tilde{z}) > 0$, then the intermediate value theorem would guarantee the existence of $\phi' \in (\tilde{\phi}, \phi^r)$ such that $z^r(\phi') = \bar{z}$, which is incompatible with the definition of $\tilde{\phi}$.

We now show that, whenever $\bar{z} > z^r(\phi^r)$, the optimal policy of the principal in the program constrained by $z \geq \bar{z}$ induces the frequency of trade $\phi = \bar{\phi}$, identified as the unique $\phi \in (\phi^r, 1)$: $z^r(\phi) = \bar{z}$, together with the contract $c^r(\bar{\phi})$.

Start by noting that, if the optimal frequency of trade is such that $z^r(\phi) \geq \bar{z}$, since $c^r(\phi)$ is then feasible, it is necessary that $c = c^r(\phi)$ is the implemented contract, thus yielding $v^r(\phi)$ to the principal. Instead, for the same argument as in the proof of Proposition 4 in case $z^r(\phi) < \bar{z}$, it must be that $(z = 0, t_b = |z^r(\phi) - \bar{z}|)$ in the optimal contract whenever the optimal trade frequency has $z^r(\phi) < \bar{z}$, thus yielding $\hat{V}(\phi) - \bar{z}$ to the principal.

Also, since $v^r(\phi)$ is strictly concave and has a global maximizer in ϕ^r, it must be increasing in the $[0, \phi^r]$ interval and strictly decreasing in $[\phi^r, 1]$.

But then, any $\phi > \bar{\phi}$ is not optimal, since $z^r(\phi) > \bar{z}$ is an implication of the fact that $z^r(\phi)$ is increasing over $[\phi^r, 1]$, implying that by inducing any $\phi \in [\phi^r, 1]$ the principal optimally obtains $v^r(\phi)$, which is maximized at $\phi = \bar{\phi}$ in this restricted domain.

Furthermore, any $\phi < \tilde{\phi}$ cannot be optimal, since inducing $\phi = \tilde{\phi}$ yields optimally $v^r(\tilde{\phi})$ to the principal, and any $\phi \in (0, \tilde{\phi}) \subset [0, \phi^r]$ yields at most $v^r(\phi) < v^r(\tilde{\phi})$ where the inequality is guaranteed by the fact that $v^r(\phi)$ is stricty increasing in that interval.

Finally, any $\phi \in (\tilde{\phi}, \bar{\phi})$ is not optimal since the associated optimal payoff $\hat{V}(\phi) - \bar{z}$ is strictly increasing in ϕ over that interval.

But then, since any choice in all these three intervals is strictly worse than $\bar{\phi}$ and the union of the intervals coincides with $[0,1] \backslash \bar{\phi}$, it follows that $\phi = \bar{\phi}$ is the optimal trade frequency for the principal to induce under the $z \geq \bar{z} > z^r(\phi^r)$ constraint.

To finally evaluate the social welfare effect of the new constraint $\bar{z} > z^r(\phi)$, we must consider two cases.

Suppose first that $\phi^* = \underline{\phi}$. This means that $z^r(\phi^r) < 0$ and thus $\bar{z} > z^r(\phi^r)$. Also, $\bar{\phi} > \underline{\phi}$ as guaranteed by the fact that $z^r(\phi)$ is strictly increasing in the $[\underline{\phi}, 1] \subset [\phi^r, 1]$ interval, and $z^r(\underline{\phi}) = 0$. But then, if $\bar{z} \leq z^r(\phi_H^*)$, the new trade frequency lies in the (ϕ^*, ϕ_H^*) interval and the utilitarian social welfare is strictly improved.[27] If $\bar{z} > z^r(\phi_H^*)$, the effect is instead ambiguous.[28]

Suppose instead that $\phi^* = \phi^r > \underline{\phi}$. In this case, a similar reasoning holds: if $\bar{z} \in (z^r(\phi^r), z^r(\phi_H^*))$, the new trade frequency $\bar{\phi}$ strictly improves social welfare, otherwise the effect ins ambiguous.

A final possibility is that $\bar{z} \in [V_b - \Pi^H, \infty]$. Then, the principal optimally induces the trade frequency $\phi = 1$ and the effect on social welfare is, again, ambiguous, which can be shown by applying exactly the same reason used above for the case in which $\bar{z} \in [z^r(\phi_H^*), V_b - \Pi^H)$. ■

Proof of Lemma 6. Note that the optimal allocation (inducing $e = H$) in the public information case solves[29]

$$\begin{aligned} \max_{(\phi,c)\in[0,1]\times[0,1]\times[0,\infty)^2} \quad & \phi(s_H q - t) + (1-\phi)(\bar{V}(\phi) - z) \\ \text{s.t.:} \quad & \phi(t - \gamma_H q) + (1-\phi)z - d \geq \phi(t - \gamma_L q) + (1-\phi)z && (ICC_H) \\ & \phi(t - \gamma_H q) + (1-\phi)z - d \geq 0 && (IRC_H) \\ & z \geq 0 && (LL_z) \end{aligned}$$

which is maximized by setting $q = 1$, $t = \gamma_H + d$, $z = d$, with (IRC_H) optimally binding and (ICC_H) slack, and $\phi = F(s_H - \gamma_H)$.[30]

But then, note that the solution also respects the additional constraint (ICC_ϕ) of the full program. Indeed, this writes as

$$s_H q - t = \hat{V}(\phi) - z$$

which, substituting the optimal values, is

$$s_H - \gamma_H - d = \hat{V}(\phi) - d$$

meaning $\hat{V}(\phi) = s_H - \gamma_H$ and thus $\phi = \hat{V}^{-1}(s_H - \gamma_H) = F(s_H - \gamma_H)$ as for the case of observable reservation payoffs. ■

Proof of Lemma 7. Suppose that V is privately observed by the agent. If the principal offers a mechanism which does not ask the agent to report V, conditionally on inducing $e = H$, she may obtain at most Π^H by offering the transfers $(\frac{d}{\Delta p}, 0)$ and trading with frequency one.

We show now that no direct, deterministic and incentive-compatible mechanism that conditions the principal's participation decision on V, meaning that $\rho(V) \neq \rho(V')$ for at least one pair of $(V, V') \in [V_a, V_b]^2$, can induce the agent to take the effort decision $e = H$.

To see it, for such a mechanism, denote V_0 the arbitrary report $V \in [V_a, V_b]$ such that $\rho(V_0) = 0$ and V_1 the analogue such that $\rho(V_1) = 1$. Note that by reporting V_0 and taking the effort decision

[27] In fact, trade is executed for a strictly larger interval of reservation payoffs for which it is ex ante efficient.

[28] In that case, the positive effect of trading when $V \in [\hat{V}(\underline{\phi}), \hat{V}(\phi_H^*)]$ may be offset by the negative effect of the ex ante inefficient trades carried out when $V \in (\hat{V}(\phi_H^*), \hat{V}(\bar{\phi})]$.

[29] Recall that a contract is $c = (q, t, z) \in [0, 1] \times [0, \infty)^2$ in this environment.

[30] Indeed, the slackness of (ICC_H) is guaranteed by the assumption that $F(s_H - \gamma_H) > \frac{d}{\gamma_L - \gamma_H}$.

$e \in \{L, H\}$ the agent obtains the utility $z - D(e)$, while, by reporting V_1 and taking $e \in \{L, H\}$, she gets $p_e t_g + (1 - p_e) t_b - D(e)$.

Now, for incentive-compatibility reasons, any $V_0 : \rho(V_0) = 0$ must be such that $z(V_0) = z$ is constant, for otherwise the agent would always strictly prefer to report the V_0 guaranteeing the higher liquidation fee, no matter what V is actually observed. For the same reason, any $V_1 : \rho(V_1) = 1$ must yield the same rent $R_1 = p_H t_g(V_1) + (1 - p_H) t_b(V_1)$ to the agent. Furthermore, again to ensure incentive-compatibility, the agent must be indifferent at the interim stage between reporting any V_0 or any V_1, meaning that $R_1 - D(e) = z - D(e)$, that is, $R_1 = z$. Indeed, since the agent's utility does not directly depend on V, the agent would otherwise always report V_1 if $R_1 > z$ or V_0 if $R_1 < z$, after having taken the effort decision $e = H$.

But then, one can argue that $e = H$ is never an agent's best response. In fact, by taking $e = H$, the agent obtains $z - d$ if she observes some $V_0 : \rho(V_0) = 0$ and truthfully reports, and $R_1 - d = z - d$ if she observes some $V_1 : \rho(V_1) = 1$ and truthfully reports, because of the requirement shown above. But then, the agent can always take the effort decision $e = L$ and report V_0, thus obtaining the payoff $z > z - d$, meaning that $e = H$ cannot be incentive-compatible together with truthful reporting whenever ρ effectively conditions on V. ■

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