ECN 7059 Macroéconomie avancée

Lecture 8: Optimal monetary policy

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Outline of this lecture

1. Review of the New Keynesian model
2. Optimal policy in the NK model
3. Time inconsistency of optimal policy in the NK model
4. Self-enforcement of a rule: suppose that the rule cannot be perfectly enforced. What is the best sustainable (or self-enforcing) rule ?
5. Tradeoff between commitment and discretion in the design of a rule: suppose that enforcement of the rule is perfect but the optimal state-contingent rule cannot be implemented. How to design a rule that balances the need for discretion and the need for discipline to address the time-inconsistency problem ?

1. Review of the New Keynesian model

"Three equations" model

IS dynamic:

$$y_t - y_t^n = E_t[y_{t+1} - y_{t+1}^n] - \frac{1}{\sigma}(i_t - E_t[\pi_{t+1}] - r_t^n)$$

New-keynesian Phillips curve:

$$\pi_t = \beta E_t[\pi_{t+1}] + \lambda(\sigma + \varphi)(y_t - y_t^n)$$

Monetary policy:

Example of interest rate rule:

$$i_t = \rho + \phi_\pi \pi_t + \phi_y(y_t - y_t^n) + v_t \qquad \text{(Taylor rule)}$$

in which $v_t = \rho_v v_{t-1} + \varepsilon_t^v$

IS dynamic:

$$y_t - y_t^n = E_t[y_{t+1} - y_{t+1}^n] - \frac{1}{\sigma}(i_t - E_t[\pi_{t+1}] - r_t^n)$$

- IS dynamic summarizes the choice of consumers in equilibrium
- IS dynamic is the log-linearized Euler condition in which consumption is replaced by production using the market clearing conditions
- In the neo-classical model (i.e., no nominal rigidities) output is at its natural level: $y_t = y_t^n$, $y_{t+1} = y_{t+1}^n$ and $r_t = r_t^n$.
- If the nominal interest rate i increases and the expected inflation $E_t[\pi_{t+1}]$ does not respond one for one, then the real interest rate $r_t = i_t - E_t[\pi_{t+1}]$ is higher that the neutral (natural) real interest rate r_t^n, which tends to lower the relative output gap between t and $t+1$.

New-keynesian Phillips curve:

$$\pi_t = \beta E_t[\pi_{t+1}] + \lambda (\sigma + \varphi) (y_t - y_t^n)$$

- New-keynesian Phillips curve summarizes the choice of prices by monopolistically competitive firms in equilibrium
- New-keynesian Phillips curve is the firms' FOC log-linearized
- The choice of firms is forward looking (i.e. $\beta E_t[\pi_{t+1}]$) since the firm anticipates not being able to update its price due to nominal rigidities
- The choice of firms depends on the output gap because the output gap is a proxy for whether inflation is, on average, higher (negative output gap) or lower (positive output gap) than it would be without nominal rigidities.

New-keynesian Phillips curve:

$$\pi_t = \beta E_t[\pi_{t+1}] + \lambda \left(\sigma + \varphi\right) \left(y_t - y_t^n\right)$$

New-keynesian Phillips curve is different from the Phillips curve (important)

- New-keynesian Phillips curve is a *theoretical* relationship between inflation, expected inflation and output gaps. There is not unemployment in the model but if the output gap is negative, then hours worked are below their natural level. The New-keynesian Phillips curve is a *structural* equation.
- Phillips curve is an empirical relationship between inflation and unemployment.

2. Optimal policy in the NK model

Optimal monetary policy with commitment

1. Social welfare function
   - Exact: U
   - Second order Taylor expansion: Loss function $\mathbb{L}(\pi^2, (y_t - y_n)^2)$
2. Planner's problem: identify two sources of inefficiency:
   2.1 markup due to monopoly pricing: $\mathcal{M} > 1$
   2.2 relative price distortions due to nominal rigidities: $\mathcal{M}_t \neq \mathcal{M}$ and $P_t(j) \neq P_t(j')$
3. Implementation of efficient allocation with labor market and monetary policy
   3.1 labor market subsidy τ that correct the monopoly pricing $\mathcal{M} > 0$ of firms: $(1 - \tau)\mathcal{M} = 1$
   3.2 inflation targeting $\pi_t = 0$. If the central bank commits to 0 inflation, there is no need to update prices and nominal rigidities are not constraining

2. Optimal policy in the NK model

Social welfare function

- Social welfare:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta_t U(C_t, N_t) Z_t$$

  Without capital, the dynamic economy is simply a repetition of the static economy so the planner can maximize utility period by period.

- Second order Taylor expansion: to simplify the algebra, it is useful to use a second-order Taylor approximation

2. Optimal policy in the NK model

Social welfare function

To simplify the algebra, it is useful to use a second-order Taylor approximation (see Gali Appendices 4.1 and 5.1)

- Let (C_t, N_t) be close to the steady state (C, N) so $U_t = U(C_t, N_t)$ is close to $U = U(C, N)$
-

$$\begin{aligned} U_t - U \approx & U_c C \tfrac{C_t - C}{C} + \tfrac{1}{2} U_{cc} C^2 \left(\tfrac{C_t - C}{C}\right)^2 + \\ & U_n N \tfrac{N_t - N}{N} + \tfrac{1}{2} U_{nn} N^2 \left(\tfrac{N_t - N}{N}\right)^2 + \\ & U_c C \tfrac{C_t - C}{C} \tfrac{Z_t - Z}{Z} + U_n N \tfrac{N_t - N}{N} \tfrac{Z_t - Z}{Z} \end{aligned}$$

where we used $U_{cn} = 0$.

2. Optimal policy in the NK model

Social welfare function

- Use market clearing conditions $C_t = Y_t$
  Use technology to relate N_t to y_t and a measure of price dispersion (see Gali Appendices 4.1 and 5.1)
- Second order approximation to social welfare:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{\epsilon}(y_t - y^n) - \frac{1}{2}\left(\frac{\epsilon}{\lambda}\pi_t^2 + (\sigma + \varphi)(y_t - y_n)^2 \right) \right]$$

  where
  - λ is an inverse measure of nominal rigidities
  - $\sigma \equiv -\frac{U_{cc}}{U_c} C$ and $\varphi \equiv \frac{U_{nn}}{U_n} N$ and $\lambda(\sigma + \varphi)$ gives the slope of the NKPC
  - ϵ is the elasticity of substitution which is an inverse measure of the market power of the firms

2. Optimal policy in the NK model

Social welfare function

Second order approximation to social welfare:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{\epsilon}(y_t - y^n) - \frac{1}{2} \left(\frac{\epsilon}{\lambda} \pi_t^2 + (\sigma + \varphi)(y_t - y_n)^2 \right) \right]$$

- $\frac{1}{\epsilon}(y_t - y^n)$ there is a first order effect of stimulating the economy above its steady state because the steady state is inefficient (monopoly distortion)
- $\frac{\epsilon}{\lambda}\pi_t^2$ there are inefficiencies due to fluctuations in prices due to nominal rigidities
- $(\sigma + \varphi)(y_t - y_n)^2$ there are inefficiencies due to output fluctuations because of concavity in the utility function of households

2. Optimal policy in the NK model

Planner's problem

For the planner's problem, we don't need to resort to the approximation of social welfare

$$\max U\left(C_t, N_t; Z_t\right)$$

subject to:

$$C_t(i) = A_t N_t(i)^{1-\alpha}, \; \textit{all} \;\; i \in [0,1]$$
$$N_t = \int_0^1 N_t(i)di$$
$$C_t \equiv \left(\int_0^1 C_t(i)^{1-\frac{1}{\epsilon}} di\right)^{\frac{\epsilon}{\epsilon-1}}$$

2. Optimal policy in the NK model

Planner's problem

Lagrangian:

$$\mathcal{L} \equiv U\left(C_t, N_t; Z_t\right) + \lambda_t \left[C_t - \left(\int_0^1 C_t(i)^{1-\frac{1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}} \right]$$
$$+ \mu_t(j) \left[C_t(i) - A_t N_t(i)^{1-\alpha} \right] + \xi \left[N_t - \int_0^1 N_t(i) di \right]$$

2. Optimal policy in the NK model

Planner's problem

FOC of the planner's problem

$$C_t(i) = C_t, \ \text{ for all} i \in [0,1]$$

$$N_t(i) = N_t, \ \text{ for all } i \in [0,1]$$

$$-\frac{U_{n,t}}{U_{c,t}} = A_t(1-\alpha)N_t^{-\alpha}$$

Symmetry in the model gives: $C_t(i) = C_t$ et $N_t(i) = N_t$

Equate MRS to marginal product of labor (we assume $\alpha = 0$ for simplicity)

2. Optimal policy in the NK model

Optimal policy

Suppose we design jointly the optimal labor market policy and monetary policy

If we set up policy instruments to make the equations characterizing an equilibrium with policy match the equations characterizing the planner's allocation, then we implement the first best and the policy is the optimal policy.

2. Optimal policy in the NK model

Optimal labor market policy

Labor market:

$$-\frac{U_{n,t}^{plan.}}{U_{c,t}^{plan.}} = A_t(1-\alpha)(N_t^{plan.})^{-\alpha}$$

Equilibrium without policy:

$$-\frac{U_{n,t}^{eq}}{U_{c,t}^{eq}} = \frac{W_t}{P_t} \qquad \text{(Household FOC)}$$

$$\frac{W_t}{P_t} = \frac{A_t(1-\alpha)(N_t^{eq})^{-\alpha}}{\mathcal{M}}$$

(Firm FOC without nominal rigidities)

2. Optimal policy in the NK model

Monopoly distortion

Claim: The natural level of output is inefficiently low due to the monopoly distortion: $N_t^{eq} < N_t^{plan.}$.

Proof.

$$-\frac{U_{n,t}^{eq}}{U_{c,t}^{eq}} = \frac{W_t}{P_t} = \frac{A_t(1-\alpha)(N_t^{eq})^{-\alpha}}{\mathcal{M}} < A_t(1-\alpha)(N_t^{plan.})^{-\alpha}$$

since the markup $\mathcal{M} > 1$

- Left-hand side $-\frac{U_{n,t}^{plan.}}{U_{c,t}^{plan.}}$ is increasing N_t
- Right-hand side $A_t(1-\alpha)(N_t^{plan.})^{-\alpha}$ is decreasing in N_t
- Hence $N_t^{eq} < N_t^{plan.}$

$\square$

2. Optimal policy in the NK model

Optimal labor market policy

Labor subsidy τ correct the monopoly distortion

- the CPO of the firm becomes

$$P_t = \mathcal{M} \times \frac{(1-\tau)W_t}{A_t(1-\alpha)N_t^{-\alpha}}$$

Hence

$$-\frac{U_{n,t}^{eq}}{U_{c,t}^{eq}} = \frac{W_t}{P_t} = \frac{A_t(1-\alpha)(N_t^{eq})^{-\alpha}}{(1-\tau)\ \mathcal{M}}$$

match the planner's FOC

$$-\frac{U_{n,t}^{plan.}}{U_{c,t}^{plan.}} = A_t(1-\alpha)(N_t^{plan.})^{-\alpha}$$

if we set

$$(1-\tau)\ \mathcal{M} = 1 \qquad \text{which is } \tau = \frac{1}{\epsilon}$$

2. Optimal policy in the NK model

Optimal monetary policy

We found the optimal labor market policy conditional on not having distortions due to nominal rigidities. Let's see if we can achieve this with monetary policy.

Two types of distortions due to nominal rigidities:

1. firms may not have their preferred markup $\mathcal{M}_t \neq \mathcal{M}$

$$\mathcal{M}_t = \frac{P_t}{(1-\tau)(W_t/(A_t(1-\alpha)(N_t^{eq})^{-\alpha})}$$
$$= \mathcal{M}\frac{P_t}{W_t/(A_t(1-\alpha)(N_t^{eq})^{-\alpha})}$$

and so

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} = A_t(1-\alpha)(N_t^{eq})^{-\alpha}\frac{\mathcal{M}}{\mathcal{M}_t} \neq A_t(1-\alpha)(N_t^{eq})^{-\alpha}$$

Two types of distortions due to nominal rigidities (continued):

2. relative prices between commodities may not be equal to 1:
   - if $P_t(j) \neq P_t(j')$, then $C_t^{eq}(j) \neq C_t^{eq}(j')$
   - planner: $C_t^{plan.}(j) \neq C_t^{plan.}(j')$

2. Optimal policy in the NK model

Optimal monetary policy

- The monetary policy can control the price level and inflation with a Taylor rule that satisfies the Taylor principle.
- If the monetary policy is set so as to perfectly stabilize the nominal marginal cost, then firms do not need to change their prices.
- If firms do not need to change their prices, then nominal rigidities are not constraining
- **Optimal monetary policy:** target 0 inflation

  This can be done with a Taylor rule with target $\pi = 0$:

$$i_t = r_t^n + \pi + \phi_\pi(\pi_t - \pi) + \phi_y(y_t - y_t^n)$$

  and set $\phi_\pi > 1$ to satisfy the Taylor principle and have price level determinacy

To summarize, if we jointly design labor and monetary policy, it is optimal to:

- $\tau = \frac{1}{\epsilon}$: subsidy on labor to correct the monopoly distortion
- $\pi = 0$: target 0 inflation to make the nominal rigidity non-constraining. This can be done with a Taylor rule.

Exercise:

1. Is this optimal policy time-consistent?

2. Optimal policy in the NK model

Optimal policy in the NK model

To summarize, if we jointly design labor and monetary policy, it is optimal to:

- $\tau = \frac{1}{\epsilon}$: subsidy on labor to correct the monopoly distortion
- $\pi = 0$: target 0 inflation to make the nominal rigidity non-constraining. This can be done with a Taylor rule.

Exercise:

1. Is this optimal policy time-consistent? Yes, we solved the problem period by period just like a recursive problem.
2. Could we do better by designing policy from time 0 (i.e. with commitment)?

2. Optimal policy in the NK model

Optimal policy in the NK model

To summarize, if we jointly design labor and monetary policy, it is optimal to:

- $\tau = \frac{1}{\epsilon}$: subsidy on labor to correct the monopoly distortion
- $\pi = 0$: target 0 inflation to make the nominal rigidity non-constraining. This can be done with a Taylor rule.

Exercise:

1. Is this optimal policy time-consistent? Yes, we solved the problem period by period just like a recursive problem.
2. Could we do better by designing policy from time 0 (i.e. with commitment)?
   No, the optimal time-consistent policy achieves the first best.

3. Time inconsistency of optimal policy in the NK model

Suppose that there no labor market policy (or it is set such that $\tau < \frac{1}{\epsilon}$).

The design of monetary policy aims to achieve two goals:

1. correct for the inefficiency of the labor market
2. limit the distortions due to nominal rigidities

This is exactly what the approximation to social welfare captures

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{\epsilon}(y_t - y^n) - \frac{1}{2} \left(\frac{\epsilon}{\lambda} \pi_t^2 + (\sigma + \varphi)(y_t - y_n)^2 \right) \right]$$

With $\tau = \frac{1}{\epsilon}$, the linear term disappears and the central bank can focus on the quadratic term. We saw that inflation targeting achieves both $\pi = 0$ and $y_t = y_n$ (the so-called "divine coincidence")

3. Time inconsistency of optimal policy in the NK model

Optimal policy with commitment

$$\max_{((y_t - y_n), \pi_t)_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{\epsilon}(y_t - y^n) - \frac{1}{2} \left(\frac{\epsilon}{\lambda} \pi_t^2 + (\sigma + \varphi)(y_t - y_n)^2 \right) \right]$$

s.t.

$$\pi_t = \beta E_t[\pi_{t+1}] + \lambda (\sigma + \varphi)(y_t - y_t^n) \qquad \text{(NKPC)}$$

Exercise: We saw that an equilibrium is characterized by three equations: (1. NKPC, 2. Dynamic IS, 3. Monetary policy).

1. Show that the NKPC is the only implementability constraint needed to formulate the Ramsey planner's problem.
2. How would you modify the implementability constraints to impose a Zero Lower Bound on the nominal interest rate $i_t \geq 0$?

3. Time inconsistency of optimal policy in the NK model

Optimal policy with commitment

Let $\beta^t \xi_t$ denote the Lagrange multiplier on the implementability constraint (i.e. the NKPC).

FOCs of the planner's problem:

$$\frac{1}{\epsilon} - (\sigma + \varphi)(y_t - y_t^n) = \xi_t \lambda (\sigma + \varphi) \qquad \text{(FOC w.r.t. } (y_t - y_t^n))$$

$$-\frac{\epsilon}{\lambda}\pi_t = \xi_{t-1} - \xi_t \qquad \text{(FOC w.r.t. } \pi_t)$$

and $\xi_{-1} = 0$.

The term in red reflect the intertemporal considerations that a central bank with the ability to commit to its policy can afford. It's a telltale sign of the time-inconsistency of the optimal policy with commitment.

3. Time inconsistency of optimal policy in the NK model

Optimal policy with commitment

Combining the FOCs of the planner's problem, we get:

$$y_0 - y_0^n = \frac{1}{\epsilon(\sigma + \varphi)} - \epsilon\pi_0 \qquad \text{(for } t = 0\text{)}$$

$$y_t - y_t^n = y_{t-1} - y_{t-1}^n - \epsilon\pi_t \qquad \text{(for } t \geq 1\text{)}$$

Again, the difference between the FOC at $t = 0$ and $t \geq 1$ is a telltale sign of time-inconsistency.

Solving it backwards, we get:

$$\begin{aligned} y_t - y_t^n &= \frac{1}{\epsilon(\sigma + \varphi)} - \epsilon \sum_{j=0}^{t} \pi_j \\ &= \frac{1}{\epsilon(\sigma + \varphi)} - \epsilon(p_t - p_{-1}) \end{aligned}$$

since $\pi_t = p_t - p_{t-1}$.

3. Time inconsistency of optimal policy in the NK model

Optimal policy with commitment

The NKPC can be rewritten:

$$p_t - p_{t-1} = \beta E_t[p_{t+1} - p_t] + \lambda\,(\sigma + \varphi)\,(y_t - y_t^n) \qquad \text{(for } t \geq 0\text{)}$$

Combining the NKPC (which is the implementability constraint) and the FOC of the planner, we get:

$$\frac{p_t - p_{t-1}}{\lambda\,(\sigma + \varphi)} - \beta \frac{E_t[p_{t+1} - p_t]}{\lambda\,(\sigma + \varphi)} = \frac{1}{\epsilon(\sigma + \varphi)} - \epsilon(p_t - p_{-1})$$

This is a second order difference equation. It can be solved as in QuantEcon: Linear Rational Expectations Models (link). Solving it gives a feedback and a feedforward part. Alternatively, one can use the method of undetermined coefficients as in Gali.

3. Time inconsistency of optimal policy in the NK model

Optimal policy with commitment

Qualitative features of the solution with commitment:

- The price level p_t increases over time to converge to a level that is strictly positive and bounded.
- The output gap is strictly positive and converges to the (inefficient) natural level.

3. Time inconsistency of optimal policy in the NK model

Optimal policy with commitment

Comparison:

- optimal monetary policy with commitment in the absence of a labor subsidy (i.e., inefficient natural equilibrium)
  - The price level p_t increases over time to converge to a level that is strictly positive and bounded.
  - The output gap is strictly positive and converges to the (inefficient) natural level.

- optimal monetary policy when the labor subsidy makes the natural equilibrium efficient
  - The price level p_t is constant.
  - The output gap is 0.

Comparison (continued):

- *Short run:* in the absence of a labor subsidy and an inefficient low natural level, the is some inflation and a positive output gap. This is the **inflation bias** that result from the first order gain from using monetary policy to correct for the inefficiency of the natural level.
- *Long run:* the optimal monetary policy with commitment implements an equilibrium with the same properties in the long run as the policy with an efficient natural equilibrium.

3. Time inconsistency of optimal policy in the NK model

Optimal policy without commitment

$$\max_{(y_t - y_n),\pi_t} \frac{1}{\epsilon}(y_t - y^n) - \frac{1}{2}\left(\frac{\epsilon}{\lambda}\pi_t^2 + (\sigma+\varphi)(y_t - y_n)^2\right)$$

s.t.

$$\pi_t = \beta E_t[\pi_{t+1}] + \lambda\left(\sigma+\varphi\right)\left(y_t - y_t^n\right) \qquad \text{(NKPC)}$$

3. Time inconsistency of optimal policy in the NK model

Optimal policy without commitment

$$\max_{(y_t - y_n), \pi_t} \frac{1}{\epsilon}(y_t - y^n) - \frac{1}{2}\left(\frac{\epsilon}{\lambda}\pi_t^2 + (\sigma + \varphi)(y_t - y_n)^2\right)$$

s.t.

$$\pi_t = \beta E_t[\pi_{t+1}] + \lambda\left(\sigma + \varphi\right)\left(y_t - y_t^n\right) \qquad \text{(NKPC)}$$

FOCs of the planner's problem:

$$\frac{1}{\epsilon} - (\sigma + \varphi)(y_t - y_t^n) = \xi_t \lambda(\sigma + \varphi) \qquad \text{(FOC w.r.t. } (y_t - y_t^n))$$

$$-\frac{\epsilon}{\lambda}\pi_t = -\xi_t \qquad \text{(FOC w.r.t. } \pi_t)$$

3. Time inconsistency of optimal policy in the NK model

Optimal policy without commitment

FOCs of the planner's problem:

$$\frac{1}{\epsilon} - (\sigma + \varphi)(y_t - y_t^n) = \xi_t \lambda(\sigma + \varphi) \qquad \text{(FOC w.r.t. } (y_t - y_t^n))$$

$$\frac{\epsilon}{\lambda}\pi_t = \xi_t \qquad \text{(FOC w.r.t. } \pi_t)$$

Combining the FOCs of the planner's problem, we get:

$$y_t - y_t^n = \frac{1}{\epsilon(\sigma + \varphi)} - \epsilon\pi_t$$

Using the NKPC to substitute $y_t - y_t^n$, we get

$$\frac{\pi_t - \beta E_t[\pi_{t+1}]}{\lambda\,(\sigma + \varphi)} = \frac{1}{\epsilon(\sigma + \varphi)} - \epsilon\pi_t.$$

This is a second order difference equation. It can be solved as in QuantEcon: Linear Rational Expectations Models (link).

Qualitative features of the solution without commitment:

- Inflation is strictly positive and constant over time.
- The output gap is strictly positive and constant over time.

The first-order gain from raising output above its natural level gives rise to an *inflation bias*. The central bank cannot commit to have inflation and the output gap converge to zero. The optimal policy with commitment is time-inconsistent.

3. Time inconsistency of optimal policy in the NK model

Further study

- Other sources of time-inconsistency: Gali Sections 5.2 (undistorted steady state but inefficient fluctuations of the natural output) and Section 5.3 (zero lower bound on the nominal interest rate)

4. Self-enforcement of a rule

Further readings

- Barro and Gordon (1983)
- Chari and Kehoe (1990)
- QuantEcon https://python-advanced.quantecon.org/calvo.html Ramsey Plans, Time Inconsistency, Sustainable Plans
- Abreu Pearce Stachetti (1990)

5. Design of a rule

Further readings

- Athey Atkeson Kehoe (2005)
- Halac and Yared (2020, 2021)
- Clayton and Schaab (2021)