ECN 7059 Macroéconomie avancée

Lecture 7: The New Keynesian Model

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Monetary economics

- So far, we abstracted from monetary considerations. All economic variables we studied were expressed in real terms.
- In this lecture, we study optimal monetary policy when prices are rigid in the short term.
- Readings : Galí, J. (2015) *Monetary Policy, Inflation, and the Business Cycle*, PUP, Chapitre 2, 3, 4

Outline :

1. Derive a demand for money
2. Empirical evidence of money non-neutrality
3. New-keynesian model (3 equations)

1. Derive a demand for money

- Money is an asset which costs 1 unit at t and delivers 1 unit at $t+1$
- Capital is an asset which costs 1 unit at t and delivers $1 + i_t > 1$ unit at $t+1$
- Money is dominated by other assets in terms of return if $i_t > 0$.
- The RBC model does not give any reason to hold money
- First step is to generate a demand for money

1. Derive a demand for money

Shortcut : money in the utility function

Other approaches :

- Shopping Time : purchasing power M/P lowers the time cost of buying consumption goods
- Cash in Advance : some goods can only be purchased with money

1. Derive a demand for money

$$\max_{C_t, M_t, N_t} \quad E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t, \frac{M_t}{P_t}, N_t\right)$$

s.t. :

$$P_t C_t + Q_t B_t + M_t \leq B_{t-1} + M_{t-1} + W_t N_t + D_t$$

$$\lim_{T \to \infty} E_t \left\{ \Lambda_{t,T} (\mathcal{A}_T / P_T) \right\} \geq 0$$

Remarks :

1. Two assets : government debt and money
2. Money is the unit of account so its price is normalized to 1.
3. D_t dividendes from firms
4. $Q_t = \frac{1}{1+i_t}$ is the price of government debt
5. Transversality constraint : $\lim_{T \to \infty} E_t \left\{ \Lambda_{t,T} (\mathcal{A}_T / P_T) \right\} \geq 0$
6. No capital (simplifying assumption)

1. Derive a demand for money

Opportunity cost of money

$$P_t C_t + Q_t B_t + M_t \leq B_{t-1} + M_{t-1} + W_t N_t + D_t$$

Equivalent to :

$$P_t C_t + B_t + M_t \leq B_{t-1}(1+i_t) + M_{t-1} + W_t N_t + D_t$$
$$P_t C_t + B_t + M_t \leq B_{t-1}(1+i_t) + M_{t-1}(1+i_t) - i_t M_{t-1} + W_t N_t + D_t$$

Define total asset : $A_t \equiv B_t + M_t$ which gives :

$$P_t C_t + A_t + i_t M_{t-1} \leq A_t(1+i_t) + W_t N_t + D_t$$

Remarks :

- $i_t M_{t-1}$ is the opportunity cost of money
- nominal interest rate and price of government debt

$$i_t \approx \ln(1+i_t) = -\ln\frac{1}{1+i_t} = -\ln Q_t$$

1. Derive a demand for money

$$\max_{C_t, M_t, N_t} \quad E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t, \frac{M_t}{P_t}, N_t\right)$$

subject to :

$$P_t C_t + Q_t B_t + M_t \leq B_{t-1} + M_{t-1} + W_t N_t + D_t$$

B_t is a bounded sequence

FOC are :

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} \qquad \text{(Euler intra-temporal } c, n\text{)}$$

$$\frac{U_{c,t}}{P_t} Q_t = \beta E_t \left[\frac{U_{c,t+1}}{P_{t+1}}\right] \qquad \text{(Euler inter-temporal)}$$

$$\frac{U_{m,t}}{U_{c,t}} = 1 - Q_t \qquad \text{(Euler intra-temporal } c, m\text{)}$$

1. Derive a demand for money

$$\frac{U_{m,t}}{U_{c,t}} = 1 - Q_t \qquad \text{(Euler intra-temporal } c, m\text{)}$$

Suppose additively separable utility function with CRRA utility index :

$$U\left(C_t, \frac{M_t}{P_t}, N_t\right) = \frac{C_t^{1-\sigma} - 1}{1-\sigma} + \frac{(M_t/P_t)^{1-\nu} - 1}{1-\nu} - \frac{N_t^{1+\varphi}}{1+\varphi}$$

Exercise : Substitute to get the following Euler intra-temporal c, m.

$$\frac{\left(\frac{M_t}{P_t}\right)^{-\nu}}{C_t^{-\sigma}} = 1 - Q_t$$

1. Derive a demand for money

With separable and isoélastic utility, we get Euler intra-temporal c, m :

$$\frac{M_t}{P_t} = C_t^{\frac{\sigma}{\nu}} \ (1 - Q_t)^{-\frac{1}{\nu}}$$

which gives us the demand for money, in log :

$$\ln M_t - \ln P_t = \frac{\sigma}{\nu} \ln C_t - \frac{1}{\nu} \ln(1 - Q_t)$$

If $\sigma = \nu$ and anticipating the equilibrium in the goods market : $C_t = Y_t$, we get :

$$m_t - p_t = y_t - \eta \ i_t \qquad \text{(Money demand equation)}$$

in which $\eta = \frac{1}{\nu}$.

1. Derive a demand for money

Elasticity of the demand for money

$$\ln\left(\frac{M_t}{P_t}\right) \equiv m_t - p_t = \frac{\sigma}{\nu} y_t - \frac{1}{\nu}\ i_t \qquad \text{(Demand for money)}$$

- Income-elasticity of the demand for money

$$\frac{\partial \ln\left(\frac{M}{P}\right)}{\partial \ln Y} = \frac{\sigma}{\nu}$$

  Estimates of this elasticity hover around 1 which motivates the hypothesis $\sigma = \nu$ (cf. Goldfel et Sichel (1990)).

- Interest-rate-semi-elasticity of the demand for money

$$\frac{\partial \ln\left(\frac{M}{P}\right)}{\partial i} = -\frac{1}{\nu}$$

  Estimates of this semi-elasticity hover around -0.2 (cf. Goldfel et Sichel (1990)).

2. Empirical evidence on nominal rigidities

Average frequency of price changes

- Taylor (1999) : around once per year
- Bils Klenow (2004) : 350 categories of products in the CPI in the USA. Frequency 4 to 6 months
- Nakamura and Steinsson (2008) : excluding price changes due to sales. Frequency 8 to 11 months
- Dhyne et al. (2006) : similar frequency as found by Nakamura and Steinsson but for the Euro zone.

2. Empirical evidence on nominal rigidities

- Heterogeneity between sectors and types of goods (services are more rigid, food/energy less rigid)

- Similar evidence of nominal rigidities for wages except that they are asymmetric : decreases in salary are very rare.

2. Empirical evidence on nominal rigidities

Identification of exogenous monetary policy shocks

Challenge in estimating the effect of monetary policy on inflation :

- if the central bank anticipates inflation, it raises the interest rate
- if the central bank raises the interest rate, it affects inflation.

The causality goes both ways since the central bank chooses its monetary policy actively. The obejctive is to *exogenous* monetary policy shocks. It should not be linked to the business cycle. For instance, a change of governor of the Bank of Canada)

2. Empirical evidence of non-neutrality of money

Time series analysis

- Step 1 : estimate an interest rate rule for the central bank's monetary policy

  interpret residuals as monetary policy shocks

- Step 2 : regress GDP, inflation, mass of money on monetary policy shocks from the first step.

2. Empirical evidence of non-neutrality of money

Impulse response

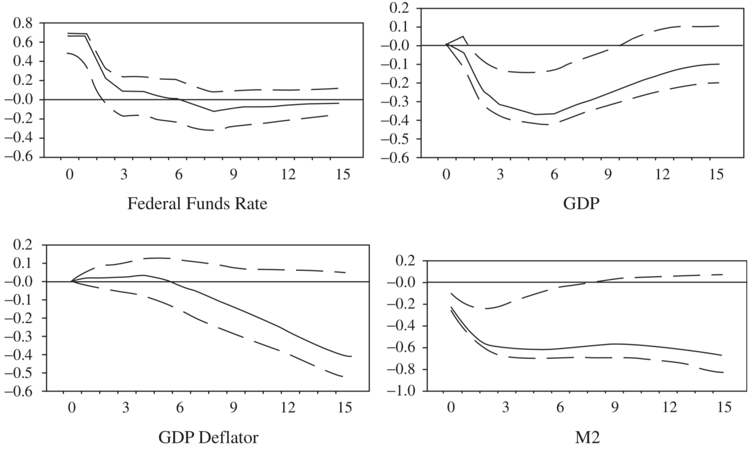

Figure 1.1 Estimated Dynamic Response to a Monetary Policy Shock

Source: Christiano, Eichenbaum, and Evans (1999).

2. Empirical evidence of non-neutrality of money

Higher (exogenous shock) nominal interest rate

- Stable inflation for 5 trimester then declines
- GDP drops in the short run and then recovers (evidence of money non-neutrality)
- M2 drops en then stays at a lower level.

  Liquidity effect : negative correlation between mass of money and nominal interest rate

New-keynesian model

Enrich the neoclassical model in two ways :

- *Monopolistic competition :* firm set prices
- *Nominal rigidities :* firm faces an adjustment cost to change their price (e.g. menu cost) or firms are constrained to adjust their price (e.g. Calvo).

New-keynesian model

Monetary policy is not neutral in the short run in the New-keynesian model. The following equation defines the real interest rate r :

$$i = E[\pi] + r \qquad \text{(Fisher equation)}$$

If the central bank changes i and there are

- no nominal rigidities, then π adjusts and r is constant. The equilibrium of the neoclassical model pins down r independently of monetary policy. This is relevant in the long-run when prices have adjusted.
- nominal rigidities, then prices can't adjust so π is roughly constant. The real interest r must change which affects the real economy (this can be seen directly from the Euler equation)

Effet de Fisher et Effet de liquidité

- **Liquidity effect** : negative correlation between the money mass and the nominal interest rate
- **Fisher effect** : positive correlation between the money mass and the nominal interest rate ($r = i - E[\pi]$)
- Liquidity effect is the opposite of the Fisher effect. We can reconcile the two effects by considering them to be relevant for different time horizons :
  - Short run : while firms haven't adjusted their price, the liquidity effect dominates
  - Long run : once firms have adjusted their price, the model dictates that the Fisher effect dominates
- Remark : this distinction is central to the classical lecture Friedman (1968) ≪ The Role of Monetary Policy ≫*AER*

Monetary policy : New-keynesian model

Description of the environment : consumers

$$\max_{C_t, N_t, B_t} E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t, N_t\right)$$

subject to :

$$P_t C_t + Q_t B_t \leq B_{t-1} + W_t N_t + D_t + T_t$$

for $t = 0, 1, 2, ...$ and the constraint :

$$\lim_{T \to \infty} E_t \left\{ \frac{B_T}{P_T} \right\} \geq 0$$

Monetary policy : New-keynesian model

Description of the environment : consumers

Basket of goods C_t :

$$C_t \equiv \left(\int_0^1 C_t(j)^{1-\frac{1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}$$

The consumer chooses the consumptions bundle $((C_t(j))_{j\in[0,1]})$ for a given level of expenditure $P_t C_t$:

$$\max_{(C_t(j))_{j\in[0,1]}} \left(\int_0^1 C_t(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}$$

subject to :

$$\int_0^1 P_t(j) C_t(j) dj = P_t C_t$$

The solution of the consumer problem gives a demand function for each good :

$$C_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} C_t$$

in which

$$P_t \equiv \left(\int_0^1 P_t(j)^{1-\epsilon} dj\right)^{\frac{1}{1-\epsilon}}$$

Monetary policy : New-keynesian model

Description of the environment : consumers

"Cashless limit" :

- utility does not depend on real money balances
- we add a money demand equation *as if* there is MiU :

$$m_t - p_t = c_t - \eta i_t$$

- Interpretation : negligible transaction services from money
- Demand for money is *ad hoc* (i.e. it does not result from the preferences of consumers). The "cashless limit" allows the New-Keynesian model to focus on the effect of nominal rigidities on the optimal conduct of monetary policy. If we kept "MiU" money in the utility, there would be an economic motive to lower the nominal interest rate as prescribed by the Friedman rule (which prescribes $i_t = 0$).

Monetary policy : New-keynesian model

Description of the environment : consumers

Log-linearised FOC of the consumer with CRRA utility :

$$w_t - p_t = \sigma c_t + \varphi n_t \qquad \text{(Euler intra-temporal } c, n\text{)}$$

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - \rho) \qquad \text{(Euler inter-temporal)}$$

$$m_t - p_t = c_t - \eta i_t \qquad \text{(Money demand)}$$

and the demand *schedule* for each commodity in the basket is :

$$C_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} C_t$$

in which

$$P_t \equiv \left(\int_0^1 P_t(j)^{1-\epsilon} dj\right)^{\frac{1}{1-\epsilon}}$$

Technology :

$$Y_t(j) = A_t N_t(j)^{1-\alpha}$$

in which

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a$$

Each firm has a monopoly on the production of a good from the basket of commodities.

Monetary policy : New-keynesian model

Description of the environment : firms

1. Monopoly : each firm $j \in [0, 1]$ produces a differentiated commodity and chooses its price $P_t(j)$ given the demand schedule

$$C_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\epsilon} C_t$$

2. *Nominal rigidities à la Calvo*
   - with probability θ, a firm cannot change its price (Calvo (1983)).
   - $\theta \in [0, 1]$: index of price rigidities
   - $1 - \theta$ probability that a firm cannot change its price
   - On average, when a firm changes its price, it has to keep the same price, irrespectively of the business cycle during $\frac{1}{1-\theta}$ periods

- Consumer Price Index is P_t defined as follows : $P_t \equiv \left(\int_0^1 P_t(j)^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}}$
- Denote by P_t^* the price that a firm which has the opportunity to change its price at t chooses. All firms are identical (same technology and same demand schedule), so by symmetry : $P_t^* = P_t^*(j)$ for all $j \in [0,1]$
- $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ and $\pi_t \equiv \ln(\Pi_t) = \ln(P_t) - \ln(P_{t-1}) = p_t - p_{t-1}$
- The index $t+k|t$ denotes a variable at time $t+k$ for a firm that hasn't had the opportunity to change its price since t

Monetary policy : New-keynesian model

Description of the environment : firms

Notation :

- A monopoly makes profits by choosing a price above its marginal cost.
- Let $\mathcal{M}_t$ denote the firm's markup ; that is : $P_t = \mathcal{M}_t \times$ marginal cost.
- In a model without nominal rigidities, let the markup be denoted by $\mathcal{M}$.

To solve the problem of a firm, we proceed in three steps :

1. determine the evolution of the consumer price index P_t as a function of the degree of nominal rigidities θ
2. find the optimal price P_t^* that any firm $j \in [0,1]$ would choose if it had the opportunity to change its price : this is a dynamic choice for which anticipation matter a lot
3. rewrite the problem of the firm in a recursive way to express P_t^* as a function of tomorrow's price P_{t+1}^* and the difference in markup $\mathcal{M}_t - \mathcal{M}$

Monetary policy : New-keynesian model

1. determine the evolution of the consumer price index P_t as a function of the degree of nominal rigidities θ

Consumer Price Index :

$$P_t \equiv \left(\int_0^1 P_t(j)^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}}$$

A share θ of consumers can't adjust their prices :

$$P_t = \left[\theta (P_{t-1})^{1-\epsilon} + (1-\theta)(P_t^*)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}}$$

where P_t^*) is the price that firms which can adjust their price at t would choose at t

After dividing on both sides by P_{t-1} :

$$\Pi_t^{1-\epsilon} = \theta + (1-\theta) \left(\frac{P_t^*}{P_{t-1}} \right)^{1-\epsilon}$$

Monetary policy : New-keynesian model

1. determine the evolution of the consumer price index P_t as a function of the degree of nominal rigidities θ

$$\Pi_t^{1-\epsilon} = \theta + (1-\theta)\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\epsilon}$$

After log-linearizing around the steady state without inflation, we get :

$$\pi_t = (1-\theta)(p_t^* - p_{t-1})$$

which can be rewritten :

$$p_t = \theta p_{t-1} + (1-\theta)p_t^*$$

Given this evolution of the aggregate price (CPI), we can now study the choice of a price by each monopolist.

Monetary policy : New-keynesian model

2. find P_t^* any firm $j \in [0,1]$ would like to set

If a monopolist j can update its price at t, then :

- it maximizes its expected profit anticipating that the price chosen at t
  - will be the same at $t+1$ with probability θ
  - and again the same at $t+2$ with probability θ^2
  - ...
- the stochastic discount factor is : $\Lambda_{t,t+k} \equiv \beta^k \frac{U_{c,t+k}}{U_{c,t}}$
- demand at $t+k$ is a function of the price set at t if the firm has not had a chance to revise its price between t and $t+k$:

$$C_{t+k|t}(j) = \left(\frac{P_t^*(j)}{P_{t+k}} \right)^{-\epsilon} C_{t+k}$$

Monetary policy : New-keynesian model

2. find P_t^* any firm $j \in [0,1]$ would like to set

The marginal cost of production is the marginal labor requirement $Y = AN^{1-\alpha}$ multiplied by the wage :

$$\mathcal{C}(Y) = W\ N(Y) = W\left(\frac{Y}{A}\right)^{\frac{1}{1-\alpha}}$$

Each monopolist maximizes the net present value of its profits :

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta^k E_t \left\{ \Lambda_{t,t+k} \frac{\left(P_t^* Y_{t+k|t} - \mathcal{C}_{t+k}(Y_{t+k|t})\right)}{P_{t+k}} \right\}$$

subject to :

$$Y_{t+k|t} = C_{t+k|t}(j) = \left(\frac{P_t^*}{P_{t+k}}\right)^{-\epsilon} C_{t+k}$$

FOC of the firm's problem without nominal rigidities (i.e. $\theta = 0$) :

$$P_t^* = \frac{\epsilon}{\epsilon - 1}\mathcal{C}_t'(Y_{t|t})$$

Define by $\mathcal{M} \equiv \frac{\epsilon}{\epsilon-1}$ as the firm's *ideal markup when there are no nominal rigidities*

Monetary policy : New-keynesian model

2. find P_t^* any firm $j \in [0, 1]$ would like to set

FOC of the firm's problem with nominal rigidities (i.e. $\theta > 0$) :

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ \Lambda_{t,t+k} Y_{t+k|t} \frac{\left(P_t^* - \mathcal{MC}'_{t+k}(Y_{t+k|t})\right)}{P_{t+k}} \right\} = 0$$

The firm chooses a price that is a weighted average of the desired price over the period during which it expects not being able to adjust its price

Monetary policy : New-keynesian model

2. find P_t^* any firm $j \in [0, 1]$ would like to set

First order Taylor approximation of the firm's CPO around the steady state without inflation gives :

$$p_t^* = \ln(\mathcal{M}) + (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t\{\ln(\mathcal{C}'_{t+k}(Y_{t+k|t}))\}$$

in which $\ln(\mathcal{M})$is the log of the ideal markup (i.e. without nominal rigidities) and $\ln(\mathcal{C}'_{t+k}(Y_{t+k|t}))$ is the log of the marginal cost

This equation characterizes the firm's optimal choice

Monetary policy : New-keynesian model

Marginal cost

We just saw that the firm exercise its monopoly power by fixing a price above its marginal cost. Now we determine the marginal cost

$$\mathcal{C}'_{t+k}(Y_{t+k|t}) = W_{t+k}\frac{\partial N_{t+k|t}(Y_{t+k})}{\partial Y_{t+k}}$$

in which $N_{t+k|t}(Y_{t+k|t}) = \left(\frac{Y_{t+k|t}}{A_{t+k}}\right)^{\frac{1}{1-\alpha}}$.

We get :

$$\mathcal{C}'_{t+k|t}(Y_{t+k|t}) = W_{t+k}\frac{1}{1-\alpha}\frac{N^{\alpha}_{t+k|t}}{A_{t+k}}$$

In log :

$$\ln(\mathcal{C}'_{t+k|t}(Y_{t+k|t})) = w_{t+k} - (a_{t+k} - \alpha n_{t+k|t} + \ln(1-\alpha))$$

Suppose that returns to scale are constant $\alpha = 0$, then the marginal cost is independent of the scale of production $n_{t+k|t}$.

$$\ln(\mathcal{C}'_{t+k|t}(Y_{t+k|t})) = w_{t+k} - a_{t+k}$$

Monetary policy : New-keynesian model

3. recursive formulation of the firm's problem

$$\begin{aligned}
p_t^* &= \ln(\mathcal{M}) + (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\{\ln(\mathcal{C}'_{t+k}(Y_{t+k|t}))\} \\
&= \ln(\mathcal{M}) + (1-\beta\theta)E_t\{\ln(\mathcal{C}'_t(Y_t))\} \\
&\qquad + \beta\theta(1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_{t+1}\{\ln(\mathcal{C}'_{t+k}(Y_{t+k|t+1}))\} \\
&= \ln(\mathcal{M}) + (1-\beta\theta)E_t\{\ln(\mathcal{C}'_t(Y_t))\} + \beta\theta E_t[p_{t+1}^* - \ln(\mathcal{M})]
\end{aligned}$$

So :

$$p_t^* = \beta\theta E_t[p_{t+1}^*] + (1-\beta\theta)p_t - (1-\beta\theta)(\ln(\mathcal{M}_t) - \ln(\mathcal{M}))$$

in which $\mathcal{M}_t$ is the effective markup : $P_t = \mathcal{M}_t\ \mathcal{C}'_t(Y_t)$

A firm chooses a price :

$$p_t^* = \beta\theta E_t[p_{t+1}^*] + (1-\beta\theta)p_t - (1-\beta\theta)(\ln(\mathcal{M}_t) - \ln(\mathcal{M}))$$

Dynamic evolution of the log of the Consumer Price Index :

$$p_t - p_{t-1} \equiv \pi_t = (1-\theta)(p_t^* - p_{t-1})$$

Combining these two equations (individual price setting and aggregate price dynamics) we can rewrite inflation as a function of anticipated inflation and the difference between the actual markup and the desired markup (actual and desired markups differ only because of nominal rigidities)

$$\begin{aligned} p_t^* - p_{t-1} &= \beta\theta E_t\{p_{t+1}^* - p_t\} + p_t - p_{t-1} - (1-\beta\theta)(\ln(\mathcal{M}_t) - \ln(\mathcal{M})) \\ p_t^* - p_{t-1} &= \beta\theta E_t\{p_{t+1}^* - p_t\} + (1-\theta)(p_t^* - p_{t-1}) \\ &\qquad - (1-\beta\theta)(\ln(\mathcal{M}_t) - \ln(\mathcal{M})) \end{aligned}$$

(continued)

$$p_t^* - p_{t-1} = \beta\theta E_t\{p_{t+1}^* - p_t\} + (1-\theta)(p_t^* - p_{t-1}) \\ - (1-\beta\theta)(\ln(\mathcal{M}_t) - \ln(\mathcal{M}))$$

We can rewrite this equation as follows :

$$\pi_t = \beta E_t\{\pi_{t+1}\} - \lambda(\ln(\mathcal{M}_t) - \ln(\mathcal{M}))$$

in which

$$\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}$$

Monetary policy : New-keynesian model

Competitive equilibrium with interest rate rule

Given an interest rate rule $(i_t)_{t=0}^{\infty}$ and transfers $(T_t)_{t=0}^{\infty}$, a competitive equilibrium is an allocation $((C_t(j), Y_t(j), N_t(j))_{j\in[0,1]}, N_t)_{t=0}^{\infty}$, a portfolio $(B_t)_{t=-1}^{\infty}$, and dividends $(D_t)_{t=0}^{\infty}$, and prices $((P_t(j))_{j\in[0,1]}, W_t, Q_t)$ such that $Q_t = \exp(-i_t)$:

- $(C_t(j)_{j\in[0,1]}, B_t, N_t)_{t=0}^{\infty}$ solves the consumer problem given dividends and prices
- $(Y_t(j), N_t(j))_{t=0}^{\infty}$ et $(P_t(j))_{t=0}^{\infty}$ solves firm j's problem given the demand for its product j and nominal rigidities. Dividends are $D_t = \int_0^1 P_t(j)Y_t(j) - W_t N_t(j)$
- Markets clear
  - goods market $C_t(j) = Y_t(j)$
  - market for debt $B_t = 0$
  - labor market $\int_0^1 N_t(j)dj = N_t$
- The government budget constraint is satisfied : $\frac{T_t}{P_t} = \frac{M_t - M_{t-1}}{P_t}$ (i.e. seignorage revenues (in real terms) are transferred back to households (through the treasury which is not modelled)

Monetary policy : New-keynesian model

Competitive equilibrium with interest rate rule

$$\begin{aligned}
w_t - p_t &= \sigma c_t + \varphi n_t && \text{(Euler intra-temporal } c, n\text{)} \\
c_t &= E_t\{c_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - \rho) && \\
& && \text{(Euler inter-temporal)} \\
m_t - p_t &= c_t - \eta i_t && \text{(Money demand)} \\
\pi_t &= \beta E_t\{\pi_{t+1}\} - \lambda(\ln(\mathcal{M}_t) - \ln(\mathcal{M})) && \text{(Firm - price)} \\
\ln(\mathcal{M}_t) &= -(w_t - p_t) + a_t && \text{(Firm - labor)} \\
c_t &= y_t && \text{(Equilibrium on the market for goods)} \\
y_t &= a_t + n_t && \text{(Firm technology)}
\end{aligned}$$

The only two equations that change compared to a neo-classical model are : (Firm-price) and (Firm-labor).

Monetary policy : New-keynesian model

Unlike the neo-classical model, we cannot solve for the equilibrium on the labor market independently of the monetary policy (this suggests that the monetary policy is not neutral)

We can reduce this system of seven equations to a system of two equations :

1. New-keynesian Phillips curve (summarize the optimal choice of the firm)
2. IS dynamic (summarize the optimal choice of consumers)

Monetary policy : New-keynesian model

Optimality conditions of firms' choice links inflation to the markup as follows :

$$\pi_t = \beta E_t\{\pi_{t+1}\} - \lambda(\ln(\mathcal{M}_t) - \ln(\mathcal{M}))$$

If the markup of firms is higher than what it would be without nominal rigidities, then prices are « too high » and firms will have a tendance to raise their price in period t given inflation expectations for $t+1$

New-keynesian Phillips curve

- replace the gap in markups $(\ln(\mathcal{M}_t) - \ln(\mathcal{M}))$ by the output gap
- define the *output gap* = production in the model with nominal rigidities - production in the model without nominal rigidities
- New-keynesian Phillips curve links inflation to expectations of inflation and the output gap

Use these equilibrium conditions to link the markup to output :

$$
\begin{aligned}
w_t - p_t &= \sigma c_t + \varphi n_t && \text{(Euler intra-temporal } c, n\text{)} \\
\ln(\mathcal{M}_t) &= -(w_t - p_t) + a_t + \ln(1-\alpha) && \text{(Firm - labor)} \\
c_t &= y_t && \text{(goods market clearing)} \\
y_t &= a_t + n_t && \text{(technology)}
\end{aligned}
$$

We get :

$$\ln(\mathcal{M}_t) = -\left(\sigma + \varphi\right) y_t + \left(1 + \varphi\right) a_t$$

Define the *natural level of output* y_t^n as production in the model without nominal rigidities, that is :

$$\ln(\mathcal{M}) = -\left(\sigma + \varphi\right) y_t^n + \left(1 + \varphi\right) a_t$$

We get the difference in markup as a function of the output gap

$$\ln(\mathcal{M}_t) - \ln(\mathcal{M}) = -\left(\sigma + \varphi\right)\left(y_t - y_t^n\right)$$

If markups (and hence prices) are above what they would be without nominal rigidities, then output is below its natural level.

Once substituted in the condition (firm-price), we get the New-keynesian Phillips curve which links output gap, inflation, and expected inflation :

New-keynesian Phillips curve :

$$\pi_t = \beta E_t[\pi_{t+1}] + \lambda\left(\sigma + \varphi\right)\left(y_t - y_t^n\right)$$

Monetary policy : New-keynesian model

New-keynesian Phillips curve

New-keynesian Phillips curve :

$$\pi_t = \beta E_t[\pi_{t+1}] + \lambda (\sigma + \varphi) (y_t - y_t^n)$$

- New-keynesian Phillips curve summarizes the choice of firms in equilibrium
- New-keynesian Phillips curve is the firms' FOC log-linearized.
- The choice of firms is forward looking (i.e. $\beta E_t[\pi_{t+1}]$) since the firm anticipates not being able to update its price (i.e. nominal rigidities)
- The choice of firms depends on the output gap because the output gap is a proxy for whether prices are too high (negative output gap) or too low (positive output gap).

New-keynesian Phillips curve :

$$\pi_t = \beta E_t[\pi_{t+1}] + \lambda (\sigma + \varphi) (y_t - y_t^n)$$

New-keynesian Phillips curve is different from the Phillips curve (important)

- New-keynesian Phillips curve is a *theoretical* relationshop between inflation, expected inflation and output gaps. There is not unemployment in the model but if the output gap is negative, then hours worked are below their natural level. The New-keynesian Phillips curve is a *structural* equation.
- Phillips curve is an empirical relationship between infaltion and unemployment.

Euler inter-temporal determines the choice of Investment-Savings of the consumer

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - \rho) \quad \text{(Euler inter-temporal)}$$

$$c_t = y_t \quad \text{(goods market clearing)}$$

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - \rho) \quad \text{(Euler inter-temporal)}$$

$$y_t^n = E_t\{y_{t+1}^n\} - \frac{1}{\sigma}(r_t^n - \rho) \quad \text{(Euler inter-temporal)}$$

IS dynamic :

$$y_t - y_t^n = E_t[y_{t+1} - y_{t+1}^n] - \frac{1}{\sigma}(i_t - E_t[\pi_{t+1}] - r_t^n)$$

IS dynamic :

$$y_t - y_t^n = E_t[y_{t+1} - y_{t+1}^n] - \frac{1}{\sigma}(i_t - E_t[\pi_{t+1}] - r_t^n)$$

- IS dynamic summarizes the choice of consumers in equilibrium
- IS dynamic is the log-linearized Euler inter-temporal in which consumption is replaced by production using the market clearing conditions
- In the neo-classical model there are no nominal rigidities so output is at its natural level : $y_t = y_t^n$, $y_{t+1} = y_{t+1}^n$ and $r_t = r_t^n$.
- If the nominal interest rate i increases and the expected inflation $E_t[\pi_{t+1}]$ does not respond one for one, then the real interest rate $r_t = i_t - E_t[\pi_{t+1}]$ is higher that the neutral (natural) real interest rate r_t^n, which tends to lower the relative output gap between t and $t+1$.

IS dynamic :

$$y_t - y_t^n = E_t[y_{t+1} - y_{t+1}^n] - \frac{1}{\sigma}(i_t - E_t[\pi_{t+1}] - r_t^n)$$

New-keynesian Phillips curve :

$$\pi_t = \beta E_t[\pi_{t+1}] + \lambda\left(\sigma + \varphi\right)\left(y_t - y_t^n\right)$$

Monetary policy :
Example of interest rate rule : the Taylor rule :

$$i_t = \rho + \phi_\pi \pi_t + \phi_y(y_t - y_t^n) + v_t$$

in which $v_t = \rho_v v_{t-1} + \varepsilon_t^v$

New-keynesian model

Solution

1. Simulate a time series of technological shocks a_t and monetary policy shocks v_t according to their stochastic processes.
2. Compute the equilibrium of the model without nominal rigidities (y_t^n, r_t^n). This can be done in closed form.
3. Solve the system of equations for (i_t, π_t, y_t)

$$\pi_t = \beta E_t[\pi_{t+1}] + \lambda(\sigma + \varphi)(y_t - y_t^n) \quad \text{(C. Phillips N-K)}$$

$$y_t - y_t^n = E_t[y_{t+1} - y_{t+1}^n] - \frac{1}{\sigma}(i_t - E_t[\pi_{t+1}] - r_t^n) \quad \text{(IS dyn.)}$$

$$i_t = \rho + \phi_\pi \pi_t + \phi_y(y_t - y_t^n) + v_t \qquad \text{(Monetary pol.)}$$