ECN 7059 Macroéconomie avancée

Lecture 5: Optimal design of rules

Guillaume Sublet

Université de Montréal et CIREQ

Recap

- So far we have looked at policy *with commitment*: the complete contingent policy plan is chosen at the beginning of time.

- Policy *without commitment*: the policy is chosen recursively. In other words, every period t the policy maker chooses period t policy anticipating that it will choose period $t+1$ policy optimally at $t+1$ in the future.

- We saw that the Ramsey plan is *time-inconsistent*: the implementation of the Ramsey plan requires commitment since every period, the government wants to revise the Ramsey plan. In other words, the Ramsey plan is optimal ex-ante but not optimal ex-post. The reason is that the supply of capital is elastic ex-ante but inelastic ex-post.

Recap

- The principle of optimality tells us that for a single decision maker with preferences $\sum_{t=0}^{\infty} \beta^t u(x_t)$, the optimal policy is time-consistent: the solution to the sequential problem chosen at $t = 0$ coincide with the solution of the recursive formulation.
- The time-inconsistency of the Ramsey plan comes for the fact that the government is not a single decision maker: the government is optimizing subject to the (implementability) constraint that citizens are also optimizing. It is like a "game". (Remark: it is possible to solve for the Ramsey plan recursively, but the commitment must be included as a constraint. cf. LS Chapter 19 " Dynamic Stackelberg Problems")

Lecture 5: Optimal design of rules

- So far we have studied optimal policy by a government that is optimizing social welfare.

- In this lecture, we introduce a *political economy* consideration to our analysis: the government is present-biased in the sense that it discounts the future at a higher rate than the population. (government discount factor $\delta\beta < \beta$ discount factor of the population)

- Since the objective of the government is not quite aligned with the objective of the population, the population wants to *impose a rule on the government*.

- In this lecture we study how to design the rule imposed on the government.

Lecture 5: Optimal design of rules

- So far we have studied optimal policy by a government that is optimizing social welfare.

- In this lecture, we introduce a *political economy* consideration to our analysis: the government is present-biased in the sense that it discounts the future at a higher rate than the population. (government discount factor $\beta\delta < \beta$ discount factor of the population)

- Since the objective of the government is not quite aligned with the objective of the population, the population wants to *impose a rule on the government*.

- In this lecture we study how to optimally design the rule to impose on the government.

Lecture 5: Optimal design of rules

Background

1. Kydland and Prescott (1977) "Rules Rather than Discretion: The Inconsistency of Optimal Plans" *JPE*, showed that the government policy may be time-inconsistent: the policy under discretion may not maximize social welfare. In other words commitment to a *state-contingent rule is better than discretion*.

2. Barro Gordon (1983) "Rules, Discretion and Reputation in a Model of Monetary Policy" *JME* cast the analysis of Kydland and Prescott in the context of a repeated game and show that repeated interactions provides opportunities for punishments which, for patient enough governments, can sustain the optimal rule as an equilibrium despite the lack of commitment. See also Chari Kehoe "Sustainable Plans" *JPE*.

3. In this lecture, we design the optimal *rule when it cannot be made contingent on the state* because the state is private information to the government.

Lecture 5: Optimal design of rules

Readings

Background:

- Chari Kehoe “Sustainable Plans” *JPE*
- QuantEcon: “Stackelberg Plans” (link) and “Ramsey Plans, Time Inconsistency, Sustainable Plans” (link)

Readings for this lecture on “*non*-state-contingent” rule

- Amador Werning Angeletos (2005) *Ecta*
- LS Section 7.6 “Markov Perfect Equilibrium”
- Amador and Bagwell (2013) *Ecta*

Lecture 5: Optimal design of rules

This lecture focuses on the design of fiscal rules.

Other applications include:

- design of monetary policy rule
  - Halac Yared (2020) Inflation targeting under political pressure
  - Halac Yared (2020) Instrument-based vs. Target-based rules
  - Athey Atkeson Kehoe (2005) *Ecta*
- design of retirement savings account
  - Moser Olea de Souza e Silva (2019) "Optimal Paternalistic Savings Policy"
  - Beshears Choi Clayton Harris Laibson Madrian (2020) "Optimal Illiquidity"

Lecture 5: Optimal design of rules

Plan

1. Time-inconsistent preferences due to present-bias (also known as quasi-hyperbolic discounting)
   1.1 Setup of the government problem
   1.2 Discretion: solution (Markov Perfect Equilibrium) without commitment
   1.3 Optimal state-contingent rule: solution with commitment

2. Design of optimal rule (when the state is private information)
   2.1 Setup of the problem of designing a rule
   2.2 Revelation principle to rewrite it as a mechanism design *without transfers*
   2.3 Solve it using Lagrangian techniques

Model

1.1 Setup of the government problem

- Finite horizon $t = 0, \dots, N$
- One small open economy
- Shocks to the economy are θ iid every period according to F on $[\underline{\theta}, \bar{\theta}]$
- θ_t is the state of the economy and it is private information to the government

Model

1.1 Setup of the government problem

- Present-biased government ($\delta < 1$)
  - Preferences at $t = N$ are $u(g_T)$
  - Preferences at $t < N$ are

$$\theta_t U(g_t) + \delta \sum_{j=1}^{N-t} \beta^j E[\theta_{t+j} \ u(g_{t+j})]$$

- Preferences of the population:

$$\sum_{t=0}^{N} \beta^t E[\theta_t \ u(g_t)]$$

- Budget constraint at t

$$g_t + \frac{x_t}{R} = T + x_{t-1}$$

- Chooses g_t, x_t at t taking the world interest rate R as given

1.2 Discretion: solution without commitment

Each government anticipates that future governments are optimizing according to their preferences. To take that into account, we proceed by backward induction.

Period N is the last period. The value functions are:

$$
\begin{aligned}
v_N(x,\theta) &= \theta u(T+x) \\
W_N(x,\theta) &= \theta u(T+x)
\end{aligned}
$$

We keep track of *two* value functions:

- v_t is the value function of the government optimizing at t (i.e. the government in power). It discounts with factor $\delta\beta$ between t and $t+1$ and with factor β between any two periods after $t+1$
- w_t is the value of the population which discounts with factor β. It is also the continuation value of government $t-1$.

1.2 Discretion: solution without commitment

The index j counts steps back from the end of time N

$$v_{N-j}(x,\theta) := \max_{x'} \theta u\left(T + x - \frac{x'}{R}\right) + \delta\beta E[W_{N-j+1}(x',\theta')]$$

$$x'_{N-j}(x,\theta) := \arg\max_{x'} \theta u\left(T + x - \frac{x'}{R}\right) + \delta\beta E[W_{N-j+1}(x',\theta')]$$

$$W_{N-j}(x,\theta) := \theta u\left(T + x - \frac{x'_{N-j}(x,\theta)}{R}\right) + \beta E[W_{N-j+1}(x'_{N-j}(x,\theta),\theta')]$$

where $x'_{N-j}(x,\theta)$ denotes the policy function from the right hand side the equation defining v_{N-j}.

1.2 Discretion: solution without commitment

(For reference, LS Section 7.6 "Markov Perfect Equilibrium")

Definition (Recursive Markov Perfect Partial Equilibrium with time inconsistent preferences)

A Markov Perfect Equilibrium is

- a sequence of $N+1$ value functions $(v_t)_{t=0}^N$ for self t at t
- a sequence of $N+1$ consumption functions $(x_t)_{t=0}^N$
- a sequence of $N+1$ continuation value functions $(W_t)_{t=0}^N$ for all other selves at t

such that ...

1.2 Discretion: solution without commitment

Definition of Recursive MPE (continued)

such that ...

- v_{N-j}, W_{N-j} are defined recursively by backward induction, starting with $v_N(x,\theta) = \theta u(T + x)$ and $W_N(x,\theta) = \theta u(T + x)$ as follows:

$$v_{N-j}(x,\theta) := \max_{x'} \theta u\left(T + x - \frac{x'}{R}\right) + \delta\beta E[W_{N-j+1}(x',\theta')]$$

$$W_{N-j}(x,\theta) := \theta u\left(T + x - \frac{x'_{N-j}(x,\theta)}{R}\right) + \beta E[W_{N-j+1}(x'_{N-j}(x,\theta),\theta')]$$

- the policy function x_{N-j} solves the right hand side of the definition of v_{N-j}

$$x'_{N-j}(x,\theta) := \arg\max_{x'} \theta u\left(T + x - \frac{x'}{R}\right) + \delta\beta E[W_{N-j+1}(x',\theta')]$$

and $x_N = 0$.

end of definition of MPE

1.2 Discretion: solution without commitment

For computations, it is convenient to define the operator $v\_x\_w(W)$ which computes one step back in the backward induction procedure.

For each output x, θ on a grid, record the value function v for period t-government, the associated policy function $x'(\cdot, \cdot)$, and the continuation value w from the perspective of all other selves as follows:

$$\begin{bmatrix} v_{N-j}(x,\theta) \\ x'_{N-j}(x,\theta) \\ W_{N-j}(x,\theta) \end{bmatrix} = \begin{bmatrix} \theta u\left(T + x - \frac{x'}{R}\right) + \delta\beta E[W_{N-j+1}(x',\theta')] \\ \text{is the policy function } v_{N-j}\text{−greedy} \\ \theta u\left(T + x - \frac{x'_{N-j}(x,\theta)}{R}\right) + \beta E[W_{N-j+1}(x'_{N-j}(x,\theta),\theta')] \end{bmatrix}$$

Exercise: Value Function Iteration

Infinite vs. finite horizon

A) First part: infinite horizon. Consider the following problem:

$$v(k,z) = \max_{c,k'} \left\{ u(c) + \beta E[v(k',z')] \right\}$$
$$c + k' - (1-\delta)k = z\ f(k)$$
$$z' = \rho z + \sigma\epsilon \qquad \text{where} \quad \epsilon \sim_{iid} \phi$$

1. Complete the following sentence: a solution to this problem is ___________________________________
2. Describe the Value Function Iteration algorithm to compute the solution.

Exercise: Value Function Iteration

Infinite vs. finite horizon

B) Second part: finite horizon $n = 1, \ldots, N$. Investment is null in the last period $v_N(k,z) = u(z\ f(k) + (1-\delta)k)$. For all $n < N$:

$$v_n(k,z) = \max_{c,k'} \left\{ u(c) + \beta E[v_{n+1}(k',z')] \right\}$$

$$c + k' - (1-\delta)k = z\ f(k)$$

$$z' = \sigma\epsilon \qquad \text{where} \quad \epsilon \sim_{iid} \phi$$

1. Complete the following sentence: a solution to this problem is ____
2. Describe an algorithm to compute the sequence of value functions: $(v_t)_{t=1}^{T}$.
3. Compare the algorithm for the finite and and the infinite horizons.

Exercise: quasi-hyperbolic discounting and self-control[1]

A household has time inconsistent preferences as proposed by Phelps and Pollak (1968) and Laibson (1994). The horizon is finite $t = 0, 1, 2, \ldots, N$. The representative household supplies one unit of labor inelastically. The economy has one sector with full depreciation.

$$c_t + k_{t+1} = \xi_t f(k_t) \tag{1}$$

with $f(k) = k^\alpha$ and $\alpha \in (0, 1)$ and ξ_t is log-normally distributed with mean μ and variance s. The household behaves as if $N + 1$ selves make decisions. The time $t-$self discounts the next period with factor $\beta\delta$ and any two subsequent periods after that with factor δ. Self N has preferences $u(c_N)$. Self $t < T$ has preferences:

$$u(c_t) + \delta \sum_{j=1}^{N-t} \beta^j E[u(c_{t+j})]$$

$u(\cdot)$ is increasing, strictly concave and twice continuously differentiable, $\beta \in (0, 1)$, and $\delta \in (0, 1]$.

[1]Based on LS Exercise 7.6 “Self-control”

Exercise: quasi-hyperbolic discounting and self-control

Continued

When $\delta < 1$, the preferences of the household are time-inconsistent (i.e. not recursive). The household chooses c_t at time t and cannot commit to future choices. A household that can commit to future choices is said to have "self-control".

1. Define a Markov Perfect Equilibrium for the $N + 1$ personalities.
2. Formulate the computation of the Markov Perfect Equilibrium recursively.
3. Write a numerical algorithm to solve this problem and implement it with the following parameters $\alpha = 0.4$, $\beta = 0.96$, $\mu = 0$, $s = 0.1$, $N = 81$ and $\delta = 0.9$.
4. Plot the policy functions at time 80 and at time 0 for $\delta = 0.9$ and for $\delta = 1$.
5. Discuss your results.

1.3. Optimal state-contingent rule: solution with commitment

Exercise Consider the economy studied in the previous exercise.

1. Characterize the optimal state-contingent rule and describe how to compute it. Note that this is the solution with commitment.

2. Is the optimal state-contingent rule (i.e. the solution with commitment) incentive compatible when θ is private information? If it is compatible with incentives, please show it. If it is not, please give an example of the perverse incentives under the optimal state-contingent rule.

2. Design of optimal rule (state is private information)

- What about the infinite horizon economy? The infinite horizon economy opens up the possibility of sustaining better outcomes than the Markov Perfect Equilibrium. Future governments could punish past governments for not following the state-contingent rule (reminiscent of Folk Theorems in repeated games). However, future governments must have an incentive to punish. We will study this later when we look at *sustainable equilibria* (Chari Kehoe (1990) JPE).

- When the horizon is finite, it is impossible to give future governments an incentive to punish: in the last period, there are no future governments that can give the government an incentive to punish previous governments for not following the rule. Reasoning by backward induction, we get the Markov Perfect Equilibrium.

2. Design of optimal rule (state is private information)

- We now study the design of a rule that is not state-contingent since θ is private (or public but non-rulable) information.

- Consider period $n < N$

$$V_n(x, \theta) = \max_{g, x'} \ \theta U(g) + \delta \ \beta E[W_{n+1}(x, \theta')]$$

$$\text{s.t.} \quad g + \frac{x'}{R} = \tilde{T} + x$$

- For the next lecture, we drop the time index n and abuse notation as follows:

$$W(x) \equiv E[W_{n+1}(x, \theta')]$$

and

$$T \equiv \tilde{T} + x$$