ECN 7059 Macroéconomie avancée

Lecture 2: Ramsey optimal taxation

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Section 1 of the syllabus : concepts and methods

1.a) Is there a role for macroeconomic policy ? Elements of welfare analysis

1.b) **Ramsey approach to optimal taxation**
References :
Kocherlakota Chapter 2 (available on StudiUM)
Chapter 16 "Optimal Taxation with Commitment" of Ljunqvist and Sargent's textbook

1.c) Mirrleesian approach

- Revelation principle
- Static (Salanié Chapter 2)
- Dynamic (Kocherlakota Chapter 3)

Main theories of government policy

There are three main normative theories of government policy :

1. *tax smoothing* : debt allows to absorb shocks, see Lucas Stokey (1983)
2. *safe asset provision* : the government issues debt to provide liquidity, see Aiyagari McGrattan (1998)
3. *dynamic efficiency* : the government issues debt in response to the over-accumulation of capital, see Diamond (1965), Blanchard (1985)

In this lecture, we study the *tax smoothing* theory.

Ramsey approach to policy design

How to design optimal policy ?

Ramsey approach :

1. Describe the environment
2. Define competitive equilibrium *given* a policy

Ramsey approach to policy design

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Ramsey approach to policy design

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2. Define competitive equilibrium *given* a policy
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4. Characterize the constraint set of the Ramsey problem by characterizing the equilibrium allocation using constraints that depend only on the allocation. These constraints are called **implementability constraints**.

Ramsey approach to policy design

How to design optimal policy ?

Ramsey approach :

1. Describe the environment
2. Define competitive equilibrium *given* a policy
3. Ramsey problem : choose policy to maximise social welfare subject to the constraint that the allocation is a competitive equilibrium allocation given the policy
4. Characterize the constraint set of the Ramsey problem by characterizing the equilibrium allocation using constraints that depend only on the allocation. These constraints are called **implementability constraints**.
5. Solve the **primal problem** : choose allocation to maximise social welfare subject to the constraint that the allocation is feasible and satisfies the implementability constraint
6. Decentralize : recover prices and optimal policy using the equivalence proved in Step 4.

Ramsey approach to policy design

- Consider the Neoclassical Growth Model with a public sector that needs to finance a stream of government expenditures
- Suppose that the government has access to linear taxes on capital and labor income.

  A **linear tax** is a tax that is a linear function of the activity taxed
  - Let $w\ \ell$ denote labor income and $T(w\ell)$ the labor income tax
  - The tax is linear iff $T(w\ \ell) = \tau\ w\ \ell$ (ad-valorem) or $T(w\ n) = \tau\ \ell$ (excise)

    Exercise : Give an example of a linear tax and a non-linear tax.
- *Should a government tax capital or labor income, or both ? When should the government levy taxes to finance public expenditure (i.e. should it issue debt and tax later, or vice versa) ?*

References : Lucas and Stokey (1983), Kocherlakota NDPF Chapter 2, Chari Kehoe (1999) Handbook chapter

Ramsey approach to policy design

1) Description of the environment : consumers

- $t = 1, \ldots \infty$
- Representative agent (unit mass of identical agents)
- $\sum_{t=1}^{\infty} \beta^{t-1} \left(u(c_t) - v(\ell_t)\right), \quad 0 < \beta < 1$
- $u', -u'', v', v''$ exist and are positive and Inada conditions
- Initial endowment of capital K_1

Ramsey approach to policy design

1) Description of the environment : technology

- δ denotes capital depreciation

- 2 sectors : consumption good and labor

  consumption at t can be invested in capital at $t+1$

  capital at t can be consumed at t

- Representative firms (unit mass of identical firms) with technology

$$y = F(k, \ell)$$

  where f exhibits Constant Returns to Scale and is concave.

  $F_k, F_l > 0$ and Inada conditions are satisfied.

Ramsey approach to policy design

1) Description of the environment : government

- Can convert consumption into public good and vice versa one for one
- Government needs to create G_t units of public good in period t

The sequence G_t is exogenous since we are not studying public expenditure. We are studying public finance : what is the best labor and capital income taxes with debts to finance $(G_t)_{t=1}^{\infty}$.

Exercise :

- What would be the best sequence $(G_t)_{t=1}^{\infty}$ in the environment described so far ?
- How would you change the model to start studying public expenditure ?

Ramsey approach to policy design

2) Define equilibrium given a policy : market structure

- Competitive market for final goods : q_t denotes the price of period t consumption relative to period 1 $q_1 \equiv 1$.
- Competitive market for labor : wage w_t denotes the relative price of leisure and period t consumption
- Competitive market for capital services : r_t denotes the rental rate of capital in terms of period t consumption

Ramsey approach to policy design

2) Define equilibrium given a policy : policy tools

The government has access to taxes and debt

- Linear labor income tax : $\tau_{\ell t}$
- Linear capital income tax : τ_{kt}
- One period debt

Ramsey approach to policy design

2) Define equilibrium given a policy : policy tools

Remarks on the restriction imposed on policy tools : *linear* taxes

- The tax rate is fixed in a given period
  $T(w_t\ell_t, r_tk_t) = \tau_{\ell t}\ w_t\ \ell_t + \tau_{kt}\ r_t\ k_t$
- The tax rate can vary across periods
- Would the government want to use a non-linear tax ?
- Exercise : what if the government could use lump-sum taxes ?
- As we will see later in the course, the Mirrleesian approach allows for non-linear tax. What prevents the government from simply using lump-sum taxes is that the government does not have the information needed to implement the desired lump-sum tax policy.

Ramsey approach to policy design

2) Define equilibrium given a policy : consumer's problem

$$
\begin{aligned}
\max_{c_t, k_t, \ell_t} \; & \sum_{t=1}^{\infty} \beta^{t-1} \left(u(c_t) - v(\ell_t) \right) \\
\text{s.t.} \; & \sum_{t=1}^{\infty} q_t (c_t + k_{t+1}) \leq \\
& \qquad \sum_{t=1}^{\infty} q_t [(1 - \tau_{\ell t}) w_t \ell_t + (1 - \tau_{kt}) r_t k_t + (1 - \delta) k_t] \\
& c_t, \ell_t, k_t \geq 0 \\
& k_1 \leq K_1
\end{aligned}
$$

Ramsey approach to policy design

2) Define equilibrium given a policy : firm's problem

$(k_t, \ell_t)_{t=0}^{\infty}$ solves the firms' problem if k_t, ℓ_t solves

$$\max_{k,\ell} \quad F(k,\ell) - r_t k - w_t \ell$$

for every t.

Firm optimality requires that factors are paid their marginal product (Firm FOCs) :

$$F_\ell(k_t, \ell_t) = w_t$$
$$F_k(k_t, \ell_t) = r_t$$

Ramsey approach to policy design

2) Define equilibrium given a policy : government budget constraint

A fiscal policy **balances the government budget** if the net present value of tax revenue must finance the net present value of government expenditures

$$\sum_{t=1}^{\infty} q_t \, [\tau_{\ell t} \; w_t \ell_t + \tau_{kt} r_t k_t] = \sum_{t=1}^{\infty} q_t \; G_t$$

Ramsey approach to policy design

2) Define equilibrium given a policy : equilibrium

An equilibrium, given fiscal policy $(\tau_{\ell t}, \tau_{kt})_{t=1}^{\infty}$ and government spending $(G_t)_{t=1}^{\infty}$, is an allocation $(c_t, \ell_t, k_t)_{t=1}^{\infty}$ and prices $(q_t, w_t, r_t)_{t=1}^{\infty}$ such that :

- $(c_t, \ell_t, k_t)_{t=1}^{\infty}$ solves the consumer problem taking as given prices $(q_t, w_t, r_t)_{t=1}^{\infty}$ and fiscal policy $(\tau_{\ell t}, \tau_{kt})_{t=1}^{\infty}$
- $(\ell_t, k_t)_{t=1}^{\infty}$ solves the firms' problem given prices
- markets clear :

$$c_t + k_{t+1} + G_t = F(k_t, \ell_t) + (1 - \delta)k_t$$

- the fiscal policy balances the government budget.

Exercise : Walras's Law : show that the government budget balance condition is redundant in the definition of equilibrium.

Ramsey approach to policy design

2) bis Computing an equilibrium given a policy

- So far, the fiscal policy is exogenous : we defined an equilibrium *given* a fiscal policy
- In step 3), the policy is endogenously determined as the solution of a planner's problem
- To conduct a quantitative evaluation of fiscal policy, it is important to calibrate the economy with the current fiscal policy. So, we need to compute an equilibrium for a given fiscal policy.
- The First and Second Welfare theorems do not apply to an economy with fiscal policies (i.e. distorting taxes).

Ramsey approach to policy design

2) bis Computing an equilibrium given a policy

- Without the First and Second Welfare theorems, we cannot rely on the equivalence between the equilibrium allocation and the solution to the planner's problem
- The equilibrium allocation and prices with a given fiscal policy solves a system of nonlinear difference equations consisting of the FOC for households and firms and resource constraints.
- A useful computational algorithm is the "Shooting algorithm"
- A useful reference is Chapter 11 "Fiscal Policies in a Growth Model" of Ljungqvist Sargent textbook *Recursive Macroeconomic Theory*

Ramsey approach to policy design

2) bis Computing an equilibrium given a policy

With inelastic labor (i.e. $v(\ell) = 0$), the equilibrium can be summarized by the following system of non-linear difference equations

$$k_{t+1} = F(k_t, 1) + (1 - \delta k_t) - g_t - c_t$$
$$u'(c_t) = \beta u'(c_{t+1})[(1 - \tau_{kt+1})(F_k(k_t, 1) - \delta) + 1]$$

- This is the same as what we would get to compute the neoclassical growth model except that now the return on capital is net of taxes.
- Suppose that the fiscal policy is constant after date T : $(\tau_{\ell t}, \tau_{kt}) = (\bar{\tau}_\ell, \bar{\tau}_k)$ for $t \geq T$. The if the allocation converges as well, we can use the Euler equation to solve (only need pen and paper) for the equilibrium value of capital at the steady state, say $\bar{k}$.
- Exercise : if $\bar{\tau}_k = 0$, what is the name that we give the value of $\bar{k}$?

Ramsey approach to policy design

2) bis Computing an equilibrium given a policy : Overview of the "Shooting algorithm"

1. Select a large time index S much larger than T. Guess an initial value c_1 for consumption. (a good guess could be the solution of a log-linear approximation of the system)
2. For $t = 1$, use the resource constraint to compute $k_{t+1} = F(k_t, 1) + (1 - \delta k_t) - g_t - c_t$
3. For $t = 1$, use the Euler equation to compute c_{t+1}

$$u'(c_t) = \beta u'(c_{t+1})[(1 - \tau_{kt+1})(F_k(k_t, 1) - \delta) + 1]$$

4. For $t = 1$, use the resource constraint to compute $k_{t+1} = F(k_t, 1) + (1 - \delta k_t) - g_t - c_t$
5. Iterate to compute $(k_t)_{t=1}^{S}$.
6. Compare k_S to $\bar{k}$
   6.1 If $k_S > \bar{k}$ (resp. $<$), raise (resp. lower) your guess of c_1
7. Iterate until $k_S \approx \bar{k}$
8. Compute initial lump sum tax for the government budget constraint hold

Ramsey approach to policy design

2) bis Computing an equilibrium given a policy

- The shooting algorithm works well for the model with a small state space as studied in this class.

- For a large state space, one could resolve to local solution method and solve for a local log-linear approximation of the equilibrium model.

Recall that the Ramsey problem is to choose policy to maximise social welfare subject to the constraint that the allocation is a competitive equilibrium allocation given the policy.

Notation : denote by $E((\tau_{\ell t}, \tau_{kt})_{t=1}^{\infty})$ the set of equilibrium allocations given fiscal policy $\tau_{\ell t}, \tau_{kt}$.

Ramsey problem (dual formulation) :

$$\max_{c_t, \ell_t, k_{t+1}, \tau_{\ell t}, \tau_{kt+1}} \sum_{t=1}^{\infty} \beta^{t-1} \left(u(c_t) - v(\ell_t)\right)$$
$$\text{s.t. } (c_t, \ell_t, k_t)_{t=1}^{\infty} \in E((\tau_{\ell t}, \tau_{kt})_{t=1}^{\infty})$$

Remark on taxing initial capital "capital levy" :

- The initial capital K_1 is special because its supply is inelastic (it is exogenous).
- Taxing initial capital ($\tau_{k1} > 0$) is like a lump sum tax in the sense that it does not distort household investment decisions.
- To keep the problem interesting, we rule out initial "capital levy" by imposing τ_{k1} small.

What if there are multiple equilibria ? If we assume that the government can coordinate private agents on the best equilibrium, then we write the Ramsey problem as follows :

$$\max_{c_t, \ell_t, k_t, \tau_{\ell t}, \tau_{kt}} \sum_{t=1}^{\infty} \beta^{t-1} \left(u(c_t) - v(\ell_t)\right)$$

$$\text{s.t. } (c_t, \ell_t, k_t)_{t=1}^{\infty} \in E((\tau_{\ell t}, \tau_{kt})_{t=1}^{\infty})$$

Ramsey approach to policy design

3) Ramsey problem : some remarks

- The Ramsey problem as formulated is very hard to solve because the constraint set is hard to work with. To check that an allocation is an equilibrium allocation, one needs to find equilibrium prices and check all equilibrium conditions.

- We will greatly simplify the problem by rewriting the constraint set as two constraints : the resource constraint and the **implementability constraint**. This is called the **primal approach**.

- The **implementability constraint** is the inter-temporal budget constraint of the representative consumer with after-tax prices replaced by the appropriate Marginal Rate of Substitutions.

Ramsey approach to policy design

4) Implementability constraints

Proposition (Implementability constraint) :
A strictly positive allocation $(c_t, \ell_t, k_t)_{t=1}^{\infty}$ with $k_1 > 0$ is in $E((\tau_{\ell t}, \tau_{kt})_{t=1}^{\infty})$
if and only if

$$c_t + k_{t+1} + G_t = F(k_t, \ell_t) + (1-\delta)k_t \quad \text{for all } t \qquad \text{(Resource)}$$

$$\sum_{t=1}^{\infty} \left[\beta^{t-1}\frac{u'(c_t)}{u'(c_1)}c_t - \beta^{t-1}\frac{v'(\ell_t)}{u'(c_1)}\ell_t\right] = K_1\ [1-\delta+F_k(k_1,\ell_1)(1-\tau_{k1})] \qquad \text{(implementability)}$$

$$k_1 = K_1 \qquad \text{(initial condition)}$$

Ramsey approach to policy design

4) Implementability constraints

Sketch of proof (Implementability constraint) :

$\Rightarrow$ (necessity)

- Market clearing implies the resource constraint.
- Since $k_1 > 0$ by assumption, it follows that $k_1 = K_1$.
- FOCs of the consumer problem are :

$$(1-\tau_{\ell t})w_t = \frac{v'(\ell_t)}{u'(c_t)}$$

$$\beta^{t-1}\frac{u'(c_t)}{u'(c_1)} = \frac{q_t}{q_1}$$

$$q_t = [1-\delta + r_t(1-\tau_{k\ t+1})]\ q_{t+1}$$

  Substitute these FOCs in the budget constraint :

$$\sum_{t=1}^{\infty}\left[\beta^{t-1}\frac{u'(c_t)}{u'(c_1)}c_t - \beta^{t-1}\frac{v'(\ell_t)}{u'(c_1)}\ell_t\right] = K_1\ [1-\delta + r_1(1-\tau_{k1})]$$

- Use firm FOC to substitute $r_1 = F_k(K_1, \ell_1)$

Problem Set 1, Exercise 1

4) Implementability constraints

Prove the sufficiency part of the Proposition (Implementability constraint) : $\Leftarrow$ (sufficiency)

Hint : you need to recover factor prices and taxes and check that all equilibrium conditions are satisfied

- *use firms' FOC to get factor prices* r_t, w_t *which guarantees firms are optimizing*
- *use consumers' FOC to recover* q_t *and tax rates* $\tau_{\ell t}, \tau_{kt}$ *by guessing the Lagrange multiplier* λ *is 1. This guarantees that consumers are optimizing so long as their budget constraint is satisfied.*
- *use consumer's FOC w.r.t.* k_{t+1} *to cancel all terms involving* k_t *except* k_1*. The resulting condition is the implementability constraint which holds by assumption.*

Ramsey approach to policy design

4) Implementability constraints

Some remarks

- Recall that initial capital is special : it is inelastically supplied so it is always optimal to finance as much of the government expenditure as possible with initial capital income tax τ_{k1}
- Again, the implementability constraint is the budget constraint where we substituted the prices by Marginal Rates of Substitution.
- We can use this proposition to rewrite the Ramsey problem in the **Primal form**

Ramsey problem in primal form :

$$\max_{(c_t,\ell_t,k_t)_{t=1}^{\infty},\tau_{k1}} \sum_{t=1}^{\infty} \beta^{t-1} \left(u(c_t) - v(\ell_t)\right)$$

s.t.

$$c_t + k_{t+1} + G_t = F(k_t, \ell_t) + (1-\delta)k_t \quad \text{for all } t \qquad \text{(Resource)}$$

$$\sum_{t=1}^{\infty} \left[\beta^{t-1} \frac{u'(c_t)}{u'(c_1)} c_t - \beta^{t-1} \frac{v'(\ell_t)}{u'(c_1)} \ell_t\right] = K_1 \left[1 - \delta + F_k(k_1, \ell_1)(1 - \tau_{k1})\right]$$

(implementability)

$$k_1 = K_1 \qquad \text{(initial condition)}$$

Remark : we substituted out prices and taxes from the constraint set (except the special τ_{k1} for inelastically supplied initial capital)

Ramsey approach to policy design

4) Implementability constraints

The only difference between a social planner's problem and the Ramsey problem in primal form is the the addition of the implementability constraint in the Ramsey problem.

The implementability constraint exactly captures the constraint that the government has to allocate resources through competitive markets with *linear* taxes.

When design its fiscal policy, the government takes into considerations that choices and prices are going to change (i.e. general equilibrium effects).

Ramsey approach to policy design

5) Solve the primal problem

- We can solve the Ramsey problem in primal form using Lagrangian techniques.
- The solution of the Ramsey problem in primal form gives us an allocation, which we call the *Ramsey plan*.
- We can recover lessons about optimal taxation using the equivalence proved in Proposition (Implementability constraint).

Ramsey approach to policy design

5) Solve the primal problem

- Let Φ denote the Lagrange multiplier on the implementability constraint
- Let $(\beta^{t-1}\theta_t)_{t=1}^{\infty}$ denote the sequence of Lagrange multipliers (scaled by β^{t-1}) on the sequence of resource constraints

Lagrangian of Ramsey problem in primal form :

$$
\begin{aligned}
\mathcal{L} \equiv \; & \sum_{t=1}^{\infty} \beta^{t-1}\left[(u(c_t) - v(\ell_t)) + \Phi(u'(c_t)c_t - v'(\ell_t)\ell_t)\right] \\
& + \sum_{t=1}^{\infty} \beta^{t-1}\theta_t\left[F(k_t, \ell_t) + (1-\delta)k_t - c_t - k_{t+1} - G_t\right] \\
& - \Phi u'(c_1)\left[K_1 \; [1 - \delta + F_k(k_1, \ell_1)(1 - \tau_{k1})]\right]
\end{aligned}
$$

Ramsey approach to policy design

5) Solve the primal problem

Remarks about the Lagrangian

- The objective of the Ramsey planner can be seen as the objective of a planner modified to account for distortionary effects of linear taxation and general equilibrium effects :

$$V(c,\ell;\Phi) = (u(c) - v(\ell)) + \underbrace{\Phi(u'(c)c - v'(\ell)\ell)}_{\text{Distortion and GE}}$$

- The special nature of initial capital (inelastic) can be seen by the last term involving only terms related to the initial period :

$$A(c_1,\ell_1,K_1;\Phi) \equiv \Phi u'(c_1)\big[K_1\ [1-\delta+F_k(k_1,\ell_1)(1-\tau_{k1})]\big]$$

Ramsey approach to policy design

5) Solve the primal problem

FOCs characterizing the Ramsey plan

$$
\begin{aligned}
V_c(t, \Phi) =& \theta_t && \text{(FOC } c_t\text{)} \\
V_\ell(t, \Phi) =& -\theta_t F_\ell(t) && \text{(FOC } \ell_t\text{)} \\
V_c(1, \Phi) =& \theta_1 + \Phi A_{c_1} && \text{(FOC } c_1\text{)} \\
V_\ell(1, \Phi) =& -\theta_1 F_\ell(1) + \Phi A_{\ell_1} && \text{(FOC } \ell_1\text{)} \\
\theta_t =& \beta\theta_{t+1}[F_k(t+1) + 1 - \delta] && \text{(FOC } k_{t+1}\text{)}
\end{aligned}
$$

where $V_c(t, \Phi)$ denotes $\frac{\partial V(c_t, \ell_t, \Phi)}{\partial c}$

Ramsey approach to policy design

5) Solve the primal problem

Combining the FOCs to eliminate θ_t gives.

$$
\begin{aligned}
V_c(t,\Phi) =& \beta V_c(t+1,\Phi)[F_k(t+1)+1-\delta] && \text{(Ramsey Euler)} \\
V_\ell(t,\Phi) =& -V_c(t,\Phi)F_\ell(t) && \text{(FOC } \ell_t\text{)} \\
V_c(1,\Phi) =& \beta V_c(2,\Phi)[F_k(2)+1-\delta] + \Phi A_{c_1} && \text{(FOC } c_1\text{)} \\
V_\ell(1,\Phi) =& -V_c(1,\Phi)\ F_\ell(1) + \Phi A_{c_1} F_\ell(1) + \Phi A_{\ell_1} && \text{(FOC } \ell_1\text{)}
\end{aligned}
$$

This set of equations together with the resource and implementability constraints characterize a Ramsey plan. We need an allocation $(c_t, \ell_t, k_{t+1})_{t=0}^{\infty}$ and a Lagrange multiplier on the implementability constraint Φ that satisfies the above equations with the resource and implementability constraints.

Ramsey approach to policy design

Optional : time inconsistency of Ramsey plan

- Let $(c_t^*, \ell_t^*, k_{t+1}^*)_{t=0}^{\infty}$ denote the solution to the Ramsey problem in primal. This Ramsey plan is the solution of the government optimizing at time 0 for the infinite future.

- The solution to a Ramsey problem planning from time 0 for the infinite future (the Ramsey plan studied in this Lecture) is a solution *with commitment.*

- The solution where the government sequentially optimizes every period is the solution *without commitment.*

- Kydland and Prescott (1977) and the large literature on Rules vs. Discretion that followed started from the observation that the solution with commitment always weakly dominates the solution without commitment.

Ramsey approach to policy design

Optional : time inconsistency of Ramsey plan

- Does the government have an incentive to revise it's Ramsey plan for fiscal policy in the future ?
- When the plan with commitment and the plan without commitment differ, the plan with commitment is said to be *time-inconsistent.*

Ramsey approach to policy design

Optional : time inconsistency of Ramsey plan

Is the Ramsey plan time-consistent ? The answer is no.

$$
\begin{aligned}
V_c(t,\Phi) &= \beta V_c(t+1,\Phi)[F_k(t+1)+1-\delta] && \text{(Ramsey Euler)}\\
V_\ell(t,\Phi) &= -V_c(t,\Phi)F_\ell(t) && \text{(FOC } \ell_t\text{)}\\
V_c(1,\Phi) &= \beta V_c(2,\Phi)[F_k(2)+1-\delta] + \Phi A_{c_1} && \text{(FOC } c_1\text{)}\\
V_\ell(1,\Phi) &= -V_c(1,\Phi)\ F_\ell(1) + \Phi A_{c_1} F_\ell(1) + \Phi A_{\ell_1} && \text{(FOC } \ell_1\text{)}
\end{aligned}
$$

The terms in red depend on the initial condition. If the government could re-optimize at t it would solve

$$
\begin{aligned}
V_c(t+j,\hat\Phi) &= \beta V_c(t+1+j,\hat\Phi)[F_k(t+1)+1-\delta]\\
V_\ell(t+j,\hat\Phi) &= -V_c(t+j,\hat\Phi)F_\ell(t)\\
V_c(t,\hat\Phi) &= \beta V_c(t+1,\hat\Phi)[F_k(t+1)+1-\delta] + \hat\Phi A_{c_t}\\
V_\ell(t,\hat\Phi) &= -V_c(t,\hat\Phi)F_\ell(t) + \hat\Phi A_{c_t} F_\ell(t) + \hat\Phi A_{\ell_t}
\end{aligned}
$$

and the Ramsey plan optimizing from t onwards does not coincide with the Ramsey plan from t onwards that results from optimizing at 0 : the Ramsey plan is time-inconsistent.

Ramsey approach to policy design

5) Solve the primal problem : long run taxation

It is easier to characterize what happens at the *steady state*. This will tell us about the *long run* properties of an optimal fiscal policy with linear taxes.

- Suppose that after period $\bar{T}$ the exogenous stream of fiscal expenditure is constant : $g_t = \bar{g}$ for $t > \bar{T}$
- Suppose that there is a solution to the Ramsey problem and that it converges to a time invariant Ramsey plan :

$$\lim_{t \to \infty} (c_t, \ell_t, k_{t+1}) = (c, \ell, k)$$

- Note that $V_c(t) \to V_c(c, \ell; \Phi))$ a constant so the Ramsey Euler equation is :

$$V_c = \beta V_c [F_k + 1 - \delta]$$

Ramsey approach to policy design

5) Solve the primal problem : long run taxation

- What does this imply about capital taxation ? We turn to the household Euler equation to get :

$$1 = \beta[(1 - \lim_{t\to\infty} \tau_{kt+1})F_k + 1 - \delta]$$

- By identification of the household Euler and Ramsey Euler equation in the long run we get that the optimal fiscal :

$$\lim_{t\to\infty} \tau_{kt+1} = 0.$$

This result is called the Chamley-Judd result.

Ramsey approach to policy design

6) Decentralize : recover prices and optimal policy

Combining consumer FOCs for the household and firm problems, we get :

$$\tau_{lt} = 1 - \frac{v'(\ell_t)}{u'(c_t)\ F_\ell(k_t, \ell_t)}$$

$$\tau_{k\ \ t+1} = 1 - \frac{\frac{u'(c_t)}{\beta u'(c_{t+1})} - 1 + \delta}{F_k(k_{t+1}, \ell_{t+1})}$$

These two equations, when evaluated at the allocation that solves the Ramsey problem in primal form, allow to recover the optimal fiscal policy $(\tau_{lt}, \tau_{k\ \ t+1})_{t=1}^{\infty}$ of the government.

Ramsey approach to policy design

Exercise : the CRRA case

Prove the following proposition for the special case with CRRA preferences

Proposition (Optimal capital taxation) :

Suppose that $u'(c) = c^{-\gamma}$ for $\gamma > 0$ and that $\tau_{k1} = 0$, then the optimal fiscal policy features no tax on capital : $\tau_{k,t} = 0$.

Hint : derive the FOC of the Ramsey problem in primal form to get

$$\frac{1}{\beta} \left(\frac{c_{t+1}}{c_t} \right)^{\gamma} = 1 - \delta + F_{k\ t+1}(k_{t+1}, \ell_{t+1}) \ .$$

which you can plug in the optimal tax rate formula to get the result.

Ramsey approach to policy design

Some conclusions

- the result of zero capital taxation in the short and long run depends on the assumption $u'(c) = c^{-\gamma}$

- Chamley-Judd result : if the allocation that solves the Ramsey problem in primal form converges to a strictly positive limit, then the implied capital tax rate is 0 in the long run :

$$\lim_{t \to \infty} \tau_{k,t+1} = 0$$

- Intuition for Chamley-Judd result : positive capital income tax consists in taxing consumption at $t+1$ at a higher rate than at t. if the capital tax rate converges to a strictly positive limit, then it consists in taxing future consumption at a rate that grows exponentially which causes distortions that are suboptimal.

Ramsey approach to policy design

Main lessons

Method :

- Define Ramsey problem
- Characterize the equilibrium using the implementability constraint
- Formulate Ramsey problem in primal form

Policy :

- Chamley-Judd : zero long-run capital income tax.
- This result depends on the restriction to *linear* taxes.
- Straub and Werning (2014) NBER show limitations of the Chamley-Judd result : the condition "converges to a strictly positive limit" is a condition on endogenous variables that need not be satisfied.
- Lack of robustness with restricted policy tools motivates the Mirrleesian approach

Computations of LQ Ramsey problem

References :

- QuantEcon : Optimal Taxation in an LQ Economy, https ://lectures.quantecon.org/py/lqramsey.html
- Sargent T., J. and Velde, F., R., (1999) "Optimal Fiscal Policy in a Linear Stochastic Economy", working paper on StudiUM
- LS Sections 16.8 "A stochastic economy" and 16.12 "A stochastic economy without capital"

Model is slightly different from the one we studied last lecture :

- "Linear Quadratic" : $u(c, \ell) = -(c-b)^2 - \ell^2$
- only linear labor income tax (no capital)
- shocks to
  - preferences : b
  - government expenditures
  - endowments

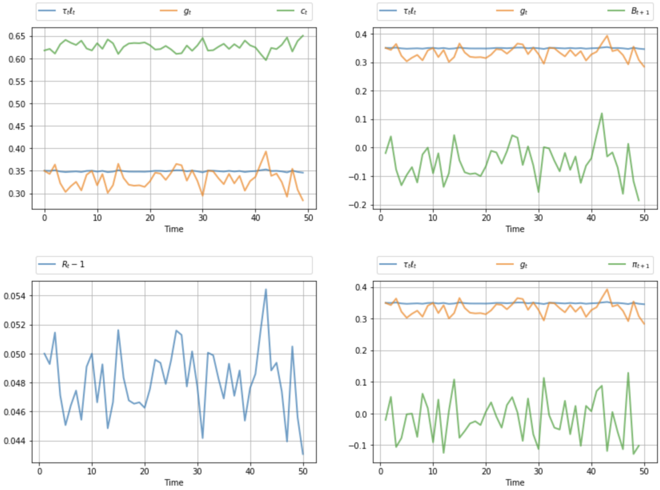
  (Note that so far we studied a deterministic economy)
- stream of coupon payment initially owed by the government

Computations of LQ Ramsey problem

Solution strategy :

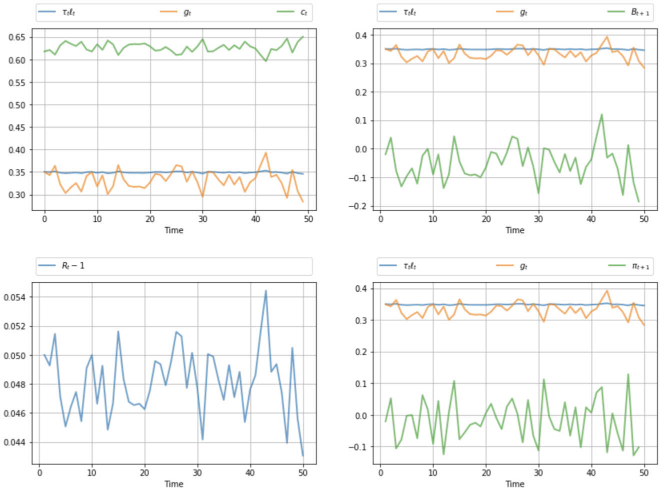
1. Rewrite the Ramsey problem in primal form
2. Characterize the solution to the Ramsey problem as the solution to a system of equations including the first order conditions, resource and implementability constraints (use Lagrangian methods so you will have two Lagrange multipliers to solve for).
3. Solve this system :
   - the usual challenge in Ramsey problem is to solve for the Lagrange multiplier on the implementability constraint which calls for the repeated evaluations of the present value of the government's surplus, which is like an asset price (see LS Section 16.12 and in particular equation (16.12.8) p. 692)
   - Linear-Quadratic model makes this task easy

Computations of LQ Ramsey problem



The government uses debt to smooth the labor income tax.

Computations of LQ Ramsey problem



Exercise : how would the blue line $\tau_t \ell_t$ look like if the government could not issue debt ?

What to read next ?

If you plan to do research in this area, I recommend the following next steps :

- LS Section 16.14 Lessons for optimal debt policy
- LS Section 16.15 Taxation without state contingent debt and the QuantEcon Lecture “Optimal Taxation without State-Contingent Debt”
- Bhandari Evans Golosov and Sargent and the associated numerical methods on QuantEcon Lectures “Fluctuating Interest Rates Deliver Fiscal Insurance” and “Fiscal Risk and Government Debt”

Guide to QuantEcon lectures on Ramsey taxation

If you plan to do research in this area, I recommend to study recent research on the topics and develop familiarity with the numerical methods on QuantEcon.

The next three slides give you a roadmap for the different QuantEcon lectures on this topic.

Guide to QuantEcon lectures on Ramsey taxation

Partial equilibrium : exogenous interest rate

- "Tax Smoothing with Complete and Incomplete Markets."
  Barro (1979) model with incomplete markets with risk free debt. The lecture also has an extension to complete markets. Exogenous constant interest rate $R = \frac{1}{\beta}$ and asset prices, LQ preferences (minimize sum of tax squarred). Need
  i) stochastic process for non-financial income
  ii) stochastic process for public expenditures
- "How to Pay for a War: Part 1" *allows for a fluctuating interest rate*. The reference is Barro (1999). The tool is Markov Jump Linear Quadratic Dynamic Programming. Addition of a
  iii) stochastic process for interest rate.
- "How to Pay for a War: Part 2" *extension to choice of maturity* Extension of Barro (1979) to allow for debt of different maturities (1 through H) with a jointly stochastic process for interest rates.
- "How to Pay for a War: Part 3" *extension to roll-over risk.* Extension of Barro (1979) to account for roll-over risk (i.e. shut down the government's ability to borrow in one state of the world)

Guide to QuantEcon lectures on Ramsey taxation

General equilibrium : endogenous interest rate

Complete markets : state contingent sovereign debt : Lucas and Stokey (1983) economy featuring complete markets and endogenous interest rate.

- "Optimal Taxation in an LQ Economy".
  LQ (linear quadratic) preferences in a Lucas and Stokey (1983) economy.

- "Optimal Taxation with State-Contingent Debt".
  Ramsey problem in a Lucas and Stokey (1983) economy with preferences not restricted to LQ. Uses recursive methods (dynamic stackelberg/Ramsey) to address the lack of tractability that is no longer accessible when we depart from the Linear-Quadratic version. References are LS Chapter 19 "Dynamic Stackelberg Problems" and 20 "Two Ramsey Problems Revisited".

Guide to QuantEcon lectures on Ramsey taxation

General equilibrium : endogenous interest rate

Incomplete markets : risk free sovereign debt

- "Optimal Taxation without State-Contingent Debt" : Aiyagari, Marcet, Sargent, and Seppälä (2002) in which only a risk free bond is traded.
- "Fluctuating Interest Rates Deliver Fiscal Insurance" : Bhandari Evans Golosov Sargent (2017) QJE. They show that with 2-states, fluctuating interest rate can provide full insurance
- "Fiscal Risk and Government Debt". Extension of previous lecture, with more than 2-states, fluctuating interest rate cannot provide full insurance

Plan for next week

1.a) Is there a role for macroeconomic policy ? Elements of welfare analysis

1.b) Ramsey approach (Kocherlakota Chapter 2)

1.c) **Mirrleesian approach**