ECN 7059 Macroéconomie avancée

Lecture 6: Optimal design of rules

Guillaume Sublet

Université de Montréal et CIREQ

2. Design of optimal rule

2.1 Setup of the problem of designing a rule

- Continuum of countries
- Present-biased ($\delta < 1$) governments

$$\theta U(g) + \delta \, \beta W(x)$$

- Budget constraint

$$g + \frac{x}{R} + \frac{\tau_f(g)}{R} = T$$

- θ: fiscal needs of the country (private/non-rulable information to the government)
- $\theta \sim F$ on $[\underline{\theta}, \bar{\theta}]$ where $\bar{\theta}$ could be infinite

2. Design of optimal rule

2.1 Setup of the problem of designing a rule

Three features highlighted in red in the previous slide:

1. present-biased governments δ (benefit of committing to a fiscal rule)
2. shocks to fiscal needs θ (benefit of discretion)
3. fiscal needs is private information to the government (tradeoff between commitment and discretion)

Tradeoff

- Present-biased governments run inefficiently large deficits: *need for commitment* to a fiscal rule that limits spending (correct the deficit bias because of δ)

- Governments use fiscal policy to respond to shocks (allow spending to increase with θ) *need discretion*

Fiscal rule

Rule is a sanction schedule $\tau_f(g) \geq 0$, *no-transfer condition*

In class discussion: in which context is the no-transfer condition a natural assumption?

Exercise: Describe a rule that effectively sets a cap on spending at $\bar{g}$, that is the choice set is restricted $[0, \bar{g}]$.

Fiscal rule

Map financial sanctions τ_f into sanctions τ on the objective

$$\tau(g) = \delta\beta \left[W(R(T-g)) - W(R(T-g) - \tau_f(g)) \right]$$

Government problem

$$\max_{g \geq 0} \; \theta U(g) + \delta\beta \; \underbrace{W(R(T-g))}_{\text{Notation: } W(g)} - \tau(g)$$

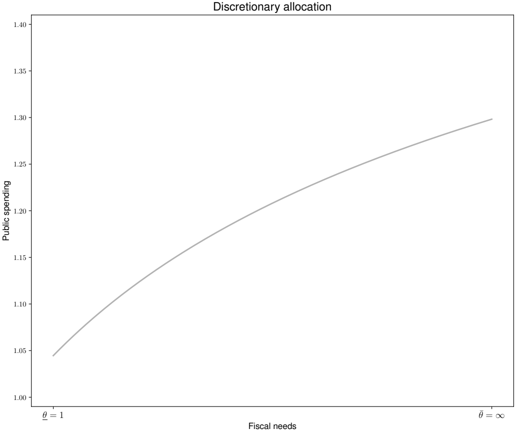
2. Design of optimal rule

Discretionary allocation (no rule)

Discretionary allocation solves the government Euler equation:

$$\theta \ U'(g_d(\theta)) = \delta \ \beta W'(R \ (T - g_d(\theta)))$$

With CRRA preferences:



2. Design of optimal rule

Rule creates wedge Δ in Euler condition:

$$U'(g)\ \Delta(g) := \delta\ \beta\ R\ W'(g, R)$$

$\Delta(g)$ denotes the fiscal need that is fulfilled by spending g.

2. Design of optimal rule

Rules and on-equilibrium sanctions

Exercise: This exercise asks you to interpret $\Delta(g)$ as a wedge.

1. What is $\Delta(g_d(\theta))$? Interpret $\Delta(g_d(\theta))$.

2. If θ was public information the rule could be made contingent on θ. Denote the the optimal state-contingent rule by $g_{fi}(\theta)$. Define the optimal state-contingent rule as the allocation that solves the Euler equation without present-bias for each fiscal need θ, that is:

$$\theta \ U'(g_{fi}(\theta)) = \beta R W'(R(T - g_{fi}(\theta))).$$

   What is the fiscal needs that the optimal state-contingent rule allows to meet $\Delta(g_{fi}(\theta))$?

Exercise: This exercise asks you to interpret $\Delta(g)$ as a wedge.

1. *Answer:* $\Delta(g_d(\theta)) = \theta$, it is the fiscal needs that is fulfilled by public spending $g_d(\theta)$. At the discretionary allocation, each government can fulfill its public spending needs.

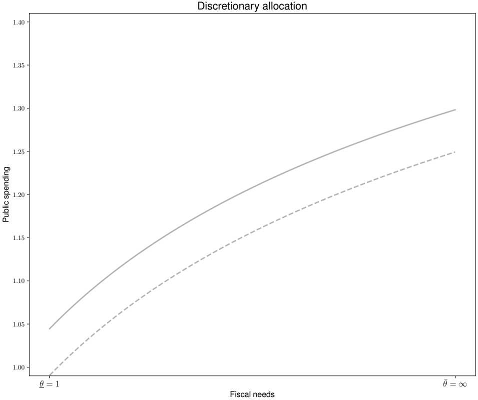
2. *Answer:* If θ was public information the rule could be made contingent on θ and the fiscal needs that the optimal state-contingent rule allows to fulfill is $\Delta(g_{fi}(\theta)) = \delta\theta < \theta$. The optimal state-contingent fiscal rule corrects for the present-bias δ while preserving discretion.

2. Design of optimal rule

Optimal state contingent rule

$$\theta \; U'(g_{fi}(\theta)) = \beta W'(R(T - g_{fi}(\theta)))$$

When θ is public information, there is no tradeoff between commitment and discretion. With CRRA preferences:

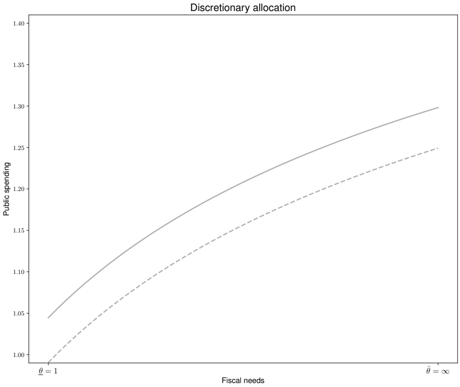


2. Design of optimal rule

Tradeoff between commitment and discretion

Exercise:

1. If the direct mechanism is $(g_{fi}(\cdot))$ and the fiscal needs θ is private information, then draw the allocation implemented by this fiscal rule on this graph:



2. What sanction schedule implements the same allocation as the allocation drawn in the previous question?

2. Design of optimal rule

Welfare of the economic union

$$\int \left[\theta U(g(\theta)) + \beta \; W(g(\theta)) - \rho \; \tau(g(\theta))\right] \, \mathrm{d}F(\theta)$$

1. Deficit bias:

$$\nu(g) \equiv (1-\delta) \; \beta \; W(g)$$

2. Asymmetry in welfare cost of sanctions measured by ρ

2. Design of optimal rule

Welfare of the union

$$\int \left[\theta U(g(\theta)) + \beta \; W(g(\theta)) - \rho \; \tau(g(\theta))\right] \, \mathrm{d}F(\theta)$$

Asymmetry in welfare cost of sanctions measure by ρ

- Single country: sanctions are completely wasteful $\rho = \frac{1}{\delta}$ (Amador Werning Angeletos (2006), Halac Yared (2019))
- Economic union: revenues from sanctions benefit the union $0 < \rho < \frac{1}{\delta}$
- This lecture $\rho = 1$ (symmetric welfare costs)
- In the paper Sublet (2021): any $\rho \in [0, \frac{1}{\delta}]$

2. Design of optimal rule

Welfare of the union

Welfare of the union:

$$\int \left[\theta U(g(\theta)) + \beta\ W(g(\theta)) - \rho\ \tau(g(\theta))\right]\, \mathrm{d}F(\theta)$$

For a single country, it is natural to consider $\rho = \frac{1}{\delta}$

1. Financial sanction (money burnt) – Amador Werning Angeletos (2006)
2. Penalties in the next period – Halac Yared (2019)

For a union of countries, it is natural to consider $\rho < \frac{1}{\delta}$

- Financial sanction with revenue collected by the union
- Revenue used to benefit the union, not the government of its members (e.g., collect revenues from sanctions on a European member and use it to finance European institutions)

2. Optimal design of a rule

Dual problem

$$\max_{\tau(\cdot)\geq 0} \int \big[\underbrace{\theta U(g(\theta)) + \delta\beta W(g(\theta))}_{\text{government welfare}} + \underbrace{\nu(g(\theta))}_{\text{bias}} - \tau(g(\theta))\big] dF(\theta).$$

where $g(\theta)$ solves, for each θ,

$$\max_{g\geq 0} \theta U(g) + \delta\beta \, W(g) - \tau(g)$$

Remark: if the economic union was a fiscal union, transfers would be possible and the problem would not be constrained by the no-transfer condition $\tau(\cdot) \geq 0$.

2. Optimal design of a rule

Primal problem

Exercise: prove a version of the revelation principle to rewrite the dual problem in primal form as follows:

$$\max_{t(\cdot)\geq 0, g(\cdot)} \quad \int \big[\underbrace{\theta U(g(\theta)) + \delta\beta W(g(\theta))}_{\text{government welfare}} + \underbrace{\nu(g(\theta))}_{\text{bias}} - \ t(\theta)\big] dF(\theta).$$

where $g(\theta)$ solves

$$\theta U(g(\theta)) + \delta\beta \ W(g(\theta)) - t(\theta) \geq \theta U(g(\hat{\theta})) + \delta\beta \ W(g(\hat{\theta})) - t(\hat{\theta})$$

Solution method

- The design of a fiscal rule maps into a mechanism design problem *without transfers*
- Global Lagrangian method to optimize over all IC allocations
- Monotonicity condition key to the design of rules
- Inspection of the Lagrangian suggest candidate solutions
- Use FOCs to derive sufficient conditions under which the candidate rule is optimal

To study the solution method: see Amador Bagwell (2013), Amador Bagwell (2020), and Sublet (2021) available on my website.

2. Optimal design of a rule

Characterization of incentive compatible allocations:

Lemma (Incentive compatible allocations)

An allocation $(g(\theta))$ is incentive compatible given a money burning schedule $(t(\theta))$ if and only if $(g(\theta))$ is non-decreasing and

$$\begin{aligned} t(\theta) = & \theta \; U(g(\theta)) + \beta\delta W(R(T - g(\theta))) \\ & - \underline{\theta} \; U(g(\underline{\theta})) - \beta\delta W(R(T - g(\underline{\theta}))) - \int_{\underline{\theta}}^{\theta} U(g(\tilde{\theta})) \, d\tilde{\theta}. \end{aligned}$$

Outline

Guess and verify

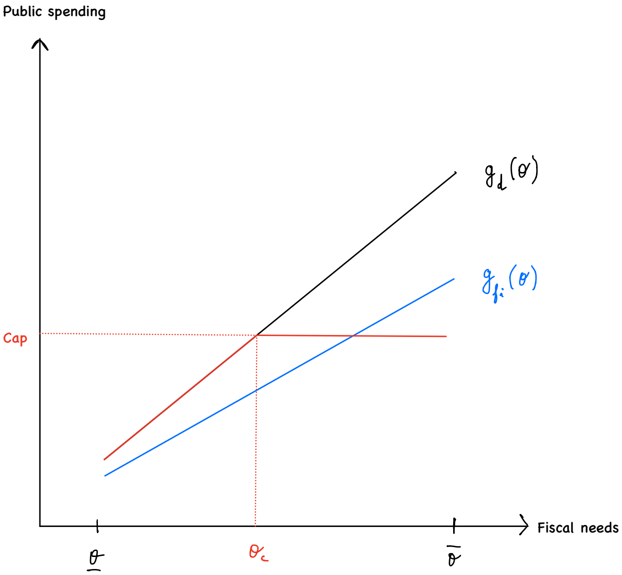
1. Determine the best off-equilibrium sanctions (i.e. best cap among caps)

2. Determine the best on-equilibrium sanctions

3. Conditions under which the best off-equilibrium sanctions are the best fiscal rule

   Conditions under which the best on-equilibrium sanctions are part of the best fiscal rule

Candidate solution: discretion and cap



Candidate solution: discretion and cap

$$\int \Big[\underbrace{\theta U(g(\theta)) + \delta\beta W(g(\theta))}_{\text{government welfare}} + \underbrace{\nu(g(\theta))}_{\text{bias}} - \tau(g(\theta))\Big] dF(\theta)$$

Equation determining stringency of the cap θ_c:

$$\underbrace{U'(g_d(\theta_c, R))\ \mathbb{E}[\theta - \theta_c | \theta \geq \theta_c]}_{\text{marginal cost limiting discretion}} = \underbrace{-\nu'(g_d(\theta_c))}_{\text{marginal bias}}$$

Candidate solution: discretion and cap

Lemma (Convex vs concave cost of limiting discretion)

- $1 - F$ *log-concave then* $\mathbb{E}[\theta - \theta_c | \theta \geq \theta_c]$ *is increasing as* θ_c *decreases*
- $1 - F$ *log-convex then* $\mathbb{E}[\theta - \theta_c | \theta \geq \theta_c]$ *is decreasing as* θ_c *decreases.*

Example

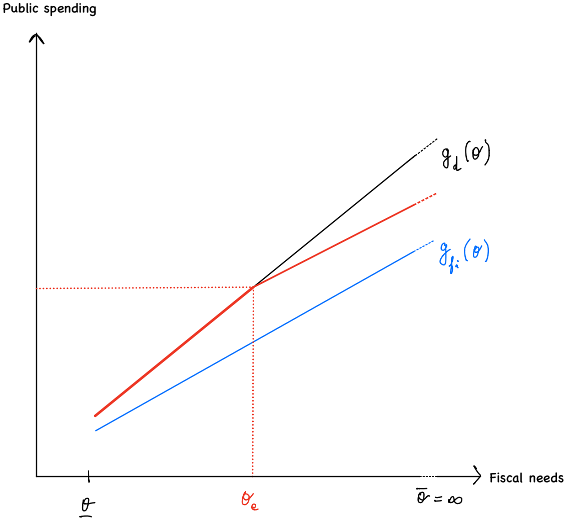
- Exponential distributions have log-concave "thin" tails

  Convex cost of limiting discretion by tightening the cap

- Pareto distributions have log-convex "thick" tails

  Concave cost of limiting discretion by tightening the cap

Candidate solution: discretion and on-equilibrium sanctions



Candidate solution: discretion and on-equilibrium sanctions

$$\int_{\Theta} \Bigg[\underbrace{\underline{\theta}\; U(g(\underline{\theta})) + \beta\delta W(g(\underline{\theta})) + U(g(\theta))\; \frac{1-F(\theta)}{f(\theta)}}_{\text{government welfare net of sanctions}} + \underbrace{\nu(g(\theta))}_{\text{bias}} \Bigg] dF(\theta)$$

Marginal sanction at $g(\theta)$ has two effects:

1. discipline θ: wedge in Euler equation
2. increases the *level* of sanctions on spending above $g(\theta)$ of which there are $1 - F(\theta)$ governments

$$\underbrace{U'(g_n(\theta))\; \frac{1-F(\theta)}{f(\theta)}}_{\text{marginal cost on all } \hat{\theta} \geq \theta} = \underbrace{-\nu'(g_n(\theta))}_{\text{marginal bias}}$$

Candidate solution: discretion and on-equilibrium sanctions

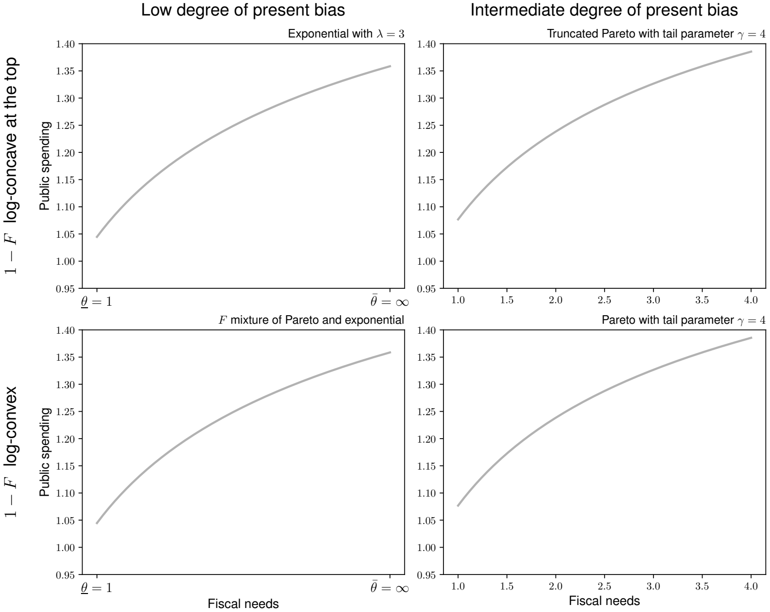
Lemma (Monotonicity with on-equilibrium sanctions)

The allocation g_n is non-decreasing if and only if $1 - F$ is log-convex.

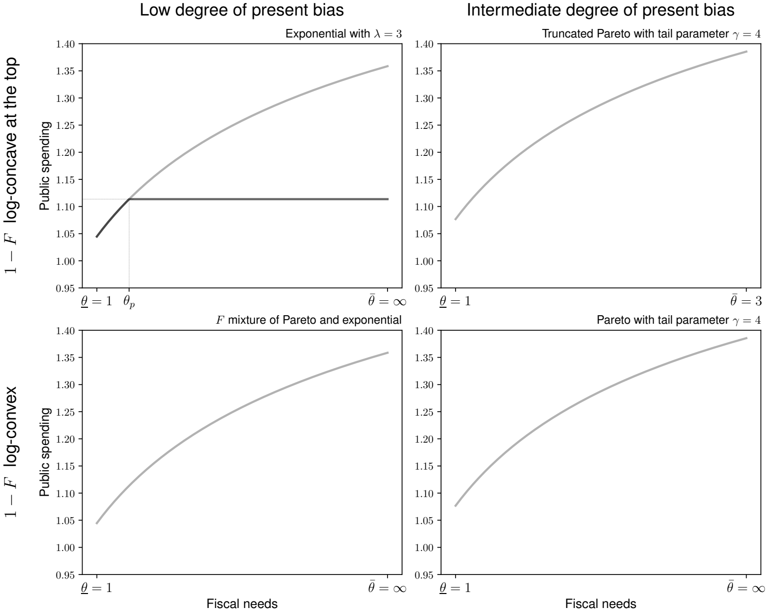
Intuition

- $\frac{1-F}{f}$ governs the marginal cost of on-equilibrium sanctions
- $1 - F$ log-convex if and only if $\frac{1-F}{f}$ is increasing
- If $1 - F$ is log-convex then cost of on-equilibrium sanctions is convex

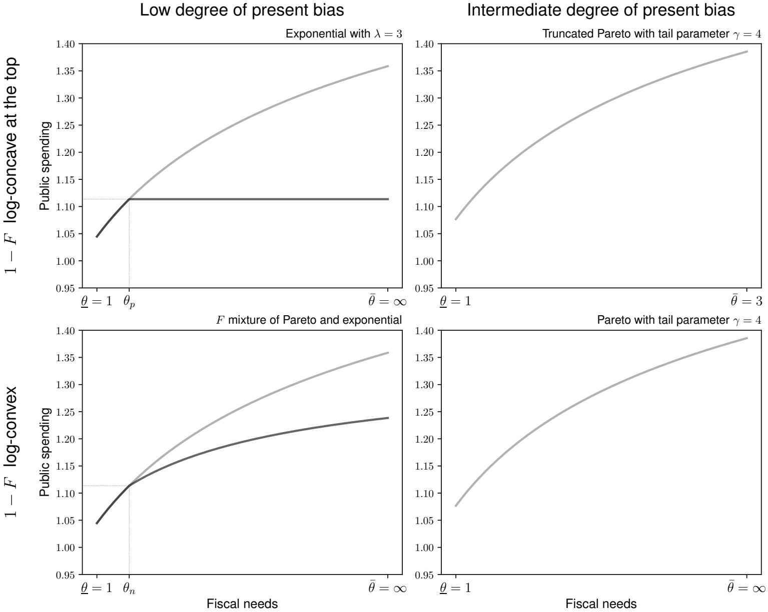
Optimal rule for an economic union



Optimal rule for an economic union



Optimal rule for an economic union



Optimal rule for low degree of present bias

1L Suppose that the present bias is low:

$$\frac{\theta f'(\theta)}{f(\theta)} \geq -\frac{1}{1-\delta} \quad \text{for} \quad \theta \leq \theta_p, \theta_n$$

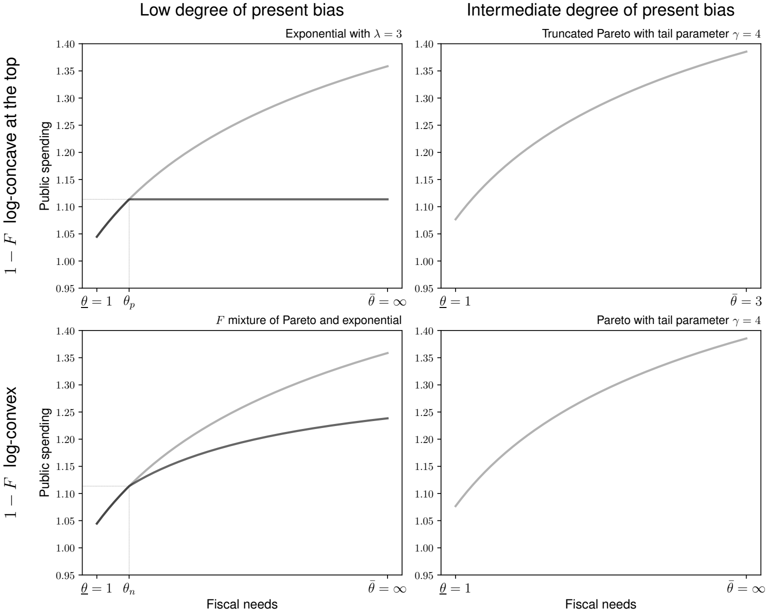
2. Condition above threshold: semi-elasticity of the tail
   2.1 Tail $1 - F$ is log-concave above θ_p
   2.2 Tail $1 - F$ is log-convex and $g_n(\theta) \leq g_d(\theta)$ above θ_n

Proposition 1

Suppose condition 1L (low present bias) is satisfied, and

- condition 2.1 is satisfied, then the optimal rule features discretion and off-equilibrium sanctions
- condition 2.2 is satisfied, then the optimal rule features discretion and on-equilibrium sanctions

Optimal rule for an economic union



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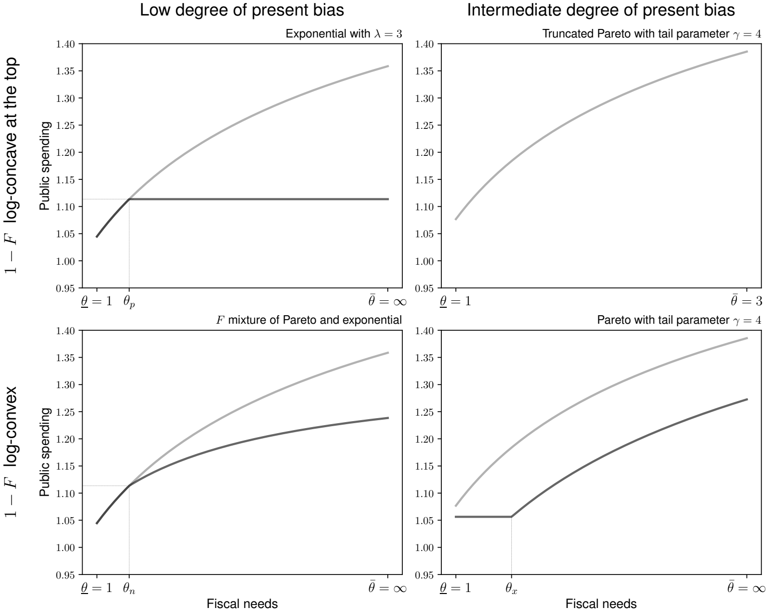
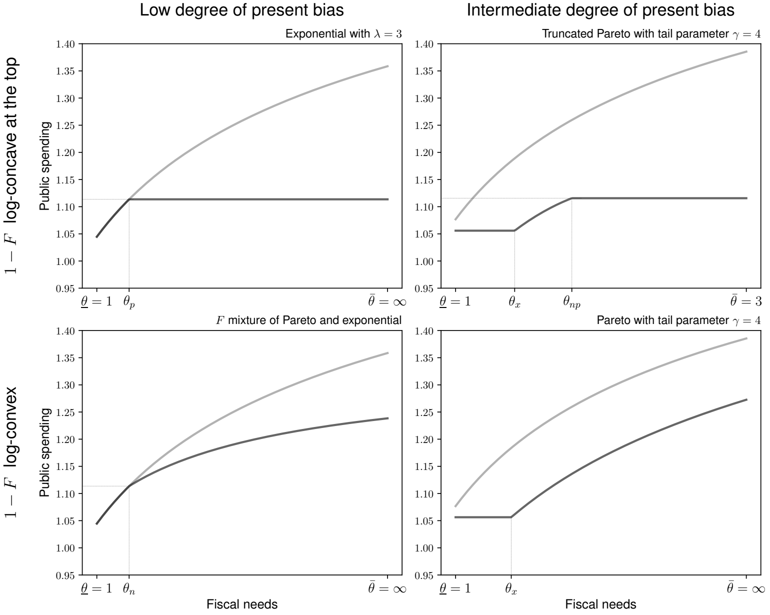


Optimal rule for an economic union



The economics of exemptions

Optimal exemption involves a trade-off:

- loss of discipline on spending below the threshold

+ lower level of sanctions above the threshold

The economics of exemptions

$$\int_{\Theta} \Bigg[\underbrace{\nu(g(\theta))}_{\text{bias}} + \underbrace{\underline{\theta}\; U(g(\underline{\theta})) + \beta\delta W(g(\underline{\theta})) + U(g(\theta))\; \frac{1-F(\theta)}{f(\theta)}}_{\text{government welfare net of sanctions}} \Bigg] dF(\theta)$$

Exemption from sanctions for low spending levels:

- threshold for exemptions θ_x:

$$\underbrace{-\nu'(g_n(\theta_x) + U'(g(\theta_x))\mathbb{E}[\theta - \theta_x | \theta \leq \theta_x]}_{\text{marginal loss discipline}} F(\theta_x) = \underbrace{U'(g(\theta_x))\frac{1-F(\theta_x)}{f(\theta_x)}}_{\text{marginal economy of sanctions}}$$

- economy of sanctions because the *level* is lower with exemptions (the *marginal* sanction above the threshold remains the same)

Optimal rule for high deficit bias

1I Suppose that the deficit bias is *intermediate* in the sense that

$$g_n(\theta) \leq g_d(\theta)$$

2. Condition above threshold: semi-elasticity of the tail
   2.1 Tail $1 - F$ is log-convex up to a point and log-concave above it
   2.1 Tail $1 - F$ is log-convex

Proposition 2

Suppose condition 1.I (intermediate deficit bias) is satisfied, and

- condition 2.1 is satisfied, then the optimal rule is *exemption*, on-equilibrium, and off-equilibrium sanctions.
- condition 2.2 is satisfied, then the optimal rule is *exemption*, and on-equilibrium sanctions.

Qualitative evaluation of the Stability and Growth Pact

1. Below 3% deficit: discretion

2. At 3% deficit: notch in sanctions from 0 to 0.2% of GDP

3. Above 3% deficit: sanctions is a non-interest-bearing deposit, which could be converted into fines

Qualitative evaluation of the Stability and Growth Pact

1. Below 3% deficit: discretion

   Discretion below a threshold is part of the optimal design

2. At 3% deficit: notch in sanctions from 0 to 0.2% of GDP

3. Above 3% deficit: sanctions is a non-interest-bearing deposit, which could be converted into fines

Qualitative evaluation of the Stability and Growth Pact

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   Notch could be optimal if meant as off-equilibrium sanction

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Qualitative evaluation of the Stability and Growth Pact

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   Otherwise, kink is optimal: express sanctions as % of deficit above the threshold, not as a % of GDP

3. Above 3% deficit: sanctions is a non-interest-bearing deposit, which could be converted into fines

Qualitative evaluation of the Stability and Growth Pact

1. Below 3% deficit: discretion

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   Otherwise, kink is optimal: express sanctions as % of deficit above the threshold, not as a % of GDP

3. Above 3% deficit: sanctions is a non-interest-bearing deposit, which could be converted into fines

   If non-interest bearing deposits are not put to best use, then a fine is better (lower cost of sanction for the economic union)

Solution method for the design of rules

1. The design of a rule maps into a mechanism design problem *without transfers*
2. Global Lagrangian method to optimize over all IC allocations while taking the no-transfer constraint into account
3. Monotonicity condition is key to interpret the optimality conditions (same purpose but different from the "ironing" approach)
4. Inspection of the FOCs suggest candidate solutions
5. Use FOCs to derive sufficient conditions under which the candidate rule is optimal

References for the solution method to design a rule

- Chapter 8 in Luenberger (1969). In particular
  - Lemma 1 p.227 for the optimality conditions associated with maximizing the Lagrangian
  - and Theorem 1 p.220 to show that maximizing the Lagangian is sufficient to obtain a solution.
- Lemma A.2 in Amador, Werning, and Angeletos (2006) and Theorem 1 in Amador and Bagwell (2013)
- Sublet (2021) Section 5 and the proofs

Solution method for the design of rules

1. The design of a rule maps into a mechanism design problem *without transfers*

$$\max_{t(\cdot)\geq 0, g(\cdot)} \int \Big[\underbrace{\theta U(g(\theta)) + \delta\beta W(g(\theta))}_{\text{government welfare}} + \underbrace{\nu(g(\theta))}_{\text{bias}} - t(\theta)\Big] dF(\theta).$$

where $g(\cdot)$ is incentive compatible given $(t(\cdot))$.

Lemma (Incentive compatible allocations)

An allocation $(g(\cdot))$ is incentive compatible given a money burning schedule $(t(\cdot))$ if and only if $(g(\cdot))$ is non-decreasing and

$$\begin{aligned} t(\theta) =& \theta\ U(g(\theta)) + \beta\delta W(R(T - g(\theta))) \\ & - \underline{\theta}\ U(g(\underline{\theta})) - \beta\delta W(R(T - g(\underline{\theta}))) - \int_{\underline{\theta}}^{\theta} U(g(\tilde{\theta}))\ d\tilde{\theta}. \end{aligned}$$

Solution method for the design of rules

2. Global Lagrangian method

- Change the variable $u(\theta) \equiv U(g(\theta))$
- Langrangian function $\Lambda : \Theta \mapsto [0, 1]$
  - non-decreasing (analogous to a positive Lagrange multiplier in Khun-Tucker theorem)
  - $\lim_{\theta \to \bar{\theta}} \Lambda(\theta) = 1$ and $1 - \Lambda$ is integrable
- Domain of definition of the Lagrangian:

  $$\Phi \equiv \{(u, \underline{t}) \mid u : \Theta \mapsto \mathbb{R}_+ \text{ is a bounded non-decreasing function}, \underline{t} \in \mathbb{R}_+\}$$

-

  $$\mathcal{L}(u, \underline{t}|\Lambda) \equiv \int \left[\theta U(g(\theta)) + \delta\beta W(g(\theta)) + \nu(g(\theta)) - \hat{t}(u(\theta), \underline{t}(\theta))\right] dF(\theta) - \int \left[\hat{t}(u(\theta), \underline{t}(\theta))\right] d\Lambda(\theta)$$

  where $\hat{t}(u(\theta), \underline{t}(\theta))$ is the $t(\theta)$ associated with $g(\theta) = U^{-1}(u(\theta))$

Solution method for the design of rules

2. Global Lagrangian method

Upon substitution of $\hat{t}$, we get:

$$\begin{aligned}\mathcal{L}(u, \underline{t}|\Lambda) \equiv & \int_{\Theta} \left[u(\theta)\ \tfrac{1-F(\theta)}{f(\theta)} - \tfrac{\nu(U^{-1}(u(\theta)))}{\rho}\right] dF(\theta) \\ & + \left(\underline{\theta}\ u(\underline{\theta}) + \beta\delta W(R(T - U^{-1}(u(\underline{\theta})))) - \underline{t}\right) \Lambda(\underline{\theta}) \\ & + \int_{\Theta} \left[\theta\ u(\theta) + \beta\delta W(R(T - U^{-1}(u(\theta))))\right] d\Lambda(\theta) \\ & - \int_{\Theta} \left[u(\theta)(1 - \Lambda(\theta))\right] d\theta.\end{aligned}$$

Define the Gateaux (in the direction of $h, h_t \in \Phi$) as follows:

$$\partial\mathcal{L}(u, \underline{t}, h, h_t|\Lambda) \equiv \tfrac{d}{d\alpha}\mathcal{L}(u + \alpha h, \underline{t} + \alpha h_t|\Lambda)\Big|_{\alpha=0} ,$$

Lemma (Lemma of optimality)

If there exists a non-decreasing $u^* \equiv U(g^*)$ *and* $\underline{t}^*$ *in the convex cone* Φ *and a non-decreasing function* $\Lambda^* : \Theta \mapsto [0, 1]$ *such that* $\lim_{\theta \to \bar{\theta}} \Lambda^*(\theta) = 1$ *and* $1 - \Lambda^*$ *is integrable, and if*

$$\partial\mathcal{L}(u^*, \underline{t}^*, u^*, \underline{t}^* | \Lambda^*) = 0, \quad \text{and} \quad \partial\mathcal{L}(u^*, \underline{t}^*, h, h_t | \Lambda^*) \leq 0 \quad \text{for all } (h, h_t) \in \Phi,$$

then $g^* \equiv U^{-1}(u^*)$ *and the associated money-burning schedule* t^* *characterized in the lemma with* $t^*(\underline{\theta}) = \underline{t}^*$ *solve the mechanism design problem without transfers formulated above.*

Solution method for the design of rules

3. Monotonicity condition

Monotonicity condition is key to interpret the optimality conditions (same purpose but different from the “ironing” approach).

The monotonicity condition is taken into consideration in considering only the directions $h, h_t \in \Phi$.

Inspection of the FOCs suggest candidate solutions

The condition 1L

$$\frac{\theta f'(\theta)}{f(\theta)} \geq -\frac{1}{1-\delta} \quad \text{for} \quad \theta \leq \theta_p, \theta_n$$

is precisely the condition that guarantees that the Lagrangian is non-decreasing.

Solution method for the design of rules

5. Use FOCs to derive sufficient conditions under which the candidate rule is optimal

Use FOCs to derive sufficient conditions under which the candidate rule is optimal

$$\partial\mathcal{L}(u^*, \underline{t}^*, u^*, \underline{t}^* | \Lambda^*) = 0$$

Equation determining stringency of the cap θ_c:

$$\underbrace{U'(g_d(\theta_c, R))\ \mathbb{E}[\theta - \theta_c | \theta \geq \theta_c]}_{\text{marginal cost limiting discretion}} = \underbrace{-\nu'(g_d(\theta_c))}_{\text{marginal bias}}$$

Solution method for the design of rules

5. Use FOCs to derive sufficient conditions under which the candidate rule is optimal

Use FOCs to derive sufficient conditions under which the candidate rule is optimal

$$\partial\mathcal{L}(u^*, \underline{t}^*, u^*, \underline{t}^* | \Lambda^*) \leq 0$$

If $1 - F$ is log-concave and the equation determining stringency of the cap θ_c is satisfied, i.e.,

$$\partial\mathcal{L}(u^*, \underline{t}^*, u^*, \underline{t}^* | \Lambda^*) = 0$$

then

$$\partial\mathcal{L}(u^*, \underline{t}^*, u^*, \underline{t}^* | \Lambda^*) \leq 0$$

is satisfied.