ECN 7059 Macroéconomie avancée

Lecture 4: Mirrleesian optimal taxation in a dynamic environment

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Lecture 4

- 1.c) Design of policy under information asymmetry : Mirrleesian approach
 - i) Intro to mechanism design and the Revelation Principle
 - ii) Static : envelope/integral condition
 - iii) Dynamic : inverse Euler equation

1) Methods for the design of policy

- Lecture 1 : role for macroeconomic policy
- ► Lecture 2 : dynamic linear tax design (Ramsey)
- Lecture 3 : static non-linear tax design (static Mirrlees)
- Lecture 4 : dynamic non-linear tax design (dynamic Mirrlees)

Readings for this lecture :

- Kocherlakota The New Dynamic Public Finance Chapter 3, 4.
- Rogerson (1985) "Repeated Moral Hazard" Ecta

Methods depend on the stochastic process for shocks

- iid shocks: recursive methods with continuation value as a state variable (more on this in future lectures)
 (References: Atkeson Lucas (1992) ReStud, LS Chapter 21 "Incentives and Insurance")
- Persistence
 - Markov shocks : recursive methods as in Fernandes Phelan (2000) JET
 - General shock structure: perturbation methods as in Kocherlakota The New Dynamic Public Finance Chapter 3, 4. and Rogerson (1985) Ecta

Preview of main takeaway from this lecture

Key finding from optimal taxation in a dynamic setting with private information (Mirrlees): the optimal allocation needs to satisfy the *Inverse* Euler equation

$$\frac{1}{u_t'} = \frac{1}{\beta R} \mathbb{E}_t \left[\frac{1}{u_{t+1}'} \right]$$

Note that if u'_{t+1} is random, this is different from the standard Euler equation $u'_t = \beta \ R \ \mathbb{E}_t \ u'_{t+1}$.

Again, the methodology is the same as in previous lectures. The technical tools are slightly different.

- 1. Describe the environment
- 2. Define competitive equilibrium given non-linear tax policy
- 3. Mirrleesian problem: choose policy to maximise social welfare subject to the constraint that the allocation is a competitive equilibrium allocation given the policy
- Use the revelation principle to derive implementability constraints
- 5. Rewrite the optimal policy problem in primal form
- Solve the primal planner problem: max social welfare subject to resource and incentive constraints. The solution is called the (constrained/incentive) efficient allocation.
- 7. Decentralize the constrained efficient allocation : find an optimal policy that implements the constrained/incentive efficient allocation, it is determined by the ex-post wedge between equilibrium and constrained/incentive efficient FOC.

- 1. Describe the environment
 - ▶ t = 1, ..., T
 - ▶ Unit mass of workers
 - Time varying heterogeneity in their skills-productivity
 - ► Heterogeneity parametrized by sequence $(\theta_t)_{t=1}^T$ and distribution Π
 - utility depends on consumption and labor :

$$\sum_{t=1}^{T} \beta^{t-1} [u(c_t) - v(\ell_t)]$$

where u, v are increasing and u is concave and v is convex

- effective units of labor : $y_t = \theta_t \ \ell_t$
- ightharpoonup initial endowment of capital : $ar{K}_1$

1. Describe the environment

- An allocation (c, y, k) maps a sequence of types $(\theta_t)_{t=1}^T$ into a sequence of consumption, output and capital (measurable w.r.t. $\theta^t \equiv (\theta_1, \dots, \theta_t)$).
- ▶ Production with CRS technology : F(k, y)
- ightharpoonup Capital depreciates at rate δ
- ightharpoonup Labor market wage : w_t
- Capital market r_t

1. Describe the environment

► Feasibility : an allocation is feasible if

$$\mathbb{E}_t\left[c_t(\theta^t) + K_{t+1}\right] \leq (1 - \delta)\mathbb{E}_t[K_t] + F(\mathbb{E}_t[K_t], \mathbb{E}_t[Y_t])$$

- ► Social welfare function : utilitarian for simplicity (analysis generalizes to other social welfare function)
- Government observes *history* of total labor income : $(\theta \ell)^T = (\theta_t \ell_t)_{t=1}^T$
- ▶ Government does not observe skills θ_t or hours worked ℓ_t
- ▶ Government has access to non-linear taxes $\tau((\theta \ell)^t, r_t k_t)$
- ► Kocherlakota NDPF Chapter 2 also has aggregate productivity shocks and government expenditures

1. Describe the environment

Consumer problem:

$$\begin{aligned} \max_{c,y,k} \ & \mathbb{E}_0 \sum_{t=1}^T \ \beta^{t-1} \left[u(c_t(\theta^T)) - v\left(\frac{y_t(\theta^T)}{\theta_t}\right) \right] \\ \text{s.t.} \\ & c_t(\theta^T) + k_{t+1}(\theta^T) + \tau(\underbrace{\left(w_s \ y_s(\theta^T)\right)_{s=1}^t}_{\text{history of labor income}}, r_t k_t(\theta^T)) \\ & \leq (1 - \delta + r_t) \ k_t(\theta^T) + w_t y_t(\theta^T) \quad \text{ for all } t, \theta^T \\ k_1 \leq \mathcal{K}_1 \end{aligned}$$

2. Define competitive equilibrium given non-linear tax policy

Given a government non-linear tax policy τ , an equilibrium in this economy is an allocation (c, y, k) and prices $(r_t, w_t)_{t=1}^T$ such that :

- (c, y, k) solves the consumer problem given prices $(r_t, w_t)_{t=1}^T$
- lacktriangle markets clear at every date $t=1,\ldots,T$

$$\mathbb{E}_{t} \left[c_{t}(\theta^{T}) + K_{t+1}(\theta^{T}) \right]$$

$$\leq (1 - \delta) \mathbb{E}_{t} [K_{t}(\theta^{T})] + F(\mathbb{E}_{t} [K_{t}(\theta^{T})], \mathbb{E}_{t} [Y_{t}(\theta^{T})])$$

the budget of the government is balanced (implied by Walras's law)

3. Mirrleesian problem

The Mirrleesian problem in this dynamic setting is :

$$\begin{aligned} \max_{\tau,c,y,k} \mathbb{E}_0 \sum_{t=1}^{\infty} \ \beta^{t-1} \left[u(c_t(\theta^T)) - v\left(\frac{y_t(\theta^T)}{\theta_t}\right) \right] \\ \text{s.t. there exists } (r_t, w_t)_{t=1}^T \text{ such that} \\ c, y, k \text{ is an equilibrium allocation given } \tau \end{aligned}$$

Exercise: Under what conditions on the tax system is the following true? An equilibrium allocation is incentive compatible.

3. Mirrleesian problem

The Mirrleesian problem in this dynamic setting is :

$$\begin{aligned} \max_{\tau,c,y,k} \mathbb{E}_0 \sum_{t=1}^{\infty} \ \beta^{t-1} \left[u(c_t(\theta^T)) - v\left(\frac{y_t(\theta^T)}{\theta_t}\right) \right] \\ \text{s.t. there exists } (r_t, w_t)_{t=1}^T \text{ such that} \\ c, y, k \text{ is an equilibrium allocation given } \tau \end{aligned}$$

Exercise: Under what conditions on the tax system is the following true? An equilibrium allocation is incentive compatible. Answer: Always (so long as taxes are only a function of an agent's choice of (y,k), not a function of the type θ)

4. Constrained/Incentive efficient allocation

Constrained/incentive efficient allocations maximizes the Social Welfare function subject to the constaints :

- ► Resource feasibility
- Incentive compatibility

The non-linear tax policy $\tau(\cdot,\cdot)$ such that the equilibrium allocation with this tax policy is a constrained/incentive efficient is an **optimal tax schedule**.

4. Constrained/Incentive efficient allocation

For simplicity:

- small open economy
- fixed interest rate R
- asset is a bond, not capital

Note that the conclusions carry to the environment with capital.

Feasibility: An allocation (c, y, k) is feasible if the net present value of consumption is no bigger than the net present value of output

$$\mathbb{E}_0 \sum_{t=1}^T \left(\frac{1}{R} \right)^t \left[c_t(\theta^T) - y_t(\theta^T) \right] \leq 0$$

4. Constrained/Incentive efficient allocation

Exercise in class: Suppose that the allocation features, for all t

$$c_t(\theta) = c_t(\theta')$$
 and $y_t(\theta) < y_t(\theta')$ for some θ, θ'

Why can't the planner implement this allocation even if it is feasible $\mathbb{E}_0 \sum_{t=1}^T \left(\frac{1}{R}\right)^t \left[c_t(\theta^T) - y_t(\theta^T)\right] \leq 0$?

Reporting strategy: A reporting strategy is a mapping from the space of sequence of types θ^T to a sequence of types $\sigma = (\sigma_t(\theta^t))_{t=1}^T$ where σ_t is θ^t -measurable.

Of particular interests are:

- ► Truth Telling strategy : $\sigma_{TT} \equiv \sigma_t(\theta^t) = \theta^t$ for all possible histories θ^t
- Mimicking strategy: we say that the strategy σ has type θ^T mimicks type $\hat{\theta}^T$ if $\sigma(\theta^T) = \hat{\theta}^T$.

4. Constrained/Incentive efficient allocation

Incentive compatibility : An allocation (c, y) is ex-ante incentive compatible if the ex-ante utility from truth telling is no less than the ex-ante utility from mimicking another type

$$V(\sigma_{TT}; c, y; \theta) \ge V(\sigma; c, y, \theta)$$
 for all σ

where

$$V(\sigma; c, y, \theta) = \mathbb{E}_0 \sum_{t=1}^{T} \beta^{t-1} \left[u(c_t(\sigma(\theta^T))) - v\left(\frac{y_t(\sigma(\theta^T))}{\theta_t}\right) \right]$$

4. Constrained/Incentive efficient allocation

Remarks:

► Ex-ante incentive compatibility presumes commitment ex-ante to the strategy since it need not imply :

$$\mathbb{E}_{t} \sum_{j=t}^{T} \beta^{j-1} \left[u(c_{j}(\sigma_{TT}(\theta^{T}))) - v\left(\frac{y_{j}(\sigma_{TT}(\theta^{T}))}{\theta_{t}}\right) \right] \geq \\ \mathbb{E}_{t} \sum_{j=t}^{T} \beta^{j-1} \left[u(c_{j}(\sigma(\theta^{T}))) - v\left(\frac{y_{j}(\sigma(\theta^{T}))}{\theta_{t}}\right) \right]$$

Measurability of a strategy

5. Rewrite the optimal policy problem in primal form

$$\max_{c,y} \mathbb{E}_0 \sum_{t=1}^{T} \beta^{t-1} \left[u(c_t(\theta^T)) - v\left(\frac{y_t(\theta^T)}{\theta_t}\right) \right]$$

subject to:

c, y is a feasible and incentive compatible allocation

5. Rewrite the optimal policy problem in primal form

$$\begin{aligned} \max_{c,y} \ \mathbb{E}_0 \sum_{t=1}^T \ \beta^{t-1} \left[u(c_t(\theta^T)) - v\left(\frac{y_t(\theta^T)}{\theta_t}\right) \right] \\ \text{subject to} \\ \mathbb{E}_0 \sum_{t=1}^T \ \left(\frac{1}{R}\right)^t \left[c_t(\theta^T) - y_t(\theta^T) \right] \leq 0 \\ V(\sigma_{TT}; c, y; \theta) \geq V(\sigma; c, y, \theta) \quad \text{for all} \ \ \sigma \end{aligned}$$

6. Solve the primal problem

Remarks on the full information case:

$$u'(c_t(\theta^T)) = \beta R u'(c_{t+1}(\theta^T))$$
 (Euler equation)

- ▶ although c may vary with time, it is constant for all θ^T : households are fully insured
- ▶ the inverse Euler equation also holds since the allocation is deterministic :

$$\frac{1}{u'(c_t(\theta^T))} = \frac{1}{\beta R} \frac{1}{u'(c_{t+1}(\theta^T))} \quad \text{(Inverse Euler equation)}$$

with private information, the allocation is not deterministic and we will show that Inverse Euler equation holds but the standard Euler equation does *not* hold. It is by comparing the inverse Euler and the Euler equations and using Jensen's inequality that we can infer wether the optimal tax system should discourage savings or not.

6. Solve the primal problem

Perturbation method : at an optimum, the Social Welfare function must not increase for any perturbation that keeps the allocation in the set of incentive feasible allocations.

- use perturbation methods since the shock process need not be Markovian
- perturbation methods consist in perturbing the utility delivered
 - set of incentive compatible allocations need not be a convex set
 - incentive constraints are linear in utility space : characterize a convex set of utilities from incentive compatible allocations

6. Solve the primal problem

Perturbation method : Suppose (c^*, y^*) is a constrained/incentive efficient allocation.

Consider a perturbation of utility $\epsilon: \theta^t \to \mathbb{R}$ and Δ a constant :

$$u(c_t'(\theta^T)) = u(c_t^*(\theta^T)) + \Delta + \epsilon(\theta^t)$$

$$u(c_{t+1}'(\theta^T)) = u(c_{t+1}^*(\theta^T)) - \frac{1}{\beta}\epsilon(\theta^t)$$

The perturbation does not violate resource feasibility :

$$\mathbb{E}_t \left[c_t'(\theta^T) - c_t^*(\theta^T) \right] + \frac{1}{R} \mathbb{E}_t \left[c_{t+1}'(\theta^T) - c_{t+1}^*(\theta^T) \right] \leq 0$$

6. Solve the primal problem

Perturbation method: Remarks:

$$u(c'_t(\theta^T)) = u(c^*_t(\theta^T)) + \Delta + \epsilon(\theta^t)$$

$$u(c'_{t+1}(\theta^T)) = u(c^*_{t+1}(\theta^T)) - \frac{1}{\beta}\epsilon(\theta^t)$$

- $ightharpoonup \Delta$ is the utility (not allocation) change which is the same for all possible type realizations
- $ightharpoonup \epsilon(\theta^t)$ reallocate resources so that the perturbation Δ is resource-feasible. $\epsilon(\theta^t)$ has no effect on lifetime utility.

Why not use Lagrangian techniques? Non-convexity of the incentive-feasible constraint set in the allocation space. Incentive constraints are linear in the utility space.

6. Solve the primal problem

Claim: If (c^*, y^*) is incentive compatible and resource feasible, then the perturbed allocation (c', y^*) is also incentive compatible and resource feasible.

Proof: Since

$$V(\sigma; c', y^*) - V(\sigma, c^*, y^*) = \Delta$$

the ranking of mimicking strategies is unchanged.

6. Solve the primal problem

Intuition : If c^*, y^* is constrained efficient, there must be no $\Delta \neq 0$ such that the resulting perturbed allocation c', y^* delivers more utility.

Exercise: Inverse Euler Equation

Derive FOC of the problem and show that the following must hold :

Inverse Euler Equation

$$\frac{1}{u'(c_t^*(\theta^T))} = \frac{1}{\beta R} \mathbb{E}_t \left[\frac{1}{u'(c_{t+1}^*(\theta^T))} \right]$$

Intuition:

marginal cost of(
$$u_t$$
) = $\frac{1}{\beta \; R} \; \mathbb{E}_t$ [marginal cost of(u_{t+1})]

6. Solve the primal problem

Lessons from the Inverse Euler Equation :

- 1. the allocation at t matters for the allocation at t+1; that is there is memory in the optimal allocation.
 - ▶ Suppose that β R = 1.
 - lacktriangle The reciprocal of the marginal utility 1/u' is a martingale

$$\frac{1}{u'(c_t^*(\theta^T))} = \mathbb{E}_t \left[\frac{1}{u'(c_{t+1}^*(\theta^T))} \right]$$

- If $\frac{1}{u'(c_i^*(\theta^T))}$ augments by 1 unit because of a shock, the expected reciprocal of the marginal utility also increases by 1 unit for every future period.
- This is different from the complete information case where consumption and hence the reciprocal of marginal utility is constant.

7. Intuition for decentralization

Lessons from the Inverse Euler Equation (continued) :

2. At a constrained efficient allocation, the agent has an incentive to save almost surely

Exercise in class: show this using the Inverse Euler Equation. Hint: use Jensen's inequality.

3. Lesson: a non-linear tax system that implements the constrained efficient allocation must discourage savings.

7. Intuition for decentralization

Intuition for the need to discourage savings (as implied inverse Euler equation) :

- ► Suppose that we implement the optimal labor income tax found by treating each period as a static economy.
- As seen in Lecture 3, the tradeoff between incentives and insurance limit the extent of insurance provided by the optimal non-linear labor income tax.
- ▶ In the face of some uninsurable risk, households engage in precautionary savings to do their best to self-insure.
- ▶ While this self-insurance is privately optimal for the household, it is socially suboptimal since it makes it harder for the government to use the non-linear labor income tax to redistribute and provide insurance. Indeed, the richer households are, the flatter is marginal utility, and the harder it is to provide incentives.
- ► The optimal capital income tax discourages savings to counteract the extent to which self-insurance is socially

4. Constrained/Incentive efficient allocation

- We derived the Inverse Euler Equation in a fairly general setting with an arbitrary stochastic process for skills θ .
- Note that consumption and the utility from consumption are both publicly observable. The utility in a given period t is

$$u(c_t) - v\left(\frac{y_t}{\theta_t}\right)$$

▶ The Inverse Euler Equation holds for economic environments where consumption and the utility from consumption are publicly observable.

4. Constrained/Incentive efficient allocation

The Inverse Euler Equation need *not* hold in economic environments where consumption or the utility of consumption is not publicly observable.

Examples where the Inverse Euler Equation need not hold :

- households can save and savings are not observed by the government: then consumption (and utility) is a function of savings as well
 - Reference: LS Chapter 21 "Incentives and Insurance".
- households receive shocks to the utility of consumption or shocks to their discount factor

$$\theta_t u(c_t) - v(y_t)$$

Reference: Atkeson and Lucas (1992) ReStud

Remarks

- Methods: iid (recursive techniques as in Atkeson Lucas (1992)), Markov (recursive techniques as in Fernandes Phelan (2000)), more general (perturbation techniques)
- How does the insights from Ramsey's optimal taxation result differ from Dynamic Mirrlees approach used in the New Dynamic Public Finance
 - Ramsey studies homogenous agents
 - Dynamic Mirrlees studies heterogenous agents subject to idiosyncratic risk
 - Ramsey finds zero capital tax in the long run (or also short run with Power utility)
 - Dynamic Mirrlees finds zero "average" capital tax but the tax rate depends on the history of labor income to make capital less of an insurance device and discourage savings so that agents with less savings are easier to incentivize in the future.

Quantitative exploration of inverse Euler equation

Reading : Farhi Werning (2012) "Capital Taxation : Quantitative Explorations of the Inverse Euler Equation" *JPE*

What are the benefits of discouraging savings as prescribed by the Inverse Euler Equation?

- benefits are measured in terms of the reduction in resource costs of achieving a given level of utility v
- compare :
 - cost of delivering utility level v in an economy with access to a risk-free bond and no capital income tax where :

$$u'(c_t^*(\theta^T)) = \beta R \mathbb{E}_t \left[u'(c_{t+1}^*(\theta^T)) \right]$$

cost of delivering utility level v in an economy with access to a risk-free bond and capital income tax such that :

$$\frac{1}{u'(c_t^*(\theta^T))} = \frac{1}{\beta R} \mathbb{E}_t \left[\frac{1}{u'(c_{t+1}^*(\theta^T))} \right]$$

Numerical exercise in two period economy

- t = 0, 1
- Unit mass of workers
- heterogeneity in their skills-productivity at time t = 1
- \blacktriangleright Skill shocks θ log-normally and independently distributed across agents with mean normalized to 1
- utility depends on consumption and labor :

$$v = u(c_0) + \beta \mathbb{E}[u(c_1) - v(n_1; \theta)]$$

where u, v is increasing and concave, convex Separability important for Inverse Euler Equation result

► Single crossing : $\frac{\partial v(n_1;\theta)}{\partial n_1}$ is strictly decreasing in θ

Numerical exercise in two period economy

- ▶ technology is linear which pins down the return on savings : $R = \frac{1}{q}$
- ▶ Production with CRS technology : F(k, y)
- c(u) denote the consumption needed to deliver u units of utility so $c(\cdot) = u^{-1}(\cdot)$
- ▶ change of variable (allocation is u_0, u_1, n_1):
 - usually : fix endowment k₀ and see what level of utility v can be achieved
 - here : fix level of utility to achieve v and see the level of endowment needed k₀

$$k_0 = c(u_0) + q \mathbb{E}[c(u_1(\theta)) - n_1(\theta)]$$

Numerical exercise in two period economy

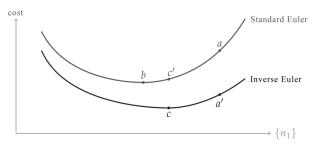


Fig. 1.—Efficiency gains as a function of η_1 for given v. Upper curve imposes free savings. Lower curve implements the inverse Euler equation with optimal savings distortions.

- a no taxes (Euler Equation)
- b optimal non-linear labor income tax, no capital income taxe
- c optimal non-linear labor and capital income tax
- ightharpoonup a' is a with capital income tax (Inverse Euler Equation)
- ightharpoonup c' is c without optimal capital income taxe

Numerical exercise in two period economy

- $ightharpoonup u(c) = \ln(c) \text{ and } V(n,\theta) = \kappa \left(\frac{n}{\theta}\right)^{1+1/\epsilon}$
- ▶ Frisch elasticity set to $\epsilon = \frac{1}{2}$
- ho $eta = 0.96^{20}$: 2 periods so a period is half of working life pprox 20 years
- ▶ $\ln(\theta) \sim N(0, 20 \times 0.0161)$ where 0.0161 is variance of the innovations in the permanent component of income estimated by Storesletten, Telmer and Yaron (2004).

Numerical exercise in two period economy

TABLE 1
EFFICIENCY GAINS FOR AN EXAMPLE ECONOMY

| Policy Experiment | Efficiency Gain (%) |
|--------------------|---------------------|
| $a \rightarrow c$ | 3.08 |
| $b \rightarrow c$ | .33 |
| $c' \rightarrow c$ | .39 |
| $a \rightarrow a'$ | 1.38 |

Note.—Gains are expressed as a percentage of aggregate consumption in autarky.

- a no taxes (Euler Equation)
- b optimal non-linear labor income tax, no capital income taxe
- c optimal non-linear labor and capital income tax
- ▶ a' is a with capital income tax (Inverse Euler Equation)
- ightharpoonup c' is c without optimal capital income taxe

Lecture 4 : recap

- 1.c) Design of policy under information asymmetry : Mirrleesian approach
 - i) Intro to mechanism design and the Revelation Principle
 - ii) Static : envelope/integral condition
 - iii) Dynamic : inverse Euler equation

Lessons for optimal non-linear capital taxation

- 1. Inverse Euler Equation tells us that asset income tax needs to deter savings
- 2. Asset income tax is a non-trivial function of labor income
- Discourage savings by making capital a worse hedge against risk
- 4. Kocherlakota (2005) New Dynamic Public Finance for
 - optimal bequest subsidy
 - individual Ricardian equivalence and social security

Optimal non-linear taxation : overview

- ► Labor income tax :
 - static economy : (see lecture 2)

$$\frac{T'}{1-T'} = \left(1 + \frac{1}{\epsilon}\right) \frac{1-F}{\theta f} \frac{G-F}{1-F}$$

- dynamic economy : see Farhi Werning (2013) 'Insurance and Taxation over the Life-Cycle' ReStud
- Capital income tax :
 - dynamic economy : (see lecture 4) Inverse Euler Equation

$$\frac{1}{u'(c_t^*(\theta^T))} = \frac{1}{\beta R} \mathbb{E}_t \left[\frac{1}{u'(c_{t+1}^*(\theta^T))} \right]$$

What are the benefits of discouraging savings as prescribed by the Inverse Euler Equation?

See Farhi Werning (2012) 'Capital Taxation : Quantitative Explorations of the Inverse Euler Equation' *JPE*

Further readings

- ► Farhi Werning (2013) "Insurance and Taxation over the Life Cycle" *ReStud* with code online on Ivan Werning's webpage (link)
- ▶ Boerma (2020) "Housing Policy Reform"
- ▶ Ndiaye (2020) "Flexible Retirement and Optimal Taxation"
- ▶ Doepke Townsend (2006) "Dynamic Mechanism Design with Hidden Income and Hidden Actions" JET with code online (link)