

Assignment or Homework

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Template based on my answers for Special Assignments 1 and 2 of ODS FS2020.

We start by showing some claims that will be reused in several assignments.

Claim 1. claim

Proof. proof □

Claim 2. claim

Proof. proof □

We will also allow ourselves to use the following standard property of Gaussian distributions.

Fact 1. Let $M, m \in \mathbb{N}$ and $g \sim \mathcal{N}(0, \Sigma)$ for any positive semi-definite matrix $\Sigma \in \mathbb{R}^{m \times m}$, and let any matrix $A \in \mathbb{R}^{M \times m}$ independent from g . Then $Ag \sim \mathcal{N}(0, A\Sigma A^T)$.

Proof. By known properties of Gaussian distributions, Ag is Gaussian and

$$\mathbb{E}Ag = A\mathbb{E}g = 0 \tag{0.1}$$

$$\mathbb{E}(Ag)(Ag)^T = \mathbb{E}Ag g^T A^T \tag{0.2}$$

$$= A (\mathbb{E}g g^T) A^T \tag{0.3}$$

$$= A\Sigma A^T \tag{0.4}$$

by linearity of expectation. So $Ag \sim \mathcal{N}(0, A\Sigma A^T)$. □

As a special case, we have a classic "isotropy" result:

Fact 2. Let $m \in \mathbb{N}$ and $g \sim \mathcal{N}(0, \sigma^2 I_m)$ for some $\sigma > 0$, and let $u \in \mathbb{R}^m$ be a vector independent from g . Then $\langle g, u \rangle = u^T g \sim \mathcal{N}(0, \sigma^2 \|u\|^2)$.

First part of the homework

Assignment 1

My answer to the question

By Claim 1, ...

Assignment 2

We consider the following algorithm, where N is to be specified and $\kappa = 2\sqrt{2}$.

Algorithm 1: Repeated Accelerated Gradient Descent

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Start with  $x_0$  s.t  $\|x_0 - x^*\| \leq R$ 
for  $k = 0 \dots N - 1$  do
    | run AGD for  $T = \kappa \sqrt{\frac{L}{\mu}}$  iterations starting at  $x_k$ 
    | let  $x_{k+1}$  the result
end
return  $x_N$ 
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We now prove that for $N \geq \log_2 \frac{\mu R^2}{2\varepsilon}$, x_N is an ε -optimal solution.
Indeed, according to the previous question, in Algorithm 1...

Assignment 3

My answer to the question

Second part of the homework

Assignment 4

My answer to the question

Assignment 5

My answer to the question