

# Assignment or Homework #

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*Template based on my answers for Special Assignments 1 and 2 of ODS FS2020.*

We start by showing some claims that will be reused in several assignments.

**Claim 1.** claim

*Proof.* proof

□

**Claim 2.** claim

*Proof.* proof

□

We will also allow ourselves to use the following standard property of Gaussian distributions.

**Fact 1.** Let  $M, m \in \mathbb{N}$  and  $g \sim \mathcal{N}(0, \Sigma)$  for any positive semi-definite matrix  $\Sigma \in \mathbb{R}^{m \times m}$ , and let any matrix  $A \in \mathbb{R}^{M \times m}$  independent from  $g$ . Then  $Ag \sim \mathcal{N}(0, A\Sigma A^T)$ .

*Proof.* By known properties of Gaussian distributions,  $Ag$  is Gaussian and

$$\mathbb{E}Ag = A\mathbb{E}g = 0 \tag{0.1}$$

$$\mathbb{E}(Ag)(Ag)^T = \mathbb{E}Ag g^T A^T \tag{0.2}$$

$$= A (\mathbb{E}g g^T) A^T \tag{0.3}$$

$$= A\Sigma A^T \tag{0.4}$$

by linearity of expectation. So  $Ag \sim \mathcal{N}(0, A\Sigma A^T)$ .

□

As a special case, we have a classic "isotropy" result:

**Fact 2.** Let  $m \in \mathbb{N}$  and  $g \sim \mathcal{N}(0, \sigma^2 I_m)$  for some  $\sigma > 0$ , and let  $u \in \mathbb{R}^m$  be a vector independent from  $g$ . Then  $\langle g, u \rangle = u^T g \sim \mathcal{N}(0, \sigma^2 \|u\|^2)$ .

## First part of the homework

### Assignment 1

My answer to the question

By Claim 1, ...

## Assignment 2

We consider the following algorithm, where  $N$  is to be specified and  $\kappa = 2\sqrt{2}$ .

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**Algorithm 1:** Repeated Accelerated Gradient Descent

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Start with  $x_0$  s.t  $\|x_0 - x^*\| \leq R$ 
for  $k = 0 \dots N - 1$  do
    | run AGD for  $T = \kappa \sqrt{\frac{L}{\mu}}$  iterations starting at  $x_k$ 
    | let  $x_{k+1}$  the result
end
return  $x_N$ 
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We now prove that for  $N \geq \log_2 \frac{\mu R^2}{2\varepsilon}$ ,  $x_N$  is an  $\varepsilon$ -optimal solution.  
Indeed, according to the previous question, in algorithm 1...

## Assignment 3

My answer to the question

## Second part of the homework

### Assignment 4

My answer to the question

### Assignment 5

My answer to the question