Assignment or Homework

Student ID: **-***-***

May 31, 2020

Template based on my answers for Special Assignments 1 and 2 of ODS FS2020.

We start by showing some claims that will be reused in several assignments.

Claim 1. claim

Proof. proof ☐

Claim 2. claim

Proof. proof ☐

We will also allow ourselves to use the following standard property of Gaussian distributions.

Fact 1. Let $M, m \in \mathbb{N}$ and $g \sim \mathcal{N}(0, \Sigma)$ for any positive semi-definite matrix $\Sigma \in \mathbb{R}^{m \times m}$, and let any matrix $A \in \mathbb{R}^{M \times m}$ independent from g. Then $Ag \sim \mathcal{N}(0, A\Sigma A^T)$.

Proof. By known properties of Gaussian distributions, Ag is Gaussian and

$$\mathbb{E}Ag = A\mathbb{E}g = 0 \tag{0.1}$$

$$\begin{aligned}
\mathbb{E}(Ag)(Ag)^T &= \mathbb{E}Agg^T A^T && (0.2)\\
&= A\left(\mathbb{E}gg^T\right) A^T && (0.3)\\
&= A\Sigma A^T && (0.4)
\end{aligned}$$

by linearity of expectation. So $Ag \sim \mathcal{N}(0, A\Sigma A^T)$. ☐

As a special case, we have a classic "isotropy" result:

Fact 2. Let $m \in \mathbb{N}$ and $g \sim \mathcal{N}(0, \sigma^2 I_m)$ for some $\sigma > 0$, and let $u \in \mathbb{R}^m$ be a vector independent from g. Then $\langle g, u \rangle = u^T g \sim \mathcal{N}(0, \sigma^2 \|u\|^2)$.

First part of the homework

Assignment 1

My answer to the question

By Claim 1, ...

Assignment 2

We consider the following algorithm, where N is to be specified and $\kappa = 2\sqrt{2}$.

Algorithm 1: Repeated Accelerated Gradient Descent

Start with x_0 s.t $\|x_0 - x^*\| \leq R$
for $k = 0...N-1$ **do**
  run AGD for $T = \kappa\sqrt{\frac{L}{\mu}}$ iterations starting at x_k
  let x_{k+1} the result
end
return x_N

We now prove that for $N \geq \log_2 \frac{\mu R^2}{2\varepsilon}$, x_N is an ε-optimal solution.
Indeed, according to the previous question, in algorithm 1...

Assignment 3

My answer to the question

Second part of the homework

Assignment 4

My answer to the question

Assignment 5

My answer to the question