

# Convergence Theorems

- A positive recurrent Markov chain  $T$  has a stationary distribution.
- If  $T$  is irreducible and has a stationary distribution, then it is unique and

$$\pi_i = \frac{1}{m_i}$$

where  $m_i$  is the mean return time of state  $i$ .

- If  $T$  is irreducible, aperiodic and has stationary distribution  $\pi$  then

$$\mathbb{P}(X_n = i) \rightarrow \pi_i \quad \text{as } n \rightarrow \infty$$

- (Ergodic Theorem): If  $T$  is irreducible with stationary distribution  $\pi$  then

$$\frac{\#\{t \leq n : X_t = i\}}{n} \rightarrow \pi_i \quad \text{as } n \rightarrow \infty$$