

# Metropolis-Hastings algorithm

- MH algorithm is a generic method to generate a  $\pi$ -reversible Markov chain.
- Introduce proposal density: for any  $\theta, \theta' \in \Theta$  we have  $q(\theta' | \theta) \geq 0$  and  $\int_{\mathbb{X}} q(\theta' | \theta) d\theta' = 1$ ; e.g.  $q(\theta' | \theta) = \mathcal{N}(\theta'; \theta, \Sigma)$ .
- Starting with  $\theta^{(1)}$  iterate for  $i = 2, 3, \dots$

❶ Sample  $\theta^* \sim q(\cdot | \theta^{(i-1)})$ .

❷ Compute

$$\alpha(\theta^* | \theta^{(i-1)}) = \min \left( 1, \frac{\pi(\theta^*) q(\theta^{(i-1)} | \theta^*)}{\pi(\theta^{(i-1)}) q(\theta^* | \theta^{(i-1)})} \right)$$

❸ Sample  $U \sim \mathcal{U}_{[0,1]}$ . If  $U \leq \alpha(\theta^* | \theta^{(i-1)})$  then set  $\theta^{(i)} = \theta^*$ , otherwise set  $\theta^{(i)} = \theta^{(i-1)}$ .