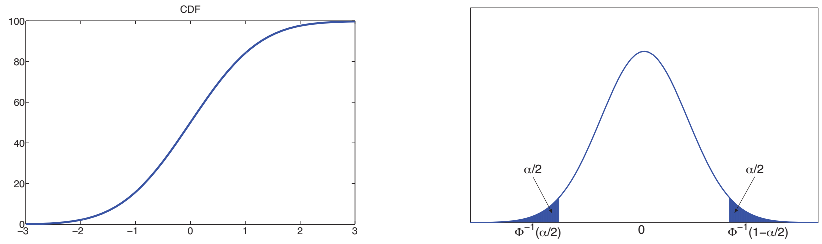

(a) (b)

Figure 2.3 (a) Plot of the cdf for the standard normal, $\mathcal{N}(0, 1)$. (b) Corresponding pdf. The shaded regions each contain $\alpha/2$ of the probability mass. Therefore the nonshaded region contains $1 - \alpha$ of the probability mass. If the distribution is Gaussian $\mathcal{N}(0, 1)$, then the leftmost cutoff point is $\Phi^{-1}(\alpha/2)$, where Φ is the cdf of the Gaussian. By symmetry, the rightost cutoff point is $\Phi^{-1}(1 - \alpha/2) = -\Phi^{-1}(\alpha/2)$. If $\alpha = 0.05$, the central interval is 95%, and the left cutoff is -1.96 and the right is 1.96. Figure generated by `quantileDemo`.

2.2.6 Quantiles

Since the cdf F is a monotonically increasing function, it has an inverse; let us denote this by F^{-1}. If F is the cdf of X, then $F^{-1}(\alpha)$ is the value of x_α such that $P(X \leq x_\alpha) = \alpha$; this is called the α **quantile** of F. The value $F^{-1}(0.5)$ is the **median** of the distribution, with half of the probability mass on the left, and half on the right. The values $F^{-1}(0.25)$ and $F^{-1}(0.75)$ are the lower and upper **quartiles**.

We can also use the inverse cdf to compute **tail area probabilities**. For example, if Φ is the cdf of the Gaussian distribution $\mathcal{N}(0, 1)$, then points to the left of $\Phi^{-1}(\alpha)/2)$ contain $\alpha/2$ probability mass, as illustrated in Figure 2.3(b). By symmetry, points to the right of $\Phi^{-1}(1-\alpha/2)$ also contain $\alpha/2$ of the mass. Hence the central interval $(\Phi^{-1}(\alpha/2), \Phi^{-1}(1 - \alpha/2))$ contains $1 - \alpha$ of the mass. If we set $\alpha = 0.05$, the central 95% interval is covered by the range

$$(\Phi^{-1}(0.025), \Phi^{-1}(0.975)) = (-1.96, 1.96) \tag{2.23}$$

If the distribution is $\mathcal{N}(\mu, \sigma^2)$, then the 95% interval becomes $(\mu - 1.96\sigma, \mu + 1.96\sigma)$. This is sometimes approximated by writing $\mu \pm 2\sigma$.

2.2.7 Mean and variance

The most familiar property of a distribution is its **mean**, or **expected value**, denoted by μ. For discrete rv's, it is defined as $\mathbb{E}[X] \triangleq \sum_{x \in \mathcal{X}} x\ p(x)$, and for continuous rv's, it is defined as $\mathbb{E}[X] \triangleq \int_{\mathcal{X}} x\ p(x) dx$. If this integral is not finite, the mean is not defined (we will see some examples of this later).

The **variance** is a measure of the "spread" of a distribution, denoted by σ^2. This is defined