Random Walk Metropolis Hastings

- In the Metropolis-Hastings, pick $q\left(\theta' \middle| \theta\right) = g\left(\theta' - \theta\right)$ with g being a *symmetric* distribution, thus

$$\theta^* = \theta^{(i-1)} + \varepsilon, \qquad \varepsilon \sim g;$$

  e.g. g is a zero-mean multivariate normal or t-student.

- Acceptance probability becomes

$$\min\left(1, \frac{\pi\left(\theta^*\right) g\left(\theta^{(i-1)} - \theta^*\right)}{\pi\left(\theta^{(i-1)}\right) g\left(\theta^* - \theta^{(i-1)}\right)}\right) = \min\left(1, \frac{\pi\left(\theta^*\right)}{\pi\left(\theta^{(i-1)}\right)}\right).$$

- We accept...
  - every move to a more probable state with probability 1.
  - moves to less probable states with probability $\pi\left(\theta^*\right) / \pi\left(\theta^{(i-1)}\right) < 1$.