



# **COMPARACIÓN DE PREFERENCIAS**

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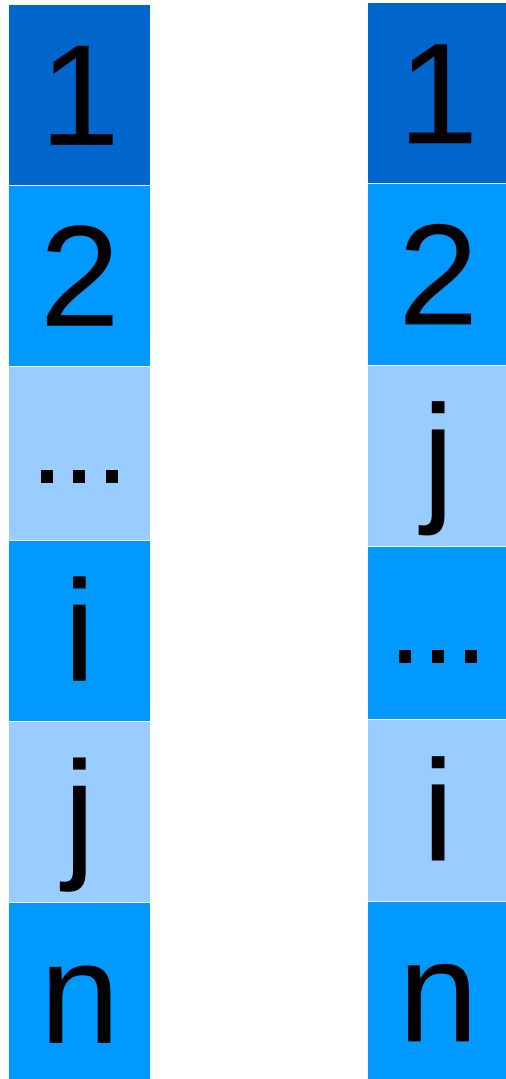
**Pablo Álvarez Cabrera**

**Adolfo Soto Werner**

**Cristina de la Torre Villaverde**

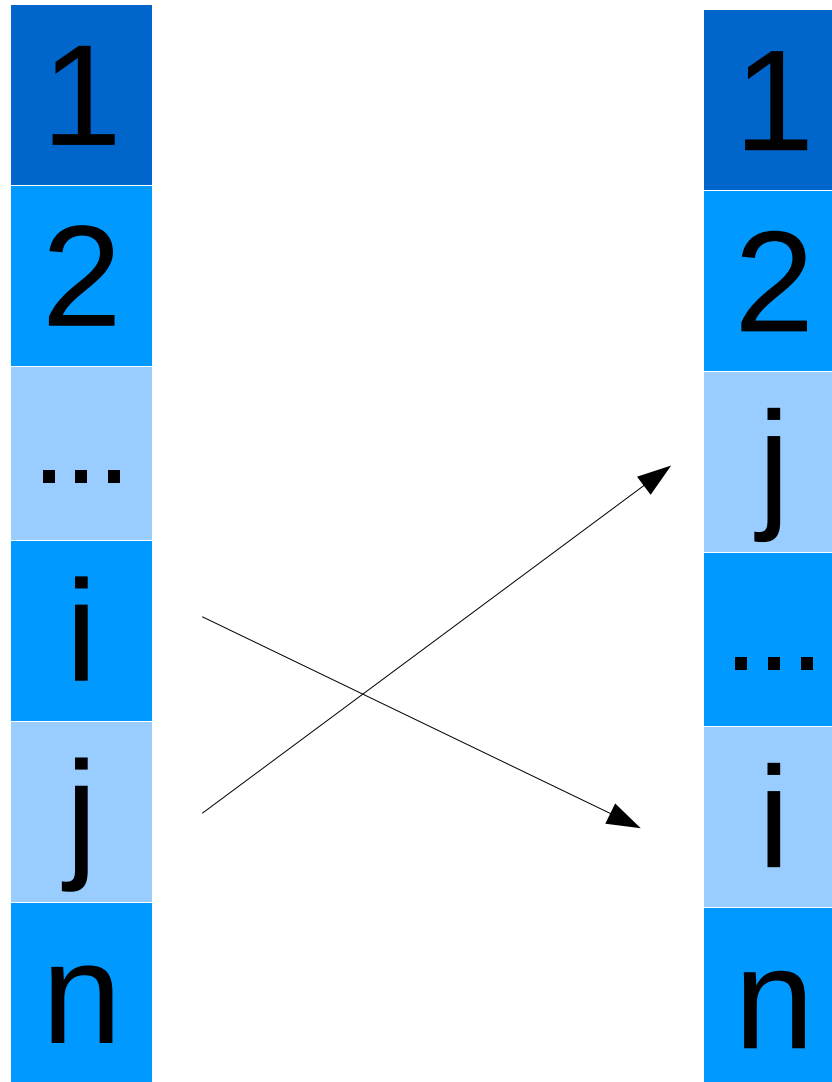


# Problema: comparación de rankings



Para resolver este problema vamos a contabilizar el número de inversiones entre los distintos elementos de dos rankings, estos elementos son los mismos en ambos casos pero puede haber diferencias en el orden en el que aparecen los mismos.

# Problema: comparación de rankings



Una inversión es un cambio de posición entre dos elementos.

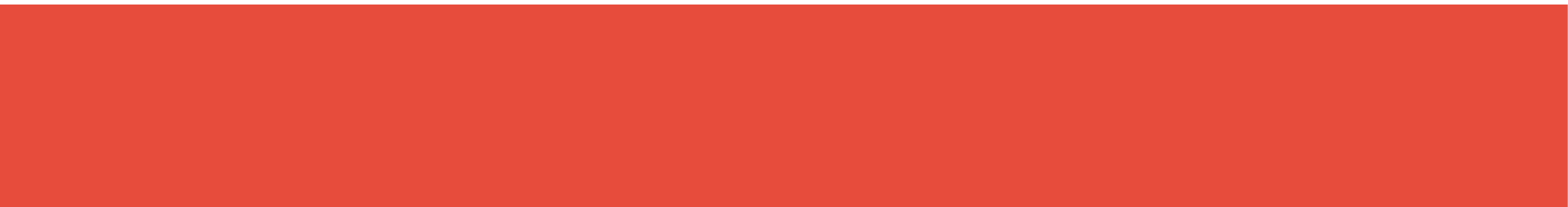
Para representar los rankings, utilizamos dos vectores de enteros.

Suponemos el primero ordenado del 1 al  $n$ .



# ALGORITMO DE FUERZA BRUTA





0	1	2	3	4	5
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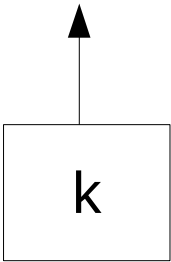
3	0	2	4	1	5
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0	1	2	3	4	5
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3	0	2	4	1	5
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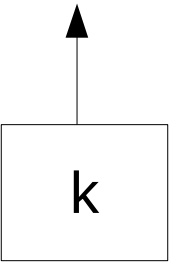
Pos i	Pos j





0	1	2	3	4	5
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3	0	2	4	1	5
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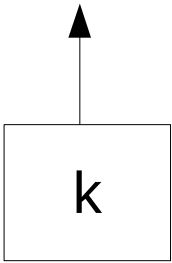
Pos i	Pos j
1	





0	1	2	3	4	5
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3	0	2	4	1	5
---	---	---	---	---	---



Pos i	Pos j
1	

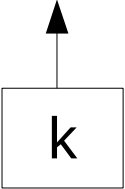






0	1	2	3	4	5
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3	0	2	4	1	5
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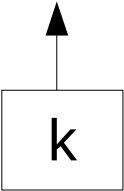
Pos i	Pos j
1	





0	1	2	3	4	5
---	---	---	---	---	---

3	0	2	4	1	5
---	---	---	---	---	---



Pos $i$	Pos $j$
1	4



i

j

Inversiones
0

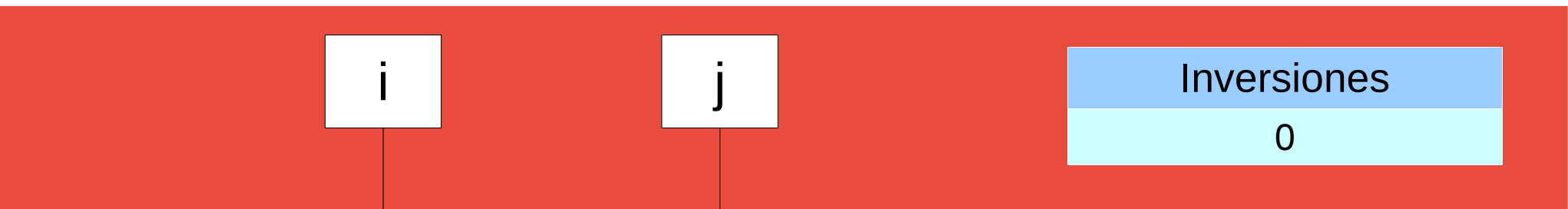
0	1	2	3	4	5
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3	0	2	4	1	5
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¿Pos j < pos i?

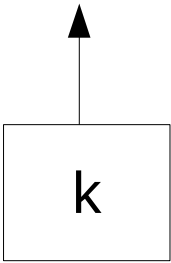
Inversiones + 1

Pos i	Pos j
1	4



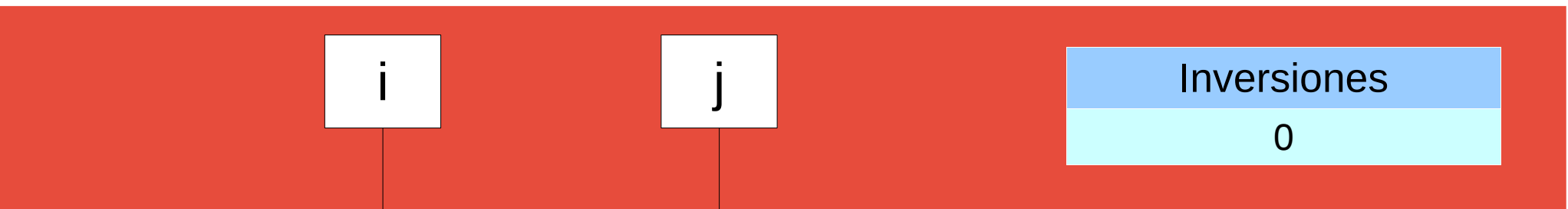
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3	0	2	4	1	5
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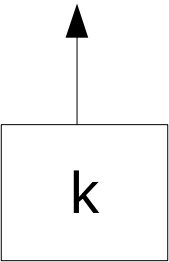
Pos i	Pos j





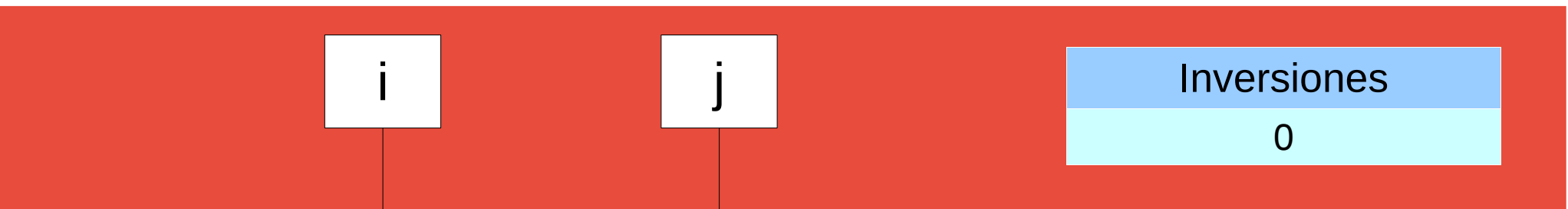
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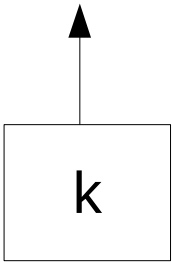
Pos i	Pos j
1	





0	1	2	3	4	5
---	---	---	---	---	---

3	0	2	4	1	5
---	---	---	---	---	---



Pos i	Pos j
1	2



i

j

Inversiones
0

0	1	2	3	4	5
---	---	---	---	---	---

3	0	2	4	1	5
---	---	---	---	---	---

¿Pos j < pos i?

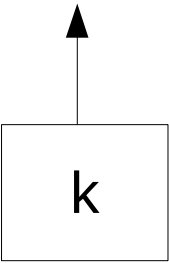
Inversiones + 1

Pos i	Pos j
1	2



0	1	2	3	4	5
---	---	---	---	---	---

3	0	2	4	1	5
---	---	---	---	---	---



Pos i	Pos j
	0

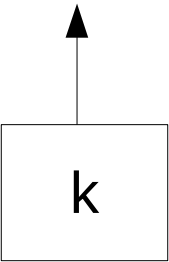






0	1	2	3	4	5
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3	0	2	4	1	5
---	---	---	---	---	---



Pos i	Pos j
1	0



i

j

Inversiones

1

0	1	2	3	4	5
---	---	---	---	---	---

3	0	2	4	1	5
---	---	---	---	---	---

¿Pos j < pos i?

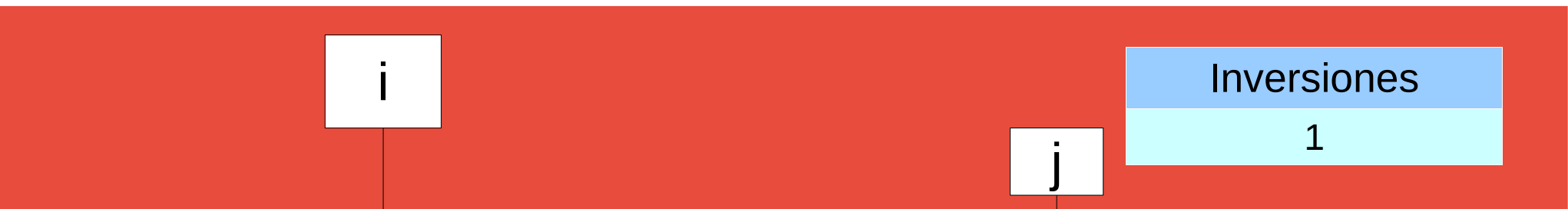
Inversiones + 1

Pos i

1

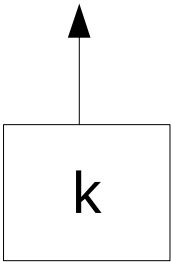
Pos j

0



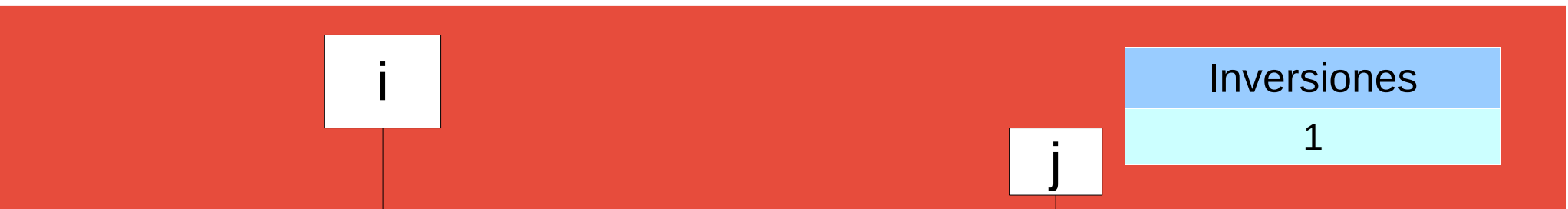
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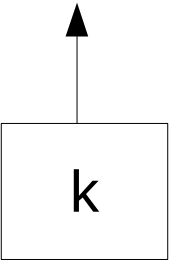
Pos i	Pos j





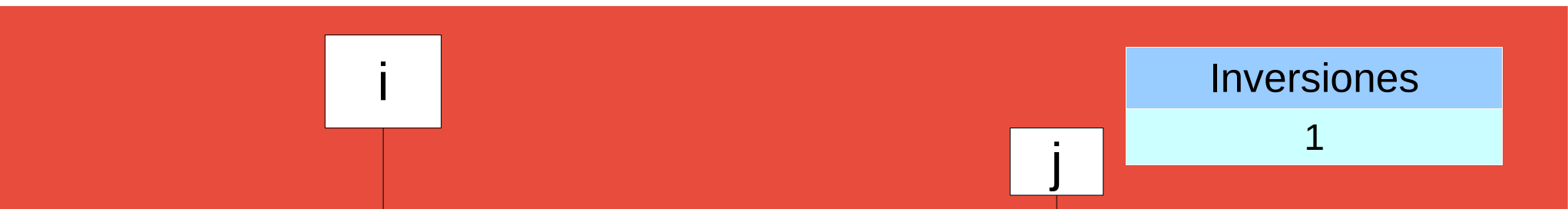
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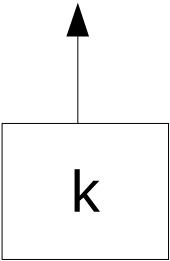
Pos i	Pos j
1	





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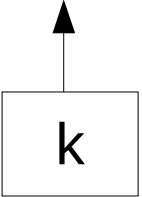
Pos i	Pos j
1	





0	1	2	3	4	5
---	---	---	---	---	---

3	0	2	4	1	5
---	---	---	---	---	---



Pos i	Pos j
1	3



i



j



Inversiones

1

0

1

2

3

4

5

3

0

2

4

1

5

¿Pos j < pos i?

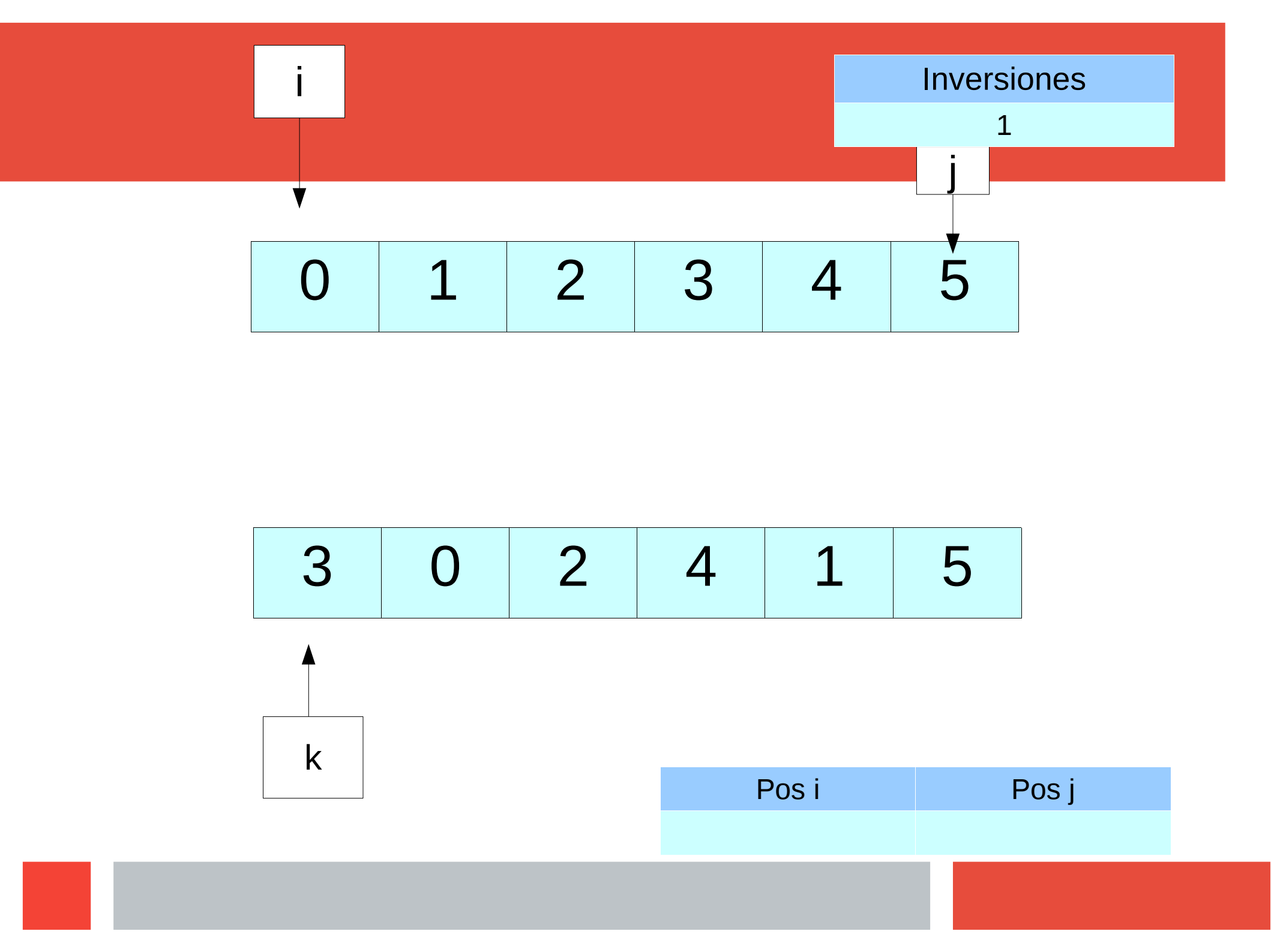
Inversiones + 1

Pos i

1

Pos j

3

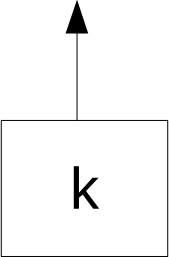






0	1	2	3	4	5
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Pos i	Pos j
1	

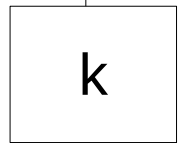




Inversiones
1

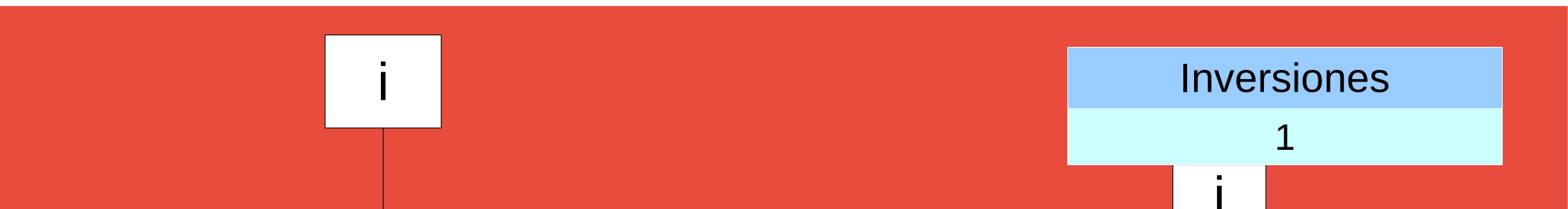
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3	0	2	4	1	5
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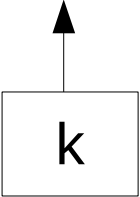
Pos i	Pos j
1	





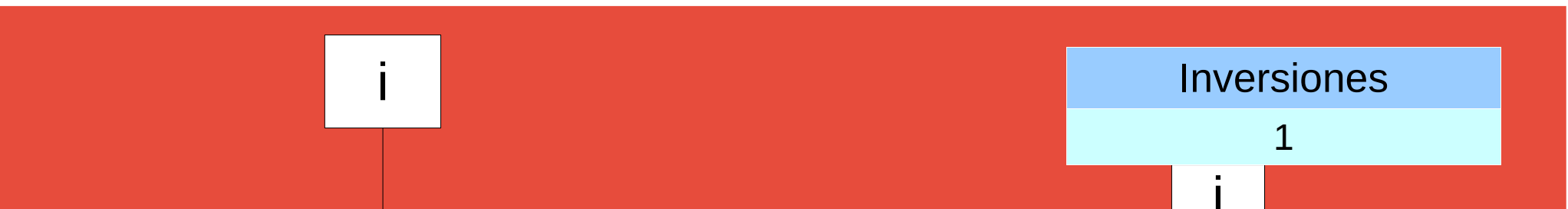
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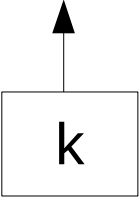
Pos i	Pos j
1	





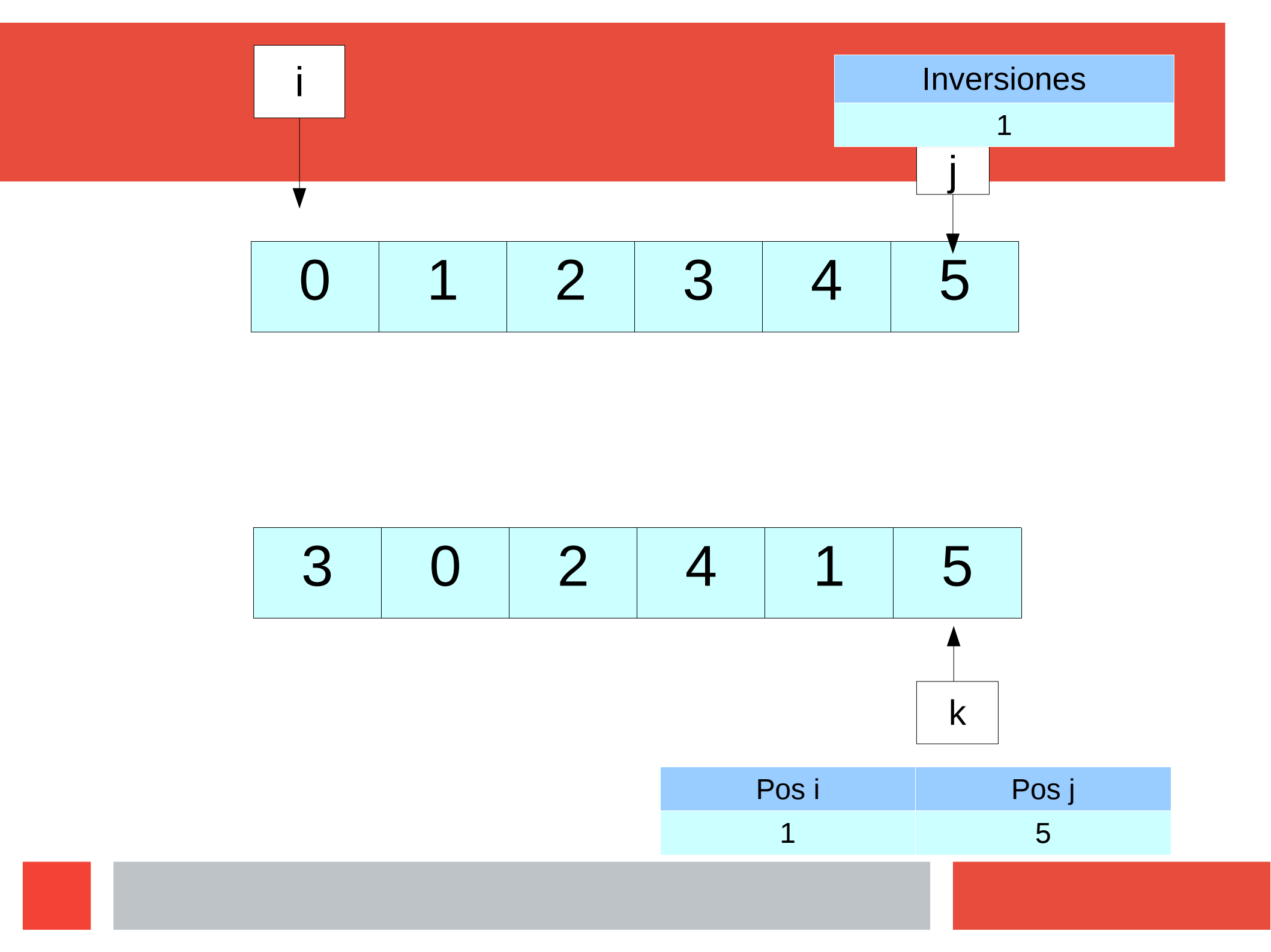
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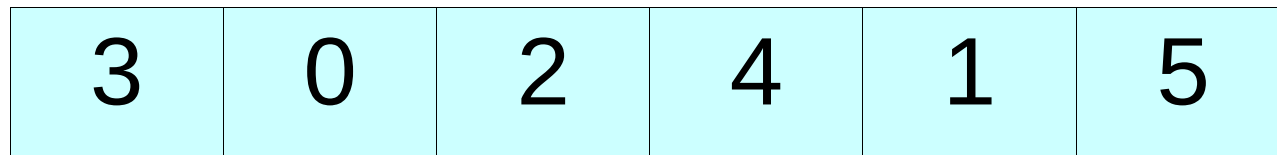
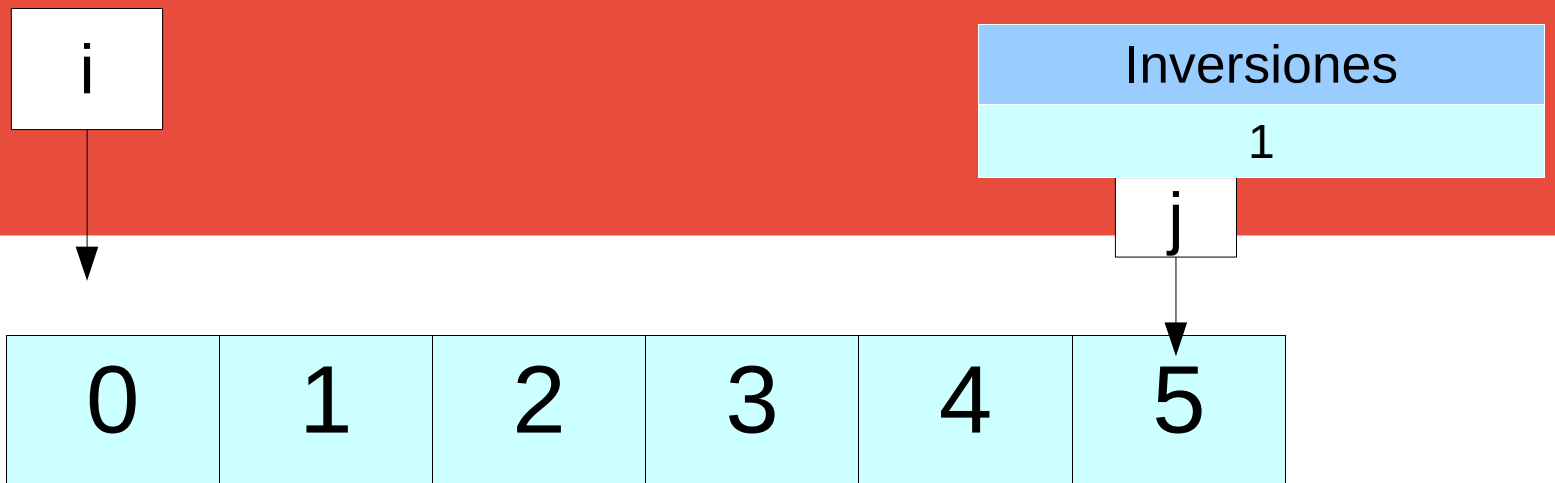
3	0	2	4	1	5
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Pos $i$	Pos $j$
1	



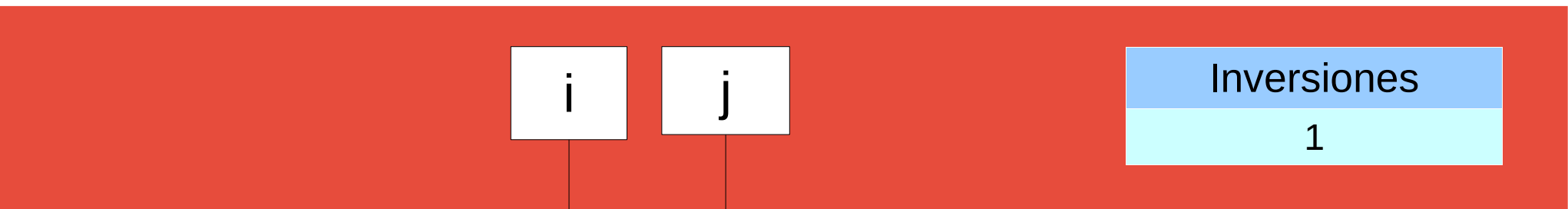




¿Pos j < pos i?

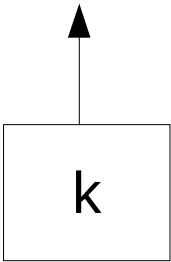
Inversiones + 1

Pos i	Pos j
1	5



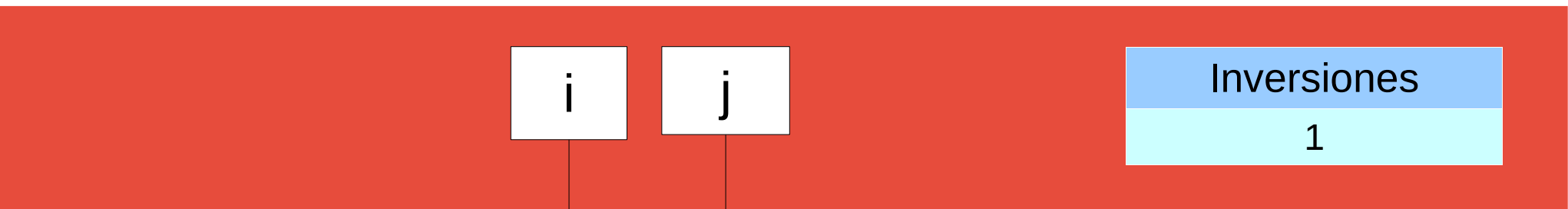
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3	0	2	4	1	5
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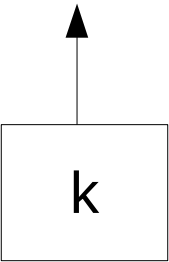
Pos i	Pos j





0	1	2	3	4	5
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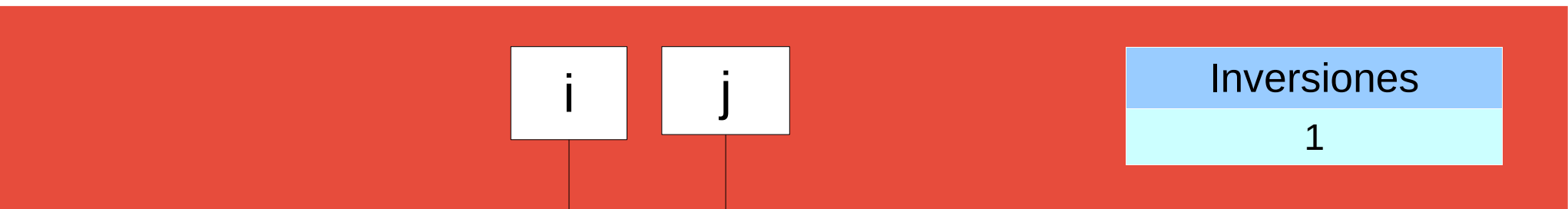
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Pos i	Pos j

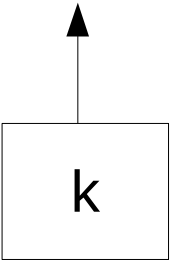






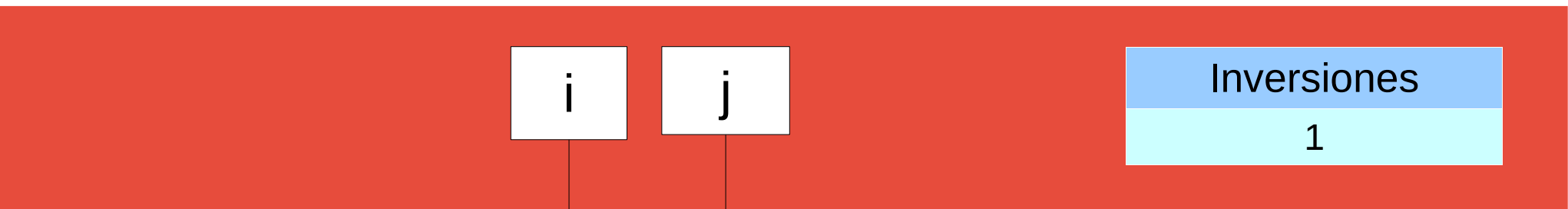
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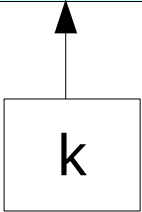
Pos i	Pos j
	2





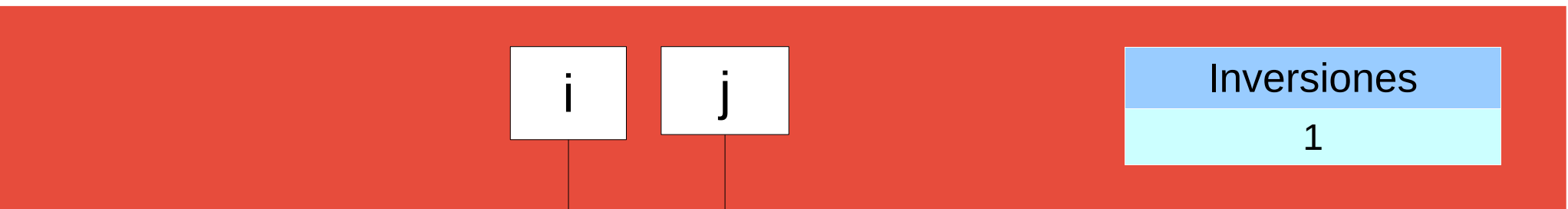
0	1	2	3	4	5
---	---	---	---	---	---

3	0	2	4	1	5
---	---	---	---	---	---



Pos i	Pos j
	2





0	1	2	3	4	5
---	---	---	---	---	---

3	0	2	4	1	5
---	---	---	---	---	---

k

Pos i	Pos j
4	2



i

j

Inversiones
2

0	1	2	3	4	5
---	---	---	---	---	---

3	0	2	4	1	5
---	---	---	---	---	---

¿Pos j < pos i?

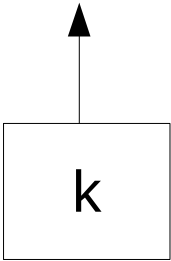
Inversiones + 1

Pos i	Pos j
4	2



0	1	2	3	4	5
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3	0	2	4	1	5
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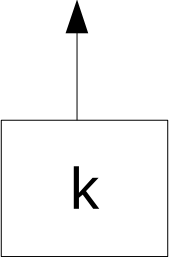
Pos i	Pos j
	0





0	1	2	3	4	5
---	---	---	---	---	---

3	0	2	4	1	5
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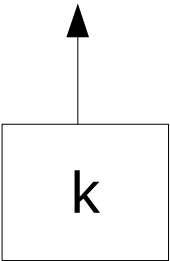
Pos i	Pos j
	0





0	1	2	3	4	5
---	---	---	---	---	---

3	0	2	4	1	5
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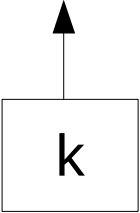
Pos i	Pos j
	0





0	1	2	3	4	5
---	---	---	---	---	---

3	0	2	4	1	5
---	---	---	---	---	---



Pos i	Pos j
	0







0	1	2	3	4	5
---	---	---	---	---	---

3	0	2	4	1	5
---	---	---	---	---	---

k

Pos i	Pos j
4	0



i

j

Inversiones
3

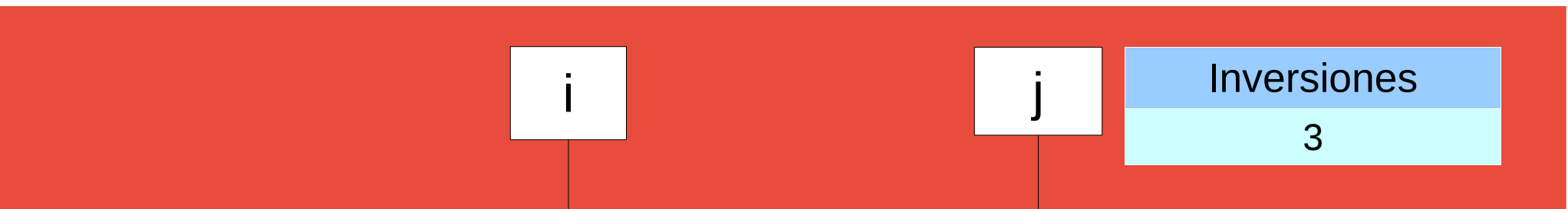
0	1	2	3	4	5
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3	0	2	4	1	5
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¿Pos j < pos i?

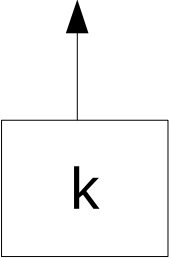
Inversiones + 1

Pos i	Pos j
4	0



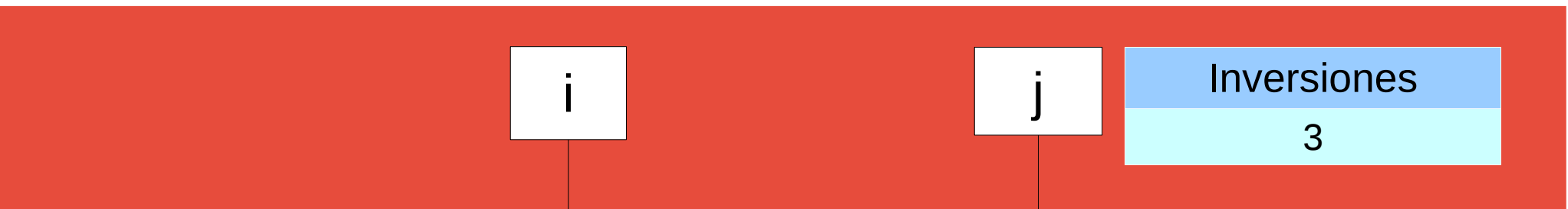
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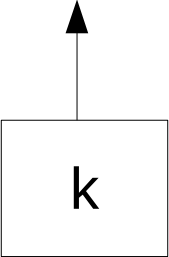
Pos $i$	Pos $j$





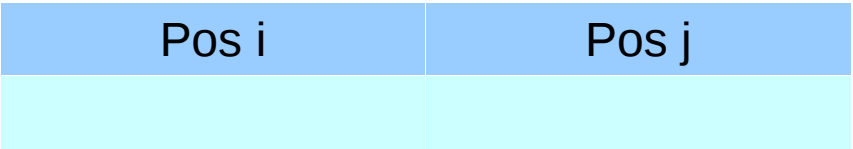
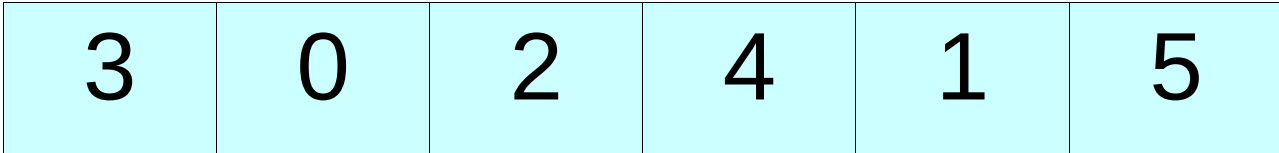
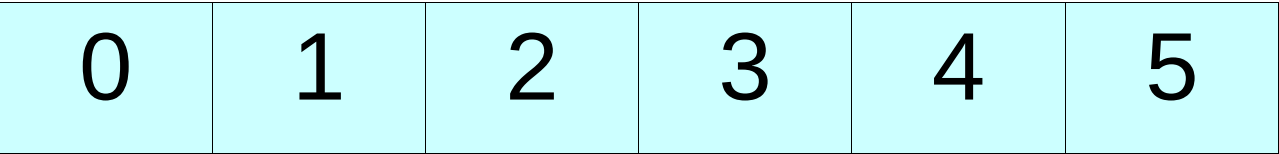
0	1	2	3	4	5
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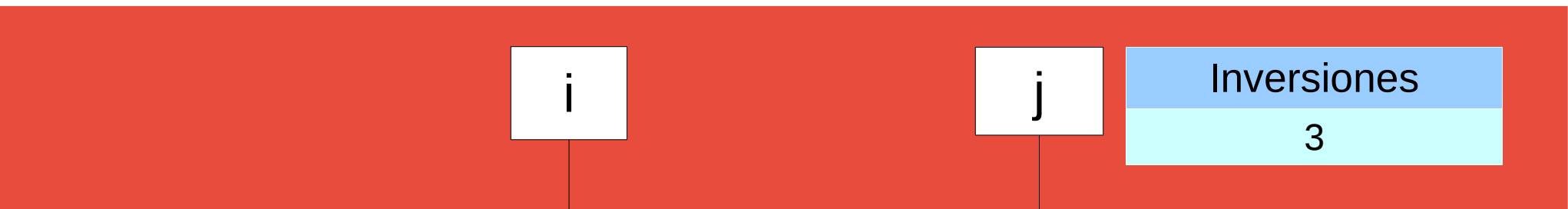
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Pos i	Pos j







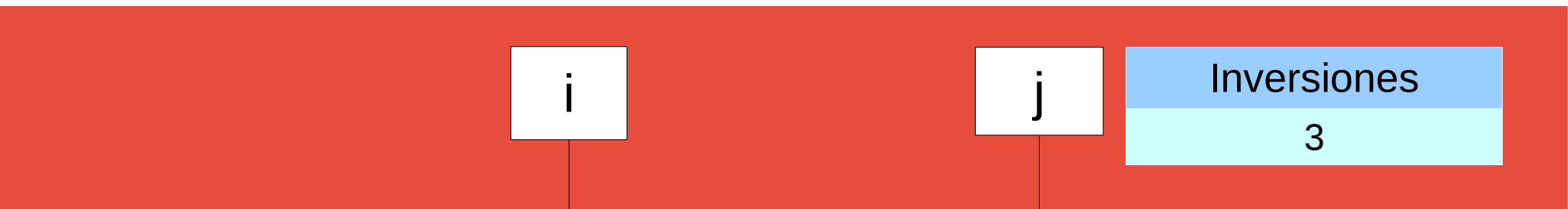
0	1	2	3	4	5
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3	0	2	4	1	5
---	---	---	---	---	---

k

Pos i	Pos j
	3





0	1	2	3	4	5
---	---	---	---	---	---

3	0	2	4	1	5
---	---	---	---	---	---

k

Pos i	Pos j
4	3



i

j

Inversiones
4

0	1	2	3	4	5
---	---	---	---	---	---

3	0	2	4	1	5
---	---	---	---	---	---

¿Pos j < pos i?

Inversiones + 1

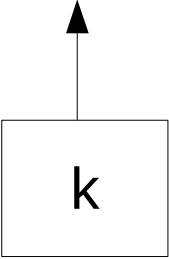
Pos i	Pos j
4	3





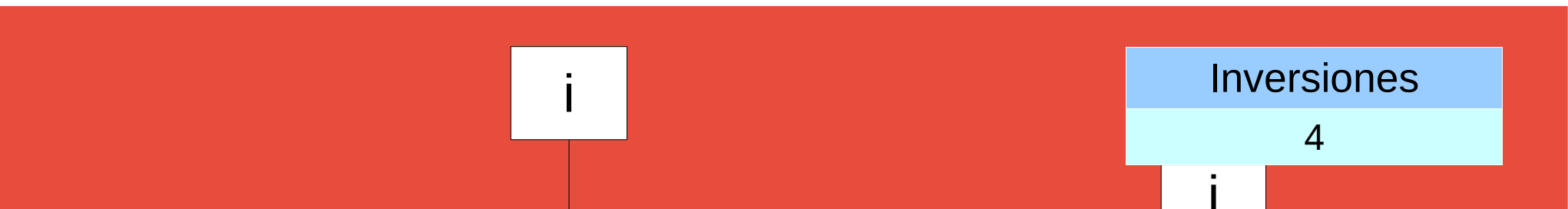
0	1	2	3	4	5
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3	0	2	4	1	5
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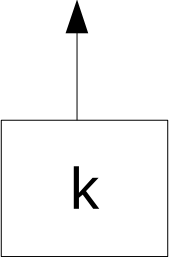
Pos $i$	Pos $j$





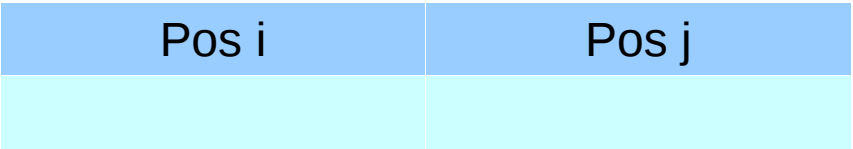
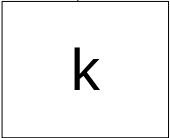
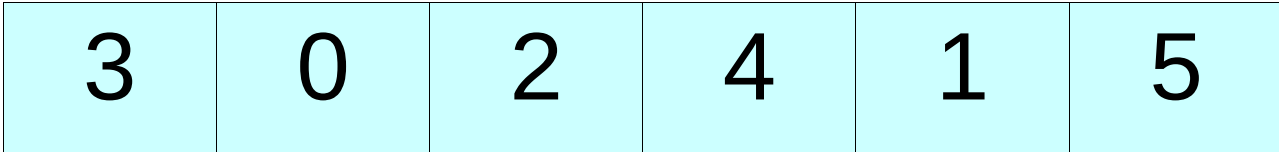
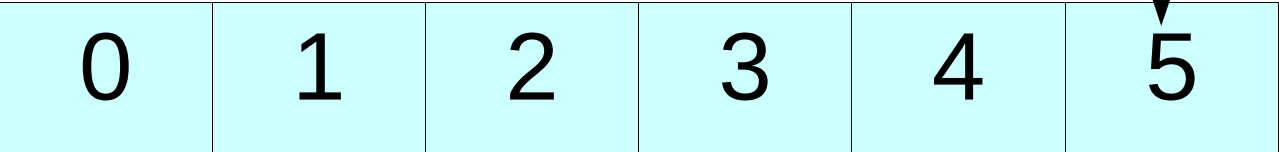
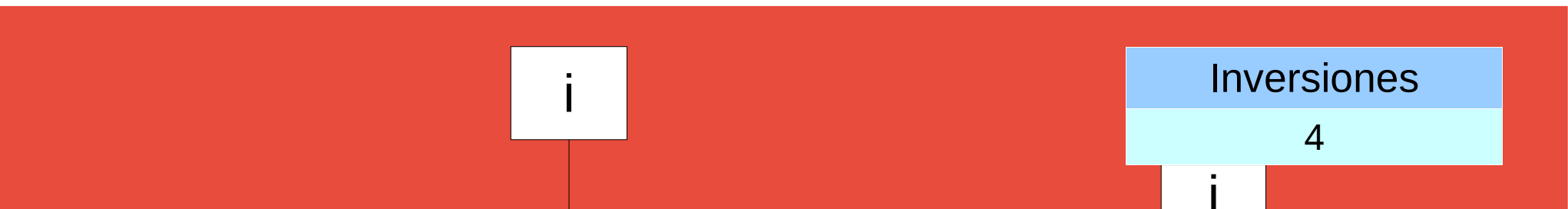
0	1	2	3	4	5
---	---	---	---	---	---

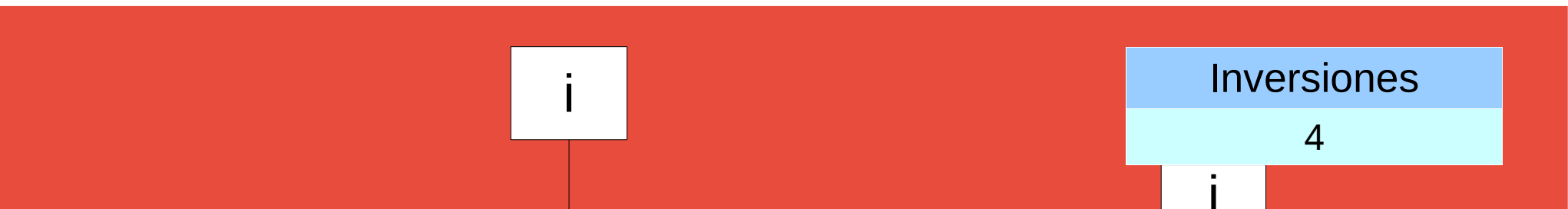
3	0	2	4	1	5
---	---	---	---	---	---



Pos i	Pos j







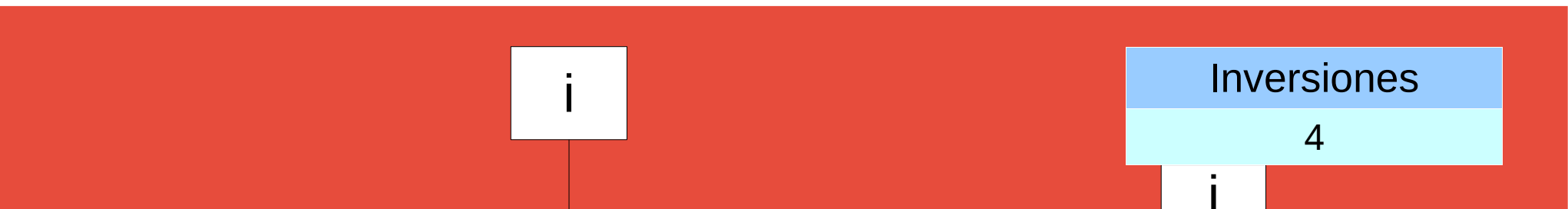
0	1	2	3	4	5
---	---	---	---	---	---

3	0	2	4	1	5
---	---	---	---	---	---

k

Pos i	Pos j





0	1	2	3	4	5
---	---	---	---	---	---

3	0	2	4	1	5
---	---	---	---	---	---

k

Pos i	Pos j
4	





0	1	2	3	4	5
---	---	---	---	---	---

3	0	2	4	1	5
---	---	---	---	---	---

k

Pos i	Pos j
4	5



i



Inversiones  
4

j



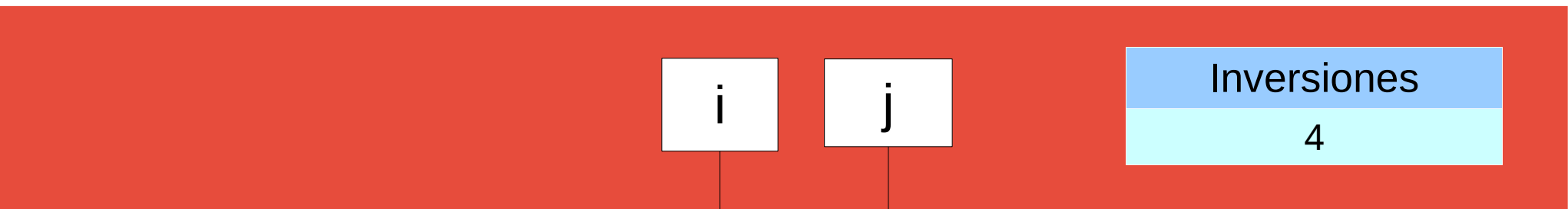
0	1	2	3	4	5
---	---	---	---	---	---

3	0	2	4	1	5
---	---	---	---	---	---

¿Pos j < pos i?

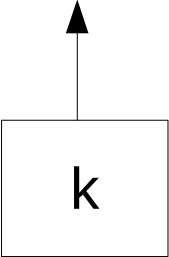
Inversiones + 1

Pos i	Pos j
4	5



0	1	2	3	4	5
---	---	---	---	---	---

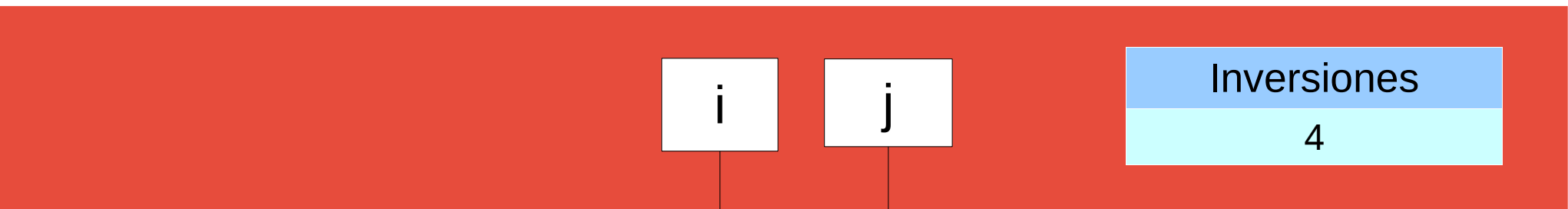
3	0	2	4	1	5
---	---	---	---	---	---



Pos i	Pos j
	0

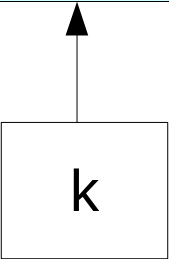






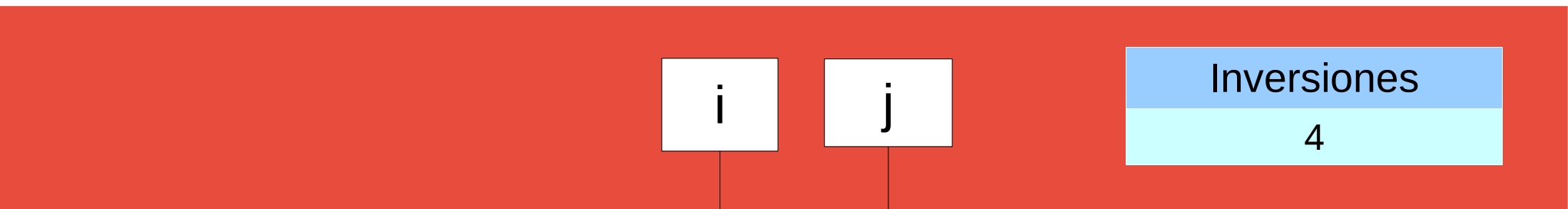
0	1	2	3	4	5
---	---	---	---	---	---

3	0	2	4	1	5
---	---	---	---	---	---



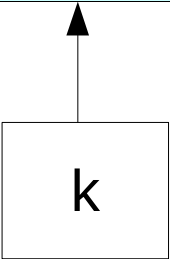
Pos i	Pos j
	0





0	1	2	3	4	5
---	---	---	---	---	---

3	0	2	4	1	5
---	---	---	---	---	---



Pos i	Pos j
2	0



i

j

Inversiones

5

0	1	2	3	4	5
---	---	---	---	---	---

3	0	2	4	1	5
---	---	---	---	---	---

¿Pos j < pos i?

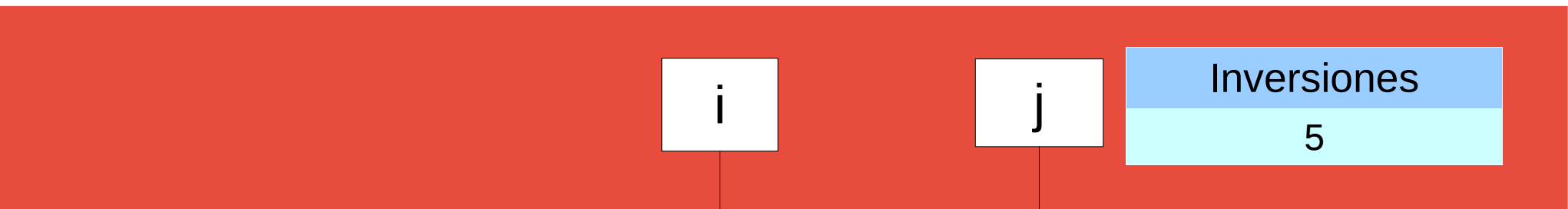
Inversiones + 1

Pos i

2

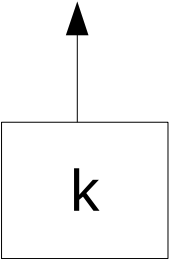
Pos j

0



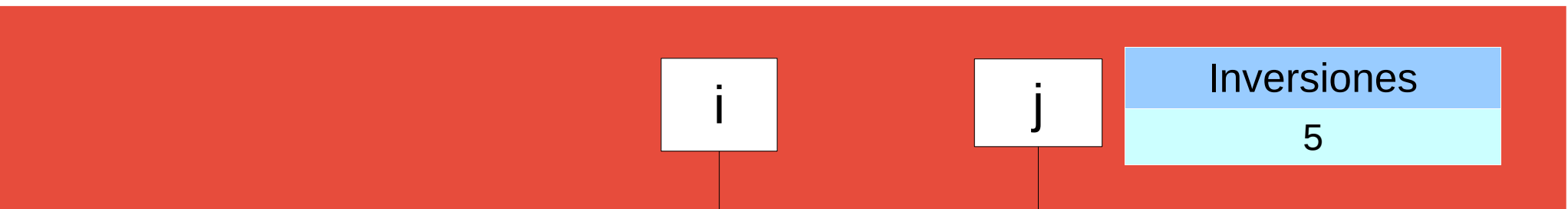
0	1	2	3	4	5
---	---	---	---	---	---

3	0	2	4	1	5
---	---	---	---	---	---



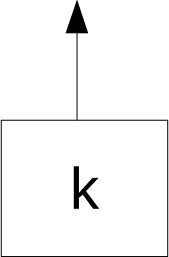
Pos i	Pos j





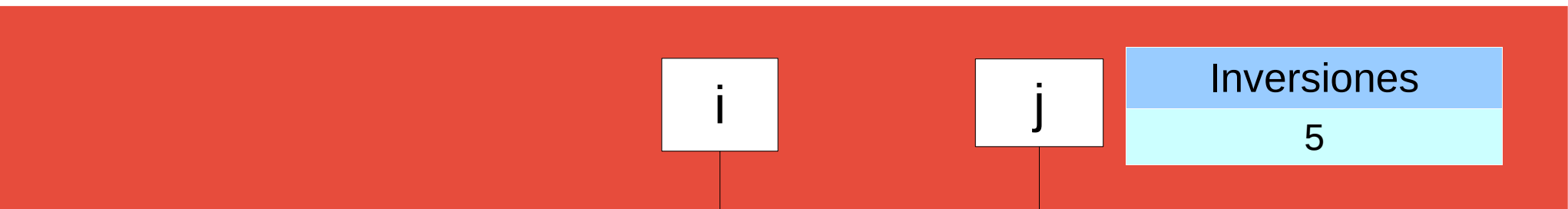
0	1	2	3	4	5
---	---	---	---	---	---

3	0	2	4	1	5
---	---	---	---	---	---



Pos i	Pos j





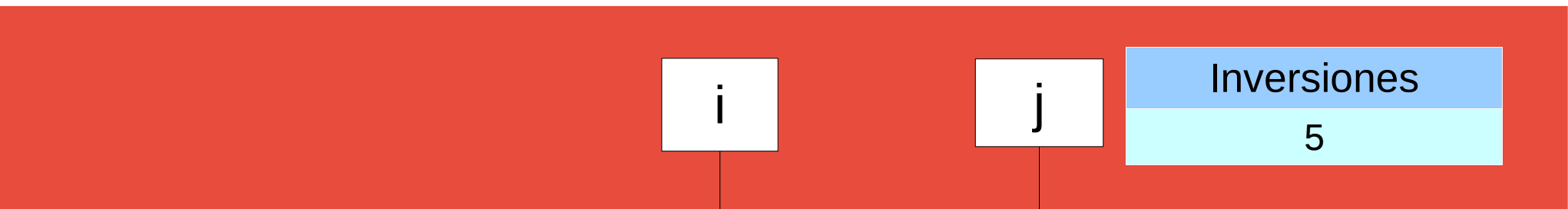
0	1	2	3	4	5
---	---	---	---	---	---

3	0	2	4	1	5
---	---	---	---	---	---

k

Pos i	Pos j
2	





0	1	2	3	4	5
---	---	---	---	---	---

3	0	2	4	1	5
---	---	---	---	---	---

k

Pos i	Pos j
2	3



i

j

Inversiones

5

0	1	2	3	4	5
---	---	---	---	---	---

3	0	2	4	1	5
---	---	---	---	---	---

¿Pos j < pos i?

Inversiones + 1

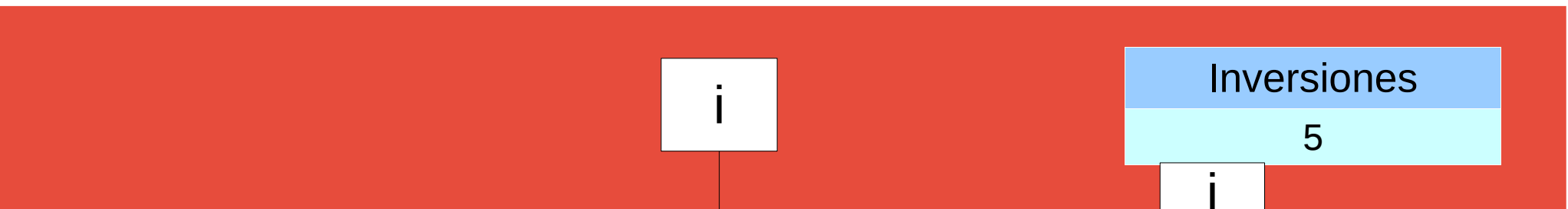
Pos i

2

Pos j

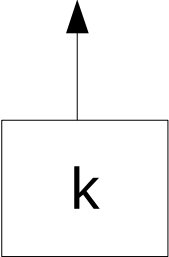
3





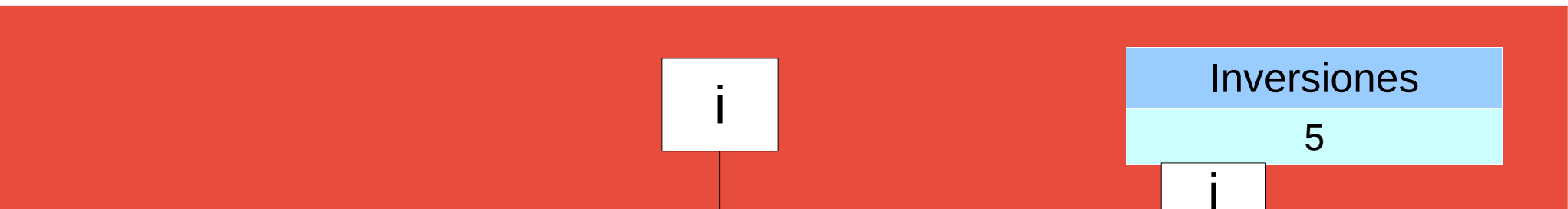
0	1	2	3	4	5
---	---	---	---	---	---

3	0	2	4	1	5
---	---	---	---	---	---



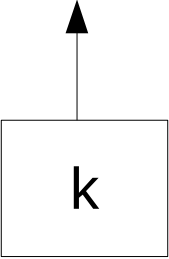
Pos $i$	Pos $j$





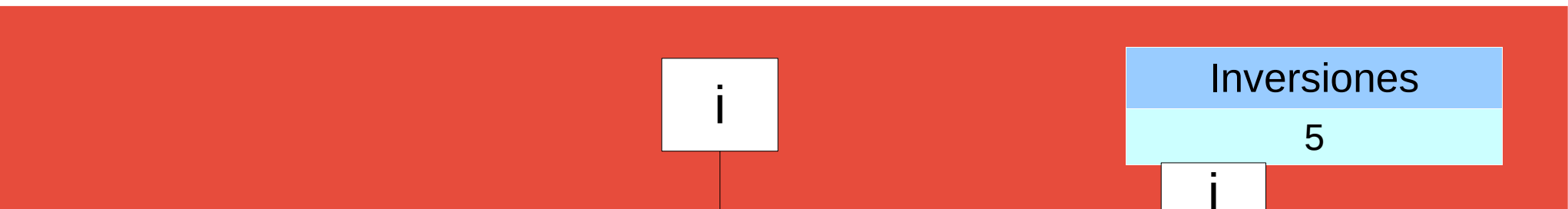
0	1	2	3	4	5
---	---	---	---	---	---

3	0	2	4	1	5
---	---	---	---	---	---



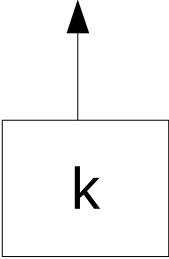
Pos $i$	Pos $j$





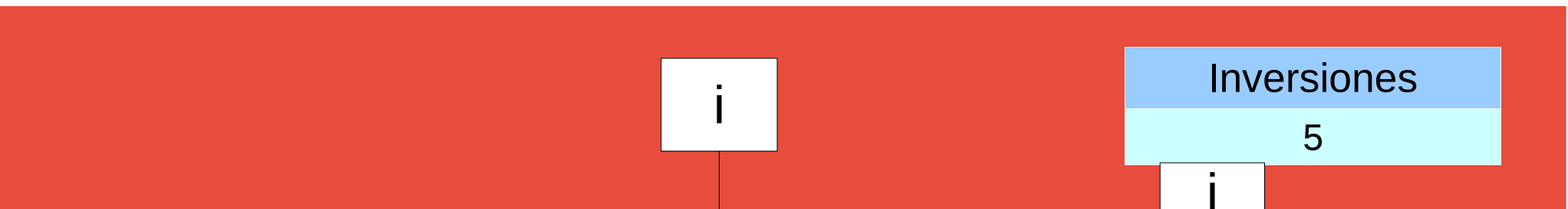
0	1	2	3	4	5
---	---	---	---	---	---

3	0	2	4	1	5
---	---	---	---	---	---



Pos i	Pos j
2	





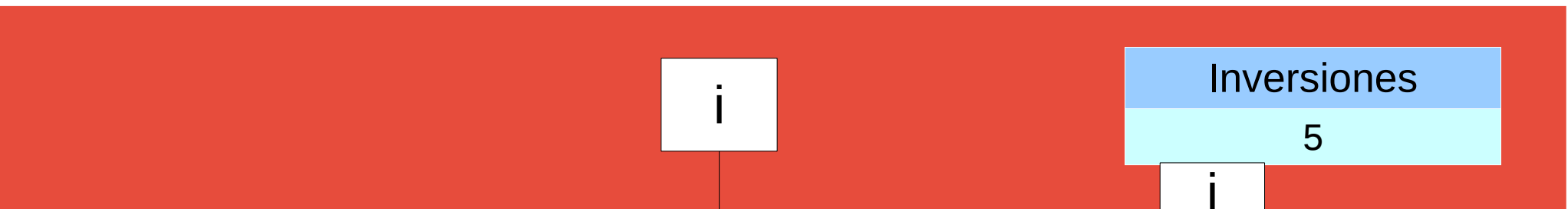
0	1	2	3	4	5
---	---	---	---	---	---

3	0	2	4	1	5
---	---	---	---	---	---

k

Pos i	Pos j
2	





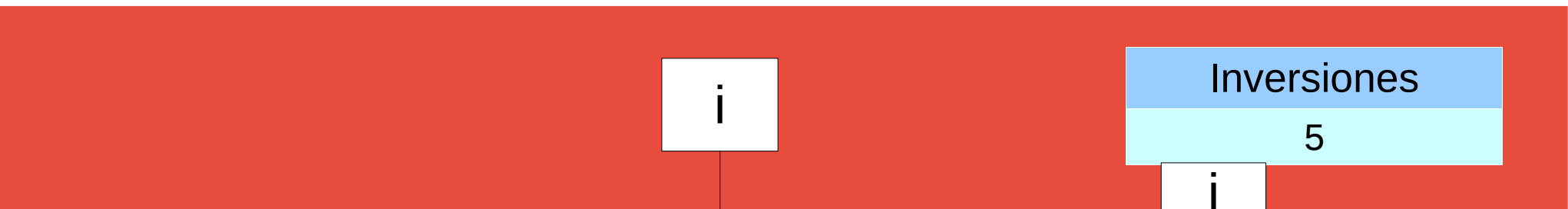
0	1	2	3	4	5
---	---	---	---	---	---

3	0	2	4	1	5
---	---	---	---	---	---

k

Pos i	Pos j
2	





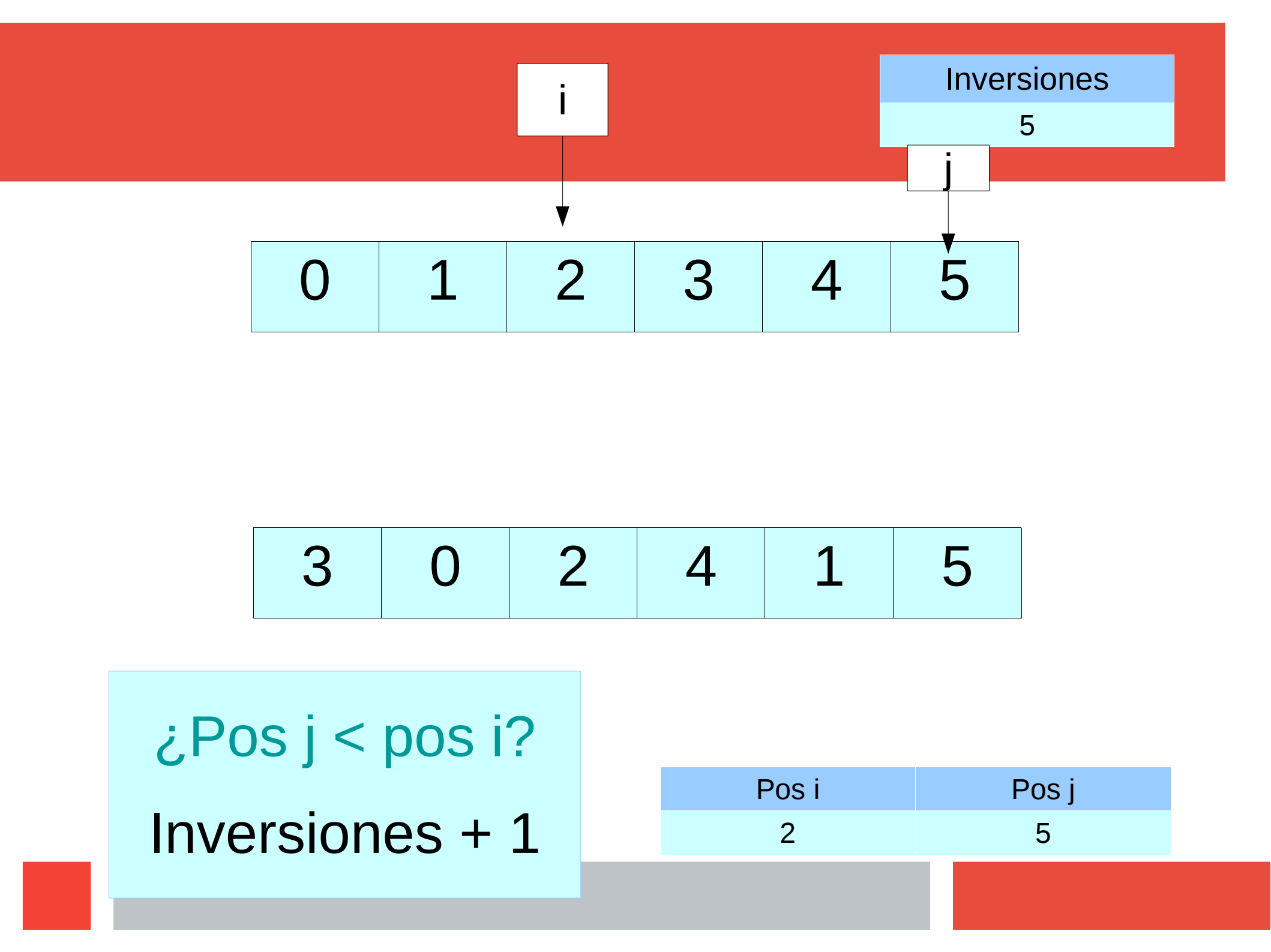
0	1	2	3	4	5
---	---	---	---	---	---

3	0	2	4	1	5
---	---	---	---	---	---

k

Pos i	Pos j
2	5

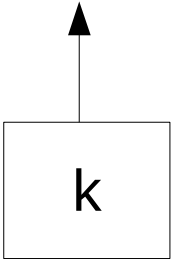






0	1	2	3	4	5
---	---	---	---	---	---

3	0	2	4	1	5
---	---	---	---	---	---



Pos i	Pos j
0	

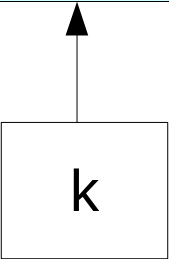






0	1	2	3	4	5
---	---	---	---	---	---

3	0	2	4	1	5
---	---	---	---	---	---



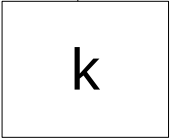
Pos i	Pos j
0	





0	1	2	3	4	5
---	---	---	---	---	---

3	0	2	4	1	5
---	---	---	---	---	---



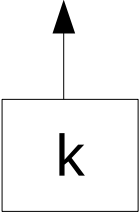
Pos i	Pos j
0	





0	1	2	3	4	5
---	---	---	---	---	---

3	0	2	4	1	5
---	---	---	---	---	---



Pos $i$	Pos $j$
0	3



i

j

Inversiones

5

0	1	2	3	4	5
---	---	---	---	---	---

3	0	2	4	1	5
---	---	---	---	---	---

¿Pos j < pos i?

Inversiones + 1

Pos i

0

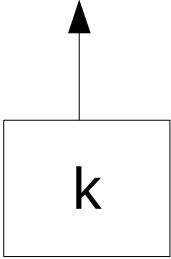
Pos j

3



0	1	2	3	4	5
---	---	---	---	---	---

3	0	2	4	1	5
---	---	---	---	---	---



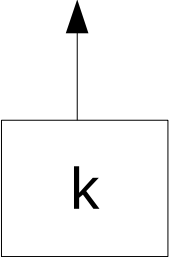
Pos i	Pos j
0	





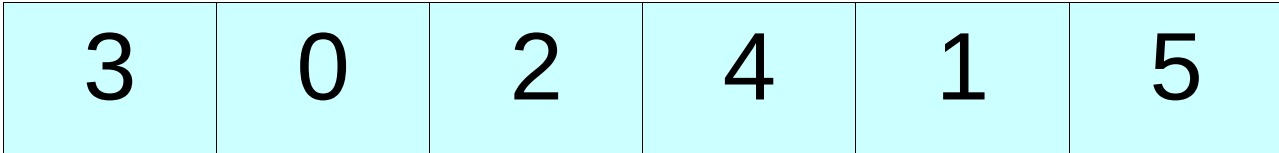
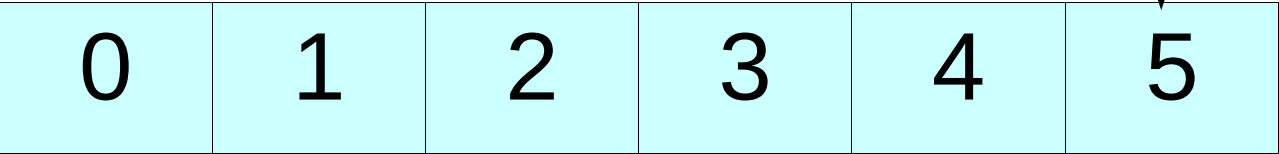
0	1	2	3	4	5
---	---	---	---	---	---

3	0	2	4	1	5
---	---	---	---	---	---



Pos i	Pos j
0	





Pos $i$	Pos $j$
0	





0	1	2	3	4	5
---	---	---	---	---	---

3	0	2	4	1	5
---	---	---	---	---	---

k

Pos i	Pos j
0	







0	1	2	3	4	5
---	---	---	---	---	---

3	0	2	4	1	5
---	---	---	---	---	---

k

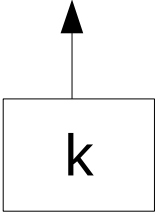
Pos i	Pos j
0	





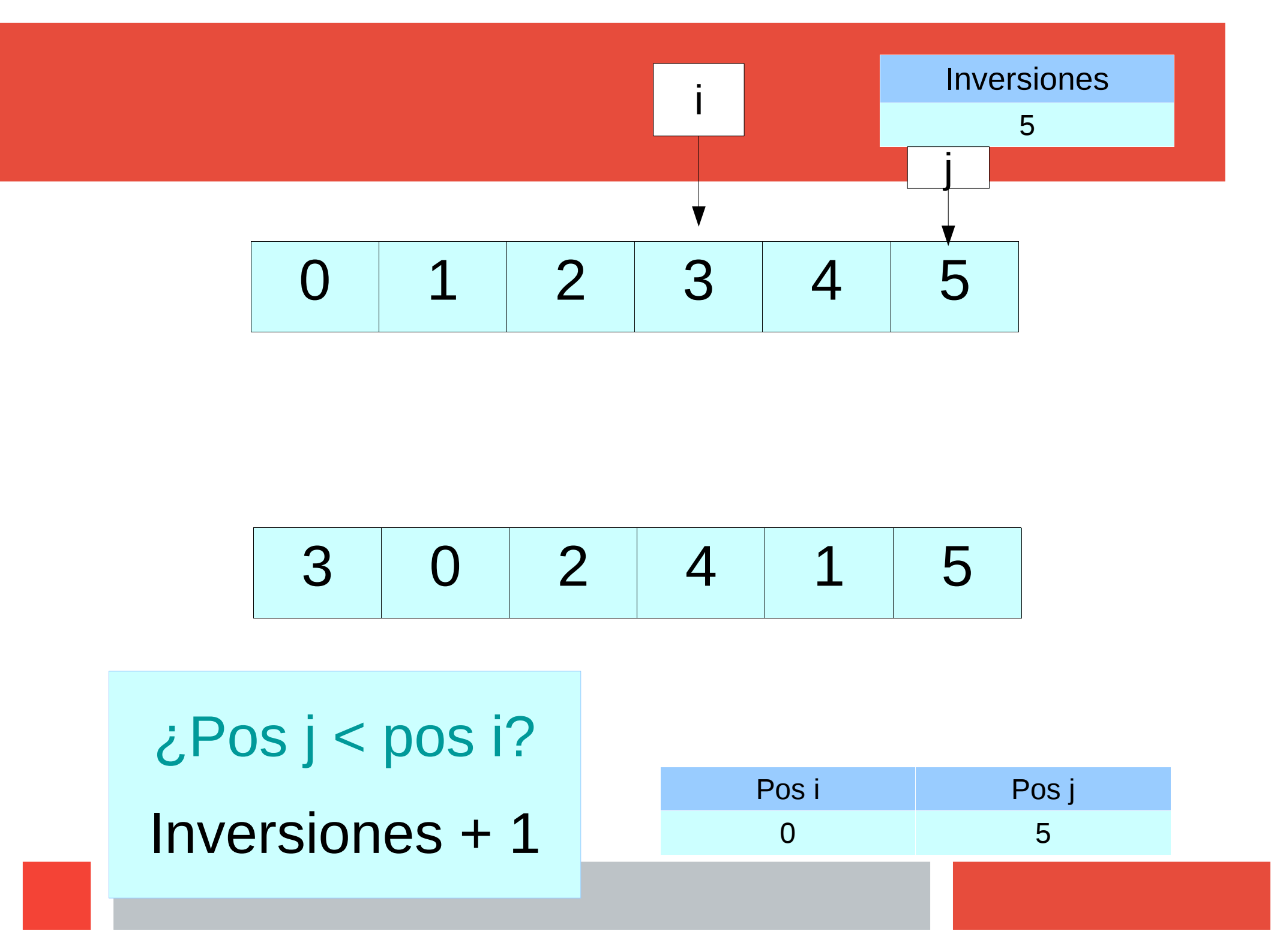
0	1	2	3	4	5
---	---	---	---	---	---

3	0	2	4	1	5
---	---	---	---	---	---



Pos $i$	Pos $j$
0	5





$i$

Inversiones  
5

$j$

0	1	2	3	4	5
---	---	---	---	---	---

3	0	2	4	1	5
---	---	---	---	---	---

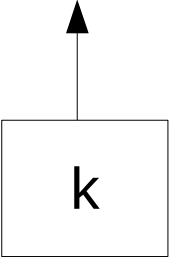
¿Pos  $j < \text{pos } i$ ?  
Inversiones + 1

Pos $i$	Pos $j$
0	5



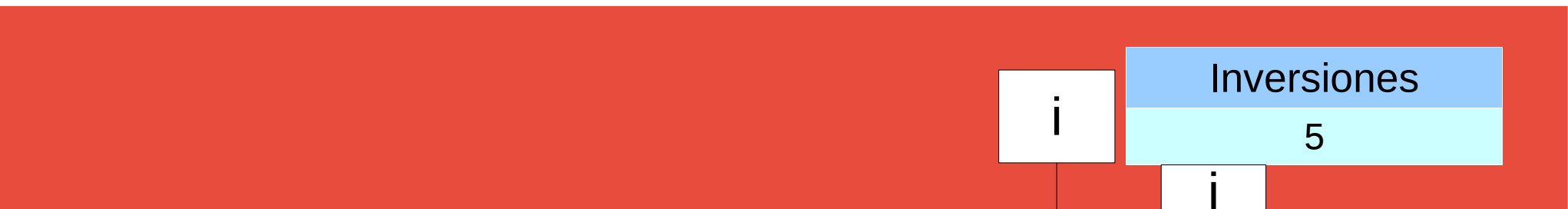
0	1	2	3	4	5
---	---	---	---	---	---

3	0	2	4	1	5
---	---	---	---	---	---



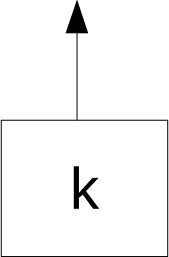
Pos i	Pos j





0	1	2	3	4	5
---	---	---	---	---	---

3	0	2	4	1	5
---	---	---	---	---	---



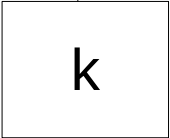
Pos i	Pos j





0	1	2	3	4	5
---	---	---	---	---	---

3	0	2	4	1	5
---	---	---	---	---	---



Pos i	Pos j





0	1	2	3	4	5
---	---	---	---	---	---

3	0	2	4	1	5
---	---	---	---	---	---

k

Pos i	Pos j
3	





0	1	2	3	4	5
---	---	---	---	---	---

3	0	2	4	1	5
---	---	---	---	---	---

k

Pos i	Pos j
3	







0	1	2	3	4	5
---	---	---	---	---	---

3	0	2	4	1	5
---	---	---	---	---	---

k

Pos i	Pos j
3	5



i

Inversiones
5

j

0	1	2	3	4	5
---	---	---	---	---	---

3	0	2	4	1	5
---	---	---	---	---	---

¿Pos j < pos i?

Inversiones + 1

Pos i	Pos j
3	5

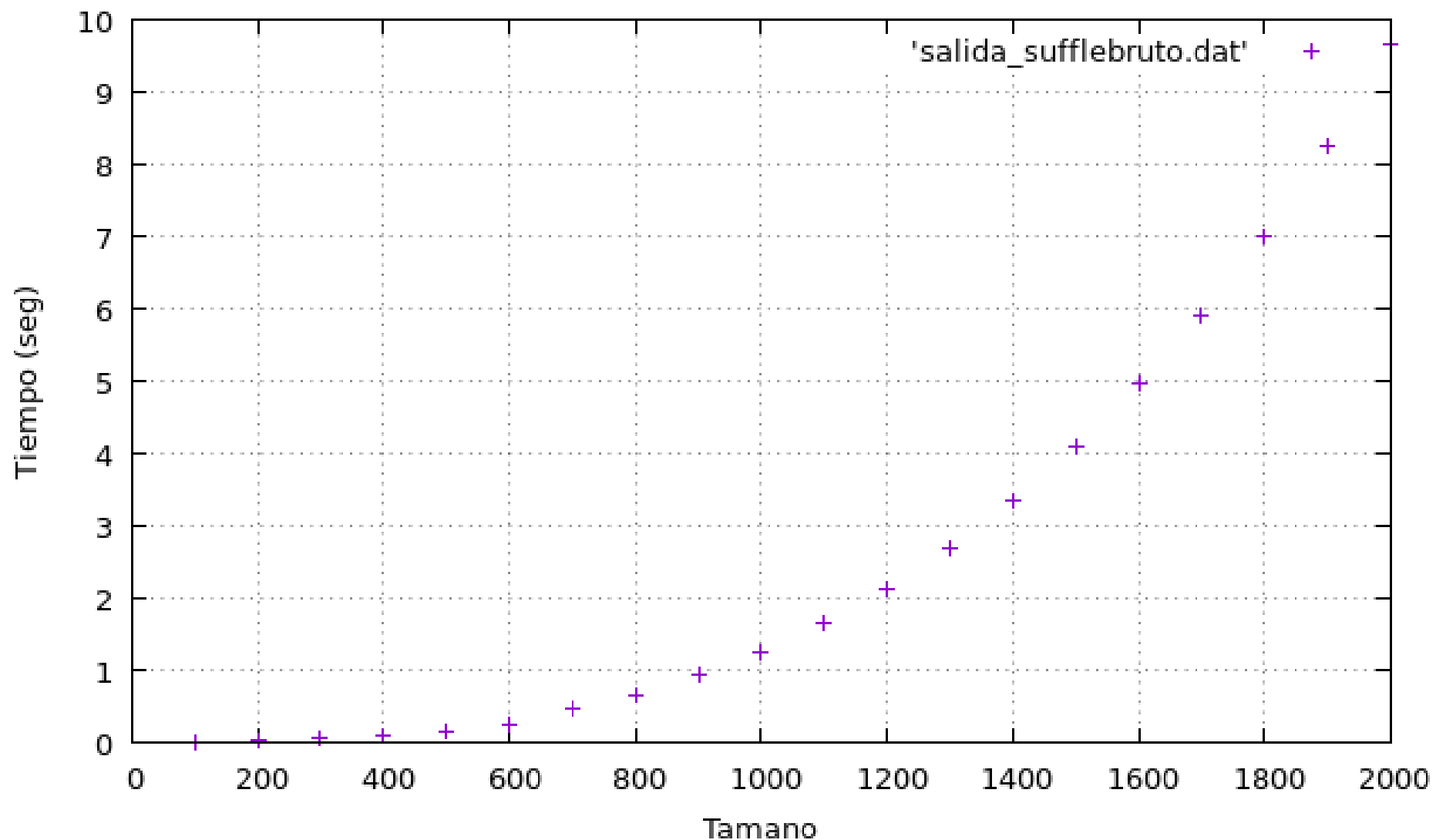
# Eficiencia teórica

Como la mayor parte del tiempo de ejecución de este algoritmo se emplea en los 3 bucles for que se encargan de buscar los elementos que se quieren comparar en el vector.

Eficiencia:  $O(n^3)$

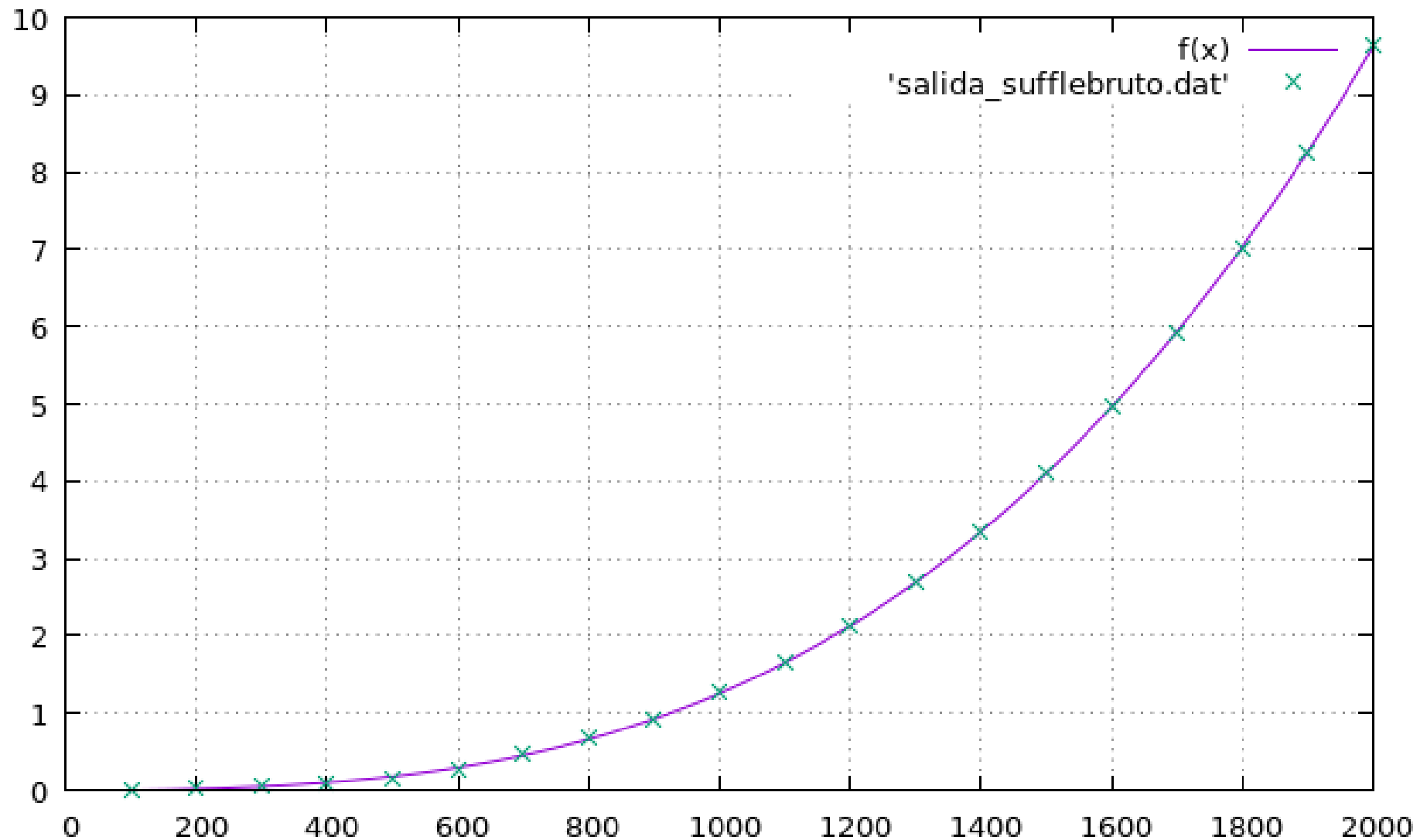


# Eficiencia empírica



# Eficiencia híbrida:

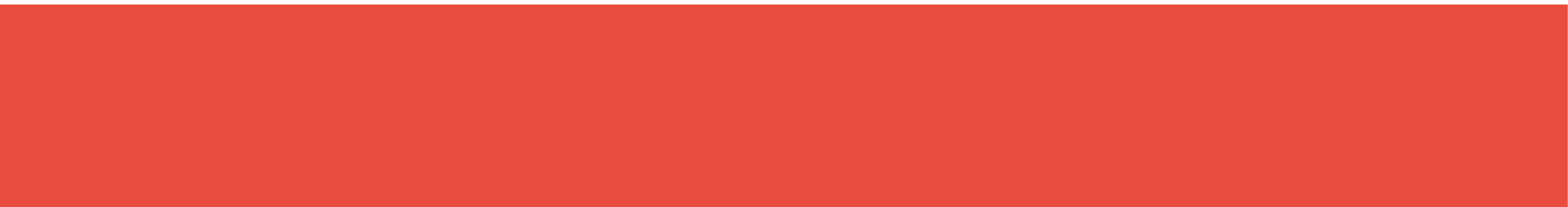
$$f(x) = 1.19599 \cdot 10^{-5} x^3 - 3.52047 \cdot 10^{-6} x^2 + 0.000102819 x - 0.00896225$$





# ALGORITMO DIVIDE Y VENCERÁS





0	1	2	3	4	5
---	---	---	---	---	---

3	0	2	4	1	5
---	---	---	---	---	---



Inversiones

0

0	1	2	3	4	5
---	---	---	---	---	---

i



3	0	2	4	1	5
---	---	---	---	---	---

j



¿ $v[i] > v[j]$ ?

Inversiones + 1



Inversiones

1

0	1	2	3	4	5
---	---	---	---	---	---

i



3	0	2	4	1	5
---	---	---	---	---	---

j

¿ $v[i] > v[j]$ ?

Inversiones + 1

Inversiones

1

0	1	2	3	4	5
---	---	---	---	---	---

i



3	0	2	4	1	5
---	---	---	---	---	---

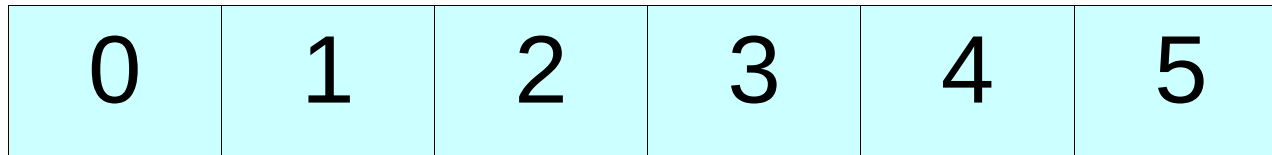
j

¿ $v[i] > v[j]$ ?

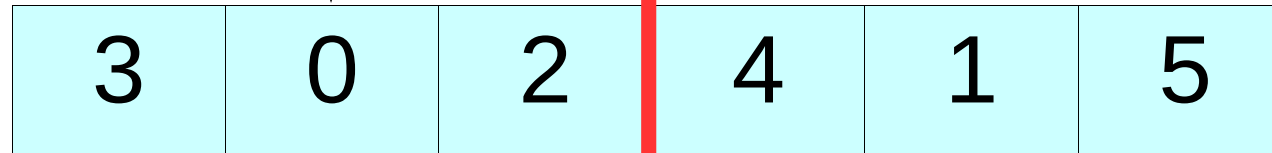
Inversiones + 1

Inversiones

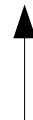
1



i



j

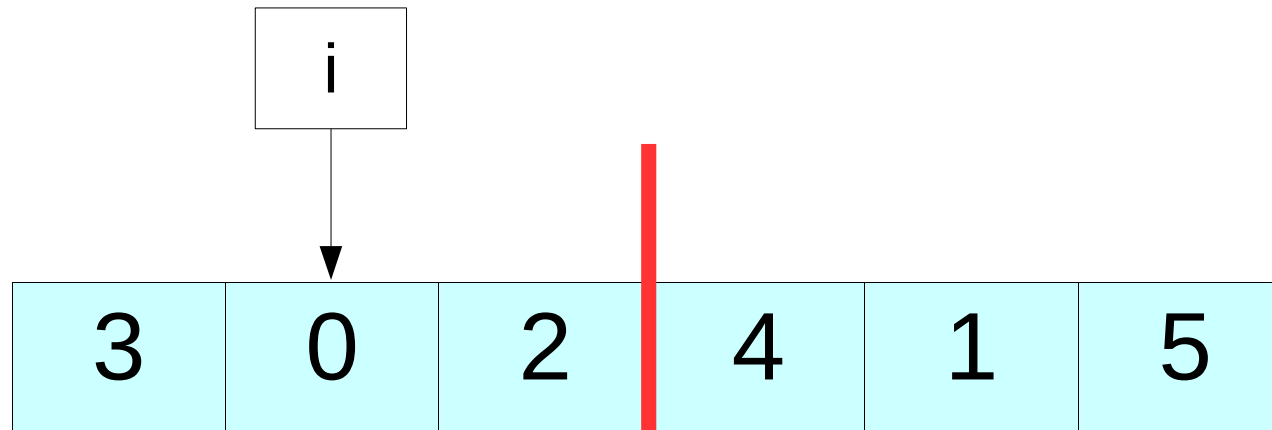
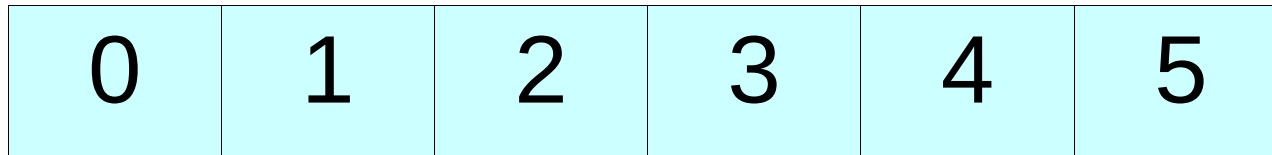


¿ $v[i] > v[j]$ ?

Inversiones + 1

Inversiones

1

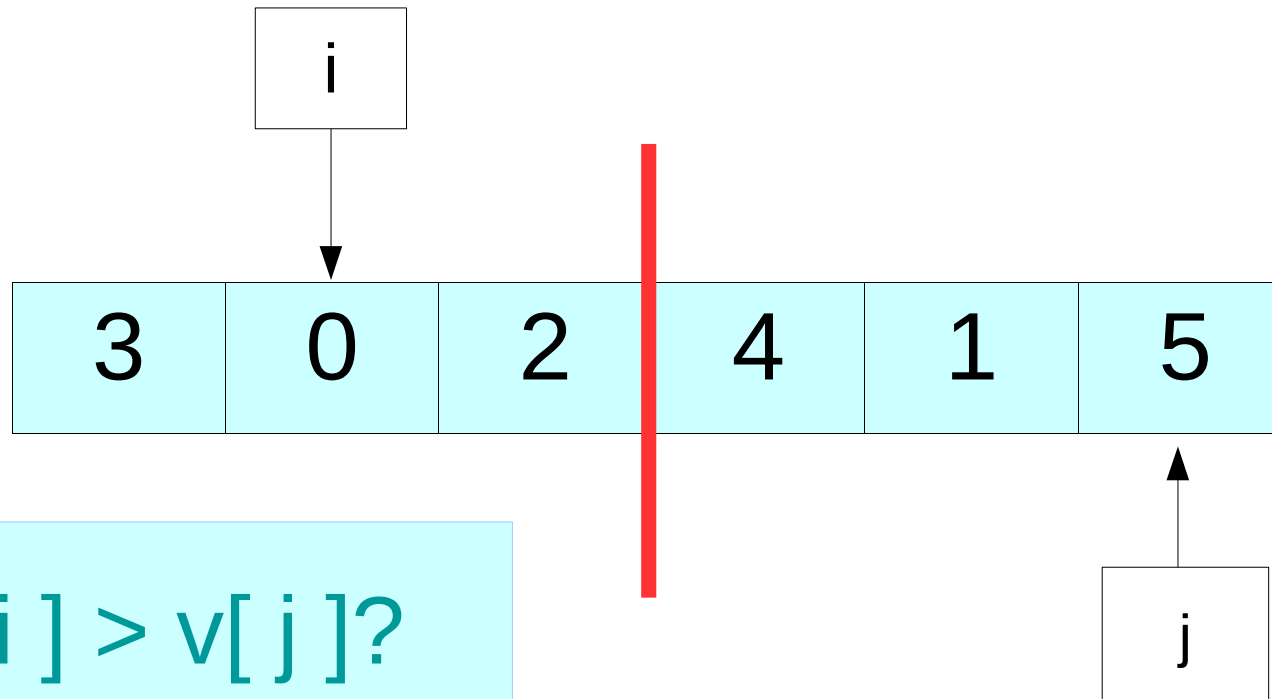
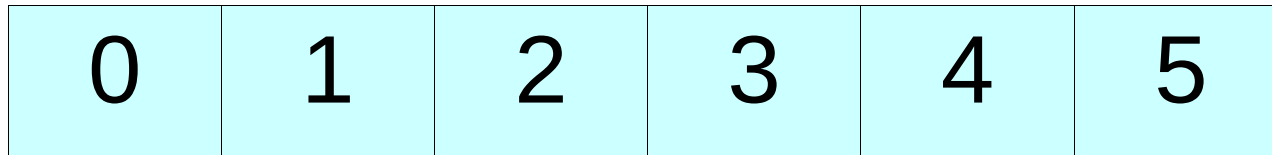


¿ $v[i] > v[j]$ ?

Inversiones + 1

Inversiones

1



¿ $v[i] > v[j]$ ?

Inversiones + 1

Inversiones

1

0	1	2	3	4	5
---	---	---	---	---	---

i

3	0	2	4	1	5
---	---	---	---	---	---

j

¿ $v[i] > v[j]$ ?

Inversiones + 1

Inversiones

2

0	1	2	3	4	5
---	---	---	---	---	---

i



3	0	2	4	1	5
---	---	---	---	---	---



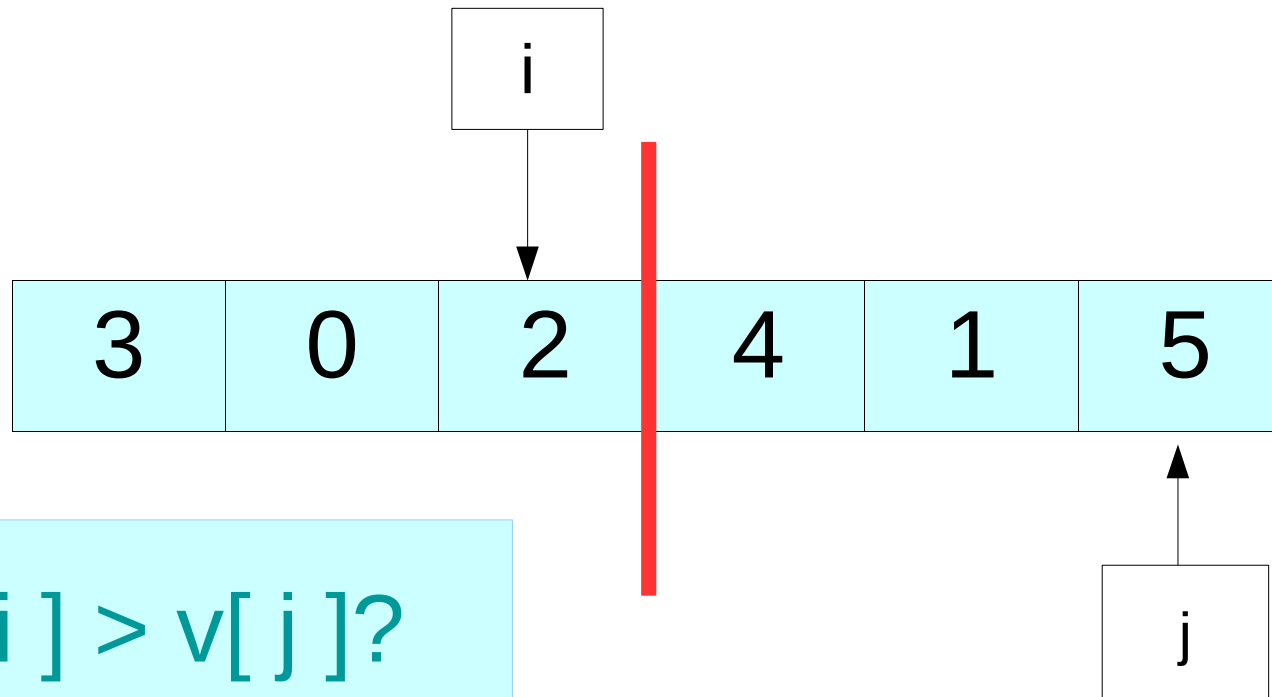
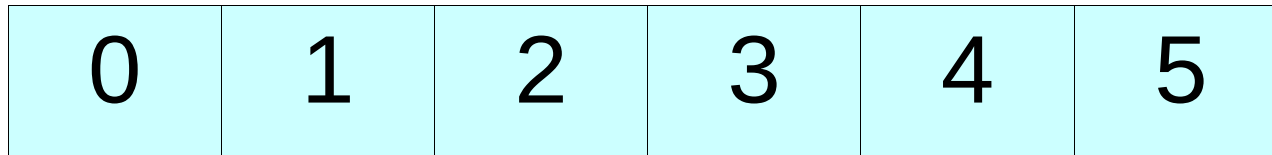
j

¿ $v[i] > v[j]$ ?

Inversiones + 1

Inversiones

2



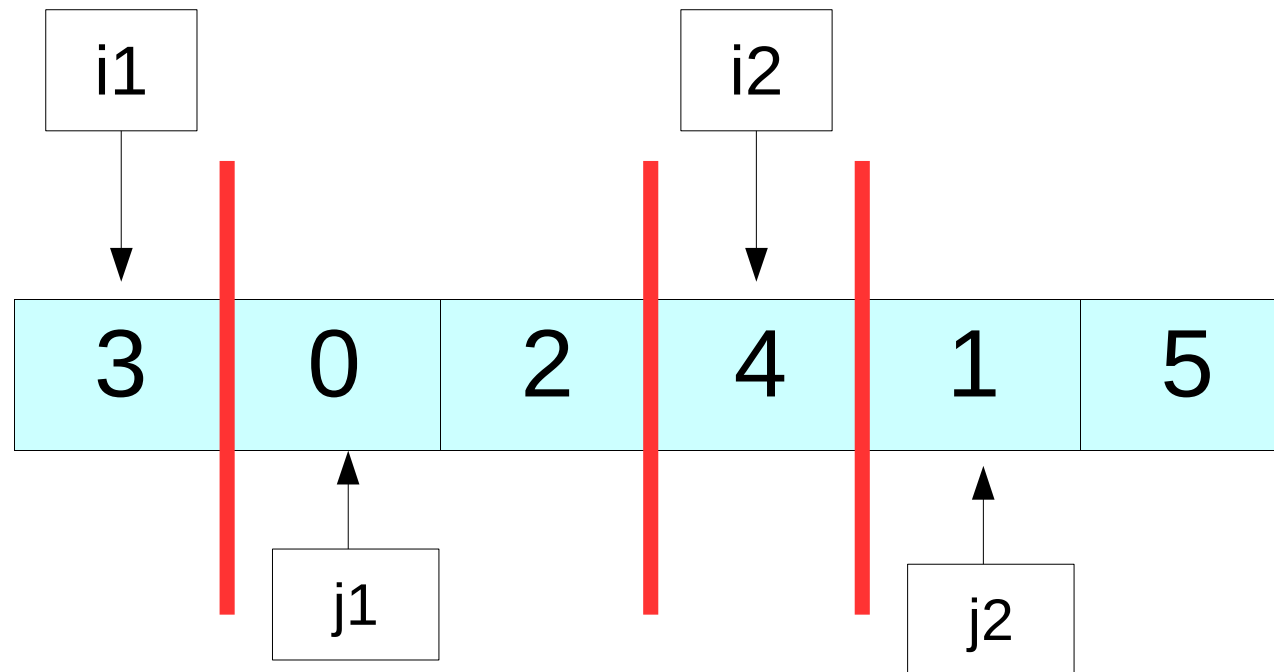
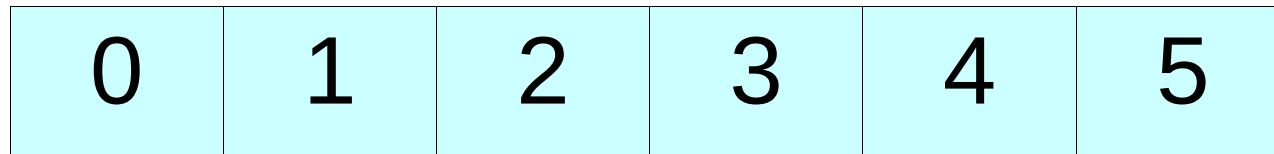
¿ $v[i] > v[j]$ ?

Inversiones + 1



Inversiones

$$2+1+1=4$$

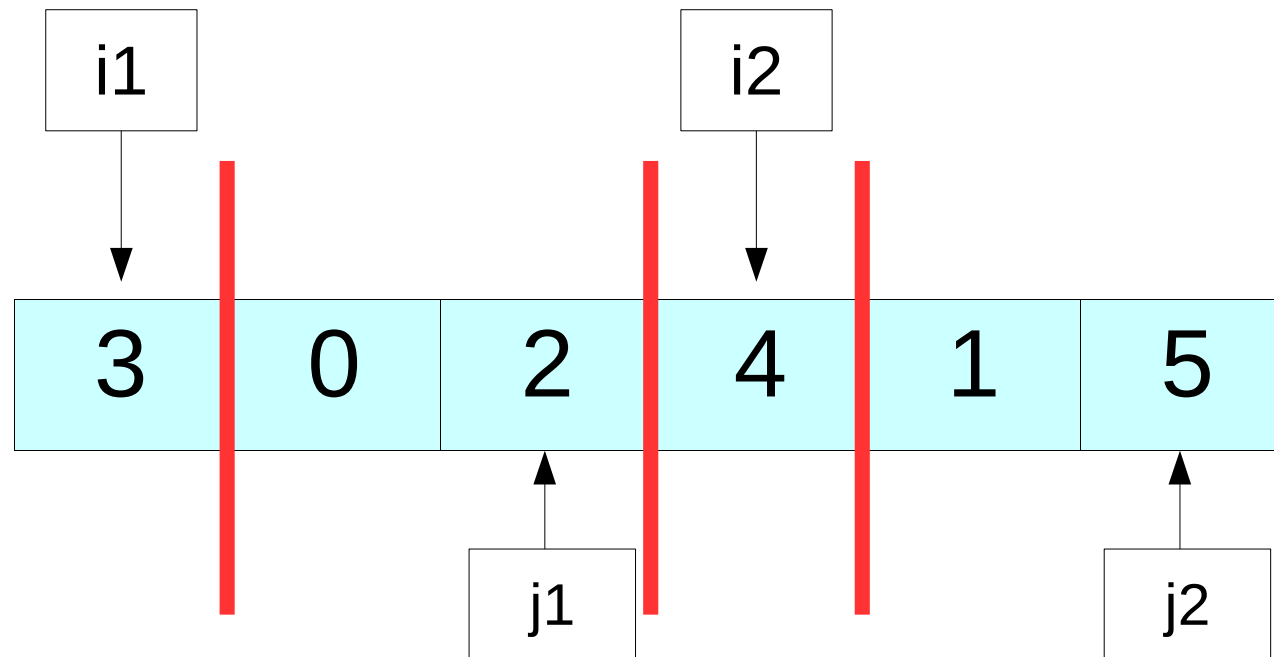
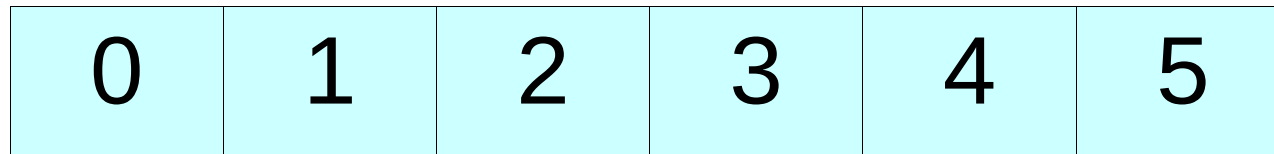


¿ $v[i] > v[j]$ ?

Inversiones + 1

Inversiones

5

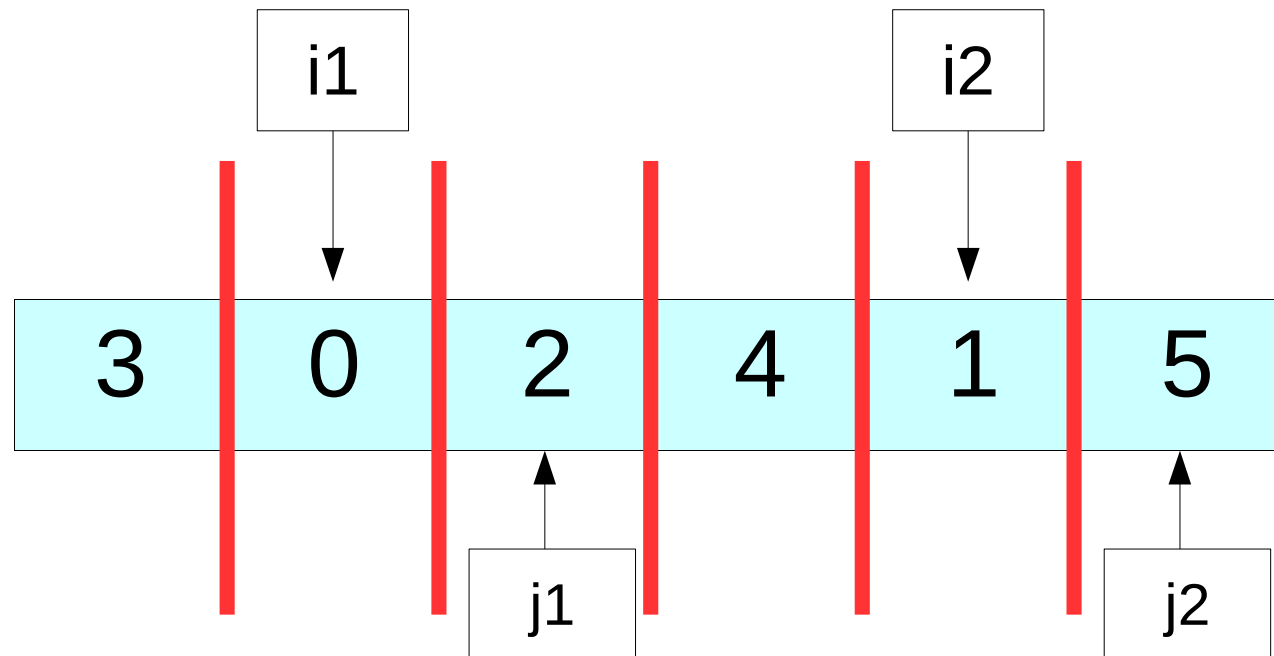
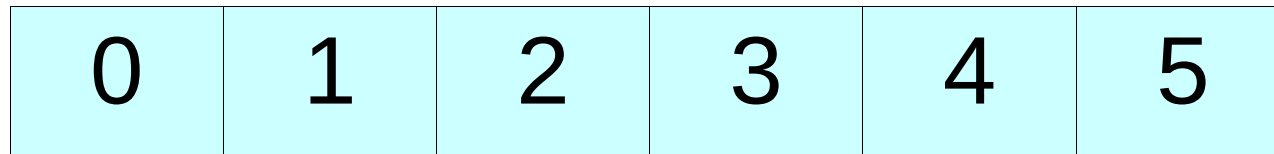


¿ $v[i] > v[j]$ ?

Inversiones + 1

Inversiones

5



¿ $v[i] > v[j]$ ?

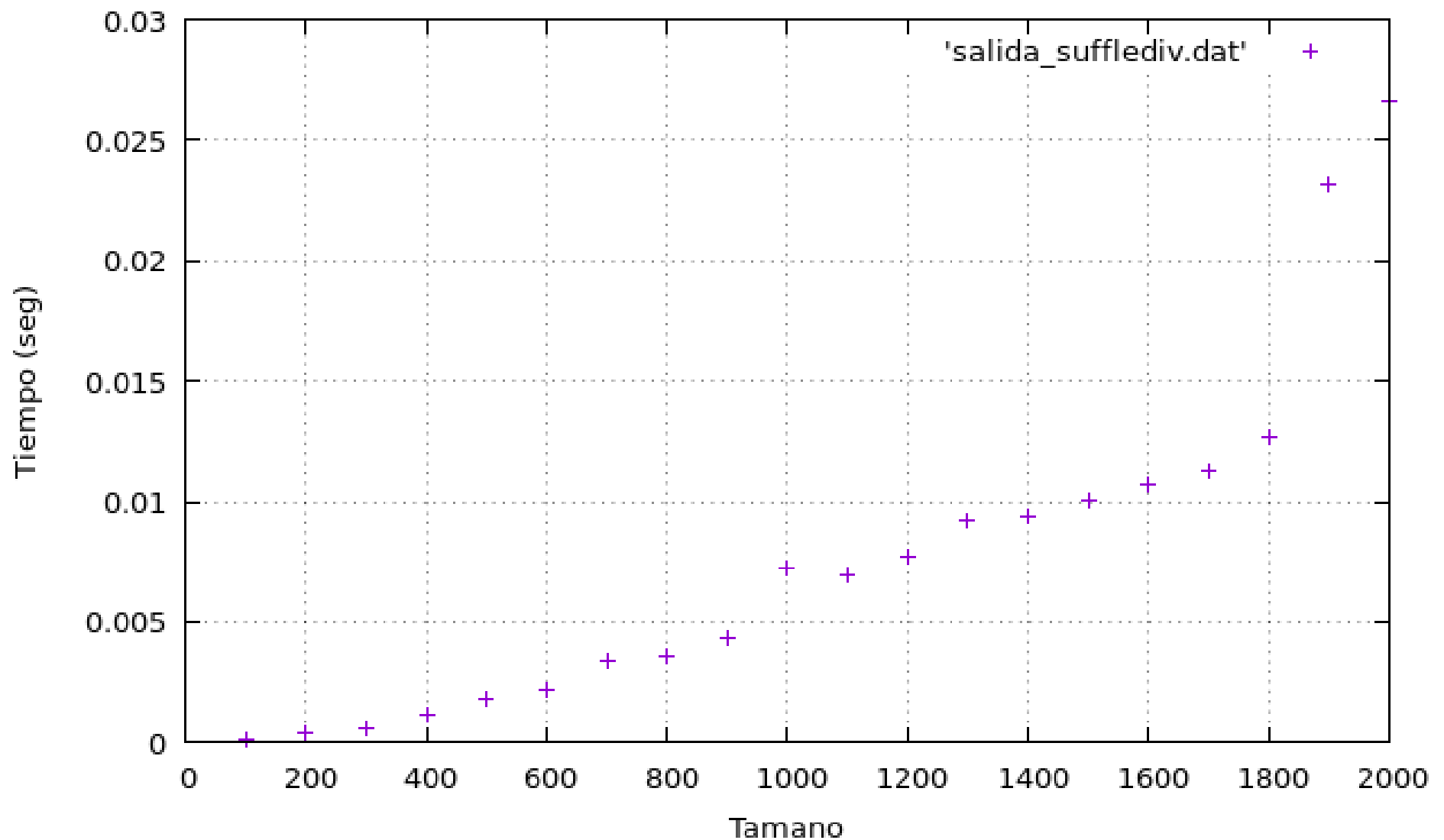
Inversiones + 1

# Eficiencia teórica

Como se trata de un algoritmo recursivo, para hallar la eficiencia teórica hay que resolver una ecuación en recurrencia.

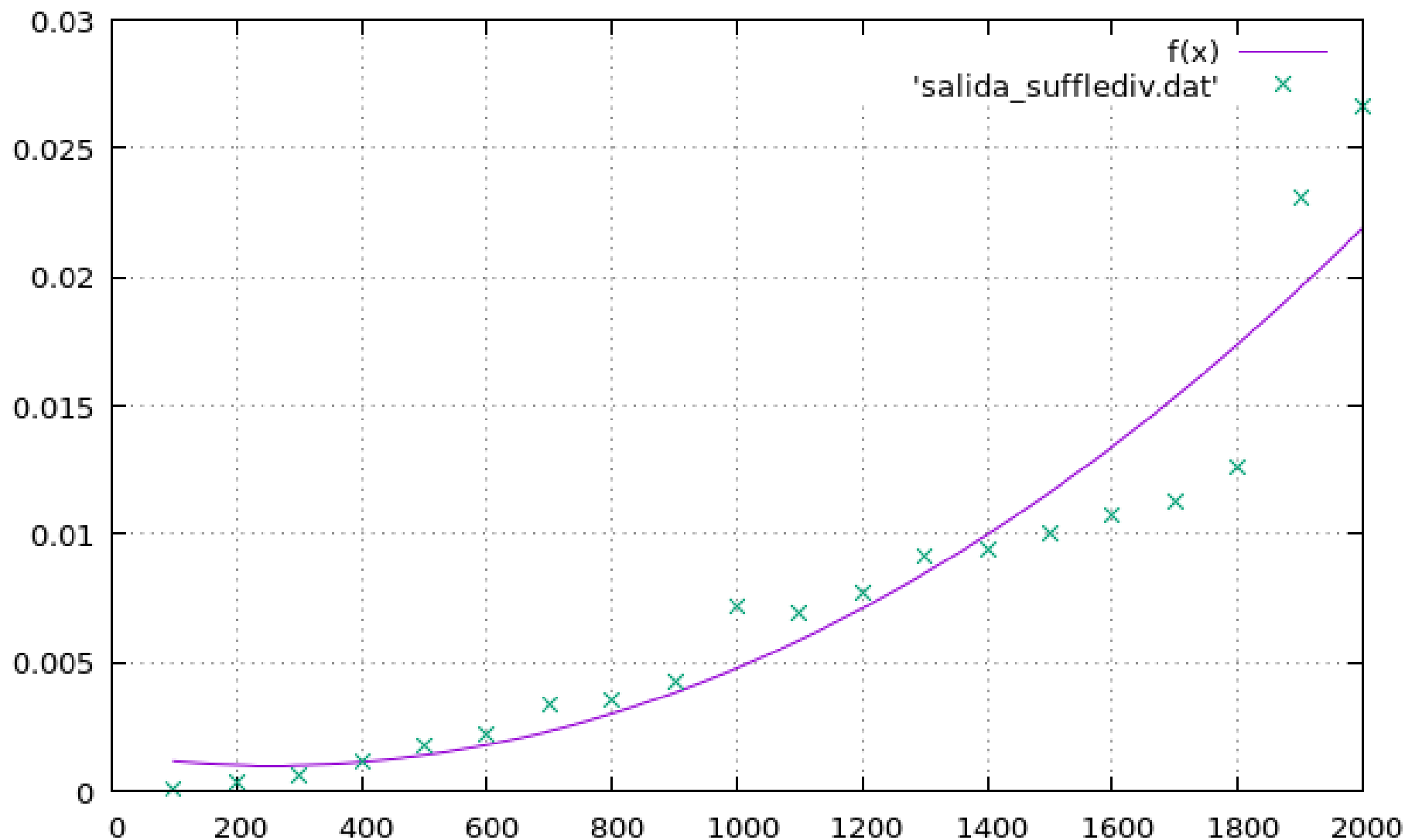
Eficiencia:  $O(n^2)$

# Eficiencia empírica



# Eficiencia híbrida:

$$f(x) = 6.89583 \cdot 10^{-5} x^2 - 3.56282 \cdot 10^{-5} x + 0.00146535$$



# Comparación de los dos algoritmos

