Artificial Intelligence HW 3: Uncertainty	Published:	2021/04/07
	Due date (volunteer):	2021/04/12, 10:00
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1. We want to develop a system that predicts the occupancy level of a restaurant. We have a dataset with the following historical data:

Weekend	End of month	Rain	Temperaturelevel	Occupancy level
yes	no	no	medium	high
yes	no	yes	medium	high
no	yes	no	low	medium
yes	no	no	high	high
no	yes	no	medium	medium
yes	yes	yes	low	medium
yes	yes	no	high	low
no	no	yes	medium	low

a) What are the hypotheses?
H1: Occupancy level=low. H2: Occupancy level=medium. H3: Occupancy level =high

b) What are the priors?
P(Occ. level=low) = 2/8, P(Occ. level=medium) = 3/8, P(Occ. level=high) = 3/8

c) What is the probability that the occupancy is high on a day at the end of the month, when it does not rain and with high temperature level?
P(Occ. level=high | end of month=yes, rain=no, temp. level=high) = 0 / 1

d) What does MAP decide for that day?
P(Occ. level=alta | end of month=yes, rain=no, temp. level=high) = 0 / 1
P(Occ. level=medium | end of month=yes, rain=no, temp. level=high) = 0 / 1
P(Occ. level=low | end of month=yes, rain=no, temp. level=high) = 1 / 1

According to MAP, Occupancy level = low

e) What does ML decide for that day?
P(end of month=yes, rain=no, high temperature | Occ. level=high) = 0 / 3
P(end of month=yes, rain=no, high temperature | Occ. level=medium) = 0 / 3
P(end of month=yes, rain=no, high temperature | Occ. level=low) = 1 / 2

According to ML, Occupancy level=low, since that is the hypothesis with highest likelihood (P(D | Hi))

f) What is the probability of the different hypotheses according to Naïve Bayes on a day at the end of the month, on weekend, without rain and with medium temperature level?
H1: Occ. level=low:
P(H1 | D) = P(D | H1) · P(H1) / P(D)
P(H1) = 2/8
According to Naïve Bayes,
P(D | H1) ~
P(end of month |Low Occ.)·P(weekend | Low Occ.)·P(rain=no | Low Occ.)·P(temp.=medium | Low Occ.)
= 1/2 · 1/2 · 1/2 · 1/2 = 1/16
Therefore, P(H1 | D) ~ 1/16 · 2/8 = 1/64 / P(D)

H2: Occ. level=medium:

P(H2 | D) = P(D | H2) · P(H2) / P(D)

P(H2) = 3/8

According to Naïve Bayes,

P(D|H2)~

P(end of month |Med.Occ.)·P(weekend | Med.Occ.)·P(rain=no | Med.Occ.)·P(temp.=medium | Med.Occ.)

= 3/3 · 1/3 · 2/3 · 1/3 = 2/27

Therefore, P(H2 | D) ~ 2/27 · 3/8 = 1/36 / P(D)

H3: Occ. level=high:

P(H3 | D) = P(D | H3) · P(H3) / P(D)

P(H3) = 3/8

According to Naïve Bayes,

P(D|H3)~

P(end of month |High occ.)·P(weekend | High occ.)·P(rain=no | High occ.)·P(temp.=medium | High occ.)

= 0/3 · 3/3 · 2/3 · 2/3 = 0

Therefore, P(H3 | D) ~ 0 · 3/8 = 0

On the other hand, P(D) = 1/64 + 1/36 + 0 = 100 / (64·36) = 25 / 576

Finally:

P(H1 | D) ~ 1 / 64 / (25 / 576) = 576 / (64·25) = 9 / 25

P(H2 | D) ~ 1 / 36 / (25 / 576) = 576 / (36·25) = 16 / 25

P(H3 | D) ~ 0

Note: symbol "~" is used to remind that Naïve Bayes is an approximation (it assumes that the evidences are independent given the hypothesis)

g) What is the probability of the different hypotheses according to Naïve Bayes, and using the Laplace estimator with μ / K = 1?

H1: Occ. level=low:
P(H1 | D) = P(D | H1) · P(H1) / P(D)
P(H1) = 2/8
According to Naïve Bayes,
P(D | H1) ~
P(end of month |Low Occ.)·P(weekend | Low Occ.)·P(rain=no | Low Occ.)·P(temp.=medium | Low Occ.)
= (1+1)/(2+2) · (1+1)/(2+2) · (1+1)/(2+2) · (1+1)/(2+3) = 1/2 · 1/2 · 1/2 · 2/5 = 1/20
Therefore, P(H1 | D) ~ 1/20 · 2/8 / P(D) = 1/80 / P(D)

H2: Occ. level=medium:
P(H2 | D) = P(D | H2) · P(H2) / P(D)
P(H2) = 3/8
According to Naïve Bayes,
P(D|H2)~
P(end of month |Med.Occ.)·P(weekend | Med.Occ.)·P(rain=no | Med.Occ.)·P(temp.=medium | Med.Occ.)
= (3+1)/(3+2) · (1+1)/(3+2) · (2+1)/(3+2) · (1+1)/(3+3) = 4/5 · 2/5 · 3/5 · 2/6 = 8 / 125
Therefore, P(H2 | D) ~ 8/125 · 3/8 / P(D) = 3/125 / P(D)

H3: Occ. level=high:
P(H3 | D) = P(D | H3) · P(H3) / P(D)
P(H3) = 3/8
According to Naïve Bayes,
P(D|H3)~
P(end of month |High occ.)·P(weekend | High occ.)·P(rain=no | High occ.)·P(temp.=medium | High occ.)
= (0+1)/(3+2) · (3+1)/(3+2) · (2+1)/(3+2) · (2+1)/(3+3) = 1/5 · 4/5 · 3/5 · 3/6 = 6/125
Therefore, P(H3 | D) ~ 6/125 · 3/8 / P(D) = 9 / 500 / P(D)

On the other hand, P(D) = 1/80 + 3/125 + 9/500 = (25 + 48 + 36) / 2000 = 109 / 2000

Finally:
P(H1 | D) ~ 1 / 80 / (109 / 2000) = 25/109
P(H2 | D) ~ 3 / 125 / (109 / 2000) = 48/109
P(H3 | D) ~ 9 / 500 / (109 / 2000) = 36/109

2. Consider the following joint probability tables, about the probabilities of winning an event that a sports team has, depending on whether it is a visitor or not, if it is the team with the highest budget or not, or if it is the team with best records:

Table 1: Joint probability of higher budget (or not) and winning(or not):

	Winner	Not winner
Higher budget	30%	A

Not higher budget	B	30%

Table 2: Joint probability of visitor (or not) and winning (or not):

	Winner	Not winner
Visitor	20%	30%
Not visitor	C	D

Table 3: Joint probability of better records (or not) and winning (or not):

	Winner	Not winner
Better records	50%	10%
Not better records	10%	E

a) What are the values of A, B, C, D, and E?

From table 3, P(Winner) = 0.5 + 0.1 = 0.6. Therefore P(not winner) = 1-0.6 = 0.4. Then, E= 0.4 - 0.1 = 0.3
Using this in table 1, B = 0.6 - 0.3 = 0.3, and A = 0.4 - 0.3 = 0.1
Finally, applying this in table 2, C = 0.6 - 0.2 = 0.4, and D = 0.4 - 0.3 = 0.1

b) What is the exact value of P (winner | visitor, better records)?

With the data provided it is not possible to compute (we do not have joint probabilities of these three variables).

c) What is the estimated value of P (winner | higher budget, better records) according to Naïve Bayes?

Hypothis H1 = winner:

P(H1 | D) = P(D | H1) · P(H1) / P(D)
P(H1) = 0.6

According to Naïve Bayes,

P(D | H1) = P(higher budget, better records | winner)
~ P(higher budget | winner)·P(better records | winner) = 0.3/0.6 · 0.5/0.6 = 5/12

Therefore, P(H1 | D) ~ 5/12 · 0.6 / P(D) = 1/4 / P(D)

Hypothesis H2 = not winner:

P(H2 | D) = P(D | H2) · P(H2) / P(D)
P(H2) = 0.4
According to Naïve Bayes,

P(D | H2) = P(higher budget, better records | not winner)
~ P(higher budget | not winner)·P(better records | not winner) = 0.1/0.4 · 0.1/0.4 = 1/16

Therefore, P(H2 | D) ~ 1/16 · 0.4 / P(D) = 1/40 / P(D)

On the other hand, P(D) = 1/4 + 1/40 = 11/40

Then:

P(H1 | D) ~ 1 / 4 / (11 / 40) = 10 / 11
P(H2 | D) ~ 1 / 40 / (11 / 40) = 1 / 11

Therefore, P(winner | higher budget, better records) ~ 10 / 11

d) What is the estimated value of P (winner | visitor, better records, higher budget) according to Naïve Bayes?

Hypothesis H1 = winner:

P(H1 | D) = P(D | H1) · P(H1) / P(D)
P(H1) = 0.6

According to Naïve Bayes,

P(D | H1) = P(visitor, higher budget, better records | winner)
~ P(visitor | winner)· P(higher budget | winner)·P(better records | winner)
= 0.2/0.6 · 0.3/0.6 · 0.5/0.6 = 5/36

Therefore, P(H1 | D) ~ 5/36 · 0.6 / P(D) = 5/60 / P(D) = 1/12 / P(D)

Hypothesis H2 = not winner:

P(H2 | D) = P(D | H2) · P(H2) / P(D)
P(H2) = 0.4

Según Naïve Bayes,

P(D | H2) = P(visitor, higher budget, better records | not winner)
~ P(visitor | not winner)·P(higher budget | not winner)·P(better records | not winner)
= 0.3/0.4 · 0.1/0.4 · 0.1/0.4 = 3/64

Therefore, P(H2 | D) ~ 3/64 · 0.4 / P(D) = 3/160 / P(D)

On the other hand, P(D) = 1/12 + 3/160 = 1/4 · (1/3 + 3/40) = 1/4 · 49/120 = 49/480

Finally:

P(H1 | D) ~ 1 / 12 / (49 / 480) = 40/49
P(H2 | D) ~ 3 / 160 / (49 / 480) = 9 / 49

Then, P(winner | visitor, higher budget, better records) ~ 40 / 49