

Computación Cuántica

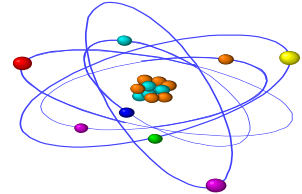
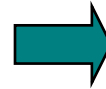
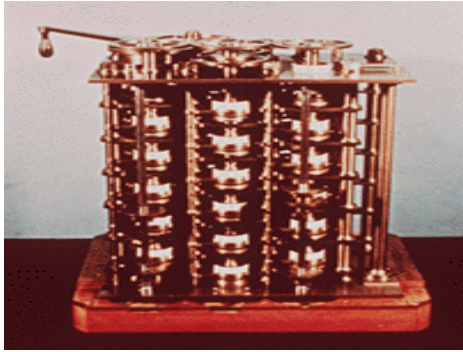
Procesamiento de Datos a Gran Escala

Contenidos

- Evolución de la tecnología: supremacía cuántica
- Bases de Mecánica Cuántica
 - Principio de Superposición.
 - Entrelazamiento, Interferencia y medida.
- Bases de la computación cuántica
 - Qubit y puertas cuánticas
 - Sistemas de n qubits : producto tensorial
 - Circuitos cuánticos: QisKit
 - Ordenador cuántico
- Algoritmos cuánticos
 - Deutsch_Jozsa
 - Bernstein-Vazirani
 - Groover
 - Shor (QFT)

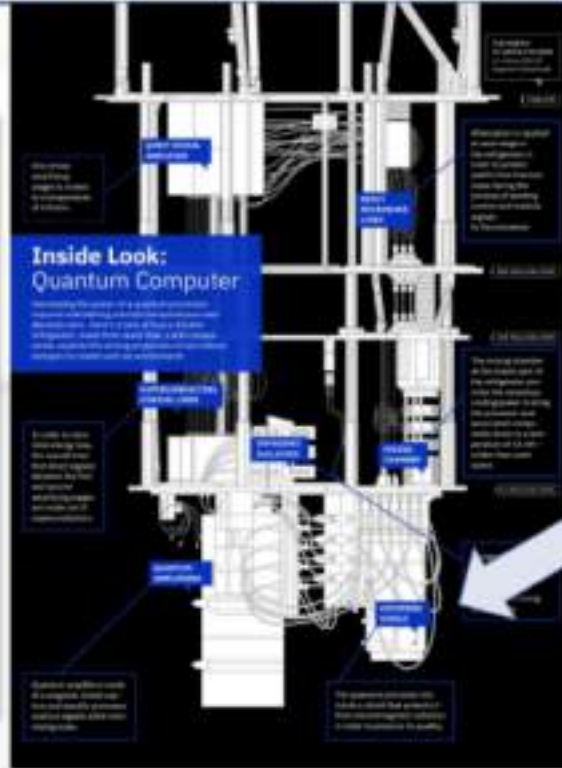
Evolución Tecnológica

La tecnología está haciendo los componentes de los computadores cada vez más pequeños...



...estamos alcanzando el punto donde la física clásica deja de ser válida => ha llegado el momento de la **física cuántica**

Yuri Manin (1980) and Richard Feynman (1981) proposed independently the concept of Quantum Computer



I'm here very "hot"!!
-273°C

Source: IBM

Supremacía cuántica



Google asegura haber logrado que un computador cuántico realice en segundos una tarea que a un ordenador clásico le llevaría miles de años

- * **ejecutar en 200 segundos** una operación para calcular números aleatorios que al superordenador más potente del mundo le hubiera llevado al menos 10.000 años.

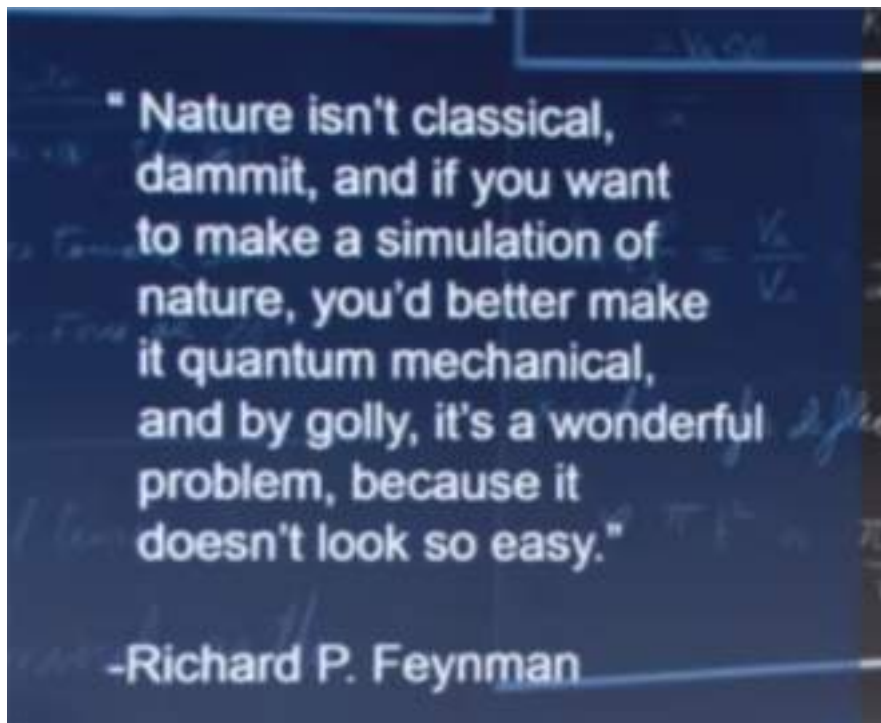
Poco después del anuncio, IBM lo ha puesto en duda, ya que considera que un superordenador basado en computación clásica (SUMMIT) podría ejecutar el mismo experimento que propone Google en dos días y medio.

Nadie entiende la mecánica cuántica

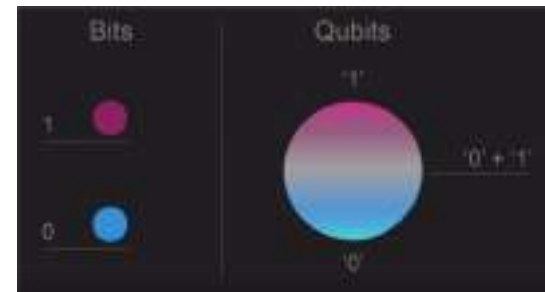
“No, you’re not going to be able to understand it. . . . You see, my physics students don’t understand it either. That is because I don’t understand it. Nobody does. ... The theory of quantum electrodynamics describes Nature as absurd from the point of view of common sense. And it agrees fully with an experiment. So I hope that you can accept Nature as She is -- absurd.

Richard Feynman

Mejor hacer un ordenador cuántico



Computación Cuántica: Qubit



Quantum bit

- Un sistema físico, con dos estados distinguibles, le denominamos **quantum bit (or qubit)**.
- Denominamos a los dos estados distinguibles con $|0\rangle$ y $|1\rangle$
- A diferencia de un bit clásico, un qubit puede estar en cualquier combinación lineal de estos estados:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \quad |\alpha|^2 + |\beta|^2 = 1$$

Donde α y β son números complejos.

- Si se realiza una medida el qubit, se encontrará en uno de los dos estados $|0\rangle$ o $|1\rangle$ con probabilidades $|\alpha|^2$ o $|\beta|^2$, y el sistema se quedará en el estado $|0\rangle$ o $|1\rangle$. Al medir se colapsa la función de onda a uno de los estados observables.

Computación Cuántica: Principios

Principio de superposición:

- La mecánica cuántica describe la información que tenemos de un sistema
 - La función de onda ψ describe la información del sistema (no describe la realidad)
- Si dos historias son diferentes pero compatibles, se suman
 - $\Psi = \psi_0 + \psi_1$



$$\Psi = \psi_{\text{vivo}} + \psi_{\text{muerto}}$$

Computación Cuántica: Principios

Evolución

- La mecánica cuántica dicta que la evolución del estado (n qubits $\rightarrow 2^n$ estados) es determinista.
- Los qubits se procesan en paralelo intrínsecamente. ¡Todo es en paralelo!

Medidas

- La medida de observables sobre este estado son probabilísticas.
- Es imposible conocer de forma exacta el estado de un sistema

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$
$$P_{S_z, |\psi\rangle}(0) = |\alpha|^2$$
$$P_{S_z, |\psi\rangle}(1) = |\beta|^2$$

Computación Cuántica: Sistema de 2 estados

2-Level Quantum System

$|\bullet\rangle$

$|\bullet\rangle$

Superposition of Quantum states (Schrödinger's Cat)



$$\Psi = \frac{1}{\sqrt{2}} (|\bullet\rangle + |\bullet\rangle)$$

Different Bash to define/measure the same quantum system

$$|\bullet\rangle = \frac{1}{\sqrt{2}} (|\bullet\rangle + |\bullet\rangle)$$
$$|\bullet\rangle = \frac{1}{\sqrt{2}} (|\bullet\rangle + |\bullet\rangle)$$
$$\Psi = \frac{1}{2} (2|\bullet\rangle + |\bullet\rangle + |\bullet\rangle)$$

Most quantum systems

Mixed Quantum State

$$\Psi = (a|\bullet\rangle + b|\bullet\rangle)$$


Computación Cuántica: Registro de múltiples Qubit

- In general, an n qubit register can represent the numbers 0 through $2^n - 1$ simultaneously.

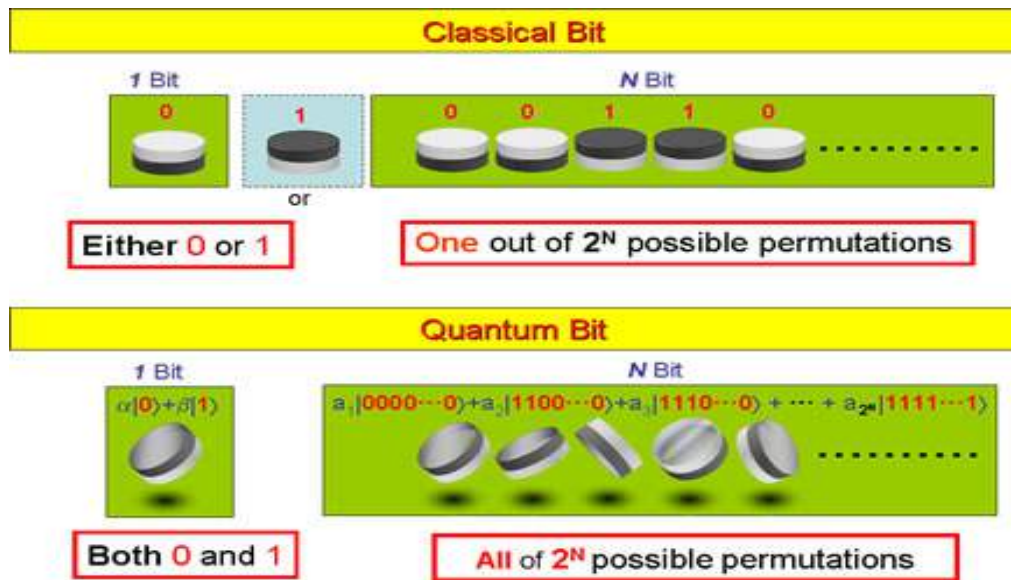
Sound too good to be true?...It is!

- If we attempt to retrieve the values represented within a superposition, the **superposition randomly collapses** to represent just one of the original values.

- Measurement Example:

$$0.316|00\rangle + 0.447|01\rangle + 0.548|10\rangle + 0.632|11\rangle$$

The probability to read the rightmost bit as 0 is $|0.316|^2 + |0.548|^2 = 0.4$



Computación Cuántica: Número de estados (Escala exponencial)

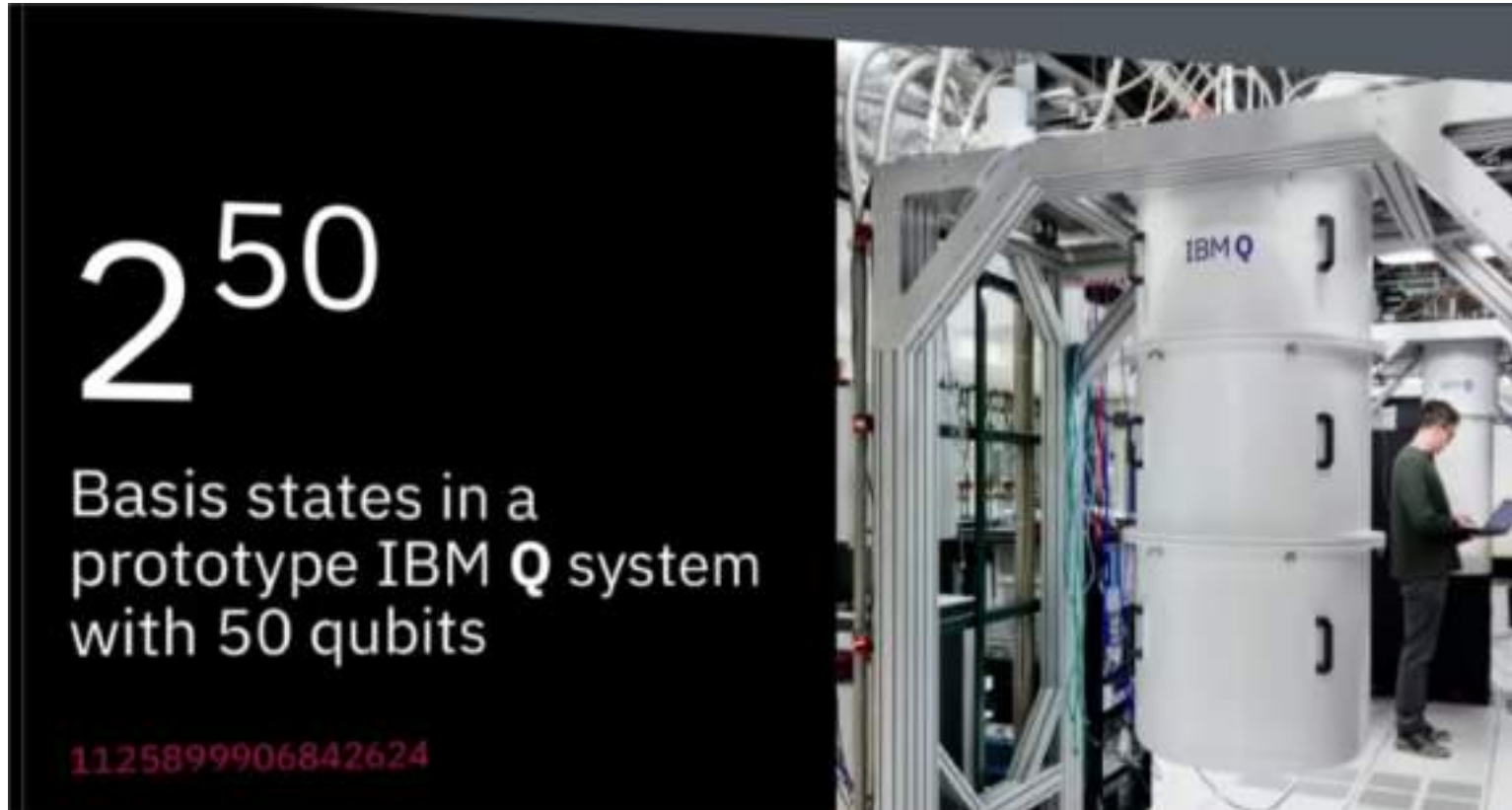


							128
256	512	1024	2048	4096	8192	16384	32768
65536	131K	262K	524K	1M	2M	4M	8M
16M	33M	67M	134M	268M	536M	1G	2G
4G	8G	17G	34G	68G	137G	274G	549G
1T	2T	4T	8T	17T	35T	70T	140T
281T	562T	1P	2P	4P	9P	18P	36P
72P	144P	288P	576P	1E	2E	4E	9E

$$T_{64} = 2^0 + 2^1 + 2^2 + \dots + 2^{63}$$

$$s = 2^{64} - 1 = 18\,446\,744\,073\,709\,551\,615$$

Computación Cuántica: Escala exponencial



Computación Cuántica: Escala exponencial



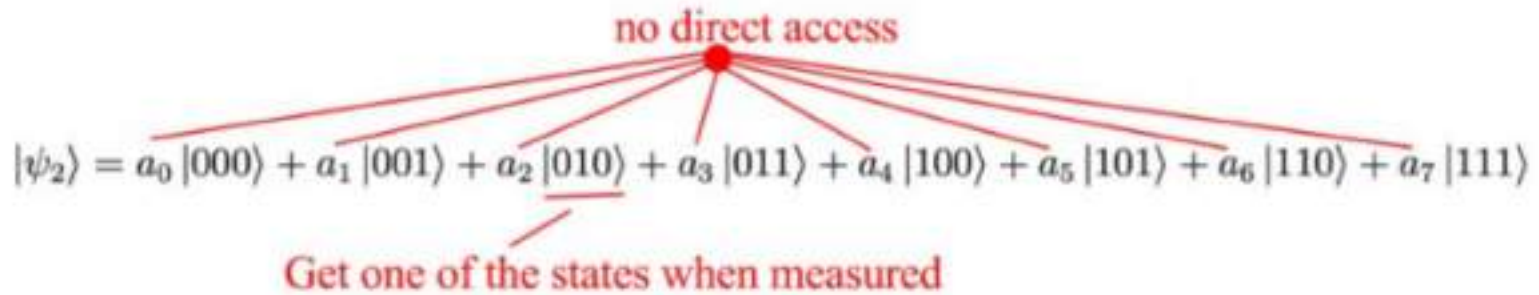
2^{275}

More basis states than
there are atoms in the
observable universe

607084028820540334662331845882349
658325752137203793600391191378043
40758912662765568

Computación Cuántica: Podemos computar pero no ver el resultado

But, there is a big catch! We can manipulate information in a very high dimensional space but we cannot read those coefficients directly. When all operations are completed, the only way to “read” the qubits is to measure it which returns one of the states only (not the coefficient).



Relaciones entre datos: Entrelazamiento de estados (**Entanglement**)

■ **Entrelazamiento** es la capacidad de un sistema cuántico de exhibir correlaciones dentro de una superposición.

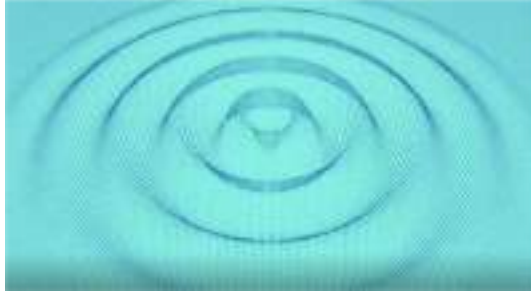
- El entrelazamiento es un fenómeno cuántico, sin equivalente clásico, en el cual los estados cuánticos de dos o más objetos se deben describir mediante un estado único que involucra a todos los objetos del sistema, aún cuando los objetos estén separados espacialmente.
- El estado de un qubit depende del estado de otro qubit.
- Observando el estado de un qubit puedo conocer el estado del otro al que está entrelazado sin necesidad de medir el segundo.



$$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

Principio de Interferencia

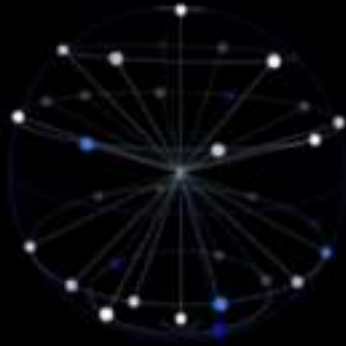
- **Interferencia** entre funciones de onda que modifica, aumentando o disminuyendo las amplitudes, incluso las puede llegar a anular.
- Es la base de los algoritmos cuánticos. QAE (Quantum Amplitud Estimation)



Bases del funcionamiento del computador cuántico

How do quantum computers work?

Universal quantum computers leverage quantum mechanical properties of superposition and entanglement to create states that scale exponentially with number of qubits, or quantum bits.



Superposition

A single quantum bit can exist in a superposition of 0 and 1, and N qubits allow for a superposition of all possible 2^N combinations.



Entanglement

The states of entangled qubits cannot be described independently of each other.

Representación de un Qubit

- Qubit - $|1\rangle$, $|0\rangle$ o superposición de ambos
- $|x\rangle$ Registro en el computador cuántico

- Superposición de estados:

$$\sum_{i=0}^{2^N-1} a_i |s_i\rangle \quad \text{con:} \quad \sum_{i=0}^{2^N-1} |a_i|^2 = 1$$

Sistema de 1Qubit

$$\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

Sistema de 2Qubits

$$\frac{1}{2}|00\rangle + \frac{1}{2}|01\rangle + \frac{1}{2}|10\rangle + \frac{1}{2}|11\rangle$$

Sistema de nQubits

$$\alpha_0 \cdot |\psi_0\rangle + \alpha_1 \cdot |\psi_1\rangle + \dots + \alpha_{2^n-1} \cdot |\psi_{2^n-1}\rangle$$

$$\sum_{i=0}^{2^n-1} |\alpha_i|^2 = 1$$

Representación de un Qubit

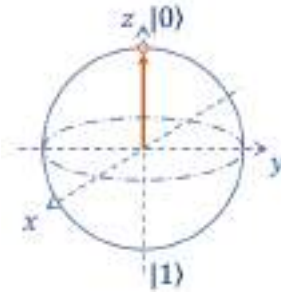
n qubits: $2^n \times 1$ matrix represents the state

1 qubit:

■ $|0\rangle$ would be represented by $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

■ $|1\rangle$ would be represented by $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

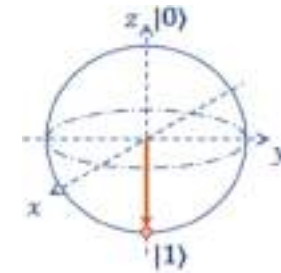
■ Equal superposition would be $\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$



$$1 \cdot |0\rangle + 0 \cdot |1\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$P(0) = 1^2 = 1 = 100\%$$

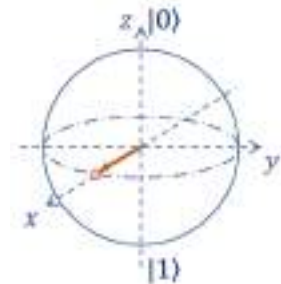
$$P(1) = 0^2 = 0 = 0\%$$



$$0 \cdot |0\rangle + 1 \cdot |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$P(0) = 0^2 = 0 = 0\%$$

$$P(1) = 1^2 = 1 = 100\%$$

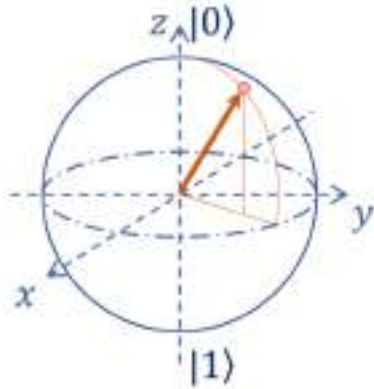


$$\frac{1}{\sqrt{2}} \cdot |0\rangle + \frac{1}{\sqrt{2}} \cdot |1\rangle = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$$

$$P(0) = \left(\frac{1}{\sqrt{2}}\right)^2 = 1/2 = 50\%$$

$$P(1) = \left(\frac{1}{\sqrt{2}}\right)^2 = 1/2 = 50\%$$

Representación de un Qubit



1 Qubit ($n=1$)

$$\alpha \cdot |0\rangle + \beta \cdot |1\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}; \quad |\alpha|^2 + |\beta|^2 = 1 \quad \longrightarrow \quad \begin{aligned} P(0) &= |\alpha|^2 \\ P(1) &= |\beta|^2 \end{aligned}$$

2 Qubits ($n=2$)

$$\alpha \cdot |00\rangle + \beta \cdot |01\rangle + \gamma \cdot |10\rangle + \delta \cdot |11\rangle = \begin{pmatrix} \alpha \\ \beta \\ \gamma \\ \delta \end{pmatrix}$$

$$P(00) = |\alpha|^2$$

$$P(01) = |\beta|^2$$

$$P(10) = |\gamma|^2$$

$$P(11) = |\delta|^2$$

$$|\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2 = 1$$

Qubit

Representación

Matemática:

Vector complejo
expresado en una
base de un
espacio de Hilbert

Un qubit es un vector de la forma $\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$ donde $\alpha, \beta \in \mathbb{C}$ y $|\alpha|^2 + |\beta|^2 = 1$.

Se considera una base del espacio de qubits, por ejemplo:

$$\left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}$$

entonces un qubit tendrá la forma

$$\alpha \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Llamaremos $|0\rangle$ al vector $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ y $|1\rangle$ al vector $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$, así, a cualquier qubit $|\psi\rangle$ lo escribiremos como

$$|\psi\rangle = \alpha |0\rangle + \beta |1\rangle$$

Puertas Cuánticas (1 qubit)

Una Compuerta Cuántica para 1 qubit será una matriz U tal que

$$UU^\dagger = U^\dagger U = I$$

donde $U^\dagger = (U^*)^T$

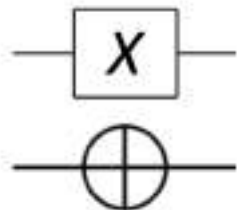
Por ejemplo:

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

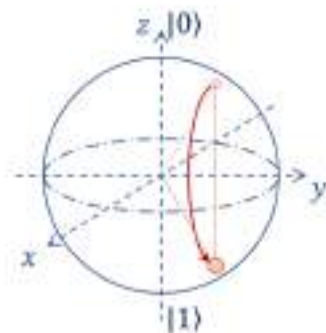
Veamos cómo actúa esta compuerta sobre un qubit $|\psi\rangle$ cualquiera:

$$\begin{aligned} X|\psi\rangle &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} (\alpha|0\rangle + \beta|1\rangle) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \\ &= \begin{pmatrix} \beta \\ \alpha \end{pmatrix} = \beta|0\rangle + \alpha|1\rangle \end{aligned}$$

Puertas Cuánticas: NOT cuántica



Y	NOT(Y)
$ 0\rangle$	$ 1\rangle$
$ 1\rangle$	$ 0\rangle$
$ \psi\rangle$	$X(\psi\rangle)$



$$\text{Pauli} - X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

En los qubits de la base canónica vemos que

$$X |0\rangle = |1\rangle, \quad X |1\rangle = |0\rangle$$

Por lo cual, la compuerta X es comunmente llamada *compuerta NOT*.

En general, la aplicación de una compuerta cuántica a un qubit se puede ver de la siguiente manera:

$$U(\alpha |0\rangle + \beta |1\rangle) = \alpha U |0\rangle + \beta U |1\rangle$$

Por lo cual, con sólo describir de qué manera actúa en una base, ya habremos descripto la compuerta completamente.

Puertas Cuánticas: Hadamard

- Simplest gate involving one qubit and is called a **Hadamard Gate**

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

- Used to put qubits into superposition.

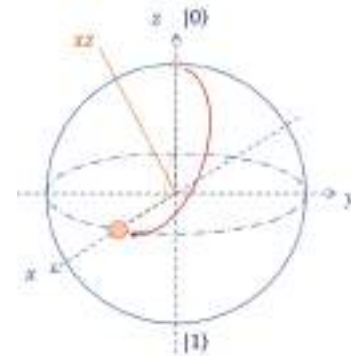
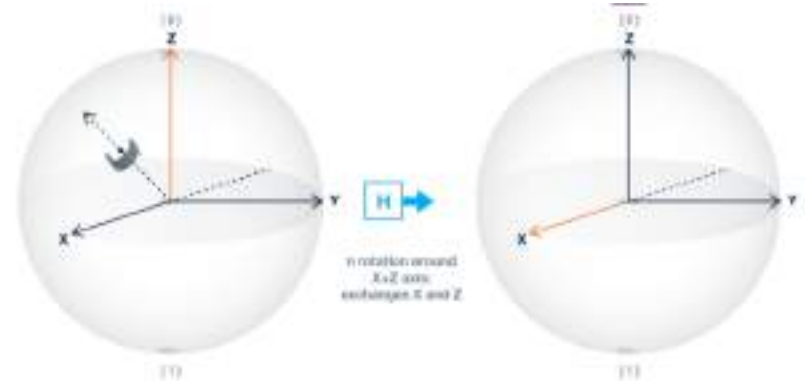
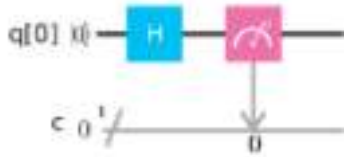
$$H|0\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) = |+\rangle$$

$$H|1\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle) = |-\rangle$$

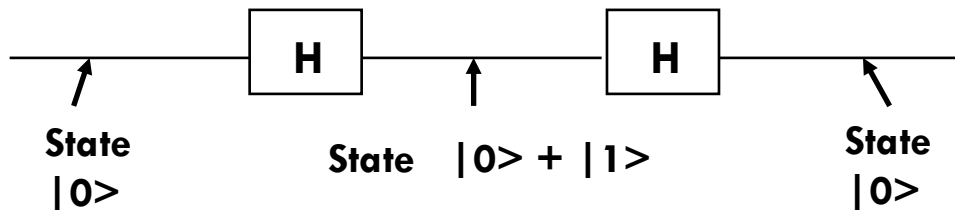
Interpretación con la Esfera de Bloch

$$H = \frac{1}{\sqrt{2}} \cdot \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

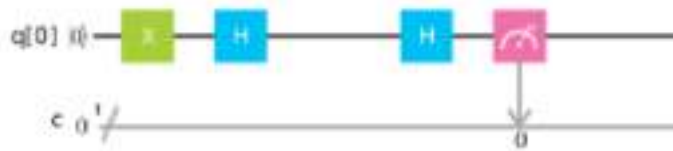
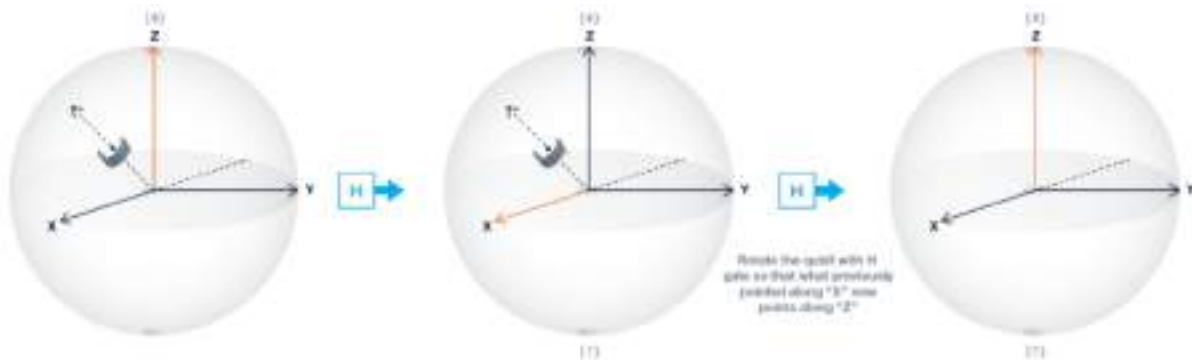
$$H|0\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$



Interpretación puerta Hadamard con la Esfera de Bloch



$$\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$



<https://qiskit.org/textbook/ch-states/single-qubit-gates.html>

Puertas Cuánticas de 1 Qubit

Hadamard		Inicial Final	$ 0\rangle$ $\frac{1}{\sqrt{2}}(0\rangle + 1\rangle)$	$ 1\rangle$ $\frac{1}{\sqrt{2}}(0\rangle - 1\rangle)$	$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$
Puerta X		Inicial Final	$ 0\rangle$ $ 1\rangle$	$ 1\rangle$ $ 0\rangle$	$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
Puerta Y		Inicial Final	$ 0\rangle$ $i 1\rangle$	$ 1\rangle$ $-i 0\rangle$	$Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$
Puerta Z		Inicial Final	$ 0\rangle$ $ 0\rangle$	$ 1\rangle$ $- 1\rangle$	$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
Puerta S		Inicial Final	$ 0\rangle$ $ 0\rangle$	$ 1\rangle$ $i 0\rangle$	$S = \begin{pmatrix} 1 & 0 \\ 0 & +i \end{pmatrix}$
Puerta T		Inicial Final	$ 0\rangle$ $ 0\rangle$	$ 1\rangle$ $e^{i\pi/4} 1\rangle$	$T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$

Puertas Cuánticas: Otras bases son posibles

$$|+\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle)$$

$$|-\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle)$$

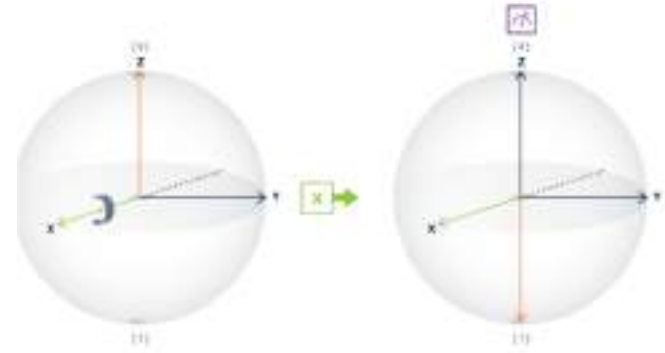
son ortogonales, por lo tanto forman base:

$$B = \{|+\rangle, |-\rangle\}$$

Ejercicio: demostrar la ortogonalidad de $|+\rangle$ y $|-\rangle$

Puertas cuánticas: Not vs CNot Gate

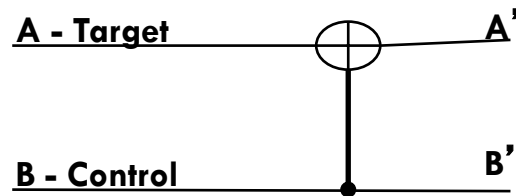
Not or X Gate: $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} b \\ a \end{bmatrix}$



Controlled Not Gate (Cnot):

A gate which operates on two qubits is called a **Controlled-NOT (CN) Gate**.
If the bit on the control line is 1, invert the bit on the target line.

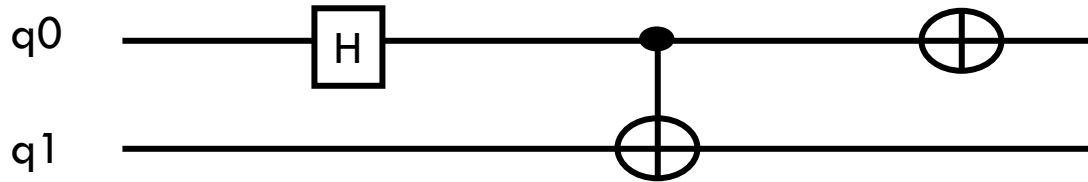
A	B	A'	B'
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	1



$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} a \\ b \\ d \\ c \end{bmatrix}$$

Visualización de un Circuito Cuántico

Quantum circuits are a way of representing unitary transformations as a composition of simple unitaries acting on one or two q-bits at a time.

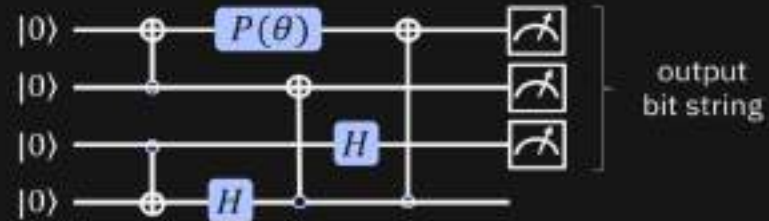


$$\begin{aligned} |00\rangle &\longrightarrow \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|01\rangle \longrightarrow \\ \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle &\longrightarrow \frac{1}{\sqrt{2}}|01\rangle + \frac{1}{\sqrt{2}}|10\rangle \end{aligned}$$

Circuito cuántico

(Static) quantum circuits

IBM Quantum



Time flows from left to right.

Each line is a qubit, width refers to the number of qubits.

Depth is the minimum number of steps required to execute the circuit.

All interesting quantum circuits can be built from a universal set of gates.

Quantum circuit

A quantum circuit is a computational routine consisting of coherent quantum operations on quantum data, such as qubits, and concurrent real-time classical computation. **It is an ordered sequence of quantum gates, measurements and resets, which may be conditioned on and use data from real-time classical computations.** A set of quantum gates is said to be universal if any unitary transformation of the quantum data can be efficiently approximated arbitrarily well as a sequence of gates in the set. Any quantum program can be represented by a sequence of quantum circuits and non-concurrent classical computation.

<http://qiskit.org/textbook/ch-circuits/defining-circuits.html>

Algoritmo cuántico

IBM Quantum

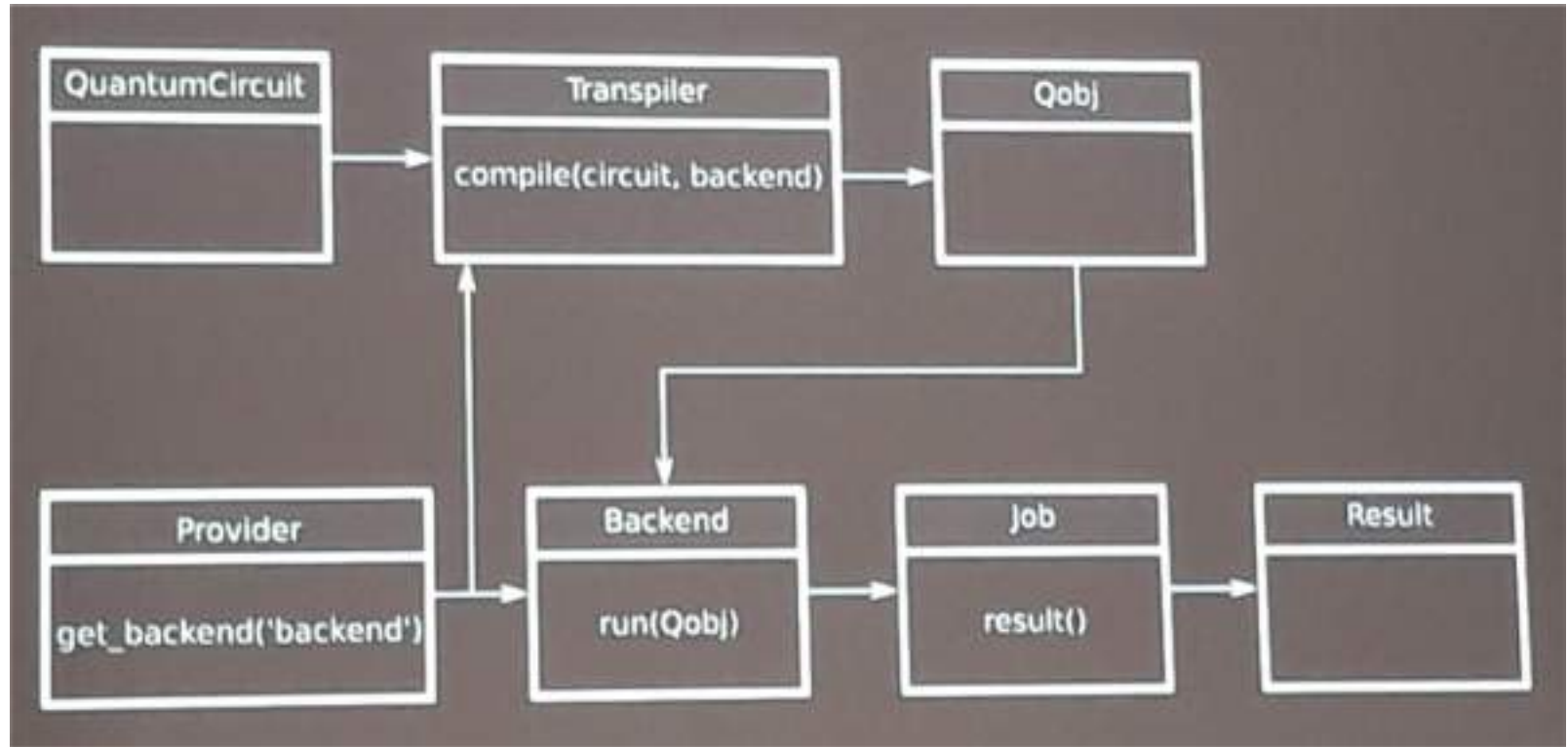
The zoo of quantum algorithms

There are a "few hundred" algorithms... <https://quantumalgorithmzoo.org>

Algebraic and number theoretic algorithms	Oracular algorithms	Approximation and simulation algorithms	Optimization, numerics, and machine-learning
<ul style="list-style-type: none">• Algorithm: factoring (superpolynomial)• Algorithm: discrete-log (superpolynomial)• Algorithm: verifying matrix products (polynomial)	<ul style="list-style-type: none">• Algorithm: searching (polynomial)• Algorithm: formula evaluation (polynomial)• Algorithm: hidden shift (superpolynomial)	<ul style="list-style-type: none">• Algorithm: quantum simulation (superpolynomial)• Algorithm: semidefinite programming (superpolynomial)	<ul style="list-style-type: none">• Algorithm: constraint satisfaction (polynomial)• Adiabatic Algorithm (unknown)• Algorithm: machine learning (varies)

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Circuito cuántico en un ordenador



Puertas lógicas cuánticas universales

- Conjunto universal de puertas:
 $\{\text{CNOT, Hadamard, fase relativa}\}$

Arbitrary relative phase

$$\begin{aligned}U_{\phi}|0\rangle &= |0\rangle \\U_{\phi}|1\rangle &= e^{i\phi}|1\rangle\end{aligned}$$

Hadamard

$$\begin{aligned}U_H|0\rangle &= \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \\U_H|1\rangle &= \frac{1}{\sqrt{2}}(|1\rangle - |0\rangle)\end{aligned}$$

CNOT (XOR)

$$\begin{aligned}U_{CNOT}|0\rangle|0\rangle &= |0\rangle|0\rangle \\U_{CNOT}|0\rangle|1\rangle &= |0\rangle|1\rangle \\U_{CNOT}|1\rangle|0\rangle &= |1\rangle|1\rangle \\U_{CNOT}|1\rangle|1\rangle &= |1\rangle|0\rangle\end{aligned}$$

Sistema de 2 Qubits

Para extender este sistema a 2 qubits haremos un “producto tensorial” entre las bases de cada sistema de 1 qubit.

$$|a\rangle = \begin{bmatrix} a_0 \\ a_1 \end{bmatrix}, \quad |b\rangle = \begin{bmatrix} b_0 \\ b_1 \end{bmatrix}$$

$$|ba\rangle = |b\rangle \otimes |a\rangle = \begin{bmatrix} b_0 \times \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} \\ b_1 \times \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} \end{bmatrix} = \begin{bmatrix} b_0 a_0 \\ b_0 a_1 \\ b_1 a_0 \\ b_1 a_1 \end{bmatrix}$$

Producto tensorial entre matrices

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix} \otimes B = \begin{pmatrix} a_{11}B & a_{12}B & \dots & a_{1n}B \\ a_{21}B & a_{22}B & \dots & a_{2n}B \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1}B & a_{m2}B & \dots & a_{mn}B \end{pmatrix}$$

Producto tensorial entre vectores: ídem matrices

$$\begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix} \otimes w = \begin{pmatrix} v_1 \cdot w \\ v_2 \cdot w \\ \vdots \\ v_n \cdot w \end{pmatrix}$$

Sistema de 2 Qubits

Para extender este sistema a 2 qubits haremos un “producto tensorial” entre las bases de cada sistema de 1 qubit.

$$|0\rangle \otimes |0\rangle \equiv |00\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$|0\rangle \otimes |1\rangle \equiv |01\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$|1\rangle \otimes |0\rangle \equiv |10\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$|1\rangle \otimes |1\rangle \equiv |11\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \otimes \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Sistema de 3 Qubits

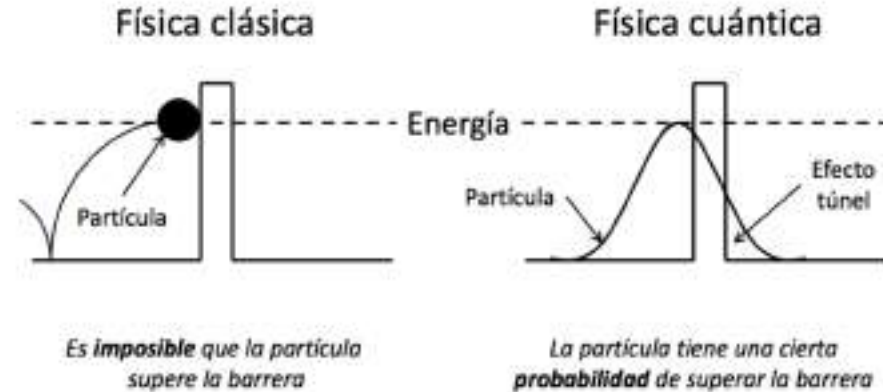
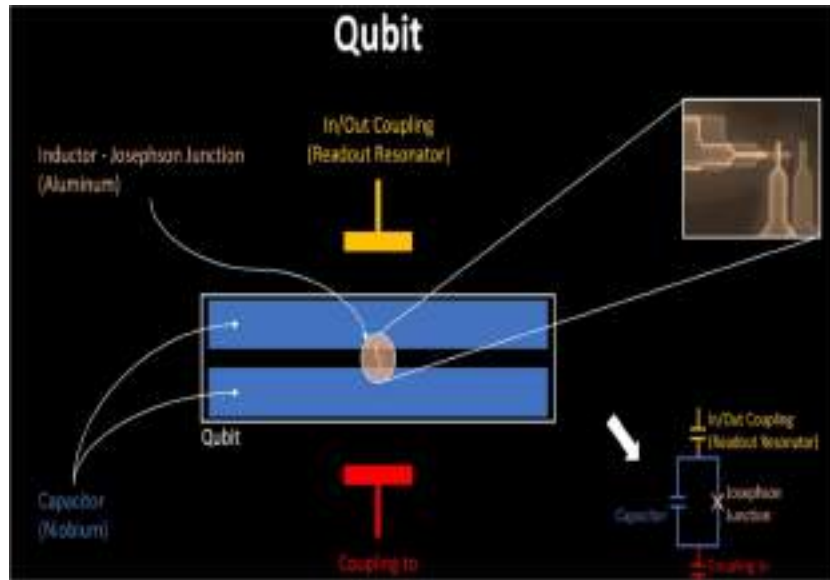
Para extender este sistema a 3 qubits haremos un “producto tensorial” entre las bases de cada sistema de 1 qubit.

$$|1\rangle \otimes |1\rangle \otimes |0\rangle = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

<https://qiskit.org/textbook/ch-gates/multiple-qubits-entangled-states.html>

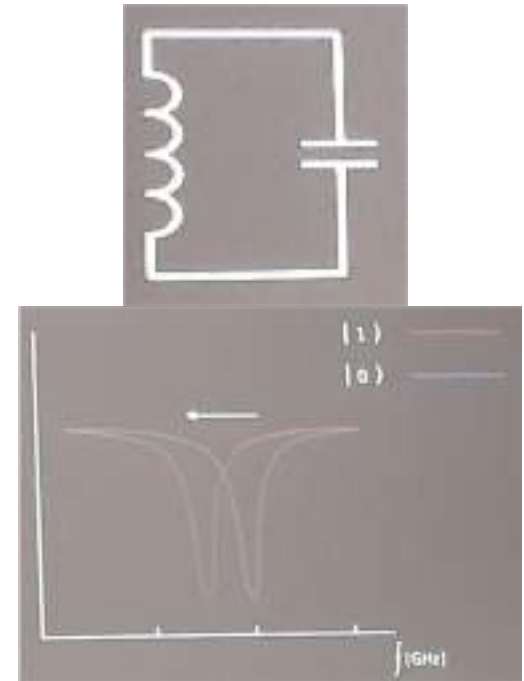
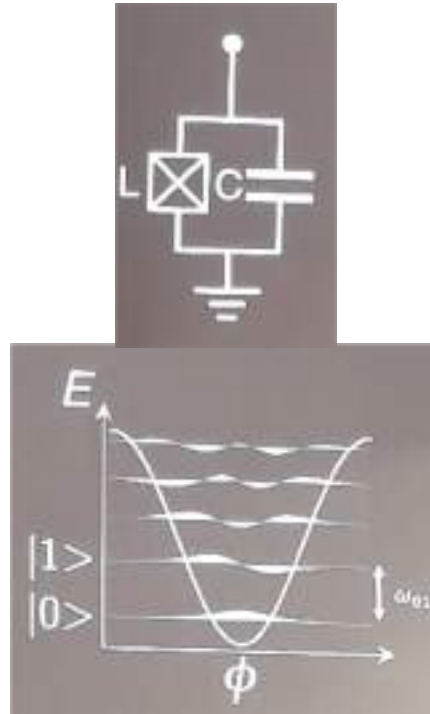
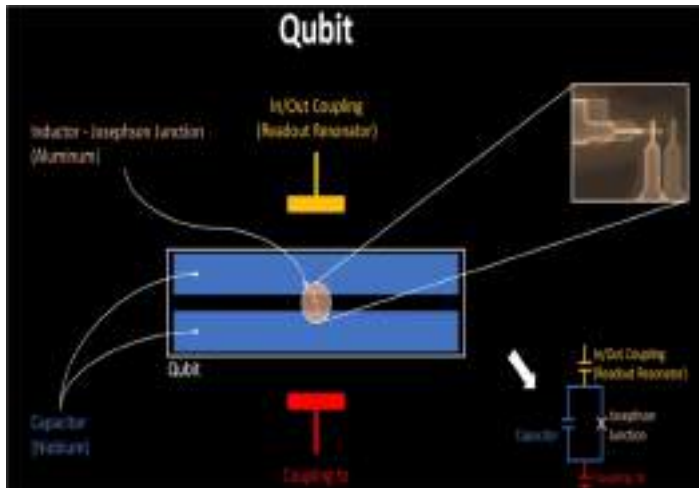
Como hacer un Qubit: Solid-state device

- Circuito superconductor basado en una unión de Josephson que crea un enlace débil entre dos superconductores.

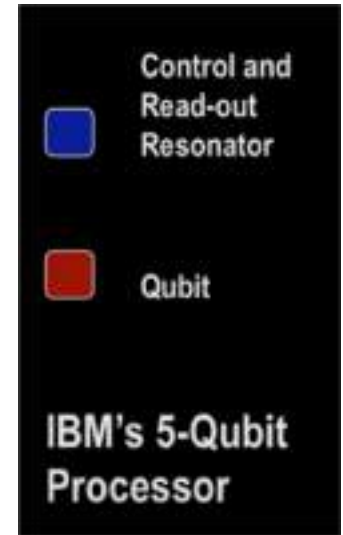
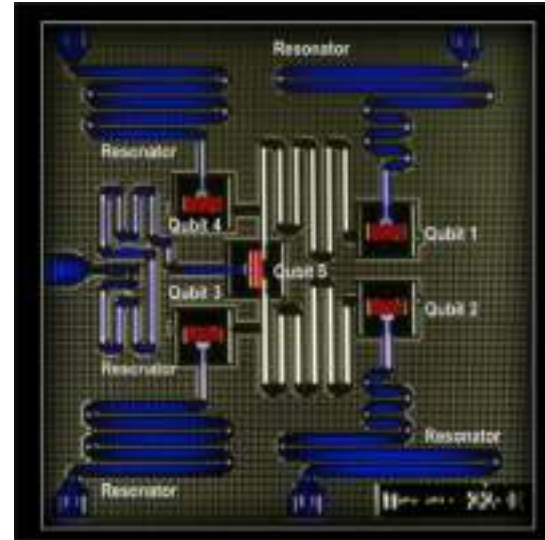
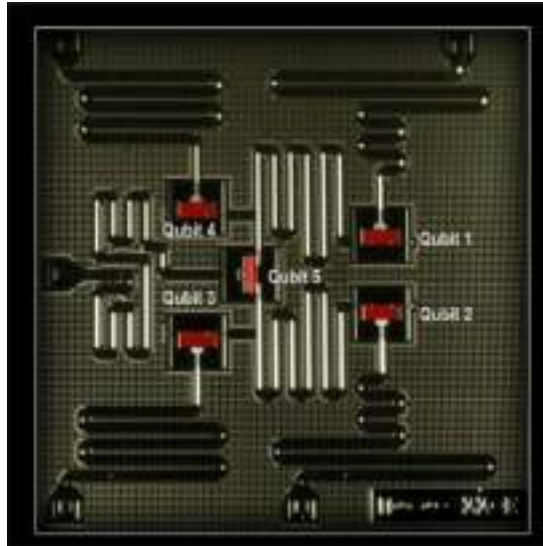


Como hacer un Qubit: Solid-state device

- Estados $|0\rangle$ y $|1\rangle$ con diferente energía



Conectando y controlando los Qubit

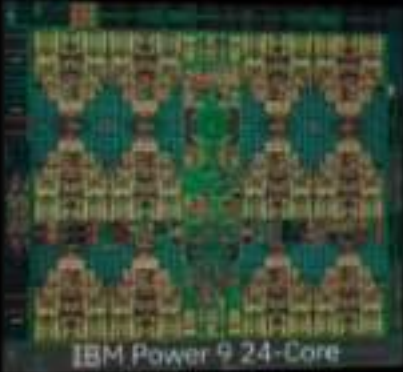


Escalado de potencia en un computador cuántico

The power of quantum computing

Classical Computers


The potential power of a classical computer doubles every time you double the number of transistors.



IBM Power 9 24-Core

Quantum Computers

The potential power of a quantum computer doubles every time you add one additional qubit.



IBM Q-16 Qubit Device

Escalado de potencia en un computador cuántico

- Quantum computers, would be based on the strange principles of quantum mechanics, in which the smallest particles of light and matter can be in different places at the same time.
- In a quantum computer, one "qubit" - quantum bit - could be both 0 and 1 at the same time. So with three qubits of data, a quantum computer could store all eight combinations of 0 and 1 **simultaneously**. That means a three-qubit quantum computer could calculate eight times faster than a three-bit digital computer.
- Typical personal computers today calculate 64 bits of data at a time. **A quantum computer with 64 qubits would be 2 to the 64th power faster, or about 18 billion billion times faster. (Note: billion billion is correct.)**

Computador clásico

Registros de bits + Puertas Lógicas y Memoria = Computador Clásico

0



+



=

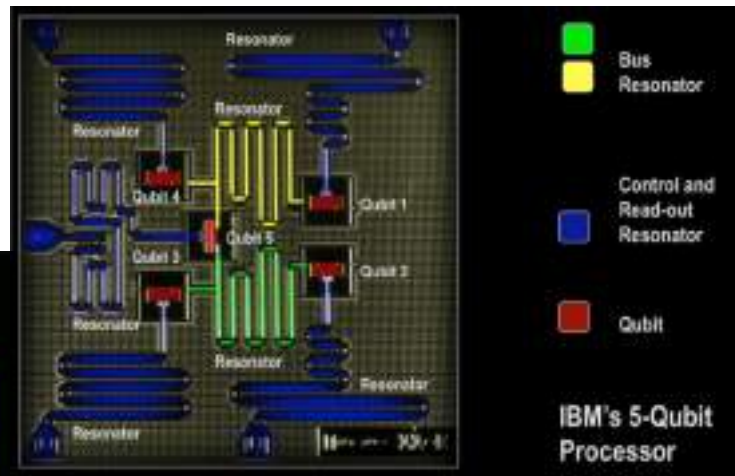
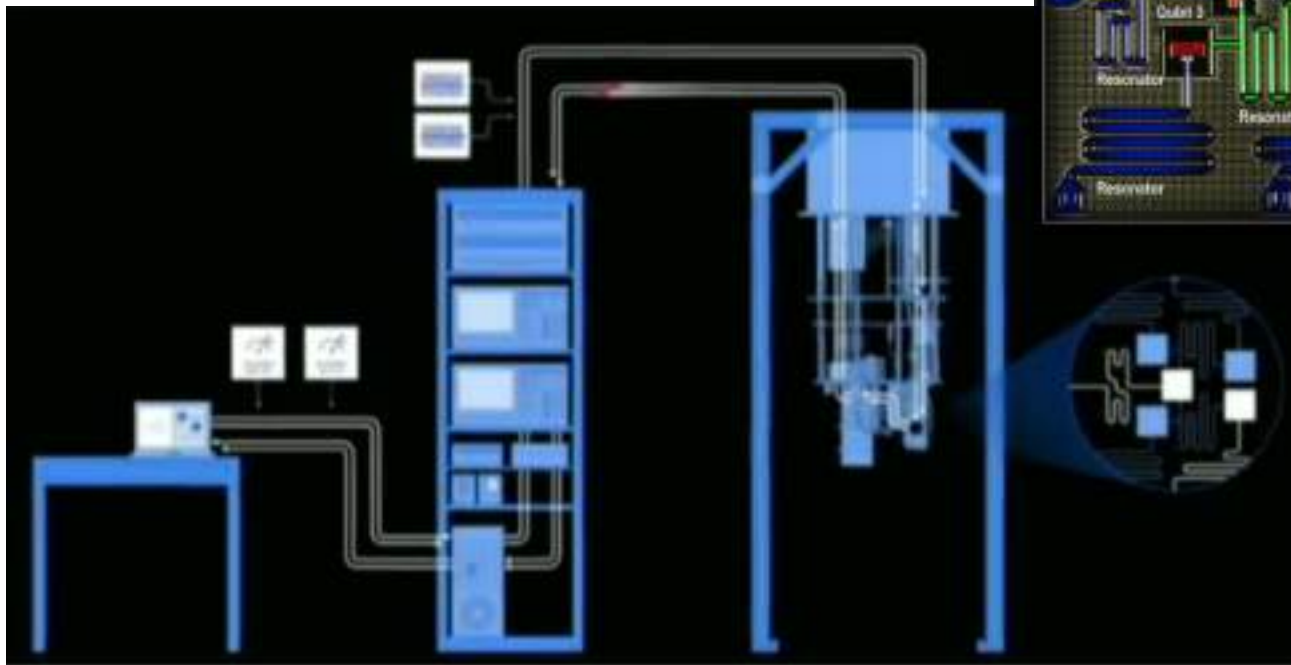


1

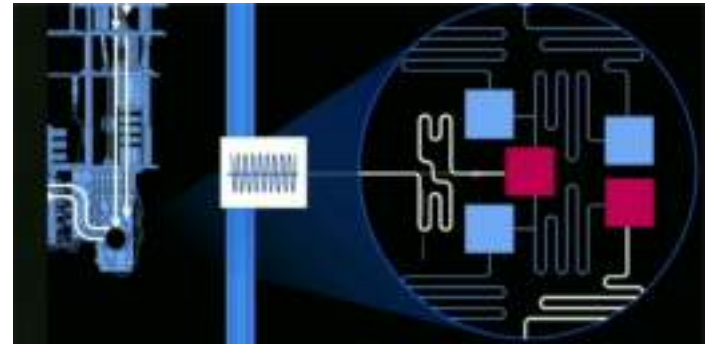
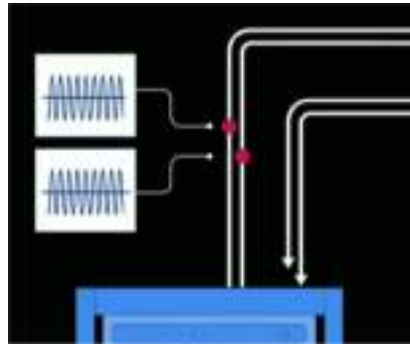
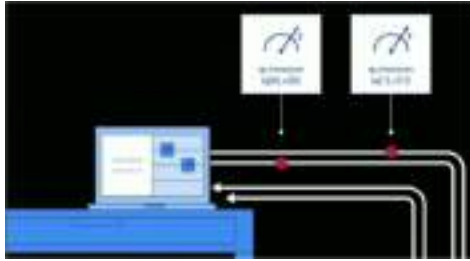


Computador cuántico de IBM

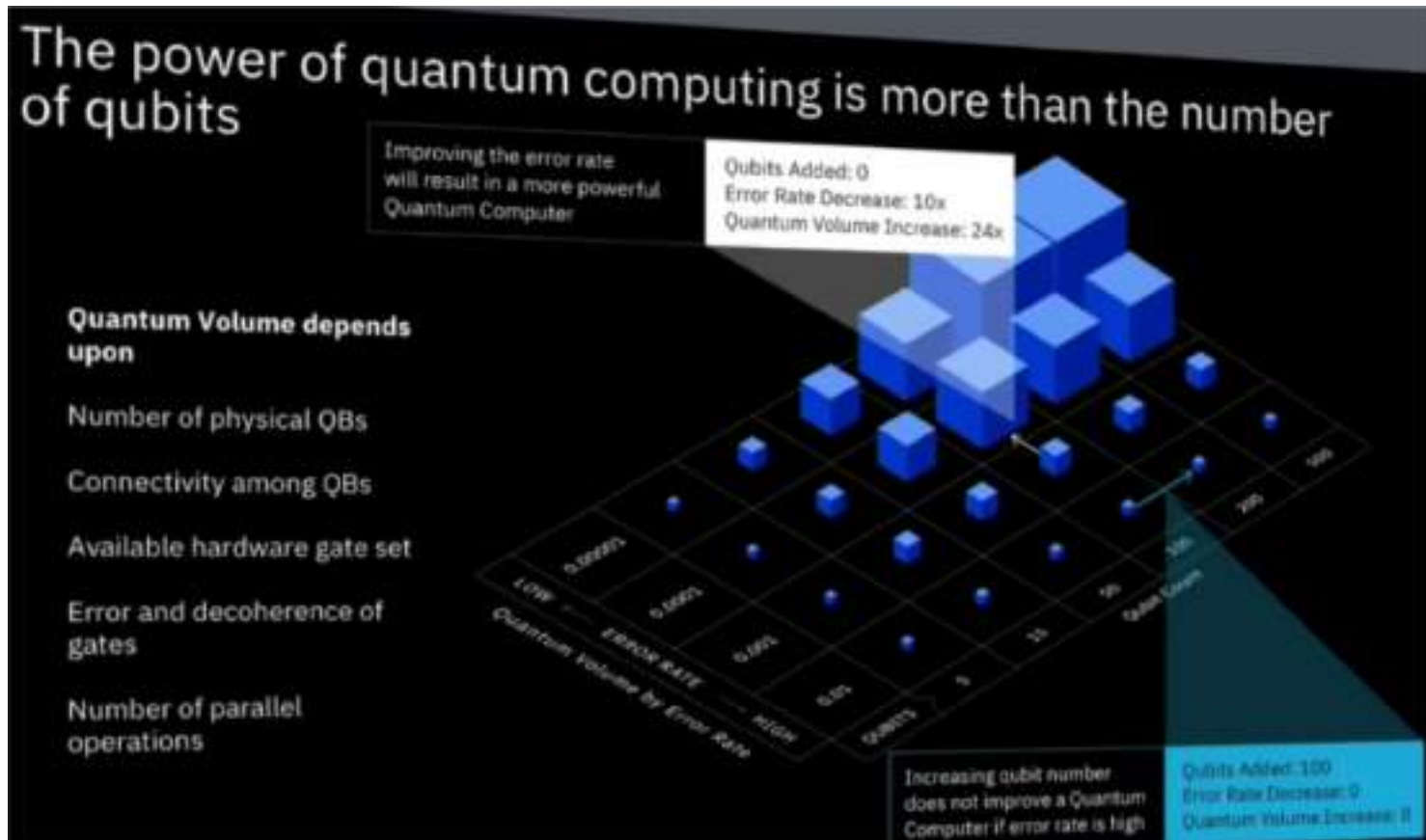
Join the IBM Q Experience Community
<https://quantumexperience.ng.bluemix.net>



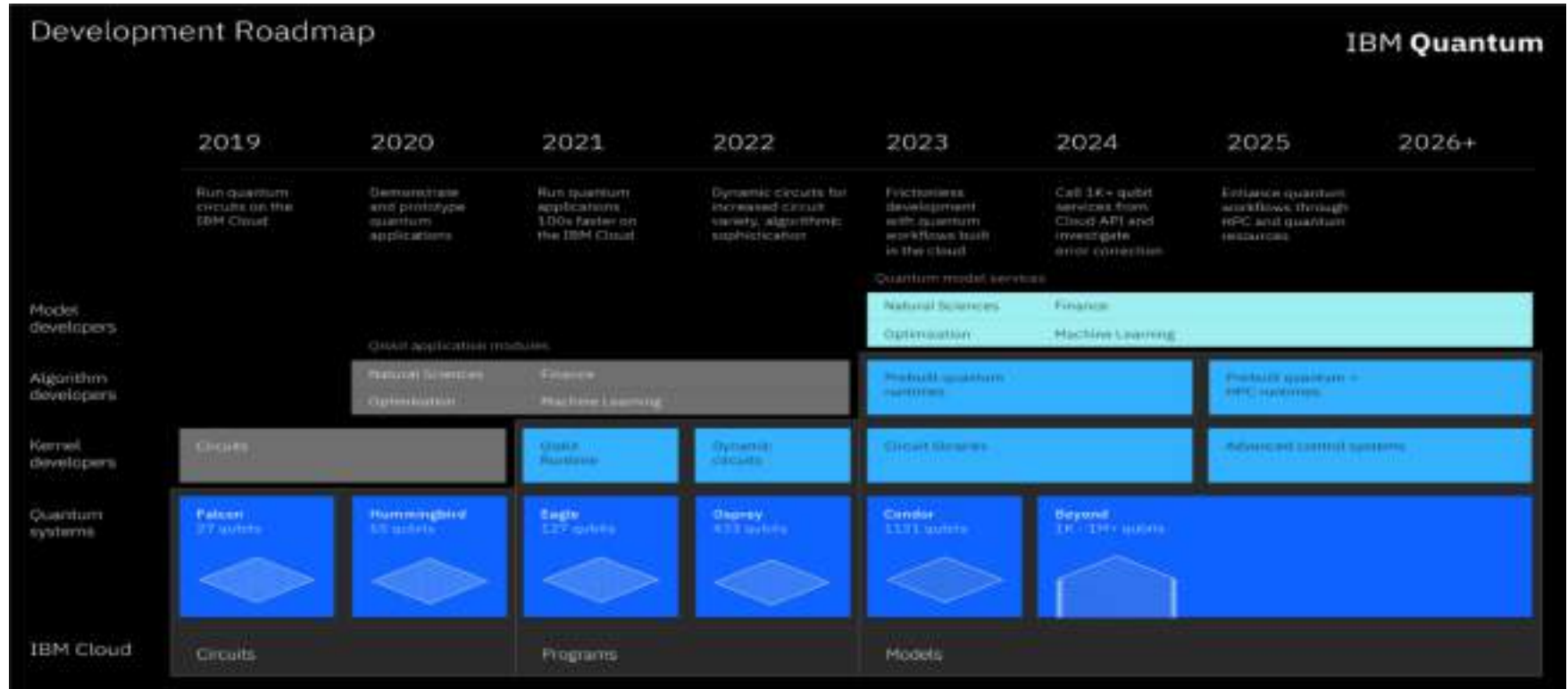
Cómo funcionan los Qbit



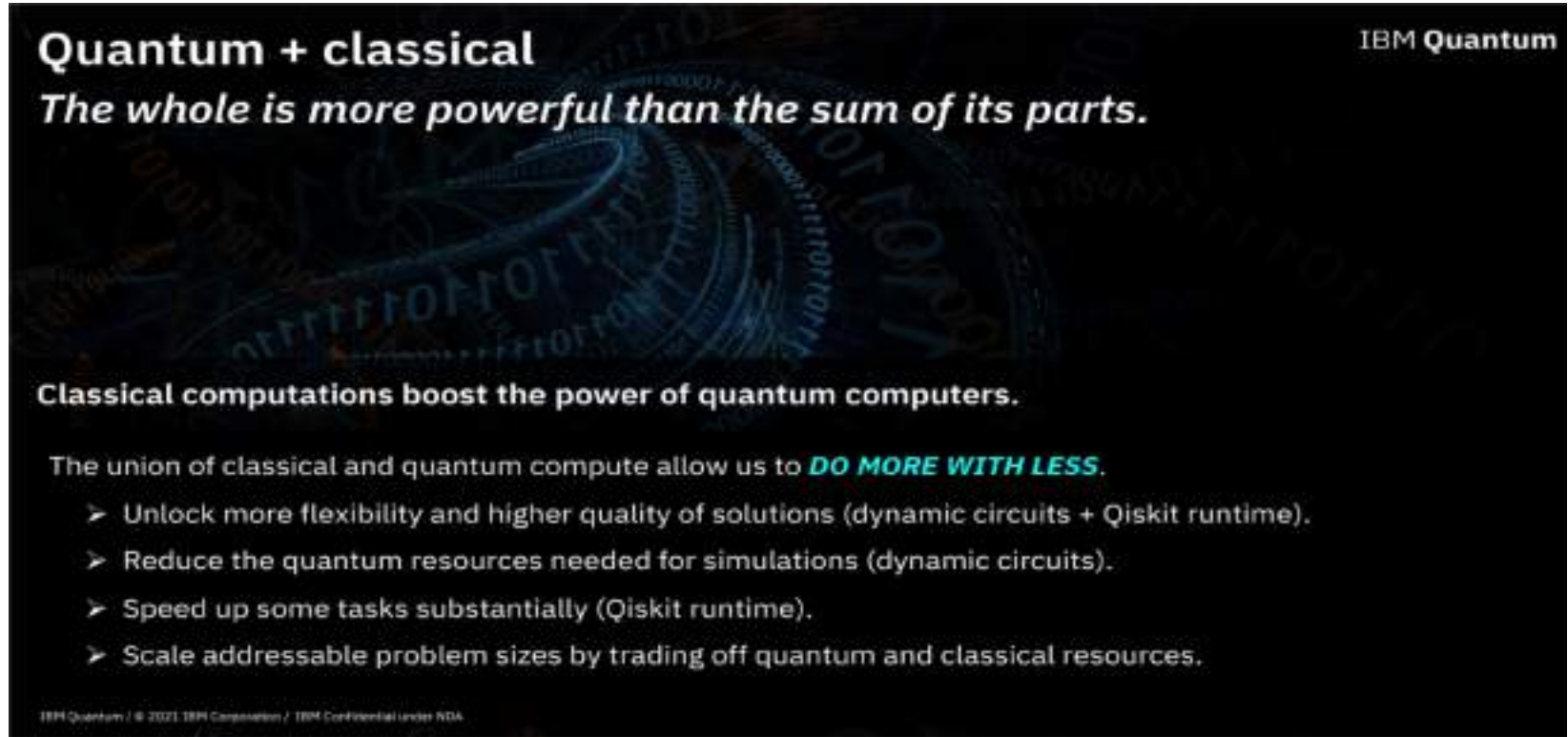
Volumen Cuántico



Roadmap Cuántico



Quantum + Classical



Quantum + classical

The whole is more powerful than the sum of its parts.

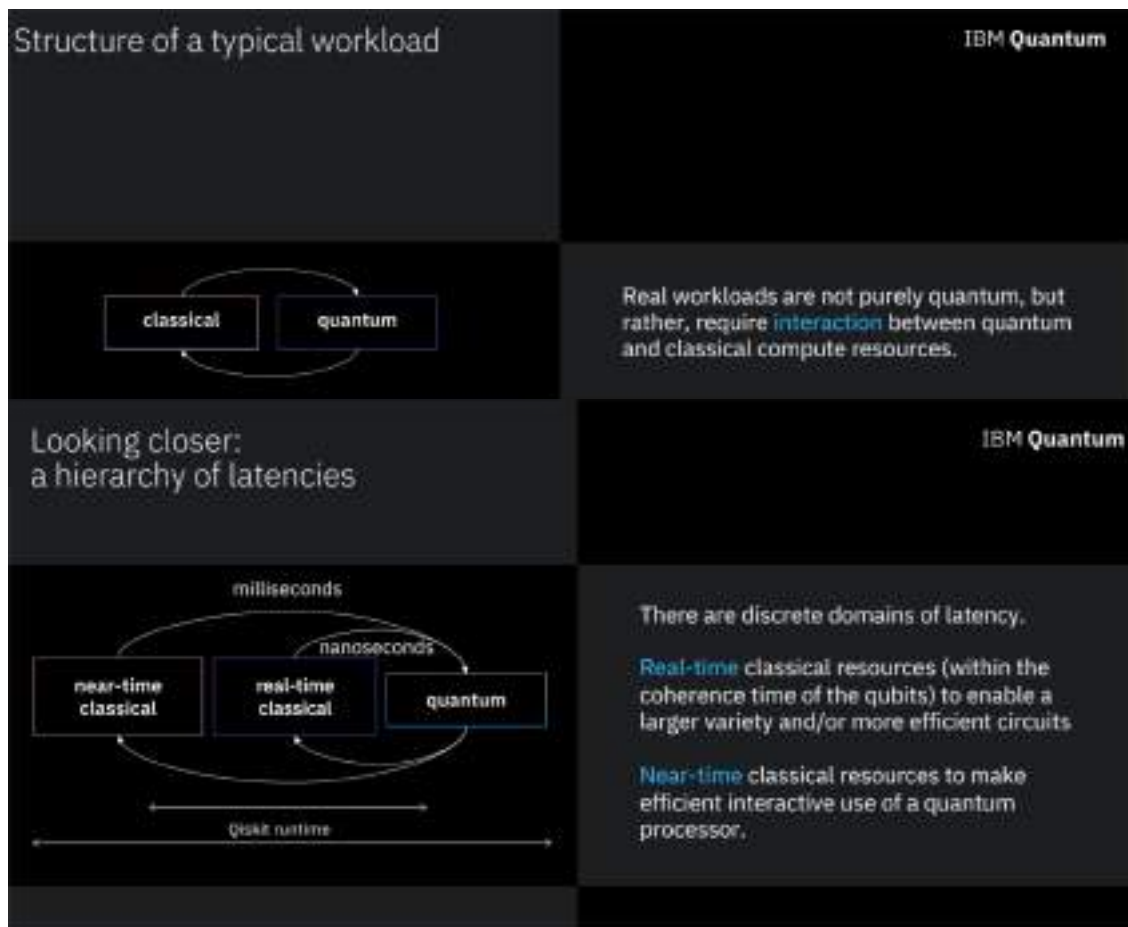
Classical computations boost the power of quantum computers.

The union of classical and quantum compute allow us to **DO MORE WITH LESS**.

- Unlock more flexibility and higher quality of solutions (dynamic circuits + Qiskit runtime).
- Reduce the quantum resources needed for simulations (dynamic circuits).
- Speed up some tasks substantially (Qiskit runtime).
- Scale addressable problem sizes by trading off quantum and classical resources.

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Real-time and near-time



Real-time and near-time

Near-time compute: Circuits for chemistry

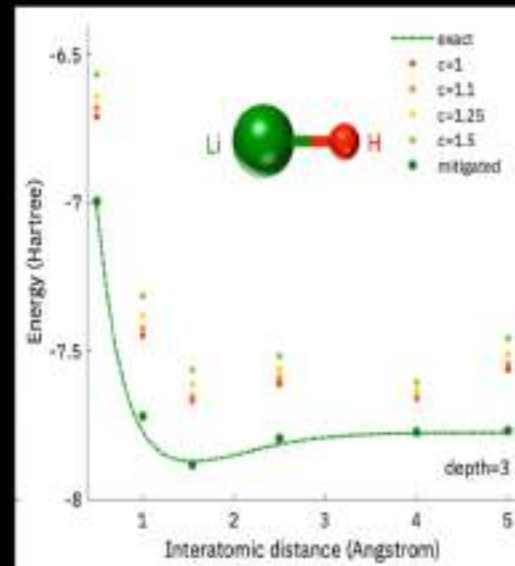
A standard chemistry problem is to solve for the energy landscape.

Computing the binding curve of LiH with error mitigation requires running **4.8 billion quantum circuits**.

Current qiskit execution time: **111 days**
(limited by today's quantum-classical execution model)

With the Qiskit Runtime: **5 days**

With Qiskit Runtime + OpenQASM3: **< 1 day**

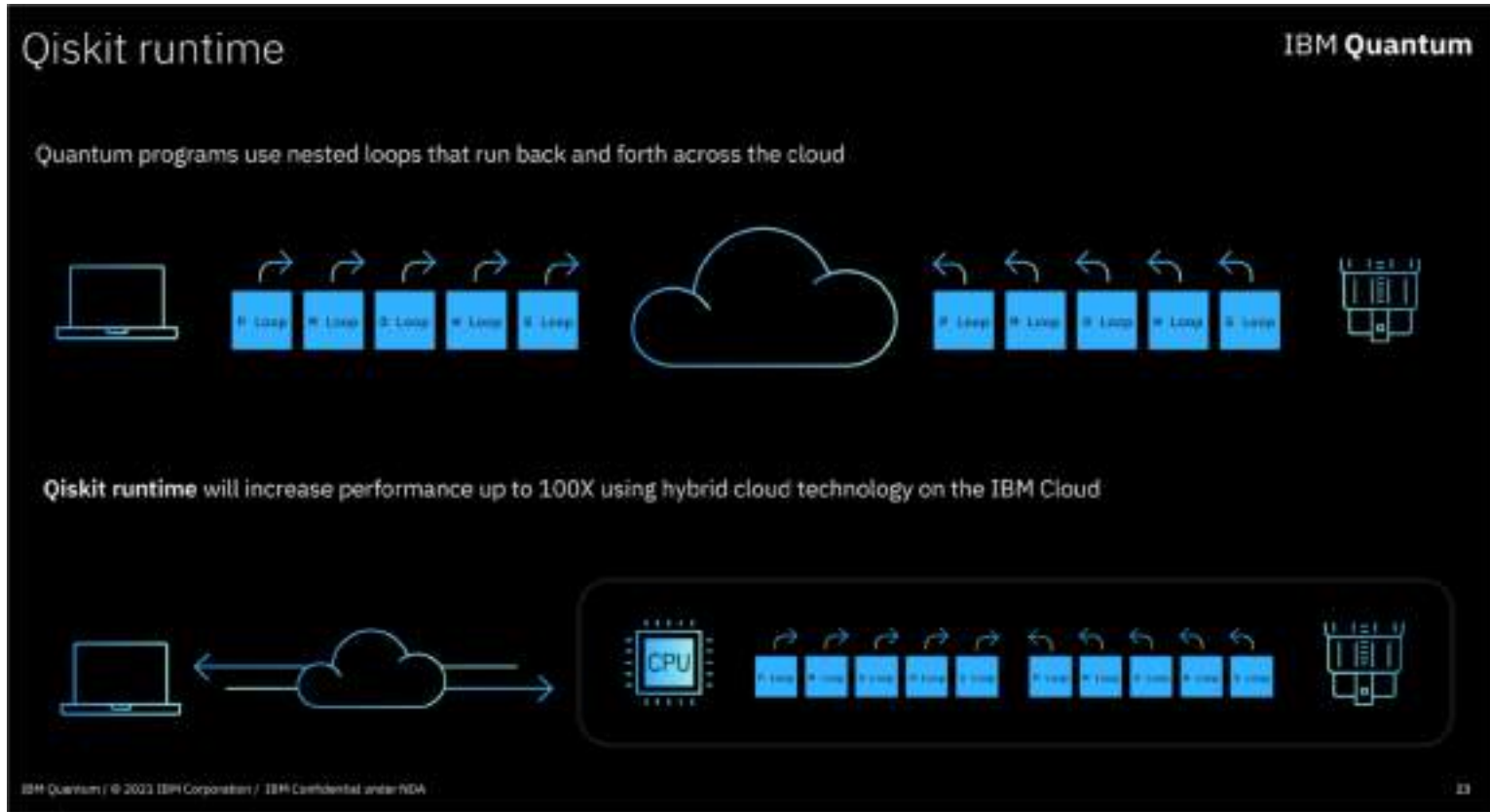


IBM Quantum / Qiskit Runtime / IBM Quantum

17/75

31

Quantum + Classical



Quantum + Classical

Dynamic circuit

IBM Quantum

A static circuit is run sequentially with the result returned to the user.



A dynamic circuit can measure qubits mid-circuit and change the flow of processing.



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35

Quantum + Classical



OpenQASM3

Features to derive the needs of the next 5-10 years in quantum computing.

Classical control flow, instructions, and data types

Quantum basic blocks

Connect pulse-level descriptions to gates

TeX-inspired timing relationships

For example

Blocks and gates in TeX

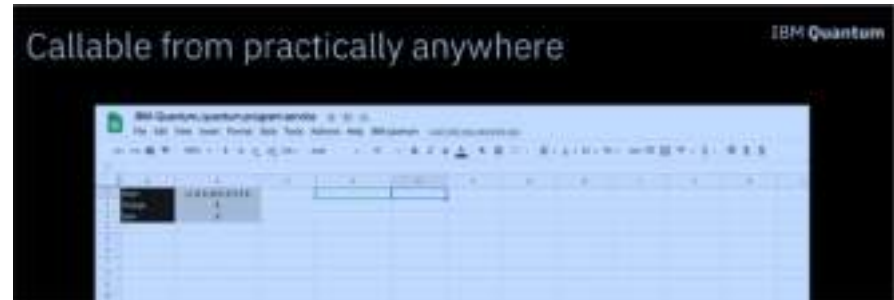
Blocks and sketches in OpenQASM3

IBM Quantum

```
qubit q; // phase estimation qubit
qubit t; // target qubit for the controlled-unitary gate
u1(2*pi) t; // phase estimation init

// prepare uniform superposition of eigenvectors of phase
H q;

// iterative phase estimation loop
var s; phase = 1;
for i in [0:n-1] {
  control q;
  CNOT q, t;
  CNOT q, phase;
  H q;
  measure q -> s[i];
  // correct measurement outcome in
  // expectation to a pi/2 phase shift
  // at the next iteration, so shift all bits of s left
  s <<= 1;
  phase = 1;
}
```

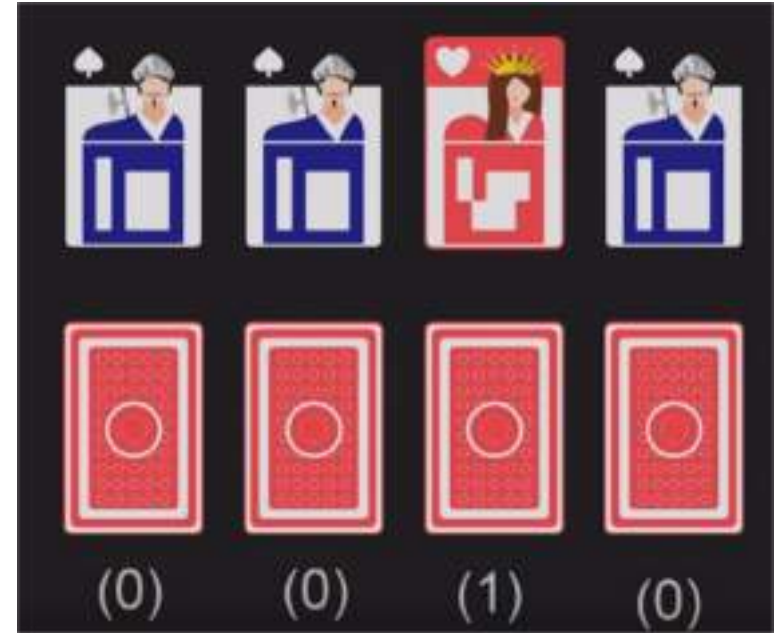
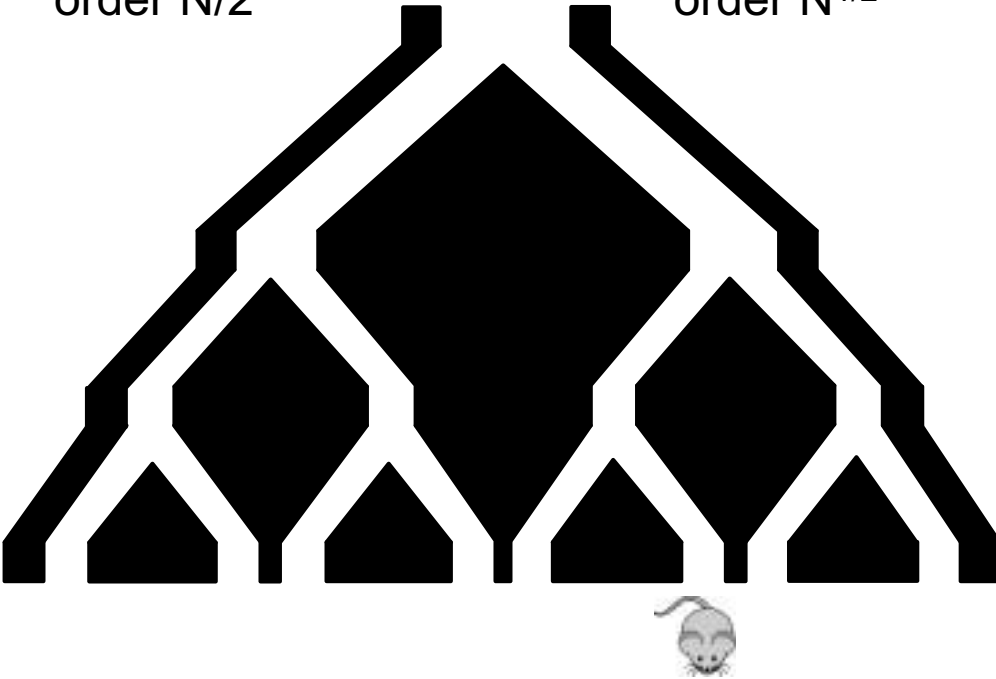


Algoritmos para computadores cuánticos: Grover

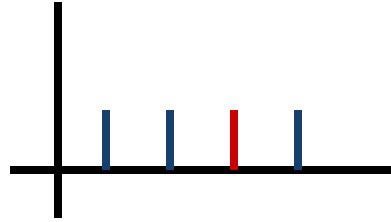
Clasical search:
order $N/2$



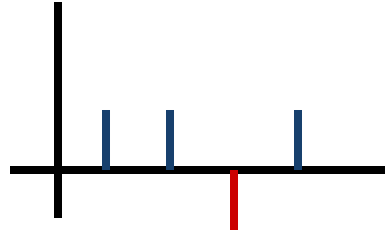
Quantum search:
order $N^{1/2}$



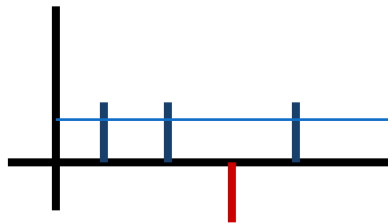
Algoritmo de Grover



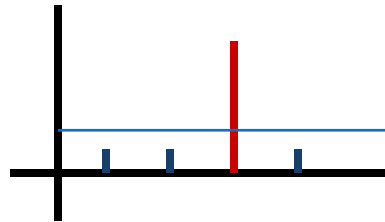
Amplitudes Originales



Invertir la amplitud que indica el “oraculo”



Amplitude media



Cambiar (Flip) Amplitudes respecto a la media

$$G = (2|\psi\rangle\langle\psi| - I)O$$

oracle function

$$|x\rangle|q\rangle \xrightarrow{O} |x\rangle|f(x) \oplus q\rangle$$

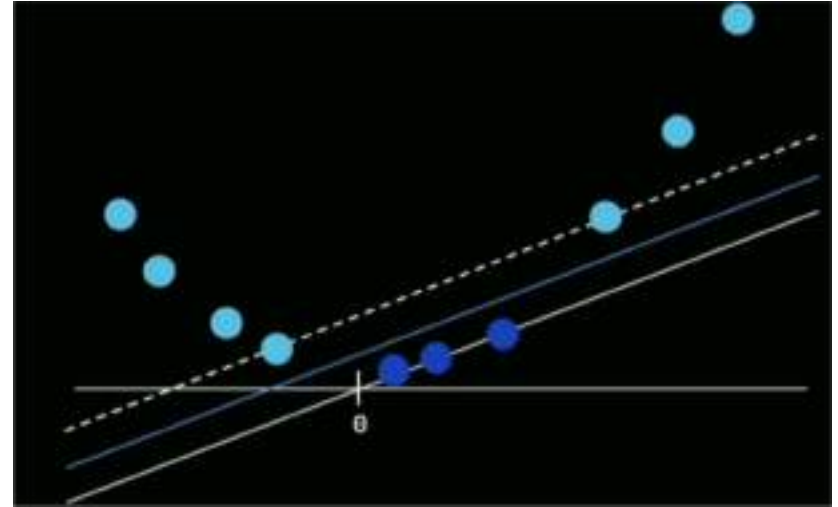
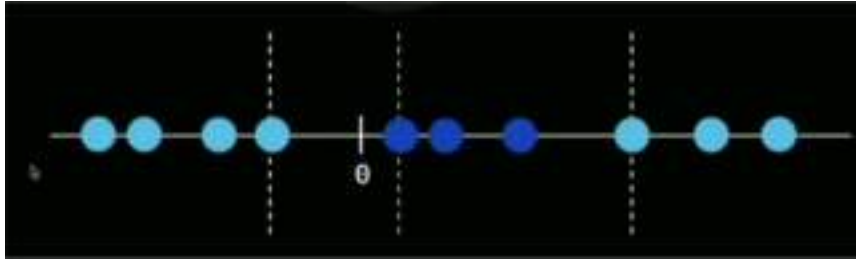
$$(2|\psi\rangle\langle\psi| - I) \sum_i a_i |i\rangle = \sum_i (2\langle a\rangle - a_i) |i\rangle$$

$$\text{where } \langle a\rangle = \frac{\sum_i a_i}{\sum_i 1}$$

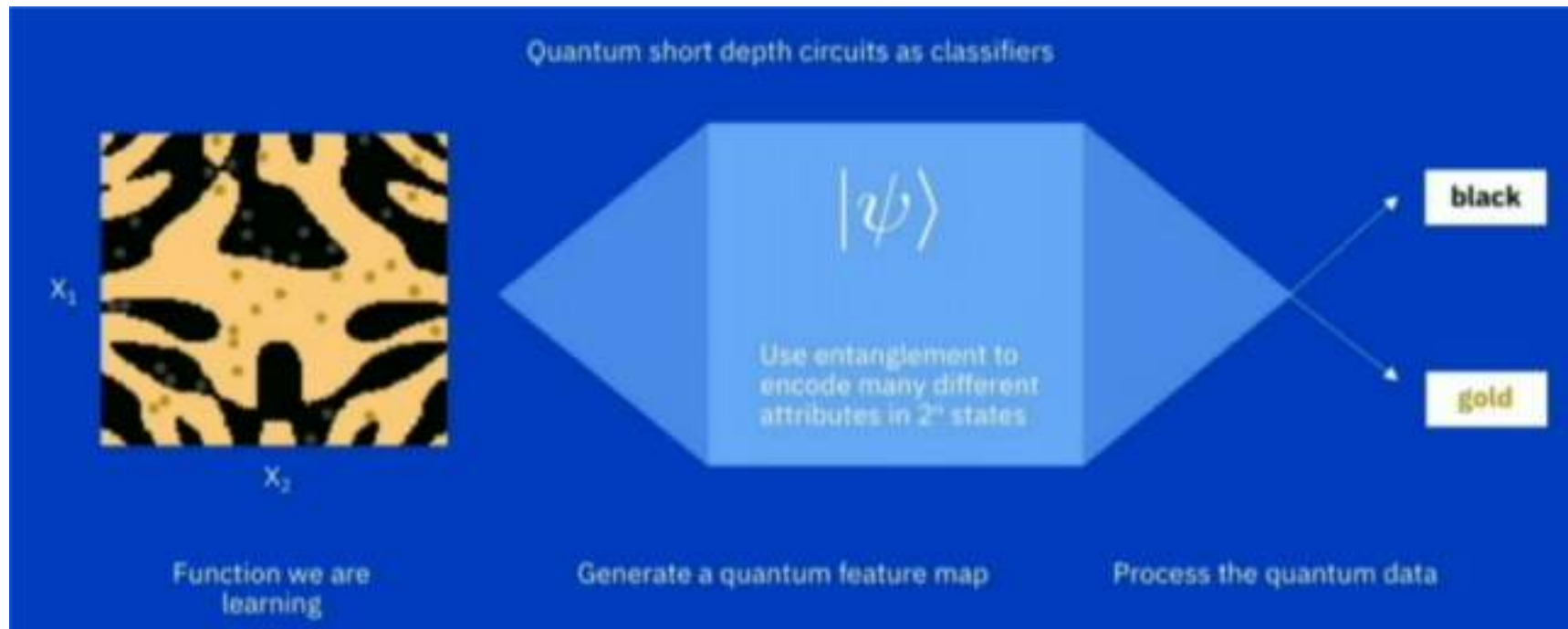
flip around the average

Machine Learning: Algoritmos de Clasificación

Idea de base: Incremento de la dimensión del espacio de búsqueda para facilitar la separación de las clases



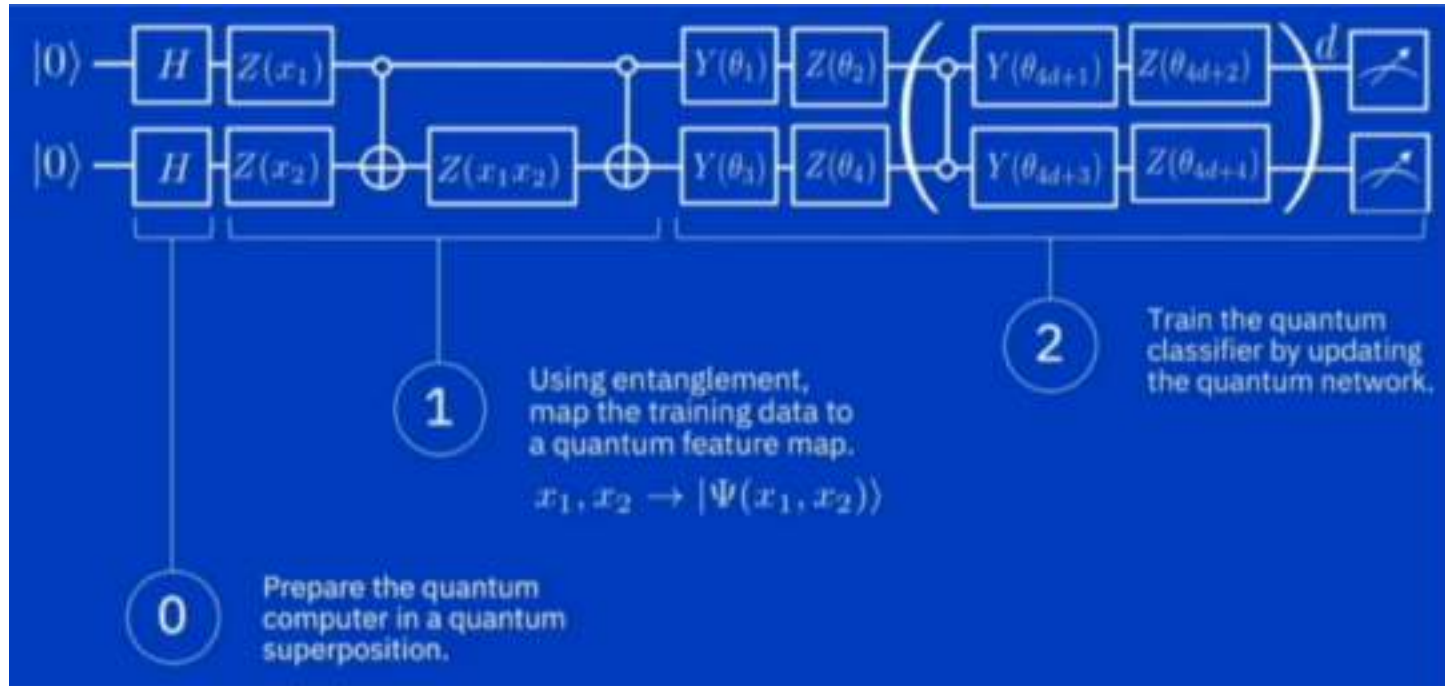
QC-ML: Algoritmos de Clasificación con entrelazamiento



Supervised learning with quantum-enhanced feature spaces

Vojtěch Havlíček, Antonio D. Córcoles, Kristan Temme, Aram W. Harrow, Abhinav Kandala, Jerry M. Chow & Jay M. Gambetta Nature volume 567, pages 209–212 (2019) <https://www.nature.com/articles/s41586-019-0980-2>

QC-ML: Clasificador cuántico



Ejemplo de Algoritmo Cuántico: Deutsch

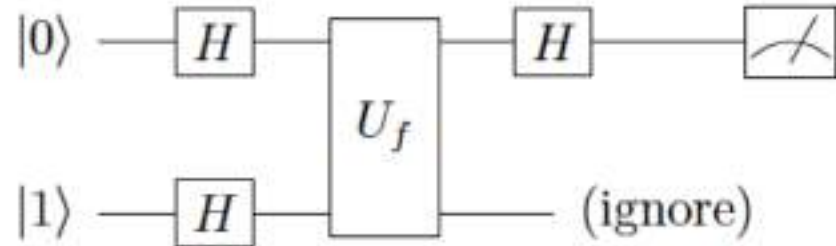
x	1
0	0
1	1

x	\neg
0	1
1	0

x	$ 0\rangle$
0	0
1	0

x	$ 1\rangle$
0	1
1	1

So exactly two of our unary ops are *constant* and the other two are *balanced* = *one-to-one* = *not constant*.

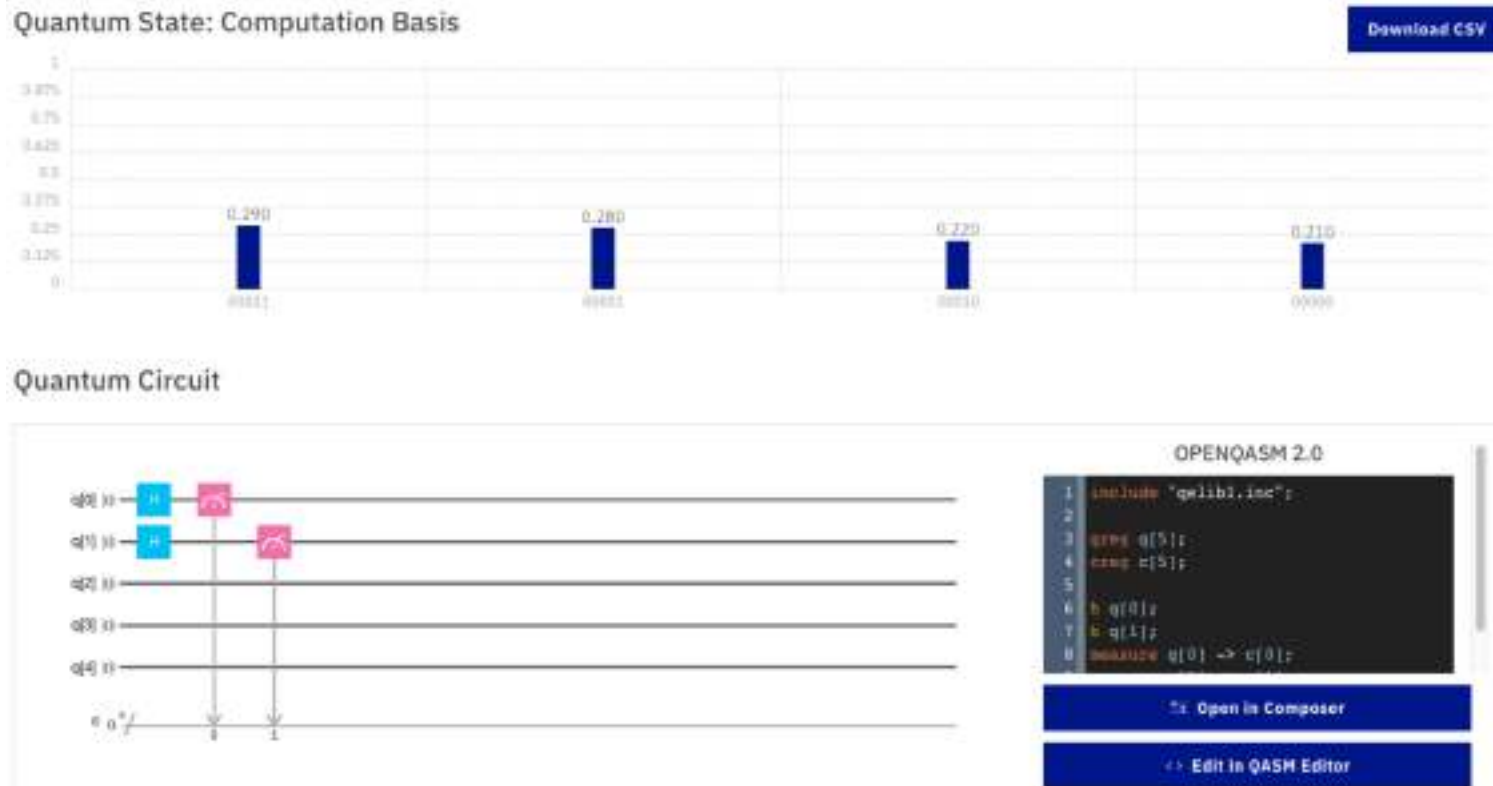


$$U_f(|x\rangle|-\rangle) = (-1)^{f(x)}|x\rangle\left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right) = \left((-1)^{f(x)}|x\rangle\right)|-\rangle,$$

Referencias

- <https://qiskit.org/textbook/preface.html>
- Quantum Computation and Quantum Information by Michael A. Nielsen & Isaac L. Chuang

Apéndice : Ejemplos de circuitos cuánticos para IBMQ



Apéndice : Ejemplos de circuitos cuánticos para IBMQ

Quantum State: Computation Basis



Download CSV

Quantum Circuit



OPENQASM 2.0

```
1 include "qelib1.inc";
2
3 nreq q[5];
4 creg c[5];
5
6 x q[0];
7 h q[0];
8 measure q[0] -> c[0];
```

Open In Composer

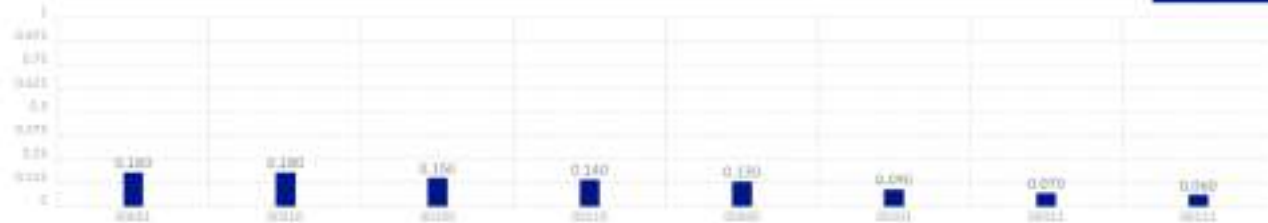
Apéndice : Ejemplos de circuitos cuánticos para IBMQ

Experiment #3qubit

Device: Simulator

Quantum State: Computation Basis

Download CSV



Quantum Circuit



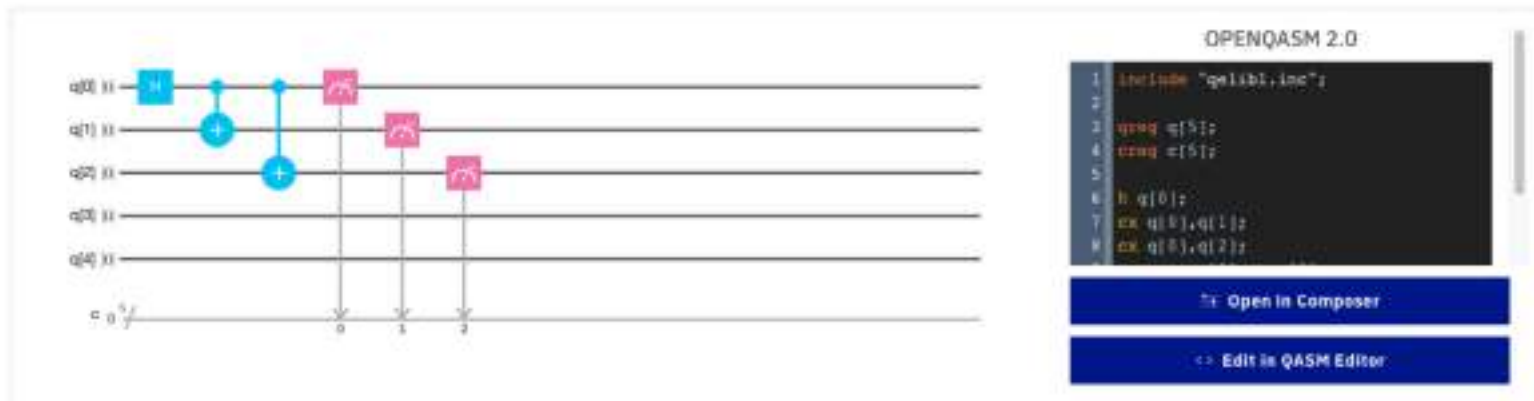
Apéndice : Ejemplos de circuitos cuánticos para IBMQ

Quantum State: Computation Basis

[Download CSV](#)



Quantum Circuit



Apéndice : Bernstein-Vazirani Algorithm

$$f: \{0, 1\}^n \rightarrow \{0, 1\}$$

$$f(x) = s \cdot x = \sum_{i=1}^n s_i \cdot x_i$$

s : contraseña secreta

$$s = 101$$

$$f(x) = s \cdot x$$



$$f(000) = 1 \cdot 0 + 0 \cdot 0 + 1 \cdot 0 = 0$$

$i?$

$$f(001) = 1 \cdot 0 + 0 \cdot 1 + 1 \cdot 1 = 1$$

$$\text{|| } s = 771$$

$$f(010) = 1 \cdot 0 + 0 \cdot 1 + 1 \cdot 0 = 0$$

$$\text{|| } s = 701$$

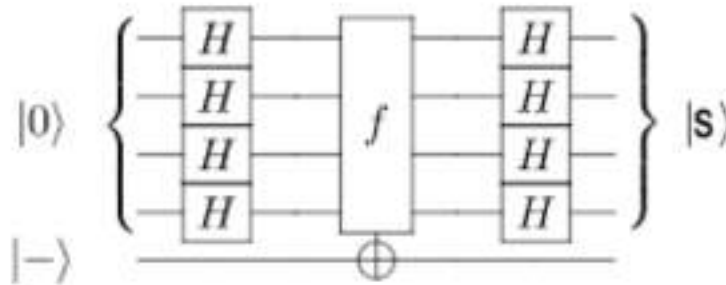
$$f(100) = 1 \cdot 1 + 0 \cdot 0 + 1 \cdot 0 = 1$$

$$\text{|| } s = 101 \text{ ||}$$

<https://www.youtube.com/watch?v=sqJlpHYI7oo>

<https://qiskit.org/textbook/ch-algorithms/bernstein-vazirani.html>

Apéndice : Bernstein-Vazirani Algorithm



$$|00 \dots 0\rangle \xrightarrow{H^{\otimes n}} \frac{1}{\sqrt{2^n}} \sum_{x \in \{0,1\}^n} |x\rangle$$

$$\frac{1}{\sqrt{2^n}} \sum_{x \in \{0,1\}^n} |x\rangle \xrightarrow{f_a} \frac{1}{\sqrt{2^n}} \sum_{x \in \{0,1\}^n} (-1)^{a \cdot x} |x\rangle$$

$$|a\rangle \xrightarrow{H^{\otimes n}} \frac{1}{\sqrt{2^n}} \sum_{x \in \{0,1\}^n} (-1)^{a \cdot x} |x\rangle.$$

$$H|a\rangle = \sum_{x \in \{0,1\}} (-1)^{a \cdot x} |x\rangle.$$

$$H^{\otimes 2}|00\rangle = |00\rangle + |01\rangle + |10\rangle + |11\rangle$$

$$H^{\otimes 2}|01\rangle = |00\rangle - |01\rangle + |10\rangle - |11\rangle$$

$$H^{\otimes 2}|10\rangle = |00\rangle + |01\rangle - |10\rangle - |11\rangle$$

$$H^{\otimes 2}|11\rangle = |00\rangle - |01\rangle - |10\rangle + |11\rangle$$

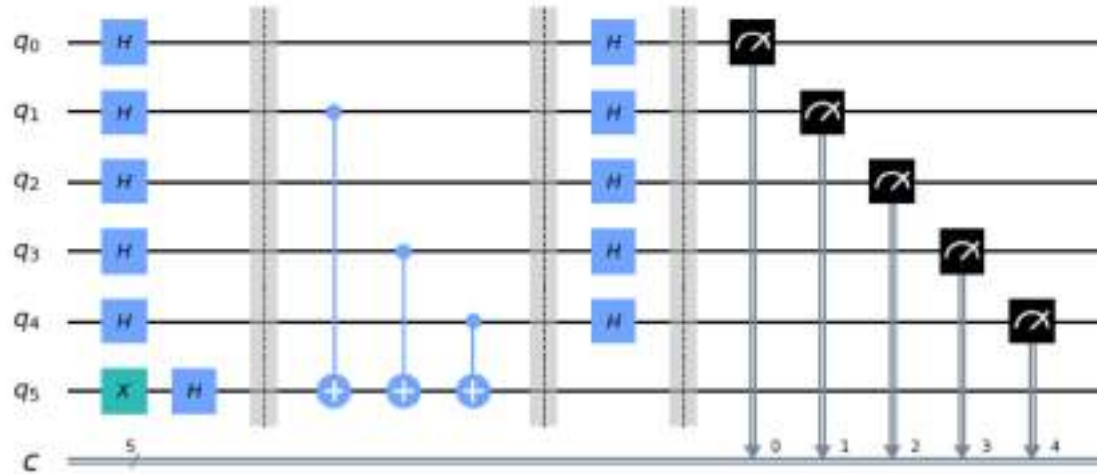
$$\frac{1}{\sqrt{2^n}} \sum_{x \in \{0,1\}^n} (-1)^{a \cdot x} |x\rangle \xrightarrow{H^{\otimes n}} |a\rangle$$

<https://qiskit.org/textbook/ch-algorithms/bernstein-vazirani.html>

Apéndice : Bernstein-Vazirani Algorithm

S = 11010

Bernstein-Vazirani Oraculo



```
simulator = Aer.get_backend('qasm_simulator')
resultado = execute(circuito, backend = simulator, shots = 1).result()
counts = resultado.get_counts()
print(counts)
```

```
{'11010': 1}
```

<https://www.youtube.com/watch?v=sqJlpHYI7oo>

<https://qiskit.org/textbook/ch-algorithms/bernstein-vazirani.html>