Analysis in the Time and Frequency Domains

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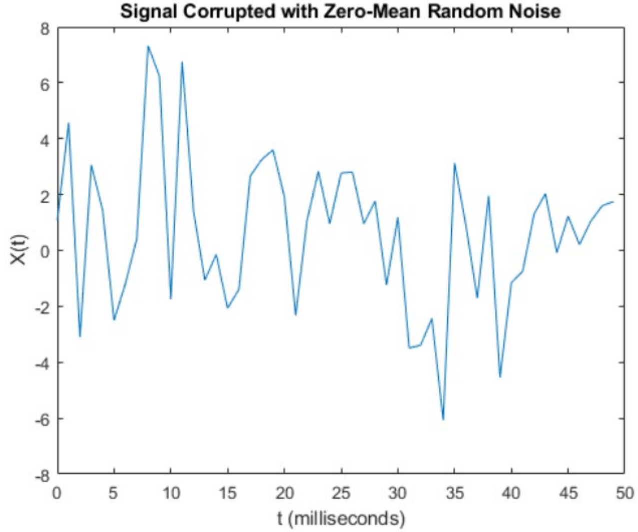
Further Frequency Transforms

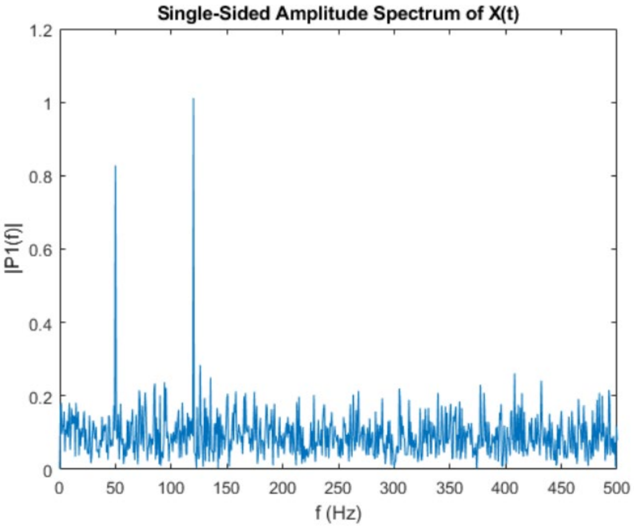
Fast Fourier Transform: FFT

Fast Fourier Transform: FFT

Implementation algorithm for the DFT (Discrete Fourier Transform), which is a frequency domain sampled version of the DTFT (Discrete Time Frequency Transform).

Example from Matlab help: https://www.mathworks.com/help/matlab/ref/fft.html





Chap 8-10: Alan V. Oppenheim, Ronald W. Schafer, “Discrete-time signal processing”, 3ª edition, 2013.

Discrete Cosine Transform: DCT

Discrete Cosine Transform: DCT

The **discrete Fourier transform (DFT)** transforms a complex signal into its complex spectrum. However, if the signal is real as in most of the applications, half of the data is redundant. In time domain, the imaginary part of the signal is all zero; in frequency domain, the real part of the spectrum is even symmetric and imaginary part odd.

In comparison, **Discrete cosine transform (DCT)** transform is a real transform that transforms a sequence of real data points into its real spectrum and therefore avoids the problem of redundancy. Also, as DCT is derived from DFT, all the desirable properties of DFT (such as the fast algorithm) are preserved.

Discrete Cosine Transform: DCT

- The DCT has the property of highly compacting the signal energy in its coefficients with low indices, much more than the FFT. For this reason, it is widely used in all types of data compression.

Energy compaction property applied to data compression:

- Concentrates energy coefficients at low indices
    - Canceling coefficients of high indices, and recovering a signal by reverse DCT closely resembles the original signal. This is a way of dimensionality reduction without training process

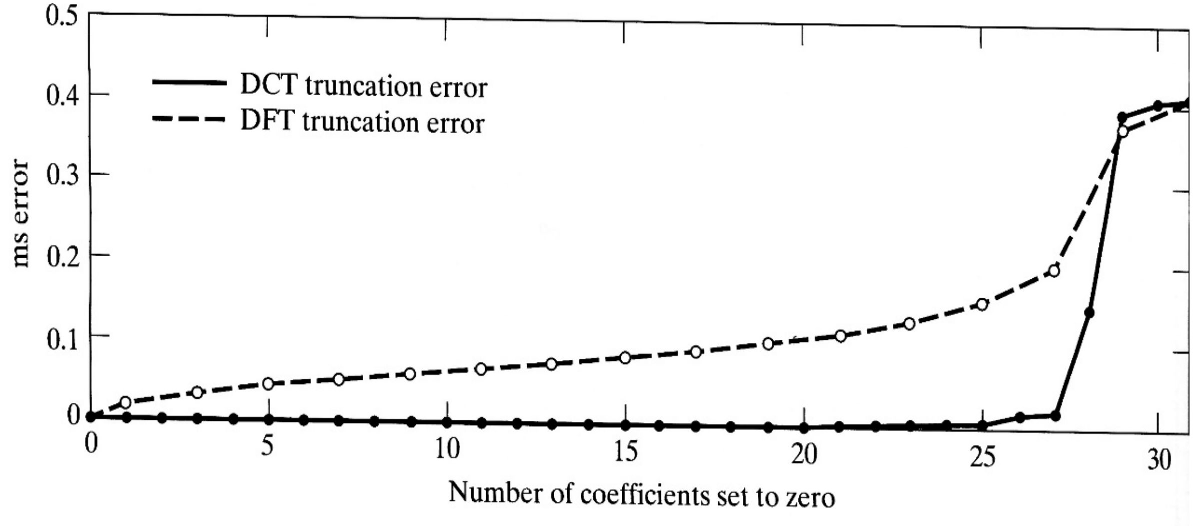
- We can represent the signal only with a few DCT coefficients!!

Discrete Cosine Transform: DCT

• DCT can be greatly truncated without producing significant errors

• The DFT, on the other hand, can hardly be truncated without significant errors appearing

Due to this property the DCT is widely used in image, audio and data compression.



- Figure: Example of comparison of the MSE (mean squared error) made when truncating the DCT and the DFT

Discrete Cosine Transform: DCT

- Example: see Matlab example to apply the DCT to the speech signal.

- https://www.mathworks.com/help/signal/ug/dct-for-speech-signal-compression.html
- openExample('signal/SpeechSignalCompressionExample')

Short Time Fourier Transform and the Spectrogram

- So far, we have assumed that the characteristics of the signal did not change with time (stationary signals. E.g.: two cosine signals). We could take the window size we wanted without the signal characteristics changing within the window.

-Completely stationary signals are rare and of little practical utility

-In practice, signals vary their characteristics (frequency components, amplitudes, phases) with time

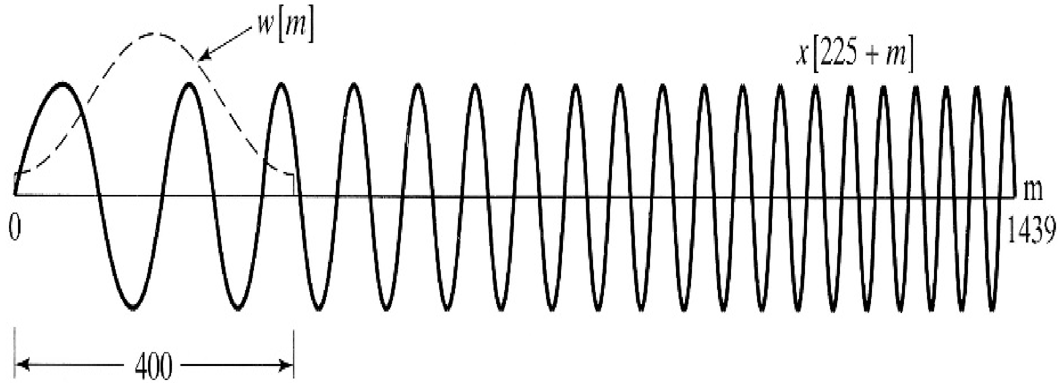
Short Time Fourier Transform and the Spectrogram

If the characteristics of the signal change over time:

- We cannot take the size of the window we want. It must be small enough to consider that in the size of the window the signal is reasonably stable
- A single DFT is not useful to describe the characteristics of the signal. We must perform a DFT every certain time interval to represent the spectral variation as a function of time
- STFT simply consists of performing a spectral analysis of the signal (a DTFT) at every certain time interval

Short Time Fourier Transform and the Spectrogram

Consider as a signal of spectral characteristics variable with time (a chirp function) defined as:



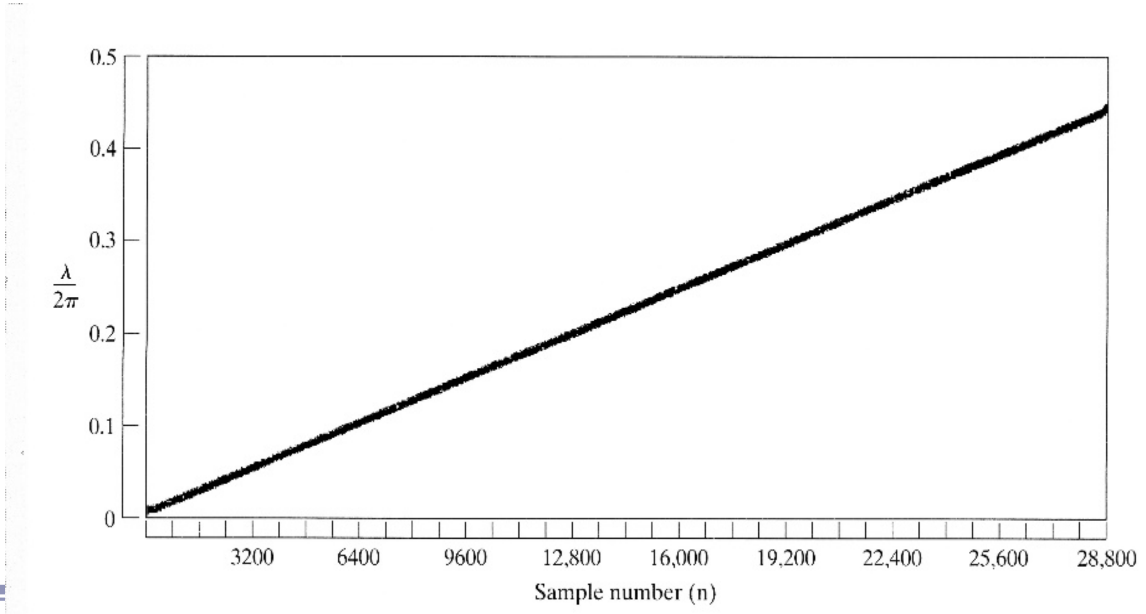
- This function is a linearly frequency modulated sinusoidal ("instantaneous frequency" increases linearly with time)

Short Time Fourier Transform and the Spectrogram

Next figure shows the STFT of the chirp function in a very common representation called a spectrogram

- The x axis represents a magnitude proportional to time
- The y axis represents a magnitude proportional to the frequency

The spectrogram clearly shows the linear variation of the frequency with time



Short Time Fourier Transform and the Spectrogram

Example: see Matlab example.

https://www.mathworks.com/help/signal/ref/spectrogram.html?s_tid=srchtitle_spectrogram_1

Default Values of Spectrogram

Generate 1024 samples of a signal that consists of a sum of sinusoids. The normalized frequencies of the sinusoids are $2\pi/5$ rad/sample and $4\pi/5$ rad/sample. The higher frequency sinusoid has 10 times the amplitude of the other sinusoid.

```
N = 1024;
n = 0:N-1;
w0 = 2*pi/5;
x = sin(w0*n)+10*sin(2*w0*n);
```

Compute the short-time Fourier transform using the function defaults. Plot the spectrogram.

```
s = spectrogram(x);
spectrogram(x,'yaxis')
```

Short Time Fourier Transform and the Spectrogram

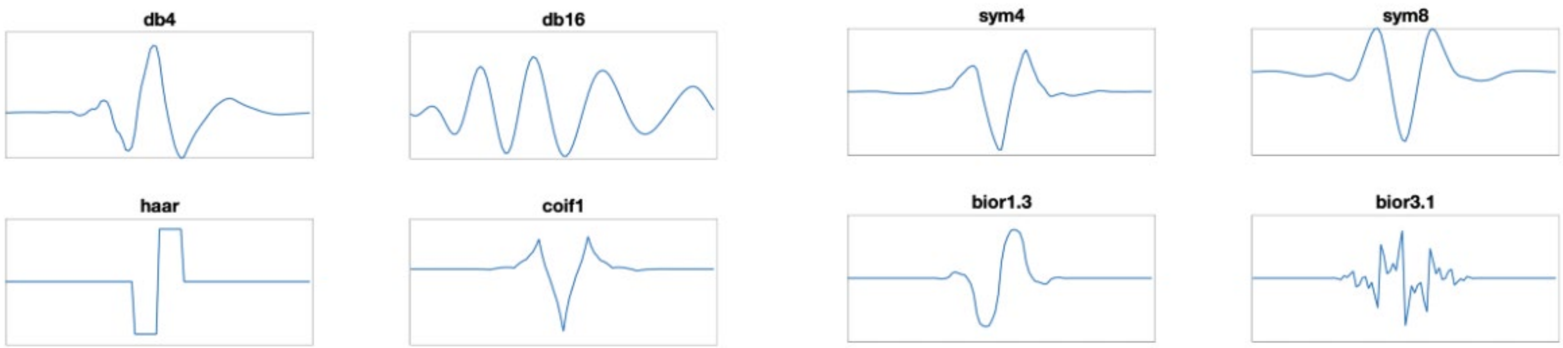
The spectrogram is very common in Audio and Speech Analysis.

The spectrogram is used to produce an image out of a time signal. From this image regular Convolutional NNs can be used for feature extraction or classification

Wavelets Transform

A major disadvantage of the Fourier Transform is it captures global frequency information, meaning frequencies that persist over an entire signal. This kind of signal decomposition may not serve all applications well, for example Electrocardiography (ECG) where signals have short intervals of characteristic oscillation. An alternative approach is the Wavelet Transform, which decomposes a function into a set of wavelets.

Wavelets are oscillation signals, such as:



Wavelets Transform

The basic idea is to compute how much of a wavelet is in a signal for a particular scale and location by convolving different types of wavelets with the signal.

Good resources:

https://towardsdatascience.com/the-wavelet-transform-e9cfa85d7b34

https://es.mathworks.com/videos/series/understanding-wavelets-121287.html

Amara Graps, "An Introduction to Wavelets" IEEE Signal & Image Processing, 1995