Black Hole Information Paradox

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Abstract

This paper examines the black hole information paradox through the lens of quantum information theory, thermodynamics, and quantum field theory. It explores the thermodynamic properties of black holes, including their temperature and entropy as dictated by the Hawking-Bekenstein entropy formula, and discusses how Hawking radiation arises as a quantum effect near the event horizon. The analysis highlights the evolution of the black hole's internal and external states, emphasizing the entanglement entropy and the Page curve, which describes the information flow during black hole evaporation. Furthermore, the paper considers various resolutions to the paradox, including the possibility that information is preserved and encoded in Hawking radiation, in accordance with the principles of unitarity and supported by the AdS/CFT correspondence.

1 BH thermodynamics

We can study the QFT (*Quantum Field Theory*) behaviour of black hole spacetime using euclidean path integrals. In Minkowski's space time this implies

$$t = i\tau \tag{1}$$

and continuing τ from imaginary to real values. In this case τ is not the proper time in a worldline but the "imaginary time".
In black hole spacetimes this leads to a continuation of Schwarzschild metric to Euclidean Schwarzschild metric

$$dS_E^2 = (1-\frac{2M}{r})d\tau^2 + \frac{dr^2}{1-\frac{2M}{r}} + r^2 d\Omega^2 \tag{2}$$

in order to study the region close to the singularity r = 2M we do

$$r - 2M = \frac{x^2}{8M} \tag{3}$$

to obtain

$$ds_E^2 \approx (kx)^2 d\tau^2 + dx^2 + \frac{1}{4\kappa^2} d\Omega^2 \tag{4}$$

We can see then that near r = 2M the metric is the product of euclidean rindler's and S^2

$$ds_E^2 = dx^2 + x^2 d(\kappa\tau)^2 \tag{5}$$

which making the periodic identification $\tau \sim \tau + \frac{2\pi}{\kappa}$ is just E^2 in polar coordinates. This means that the singularity at r = 2M is simply a coordinate singularity provided that $\tau \sim \tau + \frac{2\pi}{\kappa}$. This means that the Euclidean functional integral has to be taken over the fields $\phi(\vec{x}, t)$ that are periodic on τ with period $\frac{2\pi}{\kappa}$. Then the Euclidean functional integral is

$$Z = \int [\mathbf{D}\phi] e^{-S_E[\phi]} \tag{6}$$

were $S_E = \int dt(-ip\dot{q} + H)$ is the euclidean action. If the functional integral is taken over fields periodical on τ with periodicity $\hbar\beta$ then it can be written as

$$Z = Tr(e^{\hbar\beta}) \tag{7}$$

Which is the partition function for a quantum mechanical system with hamiltonian H at temperature T($T = \frac{k_b}{\beta}$). Since for Euclidean Schwarzschild $\hbar\beta = \frac{2\pi}{\kappa}$ a QFT can only be at equilibrium at the following temperature

$$T_H = \frac{\kappa\hbar}{2\pi k_b} \tag{8}$$

Which is the so called **Hawking temperature**, at any other temperature Euclidean Schwarzschild has a conical singularity, and therefore the system is not at equilibrium.
Now we know that black holes are thermal objects that have the capacity to radiate energy. What about entropy?
Bekenstein argued the following. Let's assume that black holes had no entropy, then an object with $S_{object} > 0$ falls into the black hole, then

$\Delta S_{total} < 0$ contradicting the second principle of thermodynamics. Then we should expect that a black hole had an entropy S_{BH} such that $S_{total} = S_{BH} + S_{object}$ such that $\Delta S_{total} \geq 0$.
Hawking then proved that the area of a black hole only can grow $\Delta A \geq 0$, which leads Bekenstein to propose that the entropy of a black hole is proportional to its mass.

$$S_{BH} = \frac{A}{4G} \tag{9}$$

The so called Hawking-Bekenstein entropy.
We have now found that black holes are thermal objects with temperature and entropy.
Black holes having a temperature implies that they emit radiation to their environment and consequently loose mass.
Nothing can travel faster than the speed of light so energy should not be able to leave the black hole, yet it does. Hawking radiation must be a quantum effect. In fact taking the classical limit($\hbar \to 0$) to the Hawking radiation equation we find that black holes appear not to have a temperature.
Obviously, Hawking radiation does not involve any object traveling faster than light; Hawking showed that it was due to particle-antiparticle pair production at the event horizon.
At this point we could ask ourselves how it that useful for quantum information?
A particle-antiparticle pair that results in Hawking radiation can be represented as follows

$$a(T)\left|1\right\rangle_A \left|1\right\rangle_B + b(T)\left|0\right\rangle_A \left|0\right\rangle_B \tag{10}$$

Where $|1\rangle$ represents the presence of a virtual particle whereas $|0\rangle$ represents the absence of a virtual particle. The A bit is outside the horizon while the B bit is inside, both bits are entangled because an observer outside of the black hole knows that if there is a particle outside of the black hole there must be the corresponding particle inside of the black hole.
We can then write the state of an instance of Hawking radiation as

$$\left|\psi_{hawking\ radiation}\right\rangle = \frac{1}{\sqrt{2}}(|1,1\rangle + |0,0\rangle) \tag{11}$$

Which we can analyze in terms of quantum information. Comparing Hawking radiation to the states inputted into a black hole leads to the black hole information paradox.

2 The paradox

Before we tackle the issue of the black hole information paradox we have to know what unitarity implies for closed systems.
Under unitary transformations pure states remain pure while mixed states remain mixed.
In other words, pure states, characterized by

$$Tr\rho^2_{pure} = 1 \tag{12}$$

remain pure, and mixed states, characterized by

$$Tr\rho^2_{mixed} \leq 1 \tag{13}$$

stay mixed.
Now that we have reviewed all the elements necessary it's time to propose the following thought experiment.
Let's consider we could throw quantum bits into a black hole at a rate such that their energy just equals that of the outgoing radiation. This way the black hole's mass and its horizon area remain constant. However, the number of possible states of the black hole grows boundlessly, and we loose the connection between this and its area given by Hawking-Bekenstein entropy.
In order to recover the connection causality should be broken by allowing the bits inside to escape with the Hawking radiation.
Even if this could be considered a kerkuffle it is not really problematic, since it can be easily solved by giving up the statistical interpretation of the black hole entropy.
Let's now suppose we start with a pure state outide of the black hole that consists of n bell states and we throw one of each pair into the black hole, we will end up with:

$$S_{inside} = S_{outside} = S_{entanglement} = nln2 \tag{14}$$

Now we allow the evaporation of the black hole proceed until completion. Causality does not allow the entanglement to decrease but the black hole completely disappears, which leaves us with half of each pair in a highly mixed state with entropy

$$S_{entanglement\ after\ evaporation} = nln2 \tag{15}$$

We could have started with the system in a pure state = 0 but it ends up with a highly mixed state. In other words, the evaporation of the black hole leads to information about the details of the initial state of the system being lost, which suggest that many physical states can evolve into the same final state, which is inconsistent with Schrödinger like evolution.

$$i\hbar\partial_t \left|\psi\right\rangle = H\left|\psi\right\rangle \tag{16}$$

Black hole evaporation takes an initial pure density matrix

$$\rho_{initial} = \left|\psi_{initial}\right\rangle \left\langle\psi_{initial}\right| \tag{17}$$

to a mixed density matrix

$$\rho_{final} = \sum_i p_i \left|\psi_i^{final}\right\rangle \left\langle\psi_i^{final}\right| \tag{18}$$

We begin by populating a black hole with a pure quantum state $|\psi_{\text{pure}}\rangle$. Since this is a pure state, we have full information about the system, which is quantified by the purity condition:

$$\mathrm{Tr}(\rho_{\text{pure}}^2) = 1 \quad \Rightarrow \quad S_{\text{pure}} = -\mathrm{Tr}(\rho_{\text{pure}} \ln \rho_{\text{pure}}) = 0 \tag{19}$$

The first pair of particles created via Hawking radiation is an entangled Bell-like pair. The joint system becomes:

$$|\Psi_1\rangle = |\psi_{\text{pure}}\rangle \otimes \frac{1}{\sqrt{2}} \left(|1\rangle_{\text{out}} |1\rangle_{\text{in}} + |0\rangle_{\text{out}} |0\rangle_{\text{in}} \right) \tag{20}$$

The corresponding density matrix is:

$$\rho_1 = |\Psi_1\rangle \langle\Psi_1| = \rho_{\text{pure}} \otimes \frac{1}{2} \left(|00\rangle \langle 00| + |00\rangle \langle 11| + |11\rangle \langle 00| + |11\rangle \langle 11| \right) \tag{21}$$

Now we examine the state accessible to an external observer — i.e., we trace over the interior (infalling) degree of freedom:

$$\rho_{\text{out},1} = \mathrm{Tr}_{\text{in}}(\rho_1) = \frac{1}{2} \left(|0\rangle \langle 0| + |1\rangle \langle 1| \right) \tag{22}$$

This reduced state is maximally mixed, and its von Neumann entropy is:

$$S_{\text{out},1} = -\mathrm{Tr}(\rho_{\text{out},1} \ln \rho_{\text{out},1}) = \ln 2 \neq 0 \tag{23}$$

This nonzero entropy doesn't yet violate unitarity, since the total system (including the black hole interior) remains pure. To fully analyze the situation, we consider the evaporation process over N Hawking pairs. Each Hawking pair contributes an entangled state of the form $\frac{1}{\sqrt{2}}(|0\rangle_{\text{out}} |0\rangle_{\text{in}} + |1\rangle_{\text{out}} |1\rangle_{\text{in}})$. After N emissions, the joint system becomes:

$$|\Psi_N\rangle = |\psi_{\text{in}}\rangle \otimes \left(\frac{1}{\sqrt{2}} \left(|0\rangle_{\text{out}} |0\rangle_{\text{in}} + |1\rangle_{\text{out}} |1\rangle_{\text{in}} \right) \right)^{\otimes N} \tag{24}$$

Now we consider the perspective of an external observer after complete black hole evaporation. Since the internal (infalling) states are no longer accessible, we trace over them to obtain the reduced density matrix:

$$\rho_{\text{rad}} = \mathrm{Tr}_{\text{in}}(|\Psi_N\rangle \langle\Psi_N|) \tag{25}$$

This final state is a mixed state. Its von Neumann entropy is:

$$S_{\text{final}} = -\mathrm{Tr}(\rho_{\text{rad}} \ln \rho_{\text{rad}}) = N \ln 2 \neq 0 \tag{26}$$

We began with a pure state of entropy $S = 0$, but after full evaporation, the radiation is described by a mixed state with entropy $S = N \ln 2$. This represents the central paradox of Hawking radiation: the *nformation loss problem. If unitarity is to be preserved, there must be a resolution that avoids this entropy increase — a central topic in quantum gravity.

There are three classical approaches to addressing the black hole information paradox:

1. **Violation of Unitarity**: This proposal suggests that unitarity—the principle that information is preserved in quantum processes—may break down in the extreme gravitational environments near black holes. This would imply that the fundamental laws of physics require modification in such regimes.
2. **Information in Hawking Radiation**: According to this view, information is not lost but is instead subtly encoded in the Hawking radiation emitted by the black hole. Over time, as the black hole evaporates, this radiation carries the information back into the universe.
3. **Stable Remnants**: This approach posits that black holes do not completely evaporate. Instead, they leave behind stable, Planck-scale remnants that retain the information that fell into the black hole.

The AdS/CFT correspondence strongly supports the second option: that information is preserved and escapes in a unitary manner during the evaporation process.

AdS/CFT posits an exact duality between a gravitational theory in Anti-de Sitter (AdS) space and a conformal field theory (CFT) on the boundary, which is manifestly unitary. Since the boundary CFT cannot lose information, the bulk gravitational dynamics must also preserve unitarity. Therefore, from the AdS/CFT perspective, black hole evaporation must be unitary, and information must be encoded in the Hawking radiation in a subtle, possibly highly scrambled way

3 The Page Curve

Are all black holes mixed states?.
Let's consider two sealed containers at 0k. We put the first container(A) in contact with a heat source until equilibrium is reached, and then, decouple the heat source. Then we heat the second(B) through a laser until it reaches the same energy density.
A is entangled with the bath and is described by a density matrix

$$\rho_A = \frac{e^{-\beta H_a}}{Tr(e^{-\beta H_a})} \tag{27}$$

and the second is in some pure state

$$\rho_B = |\psi\rangle \langle\psi| \tag{28}$$

Given only the two containers, not knowing how we have prepared them, can we tell which is which? Any measurement on A will collapse it into a random pure state so we will not be able to differenciate them.
Now let's heat those system even more until the steam in the containers collapses into a black hole. Now we can ask the same question again and we will get the same answer, we cannot tell the difference between the two, this ultimately means that black holes can be in pure states, which we will assume when talking about the Page Curve

Let's consider the three following curves. The first one corresponds to the Von Neumann entropy of the Hawking radiation, the second corresponds to the thermodynamic entropy of the blakc hole and the third one is Page Curve, that follows the smaller of both curves.
The first will be a monotonic increasing curve from zero to the order of Hawking flux, GM^2. This curve will be called the Hawking curve.

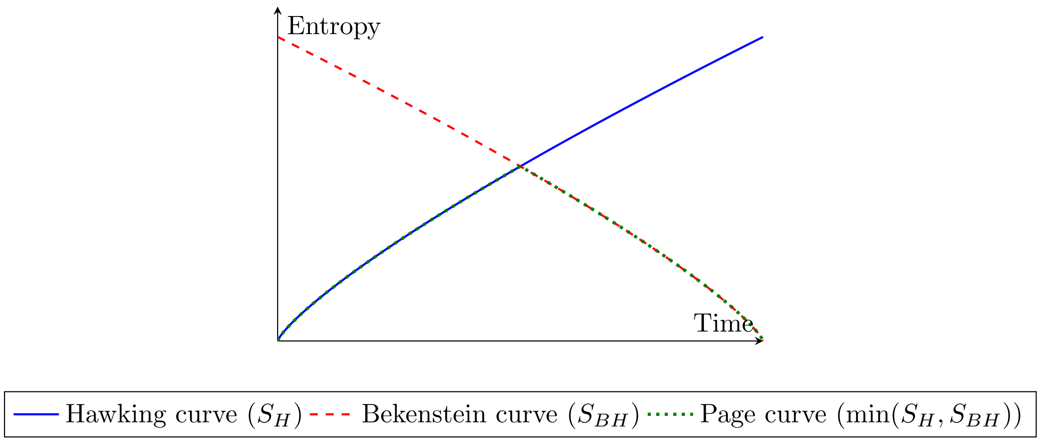
The second will decrease from Bekenstein entropy GM^2 to 0 and will be called the Bekenstein curve.
Page curve will be a parabola from 0 to 0 with a maximum of the order $GM^2/2$.

Let's consider now a black hole in a pure state, then the Von Neumann entropy of the Hawking radiation at a given time must be equal to that of the remaining black hole at that time and also to the entanglement entropy between the Hawking radiation and the black hole

$$S_H = S_{bh} = S_{entanglement} \tag{29}$$

Those three entropies follow Hawking curve.

The decreasing curve corresponds to the Bekesntein-Hawking entropy.
We can see that those two curves cross around the midpoint of the life of the black hole. Encountering again the problem that we have been discussing.
Following the curves to the end of the life of the black hole, Hawking radiation has a large Von Neumann entropy that reflects its entanglement with what remains of the black hole, and when the black hole disappears the Hawking radiation is all that is left, and its mixed state is the complete description of the system.
If we want the final state of Hawking radiation to be pure then S_H has to be 0 when the black hole disappears, so it will need to decline around the midpoint of the life of the black hole. The curve that S_H will follow is the Page curve.
The key insight is that the Hilbert space of a system grows exponentially with the number of bits. Except near the midpoint, one of the two systems (black hole or radiation) is much smaller, and therefore nearly maximally entangled. If the system is less chaotic, the entanglement entropy lies below the Page curve but must still deviate from the Hawking curve no later than the crossover.
The difference between the Page and Hawking curves is subtle in terms of measurement determining the exact curve requires complex observations of many Hawking quanta but the effect itself is large. Each Hawking pair increases entanglement by an amount of order one. To recover information, entanglement must decrease similarly something which is something that is not accounted for in Hawking's original calculation.



4 References

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