Parameter free iterative decoding metrics for non-coherent orthogonal modulation

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Metrics suited for iterative decoding of non-coherently detected bit-interleaved coded orthogonal modulation are studied. Proposed are metrics that do not require the knowledge of the signal-to-noise ratio, yet still offer very good performance.

Introduction and system model: Orthogonal modulation with non-coherent (NC) detection is a practical choice for situations where the received signal phase cannot be reliably estimated and/or tracked. Examples include military communications using fast frequency hopping, airborne communications with high Doppler shifts owing to significant relative motion of the transmitter and receiver, and high phase noise scenarios, owing to the use of inexpensive or unreliable local oscillators. A common choice for the modulator is frequency shift keying (FSK). The output alphabet of the FSK modulator is $\varepsilon = \{\mathbf{e}_b\colon b = 0, 1, \ldots, M-1\}$, where $\mathbf{e}_b$ is the canonical basis vector with a one at position b and zeros everywhere else. The channel output $\mathbf{y}[k] \in \mathbb{C}^M$ at time k is

$$\mathbf{y}[k] = \sqrt{E_s}h[k]\mathbf{x}[k] + \mathbf{n}[k], \quad k = 0, \ldots, L-1 \tag{1}$$

where E_s is the per-symbol transmit power, $h[k] \in \mathbb{C}$ is the channel gain at time k, $\mathbf{x}[k] = (x_0[k], \ldots, x_{M-1}[k])^T$, the output of the FSK modulator, is all zeros, except for a single element $x_b[k] = 1$, corresponding to transmission on a particular frequency bin $b \in \{0, 1, \ldots, M-1\}$ at time k, and $\mathbf{n}[k]$ is a vector of zero-mean circularly symmetric complex Gaussian noise samples, with variance N_0. In the cases where $|h[k]| = 1, \forall k$ or $h[k]$ are zero-mean complex Gaussian with unit variance, we have the additive white Gaussian noise (AWGN) and Rayleigh fading channels. The channel transition probabilities with non-coherent detection are [2]

$$p(\mathbf{y}[k]|\mathbf{x} = \mathbf{e}_b) = KI_0\left(2\frac{\sqrt{E_s}}{N_0}|h[k]||y_b[k]|\right) \tag{2}$$

where K is a constant independent of the hypothesis b and $I_0(\cdot)$ is the 0th order modified Bessel function of the first kind.

Bit interleaved coded modulation with iterative decoding (BICM-ID) [1] has been recently considered in [2] for the NC-FSK channel defined by (1) and (2) and a gain is demonstrated by iterating between demodulation and decoding. In BICM, the codewords $\mathbf{c} = (c_1, \ldots, c_N)$ of a binary code $\mathcal{C}$ of length $N = mL$ and rate R are interleaved and mapped over the signal alphabet, choosing frequency bin $b = \sum_{i=0}^{m-1} c_{\pi(mk+i)}2^i$ for transmission, where $m = \log_2 M$ and $\pi(\cdot)$ denotes the interleaver permutation.

An important consideration in many applications is the amount of channel state information (CSI) available at the decoder. This may range from full CSI, where the decoder knows the instantaneous fading amplitude and the average signal-to-noise ratio (SNR), to partial CSI, where only the average SNR is known, right through to no CSI, where not even the SNR is known. The latter case is of interest for partial band jamming of a fast frequency hopped system, where the resulting SNRs for each of the M frequency bins may vary with frequency and time. Valenti and Cheng [2] develop decoder metrics for both the full and partial CSI scenarios, but do not consider the complete absence of CSI.

In this Letter we develop low-complexity decoder metrics suitable for iterative decoding/demodulation with no CSI and we illustrate the corresponding effect of loss of CSI on the extrinsic information (EXIT) charts [3] of the demodulator and overall error probability.

Metrics for iterative decoding: BICM-ID consists of exchanging messages between the bitwise FSK demodulator and decoder of $\mathcal{C}$ in an iterative fashion. The bitwise FSK demodulator feeds the decoder of $\mathcal{C}$ with the log-likelihood ratios

$$\mathcal{L}(c_{\pi(mk+i)}) = \log\frac{\sum_{b\in\mathcal{B}_0^i} p(\mathbf{y}[k]|\mathbf{e}_b)q_{k,i}(b)}{\sum_{b\in\mathcal{B}_1^i} p(\mathbf{y}[k]|\mathbf{e}_b)q_{k,i}(b)} \tag{3}$$

where $\mathcal{B}_a^i \subset \{0, \ldots, M-1\}$ is the set of frequency bins that have the ith bit of the binary label equal to a (which implies that $|\mathcal{B}_a^i| = M/2$) and $q_{k,i}(b)$ are the extrinsic probabilities computed by the decoder of $\mathcal{C}$ in the previous iteration (at the first iteration these are all equal to 0.5). Substituting (2) into (3) we obtain the iterative decoder used by [2]. The summations in (3) may be undesirable from the point of view of complexity. To avoid these summations, the log-likelihood ratio (3) may be approximated in the standard way

$$\mathcal{L}(c[j(k,i)]) \simeq \max_{b\in\mathcal{B}_0^i}\log I_0\left(2\frac{\sqrt{E_s}}{N_0}|h[k]||y_b[k]|\right)q_{k,i}(b) - \max_{b\in\mathcal{B}_1^i}\log I_0\left(2\frac{\sqrt{E_s}}{N_0}|h[k]||y_b[k]|\right)q_{k,i}(b) \tag{4}$$

We shall refer to (3) and (4) as the Bessel and Bessel dual-max metrics, respectively. Note that to compute (3) and (4), E_s, N_0 or $|h[k]|$ (or accurate estimates) must be available to the receiver (full CSI).

We will now develop decoder metrics that do not depend on E_s, N_0 or $|h[k]|$. Taylor series expansion of the Bessel function $I_0(\alpha)$ around zero yields

$$I_0(\alpha) = 1 + \frac{\alpha^2}{4} + O(\alpha^4) \tag{5}$$

which motivates the following approximation of the log-likelihood ratios (3),

$$\mathcal{L}(c[j(k,i)]) \simeq \log\frac{\frac{M}{2} + \frac{E_s}{N_0^2}|h[k]|^2\sum_{b\in\mathcal{B}_0^i}|y_b[k]|^2q_{k,i}(b)}{\frac{M}{2} + \frac{E_s}{N_0^2}|h[k]|^2\sum_{b\in\mathcal{B}_1^i}|y_b[k]|^2q_{k,i}(b)} \tag{6}$$

If we further assume that $(E_s/N_0^2)\ |h[k]|^2\ \sum_{b\in\mathcal{B}_0^i}|y_b[k]|^2 \gg M/2$ we have

$$\mathcal{L}(c[j(k,i)]) \simeq \log\frac{\sum_{b\in\mathcal{B}_0^i}|y_b[k]|^2q_{k,i}(b)}{\sum_{b\in\mathcal{B}_1^i}|y_b[k]|^2q_{k,i}(b)} \tag{7}$$

which is independent of E_s, N_0 and the fading amplitudes $|h[k]|$. The interpretation of (7) is interesting. The receiver first measures the received energies at every frequency bin and computes the *empirical probability* at every bin as the fraction of the total received energy present in a given bin. Obviously, the normalisation factor (the total energy $\sum_{i=0}^{M-1}|y_i[k]|^2$) cancels in (7). We can further approximate (7) using the dual-max method as follows,

$$\mathcal{L}(c[j(k,i)]) \simeq \max_{b\in\mathcal{B}_0^i}\log(|y_b[k]|^2q_{k,i}(b)) - \max_{b\in\mathcal{B}_1^i}\log(|y_b[k]|^2q_{k,i}(b)) \tag{8}$$

which yields the corresponding parameter free dual-max metrics.

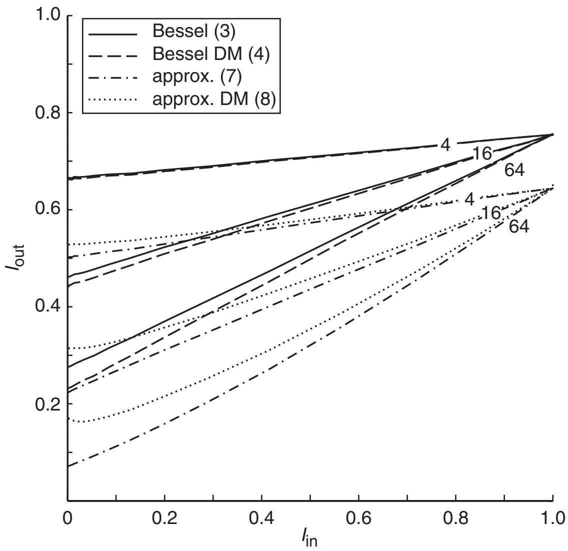

Fig. 1 *EXIT charts of Bessel and parameter free metrics for M = 4, 16, 64 on AWGN channel with SNR = 6 dB*

We now present some numerical examples which demonstrate the utility of the parameter free metrics. Since we are interested in application of the metrics to iterative decoding, it is of interest to compare the corresponding EXIT charts [3]. Fig. 1 shows EXIT charts for soft demodulation using the Bessel metrics (3) (solid), dual-max Bessel (4) (dashed), and the parameter free metrics (7) (dashed-dotted) and (8) (dotted) for $M=4$, 16, 64 in the AWGN channel. The curves exhibit an almost linear behaviour, with Bessel metrics and parameter free metrics resulting in similar slopes. This implies that at higher SNR, the parameter metrics will have the same EXIT chart, which will help in assessing the performance degradation due to the lack of CSI. Further, we observe that the parameter free metrics are information lossy, namely, when the input mutual information is $I_{\text{in}}=1$, the output mutual information is lower than that obtained with Bessel metrics. Finally, and perhaps most surprising, the parameter free dual-max metric (8) is significantly better than (7) at low I_{in}, despite the reduction in computational complexity. Application of the dual-max approximation following the Taylor approximation seems to regain some of the loss from the ideal Bessel metrics. Similar charts are obtained for the Rayleigh fading channel. From now, we concentrate in comparing the metrics (3) and (8).

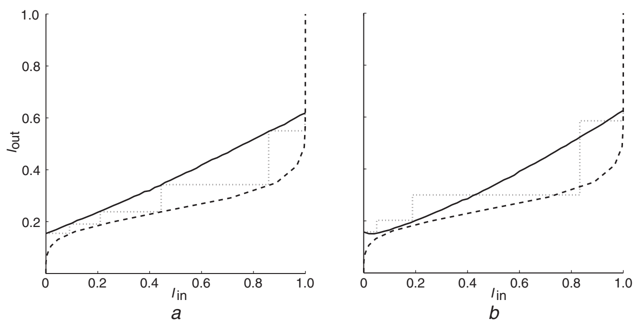

Fig. 2 *Trajectories for $M=64$ in AWGN channel with $(25, 27, 33, 37)_8$ convolutional code using Bessel (3) at $E_b/N_0=3$ dB and parameter free metrics (8) at $E_b/N_0=4$ dB*

a Bessel (3) at $E_b/N_0=3$ dB
b Parameter free metrics (8) at $E_b/N_0=4$ dB

Fig. 2 shows the EXIT charts and simulated trajectories for metrics (3) (Fig. 2*a*) and (8) (Fig. 2*b*) with the $(25, 27, 33, 37)_8$ convolutional code and 64-FSK in the AWGN channel. While the EXIT analysis predicts the threshold behaviour quite accurately for (3), the EXIT chart analysis is slightly pessimistic in the case of metrics (8). This is because the Gaussian approximation inherent in the EXIT analysis is not accurate. Recall that metrics (8) are a result of three consecutive approximations to (3) and therefore some loss in the Gaussianity of the iterative process is expected. The predicted EXIT chart thresholds are $E_b/N_0=2.5091$ dB for metrics (3) and $E_b/N_0=3.8391$ dB for (8). Simulations show that bit error rate (BER) of 10^{-5} is achieved at 4 and 4.6 dB, respectively, thus implying that the EXIT chart analysis is slightly optimistic. The penalty for not knowing the channel is 0.60 dB only. Table 1 summarises the simulated BER for BICM with an outer rate $R=1/4$ repeat-accumulate code and 4, 16 and 64-ary NC-FSK in the AWGN and Rayleigh fading channels. The simulations were performed using 10 000 information bits per codeword and 20 decoding iterations (one iteration of the RA decoder per demodulation iteration). The results in Table 1 highlight the small loss for not knowing E_s, N_0 or the fading amplitude.

Table 1: E_b/N_0 at 10^{-5} for $M=4$, 16, 64 and RA code of $R=1/4$

M	AWGN		Rayleigh fading	
	Metrics (3)	Metrics (8)	Metrics (3)	Metrics (8)
4	5.5 dB	5.9 dB	6.1 dB	8.1 dB
16	3.9 dB	4.3 dB	4.3 dB	5.8 dB
64	3.5 dB	4.1 dB	3.7 dB	5.1 dB

Conclusion: We present a low complexity method of computing metrics suited for iterative demodulation/decoding of M-ary non-coherent orthogonal modulation that does not require any knowledge of the signal-to-noise ratio or fading coefficients at the receiver. The method is based on the first-order Taylor series expansion of the Bessel function. The proposed method performs very close to the ideal metrics and enables use of methods such as BICM over non-coherent channels without side information.

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References

1 Li, X., and Ritcey, J.A.: 'Trellis-coded modulation with bit interleaving and iterative decoding', *IEEE J. Sel. Areas Commun.*, 1999, **17**, (4), pp. 715–724
2 Valenti, M.C., and Cheng, S.: 'Iterative demodulation and decoding of turbo-coded M-ary noncoherent orthogonal modulation', *IEEE J. Sel. Areas Commun.* (Special Issue on Differential and Noncoherent Wireless Communications), 2005, **23**, (9), pp. 1739–1747
3 ten Brink, S.: 'Convergence of iterative decoding', *Electron. Lett.*, 1999, **35**, (10), pp. 806–808