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Relationship between dilemma occurrence and the existence of a weakly dominant strategy in a two-player symmetric game

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Abstract

Defining the dilemma game by the proposition, *A game cannot sustain an increase of cooperation strategy in its strategy distribution*, we deduced that the substance of a dilemma can be expressed by a productive summation of the *static factor* and the *dynamic factor* independently. A static factor is an element of the game's structure that influences a possible dilemma, which relates to a game's structural deviation from a situation where the cooperation strategy can be weakly dominant over other strategies. In contrast, a dynamic factor refers to a strategy distribution's influence on the dilemma by affecting the game dynamics. In a 2×2 game, the existence of a dilemma can be determined only by a static factor. That is, whether or not a dilemma occurs is related only to the structural effect of the game. On the other hand, in a more-than-two-strategies game, both static and dynamic factors determine the occurrence of a dilemma, and the static factor cannot solely explain the occurrence of a dilemma.

Keywords: Dilemma; Cooperative mutualism; Evolutionary game; Dynamics; Prisoner's dilemma; Chicken; Hero; Leader

1. Introduction

Evolutionary game theory has been applied to various fields, such as biology, economics, and sociology (Maynard Smith (1982)), with the support of mathematical science. An interesting theme in evolutionary game theory, and even in classic game theory, is the so-called dilemma game, which can be applied to fields such as political science and environmental problems (Tayer (1987)). From a biological viewpoint, the dilemma game, or the question of how cooperation can emerge in populations where all the other players are non-mutualisms, is an enduring conundrum. This is because cooperative interactions between members play an important role in ecosystems (Scheuring (2005)). Further, the dilemma game provides a fundamental basis on which to analyze human social systems, particularly those questioning how we establish social norms to maintain mutual cooperation (Yamashita et al. (2005)).

Hundreds, and possibly thousands, of studies have involved dilemma games, including PD (Prisoners' Dilemma), n-PD (n-players PD), IPD (iterated PD), and others. An early study by Rapoport and Guyer (1966) that classified 2×2 games into several categories, highlighted the dilemma game (or "conflict game" in their terminology). Computer scientists continuing in the line of Axelrod's work (1984) have focused on how cooperation can be supported in various dilemma games, from typical examples such as PD or Chicken to more specific dilemma games. Game iteration as well as spatial structure or social network is confirmed to be effective for the emergence of cooperation (e.g., Nowak et al. (1994)). A punishing option (e.g., Boyd et al. (1992)) and a not-participating option (e.g., Schuessler (1989)) are other

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measures that support cooperation. We first address two questions: (1) what is the "dilemma" in a game? and (2) how can we measure its intensity in different game structures? Unfortunately, this kind of fundamental work has thus far lagged behind practical applications.

In this paper, we explicitly define the substance of the dilemma game. That is, we deduce that the substance of a dilemma can be expressed by a productive summation of the *static factor* and the *dynamic factor* independently. A static factor is an element of the game's structure that influences a possible dilemma, which relates to a game's structural deviation from a situation where the cooperation strategy can be weakly dominant over other strategies. In contrast, a dynamic factor refers to a strategy distribution's influence on the dilemma by affecting the game dynamics. In a 2×2 game, the existence of a dilemma can be determined only by a static factor. But in games of more than two strategies, the dilemma existence cannot be determined only by a static factor. Our discussion adopts the premise that the game has a symmetric, two-player structure based on a single population (Weibul (1995)).

2. Substance of the dilemma game

2.1. Game structure expression

In a symmetric two-player N-strategies game, a payoff matrix that determines the game structure is given as

$$\mathbf{M} = \begin{bmatrix} m_{11} & \dots & m_{1N} \\ \vdots & \ddots & \vdots \\ m_{N1} & \cdots & m_{NN} \end{bmatrix}. \tag{1}$$

Let a certain player adopt strategy i (=1, ..., N) that is expressed as $s_i \in s = \{{}^{\mathrm{T}}(1 \quad 0 \quad \cdots \quad 0), \ldots, (0 \quad \cdots \quad 0 \quad 1)\}$. By playing with another player who adopts strategy j (=1, ..., N), the player with strategy i gets π_{ij}, formulated as

$$\pi_{ij} = {}^{\mathrm{T}}\mathbf{s}_i \cdot \mathbf{M}\ \mathbf{s}_j, \tag{2}$$

where the superscript T indicates a transposition.

2.2. Cooperation strategy

A cooperation strategy can be precisely defined as follows:

Definition 1. Strategy i (=1, ..., N) is a cooperation strategy, when the following equation is established:

$$\max\,(m_{11}, m_{22}, \ldots, m_{N-1N-1}, m_{NN}) = m_{ii}.$$

2.3. Weak dominance by a cooperation strategy

In the following discussion, the game is defined as having a cooperation strategy i (=1, ..., N). According to the definition, the strategy i is weakly dominant over other strategies when $\max(m_{1j} - m_{ij}, m_{2j} - m_{ij}, \ldots, m_{Nj} - m_{ij})$ for any arbitrary strategies j (=1, ..., N). Inversely, if there is a strategy k (=1, ..., N) satisfying $m_{kj} - m_{ij} > 0$ for any arbitrary strategies j (=1, ..., N) in a game, the cooperation strategy i cannot be weakly dominant. Then, presuming the following $DL^k_{j(i)}$, we can measure a certain game's "deviation" from a situation where the cooperation strategy i is weakly dominant over other strategies.

$$DL^k_{j(i)} = m_{kj} - m_{ij} \tag{3}$$

We can say that $DL^k_{j(i)}$ is a dilemma potential (DP) contributed by the kth row and jth column of the game structure matrix, where the cooperation strategy is strategy i.

2.4. Dilemma game

Assuming the strategy distribution $\mathbf{s}={}^{\mathrm{T}}(s_1 \quad \cdots \quad s_i \quad \cdots \quad s_N)$ at any time step of the dynamics, the dynamics of cooperation strategy i can be expressed by the definition of replicator dynamics as

$$\dot{s}_i/s_i = [{}^{\mathrm{T}}\boldsymbol{s_i} \cdot \boldsymbol{M}\ \boldsymbol{s} - {}^{\mathrm{T}}\boldsymbol{s} \cdot \boldsymbol{M}\ \boldsymbol{s}] \tag{4}$$

Then, a dilemma game can be defined as follows, in a manner that seems consistent with common knowledge.

Definition 2. Let strategy i (=1, ..., N) be the cooperation strategy. It is a non-dilemma game if $\dot{s}_i \geq 0$ always holds, and a dilemma game otherwise.

2.5. Substance of dilemma

Next, we draw a mathematical relation between "dilemma" as defined in the previous section and "a deviation from a situation where the cooperation strategy can be weakly dominant over other strategies".

Replicator dynamics is transformed as follows by means of explicit expression of the game structure matrix:

$$\dot{s}_i/s_i = \sum_j m_{ij}s_j - \sum_k \sum_j m_{kj}s_js_k$$

$$= -\left(\sum_k \sum_j m_{kj}s_js_k = -\sum_j m_{ij}s_j\right)$$

$$= -\left(\sum_k \sum_j m_{kj} s_j s_k - \sum_k \sum_j m_{ij} s_j s_k\right)$$

$$= -\sum_k \sum_j DL^k_{j(i)} s_j s_k \tag{5}$$

Eq. (5) contains a significant statement: whether a game has a dilemma or not can be depicted by a productive summation of $DL^k_{j(i)}$ and $s_j s_k$. Dilemma potential $DL^k_{j(i)}$ is a deviation from a situation where the cooperation strategy i is weakly dominant over other strategies, which is actually a factor determined only by the game structure matrix. On the other hand, $s_j s_k$ is exactly a strategy distribution that can be determined by the dynamics of a game process. Hence, from a game dynamics point of view, it can be said that the former part is a *static factor* while the latter part is a *dynamic factor*. The most important implication from Eq. (5) is that the possibility of a dilemma can be clearly divided into static and dynamic factors, and an actual dilemma is determined by a productive summation of these two factors.

3. 2 × 2 games

In this section, we consider the 2×2 game world, where only the static factor determines whether or not a dilemma occurs, regardless of the game dynamics.

Let us consider a polar coordinate system to express the game structure, which is particularly effective for taking an overview of the holistic 2×2 game world. As schematically expressed in Fig. 1, the game structure can be depicted by a set of parameters; namely, x_0, r_1, r_2 and θ (deg).

$$m_{11} = x_0 - 0.5 r_1 \cos(45) \tag{6a}$$

$$m_{12} = x_0 + r_2 \sin(45 + \theta) \tag{6b}$$

$$m_{21} = x_0 + r_2 \cos(45 + \theta) \tag{6c}$$

$$m_{22} = x_0 + 0.5 r_1 \cos(45) \tag{6d}$$

If we consider two games with similar possible solutions as substantially analogous, we can normalize by using $r_2/r_1 \equiv r$. Fig. 1 shows that x_0 is independent of relative relationships among payoffs, because increasing or decreasing x_0 means only a translation of the possible solution set along the 45° line. Consequently, a single set of two parameters, r and θ, is sufficient for observing the whole 2×2 game world.

Table 1
Criteria for various typical games

General assumption: $m_{22} > m_{11}$	
PD	$m_{21} > m_{11}$ and $m_{12} > m_{22}$
Chicken	$m_{12} > m_{22}$ and $m_{21} > m_{11}$. Particularly, (□) $P_{DD}P_{DC}P_{CC}P_{CD}$ is convex
Stag Hunt	$m_{21} > m_{11}$ and $m_{12} > m_{22}$. Particularly, (□) $P_{DD}P_{DC}P_{CC}P_{CD}$ is convex
Leader	$m_{12} > m_{22}$ and $m_{21} > m_{11}$. Particularly, (□) $P_{DD}P_{DC}P_{CC}P_{CD}$ is not convex. And $m_{12} > m_{21}$
Hero	$m_{12} > m_{22}$ and $m_{21} > m_{11}$. Particularly, (□) $P_{DD}P_{CD}P_{CC}P_{DC}$ is not convex. And $m_{21} > m_{12}$
Anti-Leader	$m_{21} > m_{11}$ and $m_{12} > m_{22}$. Particularly, (□) $P_{DD}P_{DC}P_{CC}P_{CD}$ is not convex. And $m_{12} > m_{21}$
Anti-Hero	$m_{21} > m_{11}$ and $m_{12} > m_{22}$. Particularly, (□) $P_{DD}P_{CD}P_{CC}P_{DC}$ is not convex. And $m_{21} > m_{12}$
Avatamsaka	$m_{21} > m_{11}$ and $m_{12} > m_{22}$

(□) a quadrangle of a possible solution set.

3.1. Various 2 × 2 dilemma games

Let us consider the parameterized expression for a 2×2 game. Assume that $m_{22} > m_{11}$, which implies that strategy $i = 2(\mathbf{s_2} = {}^{\mathrm{T}}(0 \quad 1))$ is a cooperation strategy (C) and strategy $i = 1(\mathbf{s_1} = {}^{\mathrm{T}}(1 \quad 0))$ is a defection one (D). Further, let us assume that $x_0 = (m_{22} + m_{11})/2$ and payoffs of the players' mutual strategy pairs of (D, D), (C, C), (C, D), and (D, C) are expressed by P_{DD} (in Fig. 3), P_{CC}, P_{CD}, and P_{DC}, respectively.

According to Eq. (6), a single set of two parameters, $r_2/r_1 \equiv r$ and θ, is sufficient for observing the whole 2×2 game world, as shown in Fig. 2. An outstanding feature is that well-known typical dilemma games such as PD, Chicken, SH, Leader, and Hero can be distinctly drawn, including their occurring regions. Further, Fig. 2 gives an intelligible picture that "the larger r contains the larger dilemma" (see Fig. 4 for details). Table 1 indicates the conditions of various typical games. The game structure matrix of the Hero game is consistent with the transposed matrix of the Leader game. Thus, we can say that Leader is the reverse of Hero. "Reverse" refers to a geometric relation that is symmetric around either the $\theta = 0°$ or $\theta = 180°$ axis in Fig. 1. As mentioned in the following section, the Chicken game arises from a Pareto optimal dilemma situation, while the SH game arises from a Pareto least dilemma situation (see Appendix A for further details). We use the prefix "anti-" to describe this relationship between Chicken and SH. Thus, in a sense, SH can be said to be anti-Chicken. In the same context,

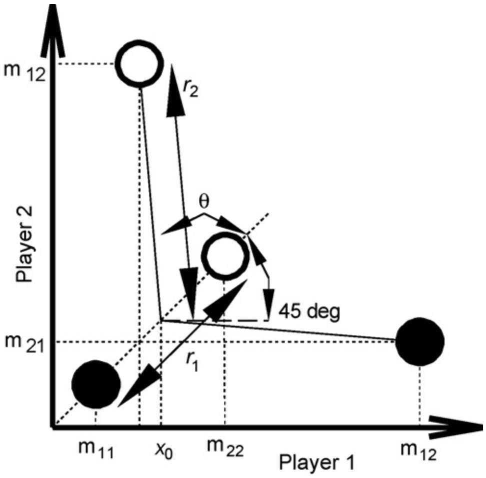

Fig. 1. General description for a 2×2 game. An explicit expression of m_{ij} by both θ and r can be drawn in Eq. (6).

we can predict that there must be anti-Leader and anti-Hero games. Such games certainly exist in Fig. 2, under the conditions listed in Table 1. Incidentally, both Leader and Hero can be categorized as versions of Chicken, broadly speaking, because their dilemmas arise from the Pareto optimal dilemma situation, as is explained further. Likewise, both anti-Leader and anti-Hero are classified as versions of SH, broadly speaking.

Fig. 3 shows examples of 10 typical games, including seven dilemma games: PD, Chicken, SH, Leader, Hero, anti-Leader, and anti-Hero. A game without a dilemma, in which cooperation (C) is regarded as the (at least weakly) dominant strategy, is called trivial. The Reverse PD game shown in Fig. 3(*j*) is obviously classified as trivial, since the cooperation strategy is dominant, as shown in the model in Fig. 3(a). In fact, Reverse PD is located

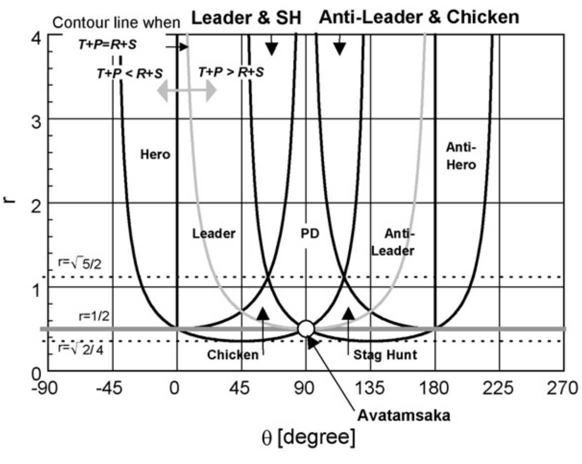

Fig. 2. Dilemma area in the case of a 2×2 game expressed by the proposed schematic description.

in an area defined by Fig. 2 as a no-dilemma zone. One specific game, called Avatamsaka, devised by Akiyama and Aruka (2004), stands on a marginal point between dilemma and trivial games, as shown in Fig. 2.

3.2. Relationship between dilemma existence in 2 × 2 games and dilemma potential

In the 2×2 game world, the dilemma potential defined by Eq. (3) can be paraphrased as

$$DL^{1}_{1(2)} = m_{11} - m_{21} = DL_1 \tag{7}$$

$$DL^{1}_{2(2)} = m_{12} - m_{22} = DL_2. \tag{8}$$

According to the Nash equilibrium, the following statements seem to be trivial:

$$DL > 0 \Leftrightarrow m_{11} - m_{21} > 0$$

$$\Leftrightarrow \text{(D, D) must be an equilibrium.}$$

$$DL_2 > 0 \Leftrightarrow m_{12} - m_{22} > 0$$

$$\Leftrightarrow \text{(C, C) cannot be an equilibrium.}$$

Each statement evokes a different DL_j. Under the conditions $m_{21} > m_{22}$ and $m_{12} > m_{22}$, we know such a game is going to be a trivial game, since (D, D) can never be an equilibrium but (C, C) becomes an equilibrium. This implies that (C, C) is strongly dominant over the other strategy pairs. Further, we know that, in a trivial game, $\dot{s}_2 \geq 0$ always holds, which implies that a no-dilemma is defined. That is, only by these two statements can we determine whether or not a dilemma exists in a 2×2 game. This is because having only two strategy alternatives in the 2×2 game leads to a specific situation where the static factor alone determines dilemma existence, even though it is generally determined by both the static and dynamic factors as expressed in Eq. (5). In other words, there is no such thing as a no-dilemma game ($\dot{s}_i > 0$) that is based on the strong dynamic factor that compensates for positive dilemma potentials. (As discussed in the next section, this type of no-dilemma game can exist in general $N > 2$ game worlds.) Furthermore, in 2×2 games, we can say that dilemma existence is determined by only whether the cooperation strategy can be dominant over other strategies. This implies that the dilemma potential is sufficient to estimate the dilemma possibility of the game structure.

Fig. 2 and Table 1 show that the condition related to the weakly dominant strategy, i.e., when $m_{21} \geq m_{11}$ and $m_{12} \geq m_{22}$, is consistent with the area for the trivial game. On the other hand, if $m_{21} \geq m_{11}$ or $m_{12} \geq m_{22}$ is not satisfied, it is confirmed that a certain type of dilemma game arises.

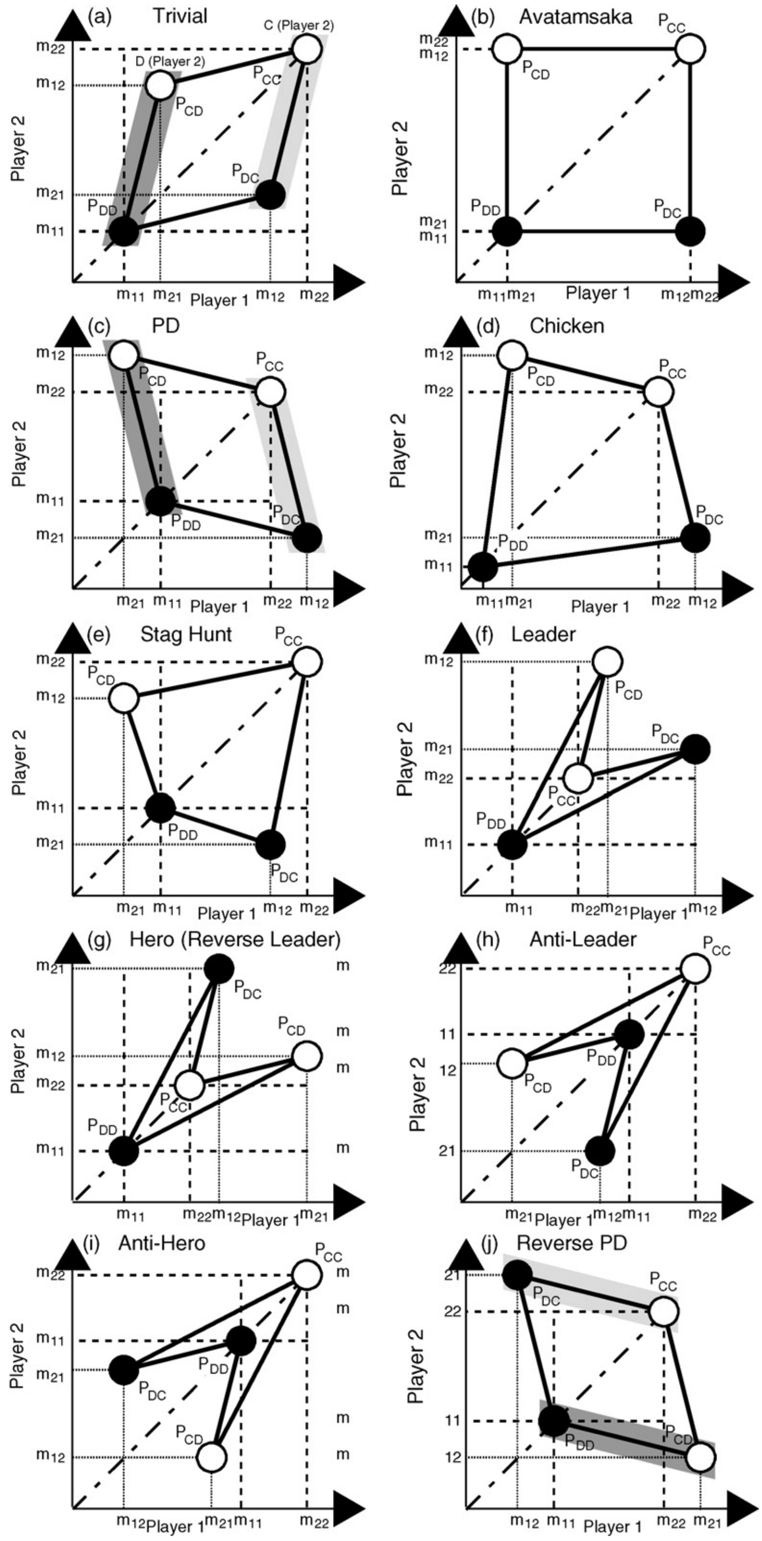

Fig. 3. Ten typical games in a 2 × 2 game world. X-axis and Y-axis indicate both players' payoffs. Open circle indicates that Player 1 adopts a cooperation strategy (C), while closed circle indicates that Player 1 defects (D). Light gray and dark gray areas designate Player 2's strategies, C and D, respectively. Hence, it is easy to understand that D is the dominant strategy in a PD game, while C is dominant in a reverse PD game.

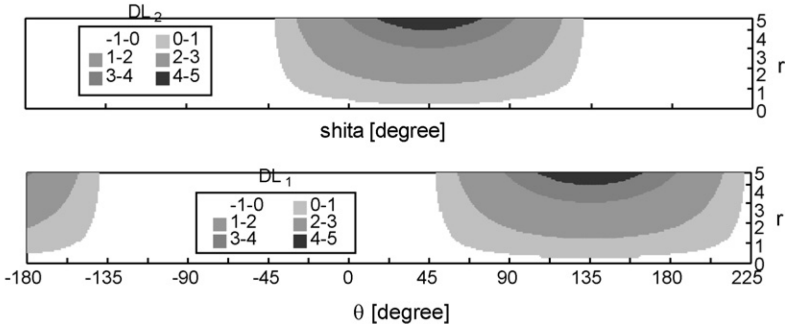

Fig. 4. DL_1 and DL_2, which are derived from a risk-averting dilemma (RAD) and a gamble-intending dilemma (GID), respectively, in the 2×2 game world. Each positive area indicates a domain-occurring RAD or GID.

Further inspection of Fig. 2, Table 1, and Fig. 3 reveals that the game is going to be Chicken, Leader, or Hero when $m_{12} \geq m_{22}$ is not satisfied, i.e., when $m_{12} \geq m_{22}$. In Chicken, as shown in Fig. 3(d), the solutions P_{CC} (i.e., where both players 1 and 2 select C), P_{CD}, and P_{DC} comprise the Pareto optimal, and we cannot determine the optimum solution among the three. In Leader and Hero (Fig. 3(f) and (g)), two solutions, P_{CD} and P_{DC}, make up the Pareto optimal, and we cannot determine the optimum solution between the two. This situation, in which no solution is optimal in terms of the upper-side payoff, inevitably leads to a particular dilemma in which equal players are inclined to exploit each other (as a result, (C, C) cannot be an equilibrium). We call this the gamble-intending dilemma (GID). The extent of a GID is estimated as DL_2.

Likewise, when $m_{21} \geq m_{11}$ is not satisfied, i.e., when $m_{21} \geq m_{11}$, the game becomes SH, anti-Leader, or anti-Hero. Observing Fig. 3(e), the solutions P_{DD}, P_{CD}, and P_{DC} establish the Pareto least, and we cannot determine the worst solution among the three. Further, in anti-Leader or anti-Hero (Fig. 3(h) and (i)), the Pareto least is composed of P_{CD} and P_{DC}. This situation, in which there is no worst payoff on the lower side, inevitably leads to another particular dilemma, wherein equal players try never to be exploited by each other (as a result, (D, D) must be an equilibrium). We call this situation the risk-averting dilemma (RAD). The extent of RAD can be estimated as DL_1.

PD is a particular kind of game, because both conditions, $m_{21} \geq m_{11}$ and $m_{12} \geq m_{22}$, are not satisfied at the same time, which means that PD has both GID and RAD at the same time; in other words, PD entails qualities of both Chicken and SH simultaneously.

Fig. 4 indicates both the areas and strengths of $DL_1 > 0$ and $DL_2 > 0$. The area of $DL_2 > 0$ consists of the areas of Chicken, Leader, and Hero, whereas that of $DL_1 > 0$ consists of the areas of SH, anti-Leader, and anti-Hero. PD (and, properly speaking, also Leader and SH, and anti-Leader and Chicken is the area overlapping both $DL_1 > 0$ and $DL_2 > 0$.

In 2×2 games, Eq. (5) can be rewritten as

$$\dot{s}_2/s_2 = -s_1 \cdot s_1 \cdot DL_1 - s_1 \cdot s_2 \cdot DL_2. \tag{9}$$

By substituting $s_1 = 1 - s_2$, we obtain Eq. (10).

$$\dot{s}_1/s_1 = s_1 \cdot s_2 \cdot DL_1 + s_2 \cdot s_2 \cdot DL_2 \tag{10}$$

Fig. 5 shows a landscape of $\dot{s}_1/s_1$ $\left(= \left\lfloor {}^{T}\boldsymbol{s_1}\,\boldsymbol{M} \cdot \boldsymbol{s} - {}^{T}\boldsymbol{s}\boldsymbol{M} \cdot \boldsymbol{s} \right\rfloor\right)$ depending on both parameters r and θ as $\mathbf{s}$ varies from $\boldsymbol{s} = {}^{T}(0.1 \quad 0.9)$ to $\boldsymbol{s} = {}^{T}(0.9 \quad 0.1)$ by increments of 0.2, while $\mathbf{s}_i$ is constant at ${}^{T}(1 \quad 0)$. This indicates whether the D-strategy is increasing ($\dot{s}_1/s_1 > 0$) or decreasing ($\dot{s}_1/s_1 < 0$) under a certain environment with a certain cooperative content of the whole society varying from a relatively cooperative $\boldsymbol{s} = {}^{T}(0.1 \quad 0.9)$ to a relatively defective $\boldsymbol{s} = {}^{T}(0.9 \quad 0.1)$ situation. Hence, in a sense, a more positive value in Fig. 5 implies a more serious dilemma tending toward defection. Alternatively, we can say that the landscape of Fig. 5 shows a holistic dilemma intensity including the dynamics point of view.

Comparing Figs. 4 and 5, we see that the positive area of the $\boldsymbol{s} = {}^{T}(0.1 \quad 0.9)$ case (Fig. 5(a)) almost completely overlaps $DL_2 > 0$, and the positive area of the $\boldsymbol{s} = {}^{T}(0.9 \quad 0.1)$ case (Fig. 5(e)) is almost consistent with $DL_1 > 0$. This implies that GID arises when most of the population of players is inclining toward cooperation, and RAD arises when most of the population becomes defective. This seems plausible because the temptation to exploit others becomes keener in a cooperative society. Conversely, the risk of being exploited by others is low in a cooperative society.

One such example is a game of $r = 2$ and $\theta = 10°$, as shown in Fig. 5, by one of two plot sets. Fig. 2 shows that

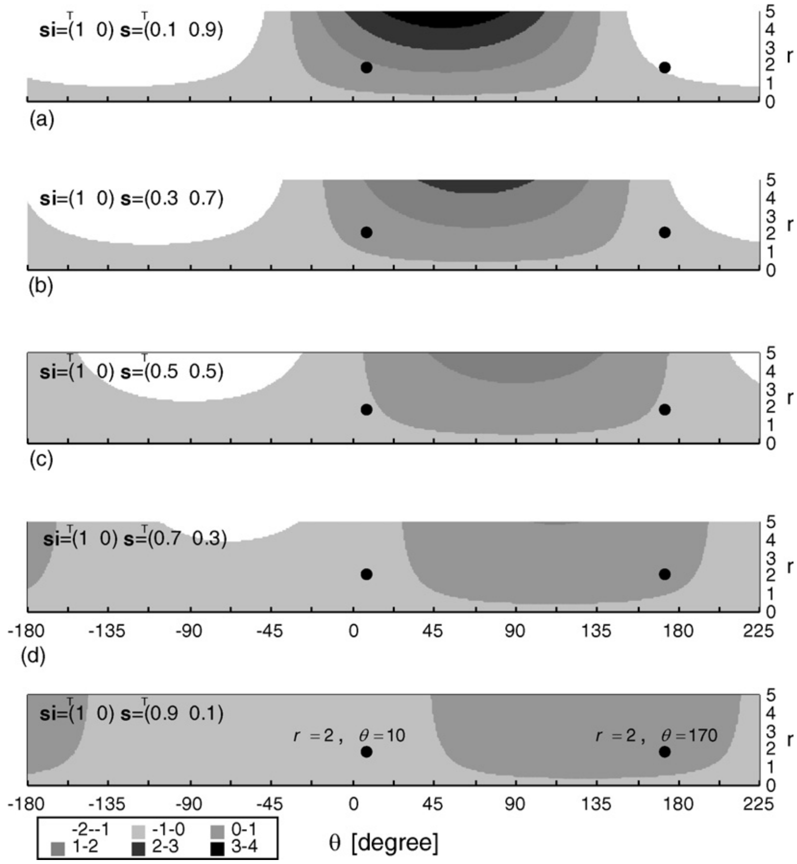

Fig. 5. Isograph of $\dot{s}_1/s_1$ in replicator dynamics. Positive indicates the area where the D-strategy is increasing.

this game is classified as a Leader with a Chicken-type dilemma, i.e., GID. As shown in Fig. 5(a), $\dot{s}_1/s_1 > 0$ at $\boldsymbol{s} = {}^{\mathrm{T}}(0.1 \quad 0.9)$, which implies that defective people are growing up. In contrast, at $\boldsymbol{s} = {}^{\mathrm{T}}(0.9 \quad 0.1)$, we see $\dot{s}_1/s_1 < 0$, which indicates that cooperative people are growing up. As consequences of these two trends, the dynamics of this case converge to an inner stationary point, $\left(\frac{-m_{12}+m_{22}}{m_{11}-m_{12}-m_{21}+m_{22}} \; \frac{m_{11}-m_{21}}{m_{11}-m_{12}-m_{21}+m_{22}}\right)$, which is well known as the inner stationary equilibrium in the Chicken game.

Another example is the case of $r=2$ and $\theta=170^\circ$, which is the anti-Leader game. Looking at each plot of this example in Fig. 5(a) through (e), under the relatively cooperative population, we see that $\dot{s}_1/s_1 < 0$, which implies that society becomes more cooperative. Under the relatively defective population, $\dot{s}_1/s_1 > 0$ is proved, which means the society becomes more defective. That is, there must be a bifurcation point at which the society goes to $\boldsymbol{s} = {}^{\mathrm{T}}(0 \quad 1)$ (all cooperative) or $\boldsymbol{s} = {}^{\mathrm{T}}(1 \quad 0)$ (all defective) depending on the initial environment: from $\boldsymbol{s} = {}^{\mathrm{T}}(0.1 \quad 0.9)$ to $\boldsymbol{s} = {}^{\mathrm{T}}(0.9 \quad 0.1)$. This particular point is actually at $\left(\frac{-m_{12}+m_{22}}{m_{11}-m_{12}-m_{21}+m_{22}} \; \frac{m_{11}-m_{21}}{m_{11}-m_{12}-m_{21}+m_{22}}\right)$, which is well known as the bifurcation point in the SH game.

It is important that Fig. 5 contains an obvious difference in contour density between (a) and (e), even when the panels are drawn to the same scale. On the other hand, DL_1 and DL_2 have the same contour lines, as shown in Fig. 4. This means that the real dilemma intensity should be grasped from not only the static but also the dynamic scope, as depicted by Eq. (10) (or Eq. (9)), which varies with the evolutionary time step.

3.3. *Comparable examples of dilemma by only the static factor*

Unequivocally, PD is the most well-known archetype for dilemma games. From a biological viewpoint, the Chicken game is sometimes called a Hawk–Dove game (Maynard Smith (1982)), which occurs when a pair of

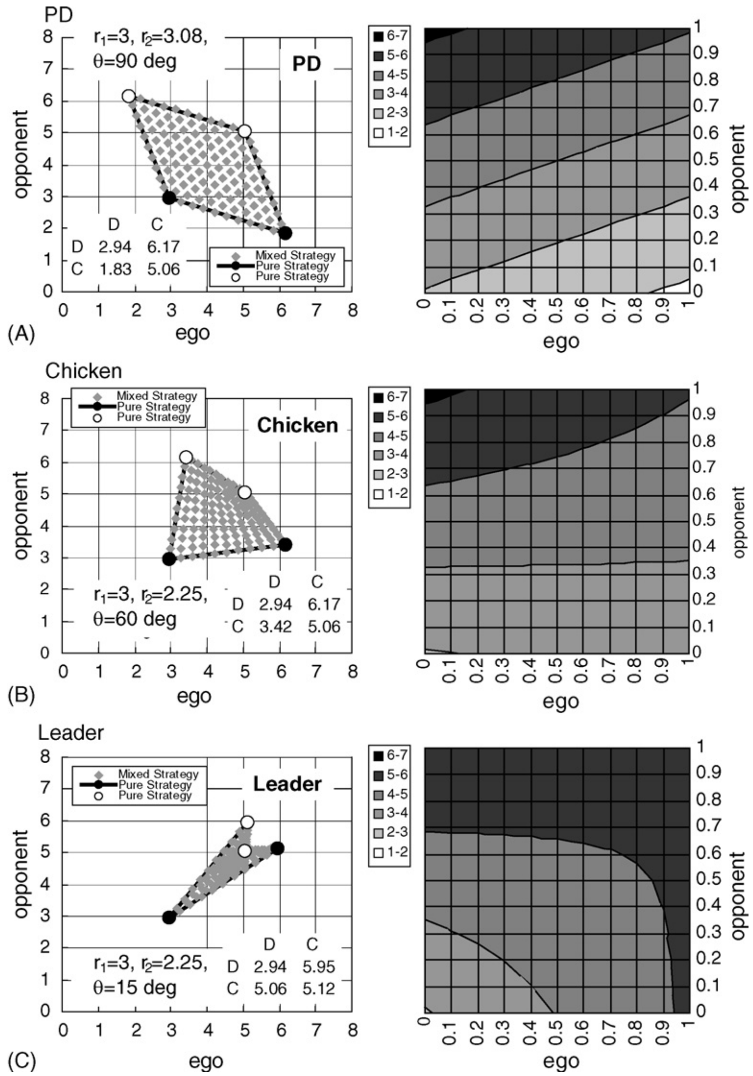

Fig. 6. A typical comparison of dilemma intensities. The right-hand side graphs show payoff contours varying the ego's and opponent's strategies, which makes it easy to understand whether or not there is a dominant strategy.

organisms contesting a resource engage in moderate fighting (C) or defect by escalated fighting (D). The Leader game might occur in nature if two organisms need to escape from a predator via an escape route through which only one of them can pass at a time. Each player can choose whether to escape in advance of another player (D) or wait for another player to escape with the intention of following immediately after (C). It might be useful if these three games are compared in terms of dilemma intensities.

Fig. 6 shows an intuitively understandable example of a relative comparison of dilemma intensities determined solely by DL_2. By comparing DL_2 among assumed games, it is proved that PD has the largest dilemma, Chicken the second largest, and Leader the smallest. PD and Chicken have the same DL_2, but PD has DL_1 as well. Hence, PD must have a more intense dilemma than Chicken, even though a quantitative comparison between the two is impossible.

4. Dilemma in general $N > 2$ game worlds

In $N > 2$ games, it is possible that a no-dilemma game ($\dot{s}_i > 0$) having positive dilemma potentials can be defined, unlike the case with 2×2 games. In short, we cannot say that a game where the cooperation strategy is not dominant over other strategies must be a dilemma game, although this specific statement is true in 2×2 games.

Let us consider the following example: a 2×3 (two-player, three-strategy) game defined as

$$\boldsymbol{M} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & -2 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{11}$$

In this game, the cooperation strategy $i=3$ is the so-called lonely strategy (or walk-away strategy) (e.g., Hauert et al. (2002)). The dilemma potential defined by Eq. (3) can be expressed as follows:

$$DL^1_{1(3)} = m_{11} - m_{31.} = -2 \tag{12a}$$

$$DL^1_{2(3)} = m_{12} - m_{32.} = 1 \tag{12b}$$

$$DL^2_{1(3)} = m_{21} - m_{31.} = 1 \tag{12c}$$

$$DL^2_{2(3)} = m_{22} - m_{32.} = -2. \tag{12d}$$

Hence, the cooperation strategy cannot be dominant over other two strategies. However, Eq. (5) gives us the following relation:

$$\dot{s}_3/s_3 = 2s_1^2 + 2s_2^2 - 2s_1s_2 \geq 0 \tag{13}$$

This obviously indicates that this is a no-dilemma game.

5. Conclusions

To clarify the substance of a dilemma, we defined a dilemma game as *one that cannot sustain an increase of cooperation strategy in its strategy distribution*. This proposition logically leads to the fact that dilemma existence can be expressed by a productive summation of both the *static factor* and the *dynamic factor* independently. The static factor refers to an influence of the game structure matrix on the existence of a dilemma, which relates to the structural deviation of a game from a situation where the cooperation strategy can be weakly dominant over other strategies. On the other hand, the dynamic factor refers to the strategy distribution's influence on game processing, which also affects the dilemma existence. In a 2×2 game, dilemma existence can be determined only by the static factor. This means that, whether or not a dilemma occurs, is related only to the game structural effect. On the other hand, in $N > 2$ games, both the static and dynamic factors determine the existence of a dilemma; the static factor alone cannot do this.

Appendix A. Pareto least

We can define the Pareto least as well as the Pareto optimal in the following way.

Definition A1. In a strategic-type n-player, N-strategy game (f_i indicates a payoff of the ith player, while s_j implies a strategy set of jth player), $G=(N, \{s_i\}_{i\in N}, \{f_i\}_{i\in N})$, if a strategy pair $s=(s_1, \ldots, s_n)$ of a strategy set $s = s_1 \times \cdots \times s_n$ is *Pareto least*, then there are no strategy pairs $(t_1, \ldots, t_n) \in s$ for any arbitrary players i (=1, ..., n) satisfying

$$f_i(t_1, \ldots, t_n) < f_i(s_1, \ldots, s_n)$$

If a certain strategy pair is Pareto least, there exist no other strategy pair that players want to avoid more.

Fig. A1 compares the concepts of Pareto least and Pareto optimal. Bold lines in the left and right panels indicate the Pareto optimal and the Pareto least, respectively. Assuming a situation where the possible solution region is a convex set, as shown, any solutions on segment P_bP_l will provide a lower payoff for Player A than for Player B; likewise, any solutions on segment P_lP_a will provide a lower payoff for Player B than for Player A. Hence, treating Players A and B equally, we cannot find the worst solution, but consider only the Pareto least.

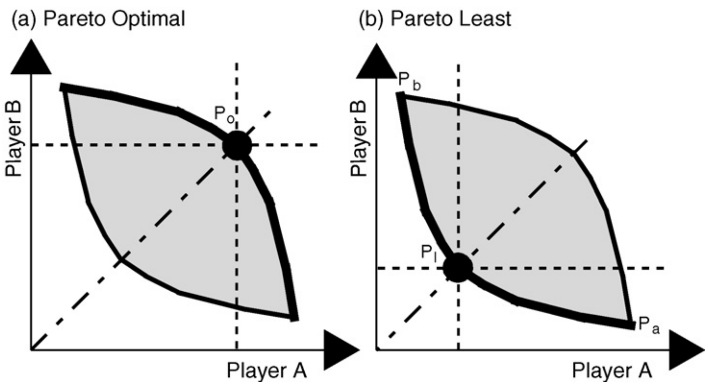

Fig. A1. Concepts of Pareto optimal and Pareto least. Gray area indicates a possible solution set.

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