Bayesian seismic wavelet and subsurface estimation via Gibbs sampling on a cyclic domain

Guillermina Senn [1] Matt Walker [2] Håkon Tjelmeland [1]

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[2]bp

Bloomington, December 9, 2024

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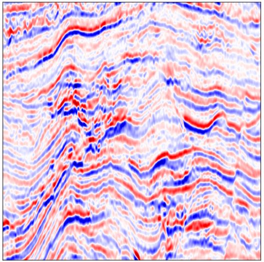

Seismic data.

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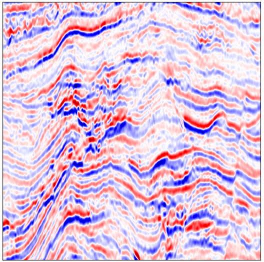

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Physical model

$$\boldsymbol{d}_j = \boldsymbol{c}_j \star \boldsymbol{w} + \boldsymbol{e}_j,$$

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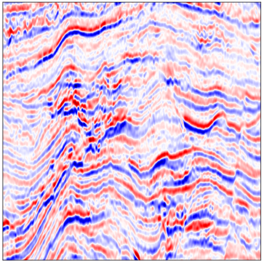

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Stochastic model

$$\begin{aligned}\boldsymbol{e} &\sim N(0, \Sigma_d),\\ \boldsymbol{c} &\sim N(0, \Sigma_c),\\ \boldsymbol{w} &\sim N(0, \Sigma_w).\end{aligned}$$

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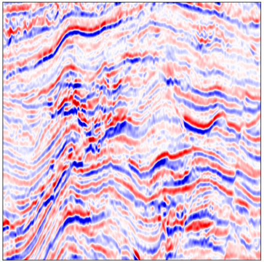

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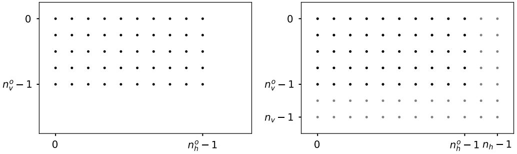
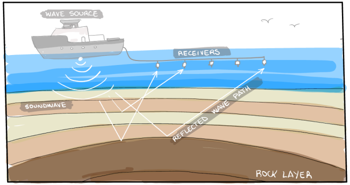


Gibbs sampler on a cyclic domain.

Seismic data and physical model



Data acquisition.

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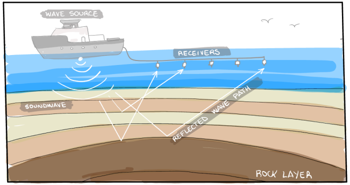

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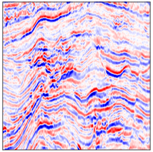

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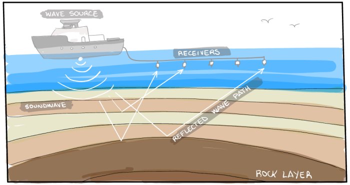

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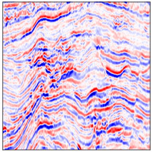

Seismic data $\boldsymbol{d}$.

d_{ij}: Seismic data at node (i, j).
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$\boldsymbol{w}$: Seismic wavelet.

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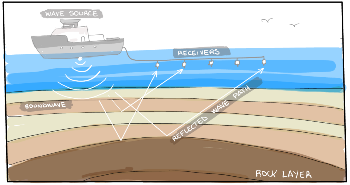

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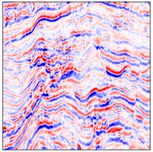

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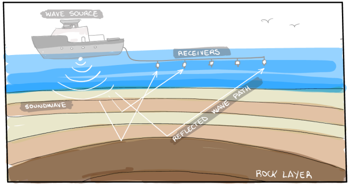

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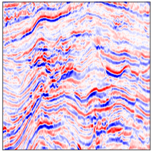

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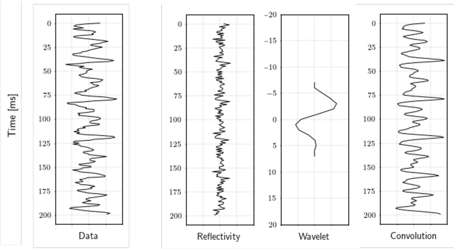
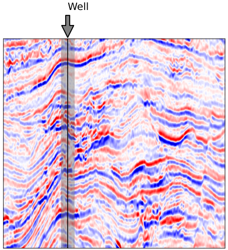
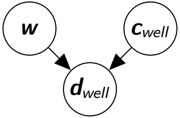


Convolutional model.

Bayesian joint wavelet and reflectivity estimation



Buland and Omre (2003a, b)



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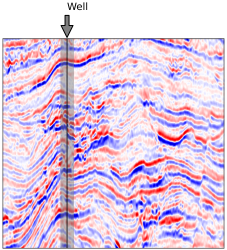

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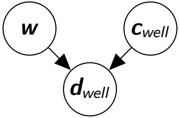
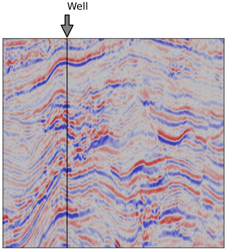


N nodes; Gibbs sampler is $\mathcal{O}(N^3)$



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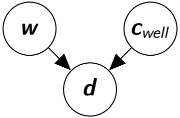

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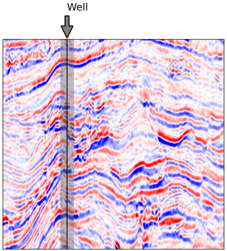

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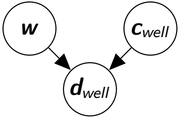

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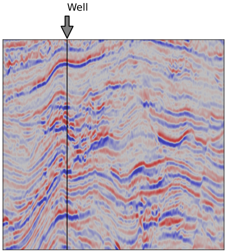

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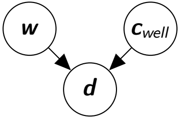

$n = NM$ nodes; cyclic Gibbs sampler is $\mathcal{O}(n \log n)$

The stochastic model

$$\boxed{\boldsymbol{d} = \boldsymbol{W}\boldsymbol{c} + \boldsymbol{e}}$$

$$\begin{aligned} p(\boldsymbol{c}|\sigma_c^2) &= N_n(\boldsymbol{c};\ 0, \sigma_c^2 \boldsymbol{R}_c) \\ p(\boldsymbol{w}|\sigma_w^2) &= N_k(\boldsymbol{w};\ 0, \sigma_w^2 \boldsymbol{R}_w) \\ p(\boldsymbol{d}|\boldsymbol{c}, \boldsymbol{w}, \sigma_c^2, \sigma_w^2, \zeta) &= N_n(\boldsymbol{d};\ \boldsymbol{W}\boldsymbol{c}, \sigma_d^2 \boldsymbol{R}_d), \quad \sigma_d^2 \propto \sigma_c^2 \sigma_w^2 \zeta \end{aligned}$$

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$$p(\sigma_c^2) = IG(\sigma_c^2;\ \alpha_c, \beta_c)$$
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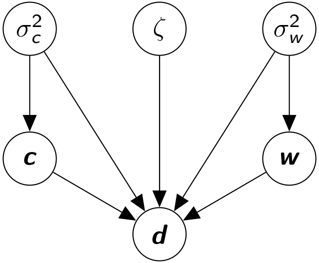

The stochastic model represented by a DAG.

The constrained stochastic model

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$$\boldsymbol{w}^{\star} = \boldsymbol{w} | \boldsymbol{A}_w \boldsymbol{w} = 0$$

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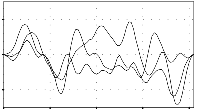

Samples from (constrained) wavelet prior.

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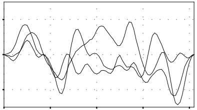

Samples from (constrained) wavelet prior.

$$\boldsymbol{c}^{\star} = \boldsymbol{c} | \boldsymbol{A}_c \boldsymbol{c} = \boldsymbol{c}_{well}$$
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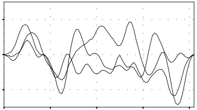

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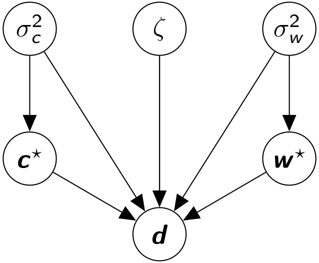

The model's DAG reflecting the constraints.

Posterior distribution

Given known data $\boldsymbol{y}$ and unknown parameters $\boldsymbol{\theta}$

$$\boldsymbol{y} = (\boldsymbol{d}, \boldsymbol{c}_{well}); \quad \boldsymbol{\theta} = (\boldsymbol{w}^\star, \boldsymbol{c}^\star, \sigma_w^2, \sigma_c^2, \zeta),$$

use Bayes' rule

$$p(\boldsymbol{\theta}|\boldsymbol{y}) = \frac{p(\boldsymbol{\theta}, \boldsymbol{y})}{p(\boldsymbol{y})} \propto p(\boldsymbol{y}, \boldsymbol{\theta})$$

to find the joint distribution

$$p(\boldsymbol{y}, \boldsymbol{\theta}) = p(\boldsymbol{d}, \boldsymbol{c}^\star, \boldsymbol{w}^\star, \sigma_c^2, \sigma_w^2, \zeta)$$

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&= N_n(\boldsymbol{d}; \boldsymbol{Wc}, \boldsymbol{\Sigma}_d) N_{n^\star}(\boldsymbol{c}^\star; 0, \boldsymbol{\Sigma}_c^\star) N_{k^\star}(\boldsymbol{w}^\star; 0, \boldsymbol{\Sigma}_w^\star) IG(\sigma_w^2; \alpha_w, \beta_w) IG(\sigma_c^2; \alpha_c, \beta_c) IG(\zeta; \alpha_\zeta, \beta_\zeta).
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Sample with Gibbs.

Full conditional distributions in the Gibbs sampler

$$p(\boldsymbol{c}^\star, \boldsymbol{d}, \boldsymbol{w}^\star, \sigma_c^2, \sigma_w^2, \zeta) = N_n(\boldsymbol{d}; \boldsymbol{Wc}, \boldsymbol{\Sigma}_d) N_{n^\star}(\boldsymbol{c}^\star; 0, \boldsymbol{\Sigma}_c^\star) N_{k^\star}(\boldsymbol{w}^\star; 0, \boldsymbol{\Sigma}_w^\star) IG(\sigma_w^2; \alpha_w, \beta_w) IG(\sigma_c^2; \alpha_c, \beta_c) IG(\zeta; \alpha_\zeta, \beta_\zeta).$$

FCs are Gaussian or inverse-Gamma.

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Example: Full conditional for the reflectivity $\boldsymbol{c}^\star$ is

$$p(\boldsymbol{c}^\star | \boldsymbol{d}, \boldsymbol{w}, \sigma_c^2, \sigma_w^2, \zeta) \propto p(\boldsymbol{d} | \boldsymbol{c}, \boldsymbol{w}, \sigma_c^2, \sigma_w^2, \zeta) p(\boldsymbol{c}^\star | \sigma_c^2)$$

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$$\begin{aligned} p(\boldsymbol{c}^\star | \boldsymbol{d}, \boldsymbol{w}, \sigma_c^2, \sigma_w^2, \zeta) &\propto p(\boldsymbol{d} | \boldsymbol{c}, \boldsymbol{w}, \sigma_c^2, \sigma_w^2, \zeta) p(\boldsymbol{c}^\star | \sigma_c^2) \\ &= N_n(\boldsymbol{d}; \boldsymbol{Wc}, \boldsymbol{\Sigma}_d) N_{n^\star}(\boldsymbol{c}^\star; 0, \boldsymbol{\Sigma}_c^\star) \\ &= N_{n^\star}(\boldsymbol{c}^\star | \boldsymbol{d}; \boldsymbol{\mu}_{c|d}, \boldsymbol{\Sigma}_{c|d}), \end{aligned}$$

with conditional parameters

$$\begin{aligned} \boldsymbol{\mu}_{c|d} &= \boldsymbol{\Sigma}_c^\star \boldsymbol{W}^T (\boldsymbol{W} \boldsymbol{\Sigma}_c^\star \boldsymbol{W}^T + \boldsymbol{\Sigma}_d)^{-1} \boldsymbol{d}; \\ \boldsymbol{\Sigma}_{c|d} &= \boldsymbol{\Sigma}_c^\star - \boldsymbol{\Sigma}_c^\star \boldsymbol{W}^T (\boldsymbol{W} \boldsymbol{\Sigma}_c^\star \boldsymbol{W}^T + \boldsymbol{\Sigma}_d)^{-1} \boldsymbol{W} \boldsymbol{\Sigma}_c^\star. \end{aligned}$$

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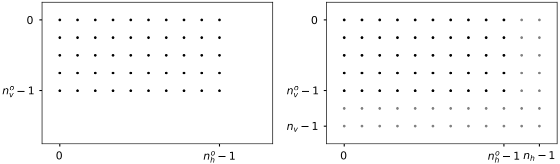

(a) Extend the seismic lattice and glue the edges.

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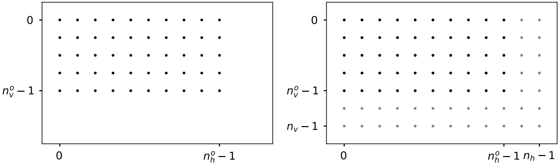

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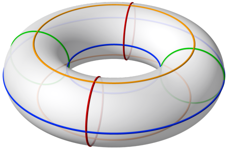

(b) The cyclic lattice.

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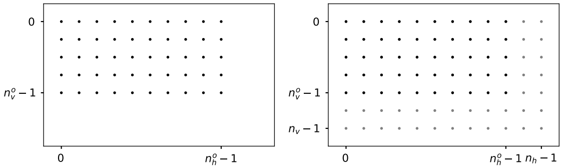

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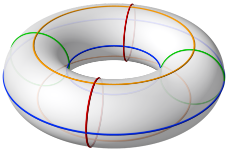

(b) The cyclic lattice.

1D distance vectors wrap around, i.e.

$$\boldsymbol{h}_h = \left(0, 1, \ldots, \frac{n_h}{2}, \frac{n_h}{2} - 1, \ldots, 1\right)^T, \quad \boldsymbol{h}_v = \left(0, 1, \ldots, \frac{n_v}{2}, \frac{n_v}{2} - 1, \ldots, 1\right)^T$$

Stationarity and separability on the cyclic lattice

The 1D stationary correlation vectors wrap around:

$$\boldsymbol{r}_h = (1, r_1, \ldots, r_{\frac{n_h}{2}}, r_{\frac{n_h}{2}-1}, \ldots, r_1)^T, \quad \boldsymbol{r}_v = (1, r_1, \ldots, r_{\frac{n_v}{2}}, r_{\frac{n_v}{2}-1}, \ldots, r_1)^T,$$

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The separable spatial correlation matrix is block-circulant with circulant blocks:

$$\underline{\underline{\boldsymbol{R}}} = \underline{\boldsymbol{R}}_h \otimes \underline{\boldsymbol{R}}_v.$$

The Fourier transform

Let $w = \exp(\frac{2\pi i}{n})$. $\boldsymbol{F}_n$ is the matrix such that

$$\boldsymbol{F}_n^H = \frac{1}{\sqrt{n}} \begin{pmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & w & w^2 & \dots & w^{n-1} \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & w^{n-1} & w^{2(n-1)} & \dots & w^{(n-1)(n-1)} \end{pmatrix}.$$

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Example: Let $\underline{\boldsymbol{A}} = \boldsymbol{F}_n^H \boldsymbol{\Lambda}_A \boldsymbol{F}_n$ and $\underline{\boldsymbol{B}} = \boldsymbol{F}_n^H \boldsymbol{\Lambda}_B \boldsymbol{F}_n$. Then

$$\underline{\boldsymbol{C}} = \underline{\boldsymbol{AB}} = (\boldsymbol{F}_n^H \boldsymbol{\Lambda}_A \boldsymbol{F}_n)(\boldsymbol{F}_n^H \boldsymbol{\Lambda}_B \boldsymbol{F}_n) = \boldsymbol{F}_n^H (\boldsymbol{\Lambda}_A \boldsymbol{\Lambda}_B) \boldsymbol{F}_n = \boldsymbol{F}_n^H \boldsymbol{\Lambda}_C \boldsymbol{F}_n.$$

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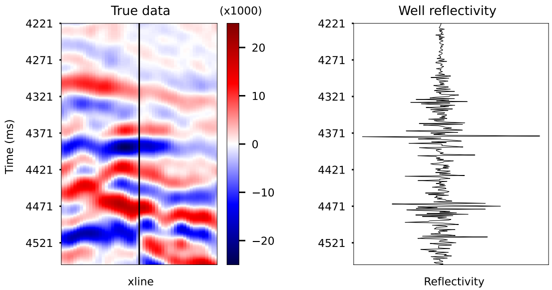
- Gaussian sampling with the DFT.

Real application: Gas reservoir in offshore Egypt



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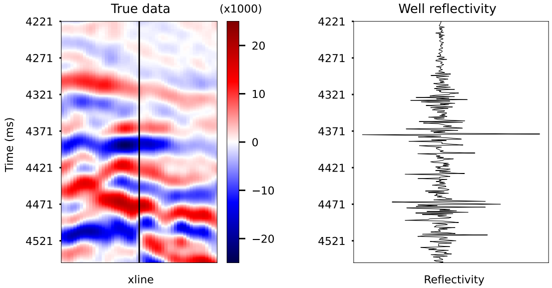
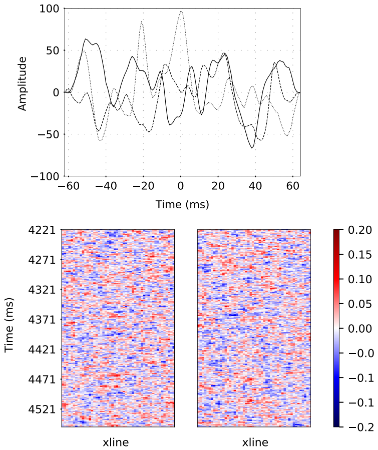
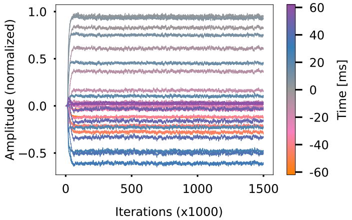


Wavelet and reflectivity prior samples:



Wavelet posterior exploration



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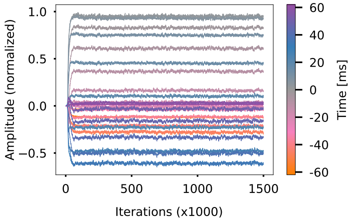

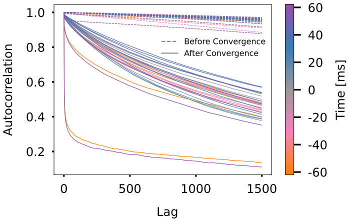

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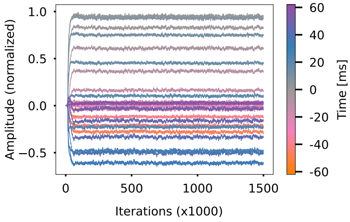
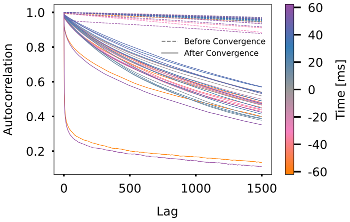
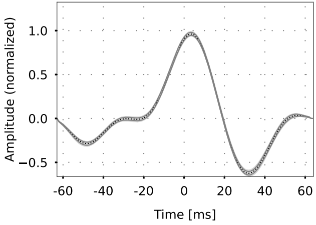
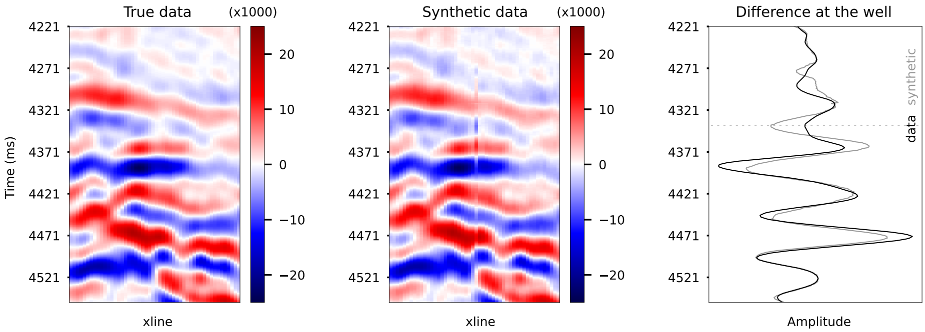






Synthetic data



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