$$\Phi_{k+3}(z) = A\Phi_{k-1}(z) + B\Phi_k(z) + C\Phi_{k+1}(z) + D\Phi_{k+2}(z)$$

$$\begin{aligned}
A &= (k+1)kz^2t_+^2 \\
B &= (k+1)z\left(z\beta^2 + (\lambda+1+2k)t_+\right) \\
C &= (k+1)(\lambda+1+k) - z^2(t_+^2 - \beta^2) + zt_+(\lambda+1) \\
D &= \lambda + 1 - 2zt_+
\end{aligned}$$