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Do Financial Markets Reward Buying or Selling Insurance and Lottery Tickets?

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Selling financial investments with insurance or lottery characteristics should earn positive long-run premiums if investors like positive skewness enough to overpay for these characteristics. The empirical evidence is unambiguous: Selling insurance and selling lottery tickets have delivered positive long-run rewards in a wide range of investment contexts. Conversely, buying financial catastrophe insurance and holding speculative lottery-like investments have delivered poor long-run rewards. Thus, bearing small risks is often well rewarded, bearing large risks not.

Economists have long puzzled over consumers' simultaneous demand for (risk-reducing) insurance and (risk-increasing) lottery tickets (Friedman and Savage 1948). Given such demand, it would seem that both selling insurance and selling lottery tickets might earn long-run rewards. These patterns appear inconsistent with rational behavior and standard risk preferences in a mean–variance world. Because both insurance and lotteries involve asymmetric payoffs, it is natural to seek explanations in skewness preferences.[1]

Financial markets are full of strategies that resemble insurance or lotteries. Thus, the title of this article asks whether investors can boost long-term expected returns by buying or selling insurance and lottery tickets—that is, financial investments that have insurance-like or lottery-like characteristics.[2]

The answer depends on the *market pricing of skewness*: how investors trade asymmetry against mean return. If most investors prefer positive skewness, investments with positively asymmetric payoffs tend to be highly priced and offer relatively low long-run returns. However, this outcome is not a foregone conclusion. Conceivably, for a given mean return, investors may prefer to suffer losses in one big hit instead of a slow trickle, implying a preference for negative skewness.

In my study, I posited that discussing separately the patterns on the left and right tails of the distribution can enhance our intuition. Positive skewness may reflect a truncated/thinner left tail or a thicker/longer right tail—that is, an eliminated/reduced likelihood of large losses or a greater possibility of outsized gains. Both characteristics can increase skewness and both can be appealing, but their psychological drivers seem quite different. On the left tail, we deal with demand for insurance, and the emphasis is on systematic risk and limiting the downside, especially in financial crises. On the right tail, we deal with demand for lottery tickets, and the emphasis is on idiosyncratic opportunities to enhance the upside. Both types of demand for positive skewness can lead to "overpricing" and thus to long-run rewards for sellers of insurance and lottery tickets.

■ *Discussion of findings.* This article surveys rational and behavioral theories on the pricing of asymmetric payoffs and presents wide-ranging empirical evidence in highly diverse contexts. Theoretical predictions differ on whether skewness is priced and, if so, with what sign. The empirical evidence is much more consistent. I found broadly similar patterns in diverse contexts:

- Selling volatility on either the left tail (insurance) or the right tail (lottery tickets) adds value in the long run. Conversely, buying option-based tail risk insurance against financial catastrophes and then holding lottery-like high-volatility investments results in poor long-run returns.
- The evidence is not restricted to option trading. Carry-seeking and other strategies with

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asymmetric payoffs are close cousins of volatility selling—all are variants of selling tail risk insurance and have earned positive long-run returns. Moreover, speculative, lottery-like investments have delivered lower risk-adjusted returns than their defensive peers in all major asset classes. Levering up low-volatility opportunities appears to boost long-run returns.

- To interpret the long-run gains from selling financial catastrophe insurance as rational risk premiums seems natural. In contrast, the gains from lottery selling seem better explained by investor irrationality or by such nonstandard preferences as lottery seeking or leverage aversion.

Theories on Skewness Pricing

Skewness pricing can be explained by rational risk-based theories and by theories involving irrational or nonstandard investor behavior.

Rational Risk Premium Theories. In the context of the capital asset pricing model (CAPM), skewness is not priced, either because return distribution is assumed to be normal (and thus symmetric) or because investors are assumed to care about only mean and variance (quadratic utility). However, Taylor expansions of many common utility functions (other than quadratic utility) imply an investor preference for positive skewness.

Extensions of the CAPM to three moments (see Kraus and Litzenberger 1976; Harvey and Siddique 2000) suggest that coskewness is priced. In three-moment CAPMs, the key aspect of skewness is not asymmetry in an asset's stand-alone returns but, rather, the contribution to the portfolio's skewness (i.e., the coskewness of an asset with the market). All else being equal, investors prefer assets that tend to earn high returns during volatile markets and thus make their portfolios more positively skewed. Such assets are rare, however, and investors may be willing to pay a high price for this characteristic. Conversely, because investors dislike negative skewness and require an extra *ex ante* reward for holding assets that perform poorly during volatile markets, they make their portfolios more negatively skewed. Other studies have stressed that if investors associate risk more asymmetrically with losses or crashes, downside betas or other measures of lower tail dependence may be more relevant risk measures than simple market betas.[3]

The ultimate determinant of required risk premiums in modern asset pricing models is the typical timing of losses—the covariance with such "bad times" as equity bear markets, financial crises, and economic recessions (periods when an extra dollar in your pocket feels especially valuable):

- Investments that perform poorly in bad times are risky and warrant high risk premiums.
- Investments with asymmetric payoffs that incur rare but large losses in bad times justify especially high risk premiums. Such investments include *selling financial catastrophe insurance* and *selling lottery tickets that pay off in bad times*.
- In contrast, safe-haven investments that perform well in bad times reduce portfolio losses when it matters most and thus warrant low or even negative risk premiums.

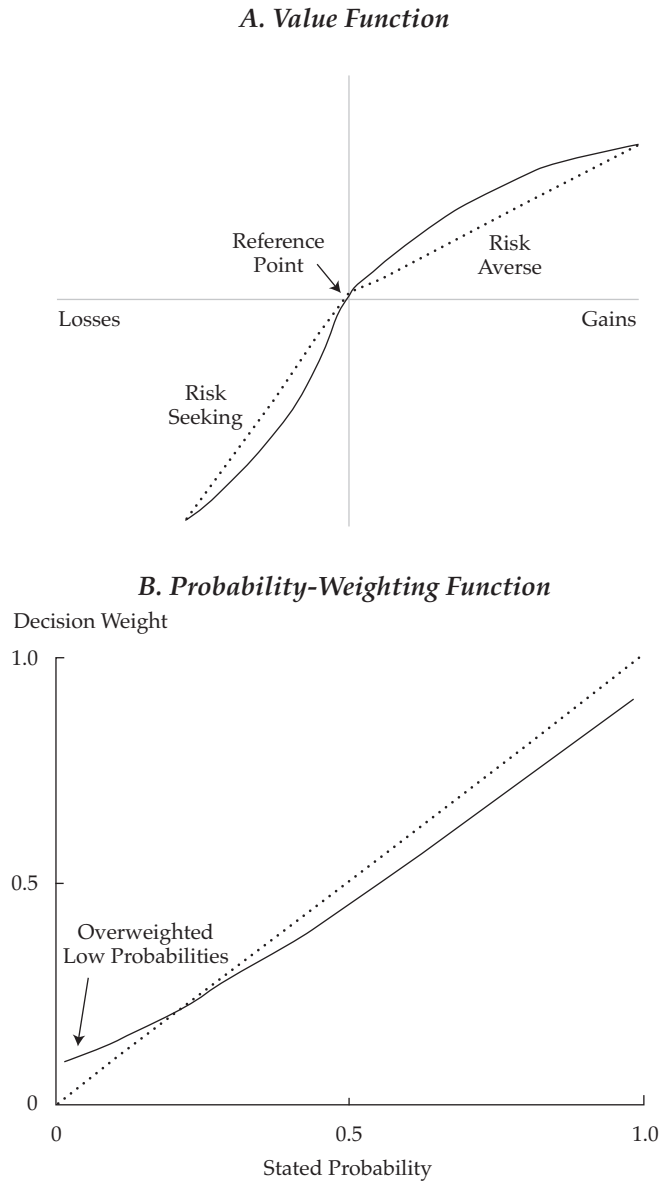
Irrational Investors and Nonstandard Preferences. To explain the pricing of *stand-alone* skewness, as opposed to coskewness, we must resort to nonstandard risk preferences or investor irrationality. There are at least three explanations for a skewness premium: investor overweighting of low-probability events, lottery preferences, and portfolio insurance preferences.

Overweighting of low-probability events. The most prominent explanation is the prospect theory of Kahneman and Tversky (1979), which can explain simultaneous demand for lottery tickets and insurance. This pattern does not follow from loss aversion or the shape of the value function (Panel A of **Figure 1**) but, rather, from the overweighting of low-probability events (Panel B of Figure 1). Such overweighting of small probabilities can be strong enough to reverse the sign of risk appetite in the value function. Lotteries that offer a small chance of very large gains can induce risk seeking, despite a general tendency toward risk aversion when gambles involve only gains. Conversely, people dislike low-probability, large-impact losses so much that they pay for insurance to avoid such losses (thus implying risk aversion), despite the general risk-seeking tendency when gambles involve only losses.[4]

In the spirit of prospect theory, Barberis and Huang (2008) developed a model in which investors overweight low-probability events. These investors are attracted to lottery-like assets with positively skewed returns because they offer a very large payoff with a small probability, which the investors overweight. In their paper, aptly titled "Stocks as Lotteries: The Implications of Probability Weighting for Security Prices," they suggested that such demand makes positively skewed securities overpriced and likely to earn low returns.[5]

The work of Nassim Taleb seems to suggest that investors *underweight* rather than *overweight*

Figure 1. The Value Function and the Probability-Weighting Function of Prospect Theory



Note: The kinks in the lines in Panel A represent loss aversion.

low-probability events. Implicit in his work on black swan events, this idea is explicit in his 2004 paper "Bleed or Blowup? Why Do We Prefer Asymmetric Payoffs?" in which he controversially argued that a preference for negative skewness is prevalent, maintaining that a combination of regular small gains and occasional large losses is psychologically more comfortable for most investors than the opposite pattern. His argument is more plausible for *delegated asset managers* who prefer frequent small gains and rare blowup losses. The alternative—to consistently underperform peers in normal times, even by small amounts—is simply bad for the business of gathering or retaining assets under management. One supporting idea is that overconfidence can lead to *disaster myopia*—the tendency to underestimate the probability of rare disasters—and the underpricing of black swan–like risks.

Finally, prospect theory postulates the reflecting ("leaning S") shape of the value function in Panel A of Figure 1. Recent novel research has proposed an alternative foundation that is based on salience and uses standard risk preferences.

According to Bordalo, Gennaioli, and Shleifer (2010), decision makers overweight the likelihood of salient states (in which lotteries have extreme, contrasting payoffs), which can explain both the reflecting shape of the value function and the overweighting of low-probability events. Bordalo et al. stressed that investors overweight only salient low-probability events—a point that may resolve the tension between the views of Kahneman and Tversky (1979) and Taleb (2004). In looser terms, events that investors think are well nigh impossible (or do not think about at all) can be underweighted unless the events are salient (i.e., vivid, attention grabbing, easily available, recent).

Other explanations simply assume some variant of skewness preferences.

- *Lottery preferences.* Mitton and Vorkink (2007) developed a model in which a subset of investors, whom they called "lotto investors," displays a direct preference for positive skewness. These lotto investors are willing to sacrifice Sharpe ratio (mean–variance efficiency) to gain higher skewness. They are deliberately underdiversified because diversification reduces the skewness they seek. Mitton and Vorkink documented that a meaningful segment of retail investors is underdiversified and exhibits "lotto" preferences (favoring highly skewed stocks and holding portfolios with positive skewness). Although such investors do not tend to outperform, they are more likely to be jackpot winners.
- *Portfolio insurance preferences.* These preferences are characterized by a need to guarantee a certain "floor" wealth. Theoretical models link the floor to a subsistence level (which can vary over time in habit formation models). In the real world, the floor may be linked to bankruptcy triggers, stop-loss limits, regulatory minimum funding ratios, and so on. Investors with such preferences may overpay for protective puts as compared with actuarially neutral prices. Perold and Sharpe (1988) discussed two mirroring investment approaches: convex and concave strategies. The former are positively skewed, achieved through dynamic trading (portfolio insurance buying[6] or directional trend following) or long option positions; the latter are negatively skewed, achieved through portfolio insurance selling, directional contrarian trading, or short option positions. Perold and Sharpe argued that the excess popularity of either approach eventually results in excess gains for the other side, a process that should balance the relative sizes of the two groups over time. This idea points to a *time-varying* reward for skewness, but the net demand for positive skewness can be so strong that insurance sellers earn a positive reward in the long run.

Let the Data Speak: Empirical Evidence on Skewness Pricing

The preceding review of pertinent theories has provided ambiguous predictions of the sign of the skewness premium. Thus, the question posed in the title of this article and the tension between the views of Kahneman and Tversky (1979) and Taleb (2004)—whether markets overweight or underweight low-probability events—can be addressed only empirically. As I will demonstrate, the evidence sides against Taleb: The selling of volatility, of lottery tickets, and of insurance all seem to enhance long-term returns.

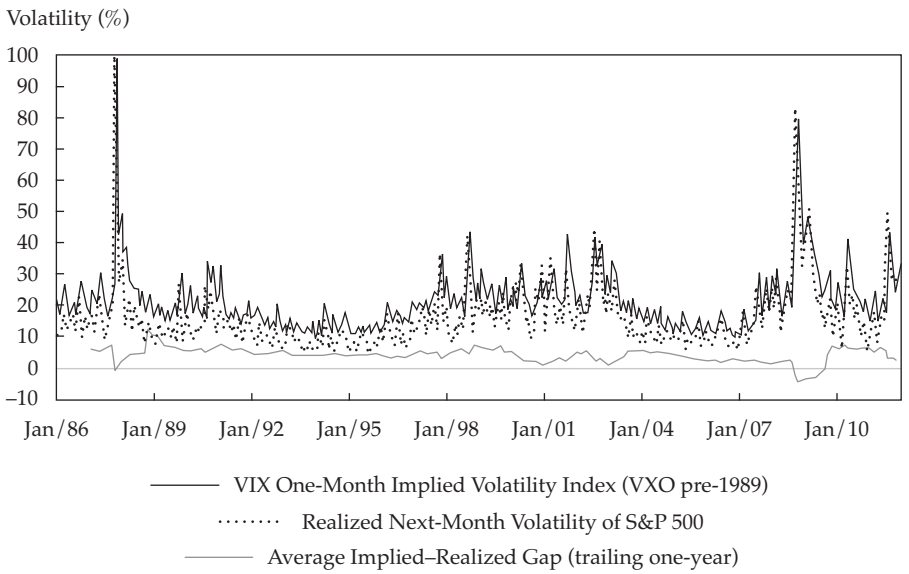
I found it convenient to structure the empirical discussion in two parts: the evidence with respect to the left tail (insurance) and the evidence with respect to the right tail (lottery tickets). For each tail, the evidence is consistent with skewness seeking. However, the underlying drivers are very different for each tail. On the left tail, insurance seeking focuses on portfolio-level downside protection and is guided by covariance with market and other systematic factors. On the right tail, lottery seeking is best served by more idiosyncratic investments and is guided by asset characteristics.[7]

Left Tail: Insurance. Equity index volatility selling and equity index put selling are archetypal strategies for selling financial catastrophe insurance. Although these strategies can consistently accrue small gains over many years, rare large losses disproportionately occur in very bad times. This terrible timing of losses is a more important reason to warrant large required risk premiums than the asymmetry of returns per se.[8]

The *ex ante* premium between implied volatility and realized volatility has almost always been positive for S&P 500 Index options, giving an edge to option sellers (**Figure 2**). This positive gap has been more consistent for equity index options than for individual stocks or other asset classes, presumably because the risk in equity market downturns is such an important systematic risk (for the same sign but weaker results with other investments, see Buraschi, Trojani, and Vedolin 2009; Duarte, Longstaff, and Yu 2007; Trolle and Schwartz 2008).

A simple strategy of holding long positions in the two nearby CBOE Volatility Index (VIX) futures (Bloomberg code SPVXSTR:IND) would have suffered annualized losses of 28% over December 2005–August 2011, despite the fact that the VIX level tripled, from 11 to 33, during that period. An initial investment of $100 would have halved by early 2007, quadrupled to more than $200 in autumn 2008 as volatility spiked, gradually melted

Figure 2. Contrasting the Implied Volatility of S&P 500 Index Options with the Realized Volatility of the Index, 1986–2012



Source: Bloomberg.

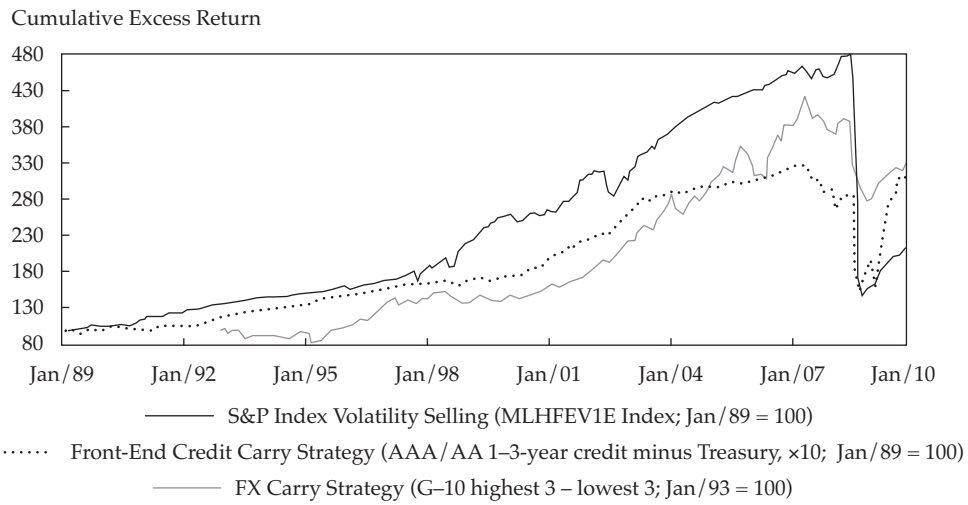
to below $10 by mid-2011, and recovered only to $15 by mid-August. The negative roll effect, which reflects the gap between realized and implied volatilities, would have more than offset the gains from rising volatility.

Moreover, skewness has been quite pronounced in index option pricing since the 1987 crash; out-of-the-money (protective) puts have especially high implied volatilities, consistent with "crash-o-phobia" and investor demand for market crash protection (rather than hedging against higher volatility per se). Such market pricing means that insurance through index option buying is expensive, albeit more reliable than insurance through dynamic strategies. In practice, tail risk insurance products look for diverse sources of insurance, and even option-based insurance tends to be based on cheaper variants than selling out-of-the-money puts, such as the collar structure whereby option-selling income partly offsets option-buying costs.

Other strategies that resemble selling financial catastrophe insurance—or selling lottery tickets that pay off in bad times—include foreign exchange (FX) carry trading, front-end credit carry trading, harvesting liquidity premiums, and buying AAA tranches of collateralized debt obligations (see Cochrane 1999; Coval, Jurek, and Stafford 2009; Burnside, Eichenbaum, Kleshchelski, and Rebelo 2010; Ilmanen 2011). All these strategies involve asymmetric payoffs in which the rare losses occur disproportionately during bad times. By design, many fixed-income investments, excluding government bonds, cap the upside and share the "up the elevator, down the escalator" payoff pattern. In the future, we should not exclude government bonds from this list, given the growing relevance of sovereign default risk. Finally, several hedge fund strategies—especially fixed-income arbitrage, convertible arbitrage, and merger arbitrage—share these characteristics, earning them the colorful description "picking up pennies in front of a steamroller." Jurek and Stafford (2011) showed that the return history of a broad hedge fund index can be closely matched by a simple strategy of out-of-the-money put writing.

Consistent with the idea that investors should require a high risk premium for such strategies, the long-run returns on these strategies have been positive—and the long-run Sharpe ratios have often been higher than that of the equity premium. **Figure 3** shows three examples of such strategies. Admittedly, these data histories are short, and with asymmetric strategies, results can be especially sample specific. All the strategies had deceivingly good Sharpe ratios before the tail event materialized in 2008. Large inflows into these strategies likely reflected more an extrapolation of past performance than a perception that such risky strategies warrant high premiums. The 2008 experience revealed to investors that the riskiness of these strategies is both insidious (hidden even over long periods) and toxic (occurring in only truly bad "states of the world").

Figure 3. Selling Financial Catastrophe Insurance Three Ways: Asymmetric Payoffs with Rare Large Losses in Bad Times, 1989–2010



Sources: Bank of America Merrill Lynch; Barclays Capital; Bloomberg; Ilmanen (2011).

The relative performance of stocks with different characteristics appears to be consistent with investors' requiring compensation for holding stocks that expose them to downside tail risks.[9] Harvey and Siddique (2000) showed that low-coskewness portfolios (which tend to perform poorly in periods of high market volatility) outperformed high-coskewness portfolios by 3.6 percentage points (pps) annually. Ang, Chen, and Xing (2006) documented that stocks with high downside betas (which covary with the market in periods of negative market returns) outperformed stocks with low downside betas by 6 pps annually. More recent studies have found even stronger results by using sharper measures of tail risk, including extreme lower tail dependence (Ruenzi and Weigert 2011) and stock-specific tail covariance (Whitelaw, Bali, and Cakici 2011). All these studies used data on a broad cross section of U.S. stocks, starting in 1963 and ending between 1993 and 2009. Using semivariance betas over 1931–2010, Post, van Vliet, and Lansdorp (2012) reported premiums of 4–6 pps.

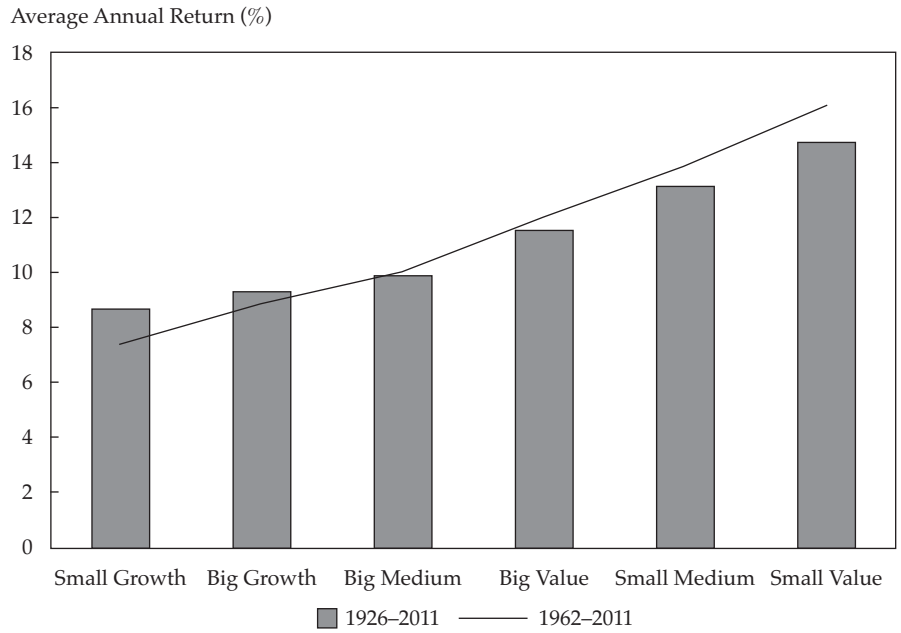
Overall, selling insurance (downside protection) against such systematic risks as financial catastrophes warrants—and apparently earns—a long-run reward. Conversely, buying such insurance (e.g., tail risk hedging, index volatility buying, portfolio insurance) is likely to be costly in the long run, resulting in lower long-run returns. Here, too, empirical evidence appears to be inconsistent with the premise of Taleb (2004). If investors really prefer bleeding to blowups, we should see volatility-selling and carry-seeking strategies earning *low* long-term returns. But that has not been the case.

I do not discuss here more traditional insurance except to note that selling insurance seems to be a reasonably profitable business—witness Warren Buffett's lifelong interest in the industry—and that it involves less systematic exposure than the selling of insurance in financial markets.

Financial investors primarily seek insurance against systematic risks. Thus, long-run risk premiums should be highest for strategies that effectively sell financial catastrophe insurance (so long as investors recognize this feature). In contrast, single-stock losses and natural catastrophes are less systematic and more diversifiable and thus warrant lower risk premiums.

Right Tail: Lotteries. The most volatile assets within each asset class offer lower rewards per risk than do low-volatility assets and, in many cases, simply lower long-run returns. Many observers thus conclude that risk is not being rewarded. The same is true for assets with high expected skewness or high betas, as documented by numerous studies (see Falkenstein 2009; Frazzini and Pedersen 2010; Ilmanen 2011). Lottery-like assets appear to be consistently overpriced and deliver poor long-run returns. Small-growth stocks and IPOs are often seen as the proverbial lottery tickets that fit the bill. **Figure 4** shows that small-growth stocks have underperformed all other subsets in the Fama–French value/size classifications.[10] Green and Hwang (2012) showed that IPOs with high expected skewness have lower long-term returns (and higher first-day returns).

Figure 4. Compound Annual Returns of Six Fama–French-Style Portfolios, 1926–2011 and 1962–2011



Source: Kenneth French's website (http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html).

Other studies have drilled into the average returns across U.S. equities sorted into portfolios on the basis of various lottery-related characteristics. **Figure 5** shows that the quintiles of stocks with the highest recent volatility, the highest upside volatility, the largest recent daily move, and the highest expected skewness have much lower average returns over the next month than do other stocks (see Ang, Hodrick, Xing, and Zhang 2006; Frieder and Jiang 2008; Bali, Cakici, and Whitelaw 2011; Boyer, Mitton, and Vorkink 2010). Lottery preferences seem to be the best common explanation for the asymmetric patterns in Figure 5. These research findings have attracted investor attention, although practical applications tend to use longer windows for volatility estimation than the past month, which is used in the academic studies. Looking at beta rather than volatility, empirical research has shown for decades that the security market line (the linear relation between average return and beta) is too flat compared with the CAPM predictions.

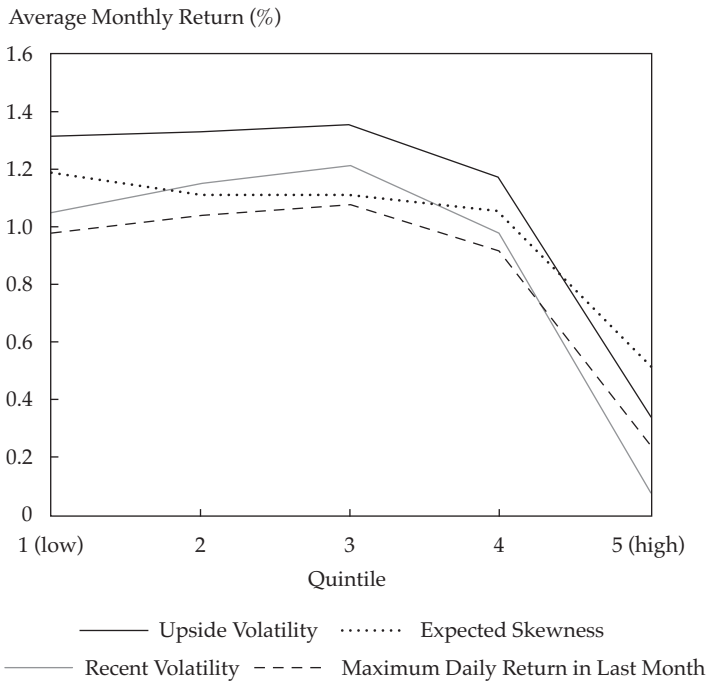
Across stock options, high expected skewness predicts very large negative returns (see Boyer and Vorkink 2011; Ni 2006). **Figure 6** shows that this pattern is strongest for options with very short maturities. With respect to midmarket prices, the expected returns on buying the most skewed options are –30% to –60% a week! Although high trading costs and market frictions prevent arbitrageurs from exploiting the richness of these single-stock options and data availability constrains the sample periods to be short, option markets are arguably a natural laboratory for identifying investor preferences regarding asymmetric return distributions.[11]

In bond markets, Sharpe ratios are much higher at short maturities and for credit risk–bearing trades than at long maturities, whereas the most speculative ratings (CCC) offer outright poor long-run returns (see Ilmanen 2011). Here, too, accepting small risks has been well rewarded but taking further large risks has been poorly rewarded.

In the more esoteric corners of financial markets, the same pattern is repeated: The more speculative the strategy variant, the worse the reward for risk. For example, this pattern has been empirically documented for subordinated distressed bonds in bankruptcy compared with senior bonds, for low-moneyness convertible bonds compared with high-moneyness bonds, for cash-financed merger arbitrage deals compared with stock-financed deals, and for the peak-peril reinsurance deals compared with a less volatile risk-balanced portfolio of deals.

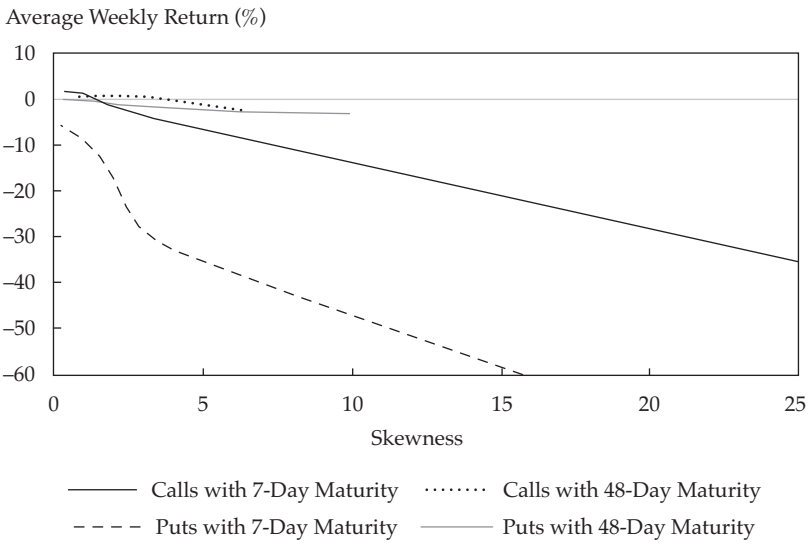
Outside financial markets, the well-known long-shot bias in racetrack betting has been widely studied (see, e.g., Golec and Tamarkin 1998;

Figure 5. Hockey-Stick Diagrams: Poor Average Returns for High-Volatility and High-Skewness Stocks



Sources: Ang, Hodrick, Xing, and Zhang (2006); Frieder and Jiang (2008); Bali, Cakici, and Whitelaw (2011); Boyer, Mitton, and Vorkink (2010)—all using different sample periods between 1962 and 2005.

Figure 6. Average Weekly Returns of Quintile Single-Stock-Option Portfolios Constructed on the Basis of Expected Skewness, 1996–2009



Source: Boyer and Vorkink (2011).

Snowberg and Wolfers 2010), as has the popularity of casino gambling (see, e.g., Barberis 2012). Both involve positively skewed payoffs and a high probability of long-term losses. And actual lottery tickets themselves are, of course, even more overpriced than their financial market brethren and offer a deeply negative expected return. Moreover, the demand for and overpricing of lottery

tickets are exacerbated when the jackpots grow exceptionally large and the payoffs become even more positively skewed (see, e.g., Garrett and Sobel 2004; Bhattacharya and Garrett 2008). Emphasizing the consistency of the evidence, Falkenstein (2009, 2010) covered many of the examples both within and outside the financial markets.

Lottery preferences are not the only explanation for the disappointing long-run performance of the most speculative investments. Several alternative—or complementary—causes have been proposed. Frazzini and Pedersen (2010) argued that leverage-constrained investors seeking high returns may substitute riskier assets for leverage, thus making these assets structurally overpriced. They showed empirically that selling high-beta assets against buying low-beta assets in a beta-neutral fashion has been a profitable strategy in a wide range of asset classes. Cowan and Wilderman (2011) noted that high-beta stocks can lose only 100% because of limited liability, whereas levered low-beta stocks can lose more. They combined the stories of asymmetric preferences and leverage, thereby linking high-beta asset richness with asymmetry on the left tail (downside protection) rather than on the right tail. Other researchers have claimed that relative status utilities (Falkenstein 2009) or active manager benchmarks (Baker, Bradley, and Wurgler 2011) make defensive deviations from the neutral stance appear as risky as aggressive deviations, thereby overturning standard risk–reward relations. Finally, Cornell (2009) proposed that overconfidence makes active investors prefer—and overpay for—volatile and positively skewed investments that offer high "bang for the buck" (apparently, the buck cannot be levered).

Conclusion

As I have shown, the pervasive empirical evidence documented in various contexts appears to be consistent with a general preference for positive skewness. Buying insurance (limiting the left tail) and buying lottery tickets (enhancing the right tail) are popular activities both outside and within financial markets and thus tend to be "overpriced" compared with actuarially neutral prices. Conversely, selling insurance and selling lottery tickets may boost long-term returns.

Among recently popular strategies, these findings suggest that tail risk insurance can be a long-run money loser unless managers can pick their battles (i.e., are skilled in active timing and selection). These strategies may still serve a purpose as portfolio diversifiers because their rare gains tend to occur when they are most valuable to investors. Litterman (2011) argued that long-horizon investors should consider selling such insurance rather than buying it.[12] Selling lottery tickets may be harder than selling insurance. Yet, various forms of low-volatility investing should benefit from underweighting or avoiding the most speculative, lottery-like investments within each asset class.

Finally, timing matters. Low-probability events are not always overweighted, and insurance is not always overpriced. New research suggests that only salient risks are overweighted. Recency is one simple indicator: Presumably, risks are more salient and risk premiums wider soon after adverse events, followed by a gradual fading of memories. After a long lull or when awash with liquidity, investors can be complacent with respect to particular risks and thus underweight them (either undervaluing them or underestimating their likelihood). Insurance-like strategies may be easier to time than strategies with symmetric distributions. Wide spreads and scarce capital after adverse events (e.g., recessions and financial crises) have often offered good entry levels for volatility-selling or carry-seeking strategies. Valuation-oriented timing/selection strategies are likely to serve investors better than, say, a blind "always sell insurance" approach.[13] If tail risk insurance is bought, well-diversified and opportunistic approaches can be much cheaper than an option-based approach. These ideas warrant further research.

I thank Andrew Ang, Nicholas Barberis, Eric Falkenstein, Knut Kjaer, and Lars Nielsen for helpful comments.

This article qualifies for 0.5 CE credit.

Notes

1. Skewness, the third moment of a (return) distribution—mean and variance are the first two—measures the asymmetry of the distribution.
2. A closely related finance question is whether buying or selling volatility boosts long-term returns. Another related question is whether investors tend to over- or underweight low-probability events. In this article, I address those questions as well, but not the more subtle question whether a given investor should buy or sell insurance and lottery tickets, the answer to which depends more on specific preferences than on market pricing.
3. Studies on multifactor asset pricing models have proposed premiums for volatility and skewness (as well as for jumps, kurtosis, correlations, etc.) and have empirically examined pricing across equities, options, and other asset classes. The results are mixed. Strictly speaking, *covariances* of asset returns with systematic risk factors should determine assets' required risk premiums. In practice, however, asset *characteristics* may be more important than such covariances. For example, stocks with low valuation ratios, low liquidity, low volatility, and low skewness tend to outperform their peers with corresponding high characteristics. These characteristics often have a better empirical ability to explain long-run average return differentials across stocks than the corresponding risk factor covariances.
4. However, Kahneman and Tversky (1979) found that *extremely* low probabilities are sometimes treated as impossibilities (rare events deemed as having essentially a zero chance). The weighting function is ambiguous near the zero probability if overweighting gives way to underweighting below some threshold.
5. In this article, overweighting refers to overvaluing unlikely events rather than misunderstanding their true probabilities. The brain's overweighting tendency may reflect both misestimating the probability of an event and overvaluing it (the possibility effect). Neuroscience research confirms that large payoff size attracts more attention than the (naturally) low probability of the payoff. The "reflexive brain" is much more responsive to the amount of reward at stake than to the likelihood of receiving the reward.
6. Portfolio insurance—selling stocks as they fall and buying them as they rise to achieve a call option–like payoff pattern without actually buying options—was a popular strategy in the mid-1980s. The strategy met its demise in the 1987 crash, when insurer selling exacerbated market moves and the strategy failed to protect a floor when the market discontinuously gapped lower on 19 October.
7. The extant literature—and evidence presented in this article—focuses on systematic negative skewness and idiosyncratic positive skewness. For completeness, I hope that future theoretical and empirical research will explore the pricing of systematic positive skewness and negative idiosyncratic skewness.
8. Note that the reward for a volatility-selling strategy is inherently skewed, which makes it hard to disentangle volatility and skewness premiums. Other distinctions that are hard to make include that between volatility and correlation premiums and that between rewards for vega and gamma exposures within volatility premiums.
9. This statement might seem trite were it not for other evidence against simple risk–reward relations. Since the 1960s, normal market beta risk has not been positively rewarded across stocks. And lottery studies have even suggested a negative relation between some volatility measures and subsequent average returns.
10. For details, see http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data_Library/six_portfolios.html.
11. Using *ex ante* measures of skewness is also elegant because recent realized skewness can be a misleading proxy for prospective skewness owing to the rarity of asymmetric large moves (see Zhang 2005; Boyer, Mitton, and Vorkink 2010). However, there is some evidence that even recent realized skewness is negatively related to future returns.
12. Even long-horizon investors may rationally want to buy tail risk insurance against the one tail risk scenario that could push the institution over the brink. For many institutions, the relevant scenario is a persistent deflationary recession. In such a "Japan" scenario, many pension funds would face a triple whammy of ills: falling asset values, rising liability values, and a troubled sponsor (needing cash). The fact that long-dated government bonds remain the most reliable safe-haven asset in this scenario goes a long way in explaining the persistent richness of U.S. Treasuries in recent years. Institutional investors believe they must hold Treasuries in this crucial tail scenario even if they expect poor performance in most other scenarios. This pattern also ties in with the theme of expensive insurance.
13. *Any* volatility-selling strategy implies governance challenges that require good communication and understanding ("buy-in" at the top level) *before* major risks materialize. Otherwise, a well-intentioned strategy of always selling volatility that is initiated in good times (when markets are complacent and the reward is low) may not be sustainable through a crisis (after which the reward is high).

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