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Globally Optimal Hierarchical Reinforcement Learning for Linearly-Solvable Markov Decision Processes

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Overview

Introduction

Introduction

- Hierachical Reinforcement Learning aims to make learning more efficient by decomposing large problems (Wen et al., 2020).
- Novel approach to HRL using Linearly-solvable Markov Decision Processes (LMDPs).
- LMDPs are computationally efficient (Todorov, 2006).
- Globally optimal value function (vs. *hierarchically optimal* or *recursively optimal*) (Dietterich, 2000).

Background

Background - LMDPs (i)

An LMDP (Kappen et al., 2012; Todorov, 2006) is as a tuple $\mathcal{L} = \langle \mathcal{S}, \mathcal{T}, \mathcal{P}, \mathcal{R}, \mathcal{J} \rangle$:

- We define $\mathcal{S}^+ = \mathcal{S} \cup \mathcal{T}$ to denote the full set of states.
- $\mathcal{P} : \mathcal{S} \to \Delta(\mathcal{S}^+)$.
- $\mathcal{R} : \mathcal{S} \to \mathbb{R}$ and $\mathcal{J} : \mathcal{T} \to \mathbb{R}$.
- Learning agent follows a policy $\pi : \mathcal{S} \to \Delta(\mathcal{S}^+)$.

Background - LMDPs (ii)

- At time-step t, the agent observes s_t and receives a reward

$$\mathcal{R}(s_t, \pi) = \mathcal{R}(s_t) - \lambda \cdot \mathrm{KL}(\pi(\cdot|s_t) \| \mathcal{P}(\cdot|s_t)).$$

- The aim of the agent is to maximize

$$v^\pi(s) = \mathbb{E}\left[\sum_{t=1}^{T-1} \mathcal{R}(S_t, \pi) + \mathcal{J}(S_T) \,\middle|\, S_1 = s\right].$$

- We obtain the following Bellman optimality equation

$$\frac{1}{\lambda} v(s) = \frac{1}{\lambda}\mathcal{R}(s) + \max_\pi \mathbb{E}_{s' \sim \pi(\cdot|s)}\left[\frac{1}{\lambda} v(s') - \log \frac{\pi(s'|s)}{\mathcal{P}(s'|s)}\right] \quad (\forall s).$$

- Taking $z(s) = e^{v(s)/\lambda}$ for each $s \in \mathcal{S}^+$ leads to

$$z(s) = e^{\mathcal{R}(s)/\lambda} \sum_{s'} \mathcal{P}(s'|s) z(s').$$

- If $\mathcal{P}$ and $\mathcal{R}$ are known, then

$$\mathbf{z} = RP\mathbf{z}^+ \text{ where } R = \mathrm{diag}(e^{\mathcal{R}(\cdot)/\lambda}).$$

Background - LMDPs (iii) - Online setting

- Alternatively, there is an online (corrected) update rule

$$\hat{z}(s_t) \leftarrow (\mathbf{1} - \boldsymbol{\alpha_t})\hat{z}(s_t) + \boldsymbol{\alpha_t} e^{r_t/\lambda}\hat{z}(s_{t+1})\frac{\mathcal{P}(s_{t+1}|s_t)}{\hat{\pi}(s_{t+1}|s_t)},$$

  samples are in the form of (s_t, r_t, s_{t+1}).

- Given a z, policies are derived using

$$\pi(s'|s) = \frac{\mathcal{P}(s'|s)z(s')}{\sum_{s''}\mathcal{P}(s''|s)z(s'')}.$$

Background - LMDPs (iv) - Compositionality

- Let $\{\mathcal{L}_1, \ldots, \mathcal{L}_n\}$ be a collection of LMDPs.
- Each LMDP $\mathcal{L}_i$ only differs in $\mathcal{J}_i(\mathsf{t})$.
- For a new LMDP $\mathcal{L}$ for which the next holds (Todorov, 2009)

$$e^{\mathcal{J}(\mathsf{t})/\lambda} = z(\mathsf{t}) = \sum_{k=1}^{n} w_k e^{\mathcal{J}_k(\mathsf{t})/\lambda} \text{ for } \mathsf{t} \in \mathcal{T}_i.$$

- Due to linearity, the following is also satisfied

$$z(s) = \sum_{k=1}^{n} w_k z_k(s) \ \forall s \in \mathcal{S}.$$

Hierarchical LMDPs

Hierarchical LMDPs (i)

- Inspired by the work of Wen et al. (Wen et al., 2020).
- For an LMDP $\mathcal{L}$, its $\mathcal{S}$ is partitioned into L subsets $\{\mathcal{S}_i\}_{i=1}^{L}$.
- Each such subset $\mathcal{S}_i$ induces a subtask, represented by an LMDP $\mathcal{L}_i = \langle \mathcal{S}_i, \mathcal{T}_i, \mathcal{P}_i, \mathcal{R}_i, \mathcal{J}_i \rangle$:
  - $\mathcal{T}_i$ includes states in $\mathcal{S}^+ \setminus \mathcal{S}_i$ that are one step away from any $s \in \mathcal{S}_i$.
  - For $\tau \in \mathcal{T}_i$,

$$\mathcal{J}_i(\tau) = \begin{cases} \mathcal{J}(\tau) \text{ if } \tau \in \mathcal{T}, \\ \hat{v}(\tau) \text{ otherwise.} \end{cases}$$

Hierarchical LMDPs (ii)

Definition

Subtask equivalence (for some $\mathcal{L}_i$ and $\mathcal{L}_j$) imples a bijective relationship $f : \mathcal{S}_i \rightarrow \mathcal{S}_j$.

- Set of *exit states* $\mathcal{E} = \cup_{i=1}^{L} \mathcal{T}_i$
- $\mathcal{E}_i = \mathcal{E} \cap \mathcal{S}_i$ denotes the set of *exit states inside* subtask $\mathcal{L}_i$.
- Set of equivalence classes $\mathcal{C} = \{\mathcal{C}_1, \ldots, \mathcal{C}_C\}$, $C \leq L$.
- A single subtask $\mathcal{L}_j = \langle \mathcal{S}_j, \mathcal{T}_j, \mathcal{P}_j, \mathcal{R}_j, \mathcal{J}_j \rangle$ per equivalence class $\mathcal{C}_j \in \mathcal{C}$.

Hierarchical LMDPs (iii) - Illustration

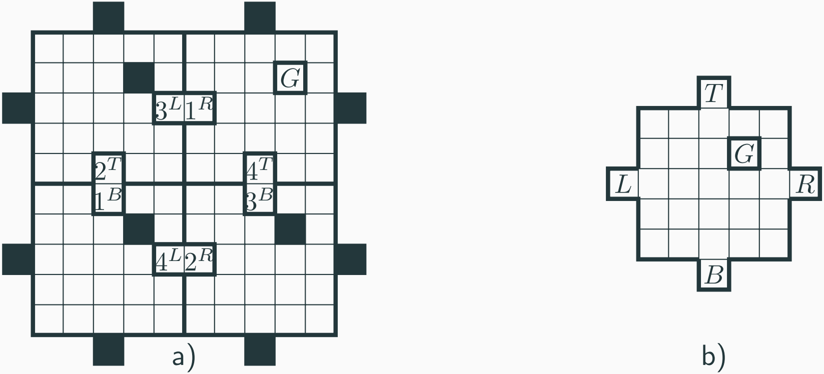


Figure 1: a) A 4-room LMDP, with all exit states highlighted; b) a single subtask with 5 terminal states G, L, R, T, B that is equivalent to all 4 room subtasks.

Hierarchical LMDPs (iv) - Subtask compositionality (i)

- Consider subtask $\mathcal{L}_j$ and its terminal set $\mathcal{T}_j = \{\tau_1, \ldots, \tau_n\}$.
- We define n base LMDPs $\mathcal{L}_j^1, \ldots, \mathcal{L}_j^n$, which only differ in $\mathcal{J}_j^k$.
- Concretely,

$$z_j^k(\tau) = \begin{cases} 1 \text{ if } \tau = \tau_k \rightarrow \mathcal{J}_j^k(\tau) = 0, \\ 0 \text{ otherwise } \rightarrow \mathcal{J}_j^k(\tau) = -\infty \end{cases}$$

- Thus, we can solve these base LMDPs to obtain $z_j^1, \ldots, z_j^n$.
- Having $\hat{z}(s)$ for each $t \in \mathcal{T}_j$, then thanks to compositionality,

$$\hat{z}(s) = \hat{z}(\tau_1) z_j^1(s) + \cdots + \hat{z}(\tau_n) z_j^n(s) \;\; \forall s \in \mathcal{S}_i, \forall \mathcal{L}_i \in \mathcal{C}_j.$$

Hierarchical LMDPs (v) - Subtask compositionality (ii)

- For all subtaks, terminal states $\tau_1 \ldots \tau_n$ are by definition in $\mathcal{E}$.
- Having access $\hat{z}_{\mathcal{E}} : \mathcal{E} \rightarrow \mathbb{R}$ and $z_j^1, \ldots, z_j^n$ for the base LMDPs is enough to represent $\hat{z}(s)\ \forall s \in \mathcal{S}$.
- Solutions for base LMDPs can be reused.

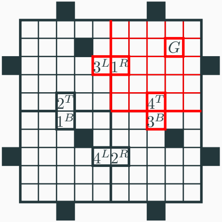
Hierarchical LMDPs (vi) - Subtask compositionality (iii)

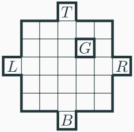
Remark

If $\hat{z}(s)$ is optimal for $s \in \mathcal{E}$, then $\hat{z}(s)$ for $s \in \mathcal{S}_i$ will also be optimal. With compositionality,

$$\hat{z}(s) = \hat{z}(3^L) * z_L(s) + \hat{z}(3^B) * z_B(s) + \hat{z}(G) * z_G(s)$$

For any s in $\mathcal{S}_i$. Thus, if $\hat{z} = z^*$ for $s \in \mathcal{E}$, then it will be optimal for the interior states as well.





Algorithms

Eigenvector algorithm (i)

- We can restrict

$$\hat{z}(s) = \hat{z}(\tau_1) z_j^1(s) + \cdots + \hat{z}(\tau_n) z_j^n(s) \;\; \forall s \in \mathcal{S}_i, \forall \mathcal{L}_i \in \mathcal{C}_j.$$

  to states $s \in \mathcal{E}$.
- Thus, we define

$$\mathbf{z}_{\mathcal{E}} = G \mathbf{z}_{\mathcal{E}}.$$

- This corresponds to an eigenvector problem.
- Value estimates for $s \in \mathcal{S} \setminus \mathcal{E}$ are obtained afterwards.

Eigenvector algorithm (ii) - Convergence proof

Lemma (1)
If the reward of each terminal state $t \in \mathcal{T}_i$ equals its optimal value in $\mathcal{L}$, i.e. $z_i(t) = z(t)$, the optimal value of each non-terminal state $s \in \mathcal{S}_i$ equals its optimal value in $\mathcal{L}$, i.e. $z_i(s) = z(s)$.

Lemma (2)
The solution to $\mathbf{z}_{\mathcal{E}} = G\mathbf{z}_{\mathcal{E}}$ is unique.

Lemma (3)
For each subtask $\mathcal{L}_i$ and state $s \in \mathcal{S}_i^+$, it holds that

$$z_i^1(s) + \cdots + z_i^n(s) \leq 1.$$

Online algorithm (i)

- We keep $\hat{z}_j^1, \ldots, \hat{z}_j^n$ for each $\mathcal{L}_j$.
- We can update all base LMDPs within a subtask $\mathcal{L}_j$ using intra-task learning (Kaelbling, 1993; Jonsson and Gómez, 2016).
- The online update rule (again, restricted to $s \in \mathcal{E}$)

$$\hat{z}_{\mathcal{E}}(s) \leftarrow (1-\alpha_\ell)\hat{z}_{\mathcal{E}}(s) + \alpha_\ell[\hat{z}_{\mathcal{E}}(t_1)\hat{z}_j^1(s) + \cdots + \hat{z}_{\mathcal{E}}(t_n)\hat{z}_j^n(s)].$$

- Estimates at any level are learned in an episodic manner.
- When shall we update states in $\mathcal{E}$?

Online algorithm (ii)

We propose the following alternatives:

V_1: Update $s \in \mathcal{E}_i$ each time the agent transitions from s.

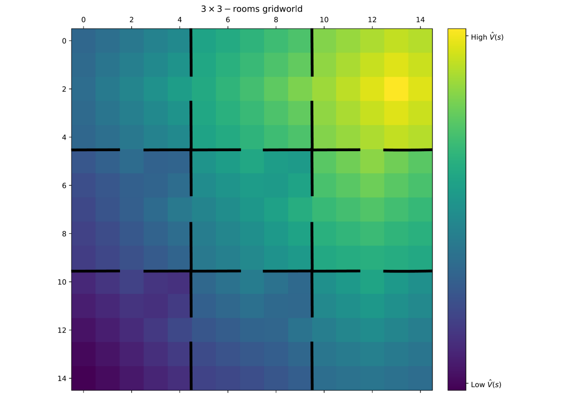
V_2: When the agent reaches $\tau \in \mathcal{T}_i$ of the subtask $\mathcal{L}_i$, update the values of every $s \in \mathcal{E}_i$.

V_3: When the agent reaches $\tau \in \mathcal{T}_i$ of the subtask $\mathcal{L}_i$, update the values of every $s \in \mathcal{E}_i$ and all exit states of subtasks in the equivalence class $\mathcal{C}_j$ of $\mathcal{L}_i$.

Experiments and results

Experiments - Rooms domain

- We varied the size of the rooms as well as the number of rooms.



Results - Rooms domain

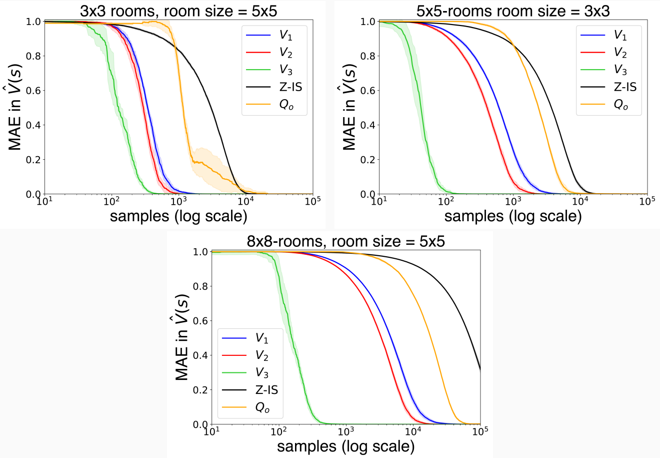
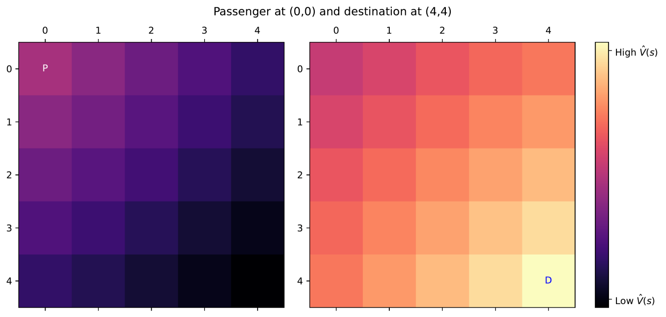


Figure 2: MAE over time for 3×3 (top-left), 5×5 (top-right) and 8×8 (bottom) room instances.

Experiments - Taxi domain

- A passenger is located at one of the four corners and he must be carried to a certain corner (excluding the pickup location).
- Base LMDPs here are going to each of the corners.



Results - Taxi domain

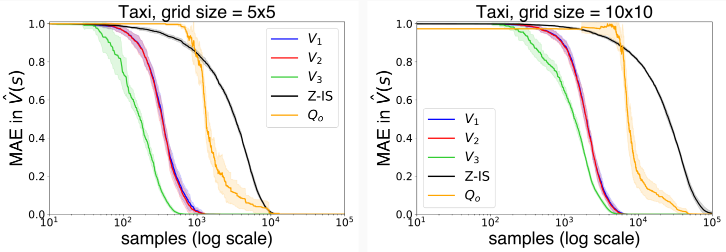


Figure 3: MAE over time for 5×5 (left) and 10×10 (right) grids of Taxi domain.

Contributions and Conclusion

Contributions

- Novel scheme based on subtask compositionality.
- The subtasks decomposition is at the level of the value function.
- Our method converges to the optimal value function.

Conclusion

- We are able to retrieve the optimal value function.
- New form of zero-shot learning.

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