

Exchange Rate Dynamics

and the Central Bank's Balance Sheet*

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Abstract

In theory, nominal exchange rates are a function of the relative difference in the supply and demand of money. In practice, some central banks also issue debt. In this study, we ask: are nominal exchange rate variations linked to the central bank's balance sheet? We use two measures of implied exchange rates using central bank balance sheet data: one is a traditional measure that includes the monetary base, and the other adds remunerated liabilities. We contextualize these measures by performing an empirical analysis and providing a simple theoretical framework. The empirical portion of the paper estimates VAR models to investigate endogenous interactions between central bank balance sheet components. Then, we use threshold cointegration techniques to compare these two measures of implied exchange rate with the spot (observed) exchange rate for a set of seven Latin American countries for the 2004:01-2019:12 period. We find that the implied exchange rates and the spot exchange rate are cointegrated for most of the set of Latin American countries, and that the implied rate with stronger patterns of cointegration is the measure that includes remunerated liabilities. The simple theoretical framework provides a link between exchange rates and remunerated liabilities which, together with the threshold cointegration results, provides evidence that central banks cannot freely choose the amount of remunerated liabilities since doing so will affect exchange rates. If a central bank accumulates remunerated liabilities, a framework that does not include remunerated liabilities tends to ignore relevant sources of exchange rate variations.

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1 Introduction

An increase in the domestic money supply, other things equal, is usually associated with a currency depreciation. However, some central banks issue debt at the same time, namely, remunerated liabilities, that offset part of the increase of the monetary base. Using theory and data from seven Latin American countries we show that there is an interplay between remunerated liabilities and spot (observed) exchange rates.

To introduce the data, consider a simplified balance sheet for a central bank with zero net worth:

| Assets | Liabilities |
|---------------------------|-------------------------|
| Foreign Exchange Reserves | Monetary Base |
| Financial Claims | Remunerated Liabilities |
| Domestic Credit | |

Table (1) shows these components ranked by remunerated liabilities as a fraction of GDP for six South American countries in 2018.¹ The stock of remunerated liabilities as a fraction of GDP averaged 10% across these countries, with considerable heterogeneity: the stock of remunerated liabilities was around 25% of GDP for Uruguay and almost non-existent in Colombia. See Appendix (A) for a details on the construction of these variables.

Table 1: Balance Sheet Components as a Fraction of GDP, 2018

| Country | Reserves | Financial Claims | Domestic Credit | Monetary Base | Remunerated Liabilities |
|----------|----------|------------------|-----------------|---------------|-------------------------|
| Uruguay | 0.30 | 0.00 | 0.10 | 0.06 | 0.25 |
| Brazil | 0.22 | 0.01 | 0.26 | 0.10 | 0.17 |
| Chile | 0.14 | 0.00 | 0.00 | 0.06 | 0.10 |
| Paraguay | 0.21 | 0.01 | 0.00 | 0.12 | 0.06 |
| Mexico | 0.15 | 0.02 | 0.00 | 0.07 | 0.02 |
| Colombia | 0.16 | 0.01 | 0.01 | 0.10 | 0.00 |

Table (A6) provides a more global perspective by extending the data to 85 countries for the period 2010-2019. Notice that Uruguay, Brazil and Chile are in the top decile with respect to the stock of remunerated liabilities as a fraction of GDP. Of the 85 countries, Uruguay is second

¹We started with all South American countries plus Mexico. Peru is excluded from our set of countries because there is no IMF IFS data for these variables. Ecuador and Bolivia are excluded because they work, respectively, under a dollarization and fixed exchange rate regime. Venezuela is excluded because IMF IFS data is not available at a monthly frequency. IMF IFS does not include data on these variables for Argentina, but Argentina is later included in our analysis because we obtain data from the UCEMA - Centro de Economía Aplicada (UCEMA-CEA) and FRED.

only to Malaysia. Mexico and Colombia belong to the large group of countries with little to no remunerated liabilities. A quick scan of table (A6) reveals that the relevant heterogeneity in our sample of Latin American countries parallels the international extremes and the relative ranking of our set of Latin American countries in terms of their remunerated liabilities is similar.

Our research investigates the extent to which remunerated liabilities are a relevant fundamental for understanding exchange rate fluctuations. We conduct empirical exercises to compare the spot exchange rate with two implied exchange rate measures that incorporate central bank balance sheet data. Both implied rates are the ratio of domestic liabilities (measured in domestic currency) to foreign assets (measured in US dollars). The first implied rate includes the monetary base while the second adds remunerated liabilities. These implied rates are sometimes used in economics and by financial market analysts (as described in for instance [Ávila \(2018\)](#)), and they will depreciate (increase) when the liabilities increase and appreciate (decrease) when there is a higher accumulation of assets. In Section (4) we provide a simple theoretical framework to derive and further understand these two implied exchange rate measures.

In this paper we often abbreviate the implied rates as the “Conversion Exchange Rate” (CER henceforth)² and define the “CER Base” as the measure that incorporates only the monetary base in the numerator. “CER Full” adds remunerated liabilities to the numerator of “CER Base”. More formally, the measures are:

$$\text{CER Base} \equiv \frac{\text{Monetary Base}}{\text{International Reserves}}, \quad (1)$$

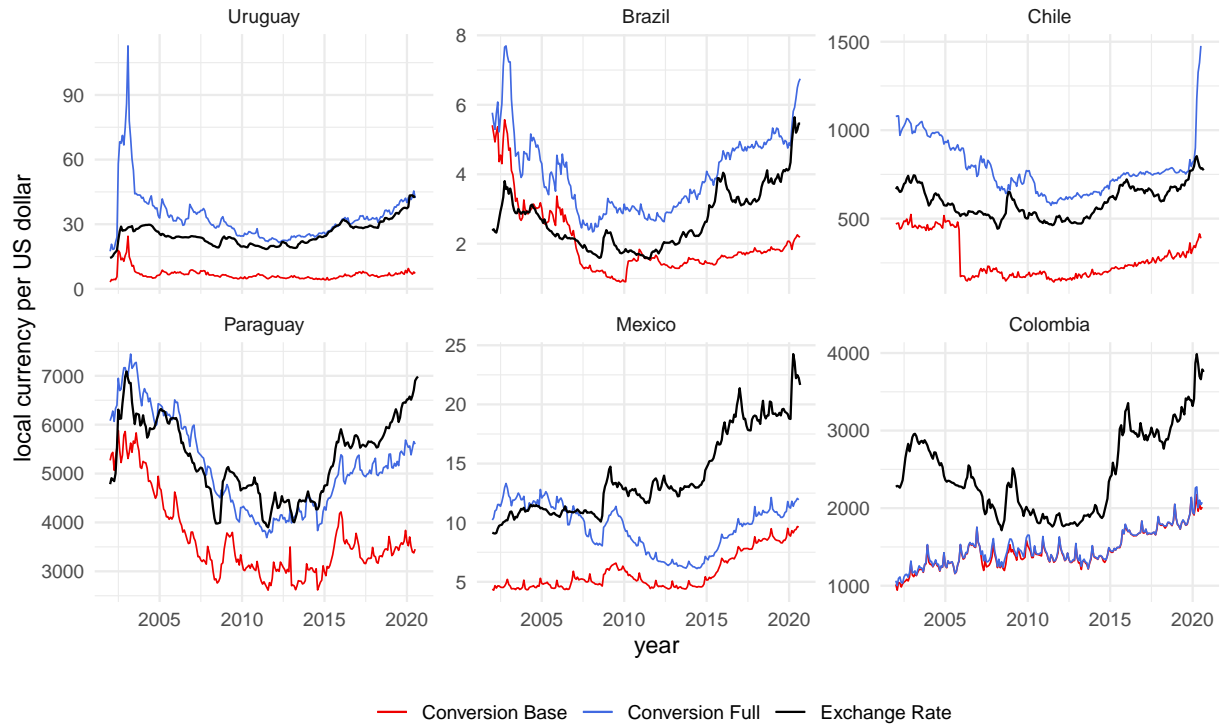
and

$$\text{CER Full} \equiv \frac{\text{Monetary Base} + \text{Remunerated Liabilities}}{\text{International Reserves}}. \quad (2)$$

Figure (1) and Figure (2) depict the CER for the seven Latin American countries. Recall, the difference between the CER Full and the CER Base is the addition of remunerated liabilities to the numerator. The larger the remunerated liabilities, the larger the difference between the two lines. The CER Full seems to track the spot exchange rate better than the CER Base, suggesting that remunerated liabilities matter. For Colombia and Mexico, however, the spot exchange rate and CER do not seem to be related. Recall from Table (1) that these two countries had little to

²In spanish this is known as “Tipo de Cambio de Conversión” ([Ávila \(2018\)](#)).

Figure 1: Monthly Spot Exchange Rate and CER Base, and CER Full, 2004-2019



Source: Author's calculations using IMF IFS data.

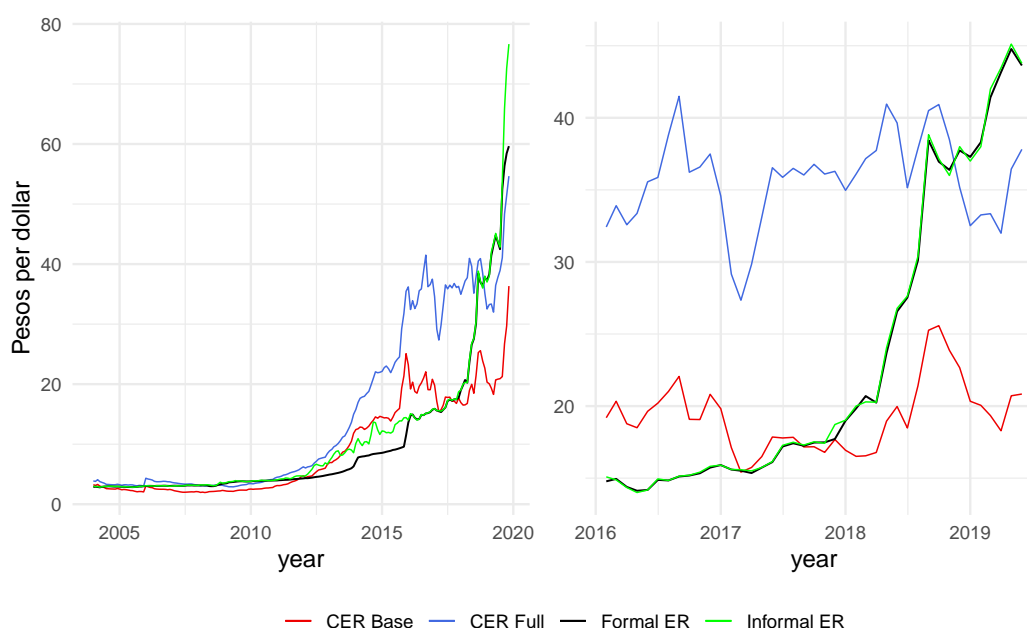
no remunerated liabilities. A pattern seems to emerge: countries with substantial remunerated liabilities (e.g., larger than 2% of GDP) have spot exchange rates that move closely with the balance sheet of the central bank, while countries without remunerated liabilities have spot exchange rates that are decoupled from the balance sheet of the central bank. We formally test the link between these implied and spot rates in a non-linear cointegration context.

Our empirical exercises also provide insights about large currency depreciations. Our case study is the 2018 currency crisis in Argentina. Between April and September 2018 the Argentine peso lost half its value, depreciating from around 20 to 40 pesos per US dollar. The central bank had accumulated around 8% of GDP in remunerated liabilities during the two years preceding the currency crisis. In the left panel of Figure (2) we observe the time series for the entire period. Notice that we have included the informal exchange rate, since in some sub-periods (2011-2015 and after mid-2019) there were currency controls that created a gap between the market exchange rate

("informal") and the official ("formal") one.³ As observed, the informal exchange rate fluctuates more in these periods of currency controls than the formal one. This would suggest a relatively tighter link between the informal exchange rate with the CER measures.

In the right panel, we focus in the January 2016 - July 2019, a period with no currency controls but with a large currency depreciation: as shown, the magnitude of this depreciation coincided with the difference in the CER Base and CER Full. During the crisis, the spot exchange rate jumped from the former to the later. This might suggest the fundamental exchange rate value is indeed given by balance sheet measures that include remunerated liabilities.⁴ Through this lens, before May 2018 the currency was overvalued, relative to the fundamentals given by the central bank's balance sheet.

Figure 2: Monthly spot Exchange Rates, CER Base, and CER Full for Argentina: full sample (left) and currency crisis episode (right).



Source: Author's calculations calculation based on [UCEMA-CEA](#).

With Figure (1) and Figure (2) in mind, we start by exploring potential endogenous relation-

³For the informal exchange rate, we use the 'Contado con Liquidación' exchange rate, a price implied from financial markets and available in the UCEMA-CEA database. For the formal exchange rate, we use data from FRED.

⁴For the period after the crisis (around May 2018), the CER Full is underestimated, since the IMF bailout transfers were temporarily recorded as international reserves by the Central Bank (thus the actual reserves were lower, which implies a higher CER Full). We thank Jorge Ávila for this insight.

ships between the different components of the balance sheet (spot exchange rate, foreign reserves, monetary base, and remunerated liabilities). We learn that these components are indeed interconnected but that their individual links to the spot exchange rate do not seem as strong as intuition suggests. To explore further, and to circumvent issues related to the disconnect of the spot exchange rate from its fundamentals, we shift our focus to comparing pairs of spot-implied exchange rates directly using non-linear cointegration to uncover potential long-run comovement between the spot exchange rate and the two balance sheet implied counterparts (CER Base and CER Full).

Results from the non-linear cointegration analysis suggests that the remunerated liabilities are a relevant determinant of the exchange rate in the long-run for most countries in our sample, and particularly for those with a longer tradition of foreign exchange interventions or that hold important amounts of remunerated liabilities. In most of these cases, the spot exchange rate co-moves more closely with the implied exchange rate measure that considers the remunerated liabilities suggesting that this type of policy instrument matters. Further, it suggests that in the long-run, the spot exchange rate is not only a measure of the relative amount of each currency (local vs. reserves) in the country, but also, the currency composition of the debt issued by the central bank can be an important determinant for its long run dynamics.

Related Literature. Monetary theory tells us that variation of the exchange rate is a function of the difference in supply and demand of money, relative to the foreign country, as exposed in for instance [Obstfeld and Rogoff \(1995\)](#). An increase in the money supply (or monetary liability), other things equal, leads to a currency depreciation. These theories, however, usually abstract from two features that are present in many emerging markets:

1. Many central banks follow intermediate exchange rate regimes (neither fixed or flexible exchange rate regimes): [Frankel \(2019\)](#) proposes to define an intermediate regime, called “systematic managed floating”, as an arrangement where the central bank regularly responds to changes in total exchange market pressure by allowing some fraction to be reflected as a change in the exchange rate and the remaining fraction to be absorbed as a change in foreign exchange reserves.

2. Many central banks issue remunerated liabilities, as reported before. Foreign exchange reserves are sometimes financed by issuing central bank debt. [Sosa-Padilla and Sturzenegger \(2023\)](#) study the link between remunerated liabilities and bond yields. We however focus on the link between remunerated liabilities and the spot exchange rate.

Our intention is not in predicting exchange rates or linking exchange rates to determinants in the short-run; a long-standing puzzle in international economics is the difficulty of tying floating exchange rates to macroeconomic fundamentals such as money supplies, outputs, and interest rates (see [Engel and West \(2005\)](#) and references therein). Instead, our contribution is an investigation into the existence of a long-run link between remunerated liabilities and the spot exchange rate using central bank balance sheet data, which provides evidence of a long-run link between remunerated liabilities and the spot exchange rate. This is particularly relevant for studies suggesting the existence of policy incentives for the accumulation of these types of debt that include the hedging of debt risks associated with strong fluctuations in the value of the domestic currency.

We support our empirical result with a simple theoretical framework that shows how we expect the issuance of remunerated liabilities and other components of central bank balance sheets to affect exchange rate dynamics. Our findings have empirical and theoretical implications for open economy macroeconomics, in particular, for the existence of the Mundell-Fleming trilemma (see [Mundell, 1963](#); [Fleming, 1962](#)), and additional trade-offs involved in the decision of a "de-facto" currency regime. Such choice, as we find, is not independent of the link between the exchange rate dynamics and other components of the central bank's balance sheet aside from the money supply.

This paper is also related to the literature on the currency composition of debt, both from a central government as well as the central bank's perspective. According to these studies, local currency debt provides a risk hedge based on the countercyclicalities of exchange rates (e.g., [Korinek, 2009](#); [Devereux and Wu, 2022](#)). Other authors explain that, although, this is the case, the capacity of indebtedness is limited by the presence of original sin (incapacity to get indebted in local currency) and potential country risk premia and policy credibility issues that imply a countervailing effect to the perceived incentives to hold larger shares of local currency debt (e.g., [Ottonello and Perez, 2019](#); [Engel and Park, 2022](#)).⁵

⁵An important aspect that we do not explore here is the role of different type of indexing arrangements for these debt instruments; when debt in local currency is indexed, the currency fluctuation effect is dampened in real terms.

The rest of the paper is organized as follows: in Section (2) we perform a vector autoregressive (VAR) analysis to investigate endogenous interactions between central bank balance sheet components, in Section (3) we move to the non-linear cointegration analysis, in Section (4) we provide a simple theory of the implied exchange rates that supports the empirical results obtained in the paper, and in Section (5) we conclude.

2 VAR Models for Balance Sheet Components

We estimate VAR models for each country with variables represented in (2) to investigate endogenous interactions between central bank balance sheet components involving the CER Full (and implicitly the CER Base). In each case our estimation equation is:

$$\begin{aligned} X_t &= \Phi X_{t-1} + u_t, \\ u_t &\sim N(0, \Sigma_u), \end{aligned} \tag{3}$$

where $X_t = [er \ h \ mb \ rl]'$, and er : exchange rate, h : foreign reserves, mb : monetary base, rl : remunerated liabilities. A separate VAR model is estimated for each of the seven countries with monthly data spanning 2004:01-2019:12. We transform the data by its logarithm and model it in levels or first differences depending on the presence of cointegration in each country, which is determined using the Johansen Multivariate cointegration by test [Johansen \(1991\)](#).

In addition, the model in (3) denotes the companion VAR(1) representation of each VAR(p) model, i.e., we allow the model in each case to have a higher lag order which is assigned according to the AIC criterion with a maximum lag length of 6, as shown in table (A1). Finally, in each country we verify the absence of autocorrelation in the reduced form errors.⁶

VAR Results. The importance of the remunerated liabilities based on the multivariate exercises that represent the structure of the balance sheet of the central banks is mixed. First, we test for multivariate cointegration, the results are shown in table (2). The results suggest that for five out of seven countries we should proceed with a VAR model in differences, i.e., abstract from

This can alter incentives to use exchange rate movements for 'hedging' purposes.

⁶Additional results related to VAR exercise are reported in Appendix B.

cointegration. We carry out the model listed in the last column of table (2).

Table 2: Johansen Cointegration Test Results, by Country

| Country | Number of Cointegration Relationships from Johansen Test | | | | | |
|----------------------|--|-------|-------|--------------------------|--------------|------------------------|
| | Deterministic Component | | | Decision by Type of Test | | |
| | None | Const | Trend | Trace | Max. Eigenv. | Final Model (selected) |
| Uruguay | 1 / 1 | 3 / 2 | 1 / 1 | 1 - None | 1 - None | VEC/VAR (Levels) |
| Brazil | 1 / 0 | 2 / 1 | 0 / 0 | 0 - Trend | 0 - None | VAR(Differences) |
| Chile | 1 / 1 | 1 / 1 | 1 / 1 | 1 - None | 1 - None | VEC/VAR (Levels) |
| Paraguay | 1 / 1 | 3 / 3 | 1 / 0 | 1 - None | 0 - Trend | VAR (Differences) |
| Mexico | 0 / 0 | 1 / 1 | 1 / 1 | 0 - None | 0 - None | VAR (Differences) |
| Colombia | 2 / 1 | 3 / 3 | 2 / 1 | 2 - None | 1 - None | VEC/VAR (Levels) |
| Argentina | 0 / 0 | 1 / 1 | 1 / 1 | 0 - None | 0 - None | VAR (Differences) |
| Argentina (Informal) | 0 / 0 | 1 / 1 | 1 / 0 | 0 - None | 0 - None | VAR (Differences) |

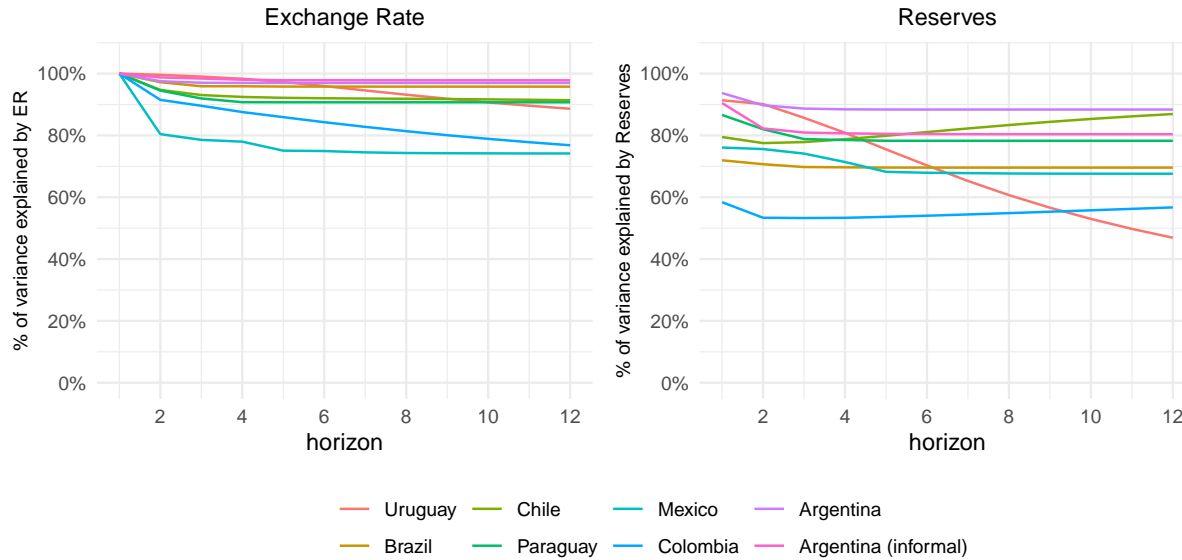
Notes: Each entry denotes the number of relations in the trace test / maximum eigenvalue test. The deterministic component refers to additional terms included in the cointegration relationship. The simplest model is selected (in number of relations and deterministic component(s)) for which the test is rejected.

Causality tests based on [Granger \(1969\)](#) and a modified Wald test to consider the contemporaneous relationships show some signs of the remunerated liabilities causing the rest of the balance sheet. Results from table (A3) find that remunerated liabilities Granger cause the other variables for Brazil and Paraguay and instantaneously cause other variables for all countries except Colombia. Furthermore, forecast variance error decomposition (FEVD) shows that for Brazil, Paraguay, Chile, and Uruguay the remunerated liabilities explain from about 2% to 10% of the spot exchange rate variance. These results provide evidence that the effect of remunerated liabilities on the dynamics of the currency is not necessarily trivial for most countries, Colombia being the exception.

To put the latter result in context, we next look at the forecast error variance decomposition to obtain percentage of the error variance of the spot exchange rate that is caused by itself. The left panel of figure (3) shows the spot exchange rate is particularly persistent and self-determined in a more marked way than the rest of the balance sheet which is shown in the right panel. On one hand, explaining 6% of the residual variance is still relevant, and on the other hand, the high degree of self-determination in this system that is displayed by the spot exchange rate poses a challenge for establishing a meaningful relationship with other fundamentals and can be traced to the exchange rate disconnect puzzle ([Obstfeld and Rogoff \(2001\)](#)). Thus, it seems exploring the

relevance of the remunerated liabilities from a multivariate balance sheet perspective is lacking. As a result, we investigate the relevance of the liabilities in spot rate dynamics by comparing pairs of spot against implied exchange rates for each country using a non-linear cointegration analysis. The analysis is used to determine the existence of long run co-movement between the spot rate and the implied exchange rate. If the CER Full is more closely related to the spot exchange rate than the CER Base, we can conclude that the way reserves accumulation are funded matters, and not only the overall level of reserves.

Figure 3: Percentage of variance of Spot Exchange Rate (left) and Reserves (right) explained by itself



3 Cointegration Analysis

We test for threshold cointegration between the 16 pairs of spot - implied exchanges rates.⁷ For each pair our econometric analysis consists of three steps. First, we transform data by its logarithm and investigate the order of integration of each series using the Augmented Dickey Fuller (ADF) unit root test (Dickey and Fuller (1979, 1981)). When the series are found to be integrated of the same order, we employ the threshold cointegration technique of Sephton and Mann (2013) which combines the threshold selection method of Gonzalo and Pitarakis (2002) with the F test developed by Seo (2008). The test for threshold cointegration examines the residuals from the cointegrating

⁷There are two pairs for each country except Argentina: spot - CER Base and spot - CER full. For Argentina there are four pairs: spot - CER Base, spot (informal) - CER Base, spot - CER Full, and spot (informal) - CER Full.

regression in equation (4), and conditional on selecting one or more thresholds (for expository purposes, we assume there are three in equation (5), denoted by τ_1 , τ_2 and τ_3) it examines the null hypothesis that all correction coefficients are zero ($\delta_1 = \delta_2 = \delta_3 = \delta_4 = 0$) against the alternative they are not jointly zero. The Heaviside indicator portion of equation (4) is the only component that changes when less than three thresholds are found. In the case of zero thresholds the equation collapses on the testing equations of the traditional cointegration test of [Engle and Granger \(1987\)](#). Finally, residual based block bootstrapping is used to draw inference when one, two, or three thresholds are found and critical values by [MacKinnon \(1996\)](#) are used to draw inference when zero thresholds are found.

$$Y_t = \beta_1 + \beta_2 Trend_t + \beta_3 X_t + \varepsilon_t, \quad (4)$$

$$\Delta \varepsilon_t = \delta_1 \varepsilon_{t-1} \mathbb{I}_{\varepsilon_{t-1} < \tau_1} + \delta_2 \varepsilon_{t-1} \mathbb{I}_{\tau_1 < \varepsilon_{t-1} < \tau_2} + \delta_3 \varepsilon_{t-1} \mathbb{I}_{\tau_2 < \varepsilon_{t-1} < \tau_3} + \delta_4 \varepsilon_{t-1} \mathbb{I}_{\tau_3 < \varepsilon_{t-1}} + \sum_{i=1}^p \alpha_i \Delta \varepsilon_{t-i} + v_t, \quad (5)$$

such that

- Y_t and X_t are the logged implied exchange rate (*Base* or *Full*) and the exchange rate (*er*).
- ε_t is the residual from the cointegrating regression from equation (4).
- $\tau_1 < \tau_2 < \tau_3$ are the thresholds that divide observations in each regime.
- $\mathbb{I}_{\text{condition}}$ is a Heaviside indicator function taking the value of 1 if the subscript condition holds, and Δ is the difference operator.

The advantage of [Sephton and Mann \(2013\)](#) over the traditional approach is that it endogenously determines both the number of thresholds and their location. Most cointegration techniques assume the absence of thresholds (e.g., Engle and Granger test); that a single threshold exists and is equal to zero (hence separating observations into two regimes with ε_{t-1} being positive in the first and negative in the second); or that two thresholds exist that are symmetric around zero. Here we allow for as many as three thresholds, and as few as zero. If the procedure chooses two thresholds, they may be symmetric about zero, but this decision is not determined a priori. For further details on the threshold cointegration technique, including size and power properties, see [Sephton and Mann \(2013\)](#).

When we reject the null hypothesis that all correction coefficients are zero ($\delta_1 = \delta_2 = \delta_3 = \delta_4 = 0$) against the alternative they are not jointly zero we estimate the error correction model (ECM) expressed in equation system (6-7). Again, the Heaviside indicator portion of the equation system is the only component that changes when less than three thresholds are found. The gamma terms from the ECM are used to determine which of the variables adjust to restore the system to its long-run relationship. If one of the series does not adjust, it is considered weakly exogenous and termed the dominant series.

$$\begin{aligned} \Delta Y_t = & \gamma_1 \varepsilon_{t-1} \mathbb{1}_{\varepsilon_{t-1} < \tau_1} + \gamma_2 \varepsilon_{t-1} \mathbb{1}_{\tau_1 < \varepsilon_{t-1} < \tau_2} + \gamma_3 \varepsilon_{t-1} \mathbb{1}_{\tau_2 < \varepsilon_{t-1} < \tau_3} + \gamma_4 \varepsilon_{t-1} \mathbb{1}_{\tau_3 < \varepsilon_{t-1}} \\ & + \sum_{i=1}^p (\alpha_i \Delta Y_{t-i} + \beta_i \Delta X_{t-i}) + u_t, \end{aligned} \quad (6)$$

$$\begin{aligned} \Delta X_t = & \gamma_5 \varepsilon_{t-1} \mathbb{1}_{\varepsilon_{t-1} < \tau_1} + \gamma_6 \varepsilon_{t-1} \mathbb{1}_{\tau_1 < \varepsilon_{t-1} < \tau_2} + \gamma_7 \varepsilon_{t-1} \mathbb{1}_{\tau_2 < \varepsilon_{t-1} < \tau_3} + \gamma_8 \varepsilon_{t-1} \mathbb{1}_{\tau_3 < \varepsilon_{t-1}} \\ & + \sum_{i=1}^p (\chi_i \Delta Y_{t-i} + \delta_i \Delta X_{t-i}) + v_t, \end{aligned} \quad (7)$$

where

- Y_t and X_t are the logged implied exchange rate (*Base* or *Full*) and the exchange rate (*er*).
- ε_t is the residual from the cointegrating regression from equation (4).
- $\tau_1 < \tau_2 < \tau_3$ are the thresholds that divide observations in each regime.
- γ_i are the regime specific speeds of adjustment to the long-run relationship.
- $\mathbb{1}_{\text{condition}}$ is a Heaviside indicator function and Δ is the difference operator.

Cointegration Results. Results from the unit root tests in table (A4) are consistent across all seven countries and provide evidence that each series is I(1). Results from the pairwise threshold cointegration tests and the coefficient estimates for the cointegrated pairs are shown in table (A5). We find cointegration between the spot and CER Base exchange rate in four out of seven countries (Chile, Paraguay, Mexico, and Colombia). The estimated parameters indicate that the implied exchange rate CER Base is doing the adjusting to the spot rate in the long-run and that it is rare for the spot rate to adjust to the CER Base. We find cointegration between the CER Full and the spot

exchange rate for five out of seven countries (Uruguay, Brazil, Chile, Paraguay, and Colombia). The estimated parameters again indicate the implied exchange rate CER Full adjusts to the spot rate in the long-run, but the spot rate also adjusts to the CER Full.

The results pattern provides an indication that an implied exchange rate measure that does not include remunerated liabilities ignores relevant sources of exchange rate variation. Finally, Argentina does not have a cointegrating relationship between the implied and spot exchange rate for either of the implied exchange rates. The results for Argentina are not surprising since in terms of monetary variables and exchange rate regimes it is an outlier. Although one would expect the informal exchange rate to reflect the balance sheet of the central bank (and thus be cointegrated), it is not. This could be due to different reasons, including the large volatility of the exchange rate in this period.

Table 3: Threshold Cointegration Results

| | | | CER Adjusts | | | | Spot Adjusts | | | |
|----------------------|------------|-----------|-------------|------------|------------|------------|--------------|------------|------------|------------|
| Case | Thresholds | Statistic | γ_1 | γ_2 | γ_3 | γ_4 | γ_5 | γ_6 | γ_7 | γ_8 |
| CER Base | | | | | | | | | | |
| Uruguay | 3 | 19.100 | - | - | - | - | - | - | - | - |
| Brazil | 2 | 11.282 | - | - | - | - | - | - | - | - |
| Chile | 3 | 23.168** | Yes | No | Yes | Yes | No | No | No | No |
| Paraguay | 2 | 60.184*** | Yes | Yes | Yes | - | Yes | No | No | - |
| Mexico | 3 | 27.190*** | Yes | No | No | Yes | No | No | No | No |
| Colombia | 1 | 15.724** | Yes | Yes | - | - | No | No | - | - |
| Argentina | 2 | 8.579 | - | - | - | - | - | - | - | - |
| Argentina (Informal) | 3 | 12.967 | - | - | - | - | - | - | - | - |
| CER Full | | | | | | | | | | |
| Uruguay | 3 | 24.835** | Yes | No | No | Yes | No | No | No | No |
| Brazil | 3 | 24.042** | Yes | No | No | Yes | No | Yes | No | Yes |
| Chile | 2 | 16.570* | Yes | No | Yes | - | Yes | No | Yes | - |
| Paraguay | 3 | 25.684** | No | Yes | No | Yes | Yes | No | No | No |
| Mexico | 3 | 17.222 | - | - | - | - | - | - | - | - |
| Colombia | 3 | 38.571*** | Yes | Yes | No | Yes | No | No | No | No |
| Argentina | 3 | 6.926 | - | - | - | - | - | - | - | - |
| Argentina (Informal) | 2 | 10.072 | - | - | - | - | - | - | - | - |

Notes: The null hypothesis is no cointegration against an alternative of (threshold) cointegration. Input specifications for the threshold cointegration tests (i.e., 1, 2, or 3 thresholds): threshold locations include the middle 90% of observations divided into 50 increments; each regime requires a minimum of 24 observations (binds for 3 of 16 pairs); AIC used throughout; critical values for the threshold cointegration test simulated following the residual-based block bootstrap methodology by Seo (2008) with a block length of 6 and 9,999 replications under the null. Significance at $\alpha = 0.10, 0.05$ and 0.01 denoted by *, **, and *** respectively.

4 Understanding the mechanism

What are the origins of equations (1) and (2)? How can we think of the relationships described previously? In this section we provide a simple theoretical link between exchange rates and remunerated liabilities.

We follow the main building blocks and notation in [Engel and West \(2005\)](#) (see references therein for earlier contributions), however we extend the framework to allow for accumulation of international reserves and remunerated liabilities.⁸

Assume that in the home country there is a money market relationship given by

$$m_t = p_t + \gamma y_t - \alpha i_t + v_{m,t}. \quad (8)$$

Here, m_t is the log of the home money supply, p_t is the log of the home price level, i_t is the level of the home interest rate, y_t is the log of output, and $v_{m,t}$ is a shock to money demand. We assume a similar equation holds in the foreign country, where the analogous foreign variables are m_t^* , p_t^* , i_t^* , y_t^* and $v_{m,t}^*$, and the parameters of the foreign money demand are identical to the home country's parameters.

The nominal exchange rate equals its purchasing power parity (PPP) value plus the real exchange rate:

$$s_t = p_t - p_t^* + q_t. \quad (9)$$

The (uncovered) interest parity relationship obtained from the financial markets is:

$$E_t s_{t+1} - s_t = i_t - i_t^* + \rho_t. \quad (10)$$

Here, ρ_t is the deviation from the rational expectations uncovered interest parity, and can be interpreted as a risk premium or an expectational error. Putting these equations together and rearranging, we get an expression for the spot rate along the lines of [Engel and West \(2005\)](#)

$$s_t = \frac{1}{1 + \alpha} \left[m_t - m_t^* - \gamma (y_t - y_t^*) + q_t - (v_{m,t} - v_{m,t}^*) - \alpha \rho_t \right] + \frac{\alpha}{1 + \alpha} E_t s_{t+1}. \quad (11)$$

⁸We use reduced form equations, but many of these equations can be derived from microfounded models, as in [Obstfeld and Rogoff \(1995\)](#) or [Vegh \(2013\)](#).

To interpret this equation, assume $\gamma = 1$, $q_t = 1$, $v_{m,t} = v_{m,t}^* = 0$, $\rho_t = 0$, and let $b = \frac{1}{1+\alpha}$, thus

$$s_t = b[m_t - m_t^* - (y_t - y_t^*)] + (1 - b)E_t s_{t+1}. \quad (12)$$

The observable fundamentals are given by $f_{1t} = m_t - m_t^* - \gamma(y_t - y_t^*)$. Notice that equation (12) holds absent intervention of the central banks. It is thus a pure flexible exchange rate “counterfactual” equilibrium. We however consider an environment where the central bank regularly responds to changes in total exchange market pressure by allowing some fraction of it to be reflected as a change in the exchange rate and the remaining to be absorbed as a change in foreign exchange reserves (Frankel (2019)).

In this context, a positive productivity shock, for instance, that leads to an increase in output y_t might be reflected as combination of an appreciation and reserve accumulation. We thus need to introduce international reserves into the picture.

Central Bank’s Balance Sheet A simplified Central Bank’s balance sheet is given by

| A | L |
|-----------|--------|
| $s_t H_t$ | MB_t |
| | RL_t |

H are international reserves (in foreign currency, “dollars”), s the exchange rate, MB the monetary base and RL_t the remunerated liabilities issued by the bank (in domestic currency, “pesos”).⁹

We consider an environment where the central bank’s net worth is zero and does not vary over time (as typically assumed, see Vegh (2013) for examples in different models). Thus, assets and liabilities vary, in dollar terms, proportionally. This implies that international reserves equals, to a first degree, total liabilities in dollar terms, $H = \frac{MB+RL}{s}$. Rearranging, we get the relationship used in our empirical exercises (we abstract from the time subindex for simplicity):

$$s = \frac{MB + RL}{H} \quad (13)$$

⁹We abstract from claims to financial institutions and from domestic credit, mainly because we focus on foreign denominated assets). We also abstract from valuation of the assets held by the central bank, a more general framework could include some of these elements, in line with Ghironi et al. (2015).

Define total liabilities (TL) as the sum of monetary base and remunerated liabilities $TL = MB + RL$ and let $\omega \equiv \frac{MB}{MB+RL}$ be the weight of the monetary base in total liabilities (and thus $1 - \omega = \frac{RL}{MB+RL}$). Then, taking logs and differentiating with respect to time, the dynamics of the exchange rate is given by¹⁰

$$\frac{\Delta s}{s} = \omega \frac{\Delta MB}{MB} + (1 - \omega) \frac{\Delta RL}{RL} - \frac{\Delta H}{H} \quad (14)$$

where Δ represents changes. Equations (14) and (12) have some similarity: other things equal, the exchange rate depreciates with increases in the monetary liabilities (which now also includes remunerated liabilities) and decreases with output growth (which tends to increase international reserves).¹¹ A framework that does not include remunerated liabilities ($\Delta RL = 0$) tends to ignore relevant sources of exchange rate variations, if the central bank indeed accumulated such type of debt. This, in fact, is what we verify empirically, the implied Base rate only considers variations stemming from MB and H , whereas the Full rate, which aligns better with the spot rate (s) over the long-run, incorporates the dynamics of RL .

The central bank can decide to intervene in the foreign exchange market. Given a shock on the exchange rate, the central bank might decide to prevent a movement in the exchange rate and let international reserves fully absorb the shock. Such a central bank, at least for this period, is fixing its exchange rate in de facto dollar terms (Levy-Yeyati and Sturzenegger (2005)). In this case, during the relevant period, international reserves are explained by variations in total liabilities (in real terms, since the exchange rate does not change), $\frac{\Delta H}{H} = \omega \frac{\Delta MB}{MB} + (1 - \omega) \frac{\Delta RL}{RL}$. In this case, given a positive shock, international reserves will increase by the sum of total liabilities.¹²

In general terms, however, given a shock, the central bank can choose it to be reflected partially in the exchange rate and partially in international reserves. Thus, in general terms, the reflection

¹⁰ Analogously, we can obtain a log-linearized version of (14) given by: $\hat{s} = \omega \hat{MB} + (1 - \omega) \hat{RL} - \hat{H}$ where hatted variables refer to the log-deviation with respect to its steady-state value. This equation would have the same interpretation, and furthermore, if the approximation at time t is made with respect to the variables in $t - 1$ these expression are identical.

¹¹ The intuition for this is in the money market: the exchange rate is the equilibrium price of supply and demand for domestic money. Output growth tends to increase money demand, which tends to appreciate the currency under flexible exchange rates. Under predetermined exchange rates however the central bank increases the supply of money -by purchasing foreign exchange reserves- and hence the exchange rate does not change but reserves do increase.

¹² Not all liabilities are the same, however. On one hand, international reserves accumulation can be explained by increases in the monetary base in real terms, which we can call as a "genuine" source of accumulation since it is endogenous under predetermined exchange rates and grows with money demand (economic growth). On the other hand, international reserves can be financed by increases in remunerated liabilities (debt accumulation). Sosa-Padilla and Sturzenegger (2023) focus on these differences.

of a shock in the balance sheet can be described by a vector $(\Delta s, \Delta H, \Delta MB, \Delta RL)$. The policy maker however cannot freely choose all of the elements of this vector.

Do remunerated liabilities give an extra degree of freedom that can be used to override the Mundell-Fleming trilemma (Mundell (1963), Fleming (1962))? To the contrary: our framework suggests that remunerated liabilities worsen the trilemma.

To see this, consider this example. A central bank can choose two of the following: the exchange rate management, autonomous monetary policy or free capital mobility. In the framework we are thinking of, an extra decision layer is added, since central banks can choose the amount of remunerated liabilities. To set ideas, assume that the central bank has chosen to have free capital mobility and a predetermined, or pegged, exchange rate regime (so the interest rate and the amount of money are endogenous and pinned down by the dynamics of the base foreign monetary policy) but the central bank has remunerated liabilities. From equation (13) it is clear that central banks in this context cannot freely choose the amount of remunerated liabilities, since doing so will affect exchange rates (since the expected exchange rate moves according to these liabilities, and hence so does the spot exchange rate). Remunerated liabilities are in this sense “endogenous”: they can only change as long as the monetary base or international reserves move to compensate them (which also are endogenous in this context due to the trilemma). If on the other hand the central bank chooses the amount of remunerated liabilities, then it cannot have a predetermined exchange rate. The Mundell-Fleming trilemma is in full force again, albeit now with remunerated liabilities.

In summary, the simplified theory laid out in this section is consistent with the empirical findings previously reported: exchange rates are related to the stock of remunerated liabilities when countries decide to make use of such policy instrument.

5 Conclusion

In this paper we investigate the extent to which remunerated liabilities are a relevant fundamental for understanding exchange rate fluctuations. Based on a threshold cointegration analysis, we compare the spot exchange rate with two implied exchange rate measures and find that the spot and implied exchange rate that incorporates only the monetary base (CER Base) is cointegrated

for four out of seven countries while the implied exchange rate that incorporates monetary base and remunerated liabilities (CER Full) is cointegrated for five out of seven countries. For the cointegrated countries, the spot rate rarely adjusts to restore the long run equilibrium with CER Base but does adjust to restore the long run equilibrium with CER Full.

This provides evidence that remunerated liabilities are a relevant fundamental for understanding exchange rate fluctuations. Consistently, a simplified balance-sheet setup delivers a link between exchange rates and remunerated liabilities and, together with threshold cointegration results, show that a framework that does not include remunerated liabilities tends to ignore relevant sources of exchange rate variation.

These results also suggest the existence of additional policy trade-offs involved when deciding on an exchange rate regime: deciding to use central bank debt or not. As we discussed, this new dimension affects the classic Mundellian Trilemma. Future research can investigate several implications of this type of monetary policy: for instance, the link between remunerated liabilities, credibility and optimal inflation targeting.

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A Data Sources

We use standardized central bank balance sheet data from the [IMF-IFS](#) statistics.¹³ The table below, from [Sosa-Padilla and Sturzenegger \(2023\)](#), summarizes the relevant balance-sheet data that we will use:

| Assets | Liabilities |
|--|---|
| Claims on non-residents (1) | Liabilities to non-residents (a) |
| Claims on others depository corporations (2) | Monetary base (b) |
| Net Claims on Central Government (3) | Other Liabilities To Other Depository Corporations (c) |
| | Deposits and Securities other than Shares Excluded from Monetary Base (d) |
| | Loans (e) |
| | Financial Derivatives (f) |
| | Shares and equity (g) |
| | Other items (h) |

Hence the CER Base is

$$\text{CER Base} \equiv \frac{\text{Monetary Base}}{\text{International Reserves}} = \frac{(b)}{(1)}$$

Following [Sosa-Padilla and Sturzenegger \(2023\)](#), we define the Remunerated Liabilities as the sum of (c), (d), (e) and (f). The CER Full is then

$$\text{CER Full} \equiv \frac{\text{Monetary Base} + \text{Remunerated Liabilities}}{\text{International Reserves}} = \frac{(b) + (c) + (d) + (e) + (f)}{(1)}$$

¹³For Argentina we use the database in [UCEMA-CEA](#) for the monetary base and remunerated liabilities. The spot exchange rate from [UCEMA-CEA](#) has missing values in 2009, thus we use the FRED database for the spot exchange rate.

B Additional Results of the Balance Sheet based VAR estimations

Table A1: Lag Selection for VAR models

| Country | AIC(n) | HQ(n) | SC(n) | FPE(n) |
|----------------------|--------|-------|-------|--------|
| Uruguay | 3 | 2 | 1 | 3 |
| Brazil | 2 | 1 | 1 | 2 |
| Chile | 2 | 2 | 1 | 2 |
| Paraguay | 2 | 1 | 1 | 2 |
| Mexico | 4 | 1 | 1 | 4 |
| Colombia | 2 | 2 | 2 | 2 |
| Argentina | 1 | 1 | 1 | 1 |
| Argentina (informal) | 2 | 1 | 1 | 2 |

Note: The lag length selected by the AIC from a maximum of $T^{1/3}$ (6 for our sample size).

Table A2: Johansen Cointegration Rest Results, by Country

| Number of Cointegration Relationships from Johansen Test | | | | | |
|--|-------------------------|-------|--------------------------|--------------|------------------------|
| Country | Deterministic component | | Decision by type of test | | |
| | Constant | Trend | Trace | Max. Eigenv. | Final Model (selected) |
| Uruguay | 3 / 2 | 1 / 1 | 1 - Trend | 1 - Trend | VEC/VAR(levels) |
| Brazil | 2 / 1 | 0 / 0 | 0 - Trend | 0 - Trend | VAR(differences) |
| Chile | 1 / 1 | 1 / 1 | 1 - Constant | 1 - Constant | VEC/VAR(levels) |
| Paraguay | 3 / 3 | 1 / 0 | 1 - Trend | 0 - Trend | VAR(differences) |
| Mexico | 1 / 1 | 1 / 1 | 1 - Constant | 1 - Constant | VEC/VAR(levels) |
| Colombia | 3 / 3 | 2 / 1 | 2 - Trend | 1 - Trend | VEC/VAR(levels) |
| Argentina | 1 / 1 | 1 / 1 | 1 - Constant | 1 - Constant | VEC/VAR(levels) |
| Argentina (informal) | 1 / 1 | 1 / 0 | 1 - Constant | 0 - Trend | VAR(differences) |

Notes: Each entry denotes the number of relations in the trace test / maximum eigenvalue test. The deterministic component refers to additional terms included in the cointegration relationship. The simplest model is selected (in number of relations and deterministic component(s)) for which the test is rejected. Here, as a robustness check, we replicate the cointegration test without 'none' as an option (no deterministic component in the cointegration relationship) to cover only cases where at least a constant is included for one or more variables. The conclusions are similar.

Table A3: Causality Tests in the VAR models, by Country

| Instantaneous causality test | | | | | Granger causality test | | | | |
|------------------------------|-------|-------|-------|-------|------------------------|-------|-------|-------|-------|
| Country | ER | H | MB | RL | Country | ER | H | MB | RL |
| Uruguay | 0.000 | 0.000 | 0.012 | 0.000 | Uruguay | 0.577 | 0.270 | 0.001 | 0.198 |
| Brazil | 0.000 | 0.000 | 0.000 | 0.000 | Brazil | 0.427 | 0.038 | 0.005 | 0.031 |
| Chile | 0.000 | 0.000 | 0.000 | 0.000 | Chile | 0.055 | 0.000 | 0.028 | 0.112 |
| Paraguay | 0.000 | 0.000 | 0.000 | 0.000 | Paraguay | 0.045 | 0.002 | 0.000 | 0.001 |
| Mexico | 0.000 | 0.000 | 0.012 | 0.005 | Mexico | 0.016 | 0.000 | 0.051 | 0.502 |
| Colombia | 0.000 | 0.000 | 0.484 | 0.326 | Colombia | 0.004 | 0.000 | 0.010 | 0.261 |
| Argentina | 0.003 | 0.000 | 0.000 | 0.000 | Argentina | 0.058 | 0.290 | 0.048 | 0.479 |
| Argentina (informal) | 0.000 | 0.000 | 0.000 | 0.000 | Argentina (informal) | 0.407 | 0.282 | 0.009 | 0.682 |

Note: The null hypothesis is the variable in the column causes (instantaneously or Granger) the rest of the system. P-values are displayed in the table.

Figure A1: FEVD for the Exchange Rate (upper left), Reserves (upper right), and Monetary Base (lower centre) (% explained by Remunerated Liabilities)

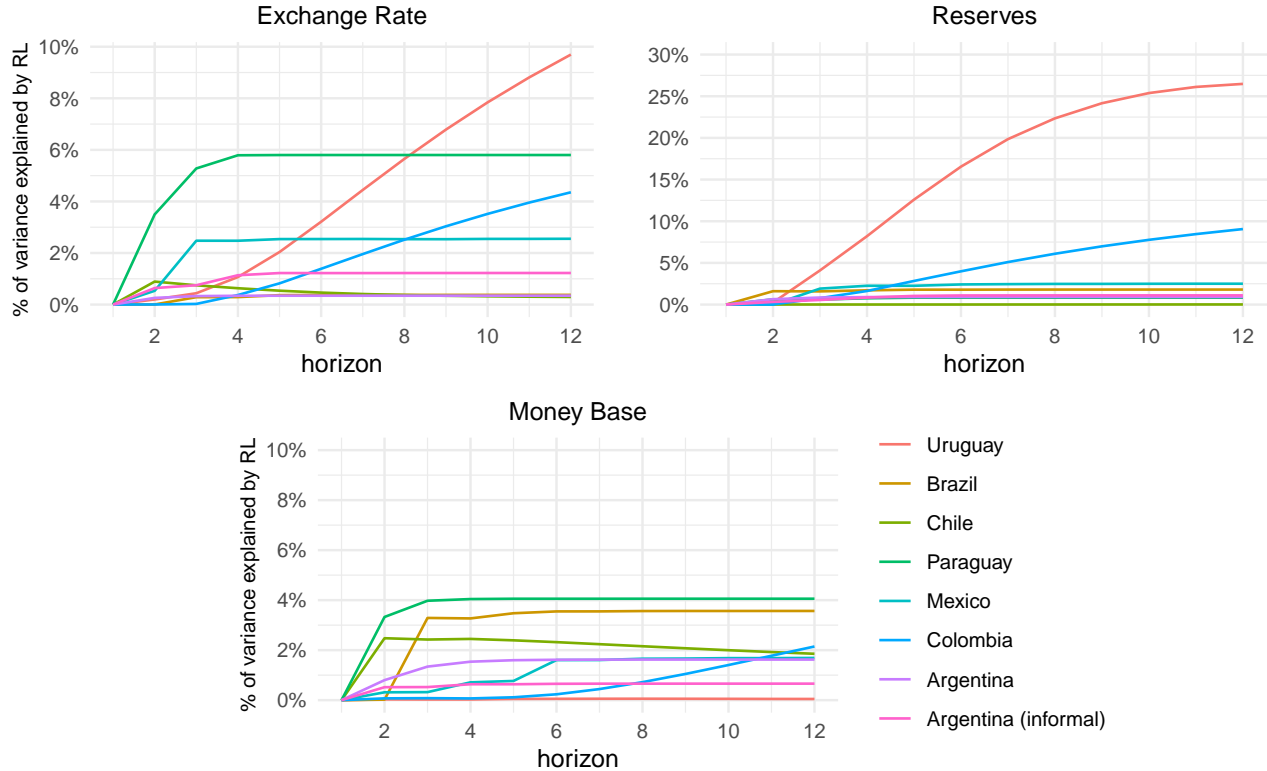
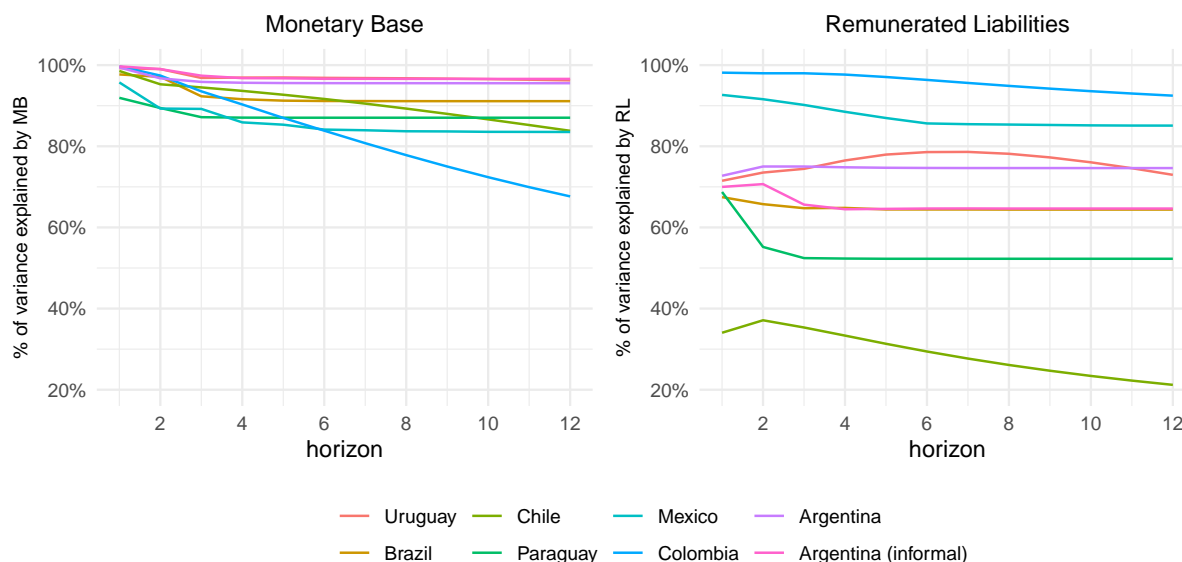


Figure A2: Percentage of Variance of Monetary Base (left) and Remunerated Liabilities (right) Explained by Itself



C Additional Results from Threshold Cointegration Exercise

Table A4: Unit Root Test Results for Spot Exchange Rate (*er*) and Implied Exchange Rates (*Base* and *Full*), 2004:1-2019:12

| Country and Exchange Rate | Level | Differenced | Country and Exchange Rate | Level | Differenced |
|---------------------------|--------|-------------|---------------------------|---------|-------------|
| Uruguay | | | Brazil | | |
| Spot | -1.759 | -7.681*** | Spot | -2.183 | -9.317*** |
| Base | -2.510 | -6.925*** | Base | -1.373 | -13.129*** |
| Full | -1.250 | -7.278*** | Full | -3.140* | -4.118*** |
| Chile | | | Paraguay | | |
| Spot | -2.076 | -10.154*** | Spot | -1.655 | -8.269*** |
| Base | -2.112 | -15.864*** | Base | -2.467 | -14.414*** |
| Full | -1.444 | -14.994*** | Full | -1.096 | -5.598*** |
| Mexico | | | Colombia | | |
| Spot | -2.551 | -9.517*** | Spot | -1.765 | -9.022*** |
| Base | -1.380 | -15.520*** | Base | -2.661 | -5.647*** |
| Full | -0.705 | -14.807*** | Full | -2.478 | -6.220*** |
| Argentina | | | | | |
| Spot | 0.423 | -8.234*** | | | |
| Spot (Informal) | 0.687 | -10.355*** | | | |
| Base | -2.317 | -11.033*** | | | |
| Full | -2.157 | -11.259*** | | | |

Notes: The null hypothesis is a unit root. Rejecting the null hypothesis for the level series and failing to reject the null hypothesis for the differenced series means the series is I(1). Input specifications for ADF unit root tests are: constant and trend for the level data; constant for the differenced data; lag length for the testing equation is selected by the AIC from a maximum of $T^{1/3}$. Significance at $\alpha = 0.10$, 0.05 and 0.01 absence of a structural break. The CER Full for Brazil is considered I(1) because the test statistic for the level series is -3.140 and the $\alpha = 0.10$ test statistic is -3.13.

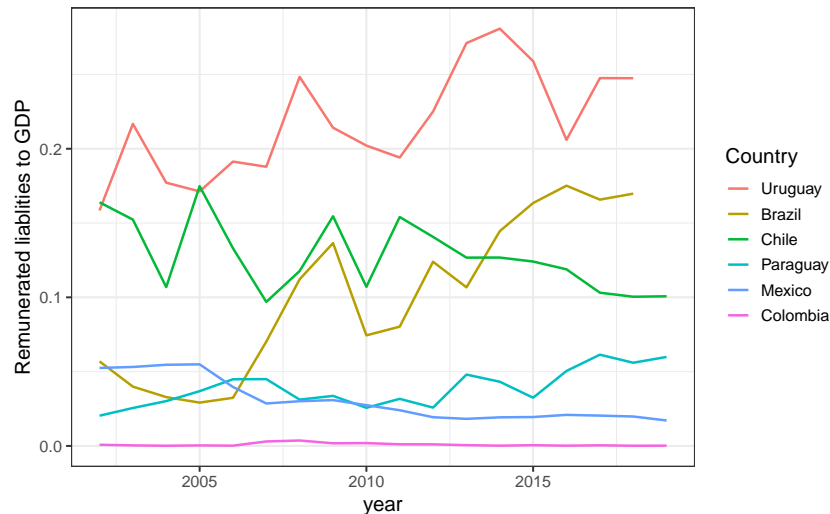
Table A5: Threshold Cointegration Results (with adjustment coefficients)

| | | | Dependent variable: ΔY_t | | | | Dependent variable: ΔX_t | | | |
|----------------------|------------|-----------|----------------------------------|------------|------------|------------|----------------------------------|------------|------------|------------|
| Case | Thresholds | Statistic | γ_1 | γ_2 | γ_3 | γ_4 | γ_5 | γ_6 | γ_7 | γ_8 |
| CER Base | | | | | | | | | | |
| Uruguay | 3 | 19.100 | - | - | - | - | - | - | - | - |
| Brazil | 2 | 11.282 | - | - | - | - | - | - | - | - |
| Chile | 3 | 23.168** | -0.062*** | 0.257 | -0.328*** | -0.192* | -0.002 | -0.007 | 0.004 | -0.008 |
| Paraguay | 2 | 60.184*** | -0.171** | -0.661*** | -0.220** | - | 0.097*** | -0.001 | -0.002 | - |
| Mexico | 3 | 27.190*** | -0.128*** | -0.035 | 0.149 | -0.191*** | 0.009 | -0.073 | 0.005 | 0.022 |
| Colombia | 1 | 15.724** | -0.431*** | -0.304*** | - | - | -0.136 | -0.097 | - | - |
| Argentina | 2 | 8.579 | - | - | - | - | - | - | - | - |
| Argentina (Informal) | 3 | 12.967 | - | - | - | - | - | - | - | - |
| CER Full | | | | | | | | | | |
| Uruguay | 3 | 24.835** | -0.130*** | 0.147 | 0.384 | -0.168*** | 0.009 | 0.040 | 0.082 | 0.026 |
| Brazil | 3 | 24.042** | -0.089* | -0.117 | 0.229 | -0.127*** | 0.036 | 0.121*** | 0.035 | 0.045* |
| Chile | 2 | 16.570* | -0.125*** | 0.153 | -0.097** | - | 0.056* | -0.199 | 0.052* | - |
| Paraguay | 3 | 25.684** | 0.049 | -0.405*** | -0.068 | -0.196*** | 0.083** | 0.005 | -0.108 | -0.009 |
| Mexico | 3 | 17.222 | - | - | - | - | - | - | - | - |
| Colombia | 3 | 38.571*** | -0.303*** | -0.741*** | 0.109 | -0.325*** | -0.099 | -0.078 | -0.092 | -0.093 |
| Argentina | 3 | 6.926 | - | - | - | - | - | - | - | - |
| Argentina (Informal) | 2 | 10.072 | - | - | - | - | - | - | - | - |

Notes: The null hypothesis is no cointegration against an alternative of (threshold) cointegration. Input specifications for the threshold cointegration tests (i.e., 1, 2, or 3 thresholds): threshold locations include the middle 90% of observations divided into 50 increments; each regime requires a minimum of 24 observations (binds for 3 of 16 pairs); AIC used throughout; critical values for the threshold cointegration test simulated following the residual-based block bootstrap methodology by Seo (2008) with a block length of 6 and 9,999 replications under the null. Significance at $\alpha = 0.10, 0.05$ and 0.01 denoted by *, **, and *** respectively.

D Additional Plots and Tables

Figure A3: Evolution of Remunerated Liabilities as a Fraction of GDP, 2004-2019



Source: Author's calculations based on IMF IFS.

Table A6: Balance Sheet Components as a Fraction of GDP, 2010-2019

| Country | Reserves | Financial Claims | Domestic Credit | Monetary Base | Remunerated Liabilities |
|-------------------------------------|----------|------------------|-----------------|---------------|-------------------------|
| Malaysia | 0.40 | 0.03 | 0.00 | 0.10 | 0.22 |
| Uruguay | 0.25 | 0.00 | 0.14 | 0.06 | 0.22 |
| China, P.R.: Macao | 0.73 | 0.01 | 0.00 | 0.08 | 0.14 |
| Jamaica | 0.18 | 0.00 | 0.10 | 0.10 | 0.14 |
| Algeria | 0.76 | 0.00 | 0.03 | 0.23 | 0.13 |
| Chile | 0.15 | 0.00 | 0.02 | 0.07 | 0.13 |
| Korea, Rep. of | 0.24 | 0.01 | 0.01 | 0.06 | 0.12 |
| Dominican Rep. | 0.07 | 0.02 | 0.12 | 0.11 | 0.11 |
| Brazil | 0.14 | 0.01 | 0.18 | 0.10 | 0.10 |
| Philippines | 0.24 | 0.00 | 0.03 | 0.13 | 0.08 |
| Bolivia | 0.37 | 0.03 | 0.07 | 0.25 | 0.07 |
| Romania | 0.22 | 0.01 | 0.00 | 0.09 | 0.06 |
| Papua New Guinea | 0.14 | 0.00 | 0.02 | 0.07 | 0.06 |
| Nigeria | 0.12 | 0.02 | 0.03 | 0.05 | 0.06 |
| Mongolia | 0.18 | 0.05 | 0.02 | 0.11 | 0.05 |
| Seychelles | 0.22 | 0.00 | 0.15 | 0.16 | 0.05 |
| Botswana | 0.63 | 0.00 | 0.00 | 0.15 | 0.04 |
| Belarus, Rep. of | 0.10 | 0.03 | 0.01 | 0.06 | 0.04 |
| Bhutan | 0.52 | 0.01 | 0.00 | 0.28 | 0.04 |
| Paraguay | 0.16 | 0.01 | 0.02 | 0.11 | 0.04 |
| Venezuela, Rep. Bolivariana de | 0.17 | 0.00 | 0.04 | 0.12 | 0.04 |
| Ghana | 0.09 | 0.01 | 0.07 | 0.07 | 0.03 |
| Mexico | 0.12 | 0.01 | 0.00 | 0.05 | 0.03 |
| Guatemala | 0.14 | 0.01 | 0.07 | 0.14 | 0.03 |
| Maldives | 0.14 | 0.00 | 0.11 | 0.18 | 0.02 |
| Sweden | 0.09 | 0.02 | 0.02 | 0.04 | 0.02 |
| Rwanda | 0.14 | 0.00 | 0.02 | 0.04 | 0.02 |
| Cambodia | 0.31 | 0.00 | 0.01 | 0.19 | 0.02 |
| Azerbaijan, Rep. of | 0.15 | 0.02 | 0.01 | 0.13 | 0.02 |
| Angola | 0.17 | 0.01 | 0.02 | 0.07 | 0.02 |
| Mozambique, Rep. of | 0.17 | 0.00 | 0.02 | 0.09 | 0.02 |
| Vanuatu | 0.24 | 0.00 | 0.03 | 0.19 | 0.02 |
| Ukraine | 0.18 | 0.04 | 0.09 | 0.17 | 0.01 |
| Namibia | 0.12 | 0.01 | 0.00 | 0.04 | 0.01 |
| Samoa | 0.14 | 0.01 | 0.00 | 0.10 | 0.01 |
| Georgia | 0.15 | 0.02 | 0.04 | 0.12 | 0.01 |
| Kyrgyz Rep. | 0.28 | 0.01 | 0.03 | 0.18 | 0.01 |
| Bangladesh | 0.08 | 0.01 | 0.03 | 0.09 | 0.01 |
| Australia | 0.05 | 0.03 | 0.00 | 0.04 | 0.01 |
| Tajikistan, Rep. of | 0.08 | 0.02 | 0.04 | 0.13 | 0.01 |
| Sri Lanka | 0.10 | 0.00 | 0.03 | 0.06 | 0.01 |
| Japan | 0.01 | 0.05 | 0.35 | 0.42 | 0.01 |
| Kenya | 0.10 | 0.01 | 0.02 | 0.07 | 0.01 |
| Tanzania, United Rep. of | 0.11 | 0.00 | 0.02 | 0.07 | 0.01 |
| Turkey | 0.13 | 0.02 | 0.02 | 0.10 | 0.01 |
| St. Kitts and Nevis | 0.24 | 0.00 | 0.00 | 0.22 | 0.01 |
| Antigua and Barbuda | 0.16 | 0.00 | 0.02 | 0.17 | 0.01 |
| Suriname | 0.14 | 0.00 | 0.06 | 0.13 | 0.01 |
| Fiji, Rep. of | 0.17 | 0.00 | 0.02 | 0.16 | 0.01 |
| Trinidad and Tobago | 0.37 | 0.00 | 0.15 | 0.15 | 0.00 |
| Uganda | 0.12 | 0.00 | 0.06 | 0.05 | 0.00 |
| Dominica | 0.20 | 0.00 | 0.00 | 0.17 | 0.00 |
| Grenada | 0.17 | 0.00 | 0.00 | 0.15 | 0.00 |
| Croatia, Rep. of | 0.26 | 0.00 | 0.00 | 0.21 | 0.00 |
| St. Lucia | 0.14 | 0.00 | 0.00 | 0.13 | 0.00 |
| Russian Federation | 0.26 | 0.04 | 0.01 | 0.14 | 0.00 |
| Armenia, Rep. of | 0.18 | 0.03 | 0.00 | 0.15 | 0.00 |
| Barbados | 0.12 | 0.00 | 0.07 | 0.16 | 0.00 |
| Guyana | 0.22 | 0.00 | 0.13 | 0.19 | 0.00 |
| Kuwait | 0.19 | 0.00 | 0.00 | 0.11 | 0.00 |
| St. Vincent and the Grenadines | 0.16 | 0.00 | 0.01 | 0.15 | 0.00 |
| Belize | 0.15 | 0.00 | 0.07 | 0.17 | 0.00 |
| Nicaragua | 0.18 | 0.00 | 0.31 | 0.16 | 0.00 |
| Anguilla | 0.16 | 0.00 | 0.01 | 0.16 | 0.00 |
| Morocco | 0.24 | 0.03 | 0.01 | 0.24 | 0.00 |
| Myanmar | 0.03 | 0.01 | 0.15 | 0.17 | 0.00 |
| Comoros, Union of the | 0.16 | 0.00 | 0.03 | 0.12 | 0.00 |
| Sierra Leone | 0.12 | 0.00 | 0.14 | 0.07 | 0.00 |
| Albania | 0.21 | 0.01 | 0.07 | 0.25 | 0.00 |
| Costa Rica | 0.14 | 0.00 | 0.01 | 0.20 | 0.00 |
| Colombia | 0.13 | 0.01 | 0.01 | 0.08 | 0.00 |
| Gambia, The | 0.11 | 0.00 | 0.06 | 0.10 | 0.00 |
| Canada | 0.00 | 0.00 | 0.04 | 0.04 | 0.00 |
| Zambia | 0.09 | 0.00 | 0.11 | 0.07 | 0.00 |
| Honduras | 0.22 | 0.00 | 0.06 | 0.20 | 0.00 |
| Sao Tomé and Príncipe, Dem. Rep. of | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Cameroon | 0.09 | 0.00 | 0.02 | 0.09 | 0.00 |
| Central African Rep. | 0.11 | 0.00 | 0.13 | 0.13 | 0.00 |
| Chad | 0.07 | 0.01 | 0.05 | 0.08 | 0.00 |
| Congo, Rep. of | 0.23 | 0.00 | 0.05 | 0.15 | 0.00 |
| Czech Rep. | 0.31 | 0.00 | 0.00 | 0.28 | 0.00 |
| Equatorial Guinea, Rep. of | 0.14 | 0.00 | 0.02 | 0.08 | 0.00 |
| Eswatini, Kingdom of | 0.14 | 0.00 | 0.01 | 0.04 | 0.00 |
| Gabon | 0.10 | 0.00 | 0.04 | 0.09 | 0.00 |
| Lesotho, Kingdom of | 0.40 | 0.00 | 0.02 | 0.06 | 0.00 |

Note: Average components of the balance sheet as a fraction of GDP for the period 2010-2019. Source: IMF-IFS database.