The Collatz Conjecture

“Mathematics may not be ready for such problems.” - Jeffery Lagarias.

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Introduction:

Mathematics could be considered as the only perfect thing humanity has invented. The way everything in mathematics falls in place in a beautiful symphony of numbers and operations is truly wondrous. From simple math rules like the relation between even powers and negative numbers to more complex things such as the pythagoras theorem, most mathematical principles are perfectly applied.

However, only most mathematical principles are perfect. There are outliers, exceptions to this rule. One of which, and the topic of this research paper, is the Collatz Conjecture. Created by Lothar Collatz in 1937, it remains one of the only unsolved math equations of all time. However, it is quite simple to understand.

The conjecture states that if we grab any number and apply the following rule: if the number is odd, multiply by 3 and add 1, if the number is even, divide by 2, all numbers will result in the number 1, which is multiplied to 4 which is divided to 2 which is divide to 1 and so on. I will refer to this loop as the 4 2 1 loop from now on. This works for negative numbers, but it becomes a -4 -2 -1 loop.

To illustrate, let's take the number 5. 5 is odd, so we will multiply by 3 and add 1. 5x3+1=16. 16 is even, so we will divide by 2. 16/2= 8, 8/2=4, 4/2=2, 2/1 =1, and so the 4 2 1 loop begins. What puzzles mathematicians about this is that every number in the world ends in the 4 2 1 loop, and nobody knows why.

Collecting data about the problem.

This equation may be near impossible to solve, but I decided to have a crack at it regardless. To be honest, I could have gotten the data online, seeing as there were many studies done on the Collatz Conjecture. However, to develop my own personal skills, I decided to collect the data myself.

To get this data, I created a python program to do the equation for me. I first made it so that the program would figure out whether the number is odd or even by dividing by 2, then if the number had any decimals, it was odd otherwise it was even.

I made a loop to do this, and if the number was odd it'd multiply by 3 and add 1, if even it would divide by 2. I added a counter so that every time the loop went, it would increase the counter by 1. The loop ends when the number reaches 1. This counter helped me figure out how many steps it took for each number to get to 1, i repeated this process for numbers 1-120, the reason why 120 is important for later.

```
def collatz(x):
 y=0
 while True:
     y+=1
     if x == 0 or x ==1:
         return None
     elif x%2 == 0:
         x = x/2
     else:
         x = x*3+1
     if x == 1:
         break
 print ("The number amounted to:",x)
 print("The number of steps taken to get there was:",y)
 return (x,y)
```

Reviewing data collected.

After collecting the data, I put it in a google sheets page to lay out in front of me. Every number was written with a colon (:) in front of them, the number past this colon is the steps taken to get there.

(The google sheet should be included with the paper, you probably should have it in front of you at this point.)

Now, the first thing you probably noticed is the highlighted red boxes. Those seemingly random red boxes are a perfect sequence of numbers to 1. What I mean by this is that if we take any number from the sequence, for example ,8, we can divide it by 2 because it's even then it becomes four which leads into the 4 2 1 loop. Every red highlighted number will be even and can be divided by 2 until we reach the 4 2 1 loop. I highlighted them because they are always going to be outliers, as their sequence will always differ from others as they are 1 straight path.

The most outstanding observation is the complete lack of a straight, uninterrupted pattern. No matter how many numbers seem to have a pattern, one random number will usually break it up. However, there is something else to note . First, I noticed that the double of every number creates an odd pattern of consecutive numbers. For example, look at the number 10. 10 has 6 next to it, while its double, 20, has 7, 40 has 8, and 80 has 9. I haven't seen this discussed very often, but from my dataset I can conclude that this is true, at least for the first 120 positive numbers.

An odd phenomenon.

There is one thing however, an observation I made, that I withheld from you until this point. It’s a very odd phenomenon that happened with numbers within this dataset, and it is the reason I chose to make the full dataset have only 120 numbers.

If you were to take a random number from the sheet, then take the steps taken next to it and look at its corresponding integer, you will be able to keep doing this until you reach 1 with every number. For example, let's take the number 10. 10 took 6 steps, so lets go look at number 6 which took 8 steps. 8 took 3 steps, 3 took 7, 7 took 16, 16 took 4, 4 took 2, 2 took 1. That formula works for every number in this dataset.

To be honest, I have no idea why this works, but I have another way of phrasing it that may help. Instead of saying each number leads to 1, we can say each number passes by the red highlighted numbers at least three times. You see, the number 16 is the ONLY number here that leads to 4. And 4 is the only number leading to 2, 2 is the only number leading to 1. For a number to reach 4, it must pass by 16, then 4 and 2 to reach 1.

Finally, the reason that I set the dataset to 120 is very simple. Every ‘steps taken’ number here is only <=120. And due to this phenomenon, I wanted a closed dataset that could all lead into each other, so that you don't have to do research to follow along with any number; all you need is that one spreadsheet.

Conclusion.

In the end, the Collatz Conjecture is simply a hypothetical mathematical equation. There's no real reason to put so much energy into a hypothetical, then why did I do this project?

The answer is very simple, it was never about the Collatz Conjecture. This project was a great way to widen my scope, to test my skills at new things. I learned how to code better, I learned how to organize and analyze data through the spreadsheet I made, I learned a lot.

Hopefully, I wasn't the only one learning throughout this research. Perhaps you learned something too, whether it was about the conjecture itself or something else from the skills I listed. A very important takeaway here, from the fact that even the most modern mathematicians haven't been able to crack this conjecture, is that no matter who you may be, you will always have something new to learn, so always look for it.

The final takeaway from this project, and perhaps the most important one, is to keep looking for new possibilities. There is always another angle, another scope to look through. I would like to close this off with one of my favorite quotes of all time and history: 'There are 360 degrees. Why stick to one?' -Zaha Hadid.