

Section-I

1. (c) 0.6
2. (c) 0.35
3. (b) stratified sampling.
4. (a) 68%.
5. (c) it will be normally distributed.
6. (b) stratified sampling.
7. (b) The new campaign ~~increases~~ ~~use~~ has no effect on website traffic.

Section-II

1A) Population:-

The entire group of individuals, objects or events that we are interested in studying. It includes all members of the group we want to learn about. can be finite or infinite.

Sample:-

A subset of the population that is selected to represent the entire population. It is used when studying the entire population is impractical or impossible.

Example:-

Considers a researcher who wants to study the average height of all students in a university.

Population:- All students enrolled in university.

Sample:- A group of 100 students selected randomly from the university.

2A) For each digit in password, we have 10 choices (0-9).
Since repetition is allowed, the total number of possible passwords is:

10 choices for the first digit * 10 choices for the second digit * 10 choices for third digit * 10 choices for fourth digit.

This equals $10^4 = 10,000$ different passwords.

3A) odd numbers: 1, 3, 5 (3 outcomes)

Numbers less than 4: 1, 2, 3 (3 outcomes)

we've counted 1 and 3 twice. so, the total number of favourable outcomes is $3 + 3 - 2 = 4$.

The total no. of possible outcomes is 6.

Therefore, the probability of rolling an odd number or a number less than 4 is:

$$P(\text{odd or less than 4}) = \frac{\text{favourable outcomes}}{\text{total outcomes}} \\ = \frac{4}{6} = \frac{2}{3}$$

6A) Probability is a measure of the likelihood of an event occurring. It's expressed as a number between 0 and 1, where 0 indicates impossibility and 1 indicates certainty.

Joint Probability refers to the probability of two or more events occurring simultaneously. It's denoted as $P(A \text{ and } B)$.

Example: The probability of drawing a red card and a king from a standard deck of cards.

Conditional Probability:- It is the probability of an event occurring given that another event has already occurred. It is denoted as $P(A|B)$, which reads

"the probability of A given B."

Ex:- The probability of rain today, given that it rained yesterday.

7A)

1. Arrange the groups:- 4 groups can be arranged in $4!$ ways.

2. Arrange books within each group:

Maths: $3!$ ways

History: $4!$ ways

Chemistry: $3!$ ways

Biology: $2!$ ways.

Now, we multiply the number of arrangements from each step to get the total no. of arrangements:-

Total arrangements: $4! \times 3! \times 4! \times 3! \times 2! = 41,472$

There are 41,472 possible arrangements for books on the shelf.

Section-III

4A) sampling is the process of selecting a subset of a population to represent the entire population in a study.

Here are some different types of sampling techniques:-

Simple random sampling:- Every member of population has an equal chance of being selected. For example, drawing names from a hat or using a random number generator.

Stratified sampling:- The population is divided into subgroups based on certain characteristics, and then a random sample is selected from each subgroup. For example, dividing a population into age groups and randomly selecting individuals from each group.

Cluster sampling:- The population is divided into clusters, and then random sample of clusters is selected.

All individuals within the selected clusters are included in the sample. For example, dividing a city into neighbourhoods & randomly selecting a few neighbourhoods to survey.

Systematic sampling:- Every n th member of population is selected. For example, selecting every 10th person on a list.

Normal Probability distributions

often referred as the bell curve, is a statistical distribution that is symmetrical about the mean.

- Curve is symmetrical
- Mean, median, mode are equal.
- Skewness & kurtosis are equal to 0.
- Area under the curve is 1.

Ex:- Heights of adult men typically follow a normal distribution with mean height of 70 inches.

Central Limit theorem

C.L.T states that if you take sufficiently large sample means approaches a normal distribution as sample size increases, regardless of underlying population distribution. This is true even if the population distribution is not normal.

$$\sigma_{\bar{x}} = \sigma / \sqrt{n}$$

Ex:- We want to estimate avg height of all adults in a country, we can take multiple random samples of adults, calculate the mean height for each sample, and plot the distribution of these sample means. According to CLT, this distribution of sample means will be approximately normal, even if the population distribution heights is not normal.

3A) To solve this, we need to consider the no. of words that can be formed starting with each letter.

1. words starting with 'A':

There are $4! = 24$ words starting with 'A'.

2. words starting with 'G':

Similarly there are $4! = 24$ words starting with 'G'.

So after the 24 words starting with 'A' and the 24 words starting with 'G', we've counted 48 words.

Therefore, 49th word will start with the letter 'I'.

The remaining letters 'A', 'G', 'N' can be arranged in $3! = 6$ ways.

So, 49th word is IAGN.

6A) Hypothesis testing is a statistical method used to determine whether a hypothesis is true or false. It involves collecting sample data, analyzing it, and making inferences about the population.

Importance:-

It helps in making informed decisions based on data driven evidence.

It is crucial for researchers to test their theories.

Difference b/n z and t Statistics:

Z-Statistics

→ Used when population standard deviation (σ) is known.

→ Assumes a normal distribution.

→ $z = (\bar{x} - \mu) / (\sigma / \sqrt{n})$ → when sample size ($n \geq 30$).

EX:- A company claims that the average lifespan of light bulbs is 10,000 hrs. A researcher wants to test this claim by collecting a sample of 100 bulbs & calculating sample mean & standard deviation. If population standard deviation is known, a z-test can be used.

T-Statistic

→ Used when the population standard deviation (σ) is unknown and is estimated by sample deviation (s).

→ Assumes a t-distribution.

EX: A researcher wants to test the effectiveness of a new teaching method. They collect a sample of students and measure their test scores. Since the population standard deviation of test scores is unknown, a t-test can be used.