Econometrics IV - Final Assignment - Code

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In [ ]:
```
import numpy as np
import pandas as pd
import os
import matplotlib.pyplot as plt
import seaborn as sns

from functions.linear_models import PCA_function, OLS_regression
from functions.number_PC import rule_thumb, informal_way, biggest_drop
```

In [ ]:
```
# hide warning messages
import warnings
warnings.filterwarnings("ignore")
```

In [ ]:
```
# better plots
sns.set(rc={'figure.figsize':(12,8)});
```

In [ ]:
```
directory = os.path.dirname(os.getcwd())
directory
```

Out[ ]:
```
'd:\\github\\AssignmentEconometricsIV'
```

Question

The first question consists of a factor analysis of a large dataset. We consider monthly close-to-close excess returns from a cross-section of 9,456 firms traded in the New York Stock Exchange. The data starts on November 1991 and runs until December 2018. There are 326 monthly observations in total.

In addition to the returns we also consider 16 monthly factors:

- Market (MKT)
- Small-minus-Big (SMB)
- High-minus-Low (HML)
- Conservative-minus-Aggressive (CMA)
- Robust-minus-Weak (RMW)
- earning/price ratio (EP)
- cash-flow/price ratio (CFP)
- dividend/price ratio
- accruals (ACC)
- market beta (BETA)
- net share issues
- daily variance (RETVOL)
- daily idiosyncratic variance (IDIOVOL)
- 1-month momentum (MOM1)
- 36-month momentum (MOM36)

The dataset is organized as an excel file named `returns.xlsx`.

In [ ]:
```
input_path = f'{directory}\\data\\returns.xlsx'
df = pd.read_excel(input_path, index_col=0)
```

In [ ]:
```
df.head()
```

Out[ ]:

dates	MKT	HML	SMB	MOM1	MOM36	ACC	BETA	CFP	CHCSH
1991-11-29	-0.041264	-0.028083	0.004779	-0.007336	-0.025496	-0.013692	0.035433	-0.015116	-0.00677
1991-12-31	0.107984	-0.022529	-0.027366	0.010963	-0.021188	-0.027887	-0.082499	-0.032122	-0.0058!
1992-01-31	-0.007668	0.051012	0.085547	0.050916	0.108588	0.021978	-0.072801	0.028117	-0.0081!
1992-02-28	0.010796	0.070501	0.002794	-0.027398	0.079286	0.003860	-0.024906	0.037363	0.01562
1992-03-31	-0.025367	0.039029	-0.015135	-0.009367	0.024631	0.004612	0.041266	0.037916	0.01711

5 rows × 9472 columns

In [ ]:
```
factors_name = df.columns[:16]
factors_name
```

Out[ ]:
```
Index(['MKT', 'HML', 'SMB', 'MOM1', 'MOM36', 'ACC', 'BETA', 'CFP', 'CHCSHO',
       'DY', 'EP', 'IDIOVOL', 'CMA', 'UMD', 'RMW', 'RETVOL'],
      dtype='object')
```

In [ ]:
```
factors = df[factors_name]
factors.head()
```

Out[ ]:

dates	MKT	HML	SMB	MOM1	MOM36	ACC	BETA	CFP	CHCSH
1991-11-29	-0.041264	-0.028083	0.004779	-0.007336	-0.025496	-0.013692	0.035433	-0.015116	-0.00677
1991-12-31	0.107984	-0.022529	-0.027366	0.010963	-0.021188	-0.027887	-0.082499	-0.032122	-0.0058!
1992-01-31	-0.007668	0.051012	0.085547	0.050916	0.108588	0.021978	-0.072801	0.028117	-0.0081!
1992-02-28	0.010796	0.070501	0.002794	-0.027398	0.079286	0.003860	-0.024906	0.037363	0.01562
1992-03-31	-0.025367	0.039029	-0.015135	-0.009367	0.024631	0.004612	0.041266	0.037916	0.01711

In [ ]:
```
returns_name = df.columns[16:]
returns_name
```

Out[ ]:
```
Index(['r_   1', 'r_   2', 'r_   3', 'r_   4', 'r_   5', 'r_   6', 'r_   7',
       'r_   8', 'r_   9', 'r_  10',
       ...
       'r_9447', 'r_9448', 'r_9449', 'r_9450', 'r_9451', 'r_9452', 'r_9453',
       'r_9454', 'r_9455', 'r_9456'],
      dtype='object', length=9456)
```

In [ ]:
```
returns = df[returns_name]
returns.head()
```

Out[ ]:

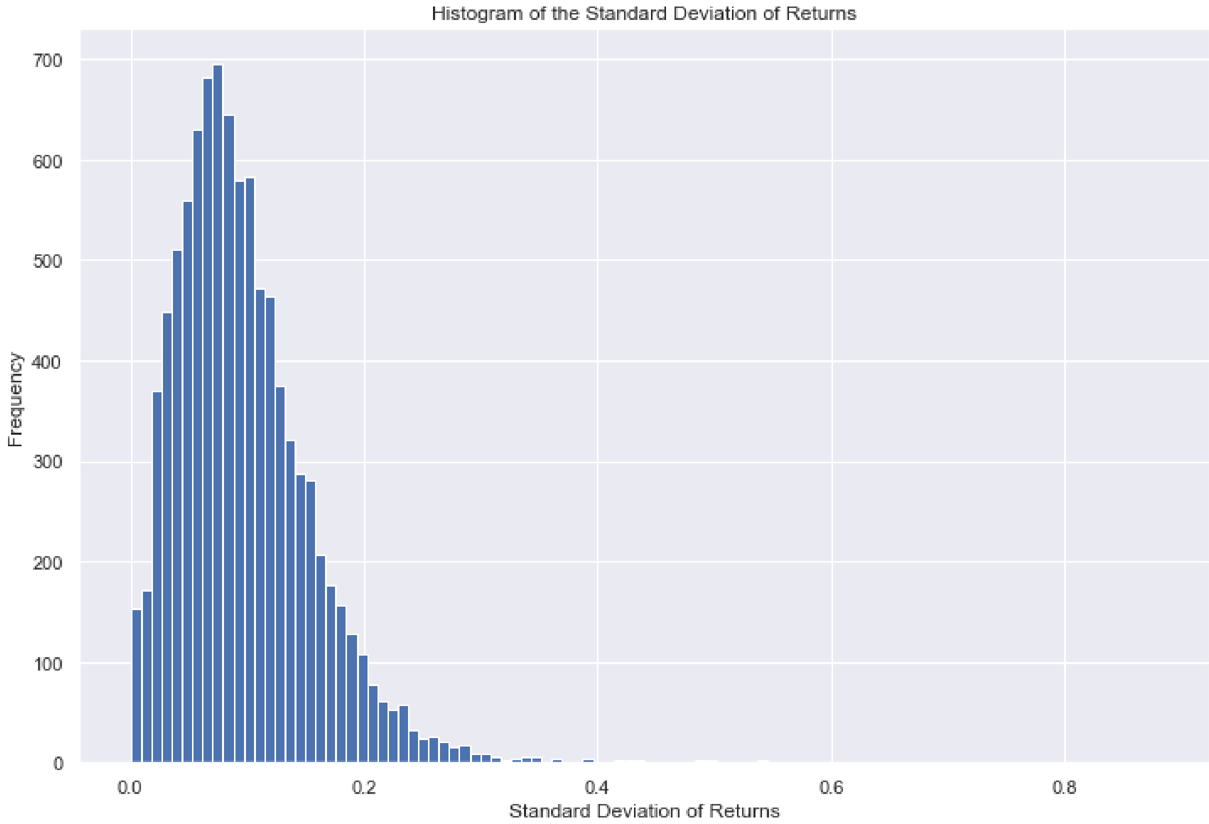
dates	r_ 1	r_ 2	r_ 3	r_ 4	r_ 5	r_ 6	r_ 7	r_ 8	r_ !
1991-11-29	0.130715	-0.053900	-0.110696	-0.043900	0.218322	0.050645	0.575047	0.008921	-0.10534!
1991-12-31	-0.010580	-0.056432	0.213591	0.183700	1.299230	0.306545	0.040644	-0.003800	0.31878
1992-01-31	-0.055124	-0.003400	-0.164114	-0.205154	-0.082347	0.075547	-0.024677	-0.066691	0.15513
1992-02-28	-0.202800	5.219422	0.039753	-0.057745	-0.117086	0.204517	-0.089757	-0.083881	0.01825
1992-03-31	0.078418	-0.181971	-0.023808	0.043112	0.415955	0.064782	-0.146257	-0.179871	-0.20958

5 rows × 9456 columns

In [ ]:
```
daterange = returns.index
daterange
```

Out[ ]:
```
DatetimeIndex(['1991-11-29', '1991-12-31', '1992-01-31', '1992-02-28',
               '1992-03-31', '1992-04-30', '1992-05-29', '1992-06-30',
               '1992-07-31', '1992-08-31',
               ...
               '2018-03-29', '2018-04-30', '2018-05-31', '2018-06-29',
               '2018-07-31', '2018-08-31', '2018-09-28', '2018-10-31',
               '2018-11-30', '2018-12-31'],
              dtype='datetime64[ns]', name='dates', length=326, freq=None)
```

In [ ]:
```
plt.hist(returns.std(axis=0), bins=100)
plt.xlabel('Standard Deviation of Returns')
plt.ylabel('Frequency')
plt.title('Histogram of the Standard Deviation of Returns');
```



In [ ]:

```
standardized_returns = (returns - returns.mean(axis=0))/returns.std(axis=0)
standardized_returns.head()
```

Out[ ]:

dates	r_ 1	r_ 2	r_ 3	r_ 4	r_ 5	r_ 6	r_ 7	r_ 8	r_
1991-11-29	1.483764	-0.219996	-0.914187	-0.665776	1.031984	0.207163	3.474317	-0.028135	-0.77523
1991-12-31	-0.223389	-0.226904	1.435410	2.066812	6.526489	1.869218	0.235135	-0.125593	2.07608
1992-01-31	-0.761584	-0.082197	-1.301224	-2.601812	-0.496389	0.368900	-0.160796	-0.607431	0.97594
1992-02-28	-2.545832	14.169328	0.175881	-0.832001	-0.672972	1.206552	-0.555266	-0.739131	0.05571
1992-03-31	0.851900	-0.569465	-0.284648	0.378895	2.036596	0.298978	-0.897734	-1.474551	-1.47598

5 rows × 9456 columns

In [ ]:

```
standardized_factors = (factors - factors.mean(axis=0))/factors.std(axis=0)
standardized_factors.head()
```

Out[ ]:

dates	MKT	HML	SMB	MOM1	MOM36	ACC	BETA	CFP	CHCSH
1991-11-29	-1.146021	-0.942950	0.110054	-0.298999	-0.981611	-0.855086	0.608329	-0.530965	-0.39099

dates	MKT	HML	SMB	MOM1	MOM36	ACC	BETA	CFP	CHCSH
1991-12-31	2.438375	-0.768570	-0.927363	0.228826	-0.830584	-1.573833	-1.458119	-1.015147	-0.35606
1992-01-31	-0.339177	1.540227	2.716591	1.381237	3.719257	0.951073	-1.288179	0.699949	-0.44761
1992-02-28	0.104274	2.152085	0.045976	-0.877648	2.691977	0.033673	-0.448956	0.963170	0.50052
1992-03-31	-0.764238	1.164027	-0.532641	-0.357583	0.775803	0.071737	0.710532	0.978913	0.55994

The model

$$
\begin{aligned}
Y_t &= \beta_0 + \beta_1 X_{1t} + \cdots + \beta_p X_{nt} + U_t, \quad t = 1, \ldots, T \\
&= \boldsymbol{\beta}' \boldsymbol{X}_t + U_t \\
\boldsymbol{Y} &= \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{U} \quad \text{(matrix notation)}.
\end{aligned}
$$

However, $n > T$ (more columns than rows in X).

The usual ordinary least squares (OLS) solution

$$
\widehat{\boldsymbol{\beta}} = \left(X'X\right)^{-1} X'Y
$$

is not valid anymore.

Solution: reduce the dimension of $\boldsymbol{X}$ by postulating that:

$$
\underset{(n\times1)}{\boldsymbol{X}_t} = \underset{(n\times k)}{\boldsymbol{\Lambda}} \underset{(k\times1)}{\boldsymbol{F}_t} + \underset{(n\times1)}{\boldsymbol{V}_t},
$$

where:

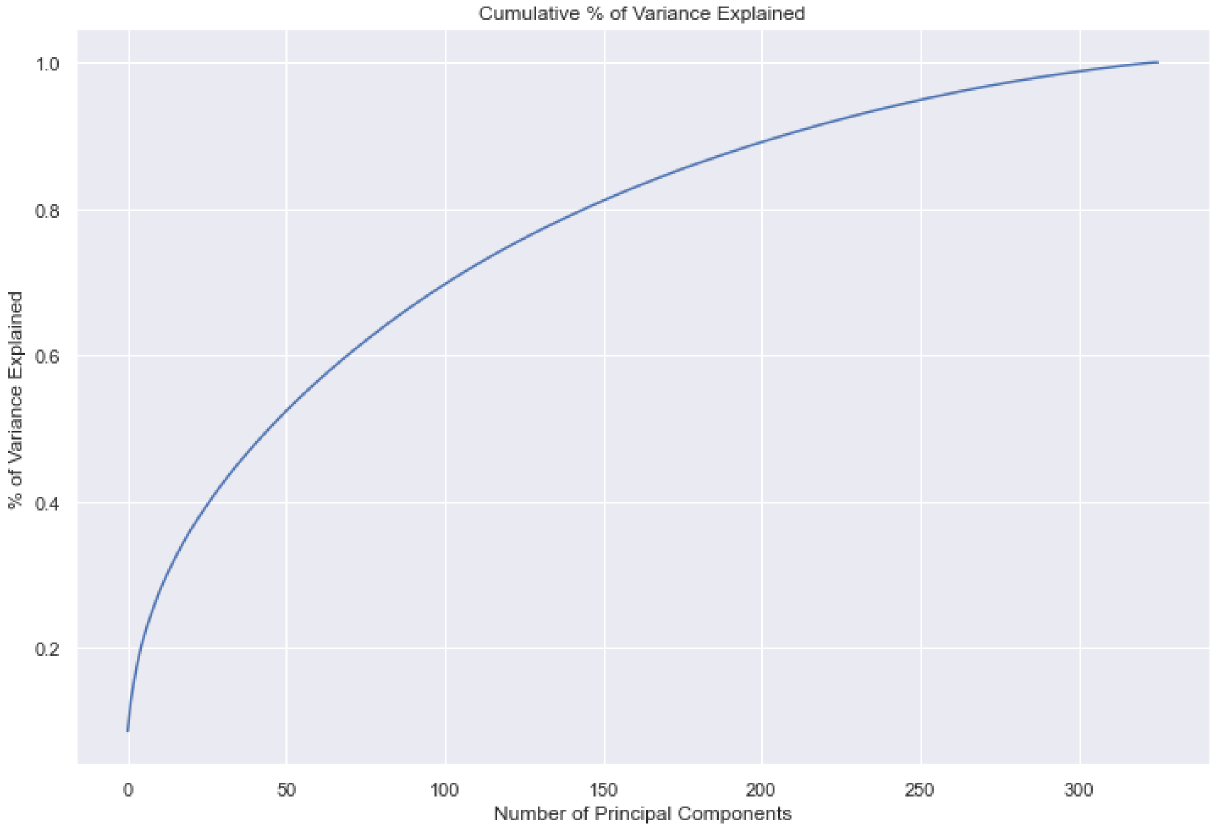
- F_t is a set of $k << n$ unobserved factors;
- $\boldsymbol{V}_t$ is the vector of idiosyncratic errors;
- $\boldsymbol{\Lambda}$ is the matrix of unobserved factor loadings.

(a) (30 points)

Compute the principal components of the returns and determine the optimal number of principal factors by one the methods described in Lecture 2. How much of the variance will the factors be able to explain?

In [ ]:

```
pcs, gammas, lambdas, alphas = PCA_function(returns)
```



The transformed dataset containing only the first k PCs is the $(T \times k)$ matrix is given by

$$\begin{aligned} Z_{(k)} &:= \boldsymbol{X}\boldsymbol{\Gamma}_k \\ &:= (Z_1, \ldots, Z_k) \,. \end{aligned}$$

In [ ]:

```
pcs
```

Out[ ]:

dates	PC 1	PC 2	PC 3	PC 4	PC 5	PC 6	PC 7	PC 8	PC
1991-11-29	-0.932311	0.655585	0.023695	-0.023417	-0.126589	0.216290	-0.287319	0.317324	0.36757
1991-12-31	0.684426	-0.766075	0.098573	0.101246	0.403129	-0.197598	0.258110	-0.600949	-0.53229
1992-01-31	2.223682	-0.458117	-0.226293	-0.012401	0.090334	-0.388412	0.722587	-1.958945	-1.08676
1992-02-28	0.562873	-0.140579	0.159400	0.006865	0.158465	-0.255173	0.436401	-0.784899	-0.00900
1992-03-31	-0.743650	0.306019	0.224420	-0.089483	-0.141921	0.050060	-0.242078	0.170550	0.23308
...	...	...	...	...	...	...	...	...	
2018-08-31	0.161871	-0.265360	-0.020674	-0.097998	0.026575	-0.056679	-0.109673	-0.051071	0.16519
2018-09-28	-0.942054	0.462936	0.124402	-0.082200	-0.051525	0.076335	-0.044931	0.278179	0.38150
2018-10-31	-2.535630	2.642636	-0.063295	0.056235	0.753975	0.545761	-0.142676	-0.170747	0.01582

	PC 1	PC 2	PC 3	PC 4	PC 5	PC 6	PC 7	PC 8	PC
dates									
2018-11-30	-0.418945	0.200803	0.180173	0.041064	0.087297	0.077100	-0.050033	0.015725	0.00710
2018-12-31	-3.014504	3.133093	-0.011031	0.038419	0.759613	0.698890	-0.215036	-0.120849	0.34036

326 rows × 326 columns

Given the desired number of PCs, say $1 \leq k \leq n$, we collect all the vectors $\gamma_1, \ldots \gamma_k$ in a $(n \times k)$ matrix

$$\Gamma_k := (\gamma_1, \ldots \gamma_k)$$

In [ ]:

```
gammas
```

Out[ ]:

	gamma 1	gamma 2	gamma 3	gamma 4	gamma 5	gamma 6	gamma 7	gamma 8	gamma
r_ 1	0.002032	-0.002777	-0.000804	0.000050	0.002960	0.001060	-0.002195	0.001189	0.0032
r_ 2	0.027143	0.015565	-0.009166	-0.002057	0.016365	0.000631	0.050304	-0.038719	0.0310
r_ 3	0.005195	-0.013071	-0.003946	-0.000183	-0.011596	-0.007778	0.006669	-0.008175	-0.0016
r_ 4	0.005707	-0.010162	-0.002122	-0.000947	0.001569	-0.003475	0.010309	-0.008786	-0.0131
r_ 5	0.012439	-0.006495	-0.005756	0.003450	0.002966	-0.002718	0.000583	0.010150	-0.0251
...	...	...	...	...	...	...	...	...	...
r_9452	0.000369	-0.000801	0.000187	0.000163	-0.000448	-0.000325	-0.000036	0.000097	-0.0000
r_9453	-0.000875	-0.000733	0.001381	-0.001983	-0.001478	-0.002800	-0.001009	0.006402	0.0069
r_9454	0.000287	-0.000691	0.000124	0.000281	-0.000370	-0.000841	0.000233	-0.000081	-0.0000
r_9455	0.002324	-0.005739	0.001035	-0.000288	-0.003415	-0.003190	0.000175	0.002162	0.0001
r_9456	0.001238	-0.003543	0.000601	-0.000469	-0.002063	-0.002022	-0.000150	0.002713	0.0014

9456 rows × 326 columns

In [ ]:

```
# PCA function centered the data before to compute the PCs
centered_returns = (returns - returns.mean(axis=0))
centered_factors = (factors - factors.mean(axis=0))
```

In [ ]:

```
# just checking
sum(round(centered_returns.dot(np.array(gammas["gamma 1"])), 10) == round(pcs["PC 1"
```

Out[ ]: 326

Also, by construction, the columns of $Z_{(k)}$ (the PCs) are orthogonal random variables, and with sample variances $\lambda_1, \ldots, \lambda_k$.

Thus, the sample covariance matrix of $Z_{(k)}$ is given by the $(k \times k)$ diagonal matrix

$$\mathbf{\Lambda}_k := \mathrm{diag}(\lambda_1, \ldots, \lambda_k)$$

In [ ]:
```
lambdas[:10]
```

Out[ ]:
```
array([10.59291481,  5.04991983,  3.33383555,  2.76500035,  2.40685564,
        1.94349613,  1.7572486 ,  1.61955915,  1.51600711,  1.45686289])
```

It is a good idea to start by running a full PCA $(k = n)$ and plotting the quantity

$$\alpha_j = \frac{\lambda_j}{\sum_{j=1}^{n} \lambda_j}$$

for $j \in \{1, \ldots, n\}$.

In [ ]:
```
alphas[:10]
```

Out[ ]:
```
array([0.0868341 , 0.04139609, 0.0273287 , 0.02266575, 0.0197299 ,
       0.01593157, 0.01440483, 0.01327614, 0.01242728, 0.01194245])
```

In [ ]:
```
Lambda = round(pcs.cov(), 2)
```

Also, by construction, the columns of $Z_{(k)}$ (the PCs) are orthogonal random variables, and with sample variances $\lambda_1, \ldots, \lambda_k$. Thus, the sample covariance matrix of $Z_{(k)}$ is given by the $(k \times k)$ diagonal matrix

$$\mathbf{\Lambda}_k := \mathrm{diag}(\lambda_1, \ldots, \lambda_k).$$

In [ ]:
```
Lambda.head(10)[['PC 1', 'PC 2', 'PC 3', 'PC 4', 'PC 5', 'PC 6', 'PC 7', 'PC 8', 'PC
```

Out[ ]:

	PC 1	PC 2	PC 3	PC 4	PC 5	PC 6	PC 7	PC 8	PC 9	PC 10
PC 1	10.59	0.00	0.00	0.00	0.00	0.00	-0.00	-0.00	-0.00	0.00
PC 2	0.00	5.05	-0.00	-0.00	0.00	-0.00	0.00	-0.00	-0.00	0.00
PC 3	0.00	-0.00	3.33	0.00	-0.00	0.00	0.00	-0.00	-0.00	0.00
PC 4	0.00	-0.00	0.00	2.77	0.00	0.00	-0.00	0.00	-0.00	0.00
PC 5	0.00	0.00	-0.00	0.00	2.41	-0.00	0.00	0.00	0.00	0.00
PC 6	0.00	-0.00	0.00	0.00	-0.00	1.94	0.00	0.00	-0.00	-0.00
PC 7	-0.00	0.00	0.00	-0.00	0.00	0.00	1.76	0.00	0.00	0.00
PC 8	-0.00	-0.00	-0.00	0.00	0.00	0.00	0.00	1.62	-0.00	-0.00
PC 9	-0.00	-0.00	-0.00	-0.00	0.00	-0.00	0.00	-0.00	1.52	-0.00
PC 10	0.00	0.00	0.00	0.00	0.00	-0.00	0.00	-0.00	-0.00	1.46

In [ ]:
```
lambdas[:10]
```

Out[ ]:
```
array([10.59291481,  5.04991983,  3.33383555,  2.76500035,  2.40685564,
        1.94349613,  1.7572486 ,  1.61955915,  1.51600711,  1.45686289])
```

Rule of Thumb

Stop at a k such that the $(k+1)$-th PC does not add much to the already explained variance (say $< 3\%$).

In [ ]:
```
n_pc_rt = rule_thumb(alphas)
```

```
The first 2 PCs explain 12.82% of the returns variance.
```

Informal Way

Choose the number of components such that a large portion (say 90\%) of the variance is explained.

In [ ]:
```
n_pc_iw = informal_way(alphas)
```

```
The first 207 PCs explain 89.91% of the returns variance.
```

Biggest Drop

Onatski (2010) suggests looking for the biggest drop computing

$$r := \arg\max_{1\leq j<n} \frac{\lambda_j}{\lambda_{j+1}}.$$

In [ ]:
```
n_pc_bd = biggest_drop(lambdas)
```

```
The first 1 PCs explain 8.68% of the returns variance.
```

(b) (30 points)

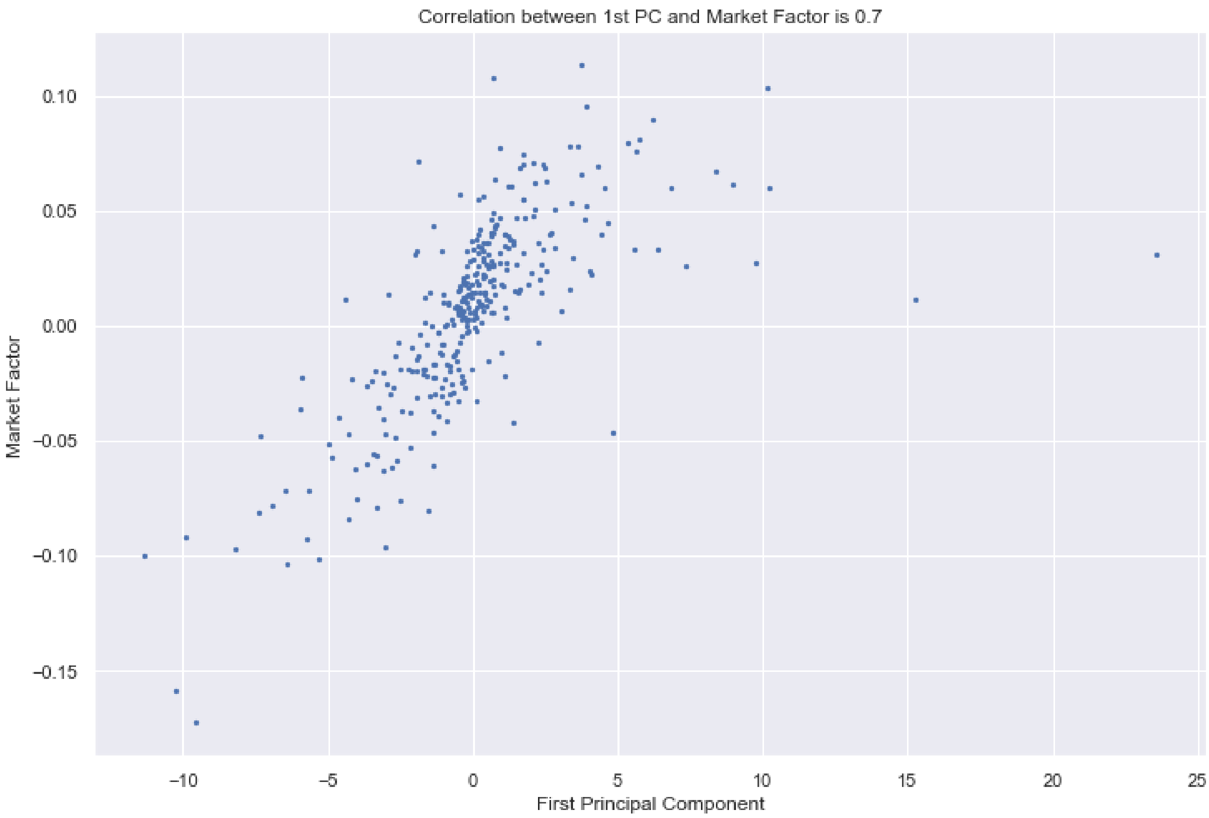
Regress the selected factors on the 16 observed "anomaly" factors described above. How do the "principal component factors" relate to the "anomaly factors"?

In [ ]:
```
# rule of thumb
pc_ret_rt = pcs.iloc[:,:n_pc_rt]
pc_ret_rt.head()
```

Out[ ]:

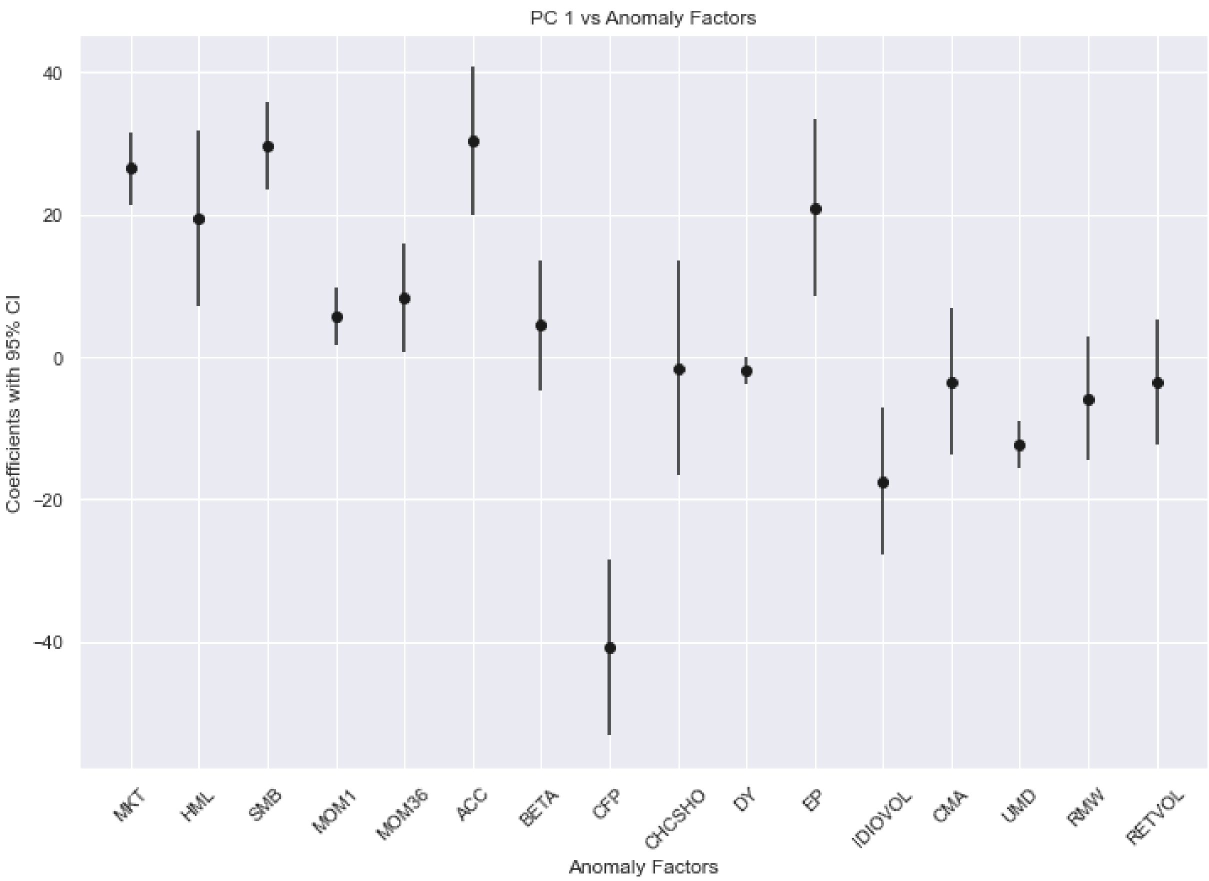
dates	PC 1	PC 2
1991-11-29	-0.932311	0.655585
1991-12-31	0.684426	-0.766075
1992-01-31	2.223682	-0.458117
1992-02-28	0.562873	-0.140579
1992-03-31	-0.743650	0.306019

In [ ]:
```
corr = round(factors['MKT'].corr(pc_ret_rt['PC 1']), 2)
plt.scatter(pc_ret_rt['PC 1'], factors['MKT'], s=5)
plt.xlabel('First Principal Component')
plt.ylabel('Market Factor')
plt.title(f'Correlation between 1st PC and Market Factor is {corr}');
```



In [ ]:

```
pc1 = OLS_regression(y = pc_ret_rt, X = factors, y_column = "PC 1", is_pc = False)
```



In [ ]:

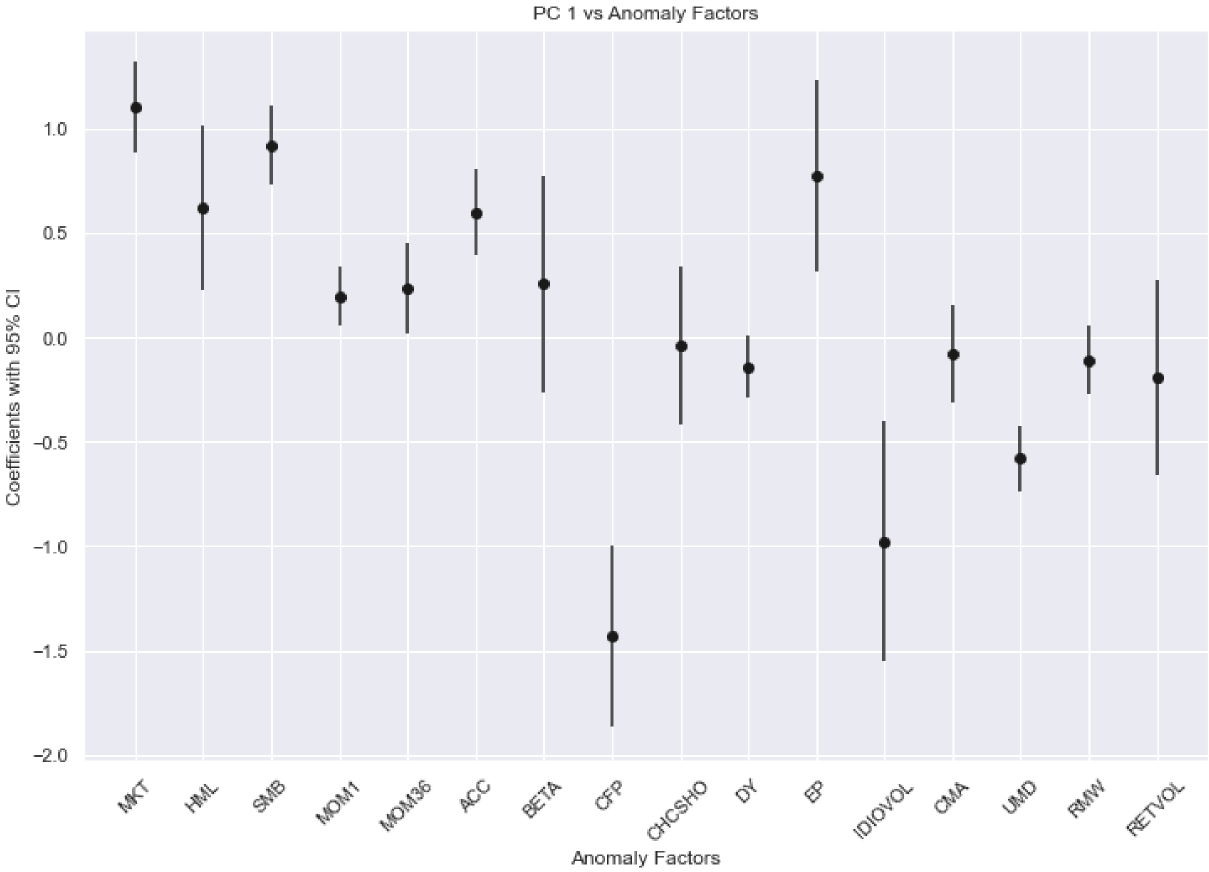
```
print(pc1.summary())
```

```
                            OLS Regression Results
==============================================================================
```

```
Dep. Variable:                   PC 1   R-squared:                       0.878
Model:                            OLS   Adj. R-squared:                  0.872
Method:                 Least Squares   F-statistic:                     139.3
Date:                Sat, 15 Jul 2023   Prob (F-statistic):          1.04e-130
Time:                        16:35:49   Log-Likelihood:                -503.55
No. Observations:                 326   AIC:                             1041.
Df Residuals:                     309   BIC:                             1105.
Df Model:                          16
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         -0.1940      0.074     -2.629      0.009      -0.339      -0.049
MKT           26.4644      2.614     10.123      0.000      21.321      31.608
HML           19.4347      6.242      3.114      0.002       7.153      31.717
SMB           29.7067      3.142      9.455      0.000      23.524      35.889
MOM1           5.6690      2.073      2.734      0.007       1.589       9.749
MOM36          8.2647      3.877      2.131      0.034       0.635      15.894
ACC           30.3074      5.272      5.748      0.000      19.933      40.682
BETA           4.4525      4.636      0.960      0.338      -4.670      13.575
CFP          -40.7554      6.249     -6.522      0.000     -53.051     -28.460
CHCSHO        -1.5690      7.657     -0.205      0.838     -16.636      13.498
DY            -1.9132      1.005     -1.903      0.058      -3.891       0.065
EP            20.9376      6.344      3.301      0.001       8.455      33.420
IDIOVOL      -17.4350      5.192     -3.358      0.001     -27.652      -7.218
CMA           -3.4515      5.189     -0.665      0.506     -13.662       6.759
UMD          -12.2663      1.692     -7.250      0.000     -15.595      -8.937
RMW           -5.8582      4.399     -1.332      0.184     -14.514       2.798
RETVOL        -3.6166      4.429     -0.817      0.415     -12.331       5.097
==============================================================================
Omnibus:                      186.517   Durbin-Watson:                   1.791
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             3153.860
Skew:                           1.974   Prob(JB):                         0.00
Kurtosis:                      17.717   Cond. No.                         147.
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly spe
cified.
```

In [ ]:

```
pc1std = OLS_regression(y = pc_ret_rt, X = standardized_factors, y_column = "PC 1",
```



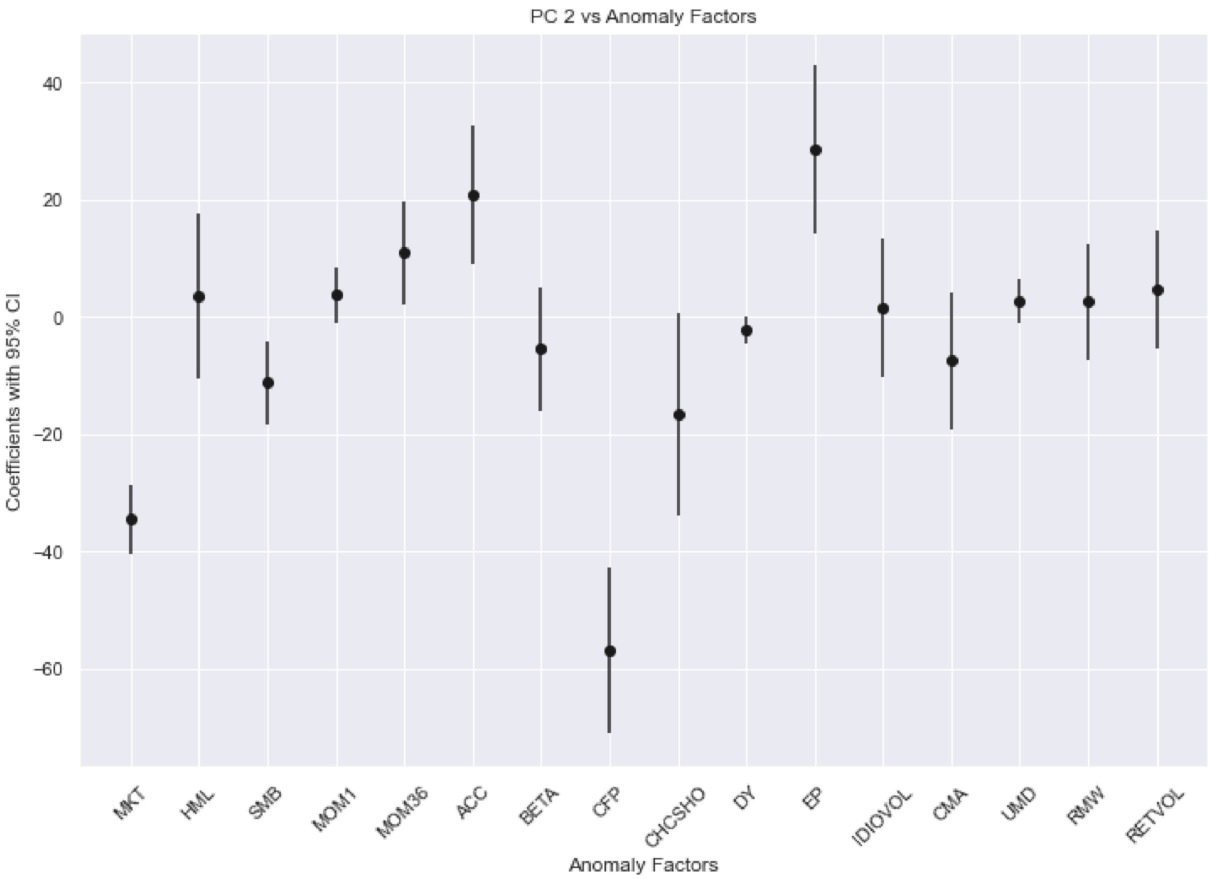
In [ ]:
```
print(pc1std.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                   PC 1   R-squared:                       0.878
Model:                            OLS   Adj. R-squared:                  0.872
Method:                 Least Squares   F-statistic:                     139.3
Date:                Sat, 15 Jul 2023   Prob (F-statistic):          1.04e-130
Time:                        16:35:50   Log-Likelihood:                -503.55
No. Observations:                 326   AIC:                             1041.
Df Residuals:                     309   BIC:                             1105.
Df Model:                          16
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const      -1.162e-16      0.065   -1.8e-15      1.000      -0.127       0.127
MKT            1.1019      0.109     10.123      0.000       0.888       1.316
HML            0.6190      0.199      3.114      0.002       0.228       1.010
SMB            0.9205      0.097      9.455      0.000       0.729       1.112
MOM1           0.1965      0.072      2.734      0.007       0.055       0.338
MOM36          0.2357      0.111      2.131      0.034       0.018       0.453
ACC            0.5986      0.104      5.748      0.000       0.394       0.803
BETA           0.2541      0.265      0.960      0.338      -0.266       0.775
CFP           -1.4315      0.219     -6.522      0.000      -1.863      -1.000
CHCSHO        -0.0394      0.192     -0.205      0.838      -0.418       0.339
DY            -0.1430      0.075     -1.903      0.058      -0.291       0.005
EP             0.7719      0.234      3.301      0.001       0.312       1.232
IDIOVOL       -0.9774      0.291     -3.358      0.001      -1.550      -0.405
CMA           -0.0798      0.120     -0.665      0.506      -0.316       0.156
UMD           -0.5809      0.080     -7.250      0.000      -0.739      -0.423
RMW           -0.1117      0.084     -1.332      0.184      -0.277       0.053
RETVOL        -0.1943      0.238     -0.817      0.415      -0.663       0.274
==============================================================================
Omnibus:                      186.517   Durbin-Watson:                   1.791
```

```
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             3153.860
Skew:                           1.974   Prob(JB):                         0.00
Kurtosis:                      17.717   Cond. No.                         14.8
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly spe
cified.
```

In [ ]:

```
pc2 = OLS_regression(y = pc_ret_rt, X = factors, y_column = "PC 2", is_pc = False)
```



In [ ]:

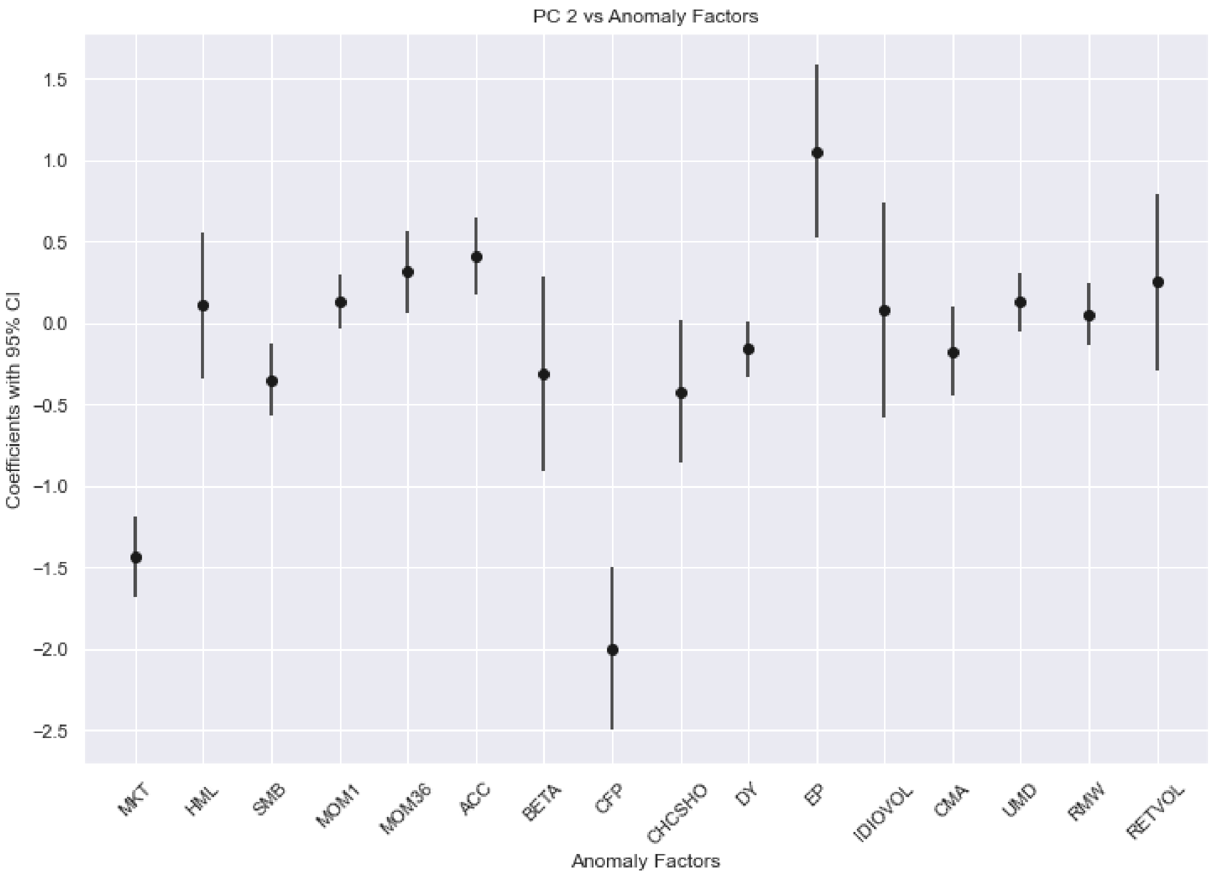
```
print(pc2.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                   PC 2   R-squared:                       0.662
Model:                            OLS   Adj. R-squared:                  0.645
Method:                 Least Squares   F-statistic:                     37.87
Date:                Sat, 15 Jul 2023   Prob (F-statistic):           4.19e-63
Time:                        16:35:50   Log-Likelihood:                -549.10
No. Observations:                 326   AIC:                             1132.
Df Residuals:                     309   BIC:                             1197.
Df Model:                          16
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.3155      0.085      3.718      0.000       0.149       0.482
MKT          -34.5536      3.006    -11.494      0.000     -40.469     -28.638
HML            3.3906      7.178      0.472      0.637     -10.733      17.514
SMB          -11.3074      3.613     -3.130      0.002     -18.417      -4.198
MOM1           3.7126      2.384      1.557      0.120      -0.979       8.404
MOM36         10.9439      4.459      2.454      0.015       2.170      19.718
ACC           20.8124      6.063      3.433      0.001       8.883      32.742
```

```
BETA          -5.5253      5.331     -1.036      0.301     -16.015       4.965
CFP          -56.8628      7.186     -7.913      0.000     -71.002     -42.724
CHCSHO       -16.7244      8.805     -1.899      0.058     -34.050       0.601
DY            -2.1662      1.156     -1.874      0.062      -4.440       0.108
EP            28.5162      7.295      3.909      0.000      14.162      42.870
IDIOVOL        1.4526      5.971      0.243      0.808     -10.296      13.201
CMA           -7.5439      5.967     -1.264      0.207     -19.286       4.198
UMD            2.6894      1.946      1.382      0.168      -1.139       6.518
RMW            2.6344      5.059      0.521      0.603      -7.320      12.589
RETVOL         4.6764      5.093      0.918      0.359      -5.344      14.697
==============================================================================
Omnibus:                      257.183   Durbin-Watson:                   1.685
Prob(Omnibus):                  0.000   Jarque-Bera (JB):            11439.574
Skew:                           2.753   Prob(JB):                         0.00
Kurtosis:                      31.493   Cond. No.                         147.
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly spe
cified.
```

In [ ]:

```python
pc2std = OLS_regression(y = pc_ret_rt, X = standardized_factors, y_column = "PC 2",
```



In [ ]:

```python
print(pc2std.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                   PC 2   R-squared:                       0.662
Model:                            OLS   Adj. R-squared:                  0.645
Method:                 Least Squares   F-statistic:                     37.87
Date:                Sat, 15 Jul 2023   Prob (F-statistic):           4.19e-63
Time:                        16:35:50   Log-Likelihood:                -549.10
No. Observations:                 326   AIC:                             1132.
Df Residuals:                     309   BIC:                             1197.
```

```
Df Model:                          16
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const       4.684e-17      0.074   6.31e-16      1.000      -0.146       0.146
MKT           -1.4387      0.125    -11.494      0.000      -1.685      -1.192
HML            0.1080      0.229      0.472      0.637      -0.342       0.558
SMB           -0.3504      0.112     -3.130      0.002      -0.571      -0.130
MOM1           0.1287      0.083      1.557      0.120      -0.034       0.291
MOM36          0.3122      0.127      2.454      0.015       0.062       0.562
ACC            0.4110      0.120      3.433      0.001       0.175       0.647
BETA          -0.3153      0.304     -1.036      0.301      -0.914       0.283
CFP           -1.9972      0.252     -7.913      0.000      -2.494      -1.501
CHCSHO        -0.4201      0.221     -1.899      0.058      -0.855       0.015
DY            -0.1619      0.086     -1.874      0.062      -0.332       0.008
EP             1.0513      0.269      3.909      0.000       0.522       1.581
IDIOVOL        0.0814      0.335      0.243      0.808      -0.577       0.740
CMA           -0.1744      0.138     -1.264      0.207      -0.446       0.097
UMD            0.1274      0.092      1.382      0.168      -0.054       0.309
RMW            0.0502      0.096      0.521      0.603      -0.140       0.240
RETVOL         0.2513      0.274      0.918      0.359      -0.287       0.790
==============================================================================
Omnibus:                      257.183   Durbin-Watson:                   1.685
Prob(Omnibus):                  0.000   Jarque-Bera (JB):            11439.574
Skew:                           2.753   Prob(JB):                         0.00
Kurtosis:                      31.493   Cond. No.                         14.8
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly spe
cified.
```

In [ ]:

```
corr_matrix = round(pd.concat([pc_ret_rt, factors], axis=1).corr(), 2)
corr_matrix[factors.columns][:2]
```

Out[ ]:

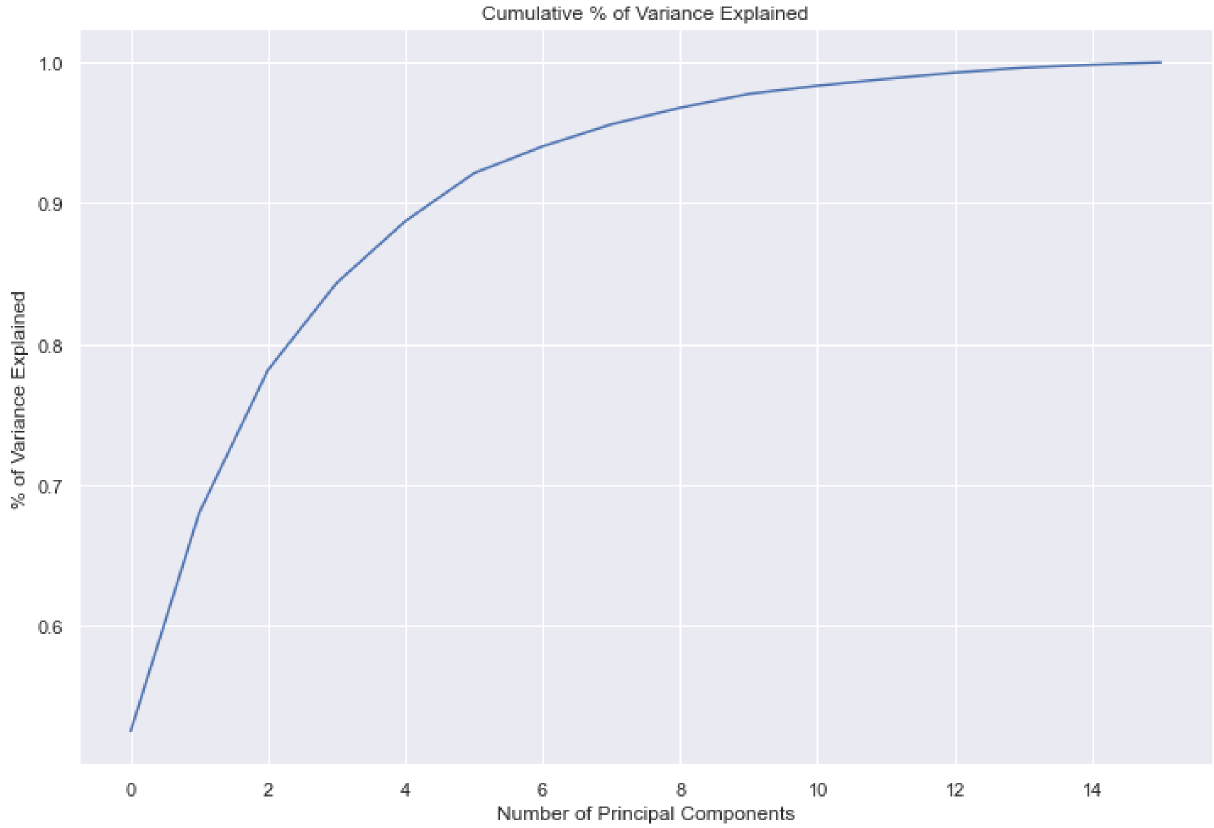
	MKT	HML	SMB	MOM1	MOM36	ACC	BETA	CFP	CHCSHO	DY	EP	IDIOVOL	CMA
PC 1	0.70	-0.38	0.70	0.31	0.16	0.40	-0.87	-0.54	-0.73	-0.39	-0.69	-0.85	-0.36
PC 2	-0.47	-0.46	-0.03	-0.11	-0.19	0.21	0.11	-0.47	-0.15	-0.12	-0.24	0.00	-0.24

(c) (30 points)

Now, run a principal component analysis on the 16 "anomaly factors" and select the optimal number of principal components using the same criterion adopted in the first item of the exercise. By inspecting the principal eigenvectors can you identify a dominating "anomaly"?

In [ ]:

```
pcs, gammas, lambdas, alphas = PCA_function(factors)
```



The transformed dataset containing only the first k PCs is the $(T \times k)$ matrix is given by

$$\begin{aligned} Z_{(k)} &:= \boldsymbol{X\Gamma}_k \\ &:= (Z_1, \ldots, Z_k)\,. \end{aligned}$$

In [ ]:

```
pcs
```

Out[ ]:

	PC 1	PC 2	PC 3	PC 4	PC 5	PC 6	PC 7	PC 8	PC
dates									
1991-11-29	-0.040773	-0.040907	-0.034789	-0.042608	0.021649	0.016281	-0.002945	0.013785	-0.01715
1991-12-31	0.112512	-0.012316	-0.038009	0.025986	-0.097625	-0.067016	0.003296	0.040766	-0.00733
1992-01-31	0.121870	0.061630	0.066577	0.094953	0.039049	0.094894	-0.019855	-0.018094	0.02758
1992-02-28	-0.005106	0.043395	0.047053	0.084682	0.035254	-0.009129	0.001453	-0.009655	0.01877
1992-03-31	-0.108007	0.012929	0.026053	0.021665	0.019578	0.005955	0.008637	-0.012087	-0.00738
...	...	...	...	...	...	...	...	...	...
2018-08-31	0.070953	-0.020014	-0.064739	-0.014552	-0.031384	0.001708	0.000216	0.006447	0.03309
2018-09-28	0.006513	-0.057196	0.004248	-0.013213	0.005573	-0.003490	0.028139	-0.015625	0.00580
2018-10-31	-0.167718	-0.033319	0.045695	-0.010240	0.027620	0.030816	0.020208	-0.008359	-0.03652

	PC 1	PC 2	PC 3	PC 4	PC 5	PC 6	PC 7	PC 8	PC
dates									
2018-11-30	-0.058530	0.037814	-0.010477	-0.049877	-0.016665	-0.006877	0.009479	0.020282	0.0027
2018-12-31	-0.101082	0.010640	-0.083628	-0.054505	0.049028	0.031435	0.002180	-0.014540	-0.0335

326 rows × 16 columns

Given the desired number of PCs, say $1 \leq k \leq n$, we collect all the vectors $\gamma_1, \ldots \gamma_k$ in a $(n \times k)$ matrix

$$\Gamma_k := (\gamma_1, \ldots \gamma_k)$$

In [ ]:

```
gammas
```

Out[ ]:

	gamma 1	gamma 2	gamma 3	gamma 4	gamma 5	gamma 6	gamma 7	gamma 8	gam
MKT	0.228449	0.191721	0.189615	0.091736	-0.553227	-0.518328	0.175670	0.206177	0.36
HML	-0.154749	0.062009	0.342674	0.383140	-0.006026	0.005076	-0.142897	-0.089115	-0.18
SMB	0.168694	0.058650	0.028389	0.137712	0.194181	0.452578	-0.387099	0.476201	0.35
MOM1	0.066700	0.065963	0.211633	-0.201040	-0.644130	0.663915	0.070393	-0.177102	-0.05
MOM36	0.003330	0.127738	0.096021	0.497916	0.209302	0.227409	0.441680	-0.126307	0.40
ACC	0.081187	0.003743	-0.132634	-0.012960	0.050161	0.014296	-0.020514	-0.583009	0.32
BETA	-0.447878	-0.218960	-0.081117	-0.134744	0.020983	0.067217	0.129617	0.021959	0.16
CFP	-0.204639	0.012831	0.344791	0.252459	-0.054861	-0.105223	-0.433638	-0.350681	0.13
CHCSHO	-0.182823	-0.029610	0.115116	0.107098	0.027947	0.004601	0.053857	0.082061	0.13
DY	-0.388018	0.862257	-0.301306	-0.070640	-0.025850	0.012490	-0.094100	0.008833	-0.00
EP	-0.256139	-0.056180	0.290519	0.065395	-0.105811	-0.090116	-0.339623	0.191388	-0.12
IDIOVOL	-0.447106	-0.141603	0.120198	-0.128713	-0.067057	0.024197	0.211033	0.162011	0.03
CMA	-0.105896	0.053198	0.110473	0.264680	-0.007434	0.081863	0.393200	0.296365	-0.20
UMD	-0.081444	-0.254533	-0.653139	0.526627	-0.407090	0.031903	-0.176837	0.052899	-0.06
RMW	0.011606	-0.055604	-0.054078	-0.243840	-0.045945	-0.031349	-0.182397	0.216336	0.46
RETVOL	-0.417622	-0.221104	-0.064418	-0.106175	-0.080566	-0.024605	0.052871	-0.070598	0.28

Also, by construction, the columns of $Z_{(k)}$ (the PCs) are orthogonal random variables, and with sample variances $\lambda_1, \ldots, \lambda_k$.

Thus, the sample covariance matrix of $Z_{(k)}$ is given by the $(k \times k)$ diagonal matrix

$$\mathbf{\Lambda}_k := \mathrm{diag}(\lambda_1, \ldots, \lambda_k)$$

In [ ]:

```
lambdas[:10]
```

Out[ ]:

```
array([0.0143643 , 0.00424359, 0.00277031, 0.00168956, 0.00119888,
       0.00093209, 0.00052007, 0.00042722, 0.00032125, 0.00027015])
```

It is a good idea to start by running a full PCA $(k = n)$ and plotting the quantity

$$\alpha_j = \frac{\lambda_j}{\sum_{j=1}^{n} \lambda_j}$$

for $j \in \{1, \ldots, n\}$.

In [ ]:

```
alphas[:10]
```

Out[ ]:

```
array([0.52522946, 0.15516658, 0.10129623, 0.06177849, 0.04383691,
       0.03408185, 0.01901649, 0.01562144, 0.01174643, 0.00987803])
```

In [ ]:

```
Lambda = round(pcs.cov(), 4)
```

Also, by construction, the columns of $Z_{(k)}$ (the PCs) are orthogonal random variables, and with sample variances $\lambda_1, \ldots, \lambda_k$. Thus, the sample covariance matrix of $Z_{(k)}$ is given by the $(k \times k)$ diagonal matrix

$$\mathbf{\Lambda}_k := \mathrm{diag}(\lambda_1, \ldots, \lambda_k).$$

In [ ]:

```
Lambda.head(10)[['PC 1', 'PC 2', 'PC 3', 'PC 4', 'PC 5', 'PC 6', 'PC 7', 'PC 8', 'PC
```

Out[ ]:

	PC 1	PC 2	PC 3	PC 4	PC 5	PC 6	PC 7	PC 8	PC 9	PC 10
PC 1	0.0144	0.0000	0.0000	0.0000	-0.0000	-0.0000	0.0000	0.0000	0.0000	0.0000
PC 2	0.0000	0.0042	0.0000	0.0000	-0.0000	0.0000	-0.0000	0.0000	-0.0000	-0.0000
PC 3	0.0000	0.0000	0.0028	-0.0000	0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000
PC 4	0.0000	0.0000	-0.0000	0.0017	0.0000	0.0000	-0.0000	-0.0000	0.0000	0.0000
PC 5	-0.0000	-0.0000	0.0000	0.0000	0.0012	-0.0000	0.0000	-0.0000	-0.0000	-0.0000
PC 6	-0.0000	0.0000	-0.0000	0.0000	-0.0000	0.0009	0.0000	0.0000	0.0000	0.0000
PC 7	0.0000	-0.0000	-0.0000	-0.0000	0.0000	0.0000	0.0005	0.0000	-0.0000	-0.0000
PC 8	0.0000	0.0000	-0.0000	-0.0000	-0.0000	0.0000	0.0000	0.0004	0.0000	0.0000
PC 9	0.0000	-0.0000	-0.0000	0.0000	-0.0000	0.0000	-0.0000	0.0000	0.0003	0.0000
PC 10	0.0000	-0.0000	-0.0000	0.0000	-0.0000	0.0000	-0.0000	0.0000	0.0000	0.0003

In [ ]:

```
lambdas[:10]
```

Out[ ]:

```
array([0.0143643 , 0.00424359, 0.00277031, 0.00168956, 0.00119888,
       0.00093209, 0.00052007, 0.00042722, 0.00032125, 0.00027015])
```

Rule of Thumb

Stop at a k such that the $(k+1)$-th PC does not add much to the already explained variance (say $< 3\%$).

In [ ]:
```
n_pc_rt = rule_thumb(alphas)
```

```
The first 6 PCs explain 92.14% of the returns variance.
```

Informal Way

Choose the number of components such that a large portion (say 90\%) of the variance is explained.

In [ ]:
```
n_pc_iw = informal_way(alphas)
```

```
The first 5 PCs explain 88.73% of the returns variance.
```

Biggest Drop

Onatski (2010) suggests looking for the biggest drop computing

$$r := \arg\max_{1\leq j<n} \frac{\lambda_j}{\lambda_{j+1}}.$$

In [ ]:
```
n_pc_bd = biggest_drop(lambdas)
```

```
The first 1 PCs explain 52.52% of the returns variance.
```

In [ ]:
```
# rule of thumb
pc_fac_rt = pcs.iloc[:,:n_pc_rt]
pc_fac_rt.head()
```

Out[ ]:

	PC 1	PC 2	PC 3	PC 4	PC 5	PC 6
dates						
1991-11-29	-0.040773	-0.040907	-0.034789	-0.042608	0.021649	0.016281
1991-12-31	0.112512	-0.012316	-0.038009	0.025986	-0.097625	-0.067016
1992-01-31	0.121870	0.061630	0.066577	0.094953	0.039049	0.094894
1992-02-28	-0.005106	0.043395	0.047053	0.084682	0.035254	-0.009129
1992-03-31	-0.108007	0.012929	0.026053	0.021665	0.019578	0.005955

By inspecting the principal eigenvectors can you identify a dominating anomaly?

In [ ]:
```
gammas
```

Out[ ]:

	gamma 1	gamma 2	gamma 3	gamma 4	gamma 5	gamma 6	gamma 7	gamma 8	gam
MKT	0.228449	0.191721	0.189615	0.091736	-0.553227	-0.518328	0.175670	0.206177	0.36
HML	-0.154749	0.062009	0.342674	0.383140	-0.006026	0.005076	-0.142897	-0.089115	-0.18
SMB	0.168694	0.058650	0.028389	0.137712	0.194181	0.452578	-0.387099	0.476201	0.35
MOM1	0.066700	0.065963	0.211633	-0.201040	-0.644130	0.663915	0.070393	-0.177102	-0.05
MOM36	0.003330	0.127738	0.096021	0.497916	0.209302	0.227409	0.441680	-0.126307	0.40

	gamma 1	gamma 2	gamma 3	gamma 4	gamma 5	gamma 6	gamma 7	gamma 8	gam
ACC	0.081187	0.003743	-0.132634	-0.012960	0.050161	0.014296	-0.020514	-0.583009	0.32
BETA	-0.447878	-0.218960	-0.081117	-0.134744	0.020983	0.067217	0.129617	0.021959	0.16
CFP	-0.204639	0.012831	0.344791	0.252459	-0.054861	-0.105223	-0.433638	-0.350681	0.13
CHCSHO	-0.182823	-0.029610	0.115116	0.107098	0.027947	0.004601	0.053857	0.082061	0.13
DY	-0.388018	0.862257	-0.301306	-0.070640	-0.025850	0.012490	-0.094100	0.008833	-0.00
EP	-0.256139	-0.056180	0.290519	0.065395	-0.105811	-0.090116	-0.339623	0.191388	-0.12
IDIOVOL	-0.447106	-0.141603	0.120198	-0.128713	-0.067057	0.024197	0.211033	0.162011	0.03
CMA	-0.105896	0.053198	0.110473	0.264680	-0.007434	0.081863	0.393200	0.296365	-0.20
UMD	-0.081444	-0.254533	-0.653139	0.526627	-0.407090	0.031903	-0.176837	0.052899	-0.06
RMW	0.011606	-0.055604	-0.054078	-0.243840	-0.045945	-0.031349	-0.182397	0.216336	0.46
RETVOL	-0.417622	-0.221104	-0.064418	-0.106175	-0.080566	-0.024605	0.052871	-0.070598	0.28

In [ ]:

```
sum(round(centered_factors.dot(np.array(gammas["gamma 1"])), 10) == round(pcs["PC 1"
```

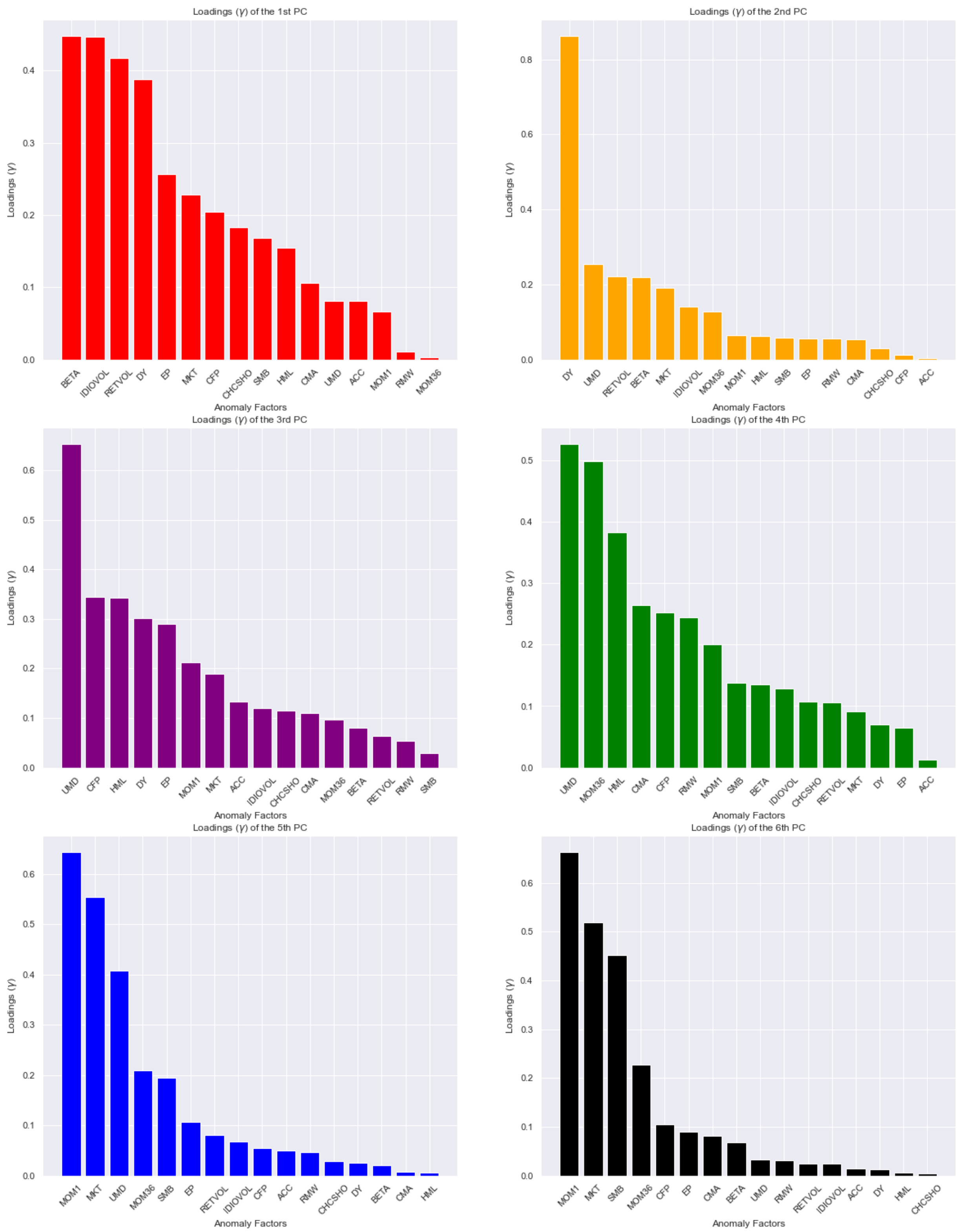
Out[ ]:

```
326
```

In [ ]:

```
fig, ax = plt.subplots(nrows=3, ncols=2, figsize=(20, 26))

aux1 = gammas["gamma 1"].abs().sort_values(ascending=False)
ax[0,0].bar(aux1.index, aux1.values, color='red')
ax[0,0].tick_params(axis='x', rotation=45)
ax[0,0].set_xlabel('Anomaly Factors')
ax[0,0].set_ylabel('Loadings ($\gamma$)')
ax[0,0].set_title(f'Loadings ($\gamma$) of the 1st PC');

aux2 = gammas["gamma 2"].abs().sort_values(ascending=False)
ax[0,1].bar(aux2.index, aux2.values, color='orange')
ax[0,1].tick_params(axis='x', rotation=45)
ax[0,1].set_xlabel('Anomaly Factors')
ax[0,1].set_ylabel('Loadings ($\gamma$)')
ax[0,1].set_title(f'Loadings ($\gamma$) of the 2nd PC');

aux3 = gammas["gamma 3"].abs().sort_values(ascending=False)
ax[1,0].bar(aux3.index, aux3.values, color='purple')
ax[1,0].tick_params(axis='x', rotation=45)
ax[1,0].set_xlabel('Anomaly Factors')
ax[1,0].set_ylabel('Loadings ($\gamma$)')
ax[1,0].set_title(f'Loadings ($\gamma$) of the 3rd PC');

aux4 = gammas["gamma 4"].abs().sort_values(ascending=False)
ax[1,1].bar(aux4.index, aux4.values, color='green')
ax[1,1].tick_params(axis='x', rotation=45)
ax[1,1].set_xlabel('Anomaly Factors')
ax[1,1].set_ylabel('Loadings ($\gamma$)')
ax[1,1].set_title(f'Loadings ($\gamma$) of the 4th PC');

aux5 = gammas["gamma 5"].abs().sort_values(ascending=False)
ax[2,0].bar(aux5.index, aux5.values, color='blue')
ax[2,0].tick_params(axis='x', rotation=45)
ax[2,0].set_xlabel('Anomaly Factors')
```

```
ax[2,0].set_ylabel('Loadings ($\gamma$)')
ax[2,0].set_title(f'Loadings ($\gamma$) of the 5th PC');

aux6 = gammas["gamma 6"].abs().sort_values(ascending=False)
ax[2,1].bar(aux6.index, aux6.values, color='black')
ax[2,1].tick_params(axis='x', rotation=45)
ax[2,1].set_xlabel('Anomaly Factors')
ax[2,1].set_ylabel('Loadings ($\gamma$)')
ax[2,1].set_title(f'Loadings ($\gamma$) of the 6th PC');
```



In [ ]:
```
gammas["gamma 1"]
```

Out[ ]:
```
MKT       0.228449
HML      -0.154749
```

```
SMB         0.168694
MOM1        0.066700
MOM36       0.003330
ACC         0.081187
BETA       -0.447878
CFP        -0.204639
CHCSHO     -0.182823
DY         -0.388018
EP         -0.256139
IDIOVOL    -0.447106
CMA        -0.105896
UMD        -0.081444
RMW         0.011606
RETVOL     -0.417622
Name: gamma 1, dtype: float64
```

In [ ]:

```
(gammas["gamma 1"].abs()).nlargest(2)
```

Out[ ]:

```
BETA        0.447878
IDIOVOL     0.447106
Name: gamma 1, dtype: float64
```

In [ ]:

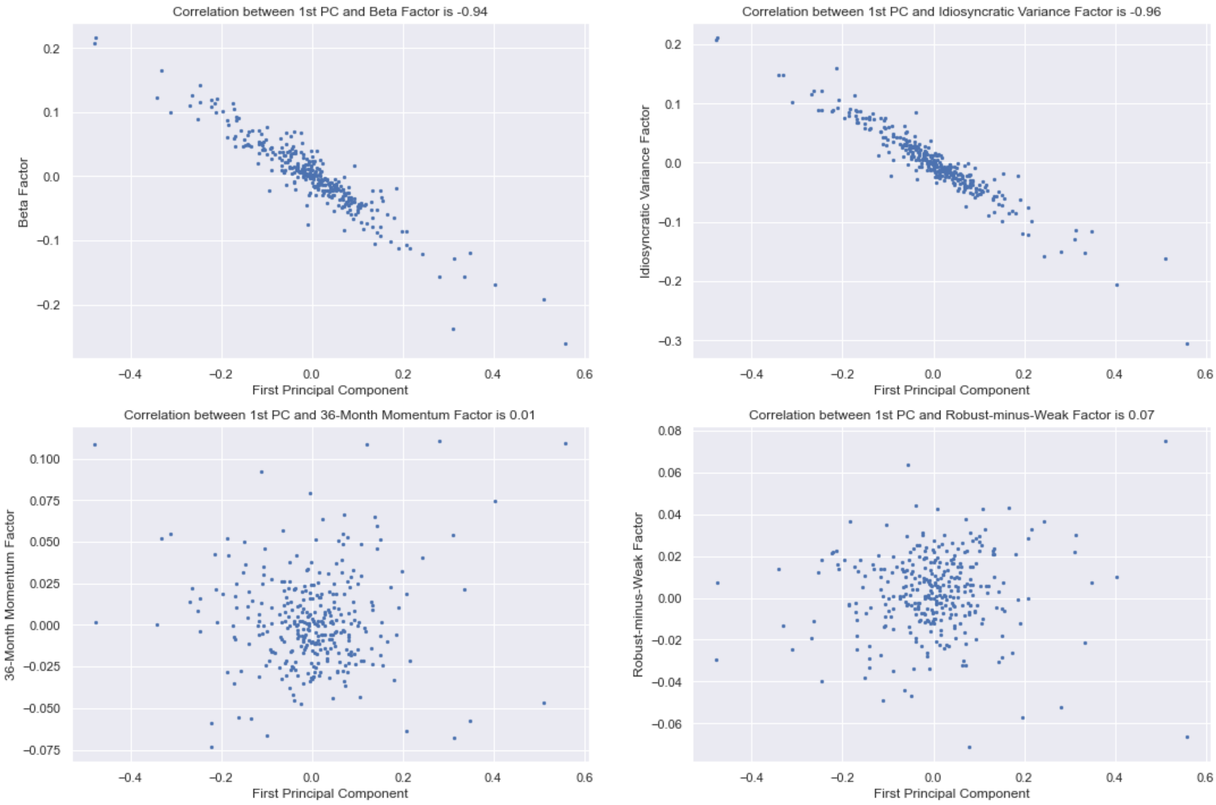
```
(gammas["gamma 1"].abs()).nsmallest(2)
```

Out[ ]:

```
MOM36     0.003330
RMW       0.011606
Name: gamma 1, dtype: float64
```

In [ ]:

```
fig, ax = plt.subplots(nrows=2, ncols=2, figsize=(18, 12))

corr_1 = round(factors['BETA'].corr(pc_fac_rt['PC 1']), 2)
ax[0,0].scatter(pc_fac_rt['PC 1'], factors['BETA'], s=5)
ax[0,0].set_xlabel('First Principal Component')
ax[0,0].set_ylabel('Beta Factor')
ax[0,0].set_title(f'Correlation between 1st PC and Beta Factor is {corr_1}');

corr_2 = round(factors['IDIOVOL'].corr(pc_fac_rt['PC 1']), 2)
ax[0,1].scatter(pc_fac_rt['PC 1'], factors['IDIOVOL'], s=5)
ax[0,1].set_xlabel('First Principal Component')
ax[0,1].set_ylabel('Idiosyncratic Variance Factor')
ax[0,1].set_title(f'Correlation between 1st PC and Idiosyncratic Variance Factor is

corr_3 = round(factors['MOM36'].corr(pc_fac_rt['PC 1']), 2)
ax[1,0].scatter(pc_fac_rt['PC 1'], factors['MOM36'], s=5)
ax[1,0].set_xlabel('First Principal Component')
ax[1,0].set_ylabel('36-Month Momentum Factor')
ax[1,0].set_title(f'Correlation between 1st PC and 36-Month Momentum Factor is {corr

corr_4 = round(factors['RMW'].corr(pc_fac_rt['PC 1']), 2)
ax[1,1].scatter(pc_fac_rt['PC 1'], factors['RMW'], s=5)
ax[1,1].set_xlabel('First Principal Component')
ax[1,1].set_ylabel('Robust-minus-Weak Factor')
ax[1,1].set_title(f'Correlation between 1st PC and Robust-minus-Weak Factor is {corr
```



(d) (30 points)

How do the "anomaly-based principal factors" related to the "return-based principal factors"?

In [ ]:

```
pc_ret_rt.head()
```

Out[ ]:

dates	PC 1	PC 2
1991-11-29	-0.932311	0.655585
1991-12-31	0.684426	-0.766075
1992-01-31	2.223682	-0.458117
1992-02-28	0.562873	-0.140579
1992-03-31	-0.743650	0.306019

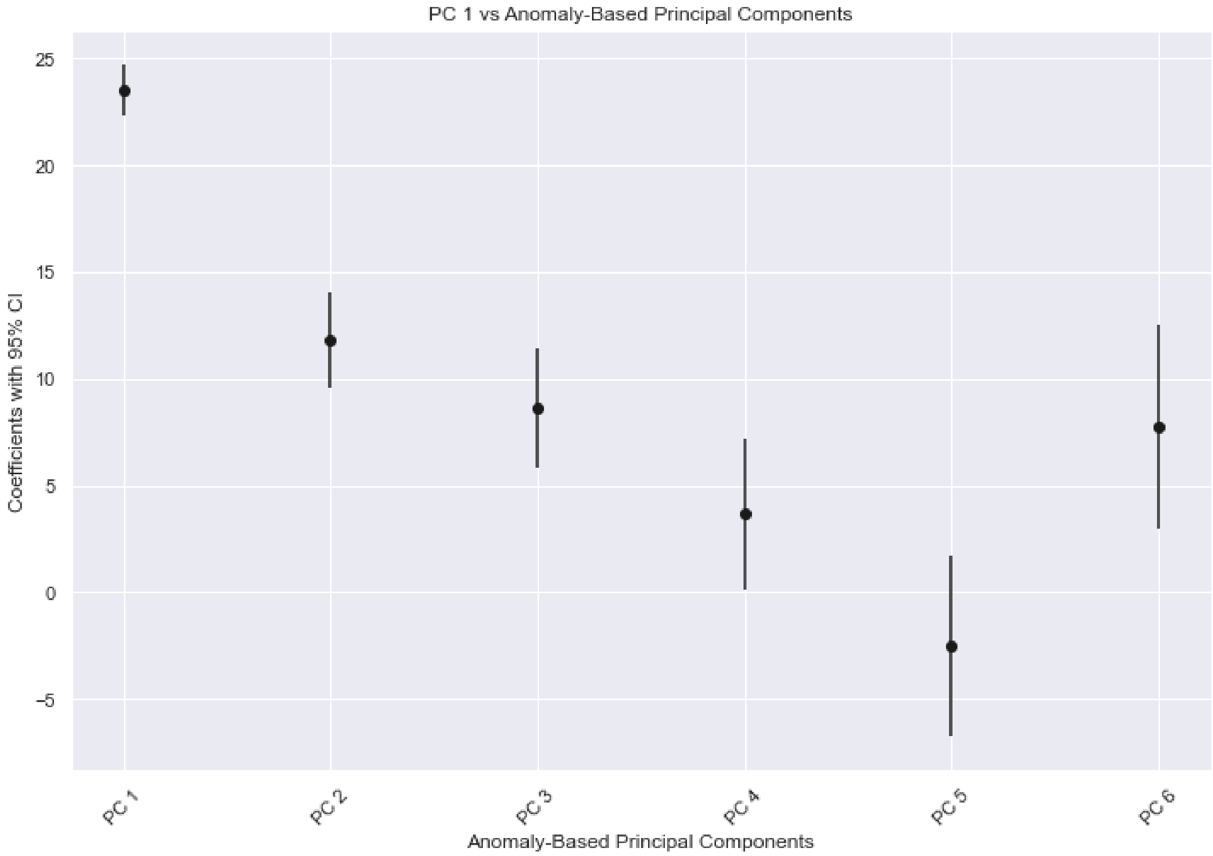
In [ ]:

```
pc_fac_rt.head()
```

Out[ ]:

dates	PC 1	PC 2	PC 3	PC 4	PC 5	PC 6
1991-11-29	-0.040773	-0.040907	-0.034789	-0.042608	0.021649	0.016281
1991-12-31	0.112512	-0.012316	-0.038009	0.025986	-0.097625	-0.067016
1992-01-31	0.121870	0.061630	0.066577	0.094953	0.039049	0.094894
1992-02-28	-0.005106	0.043395	0.047053	0.084682	0.035254	-0.009129
1992-03-31	-0.108007	0.012929	0.026053	0.021665	0.019578	0.005955

In [ ]:
```
pc1 = OLS_regression(y = pc_ret_rt, X = pc_fac_rt, y_column = "PC 1", is_pc = True)
```



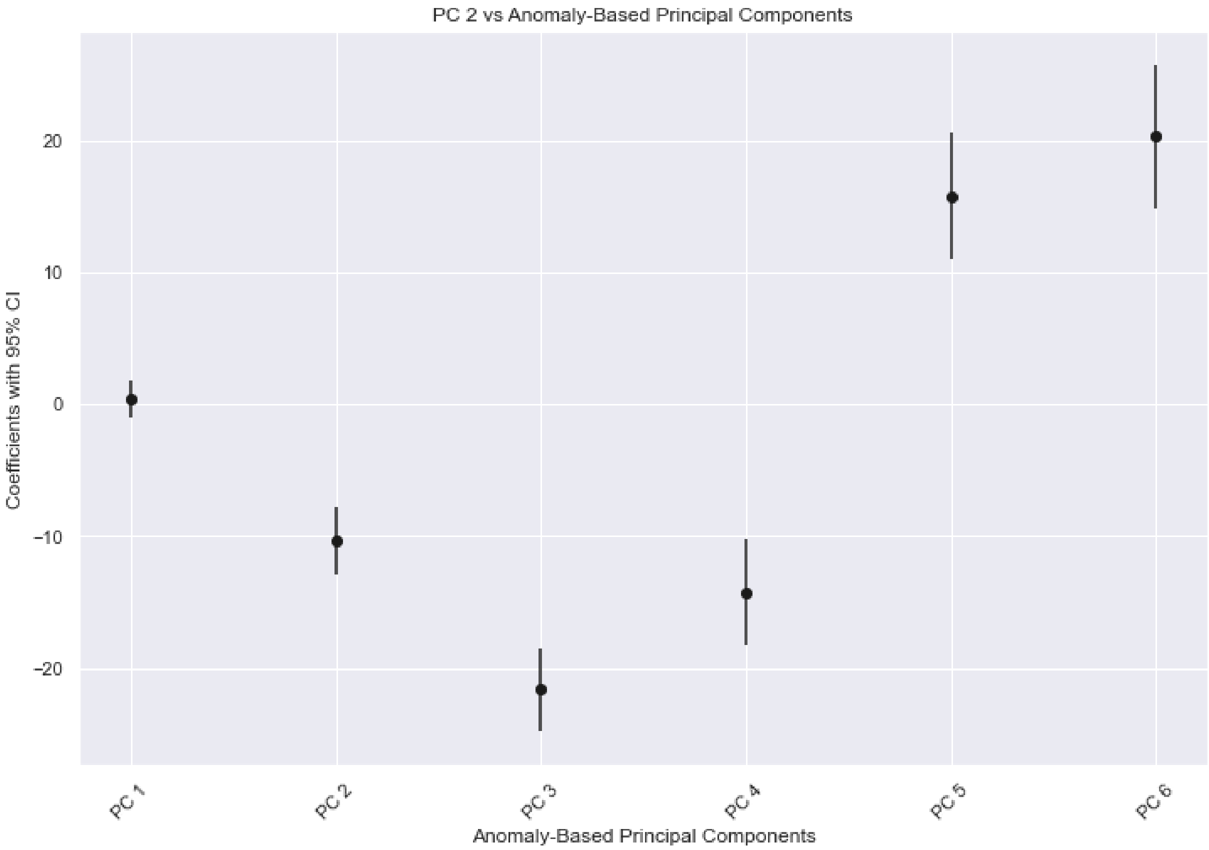
In [ ]:
```
print(pc1.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                   PC 1   R-squared:                       0.834
Model:                            OLS   Adj. R-squared:                  0.831
Method:                 Least Squares   F-statistic:                     268.0
Date:                Sat, 15 Jul 2023   Prob (F-statistic):          2.33e-121
Time:                        17:02:03   Log-Likelihood:                -553.61
No. Observations:                 326   AIC:                             1121.
Df Residuals:                     319   BIC:                             1148.
Df Model:                           6
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const      -1.162e-16      0.074  -1.57e-15      1.000      -0.146       0.146
PC 1          23.5303      0.619     38.038      0.000      22.313      24.747
PC 2          11.8198      1.138     10.385      0.000       9.581      14.059
PC 3           8.6444      1.409      6.137      0.000       5.873      11.416
PC 4           3.6781      1.804      2.039      0.042       0.129       7.227
PC 5          -2.5226      2.141     -1.178      0.240      -6.735       1.690
PC 6           7.7491      2.428      3.191      0.002       2.971      12.527
==============================================================================
Omnibus:                      171.684   Durbin-Watson:                   1.764
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             2422.522
Skew:                           1.820   Prob(JB):                         0.00
Kurtosis:                      15.849   Cond. No.                         32.8
==============================================================================

Notes:
```

```
[1] Standard Errors assume that the covariance matrix of the errors is correctly spe
cified.
```

In [ ]:

```
pc2 = OLS_regression(y = pc_ret_rt, X = pc_fac_rt, y_column = "PC 2", is_pc = True)
```



In [ ]:

```
print(pc2.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                   PC 2   R-squared:                       0.550
Model:                            OLS   Adj. R-squared:                  0.542
Method:                 Least Squares   F-statistic:                     65.07
Date:                Sat, 15 Jul 2023   Prob (F-statistic):           1.72e-52
Time:                        17:02:05   Log-Likelihood:                -595.75
No. Observations:                 326   AIC:                             1206.
Df Residuals:                     319   BIC:                             1232.
Df Model:                           6
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const       4.684e-17      0.084   5.56e-16      1.000      -0.166       0.166
PC 1           0.3608      0.704      0.513      0.609      -1.024       1.746
PC 2         -10.3232      1.295     -7.971      0.000     -12.871      -7.775
PC 3         -21.6396      1.603    -13.500      0.000     -24.793     -18.486
PC 4         -14.2854      2.053     -6.960      0.000     -18.324     -10.247
PC 5          15.7840      2.437      6.478      0.000      10.990      20.578
PC 6          20.3048      2.764      7.347      0.000      14.868      25.742
==============================================================================
Omnibus:                      233.976   Durbin-Watson:                   1.704
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             7600.490
Skew:                           2.480   Prob(JB):                         0.00
Kurtosis:                      26.129   Cond. No.                         32.8
==============================================================================
```

```
Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly spe
cified.
```

In [ ]:

```
pc_ret_rt.rename(columns={'PC 1': 'PC 1 (returns)', 'PC 2': 'PC 2 (returns)'}, inpla
pc_fac_rt.rename(columns={'PC 1': 'PC 1 (factors)', 'PC 2': 'PC 2 (factors)', 'PC 3'
```

In [ ]:

```
corr_matrix2 = round(pd.concat([pc_ret_rt, pc_fac_rt], axis=1).corr(), 2)
corr_matrix2[pc_fac_rt.columns][:2]
```

Out[ ]:

	PC 1 (factors)	PC 2 (factors)	PC 3 (factors)	PC 4 (factors)	PC 5 (factors)	PC 6 (factors)
PC 1 (returns)	0.87	0.24	0.14	0.05	-0.03	0.07
PC 2 (returns)	0.02	-0.30	-0.51	-0.26	0.24	0.28

Trabalho - Econometria IV

Guilherme Luz, Guilherme Masuko, Caio Garzeri

August 2023

```r
library(lubridate)  # for handling dates
library(zoo)  # for time series
library(dynlm)  # for time series regressions
library(forecast)  # for the improved Pacf function
library(glmnet)  # for shrinkage methods
library(HDeconometrics)  # IC for glmnet

# Packages for parallel computation
library(future)
library(foreach)
library(doFuture)
library(doRNG)
```

Question 2

First of all, we must do some data wrangling.

```r
# Import the data
raw_data = read_csv("data/2021-12.csv")
# raw_data = read_csv('C:/Users/Caio Garzeri/OneDrive -
# puc-rio.br/Econometria
# IV/AssignmentEconometricsIV/data/2021-12.csv')

data0 = raw_data[-1, ] %>%
    select_if(~!any(is.na(.)))
transformation = raw_data[1, ]
```

The suggested transformations (in order to make the series stationary) are indicated according to the following numeration.

Transformation codes (from FRED):

1. no transformation
2. Δx_t
3. $\Delta^2 x_t$
4. $\log(x_t)$
5. $\Delta \log(x_t)$
6. $\Delta^2 \log(x_t)$
7. $\Delta(x_t/x_{t-1} - 1)$

For the CPI, we apply a specific transformation to turn it into an inflation series.

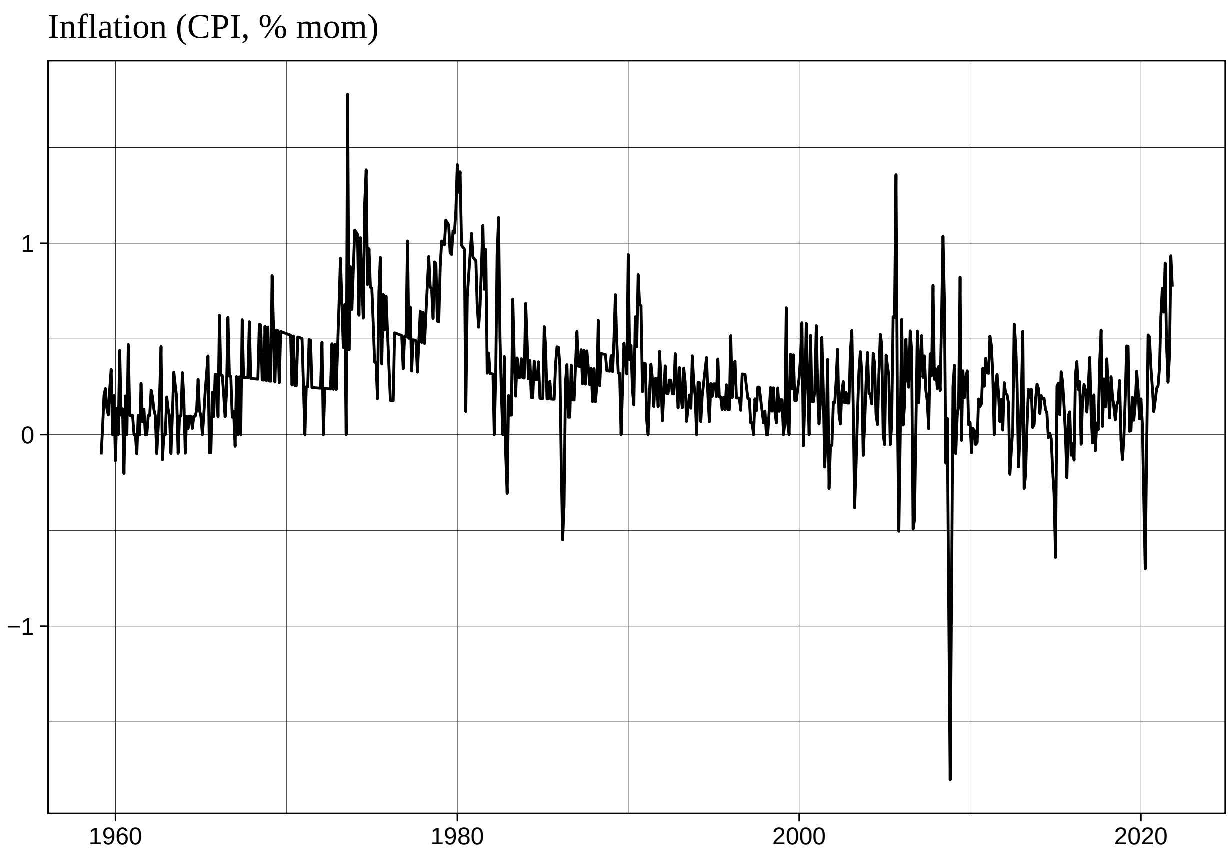
```r
# Data transformations based on the FRED transformation
# codes
```

```r
data = data0 %>%
    select(-sasdate) %>%
    rename(SP500 = "S&P 500", SPINDUST = "S&P: indust") %>%
    BVAR::fred_transform(type = "fred_md") %>%
    bind_cols(tibble(date = data0$sasdate[3:length(data0$sasdate)])) %>%
    mutate(date = as.Date(date, format = "%m/%d/%Y"))

# For the CPI, we transform into an inflation series
data = mutate(data, CPIAUCSL = 100 * (diff(data0$CPIAUCSL, differences = 1)/data0$CPIAUCSL[-1])[-1])
```

```r
# Inflation as time series
inflation = data$CPIAUCSL %>%
    ts(start = c(year(data$date[1]), month(data$date[1])), frequency = 12)
```

The resulting inflation series, which we want to forecast is shown below.

```r
# plot inflation

data %>%
    select(date, CPIAUCSL) %>%
    mutate(date = as.Date(date, format = "%m/%d/%Y")) %>%
    ggplot(aes(x = date, y = CPIAUCSL)) + geom_line() + labs(title = "Inflation (CPI, % mom)",
    x = NULL, y = NULL)
```

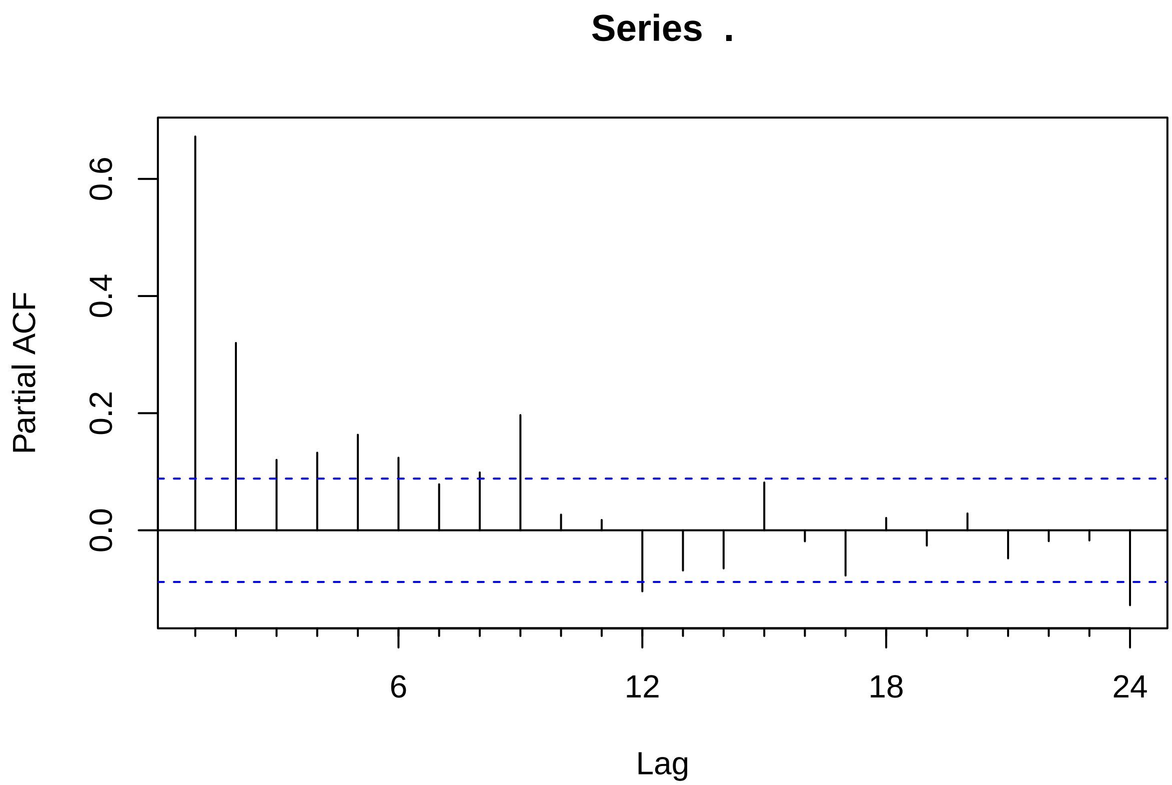


Now, we shall start the estimations.

AR

In order to get some idea of what the order of our AR(p) process is, we plot the partial autocorrelation of the inflation series for a particular window.

```
# Partial autocorrelation
inflation %>%
    window(start = start(inflation), end = start(inflation) +
        c(0, 492)) %>%
    Pacf(lag.max = 24, plot = T)
```



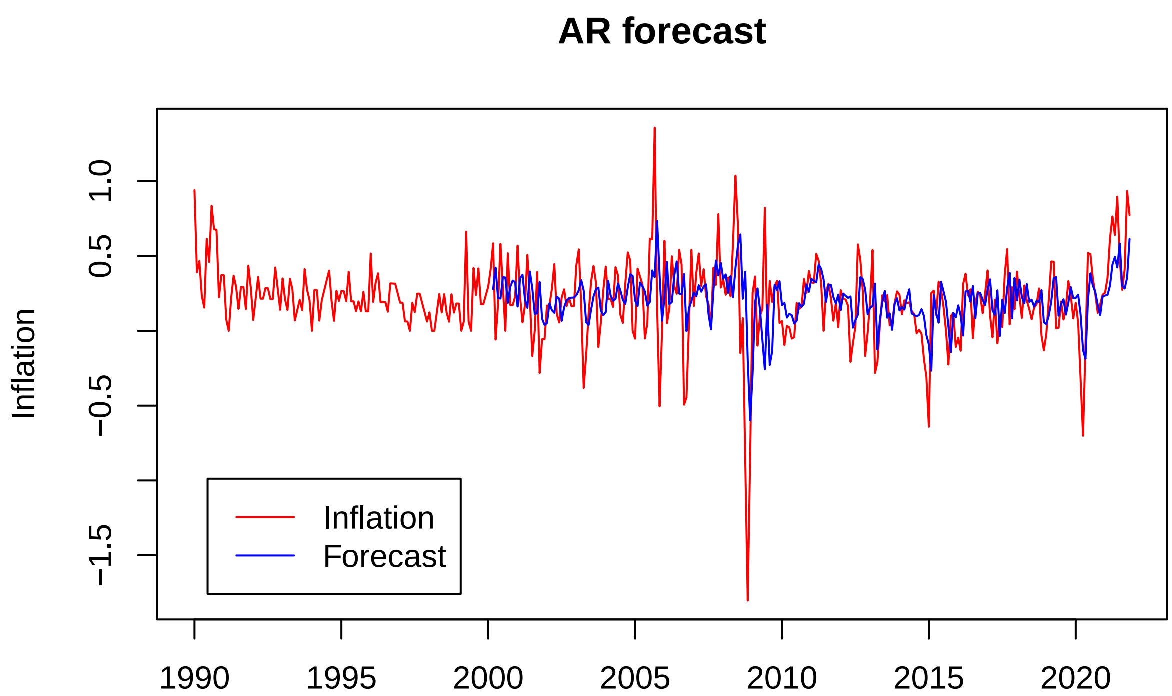
We believe that a maximum lag of 24 is more than reasonable. Then, the determine the actual order p based on the BIC.

```
# Function for calculating the BIC for AR models
BIC.ar <- function(model) {

    ssr <- sum(model$resid^2, na.rm = T)
    t <- sum(!is.na(model$resid))
    npar <- length(model$ar) + 1

    return(c(p = model$order, BIC = log(ssr/t) + npar * log(t)/t))
}
```

We proceed with a rolling window one-step-ahead forecast, in which we choose the optimal order of the AR in each window of estimation.

```
# Rolling window forecasting
rolling_window <- 492
p.max <- 24

forecast1 = list()
```

```r
popt_AR = data.frame(popt = numeric(261))

for (a in 0:(length(inflation) - rolling_window - 1)) {

    # get the window for training the model
    train = window(inflation, start = start(inflation) + c(0,
        a), end = start(inflation) + c(0, a + rolling_window -
        1))

    bic.table = c()

    for (p in 0:p.max) {
        # calculating the BIC for different orders of the
        # AR(p)
        AR = ar(train, order.max = p, method = "ols", aic = F)
        bic.line = BIC.ar(AR)
        bic.table = rbind(bic.table, bic.line)
    }
    bic.table = data.frame(bic.table)

    p.opt = bic.table$p[which.min(bic.table$BIC)]  # pick the optimal p

    popt_AR$popt[a + 1] <- p.opt

    AR = ar(train, order.max = p.opt, method = "ols", aic = F)  # run the AR model with the optimal p

    forecast1[[a + 1]] = predict(AR, n.ahead = 1)$pred  # one-step-ahead forecast
}

forecasts = forecast1 %>%
    unlist() %>%
    ts(start = start(forecast1[[1]]), frequency = frequency(forecast1[[1]]))
```



AR + PC

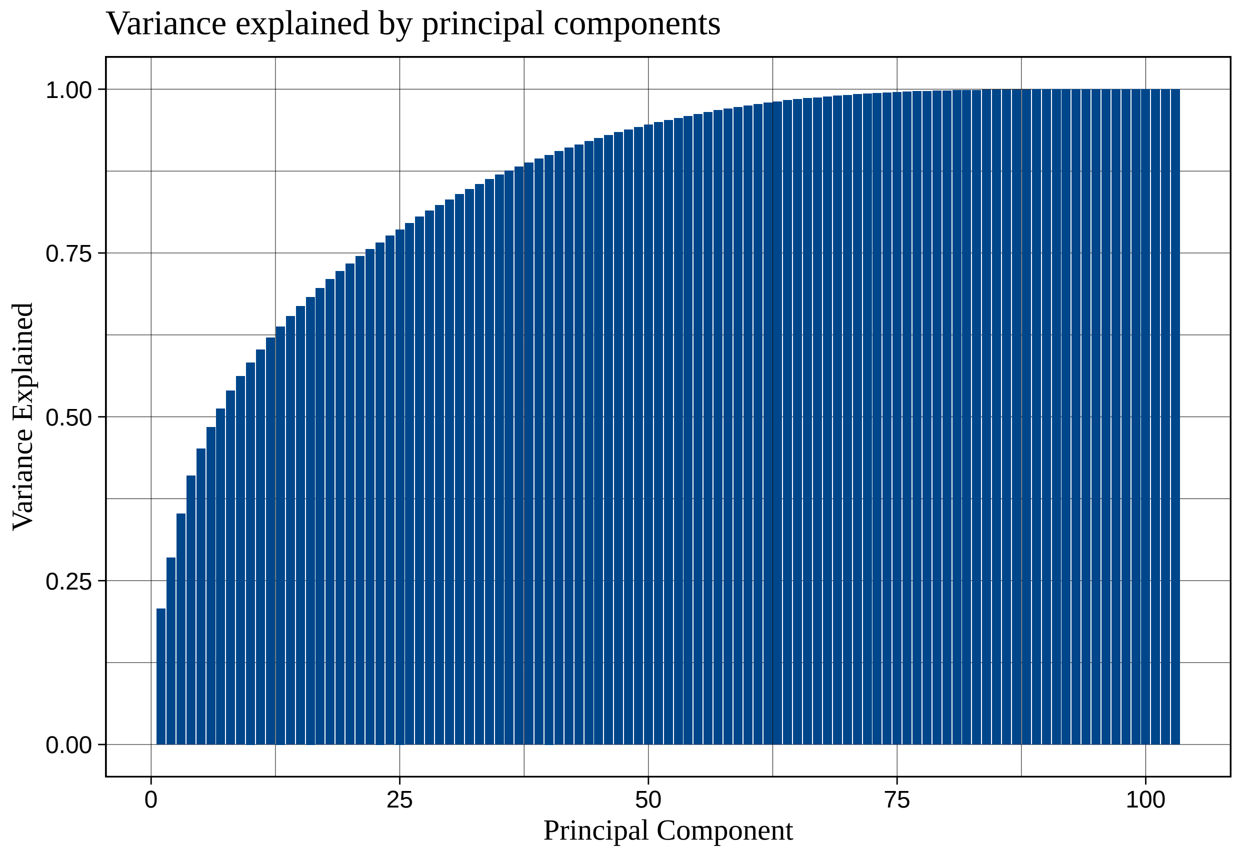
1. PCA

We do a Principal Component Analysis (PCA). Note that we must center and scale the data, since the series are in different scales.

```
# PCA
pca = data %>%
    select(-CPIAUCSL, -date) %>%
    prcomp(center = TRUE, scale = TRUE)
```

2. Select PCs

We can, then, choose the number of factors k and select the first k PCs. As seen in Question 1, there are different ways to choose the number of factors. We look at 3 common criterion (rule of thumb, informal way and biggest drop), but we opt for the rule of thumb as it seems to be the most parsimonious in this case.




```
# Choosing the number of PCs

# Rule of thumb (3%)

pca.var.prop %>%
    filter(var.prop >= 0.03) %>%
    nrow() %>%
    paste("(rule of thumb)")
```

```
## [1] "6 (rule of thumb)"
```

```
# Informal way (90%)

pca.var.prop %>%
    filter(var.prop.cum <= 0.9) %>%
    nrow() %>%
    paste("(informal way)")
```

```
## [1] "40 (informal way)"
```

```
# Biggest drop

(lag(pca.var.prop$var.prop)/pca.var.prop$var.prop) %>%
    which.max() %>%
    -1 %>%
    paste("(biggest drop)")
```

```
## [1] "102 (biggest drop)"
```

```r
# Using the rule of thumb
n_pc = pca.var.prop %>%
    filter(var.prop >= 0.03) %>%
    nrow()
```

3. Regression

Given the number of factors, the order of the autoregressive component is determined by BIC in each rolling window.

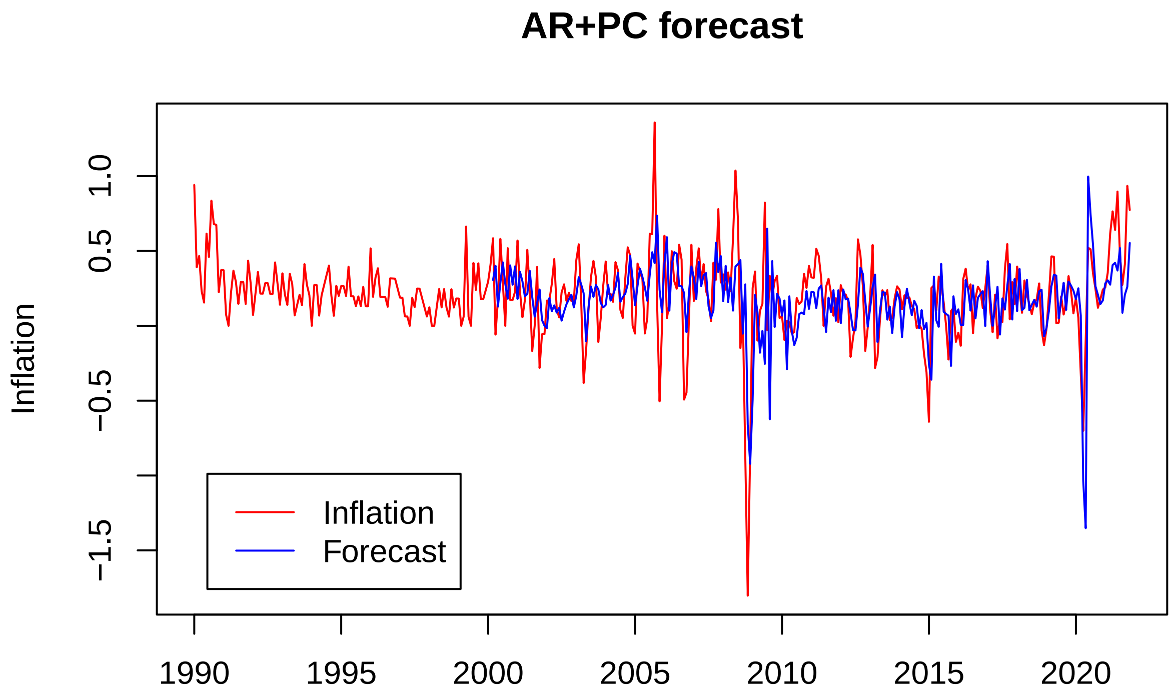
```r
# Get the factor from the PCA
Factors = pca$x[, 1:n_pc]

# Create the data matrix with the factors
variables = cbind(inflation, Factors)

variables_withdate = variables %>%
    bind_cols(date = as.Date.yearmon(time(inflation))) %>%
    setNames(c("inflation", colnames(Factors), "date"))
```

We proceed with the rolling window one-step-ahead forecast.

```r
# Function for creating the proper data matrix based on the
# regression formula

# Instead of manually creating data matrix, we use the
# dynlm() function and get only the $model component
create_datamatrix = function(train, p.opt) {
    new = dynlm(inflation ~ L(inflation, 1:p.opt) + L(train[,
        -1], 1), data = ts(rbind(train, 0), start = start(train),
        frequency = frequency(train)))
    new = new$model %>%
        tail(1) %>%
        select(-inflation) %>%
        as.matrix()
    return(new)
}
```

```r
# Rolling window forecasting
rolling_window <- 492
p.max <- 24

forecast1 = list()
coefficients_pc1 <- list()

# set up parallel computation
registerDoFuture()
plan("multisession", workers = 3)  # use 3 cores

# Loop
forecast1 = foreach(a = 0:(length(inflation) - rolling_window -
    1)) %dorng% {

    train = window(variables, start = start(inflation) + c(0,
        a), end = start(inflation) + c(0, a + rolling_window -
        1))
```

```
    bic.table = rep(NA, p.max)

    for (p in 1:p.max) {
        # calculating the BIC for different orders of the
        # AR(p)
        AR_PC = dynlm(inflation ~ L(inflation, 1:p) + L(train[,
            -1], 1), data = train)
        bic.table[p] = BIC(AR_PC)
    }

    p.opt = which.min(bic.table)  # pick the optimal p

    AR_PC = dynlm(inflation ~ L(inflation, 1:p.opt) + L(train[,
        -1], 1), data = train)  # run the AR-PC model with the optimal p

    new = create_datamatrix(train, p.opt)

    result = AR_PC$coefficients %*% c(1, new)  # one-step-ahead forecast
    result
}

forecast1 = forecast1 %>%
    unlist() %>%
    ts(start = start(inflation) + c(0, rolling_window), frequency = frequency(inflation))


# Loop to get coefficients
coefficients_pc1 = foreach(a = 0:(length(inflation) - rolling_window -
    1)) %dorng% {

    train = window(variables, start = start(inflation) + c(0,
        a), end = start(inflation) + c(0, a + rolling_window -
        1))

    bic.table = rep(NA, p.max)

    for (p in 1:p.max) {
        # calculating the BIC for different orders of the
        # AR(p)
        AR_PC = dynlm(inflation ~ L(inflation, 1:p) + L(train[,
            -1], 1), data = train)
        bic.table[p] = BIC(AR_PC)
    }

    p.opt = which.min(bic.table)  # pick the optimal p

    AR_PC = dynlm(inflation ~ L(inflation, 1:p.opt) + L(train[,
        -1], 1), data = train)  # run the AR-PC model with the optimal p

    result = AR_PC[[1]]  # one-step-ahead forecast
    result
}
```

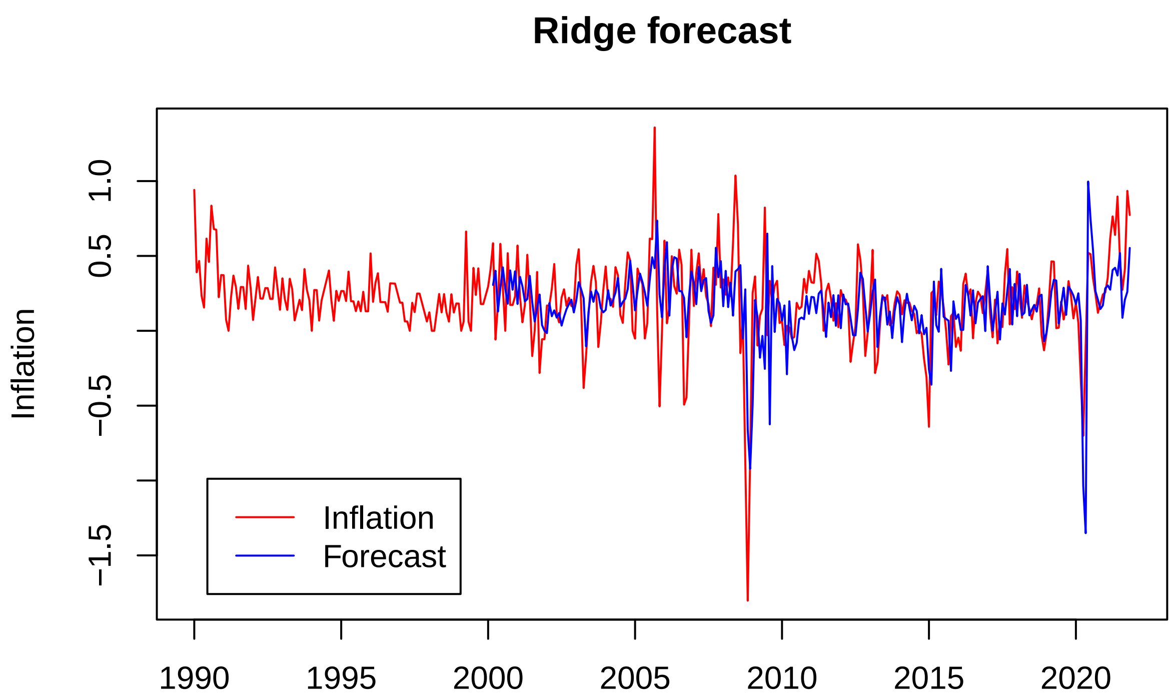



```
# Save forecasts
forecasts = cbind(AR = forecasts, AR_PC = forecast1) %>%
    as.ts()
```

Ridge Regression

We will choose penalty term according to the BIC. However, we must decide on the number of lags in the model and this criterion is obviously silent about this issue. Our strategy will be to run the models with 1, 2, 3 and 4 lags and choose the model with the smallest MSE.

```
# Embedding function that creates n_lags of all variables
# of a given data frame
my_embed = function(df, n_lags = 4) {
    Lags = list()
    Lags[[1]] = df %>%
        select(-contains("date"))
    if (n_lags >= 1) {
        for (i in 1:n_lags) {
            Lags[[i + 1]] = df %>%
                select(-contains("date")) %>%
                mutate_all(function(x) lag(x, n = i))
        }
    }
    lagged_data = reduce(Lags, function(x, y) {
        bind_cols(x, y, .name_repair = ~make.unique(.x))
    })

    return(lagged_data)
}
```



Using 4 lags of all variables

```
# Save forecasts
forecasts = cbind.zoo(forecasts, Ridge_4lags = forecast1) %>%
    as.ts()
```

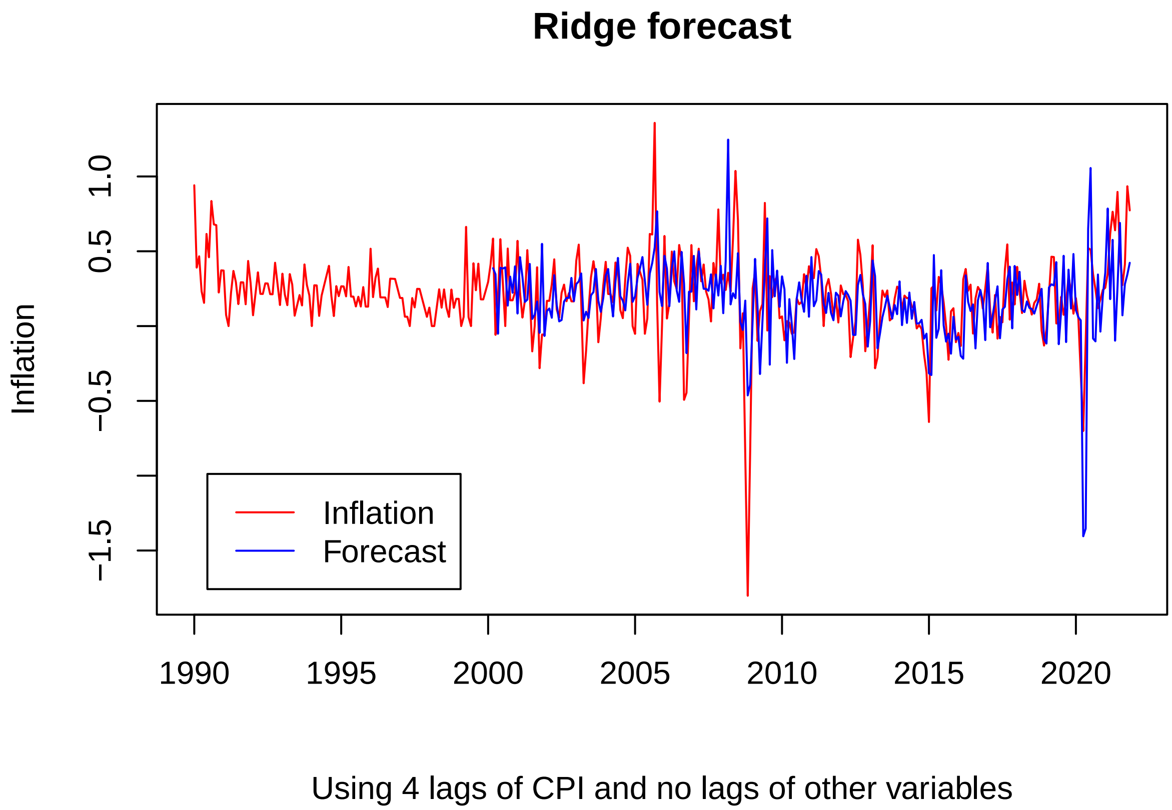
The forecast of the Ridge regression with 4 lags has a notably bad fit to the actual inflation series. We noticed that, since the ridge is not able to give a sparse solution, when there are too many variables, the estimated model becomes basically an intercept and almost all the other coefficients are very close to zero (but not zero). Hence, we tested other (more parsimonious) specifications. When we include the all the macroeconomics variables - without any lags - and lags of the CPI, we get a more reasonable result. The results are very robust to the number of CPI lags, so we keep 4 lags, as initially intended.

```
tic()
# Rolling window forecasting
rolling_window <- 492

# glmnet parameter
my_alpha = 0  # Ridge

forecast1 = list()

# set up parallel computation
registerDoFuture()
plan("multisession", workers = 3)  # use 3 cores

last_fcst = (length(inflation) - rolling_window)

output = foreach(a = 1:last_fcst) %dorng% {
    # get the window for training the model
    train = data[a:(a + rolling_window - 1), ] %>%
```

```r
        select(-CPIAUCSL)
    train_cpi = data[a:(a + rolling_window - 1), ] %>%
        select(CPIAUCSL)
    # embed
    reg_data = my_embed(train, n_lags = 0)
    cpi_lags = my_embed(train_cpi, n_lags = 4)
    # bind the embeded columns with the one-step-ahead
    # inflation
    reg_data = bind_cols(inflation.ahead = lead(inflation[a:(a +
        rolling_window - 1)]), cpi_lags, reg_data)

    # Ridge estimation
    ic_ridge <- ic.glmnet(x = reg_data %>%
        na.omit() %>%
        select(-inflation.ahead), y = reg_data %>%
        na.omit() %>%
        select(inflation.ahead) %>%
        data.matrix(), crit = "bic", alpha = my_alpha)
    ridge <- glmnet(x = reg_data %>%
        na.omit() %>%
        select(-inflation.ahead), y = reg_data %>%
        na.omit() %>%
        select(inflation.ahead) %>%
        data.matrix(), alpha = my_alpha, lambda = ic_ridge$lambda)

    # Prediction
    new = reg_data %>%
        select(-inflation.ahead) %>%
        tail(1)
    result1 = predict(ridge, newx = data.matrix(new), s = ic_ridge$lambda)

    # Coeficients
    result2 = coef(ridge, s = ic_ridge$lambda)

    result = list(forecast1 = result1, coef = result2)
    result
}

output = output %>%
    transpose()

forecast1 = output$forecast1 %>%
    unlist() %>%
    ts(start = start(inflation) + c(0, rolling_window), frequency = frequency(inflation))

ridge_coeficients = output$coef %>%
    reduce(cbind) %>%
    as.matrix()

toc()
```

```
## 115.872 sec elapsed
```

```r
beepr::beep()
```




```r
# Save forecasts
forecasts = cbind.zoo(forecasts, Ridge = forecast1) %>%
    as.ts()
```

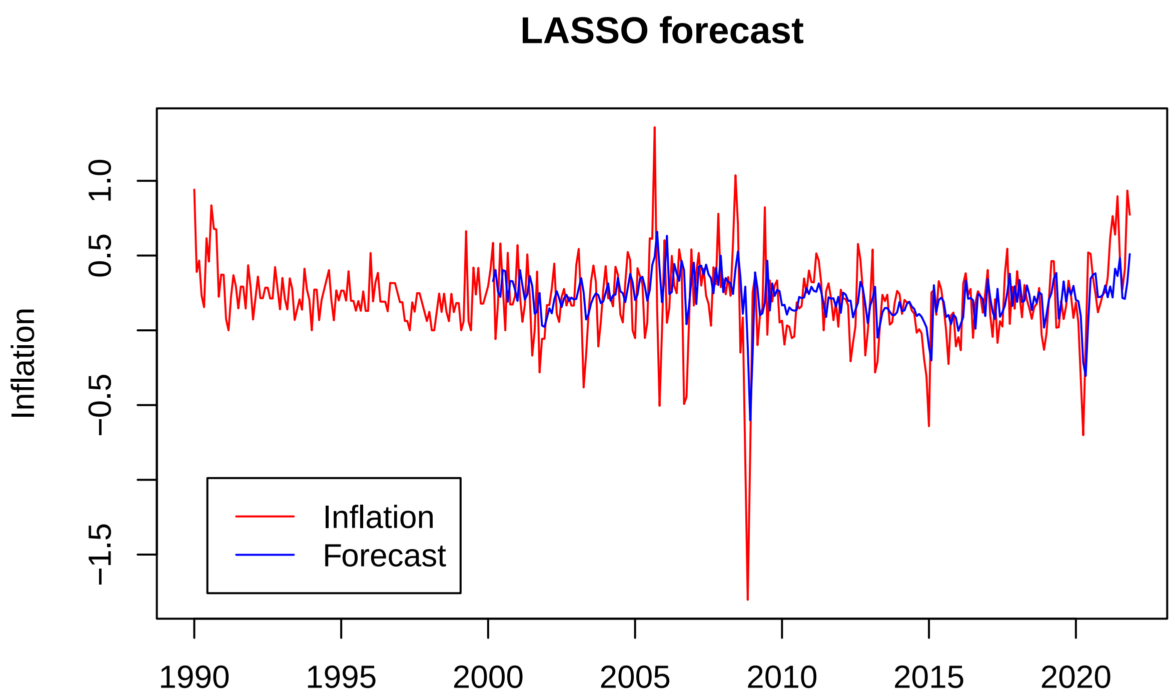
LASSO Regression

```r
tic()
# Rolling window forecasting
rolling_window <- 492

# glmnet parameter
my_alpha = 1  # LASSO

forecast1 = list()
coefficients_lasso = list()


for (a in 1:(length(inflation) - rolling_window)) {
    # get the window for training the model
    train = data[a:(a + rolling_window - 1), ]
    # embed
    reg_data = my_embed(train)
    # bind the embeded columns with the one-step-ahead
    # inflation
    reg_data = bind_cols(inflation.ahead = lead(inflation[a:(a +
        rolling_window - 1)]), reg_data)
```

```r
    # Ridge estimation
    ic_lasso <- ic.glmnet(x = reg_data %>%
        na.omit() %>%
        select(-inflation.ahead), y = reg_data %>%
        na.omit() %>%
        select(inflation.ahead) %>%
        data.matrix(), crit = "bic", alpha = my_alpha)
    lasso <- glmnet(x = reg_data %>%
        na.omit() %>%
        select(-inflation.ahead), y = reg_data %>%
        na.omit() %>%
        select(inflation.ahead) %>%
        data.matrix(), alpha = my_alpha, lambda = ic_lasso$lambda)

    # Prediction
    new = reg_data %>%
        select(-inflation.ahead) %>%
        tail(1)
    forecast1[a] = predict(lasso, newx = data.matrix(new), s = ic_lasso$lambda)

    # Coefficients
    coefficients_lasso[a] = coef(lasso)

}

forecast1 = forecast1 %>%
    unlist() %>%
    ts(start = start(inflation) + c(0, rolling_window), frequency = frequency(inflation))

toc()
```

```
## 37.407 sec elapsed
```

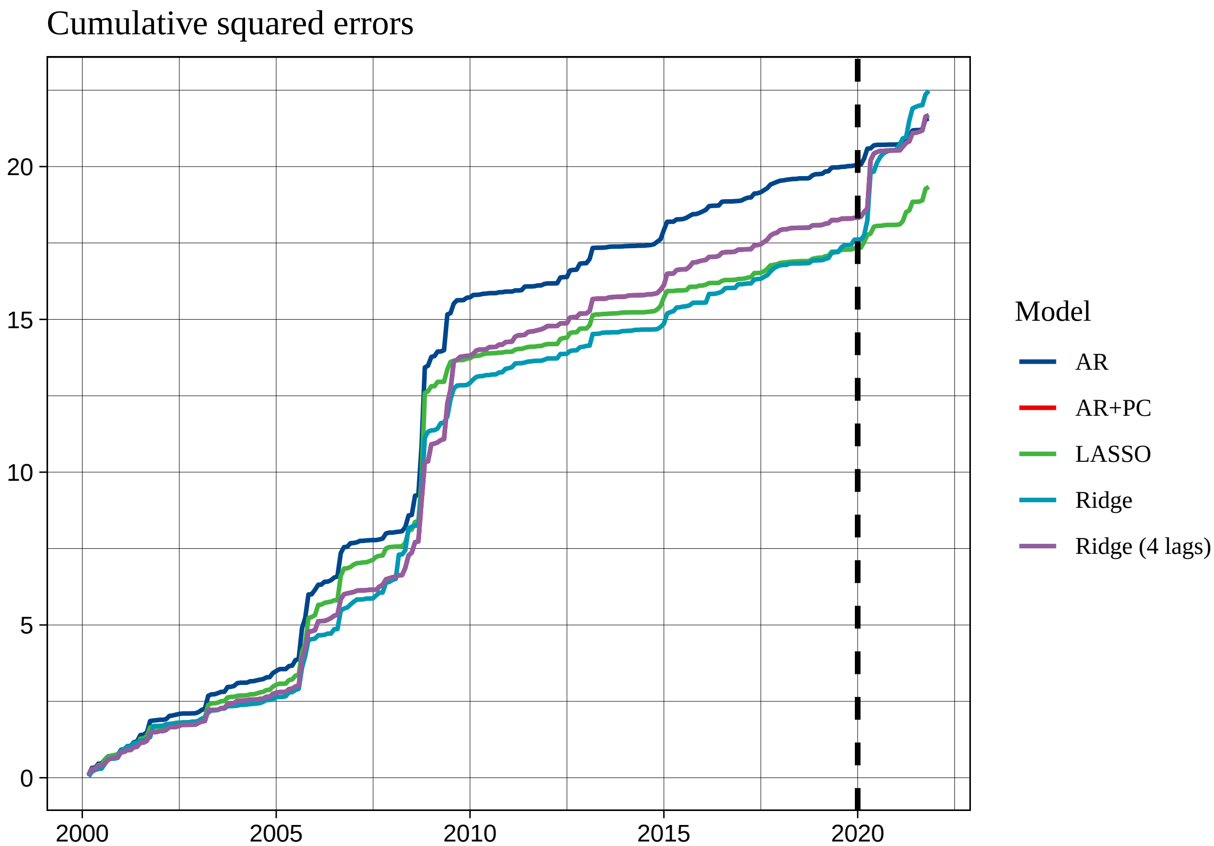
```r
beepr::beep()
```




```
# Save forecasts
forecasts = cbind.zoo(forecasts, LASSO = forecast1) %>%
    as.ts()
```

Item A

```
# Forecasting error
error = inflation - forecasts
cum_error = sapply(error, function(x) {
    x^2 %>%
        cumsum()
}) %>%
    bind_cols(date = as.Date.yearmon(time(error))) %>%
    setNames(c("AR", "AR+PC", "Ridge (4 lags)", "Ridge", "LASSO",
        "date"))


# cum_error = sapply(error, function(x){x^2 %>% cumsum()})
# %>% bind_cols(date = as.Date.yearmon(time(error))) %>%
# setNames( c('AR', 'AR+PC', 'Ridge', 'LASSO', 'date' ) )
```



Item B

We will follow the FRED-MD classification of variables into 8 groups: (i) output and income; (ii) labor market; (iii) housing; (iv) consumption,orders and inventories; (v) money and credit; (vi) interest and exchange rates; (vii) prices; and (viii) stock market. We are adding a ninth group called (ix) lags, with the lagged inflation series.

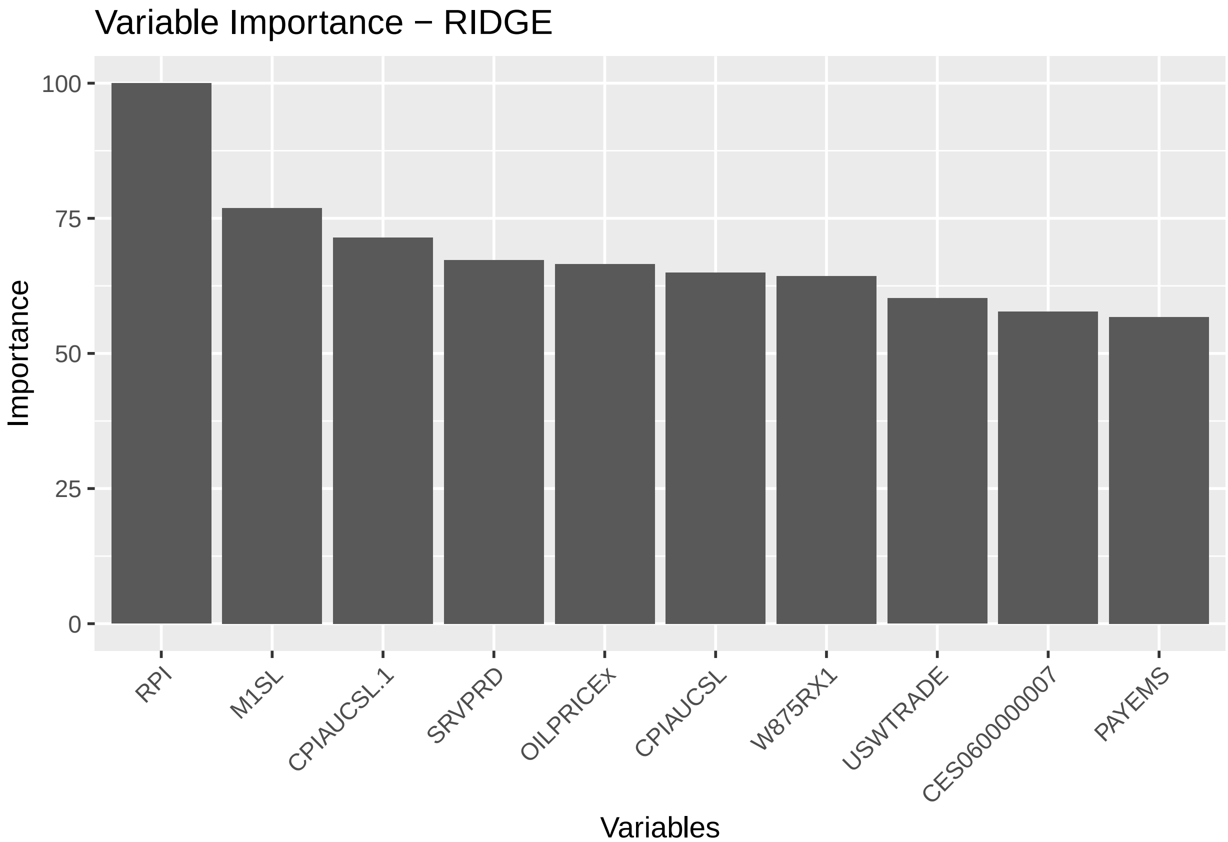
```
# Get FRED groups groups = read_xlsx('C:\\Users\\Caio
# Garzeri\\OneDrive - puc-rio.br\\Econometria
# IV\\AssignmentEconometricsIV\\data\\FRED-MD_updated_appendix.xlsx')
groups = read_xlsx("data/FRED-MD_updated_appendix.xlsx")
groups <- groups %>%
    select(fred, group)


# Change some names manually because they have minor
# differences with the variable names in existing dataframe
groups$fred[groups$fred == "S&P 500"] <- "SP500"
groups$fred[groups$fred == "IPB51222s"] <- "IPB51222S"
groups$fred[groups$fred == "S&P: indust"] <- "SPINDUST"

names <- function(base_name, n) {
    new_name = paste0(base_name, ".", n)
    return(new_name)
}

# Expand group_df with new variable names
```

```
expandgroup <- groups %>%
    rowwise() %>%
    mutate(NewVariables = list(names(fred, 1:4)), NewGroups = list(rep(group,
        length(NewVariables)))) %>%
    unnest(c(NewVariables, NewGroups)) %>%
    select(c(NewVariables, NewGroups)) %>%
    rename(fred = "NewVariables", group = "NewGroups")

# Merge with original group_df
endgroups <- bind_rows(groups, expandgroup)

# Sort by variable name
endgroups <- endgroups %>%
    arrange(fred)

# Change CPI lags to group 'lags' (9)
endgroups$group[endgroups$fred == "CPIAUCSL"] <- 9
endgroups$group[endgroups$fred == "CPIAUCSL.1"] <- 9
endgroups$group[endgroups$fred == "CPIAUCSL.2"] <- 9
endgroups$group[endgroups$fred == "CPIAUCSL.3"] <- 9
endgroups$group[endgroups$fred == "CPIAUCSL.4"] <- 9

groups <- endgroups

rm(endgroups, expandgroup, names)
```

We compute variable importance for Ridge and pick the top 10 most important overtime.

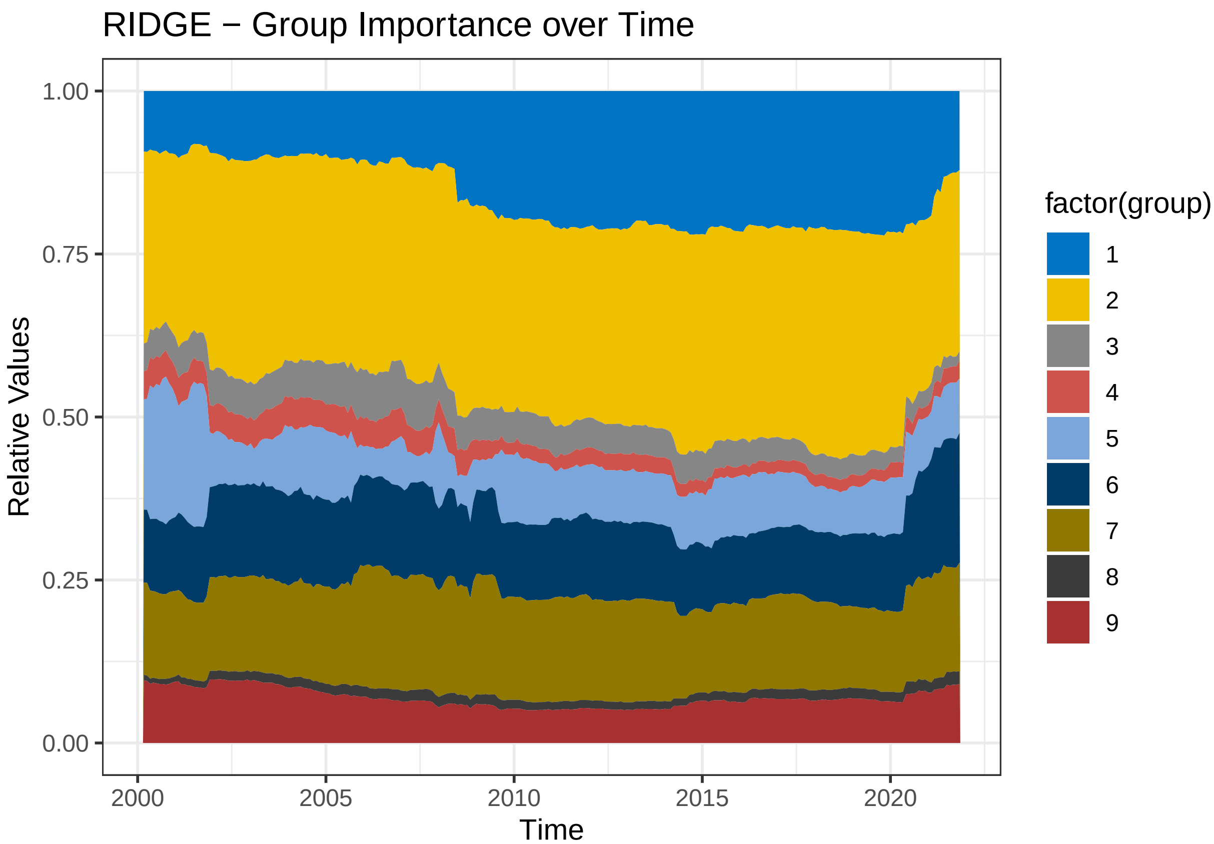
```
# Computing variable importance for RIDGE

ridge_coeff <- as.data.frame(ridge_coeficients)
colnames(ridge_coeff) <- NULL
ridge_coeff <- ridge_coeff[2:109, ]

ridge_names <- ridge_coeff %>%
    row.names(.)
names <- as.data.frame(ridge_names)
ridge_coeff <- cbind(names, ridge_coeff)
reg_data2 <- reg_data %>%
    select(-inflation.ahead)
std_deviations <- apply(reg_data2, 2, sd)
std_dev_df <- data.frame(Column_Names = colnames(reg_data2),
    Standard_Deviation = std_deviations)
std_dev_df <- std_dev_df %>%
    rename(ridge_names = "Column_Names")

ridge_coeff <- merge(ridge_coeff, std_dev_df, by = "ridge_names",
    all.x = TRUE)

ridge_coeff$Standard_Deviation[ridge_coeff$ridge_names == "CPIAUCSL.1"] <- ridge_coeff$Standard_Deviati
    "CPIAUCSL"]
ridge_coeff$Standard_Deviation[ridge_coeff$ridge_names == "CPIAUCSL.2"] <- ridge_coeff$Standard_Deviati
    "CPIAUCSL"]
ridge_coeff$Standard_Deviation[ridge_coeff$ridge_names == "CPIAUCSL.3"] <- ridge_coeff$Standard_Deviati
```

```
    "CPIAUCSL"]
ridge_coeff$Standard_Deviation[ridge_coeff$ridge_names == "CPIAUCSL.4"] <- ridge_coeff$Standard_Deviati
    "CPIAUCSL"]

ridge_coeff_std <- ridge_coeff

for (col in 2:262) {
    ridge_coeff_std[[col]] <- ridge_coeff_std[[col]] * ridge_coeff_std$Standard_Deviation
}

top10_ridge <- ridge_coeff_std %>%
    mutate(Mean_Value = rowMeans(across(2:262, ~abs(.)))) %>%
    select(ridge_names, Mean_Value) %>%
    arrange(desc(Mean_Value)) %>%
    head(10)

top10_ridge <- top10_ridge %>%
    mutate(importance = 100 * Mean_Value/Mean_Value[1]) %>%
    arrange(desc(importance))

ggplot(top10_ridge, aes(x = reorder(ridge_names, -importance),
    y = importance)) + geom_bar(stat = "identity") + labs(title = "Variable Importance - RIDGE",
    x = "Variables", y = "Importance") + theme(axis.text.x = element_text(angle = 45,
    hjust = 1))
```



We now compute importance by group in Ridge.

```
# Get sum over groups Sum over cells based on groups

ridge_coeff_std <- ridge_coeff_std %>%
    rename(fred = "ridge_names")

ridge_coeff_std <- merge(ridge_coeff_std, groups, by = "fred",
    all.x = TRUE)

ridge_group <- ridge_coeff_std

for (i in 2:262) {
    for (j in 1:108) {
        ridge_group[j, i] <- abs(ridge_coeff_std[j, i])/sum(abs((ridge_coeff_std[,
            i])))
    }
}

group_sums <- ridge_group %>%
    group_by(group) %>%
    summarize(across(2:262, ~sum(.)))

colnames(group_sums)[2:262] <- as.Date(time(forecast1))

group_sums_long <- pivot_longer(group_sums, cols = -group, names_to = "Time",
    values_to = "Value")
group_sums_long$Time <- as.integer(group_sums_long$Time)
group_sums_long$date <- as.Date(group_sums_long$Time)

ggplot(group_sums_long, aes(x = date, y = Value, fill = factor(group))) +
    geom_area() + labs(title = "RIDGE - Group Importance over Time",
    x = "Time", y = "Relative Values") + scale_fill_discrete(name = "Groups") +
    theme_bw() + scale_fill_jco()
```



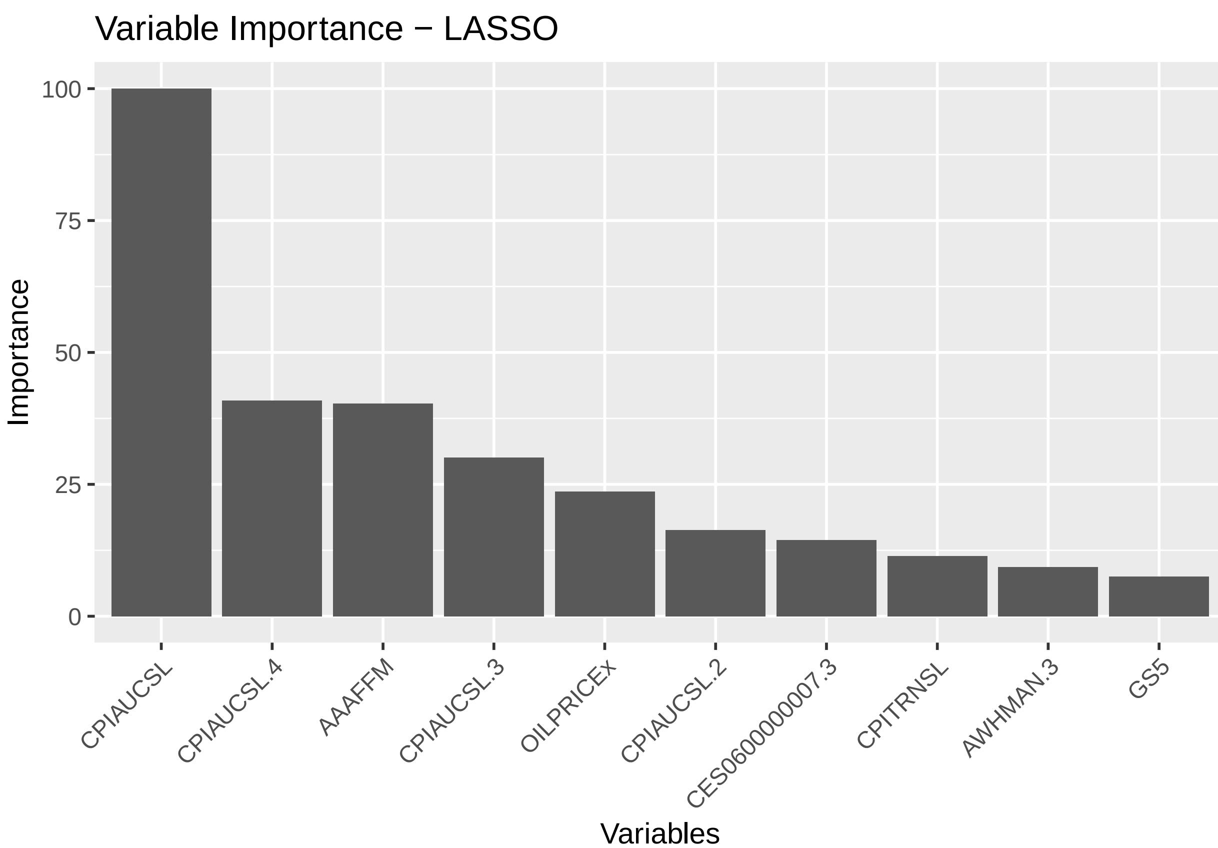
We repeat the exercise for LASSO. First selecting the top 10 most important variables.

```
# Computing variable importance for LASSO

# Create a matrix to store coefficients

coeff_lasso <- data.frame(matrix(ncol = ncol(reg_data2), nrow = length(forecast1)))
colnames(coeff_lasso) <- colnames(reg_data2)

# Retrieve coefficients and variable identifiers from lists
var_lasso = modify_depth(coefficients_lasso, 1, "i")
co_lasso = modify_depth(coefficients_lasso, 1, "x")

for (i in 1:length(forecast1)) {
    a = var_lasso[[i]] %>%
        unlist()
    b = co_lasso[[i]] %>%
        unlist()
    for (c in 2:length(a)) {
        coeff_lasso[i, a[c]] <- b[c]
    }
}

rm(var_lasso, co_lasso)

# Multiply for sd
for (i in 1:length(forecast1)) {
    coeff_lasso[, i] = coeff_lasso[, i] * sd(reg_data2[, i])
```

```
}
```

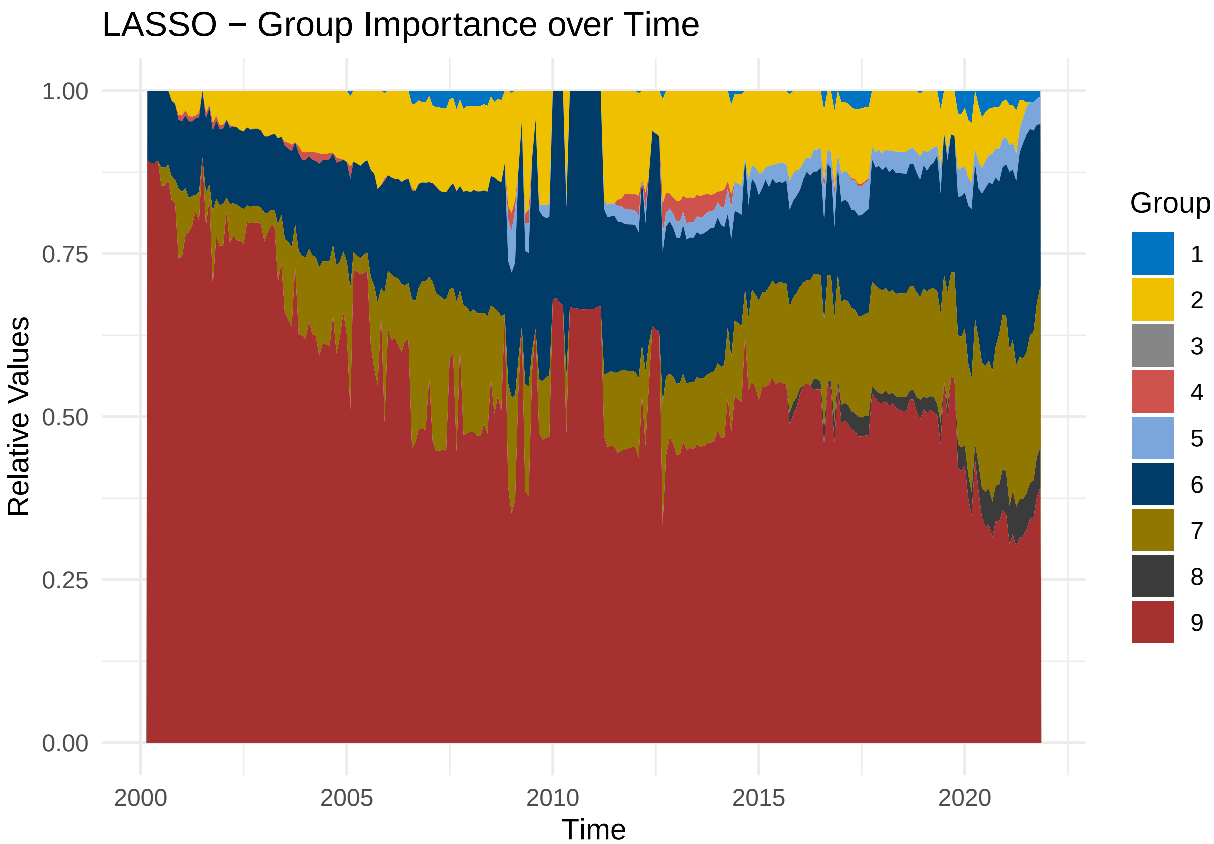
We compute the 10 most relevant predictors considering the mean absolute value of the coefficients over all estimation windows.

```
coeff_lasso <- coeff_lasso %>%
    mutate_all(~replace_na(., 0))

top10_lasso <- colMeans(abs(coeff_lasso))

top_10_lasso <- coeff_lasso %>%
    summarise_all(~mean(abs(.))) %>%
    pivot_longer(everything()) %>%
    arrange(desc(value)) %>%
    head(10)

top_10_lasso <- top_10_lasso %>%
    mutate(importance = 100 * value/value[1]) %>%
    arrange(desc(importance))

ggplot(top_10_lasso, aes(x = reorder(name, -importance), y = importance)) +
    geom_bar(stat = "identity") + labs(title = "Variable Importance - LASSO",
    x = "Variables", y = "Importance") + theme(axis.text.x = element_text(angle = 45,
    hjust = 1))
```



We now present results for groups. Group 9 (previous inflation) is consistently the most important though less important throught the sample. Groups 6 (bond and exchange rates) and 7 (prices) are important. Group (2) labor market used to be relevant. For the most recent windows, not so much.

```r
# Get sum over groups Sum over cells based on groups
coeff_long <- data.frame(variable = rep(colnames(coeff_lasso),
    each = nrow(coeff_lasso)), row_index = rep(1:nrow(coeff_lasso),
    times = ncol(coeff_lasso)), value = as.vector(as.matrix(coeff_lasso)))

coeff_long <- coeff_long %>%
    arrange(row_index)
groups <- groups %>%
    rename(variable = "fred")
merged_data <- merge(coeff_long, groups, by = "variable", all.x = TRUE)
merged_data <- merged_data %>%
    arrange(row_index)

groupfinal_lasso <- merged_data %>%
    group_by(row_index, group) %>%
    summarise(total = sum(abs(value)))

wide_group_lasso <- groupfinal_lasso %>%
    pivot_wider(names_from = group, values_from = total) %>%
    ungroup()

wide_group_lasso_rel <- wide_group_lasso %>%
    mutate(across(-1, ~./rowSums(across(-1))))

wide_group_lasso_rel$dates <- as.Date(time(forecast1))

# Melt the dataframe to long format for plotting
melted_df <- melt(wide_group_lasso_rel, id.vars = "dates", variable.name = "Column")
melted_df <- melted_df %>%
    filter(Column != "row_index")
melted_df$Group <- as.integer(melted_df$Column) - 1
melted_df$Group <- as.character(melted_df$Group)


# Create a stacked column plot
ggplot(data = melted_df, aes(x = dates, y = value, fill = Group)) +
    geom_area() + labs(title = "LASSO - Group Importance over Time",
    x = "Time", y = "Relative Values") + scale_fill_discrete(name = "Groups") +
    theme_minimal() + scale_fill_jco()
```



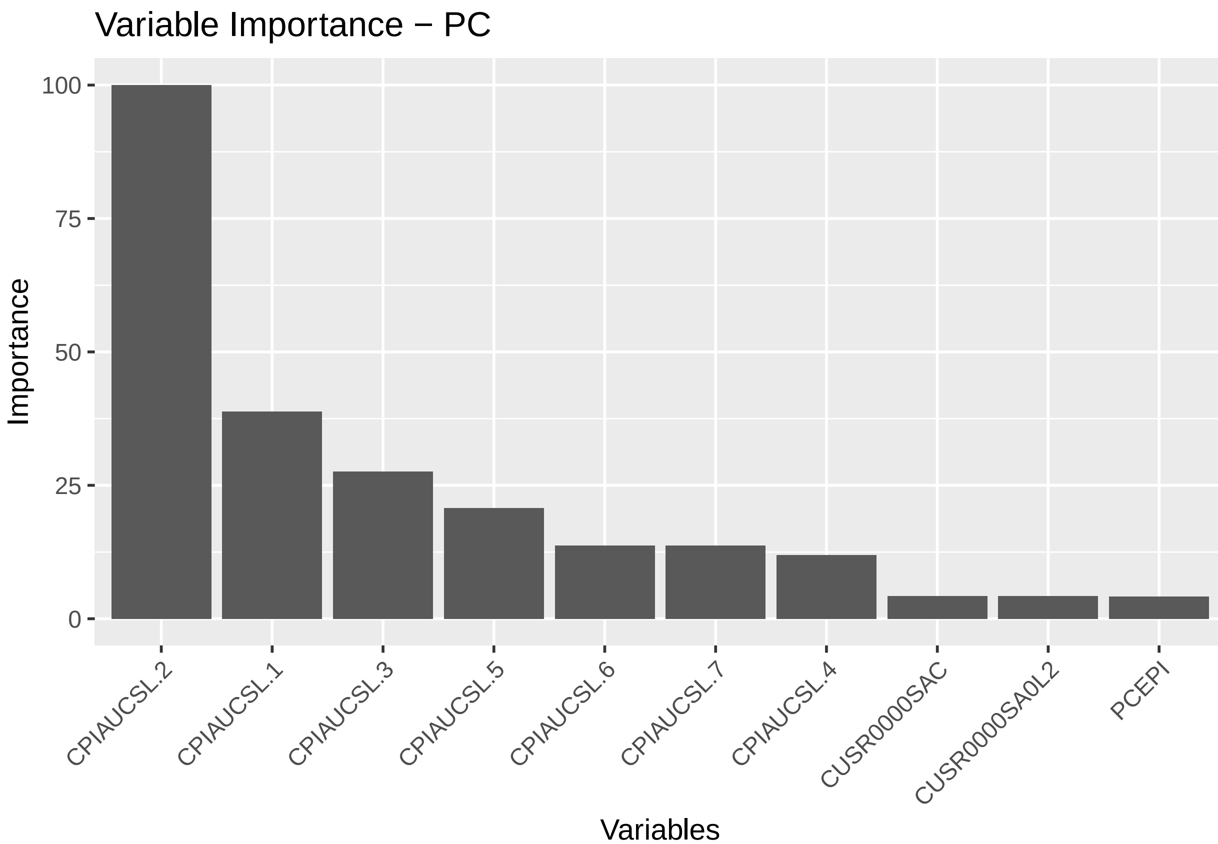
Finally, we do that for the AR+PC model, computing variable importance of the PC block of the model. This is slightly more complicated than LASSO and Ridge. We retrieve the alphas from the Factor on Variables and then multiply them be the coefficients in the model regression.

```
# Computing variable importance for PC

# Get the alphas
alpha = as.matrix(pca$rotation[, 1:n_pc])

# Get the lambdas (coefficients of the Factors) and the
# phis
lambdas = matrix(NA, 261, 6)
phis = matrix(NA, 261, 24)

for (i in 1:261) {
    size = length(coefficients_pc1[[i]])
    lags = size - 1 - 6  # intercept and 6 factors
    for (j in 1:6) {
        lambdas[i, j] <- coefficients_pc1[[i]][size - 6 + j]
    }
    for (l in 1:24) {
        phis[i, l] <- coefficients_pc1[[i]][l + 1]
    }
}

# Multiply alpha by lambdas to get 'coefficient' of each
# variable in each window
importpc = as.data.frame(alpha %*% t(lambdas))
```

```
phist = as.data.frame(t(phis))
row_names <- paste("CPIAUCSL", seq(1, 24), sep = ".")
importpc$fred = rownames(importpc)
phist$fred = row_names
importpc <- rbind(importpc, phist)

groups <- groups %>%
    rename(fred = "variable")
importpc = merge(importpc, groups, by = "fred", all.x = TRUE)
importpc$group <- ifelse(is.na(importpc$group), 9, importpc$group)  # giving all lags of inflation grou


# Get the number of lags - we use this in item A
lags_PC_AR <- table(colSums(!is.na(phist)))

lags_PC_AR <- data.frame(lags = as.numeric(names(lags_PC_AR)),
    count = as.numeric(lags_PC_AR))

lags_PC_AR = lags_PC_AR[1:13, ]
lags_PC_AR <- lags_PC_AR %>%
    arrange(desc(count))
```

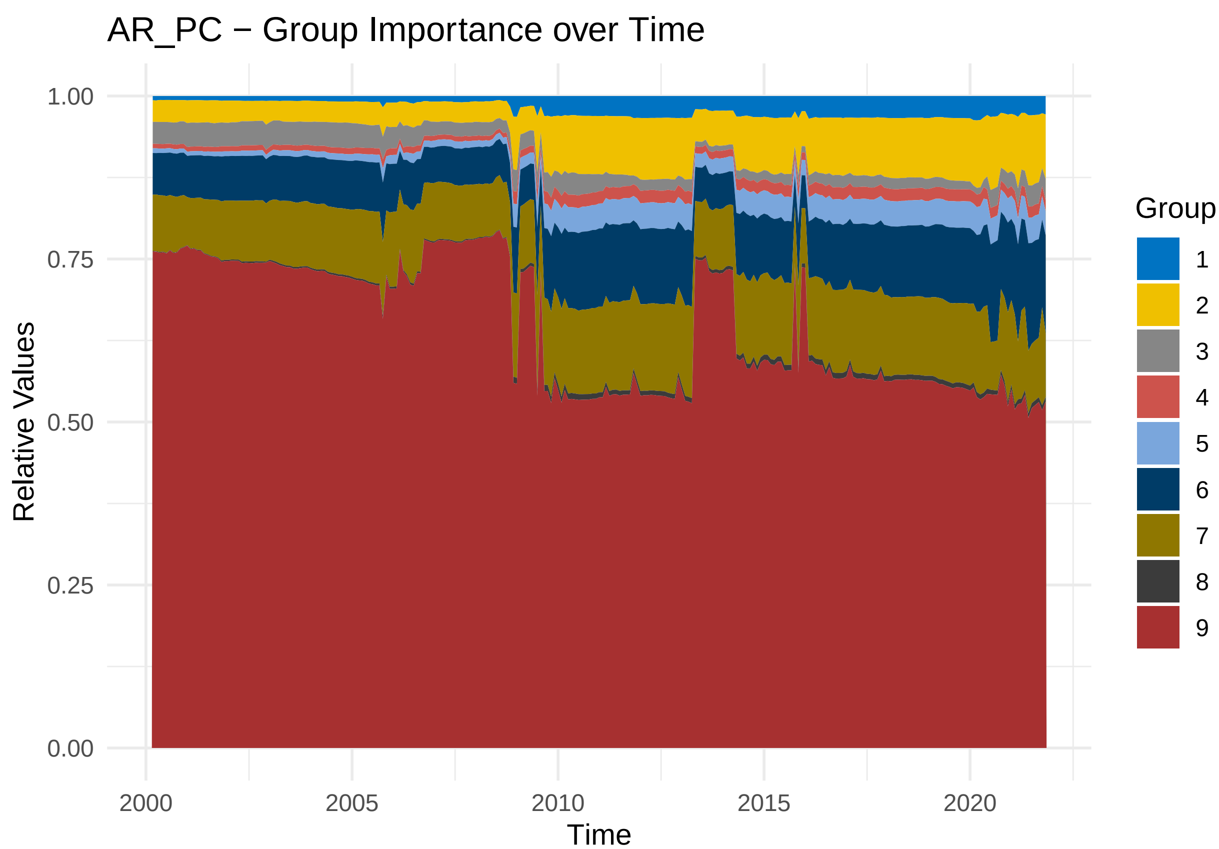
Top 10 most relevant variables

```
top_10_pc <- importpc %>%
    rowwise() %>%
    mutate(mean_abs = mean(abs(c_across(-c(fred, group))))) %>%
    ungroup() %>%
    select(fred, mean_abs) %>%
    arrange(desc(mean_abs)) %>%
    head(10)

top_10_pc <- top_10_pc %>%
    mutate(importance = 100 * mean_abs/mean_abs[1]) %>%
    arrange(desc(importance))

ggplot(top_10_pc, aes(x = reorder(fred, -importance), y = importance)) +
    geom_bar(stat = "identity") + labs(title = "Variable Importance - PC",
    x = "Variables", y = "Importance") + theme(axis.text.x = element_text(angle = 45,
    hjust = 1))
```



We again compute importance by group. Pattern is very close to that of LASSO: Groups 9, 7, 6, 2

```
result <- importpc %>%
    mutate(across(starts_with("V"), ~abs(.), .names = "abs_{.col}")) %>%
    group_by(group) %>%
    summarise(across(starts_with("abs_V"), ~sum(., na.rm = TRUE)))

pc_rel <- result %>%
    mutate(across(starts_with("abs_V"), ~./sum(., na.rm = TRUE),
        .names = "rel_{.col}")) %>%
    select(starts_with("rel_"))

pc_rel_transposed <- as.data.frame((t(pc_rel)))

pc_rel_transposed <- pc_rel_transposed %>%
    mutate(date = as.Date(time(forecast1)))

importpc_long <- pc_rel_transposed %>%
    pivot_longer(cols = starts_with("V"), names_to = "variable",
        values_to = "value")

importpc_long$Group <- as.character(gsub("\\D", "", importpc_long$variable))

ggplot(importpc_long, aes(x = date, y = value, fill = Group)) +
    geom_area() + labs(title = "AR_PC - Group Importance over Time",
    x = "Time", y = "Relative Values") + scale_fill_discrete(name = "Groups") +
    theme_minimal() + scale_fill_jco()
```


AR_PC − Group Importance over Time
Relative Values
1.00
0.75
0.50
0.25
0.00
2000
2005
2010
2015
2020
Time
Group
1
2
3
4
5
6
7
8
9

Trabalho - Econometria IV

Guilherme Luz, Guilherme Masuko, Caio Garzeri

August 2023

```r
library(lubridate)  # for handling dates
library(randomForest)  # Random Forest implementation of the original Fortran code by Brieman (2001)
library(ranger)  # Faster implementation of Random Forest
```

Question 3

Item D

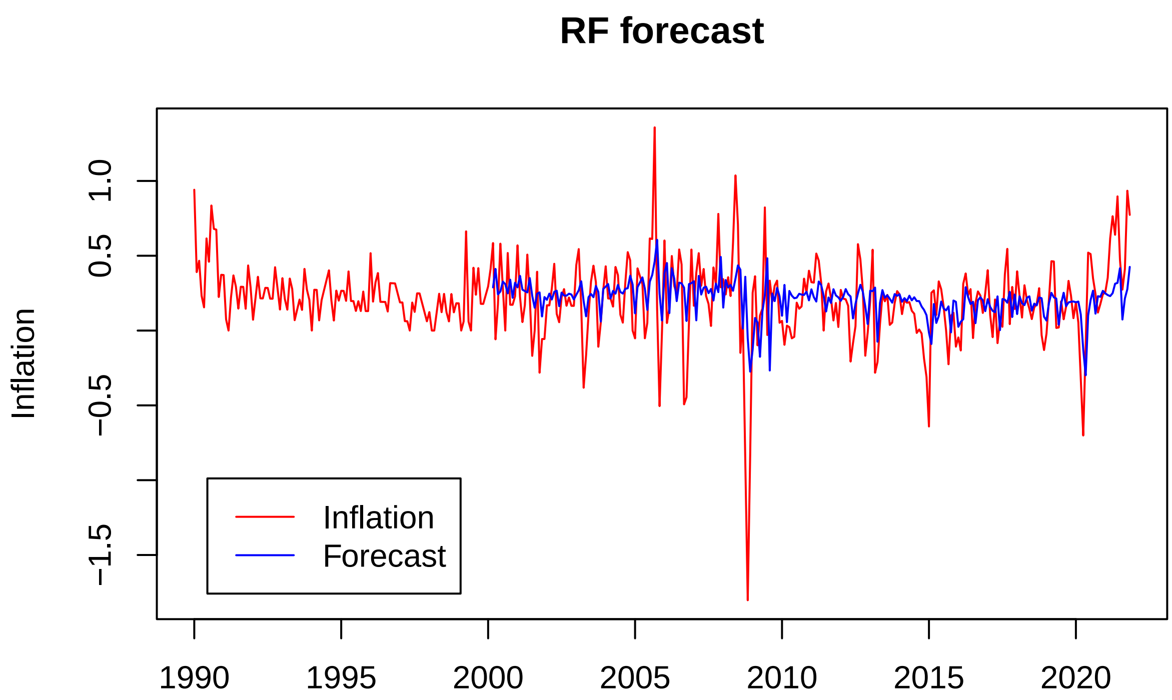
In order to include the lags of the variables as covariates, we need to do an embedding process. explicar. (We do this inside the rolling window loop to avoid 'cheating').

After this process, we can use the usual IID bootstrap, since we are interested in direct forecasting.

```r
# Embedding function that creates n_lags of all variables
# of a given data frame
my_embed = function(df, n_lags = 4) {
    Lags = list()
    Lags[[1]] = df %>%
        select(-contains("date"))
    if (n_lags >= 1) {
        for (i in 1:n_lags) {
            Lags[[i + 1]] = df %>%
                select(-contains("date")) %>%
                mutate_all(function(x) lag(x, n = i))
        }
    }
    lagged_data = reduce(Lags, function(x, y) {
        bind_cols(x, y, .name_repair = ~make.unique(.x))
    })

    return(lagged_data)
}
```

```r
n_lags = 4

# Rolling window forecasting
rolling_window <- 492

# Random Forest parameters
p = (1+n_lags)*ncol(data) # number of variables
mtry = ((1/3)*p) %>% round() # number of variables randomly selected
num.trees = 500 # number of trees
min.bucket = 5 # minimal number of observations in each leave (terminal node)
```

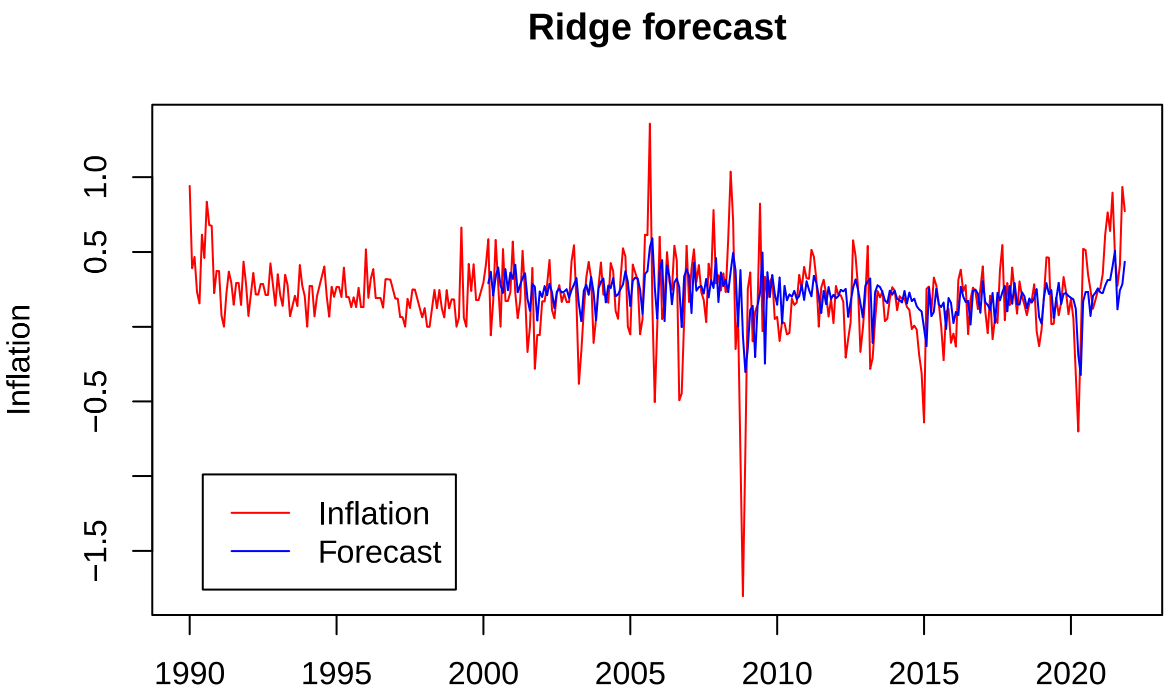
```r
set.seed(1430)

forecast1 = list()

for(a in 1:(length(inflation)-rolling_window)){
  # get the window for training the model
  train = data[a:(a+rolling_window-1), ]
  # embed
  RF_data = my_embed(train, n_lags = n_lags)
  # bind the embeded columns with the one-step-ahead inflation
  RF_data = bind_cols(inflation.ahead = lead(inflation[a:(a+rolling_window-1)]), RF_data)

  # Random forest estimation
  RF = ranger(inflation.ahead ~.,
              data = RF_data %>% na.omit(),
              oob.error = T,
              # Parameters below are set previously
              mtry = mtry,
              num.trees = num.trees,
              min.bucket = min.bucket)

  # Prediction
  new = RF_data %>% select(-inflation.ahead) %>% tail(1)
  forecast1[a] = predict(RF, data = new)
}

forecast1 = forecast1 %>% unlist() %>%
  ts(start = start(inflation)+c(0,rolling_window), frequency =  frequency(inflation) )
```



Using 4 lags of all variables

```
n_lags = 0

# Rolling window forecasting
rolling_window <- 492

# Random Forest parameters
p = (1+n_lags)*ncol(data) # number of variables
mtry = ((1/3)*p) %>% round() # number of variables randomly selected
num.trees = 500 # number of trees
min.bucket = 5 # minimal number of observations in each leave (terminal node)


set.seed(1430)

forecast1 = list()

for(a in 1:(length(inflation)-rolling_window)){
  # get the window for training the model
  train = data[a:(a+rolling_window-1), ]
  # embed
  RF_data = my_embed(train)
  # bind the embeded columns with the one-step-ahead inflation
  RF_data = bind_cols(inflation.ahead = lead(inflation[a:(a+rolling_window-1)]), RF_data)


  # get the window for training the model
  train = data[a:(a+rolling_window-1), ] %>% select(-CPIAUCSL)
  train_cpi = data[a:(a+rolling_window-1), ] %>% select(CPIAUCSL)
```

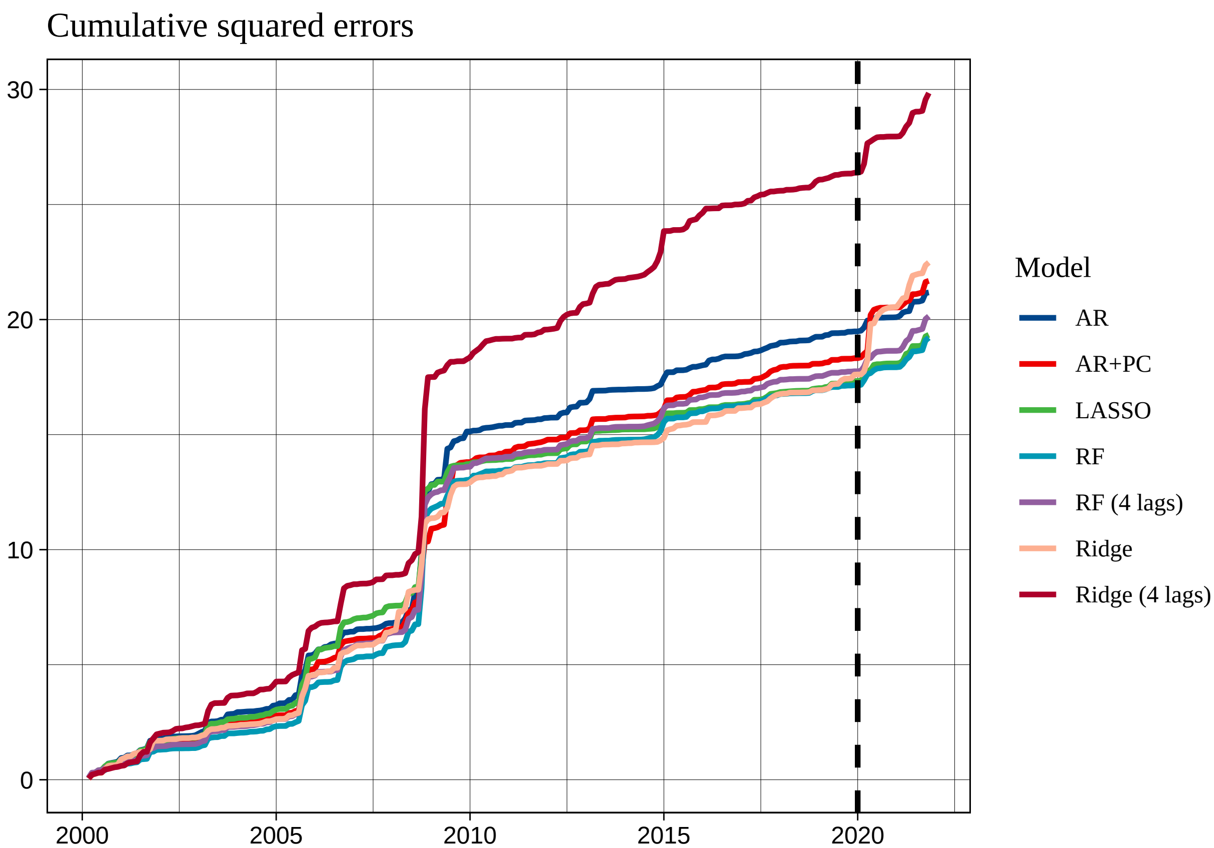
```r
  # embed
  RF_data = my_embed(train, n_lags = n_lags)
  cpi_lags = my_embed(train_cpi, n_lags = 4)
  # bind the embeded columns with the one-step-ahead inflation
  RF_data = bind_cols(inflation.ahead = lead(inflation[a:(a+rolling_window-1)]),
                      cpi_lags, RF_data)

  # Random forest estimation
  RF = ranger(inflation.ahead ~.,
              data = RF_data %>% na.omit(),
              oob.error = T,
              # Parameters below are set previously
              mtry = mtry,
              num.trees = num.trees,
              min.bucket = min.bucket)

  # Prediction
  new = RF_data %>% select(-inflation.ahead) %>% tail(1)
  forecast1[a] = predict(RF, data = new)
}

forecast1 = forecast1 %>% unlist() %>%
  ts(start = start(inflation)+c(0,rolling_window), frequency =  frequency(inflation) )
```



Using 4 lags of CPI and no lags of other variables

Item E

```r
# Forecasting error
error = inflation - forecasts
cum_error = error %>%
    data.frame() %>%
    mutate_all(function(x) {
        (x^2) %>%
            cumsum()
    }) %>%
    bind_cols(date = zoo::as.Date.yearmon(time(error))) %>%
    setNames(c("AR", "AR+PC", "Ridge (4 lags)", "Ridge", "LASSO",
        "RF (4 lags)", "RF", "date"))
```




```r
# Save
write.csv(forecasts, file = "output/forecasts.csv")
write.csv(cum_error, file = "output/cum_error.csv")
```