In [ ]:
```
import numpy as np
import pandas as pd
import os
import matplotlib.pyplot as plt
import seaborn as sns

from functions.linear_models import PCA_function, OLS_regression
from functions.number_PC import rule_thumb, informal_way, biggest_drop
```

In [ ]:
```
# hide warning messages
import warnings
warnings.filterwarnings("ignore")
```

In [ ]:
```
# better plots
sns.set(rc={'figure.figsize':(12,8)});
```

In [ ]:
```
directory = os.path.dirname(os.getcwd())
directory
```

Out[ ]:
```
'd:\\github\\AssignmentEconometricsIV'
```

Question

The first question consists of a factor analysis of a large dataset. We consider monthly close-to-close excess returns from a cross-section of 9,456 firms traded in the New York Stock Exchange. The data starts on November 1991 and runs until December 2018. There are 326 monthly observations in total.

In addition to the returns we also consider 16 monthly factors:

- Market (MKT)
- Small-minus-Big (SMB)
- High-minus-Low (HML)
- Conservative-minus-Aggressive (CMA)
- Robust-minus-Weak (RMW)
- earning/price ratio (EP)
- cash-flow/price ratio (CFP)
- dividend/price ratio
- accruals (ACC)
- market beta (BETA)
- net share issues
- daily variance (RETVOL)
- daily idiosyncratic variance (IDIOVOL)
- 1-month momentum (MOM1)
- 36-month momentum (MOM36)

The dataset is organized as an excel file named `returns.xlsx` .

In [ ]:
```
input_path = f'{directory}\\data\\returns.xlsx'
df = pd.read_excel(input_path, index_col=0)
```

In [ ]:
```
df.head()
```

Out[ ]:

dates	MKT	HML	SMB	MOM1	MOM36	ACC	BETA	CFP	CHCSH
1991-11-29	-0.041264	-0.028083	0.004779	-0.007336	-0.025496	-0.013692	0.035433	-0.015116	-0.00677
1991-12-31	0.107984	-0.022529	-0.027366	0.010963	-0.021188	-0.027887	-0.082499	-0.032122	-0.00589
1992-01-31	-0.007668	0.051012	0.085547	0.050916	0.108588	0.021978	-0.072801	0.028117	-0.00819
1992-02-28	0.010796	0.070501	0.002794	-0.027398	0.079286	0.003860	-0.024906	0.037363	0.01562
1992-03-31	-0.025367	0.039029	-0.015135	-0.009367	0.024631	0.004612	0.041266	0.037916	0.01711

5 rows × 9472 columns

In [ ]:
```
factors_name = df.columns[:16]
factors_name
```

Out[ ]:
```
Index(['MKT', 'HML', 'SMB', 'MOM1', 'MOM36', 'ACC', 'BETA', 'CFP', 'CHCSHO',
       'DY', 'EP', 'IDIOVOL', 'CMA', 'UMD', 'RMW', 'RETVOL'],
      dtype='object')
```

In [ ]:
```
factors = df[factors_name]
factors.head()
```

Out[ ]:

dates	MKT	HML	SMB	MOM1	MOM36	ACC	BETA	CFP	CHCSH
1991-11-29	-0.041264	-0.028083	0.004779	-0.007336	-0.025496	-0.013692	0.035433	-0.015116	-0.00677
1991-12-31	0.107984	-0.022529	-0.027366	0.010963	-0.021188	-0.027887	-0.082499	-0.032122	-0.00589
1992-01-31	-0.007668	0.051012	0.085547	0.050916	0.108588	0.021978	-0.072801	0.028117	-0.00819
1992-02-28	0.010796	0.070501	0.002794	-0.027398	0.079286	0.003860	-0.024906	0.037363	0.01562
1992-03-31	-0.025367	0.039029	-0.015135	-0.009367	0.024631	0.004612	0.041266	0.037916	0.01711

In [ ]:
```
returns_name = df.columns[16:]
returns_name
```

Out[ ]:
```
Index(['r_   1', 'r_   2', 'r_   3', 'r_   4', 'r_   5', 'r_   6', 'r_   7',
       'r_   8', 'r_   9', 'r_  10',
       ...
       'r_9447', 'r_9448', 'r_9449', 'r_9450', 'r_9451', 'r_9452', 'r_9453',
       'r_9454', 'r_9455', 'r_9456'],
      dtype='object', length=9456)
```

In [ ]:
```
returns = df[returns_name]
returns.head()
```

Out[ ]:

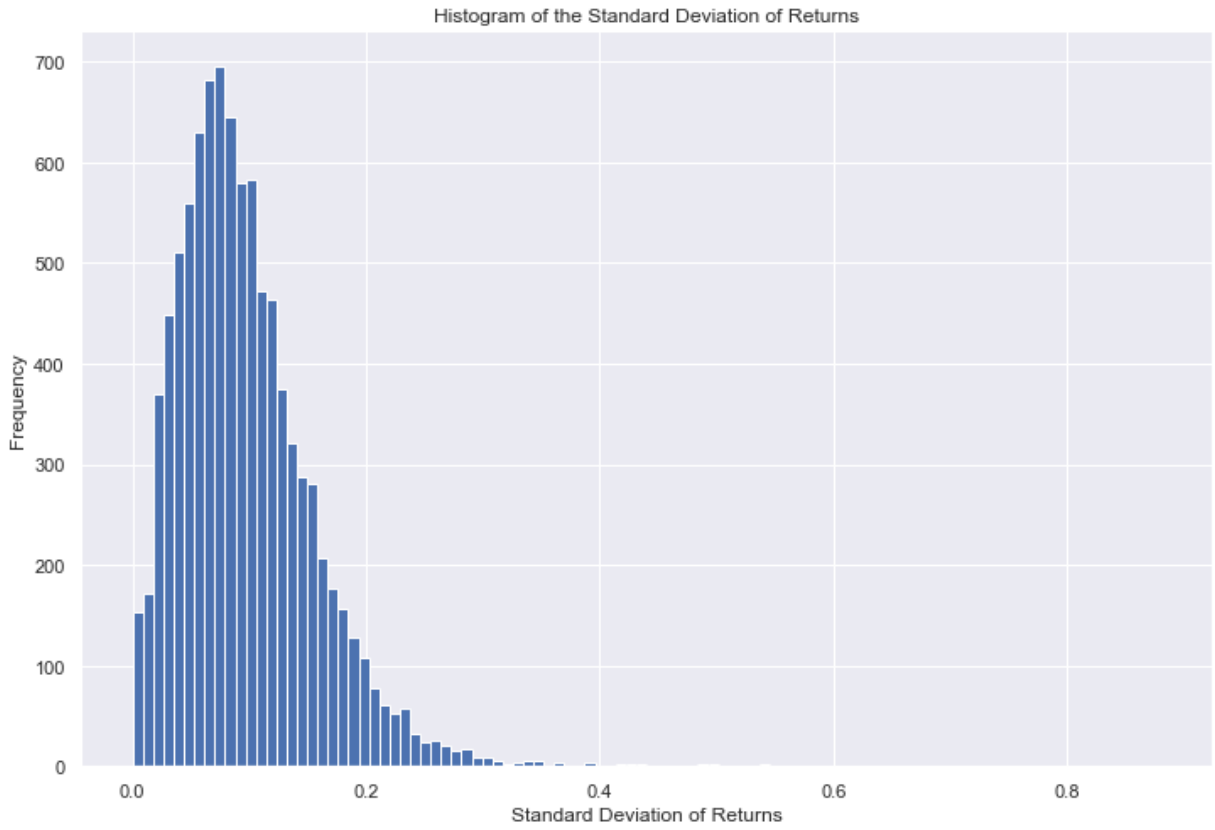
dates	r_1	r_2	r_3	r_4	r_5	r_6	r_7	r_8	r_9
1991-11-29	0.130715	-0.053900	-0.110696	-0.043900	0.218322	0.050645	0.575047	0.008921	-0.10534
1991-12-31	-0.010580	-0.056432	0.213591	0.183700	1.299230	0.306545	0.040644	-0.003800	0.31878
1992-01-31	-0.055124	-0.003400	-0.164114	-0.205154	-0.082347	0.075547	-0.024677	-0.066691	0.15513
1992-02-28	-0.202800	5.219422	0.039753	-0.057745	-0.117086	0.204517	-0.089757	-0.083881	0.01825
1992-03-31	0.078418	-0.181971	-0.023808	0.043112	0.415955	0.064782	-0.146257	-0.179871	-0.20958

5 rows × 9456 columns

In [ ]:
```
daterange = returns.index
daterange
```

Out[ ]:
```
DatetimeIndex(['1991-11-29', '1991-12-31', '1992-01-31', '1992-02-28',
               '1992-03-31', '1992-04-30', '1992-05-29', '1992-06-30',
               '1992-07-31', '1992-08-31',
               ...
               '2018-03-29', '2018-04-30', '2018-05-31', '2018-06-29',
               '2018-07-31', '2018-08-31', '2018-09-28', '2018-10-31',
               '2018-11-30', '2018-12-31'],
              dtype='datetime64[ns]', name='dates', length=326, freq=None)
```

In [ ]:
```
plt.hist(returns.std(axis=0), bins=100)
plt.xlabel('Standard Deviation of Returns')
plt.ylabel('Frequency')
plt.title('Histogram of the Standard Deviation of Returns');
```



In [ ]:
```
standardized_returns = (returns - returns.mean(axis=0))/returns.std(axis=0)
standardized_returns.head()
```

Out[ ]:

dates	r_1	r_2	r_3	r_4	r_5	r_6	r_7	r_8	r_
1991-11-29	1.483764	-0.219996	-0.914187	-0.665776	1.031984	0.207163	3.474317	-0.028135	-0.77523
1991-12-31	-0.223389	-0.226904	1.435410	2.066812	6.526489	1.869218	0.235135	-0.125593	2.07608
1992-01-31	-0.761584	-0.082197	-1.301224	-2.601812	-0.496389	0.368900	-0.160796	-0.607431	0.97594
1992-02-28	-2.545832	14.169328	0.175881	-0.832001	-0.672972	1.206552	-0.555266	-0.739131	0.05571
1992-03-31	0.851900	-0.569465	-0.284648	0.378895	2.036596	0.298978	-0.897734	-1.474551	-1.47598

5 rows × 9456 columns

In [ ]:
```
standardized_factors = (factors - factors.mean(axis=0))/factors.std(axis=0)
standardized_factors.head()
```

Out[ ]:

dates	MKT	HML	SMB	MOM1	MOM36	ACC	BETA	CFP	CHCSH
1991-11-29	-1.146021	-0.942950	0.110054	-0.298999	-0.981611	-0.855086	0.608329	-0.530965	-0.39095

dates	MKT	HML	SMB	MOM1	MOM36	ACC	BETA	CFP	CHCSH
1991-12-31	2.438375	-0.768570	-0.927363	0.228826	-0.830584	-1.573833	-1.458119	-1.015147	-0.3560
1992-01-31	-0.339177	1.540227	2.716591	1.381237	3.719257	0.951073	-1.288179	0.699949	-0.4476
1992-02-28	0.104274	2.152085	0.045976	-0.877648	2.691977	0.033673	-0.448956	0.963170	0.5005
1992-03-31	-0.764238	1.164027	-0.532641	-0.357583	0.775803	0.071737	0.710532	0.978913	0.5599

The model

$$\begin{aligned} Y_t &= \beta_0 + \beta_1 X_{1t} + \cdots + \beta_p X_{nt} + U_t, \quad t = 1, \ldots, T \\ &= \boldsymbol{\beta}' \boldsymbol{X}_t + U_t \\ \boldsymbol{Y} &= \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{U} \quad (\text{matrix notation}). \end{aligned}$$

However, $n > T$ (more columns than rows in X).

The usual ordinary least squares (OLS) solution

$$\widehat{\boldsymbol{\beta}} = \left(X'X\right)^{-1} X'Y$$

is not valid anymore.

Solution: reduce the dimension of $\boldsymbol{X}$ by postulating that:

$$\underset{(n\times 1)}{\boldsymbol{X}_t} = \underset{(n\times k)}{\boldsymbol{\Lambda}} \underset{(k\times 1)}{\boldsymbol{F}_t} + \underset{(n\times 1)}{\boldsymbol{V}_t},$$

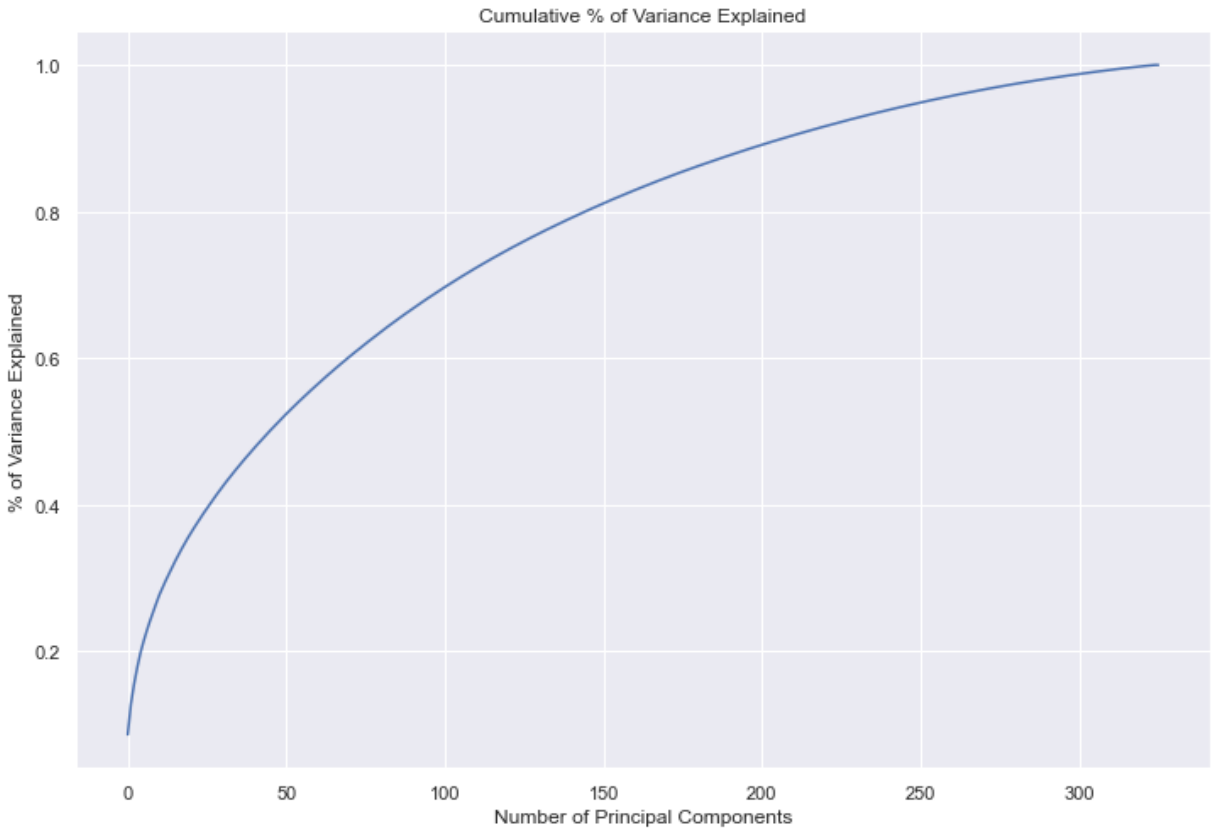
where:

- F_t is a set of $k << n$ unobserved factors;
- $\boldsymbol{V}_t$ is the vector of idiosyncratic errors;
- $\boldsymbol{\Lambda}$ is the matrix of unobserved factor loadings.

(a) (30 points)

Compute the principal components of the returns and determine the optimal number of principal factors by one the methods described in Lecture 2. How much of the variance will the factors be able to explain?

In [ ]:
```
pcs, gammas, lambdas, alphas = PCA_function(returns)
```



The transformed dataset containing only the first k PCs is the $(T \times k)$ matrix is given by

$$\begin{aligned} Z_{(k)} &:= \boldsymbol{X}\boldsymbol{\Gamma}_k \\ &:= (Z_1, \ldots, Z_k)\,. \end{aligned}$$

In [ ]:
```
pcs
```

Out[ ]:

dates	PC 1	PC 2	PC 3	PC 4	PC 5	PC 6	PC 7	PC 8	PC
1991-11-29	-0.932311	0.655585	0.023695	-0.023417	-0.126589	0.216290	-0.287319	0.317324	0.36757
1991-12-31	0.684426	-0.766075	0.098573	0.101246	0.403129	-0.197598	0.258110	-0.600949	-0.53229
1992-01-31	2.223682	-0.458117	-0.226293	-0.012401	0.090334	-0.388412	0.722587	-1.958945	-1.08676
1992-02-28	0.562873	-0.140579	0.159400	0.006865	0.158465	-0.255173	0.436401	-0.784899	-0.00900
1992-03-31	-0.743650	0.306019	0.224420	-0.089483	-0.141921	0.050060	-0.242078	0.170550	0.23308
...	...	...	...	...	...	...	...	...	
2018-08-31	0.161871	-0.265360	-0.020674	-0.097998	0.026575	-0.056679	-0.109673	-0.051071	0.16519
2018-09-28	-0.942054	0.462936	0.124402	-0.082200	-0.051525	0.076335	-0.044931	0.278179	0.38150
2018-10-31	-2.535630	2.642636	-0.063295	0.056235	0.753975	0.545761	-0.142676	-0.170747	0.01582

dates	PC 1	PC 2	PC 3	PC 4	PC 5	PC 6	PC 7	PC 8	PC
2018-11-30	-0.418945	0.200803	0.180173	0.041064	0.087297	0.077100	-0.050033	0.015725	0.0071(
2018-12-31	-3.014504	3.133093	-0.011031	0.038419	0.759613	0.698890	-0.215036	-0.120849	0.3403(

326 rows × 326 columns

Given the desired number of PCs, say $1 \leq k \leq n$, we collect all the vectors $\gamma_1, \ldots \gamma_k$ in a $(n \times k)$ matrix

$$\Gamma_k := (\gamma_1, \ldots \gamma_k)$$

In [ ]:
```
gammas
```

Out[ ]:

	gamma 1	gamma 2	gamma 3	gamma 4	gamma 5	gamma 6	gamma 7	gamma 8	gamma
r_ 1	0.002032	-0.002777	-0.000804	0.000050	0.002960	0.001060	-0.002195	0.001189	0.0032
r_ 2	0.027143	0.015565	-0.009166	-0.002057	0.016365	0.000631	0.050304	-0.038719	0.0310
r_ 3	0.005195	-0.013071	-0.003946	-0.000183	-0.011596	-0.007778	0.006669	-0.008175	-0.0016
r_ 4	0.005707	-0.010162	-0.002122	-0.000947	0.001569	-0.003475	0.010309	-0.008786	-0.0131
r_ 5	0.012439	-0.006495	-0.005756	0.003450	0.002966	-0.002718	0.000583	0.010150	-0.0251
...	...	...	...	...	...	...	...	...	...
r_9452	0.000369	-0.000801	0.000187	0.000163	-0.000448	-0.000325	-0.000036	0.000097	-0.0000
r_9453	-0.000875	-0.000733	0.001381	-0.001983	-0.001478	-0.002800	-0.001009	0.006402	0.0069
r_9454	0.000287	-0.000691	0.000124	0.000281	-0.000370	-0.000841	0.000233	-0.000081	-0.0000
r_9455	0.002324	-0.005739	0.001035	-0.000288	-0.003415	-0.003190	0.000175	0.002162	0.0001
r_9456	0.001238	-0.003543	0.000601	-0.000469	-0.002063	-0.002022	-0.000150	0.002713	0.0014

9456 rows × 326 columns

In [ ]:
```
# PCA function centered the data before to compute the PCs
centered_returns = (returns - returns.mean(axis=0))
centered_factors = (factors - factors.mean(axis=0))
```

In [ ]:
```
# just checking
sum(round(centered_returns.dot(np.array(gammas["gamma 1"])), 10) == round(pcs["PC 1"
```

Out[ ]:
```
326
```

Also, by construction, the columns of $Z_{(k)}$ (the PCs) are orthogonal random variables, and with sample variances $\lambda_1, \ldots, \lambda_k$.

Thus, the sample covariance matrix of $Z_{(k)}$ is given by the $(k \times k)$ diagonal matrix

$$\mathbf{\Lambda}_k := \text{diag}(\lambda_1, \ldots, \lambda_k)$$

In [ ]:
```
lambdas[:10]
```

Out[ ]:
```
array([10.59291481,  5.04991983,  3.33383555,  2.76500035,  2.40685564,
        1.94349613,  1.7572486 ,  1.61955915,  1.51600711,  1.45686289])
```

It is a good idea to start by running a full PCA $(k = n)$ and plotting the quantity

$$\alpha_j = \frac{\lambda_j}{\sum_{j=1}^{n} \lambda_j}$$

for $j \in \{1, \ldots, n\}$.

In [ ]:
```
alphas[:10]
```

Out[ ]:
```
array([0.0868341 , 0.04139609, 0.0273287 , 0.02266575, 0.0197299 ,
       0.01593157, 0.01440483, 0.01327614, 0.01242728, 0.01194245])
```

In [ ]:
```
Lambda = round(pcs.cov(), 2)
```

Also, by construction, the columns of $Z_{(k)}$ (the PCs) are orthogonal random variables, and with sample variances $\lambda_1, \ldots, \lambda_k$. Thus, the sample covariance matrix of $Z_{(k)}$ is given by the $(k \times k)$ diagonal matrix

$$\mathbf{\Lambda}_k := \text{diag}(\lambda_1, \ldots, \lambda_k).$$

In [ ]:
```
Lambda.head(10)[['PC 1', 'PC 2', 'PC 3', 'PC 4', 'PC 5', 'PC 6', 'PC 7', 'PC 8', 'PC
```

Out[ ]:

	PC 1	PC 2	PC 3	PC 4	PC 5	PC 6	PC 7	PC 8	PC 9	PC 10
PC 1	10.59	0.00	0.00	0.00	0.00	0.00	-0.00	-0.00	-0.00	0.00
PC 2	0.00	5.05	-0.00	-0.00	0.00	-0.00	0.00	-0.00	-0.00	0.00
PC 3	0.00	-0.00	3.33	0.00	-0.00	0.00	0.00	-0.00	-0.00	0.00
PC 4	0.00	-0.00	0.00	2.77	0.00	0.00	-0.00	0.00	-0.00	0.00
PC 5	0.00	0.00	-0.00	0.00	2.41	-0.00	0.00	0.00	0.00	0.00
PC 6	0.00	-0.00	0.00	0.00	-0.00	1.94	0.00	0.00	-0.00	-0.00
PC 7	-0.00	0.00	0.00	-0.00	0.00	0.00	1.76	0.00	0.00	0.00
PC 8	-0.00	-0.00	-0.00	0.00	0.00	0.00	0.00	1.62	-0.00	-0.00
PC 9	-0.00	-0.00	-0.00	-0.00	0.00	-0.00	0.00	-0.00	1.52	-0.00
PC 10	0.00	0.00	0.00	0.00	0.00	-0.00	0.00	-0.00	-0.00	1.46

In [ ]:
```
lambdas[:10]
```

Out[ ]:
```
array([10.59291481,  5.04991983,  3.33383555,  2.76500035,  2.40685564,
        1.94349613,  1.7572486 ,  1.61955915,  1.51600711,  1.45686289])
```

Rule of Thumb

Stop at a k such that the $(k+1)$-th PC does not add much to the already explained variance (say $< 3\%$).

In [ ]:
```
n_pc_rt = rule_thumb(alphas)
```

```
The first 2 PCs explain 12.82% of the returns variance.
```

Informal Way

Choose the number of components such that a large portion (say 90\%) of the variance is explained.

In [ ]:
```
n_pc_iw = informal_way(alphas)
```

```
The first 207 PCs explain 89.91% of the returns variance.
```

Biggest Drop

Onatski (2010) suggests looking for the biggest drop computing

$$r := \arg\max_{1 \le j < n} \frac{\lambda_j}{\lambda_{j+1}}.$$

In [ ]:
```
n_pc_bd = biggest_drop(lambdas)
```

```
The first 1 PCs explain 8.68% of the returns variance.
```

(b) (30 points)

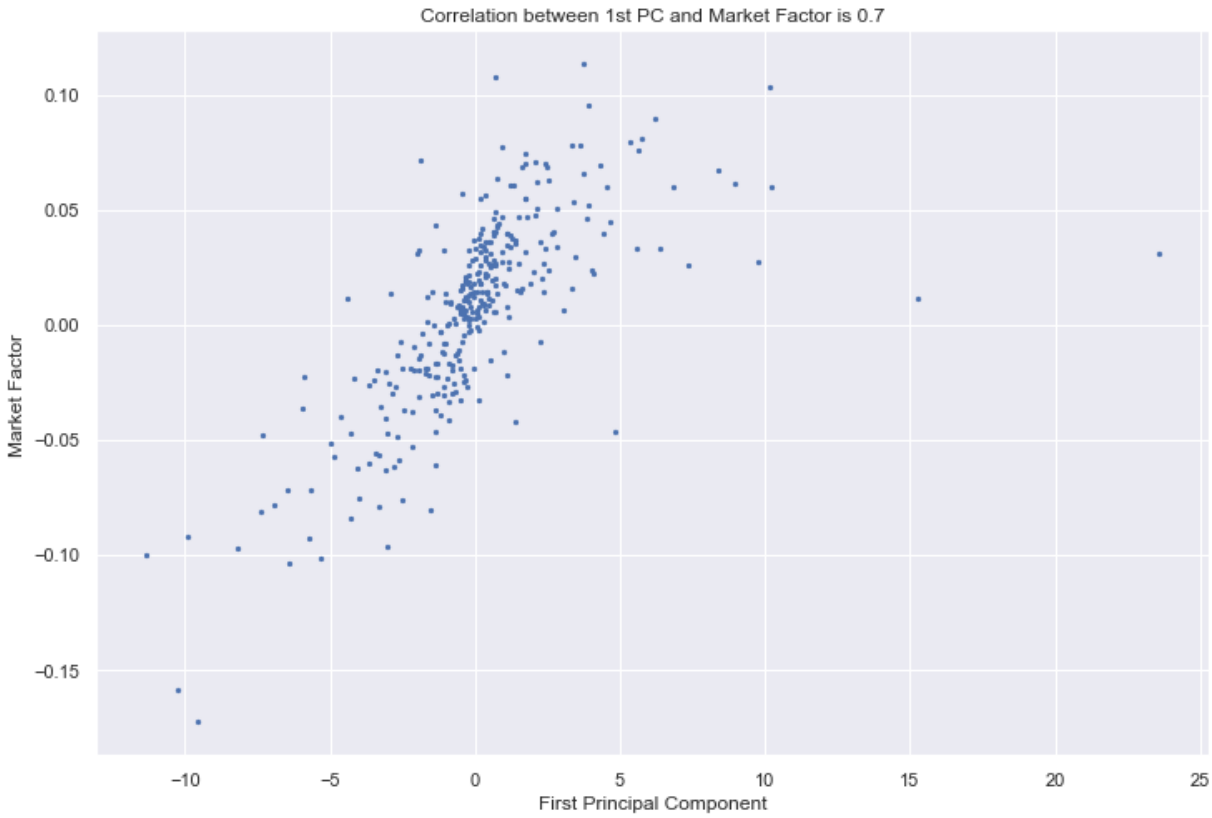
Regress the selected factors on the 16 observed "anomaly" factors described above. How do the "principal component factors" relate to the "anomaly factors"?

In [ ]:
```
# rule of thumb
pc_ret_rt = pcs.iloc[:,:n_pc_rt]
pc_ret_rt.head()
```

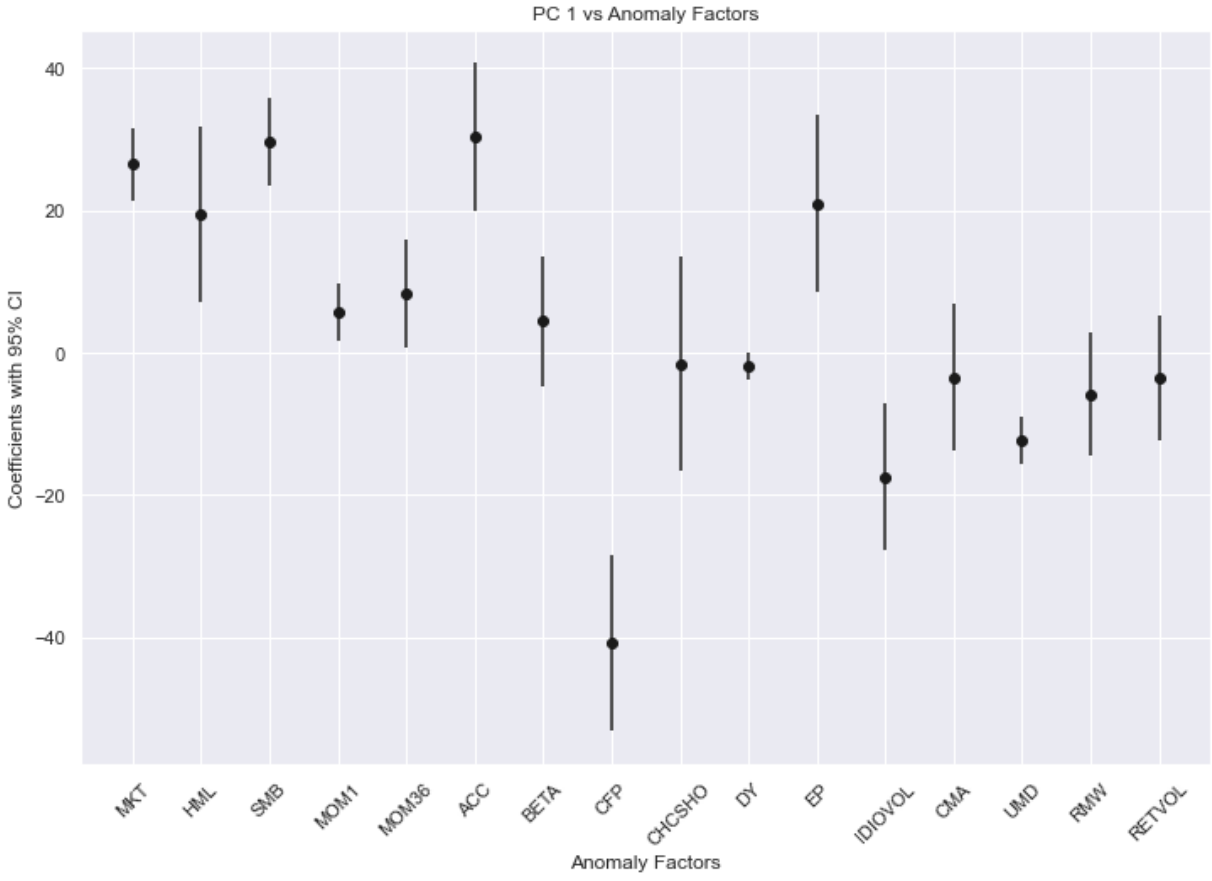
Out[ ]:

	PC 1	PC 2
dates		
1991-11-29	-0.932311	0.655585
1991-12-31	0.684426	-0.766075
1992-01-31	2.223682	-0.458117
1992-02-28	0.562873	-0.140579
1992-03-31	-0.743650	0.306019

In [ ]:
```
corr = round(factors['MKT'].corr(pc_ret_rt['PC 1']), 2)
plt.scatter(pc_ret_rt['PC 1'], factors['MKT'], s=5)
plt.xlabel('First Principal Component')
plt.ylabel('Market Factor')
plt.title(f'Correlation between 1st PC and Market Factor is {corr}');
```



In [ ]:
```
pc1 = OLS_regression(y = pc_ret_rt, X = factors, y_column = "PC 1", is_pc = False)
```



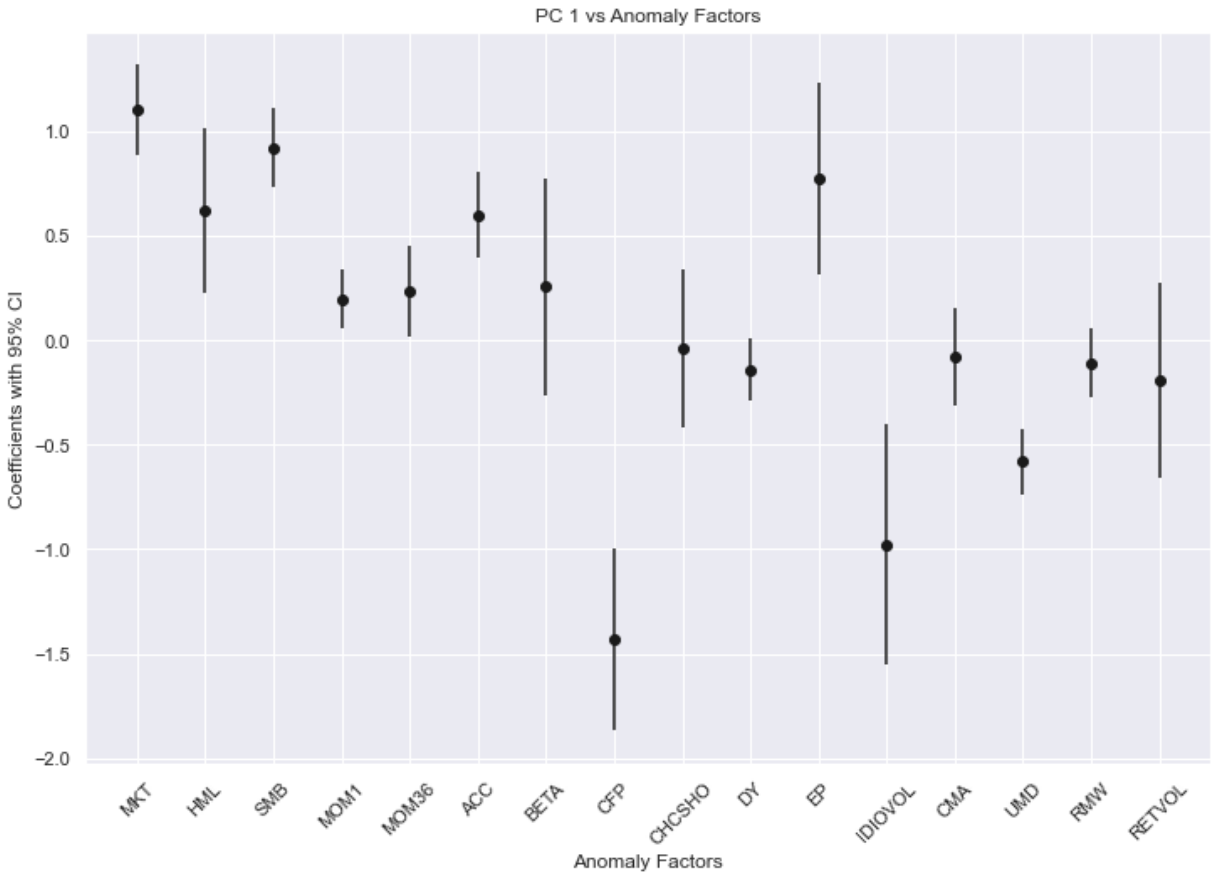
In [ ]:
```
print(pc1.summary())
```

```
                            OLS Regression Results
==============================================================================
```

```
Dep. Variable:                   PC 1   R-squared:                       0.878
Model:                           OLS   Adj. R-squared:                  0.872
Method:                Least Squares   F-statistic:                     139.3
Date:               Sat, 15 Jul 2023   Prob (F-statistic):          1.04e-130
Time:                       16:35:49   Log-Likelihood:                -503.55
No. Observations:                326   AIC:                             1041.
Df Residuals:                    309   BIC:                             1105.
Df Model:                         16
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         -0.1940      0.074     -2.629      0.009      -0.339      -0.049
MKT           26.4644      2.614     10.123      0.000      21.321      31.608
HML           19.4347      6.242      3.114      0.002       7.153      31.717
SMB           29.7067      3.142      9.455      0.000      23.524      35.889
MOM1           5.6690      2.073      2.734      0.007       1.589       9.749
MOM36          8.2647      3.877      2.131      0.034       0.635      15.894
ACC           30.3074      5.272      5.748      0.000      19.933      40.682
BETA           4.4525      4.636      0.960      0.338      -4.670      13.575
CFP          -40.7554      6.249     -6.522      0.000     -53.051     -28.460
CHCSHO        -1.5690      7.657     -0.205      0.838     -16.636      13.498
DY            -1.9132      1.005     -1.903      0.058      -3.891       0.065
EP            20.9376      6.344      3.301      0.001       8.455      33.420
IDIOVOL      -17.4350      5.192     -3.358      0.001     -27.652      -7.218
CMA           -3.4515      5.189     -0.665      0.506     -13.662       6.759
UMD          -12.2663      1.692     -7.250      0.000     -15.595      -8.937
RMW           -5.8582      4.399     -1.332      0.184     -14.514       2.798
RETVOL        -3.6166      4.429     -0.817      0.415     -12.331       5.097
==============================================================================
Omnibus:                      186.517   Durbin-Watson:                   1.791
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             3153.860
Skew:                           1.974   Prob(JB):                         0.00
Kurtosis:                      17.717   Cond. No.                         147.
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly spe
cified.
```

In [ ]:

```
pc1std = OLS_regression(y = pc_ret_rt, X = standardized_factors, y_column = "PC 1",
```



In [ ]:
```
print(pc1std.summary())
```

OLS Regression Results

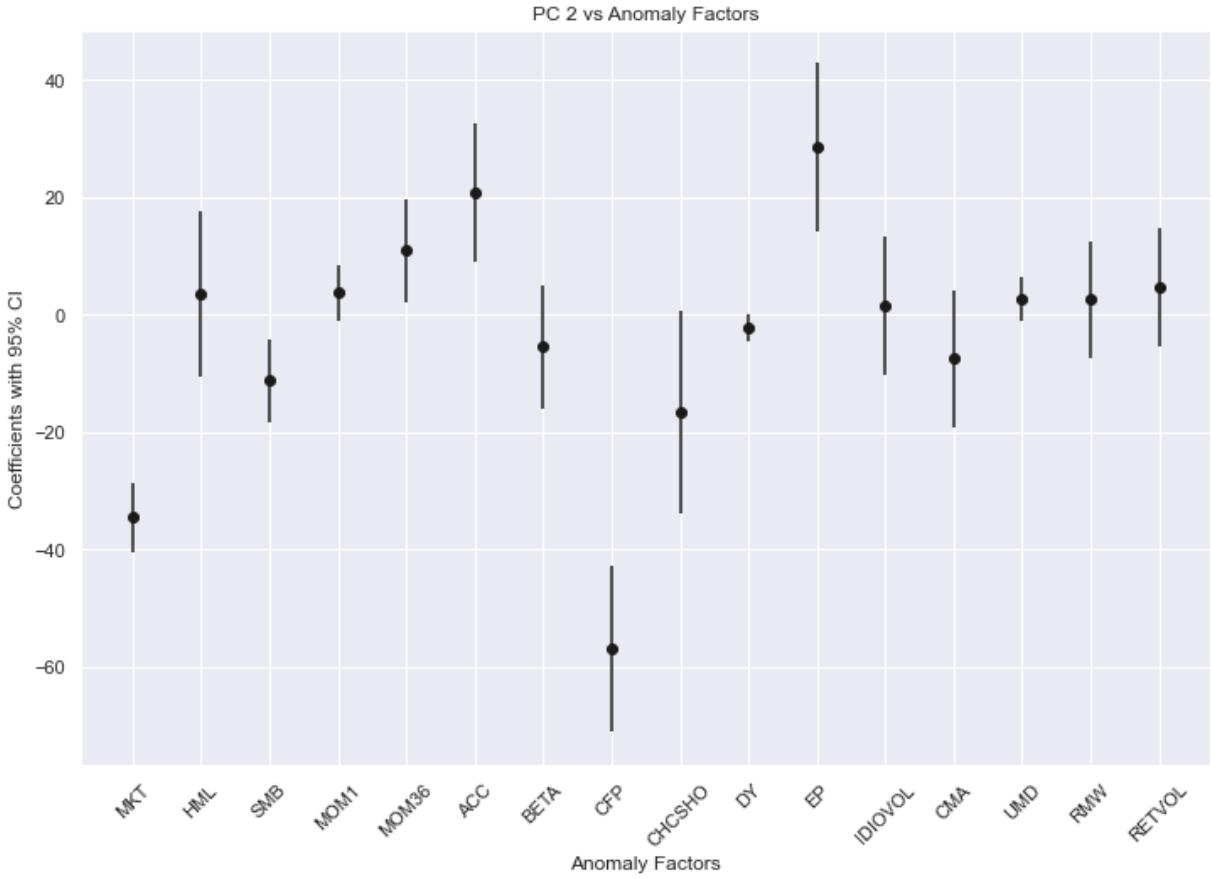
Dep. Variable:	PC 1	R-squared:	0.878
Model:	OLS	Adj. R-squared:	0.872
Method:	Least Squares	F-statistic:	139.3
Date:	Sat, 15 Jul 2023	Prob (F-statistic):	1.04e-130
Time:	16:35:50	Log-Likelihood:	-503.55
No. Observations:	326	AIC:	1041.
Df Residuals:	309	BIC:	1105.
Df Model:	16		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
const	-1.162e-16	0.065	-1.8e-15	1.000	-0.127	0.127
MKT	1.1019	0.109	10.123	0.000	0.888	1.316
HML	0.6190	0.199	3.114	0.002	0.228	1.010
SMB	0.9205	0.097	9.455	0.000	0.729	1.112
MOM1	0.1965	0.072	2.734	0.007	0.055	0.338
MOM36	0.2357	0.111	2.131	0.034	0.018	0.453
ACC	0.5986	0.104	5.748	0.000	0.394	0.803
BETA	0.2541	0.265	0.960	0.338	-0.266	0.775
CFP	-1.4315	0.219	-6.522	0.000	-1.863	-1.000
CHCSHO	-0.0394	0.192	-0.205	0.838	-0.418	0.339
DY	-0.1430	0.075	-1.903	0.058	-0.291	0.005
EP	0.7719	0.234	3.301	0.001	0.312	1.232
IDIOVOL	-0.9774	0.291	-3.358	0.001	-1.550	-0.405
CMA	-0.0798	0.120	-0.665	0.506	-0.316	0.156
UMD	-0.5809	0.080	-7.250	0.000	-0.739	-0.423
RMW	-0.1117	0.084	-1.332	0.184	-0.277	0.053
RETVOL	-0.1943	0.238	-0.817	0.415	-0.663	0.274

Omnibus:	186.517	Durbin-Watson:	1.791

```
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             3153.860
Skew:                           1.974   Prob(JB):                         0.00
Kurtosis:                      17.717   Cond. No.                         14.8
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly spe
cified.
```

In [ ]:
```
pc2 = OLS_regression(y = pc_ret_rt, X = factors, y_column = "PC 2", is_pc = False)
```



In [ ]:
```
print(pc2.summary())
```

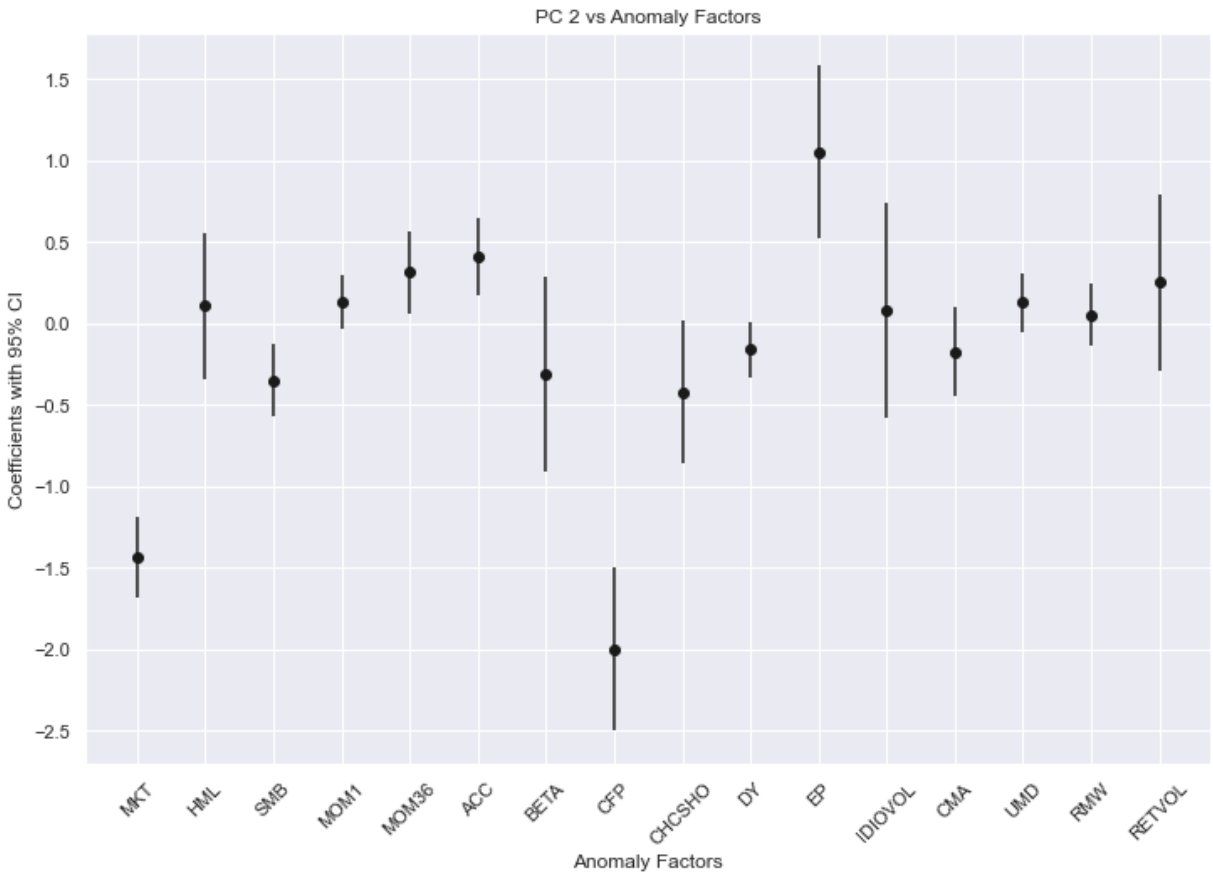
```
                            OLS Regression Results
==============================================================================
Dep. Variable:                   PC 2   R-squared:                       0.662
Model:                           OLS   Adj. R-squared:                  0.645
Method:                Least Squares   F-statistic:                     37.87
Date:               Sat, 15 Jul 2023   Prob (F-statistic):           4.19e-63
Time:                       16:35:50   Log-Likelihood:                -549.10
No. Observations:                326   AIC:                             1132.
Df Residuals:                    309   BIC:                             1197.
Df Model:                         16
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.3155      0.085      3.718      0.000       0.149       0.482
MKT          -34.5536      3.006    -11.494      0.000     -40.469     -28.638
HML            3.3906      7.178      0.472      0.637     -10.733      17.514
SMB          -11.3074      3.613     -3.130      0.002     -18.417      -4.198
MOM1           3.7126      2.384      1.557      0.120      -0.979       8.404
MOM36         10.9439      4.459      2.454      0.015       2.170      19.718
ACC           20.8124      6.063      3.433      0.001       8.883      32.742
```

BETA	-5.5253	5.331	-1.036	0.301	-16.015	4.965
CFP	-56.8628	7.186	-7.913	0.000	-71.002	-42.724
CHCSHO	-16.7244	8.805	-1.899	0.058	-34.050	0.601
DY	-2.1662	1.156	-1.874	0.062	-4.440	0.108
EP	28.5162	7.295	3.909	0.000	14.162	42.870
IDIOVOL	1.4526	5.971	0.243	0.808	-10.296	13.201
CMA	-7.5439	5.967	-1.264	0.207	-19.286	4.198
UMD	2.6894	1.946	1.382	0.168	-1.139	6.518
RMW	2.6344	5.059	0.521	0.603	-7.320	12.589
RETVOL	4.6764	5.093	0.918	0.359	-5.344	14.697

Omnibus:	257.183	Durbin-Watson:	1.685
Prob(Omnibus):	0.000	Jarque-Bera (JB):	11439.574
Skew:	2.753	Prob(JB):	0.00
Kurtosis:	31.493	Cond. No.	147.

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly spe
cified.

In [ ]:
```
pc2std = OLS_regression(y = pc_ret_rt, X = standardized_factors, y_column = "PC 2",
```



In [ ]:
```
print(pc2std.summary())
```

OLS Regression Results

Dep. Variable:	PC 2	R-squared:	0.662
Model:	OLS	Adj. R-squared:	0.645
Method:	Least Squares	F-statistic:	37.87
Date:	Sat, 15 Jul 2023	Prob (F-statistic):	4.19e-63
Time:	16:35:50	Log-Likelihood:	-549.10
No. Observations:	326	AIC:	1132.
Df Residuals:	309	BIC:	1197.

```
Df Model:                           16
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const       4.684e-17      0.074   6.31e-16      1.000      -0.146       0.146
MKT            -1.4387      0.125    -11.494      0.000      -1.685      -1.192
HML             0.1080      0.229      0.472      0.637      -0.342       0.558
SMB            -0.3504      0.112     -3.130      0.002      -0.571      -0.130
MOM1            0.1287      0.083      1.557      0.120      -0.034       0.291
MOM36           0.3122      0.127      2.454      0.015       0.062       0.562
ACC             0.4110      0.120      3.433      0.001       0.175       0.647
BETA           -0.3153      0.304     -1.036      0.301      -0.914       0.283
CFP            -1.9972      0.252     -7.913      0.000      -2.494      -1.501
CHCSHO         -0.4201      0.221     -1.899      0.058      -0.855       0.015
DY             -0.1619      0.086     -1.874      0.062      -0.332       0.008
EP              1.0513      0.269      3.909      0.000       0.522       1.581
IDIOVOL         0.0814      0.335      0.243      0.808      -0.577       0.740
CMA            -0.1744      0.138     -1.264      0.207      -0.446       0.097
UMD             0.1274      0.092      1.382      0.168      -0.054       0.309
RMW             0.0502      0.096      0.521      0.603      -0.140       0.240
RETVOL          0.2513      0.274      0.918      0.359      -0.287       0.790
==============================================================================
Omnibus:                      257.183   Durbin-Watson:                   1.685
Prob(Omnibus):                  0.000   Jarque-Bera (JB):            11439.574
Skew:                           2.753   Prob(JB):                         0.00
Kurtosis:                      31.493   Cond. No.                         14.8
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly spe
cified.
```

In [ ]:
```
corr_matrix = round(pd.concat([pc_ret_rt, factors], axis=1).corr(), 2)
corr_matrix[factors.columns][:2]
```

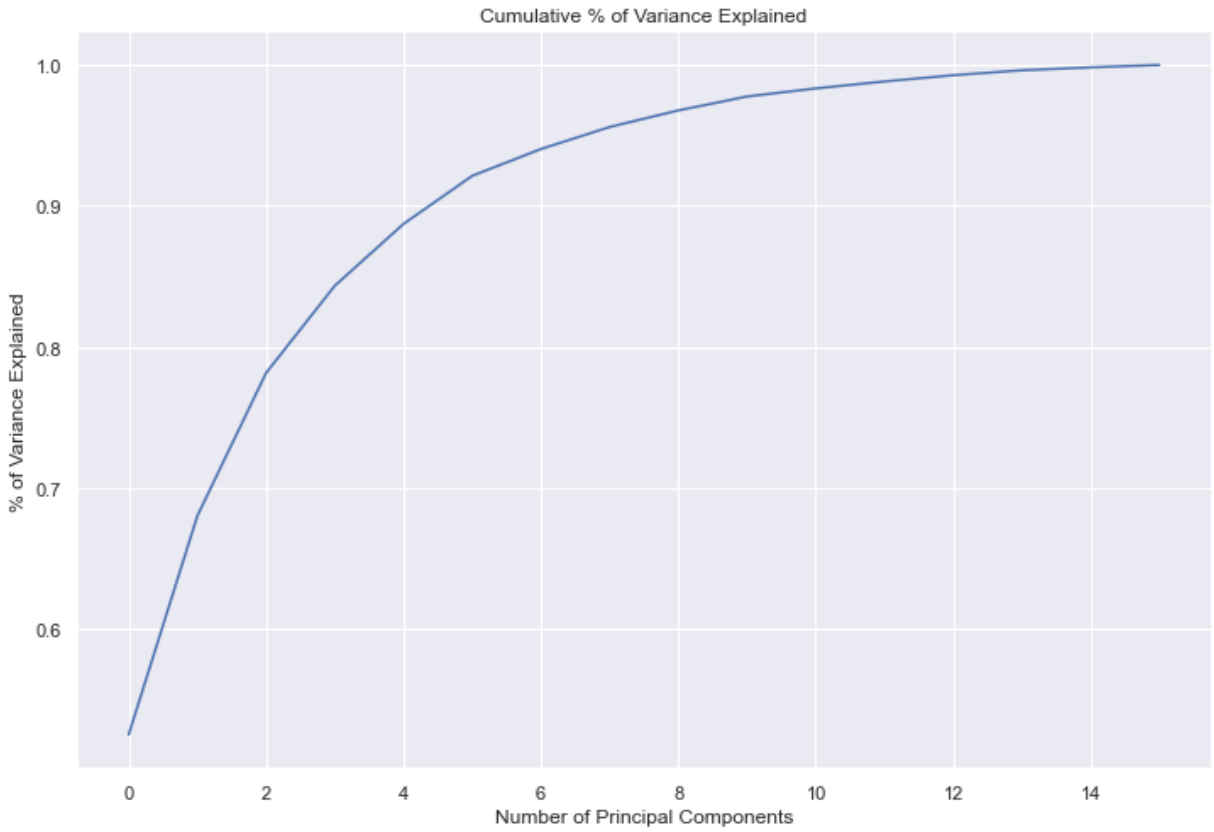
Out[ ]:

	MKT	HML	SMB	MOM1	MOM36	ACC	BETA	CFP	CHCSHO	DY	EP	IDIOVOL	CMA
PC 1	0.70	-0.38	0.70	0.31	0.16	0.40	-0.87	-0.54	-0.73	-0.39	-0.69	-0.85	-0.36
PC 2	-0.47	-0.46	-0.03	-0.11	-0.19	0.21	0.11	-0.47	-0.15	-0.12	-0.24	0.00	-0.24

(c) (30 points)

Now, run a principal component analysis on the 16 "anomaly factors" and select the optimal number of principal components using the same criterion adopted in the first item of the exercise. By inspecting the principal eigenvectors can you identify a dominating "anomaly"?

In [ ]:
```
pcs, gammas, lambdas, alphas = PCA_function(factors)
```



The transformed dataset containing only the first k PCs is the $(T \times k)$ matrix is given by

$$\begin{aligned} Z_{(k)} &:= \boldsymbol{X}\boldsymbol{\Gamma}_k \\ &:= (Z_1, \ldots, Z_k)\,. \end{aligned}$$

In [ ]:
```
pcs
```

Out[ ]:

dates	PC 1	PC 2	PC 3	PC 4	PC 5	PC 6	PC 7	PC 8	PC
1991-11-29	-0.040773	-0.040907	-0.034789	-0.042608	0.021649	0.016281	-0.002945	0.013785	-0.01715
1991-12-31	0.112512	-0.012316	-0.038009	0.025986	-0.097625	-0.067016	0.003296	0.040766	-0.00733
1992-01-31	0.121870	0.061630	0.066577	0.094953	0.039049	0.094894	-0.019855	-0.018094	0.02758
1992-02-28	-0.005106	0.043395	0.047053	0.084682	0.035254	-0.009129	0.001453	-0.009655	0.01877
1992-03-31	-0.108007	0.012929	0.026053	0.021665	0.019578	0.005955	0.008637	-0.012087	-0.00738
...	...	...	...	...	...	...	...	...	
2018-08-31	0.070953	-0.020014	-0.064739	-0.014552	-0.031384	0.001708	0.000216	0.006447	0.03309
2018-09-28	0.006513	-0.057196	0.004248	-0.013213	0.005573	-0.003490	0.028139	-0.015625	0.00580
2018-10-31	-0.167718	-0.033319	0.045695	-0.010240	0.027620	0.030816	0.020208	-0.008359	-0.03652

dates	PC 1	PC 2	PC 3	PC 4	PC 5	PC 6	PC 7	PC 8	PC
2018-11-30	-0.058530	0.037814	-0.010477	-0.049877	-0.016665	-0.006877	0.009479	0.020282	0.00273
2018-12-31	-0.101082	0.010640	-0.083628	-0.054505	0.049028	0.031435	0.002180	-0.014540	-0.03352

326 rows × 16 columns

Given the desired number of PCs, say $1 \leq k \leq n$, we collect all the vectors $\gamma_1, \ldots \gamma_k$ in a $(n \times k)$ matrix

$$\Gamma_k := (\gamma_1, \ldots \gamma_k)$$

In [ ]:
```
gammas
```

Out[ ]:

	gamma 1	gamma 2	gamma 3	gamma 4	gamma 5	gamma 6	gamma 7	gamma 8	gam
MKT	0.228449	0.191721	0.189615	0.091736	-0.553227	-0.518328	0.175670	0.206177	0.36
HML	-0.154749	0.062009	0.342674	0.383140	-0.006026	0.005076	-0.142897	-0.089115	-0.18
SMB	0.168694	0.058650	0.028389	0.137712	0.194181	0.452578	-0.387099	0.476201	0.35
MOM1	0.066700	0.065963	0.211633	-0.201040	-0.644130	0.663915	0.070393	-0.177102	-0.05
MOM36	0.003330	0.127738	0.096021	0.497916	0.209302	0.227409	0.441680	-0.126307	0.40
ACC	0.081187	0.003743	-0.132634	-0.012960	0.050161	0.014296	-0.020514	-0.583009	0.32
BETA	-0.447878	-0.218960	-0.081117	-0.134744	0.020983	0.067217	0.129617	0.021959	0.16
CFP	-0.204639	0.012831	0.344791	0.252459	-0.054861	-0.105223	-0.433638	-0.350681	0.13
CHCSHO	-0.182823	-0.029610	0.115116	0.107098	0.027947	0.004601	0.053857	0.082061	0.13
DY	-0.388018	0.862257	-0.301306	-0.070640	-0.025850	0.012490	-0.094100	0.008833	-0.00
EP	-0.256139	-0.056180	0.290519	0.065395	-0.105811	-0.090116	-0.339623	0.191388	-0.12
IDIOVOL	-0.447106	-0.141603	0.120198	-0.128713	-0.067057	0.024197	0.211033	0.162011	0.03
CMA	-0.105896	0.053198	0.110473	0.264680	-0.007434	0.081863	0.393200	0.296365	-0.20
UMD	-0.081444	-0.254533	-0.653139	0.526627	-0.407090	0.031903	-0.176837	0.052899	-0.06
RMW	0.011606	-0.055604	-0.054078	-0.243840	-0.045945	-0.031349	-0.182397	0.216336	0.46
RETVOL	-0.417622	-0.221104	-0.064418	-0.106175	-0.080566	-0.024605	0.052871	-0.070598	0.28

Also, by construction, the columns of $Z_{(k)}$ (the PCs) are orthogonal random variables, and with sample variances $\lambda_1, \ldots, \lambda_k$.

Thus, the sample covariance matrix of $Z_{(k)}$ is given by the $(k \times k)$ diagonal matrix

$$\mathbf{\Lambda}_k := \mathrm{diag}(\lambda_1, \ldots, \lambda_k)$$

In [ ]:
```
lambdas[:10]
```

Out[ ]:
```
array([0.0143643 , 0.00424359, 0.00277031, 0.00168956, 0.00119888,
       0.00093209, 0.00052007, 0.00042722, 0.00032125, 0.00027015])
```

It is a good idea to start by running a full PCA $(k = n)$ and plotting the quantity

$$\alpha_j = \frac{\lambda_j}{\sum_{j=1}^{n} \lambda_j}$$

for $j \in \{1, \ldots, n\}$.

In [ ]:
```
alphas[:10]
```

Out[ ]:
```
array([0.52522946, 0.15516658, 0.10129623, 0.06177849, 0.04383691,
       0.03408185, 0.01901649, 0.01562144, 0.01174643, 0.00987803])
```

In [ ]:
```
Lambda = round(pcs.cov(), 4)
```

Also, by construction, the columns of $Z_{(k)}$ (the PCs) are orthogonal random variables, and with sample variances $\lambda_1, \ldots, \lambda_k$. Thus, the sample covariance matrix of $Z_{(k)}$ is given by the $(k \times k)$ diagonal matrix

$$\mathbf{\Lambda}_k := \mathrm{diag}(\lambda_1, \ldots, \lambda_k).$$

In [ ]:
```
Lambda.head(10)[['PC 1', 'PC 2', 'PC 3', 'PC 4', 'PC 5', 'PC 6', 'PC 7', 'PC 8', 'PC
```

Out[ ]:

	PC 1	PC 2	PC 3	PC 4	PC 5	PC 6	PC 7	PC 8	PC 9	PC 10
PC 1	0.0144	0.0000	0.0000	0.0000	-0.0000	-0.0000	0.0000	0.0000	0.0000	0.0000
PC 2	0.0000	0.0042	0.0000	0.0000	-0.0000	0.0000	-0.0000	0.0000	-0.0000	-0.0000
PC 3	0.0000	0.0000	0.0028	-0.0000	0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000
PC 4	0.0000	0.0000	-0.0000	0.0017	0.0000	0.0000	-0.0000	-0.0000	0.0000	0.0000
PC 5	-0.0000	-0.0000	0.0000	0.0000	0.0012	-0.0000	0.0000	-0.0000	-0.0000	-0.0000
PC 6	-0.0000	0.0000	-0.0000	0.0000	-0.0000	0.0009	0.0000	0.0000	0.0000	0.0000
PC 7	0.0000	-0.0000	-0.0000	-0.0000	0.0000	0.0000	0.0005	0.0000	-0.0000	-0.0000
PC 8	0.0000	0.0000	-0.0000	-0.0000	-0.0000	0.0000	0.0000	0.0004	0.0000	0.0000
PC 9	0.0000	-0.0000	-0.0000	0.0000	-0.0000	0.0000	-0.0000	0.0000	0.0003	0.0000
PC 10	0.0000	-0.0000	-0.0000	0.0000	-0.0000	0.0000	-0.0000	0.0000	0.0000	0.0003

In [ ]:
```
lambdas[:10]
```

Out[ ]:
```
array([0.0143643 , 0.00424359, 0.00277031, 0.00168956, 0.00119888,
       0.00093209, 0.00052007, 0.00042722, 0.00032125, 0.00027015])
```

Rule of Thumb

Stop at a k such that the $(k+1)$-th PC does not add much to the already explained variance (say $< 3\%$).

In [ ]:
```
n_pc_rt = rule_thumb(alphas)
```

```
The first 6 PCs explain 92.14% of the returns variance.
```

Informal Way

Choose the number of components such that a large portion (say 90\%) of the variance is explained.

In [ ]:
```
n_pc_iw = informal_way(alphas)
```

```
The first 5 PCs explain 88.73% of the returns variance.
```

Biggest Drop

Onatski (2010) suggests looking for the biggest drop computing

$$r := \arg\max_{1 \le j < n} \frac{\lambda_j}{\lambda_{j+1}}.$$

In [ ]:
```
n_pc_bd = biggest_drop(lambdas)
```

```
The first 1 PCs explain 52.52% of the returns variance.
```

In [ ]:
```
# rule of thumb
pc_fac_rt = pcs.iloc[:,:n_pc_rt]
pc_fac_rt.head()
```

Out[ ]:

dates	PC 1	PC 2	PC 3	PC 4	PC 5	PC 6
1991-11-29	-0.040773	-0.040907	-0.034789	-0.042608	0.021649	0.016281
1991-12-31	0.112512	-0.012316	-0.038009	0.025986	-0.097625	-0.067016
1992-01-31	0.121870	0.061630	0.066577	0.094953	0.039049	0.094894
1992-02-28	-0.005106	0.043395	0.047053	0.084682	0.035254	-0.009129
1992-03-31	-0.108007	0.012929	0.026053	0.021665	0.019578	0.005955

By inspecting the principal eigenvectors can you identify a dominating anomaly?

In [ ]:
```
gammas
```

Out[ ]:

	gamma 1	gamma 2	gamma 3	gamma 4	gamma 5	gamma 6	gamma 7	gamma 8	gam
MKT	0.228449	0.191721	0.189615	0.091736	-0.553227	-0.518328	0.175670	0.206177	0.36
HML	-0.154749	0.062009	0.342674	0.383140	-0.006026	0.005076	-0.142897	-0.089115	-0.18
SMB	0.168694	0.058650	0.028389	0.137712	0.194181	0.452578	-0.387099	0.476201	0.35
MOM1	0.066700	0.065963	0.211633	-0.201040	-0.644130	0.663915	0.070393	-0.177102	-0.05
MOM36	0.003330	0.127738	0.096021	0.497916	0.209302	0.227409	0.441680	-0.126307	0.40

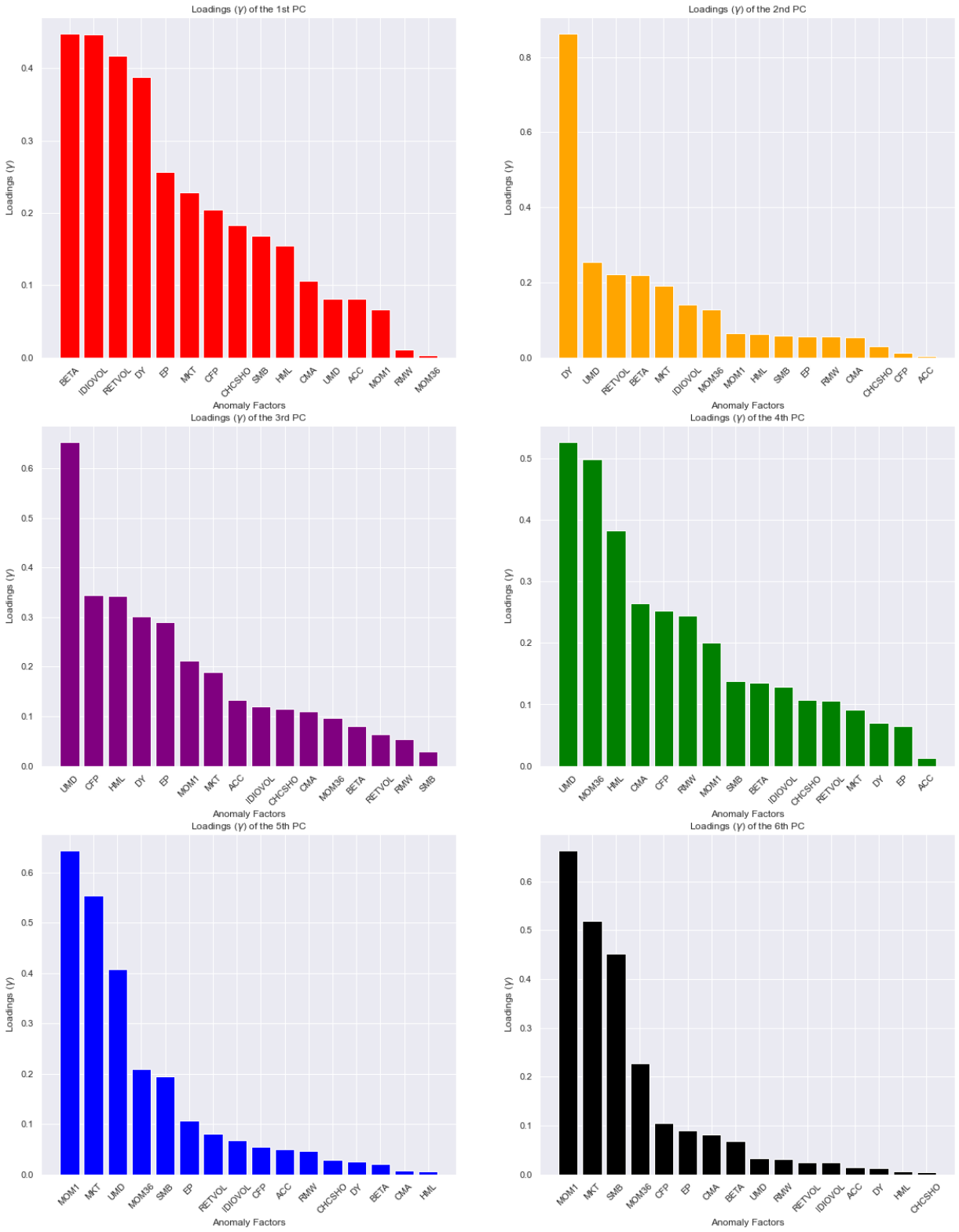
	gamma 1	gamma 2	gamma 3	gamma 4	gamma 5	gamma 6	gamma 7	gamma 8	gam
ACC	0.081187	0.003743	-0.132634	-0.012960	0.050161	0.014296	-0.020514	-0.583009	0.32
BETA	-0.447878	-0.218960	-0.081117	-0.134744	0.020983	0.067217	0.129617	0.021959	0.16
CFP	-0.204639	0.012831	0.344791	0.252459	-0.054861	-0.105223	-0.433638	-0.350681	0.13
CHCSHO	-0.182823	-0.029610	0.115116	0.107098	0.027947	0.004601	0.053857	0.082061	0.13
DY	-0.388018	0.862257	-0.301306	-0.070640	-0.025850	0.012490	-0.094100	0.008833	-0.00
EP	-0.256139	-0.056180	0.290519	0.065395	-0.105811	-0.090116	-0.339623	0.191388	-0.12
IDIOVOL	-0.447106	-0.141603	0.120198	-0.128713	-0.067057	0.024197	0.211033	0.162011	0.03
CMA	-0.105896	0.053198	0.110473	0.264680	-0.007434	0.081863	0.393200	0.296365	-0.20
UMD	-0.081444	-0.254533	-0.653139	0.526627	-0.407090	0.031903	-0.176837	0.052899	-0.06
RMW	0.011606	-0.055604	-0.054078	-0.243840	-0.045945	-0.031349	-0.182397	0.216336	0.46
RETVOL	-0.417622	-0.221104	-0.064418	-0.106175	-0.080566	-0.024605	0.052871	-0.070598	0.28

In [ ]:
```
sum(round(centered_factors.dot(np.array(gammas["gamma 1"])), 10) == round(pcs["PC 1"
```

Out[ ]:
```
326
```

In [ ]:
```
fig, ax = plt.subplots(nrows=3, ncols=2, figsize=(20, 26))

aux1 = gammas["gamma 1"].abs().sort_values(ascending=False)
ax[0,0].bar(aux1.index, aux1.values, color='red')
ax[0,0].tick_params(axis='x', rotation=45)
ax[0,0].set_xlabel('Anomaly Factors')
ax[0,0].set_ylabel('Loadings ($\gamma$)')
ax[0,0].set_title(f'Loadings ($\gamma$) of the 1st PC');

aux2 = gammas["gamma 2"].abs().sort_values(ascending=False)
ax[0,1].bar(aux2.index, aux2.values, color='orange')
ax[0,1].tick_params(axis='x', rotation=45)
ax[0,1].set_xlabel('Anomaly Factors')
ax[0,1].set_ylabel('Loadings ($\gamma$)')
ax[0,1].set_title(f'Loadings ($\gamma$) of the 2nd PC');

aux3 = gammas["gamma 3"].abs().sort_values(ascending=False)
ax[1,0].bar(aux3.index, aux3.values, color='purple')
ax[1,0].tick_params(axis='x', rotation=45)
ax[1,0].set_xlabel('Anomaly Factors')
ax[1,0].set_ylabel('Loadings ($\gamma$)')
ax[1,0].set_title(f'Loadings ($\gamma$) of the 3rd PC');

aux4 = gammas["gamma 4"].abs().sort_values(ascending=False)
ax[1,1].bar(aux4.index, aux4.values, color='green')
ax[1,1].tick_params(axis='x', rotation=45)
ax[1,1].set_xlabel('Anomaly Factors')
ax[1,1].set_ylabel('Loadings ($\gamma$)')
ax[1,1].set_title(f'Loadings ($\gamma$) of the 4th PC');

aux5 = gammas["gamma 5"].abs().sort_values(ascending=False)
ax[2,0].bar(aux5.index, aux5.values, color='blue')
ax[2,0].tick_params(axis='x', rotation=45)
ax[2,0].set_xlabel('Anomaly Factors')
```

```
ax[2,0].set_ylabel('Loadings ($\gamma$)')
ax[2,0].set_title(f'Loadings ($\gamma$) of the 5th PC');

aux6 = gammas["gamma 6"].abs().sort_values(ascending=False)
ax[2,1].bar(aux6.index, aux6.values, color='black')
ax[2,1].tick_params(axis='x', rotation=45)
ax[2,1].set_xlabel('Anomaly Factors')
ax[2,1].set_ylabel('Loadings ($\gamma$)')
ax[2,1].set_title(f'Loadings ($\gamma$) of the 6th PC');
```



In [ ]:

```
gammas["gamma 1"]
```

Out[ ]:

```
MKT      0.228449
HML     -0.154749
```

```
SMB         0.168694
MOM1        0.066700
MOM36       0.003330
ACC         0.081187
BETA       -0.447878
CFP        -0.204639
CHCSHO     -0.182823
DY         -0.388018
EP         -0.256139
IDIOVOL    -0.447106
CMA        -0.105896
UMD        -0.081444
RMW         0.011606
RETVOL     -0.417622
Name: gamma 1, dtype: float64
```

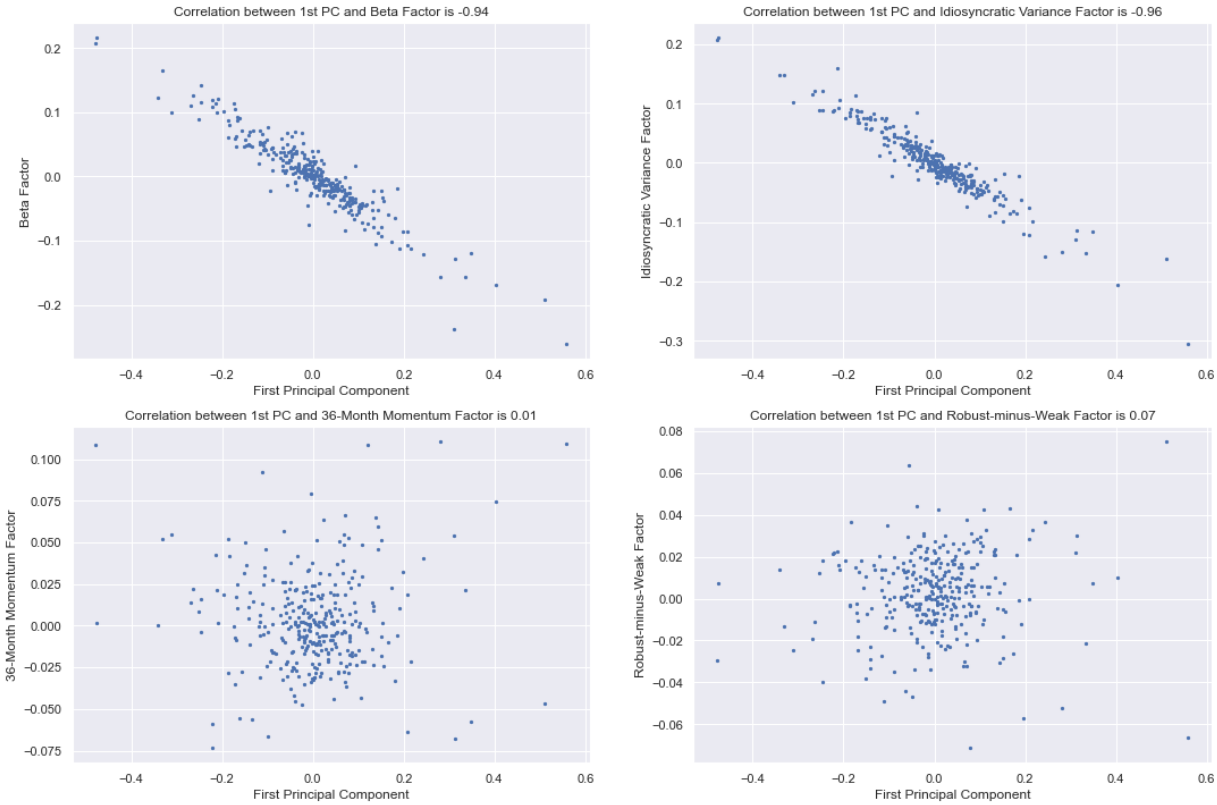
In [ ]:
```
(gammas["gamma 1"].abs()).nlargest(2)
```

Out[ ]:
```
BETA       0.447878
IDIOVOL    0.447106
Name: gamma 1, dtype: float64
```

In [ ]:
```
(gammas["gamma 1"].abs()).nsmallest(2)
```

Out[ ]:
```
MOM36    0.003330
RMW      0.011606
Name: gamma 1, dtype: float64
```

In [ ]:
```
fig, ax = plt.subplots(nrows=2, ncols=2, figsize=(18, 12))

corr_1 = round(factors['BETA'].corr(pc_fac_rt['PC 1']), 2)
ax[0,0].scatter(pc_fac_rt['PC 1'], factors['BETA'], s=5)
ax[0,0].set_xlabel('First Principal Component')
ax[0,0].set_ylabel('Beta Factor')
ax[0,0].set_title(f'Correlation between 1st PC and Beta Factor is {corr_1}');

corr_2 = round(factors['IDIOVOL'].corr(pc_fac_rt['PC 1']), 2)
ax[0,1].scatter(pc_fac_rt['PC 1'], factors['IDIOVOL'], s=5)
ax[0,1].set_xlabel('First Principal Component')
ax[0,1].set_ylabel('Idiosyncratic Variance Factor')
ax[0,1].set_title(f'Correlation between 1st PC and Idiosyncratic Variance Factor is

corr_3 = round(factors['MOM36'].corr(pc_fac_rt['PC 1']), 2)
ax[1,0].scatter(pc_fac_rt['PC 1'], factors['MOM36'], s=5)
ax[1,0].set_xlabel('First Principal Component')
ax[1,0].set_ylabel('36-Month Momentum Factor')
ax[1,0].set_title(f'Correlation between 1st PC and 36-Month Momentum Factor is {corr

corr_4 = round(factors['RMW'].corr(pc_fac_rt['PC 1']), 2)
ax[1,1].scatter(pc_fac_rt['PC 1'], factors['RMW'], s=5)
ax[1,1].set_xlabel('First Principal Component')
ax[1,1].set_ylabel('Robust-minus-Weak Factor')
ax[1,1].set_title(f'Correlation between 1st PC and Robust-minus-Weak Factor is {corr
```



(d) (30 points)

How do the "anomaly-based principal factors" related to the "return-based principal factors"?

In [ ]:
```
pc_ret_rt.head()
```

Out[ ]:

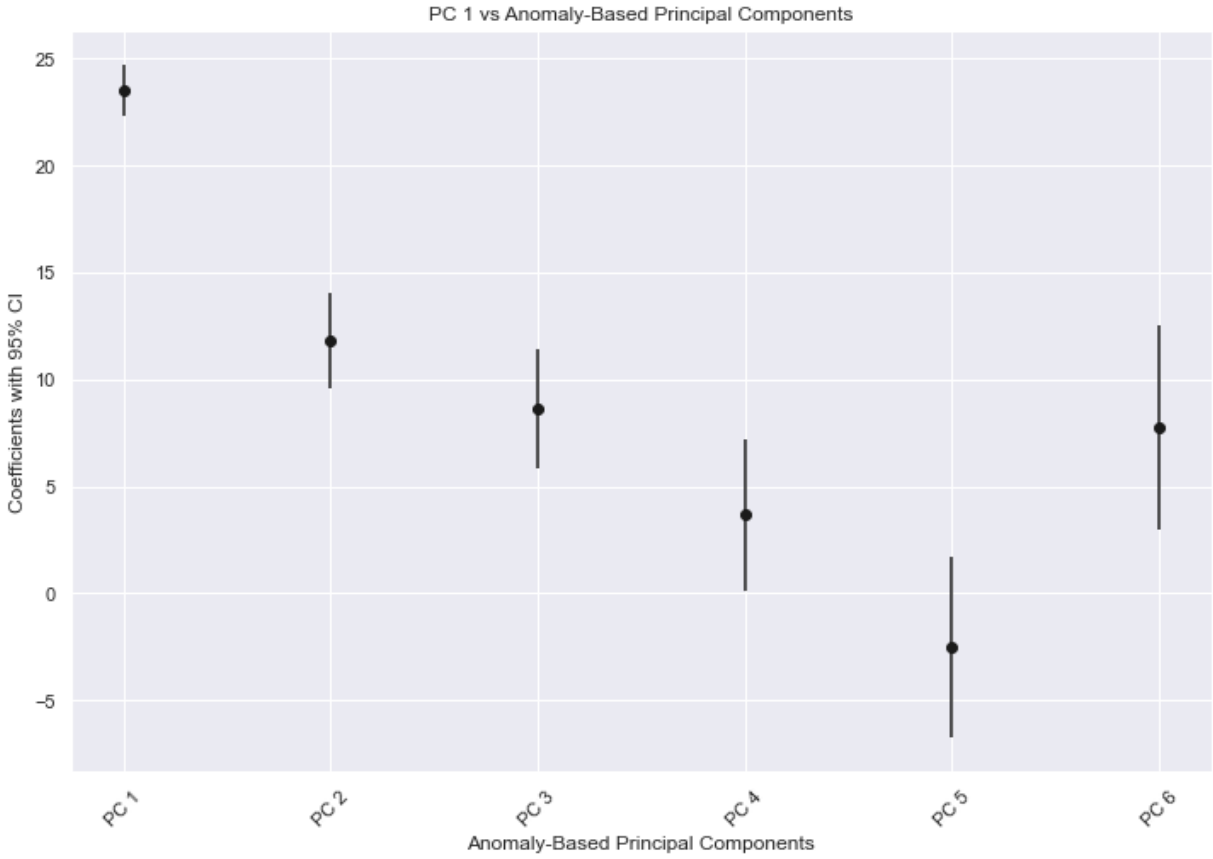
dates	PC 1	PC 2
1991-11-29	-0.932311	0.655585
1991-12-31	0.684426	-0.766075
1992-01-31	2.223682	-0.458117
1992-02-28	0.562873	-0.140579
1992-03-31	-0.743650	0.306019

In [ ]:
```
pc_fac_rt.head()
```

Out[ ]:

dates	PC 1	PC 2	PC 3	PC 4	PC 5	PC 6
1991-11-29	-0.040773	-0.040907	-0.034789	-0.042608	0.021649	0.016281
1991-12-31	0.112512	-0.012316	-0.038009	0.025986	-0.097625	-0.067016
1992-01-31	0.121870	0.061630	0.066577	0.094953	0.039049	0.094894
1992-02-28	-0.005106	0.043395	0.047053	0.084682	0.035254	-0.009129
1992-03-31	-0.108007	0.012929	0.026053	0.021665	0.019578	0.005955

In [ ]:
```
pc1 = OLS_regression(y = pc_ret_rt, X = pc_fac_rt, y_column = "PC 1", is_pc = True)
```



In [ ]:
```
print(pc1.summary())
```

OLS Regression Results

Dep. Variable:	PC 1	R-squared:	0.834
Model:	OLS	Adj. R-squared:	0.831
Method:	Least Squares	F-statistic:	268.0
Date:	Sat, 15 Jul 2023	Prob (F-statistic):	2.33e-121
Time:	17:02:03	Log-Likelihood:	-553.61
No. Observations:	326	AIC:	1121.
Df Residuals:	319	BIC:	1148.
Df Model:	6		
Covariance Type:	nonrobust		

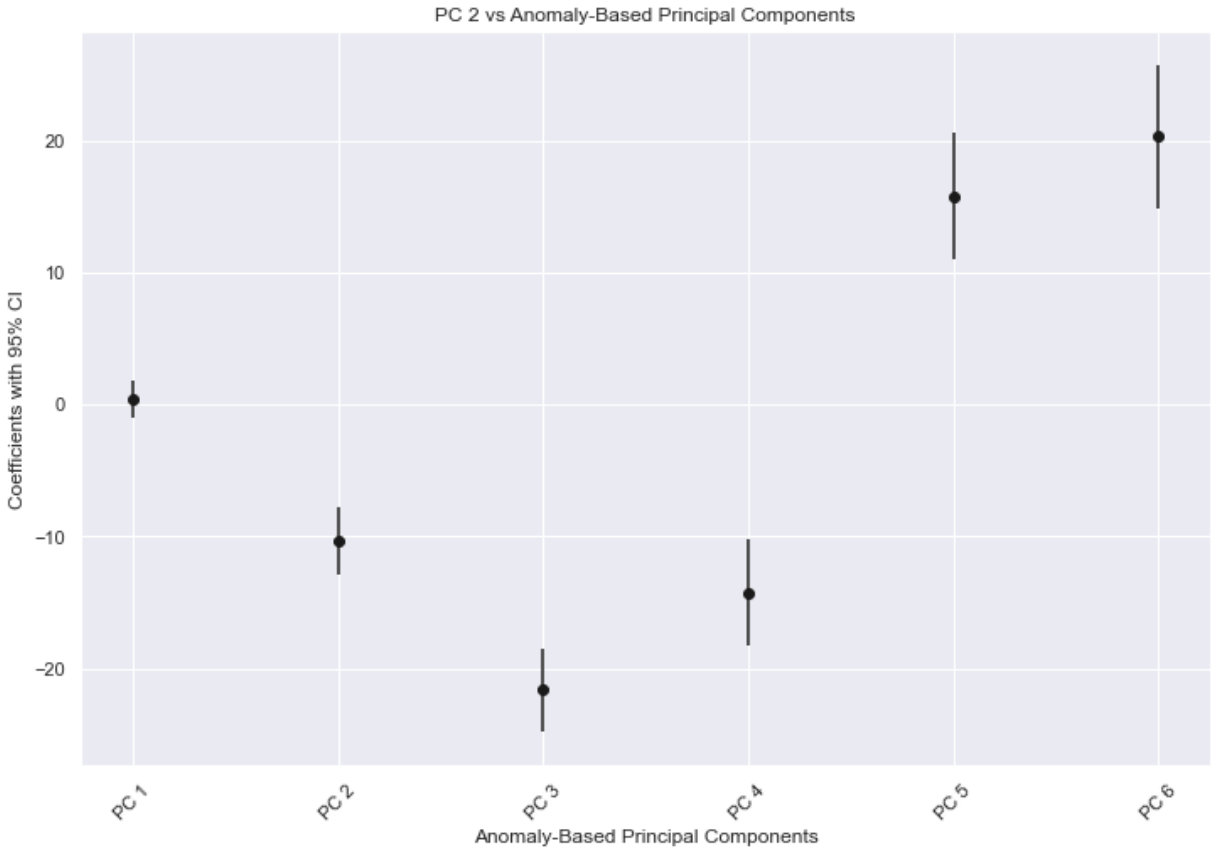
	coef	std err	t	P>|t|	[0.025	0.975]
const	-1.162e-16	0.074	-1.57e-15	1.000	-0.146	0.146
PC 1	23.5303	0.619	38.038	0.000	22.313	24.747
PC 2	11.8198	1.138	10.385	0.000	9.581	14.059
PC 3	8.6444	1.409	6.137	0.000	5.873	11.416
PC 4	3.6781	1.804	2.039	0.042	0.129	7.227
PC 5	-2.5226	2.141	-1.178	0.240	-6.735	1.690
PC 6	7.7491	2.428	3.191	0.002	2.971	12.527

Omnibus:	171.684	Durbin-Watson:	1.764
Prob(Omnibus):	0.000	Jarque-Bera (JB):	2422.522
Skew:	1.820	Prob(JB):	0.00
Kurtosis:	15.849	Cond. No.	32.8

Notes:

```
[1] Standard Errors assume that the covariance matrix of the errors is correctly spe
cified.
```

In [ ]:
```
pc2 = OLS_regression(y = pc_ret_rt, X = pc_fac_rt, y_column = "PC 2", is_pc = True)
```



In [ ]:
```
print(pc2.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                   PC 2   R-squared:                       0.550
Model:                           OLS   Adj. R-squared:                  0.542
Method:                Least Squares   F-statistic:                     65.07
Date:               Sat, 15 Jul 2023   Prob (F-statistic):           1.72e-52
Time:                       17:02:05   Log-Likelihood:                -595.75
No. Observations:                326   AIC:                             1206.
Df Residuals:                    319   BIC:                             1232.
Df Model:                          6
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const       4.684e-17      0.084   5.56e-16      1.000      -0.166       0.166
PC 1           0.3608      0.704      0.513      0.609      -1.024       1.746
PC 2         -10.3232      1.295     -7.971      0.000     -12.871      -7.775
PC 3         -21.6396      1.603    -13.500      0.000     -24.793     -18.486
PC 4         -14.2854      2.053     -6.960      0.000     -18.324     -10.247
PC 5          15.7840      2.437      6.478      0.000      10.990      20.578
PC 6          20.3048      2.764      7.347      0.000      14.868      25.742
==============================================================================
Omnibus:                      233.976   Durbin-Watson:                   1.704
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             7600.490
Skew:                           2.480   Prob(JB):                         0.00
Kurtosis:                      26.129   Cond. No.                         32.8
==============================================================================
```

```
Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly spe
cified.
```

In [ ]:
```
pc_ret_rt.rename(columns={'PC 1': 'PC 1 (returns)', 'PC 2': 'PC 2 (returns)'}, inpla
pc_fac_rt.rename(columns={'PC 1': 'PC 1 (factors)', 'PC 2': 'PC 2 (factors)', 'PC 3'
```

In [ ]:
```
corr_matrix2 = round(pd.concat([pc_ret_rt, pc_fac_rt], axis=1).corr(), 2)
corr_matrix2[pc_fac_rt.columns][:2]
```

Out[ ]:

	PC 1 (factors)	PC 2 (factors)	PC 3 (factors)	PC 4 (factors)	PC 5 (factors)	PC 6 (factors)
PC 1 (returns)	0.87	0.24	0.14	0.05	-0.03	0.07
PC 2 (returns)	0.02	-0.30	-0.51	-0.26	0.24	0.28