Topics to be covered

The Convolution Sum

- Definition and Equation of Convolution
- Steps for calculating Convolution
- Numerical Example on Convolution
- The Convolution sum in Python

Applications of Convolution

- Signal Denoising
- Edge detection in a Signal

The Convolution Theorem

Representing a discrete time signal

Following are the two commonly used methods for representing a signal.

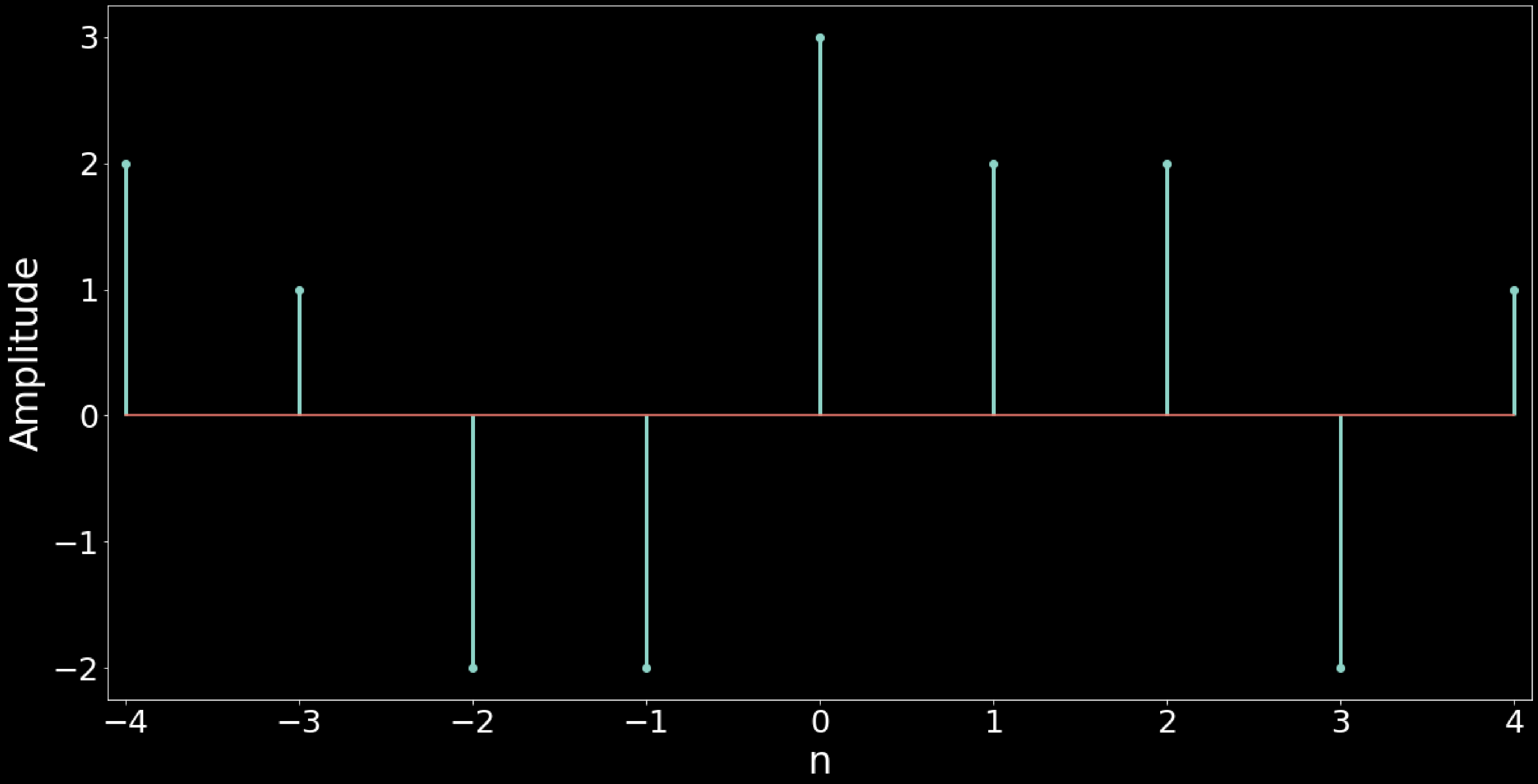
1. Sequential method.
2. Graphical method.

Sequential Representation of discrete time signal

Sequential representation of discrete time signal is shown below

$$x(n) = [2, 1, -2, -2, \underset{\uparrow}{3}, 2, 2, -2, 1]$$

Graphical Representation of discrete time signal



The Convolution Sum

The response or the convolution sum $y(n)$ of the two input signals $x_1(n)$ and $x_2(n)$ is defined by the following equation.

$$y(n) = x_1(n) \circledast x_2(n)$$

$x_2(n)$ is called kernel or filter.

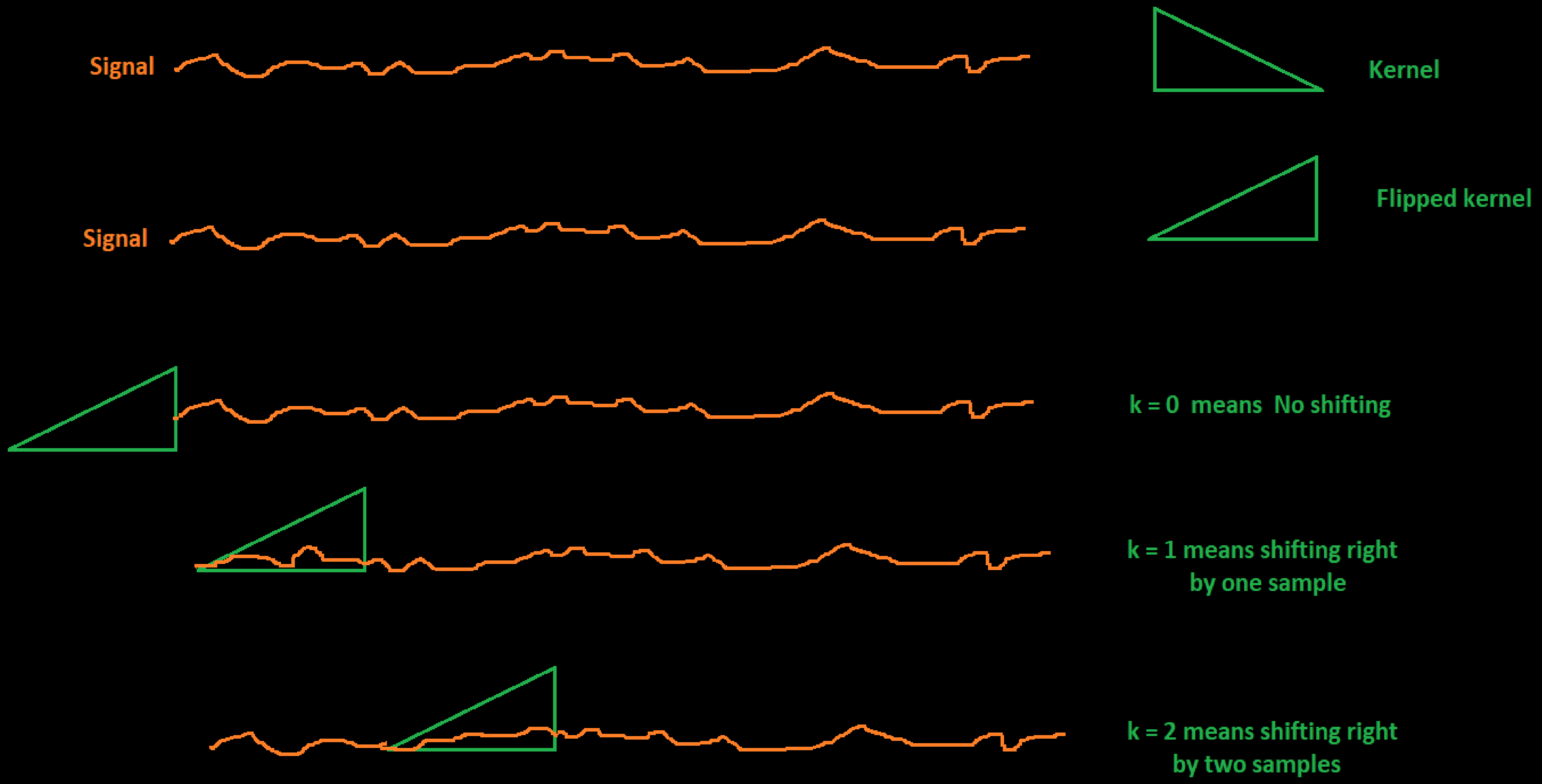
Steps for performing convolution sum

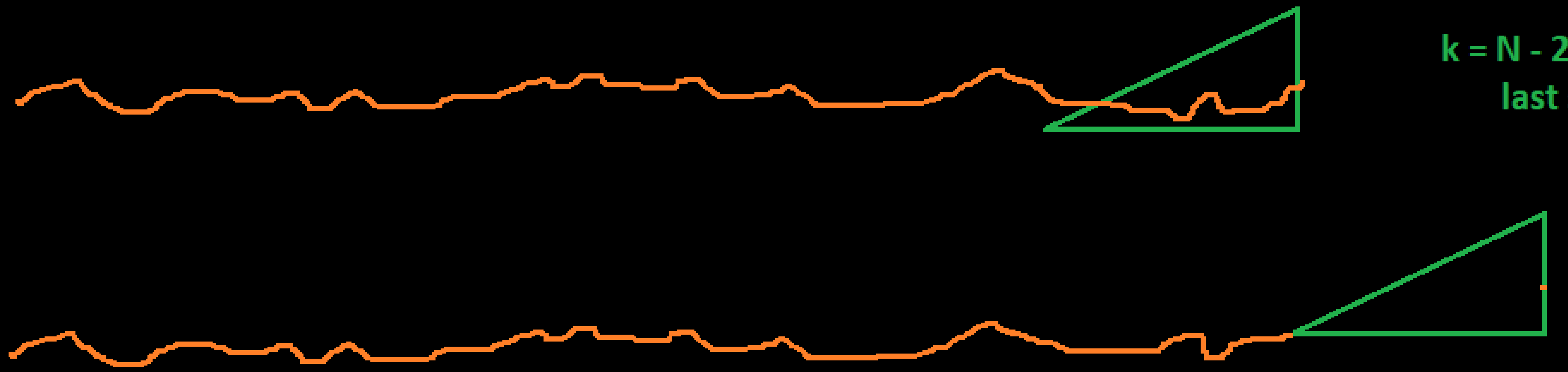
1. Flipping / Folding.
2. Shifting.
3. Multiplication.
4. Addition.

$$y(n) = x_1(n) \circledast x_2(n)$$

$$y(n) = \sum_{k=0}^{N-1} x_1(k) x_2(n-k)$$

$x_2(n)$ is called kernel or filter.


Signal
Kernel
Signal
Flipped kernel
k = 0 means No shifting
k = 1 means shifting right
by one sample
k = 2 means shifting right
by two samples

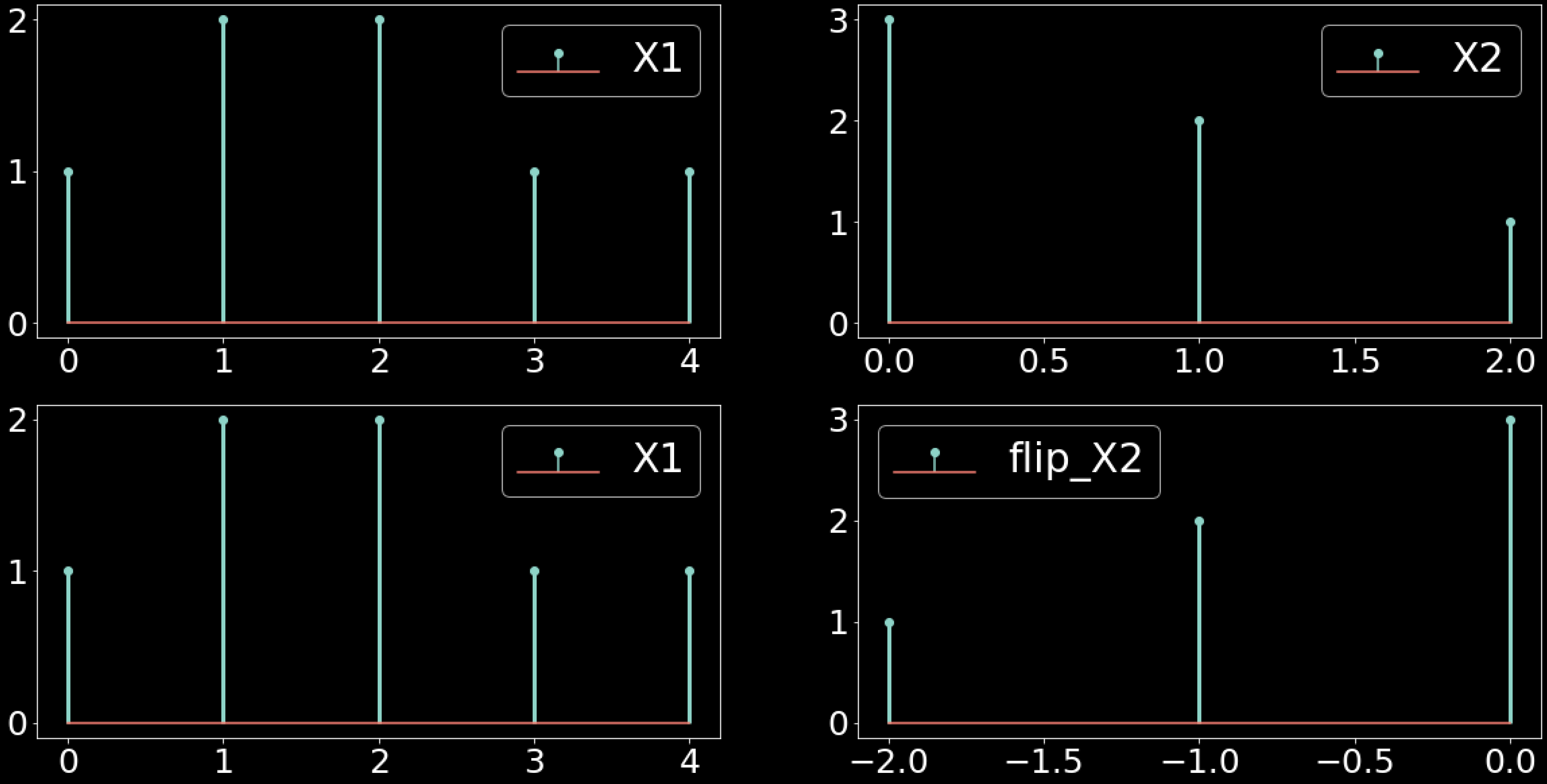


k = N - 2 means second last shifting

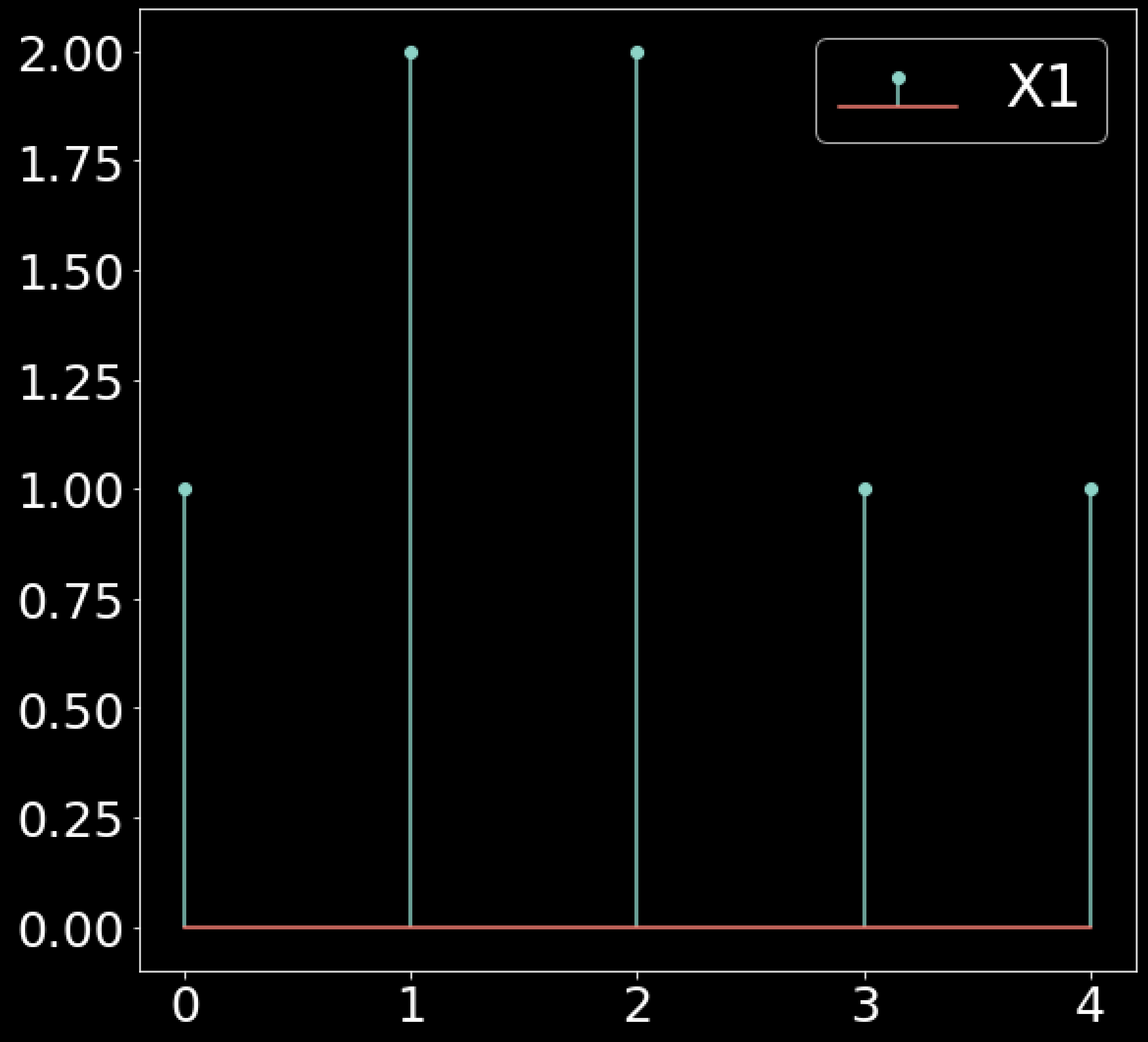
k = N -1 means last shifting

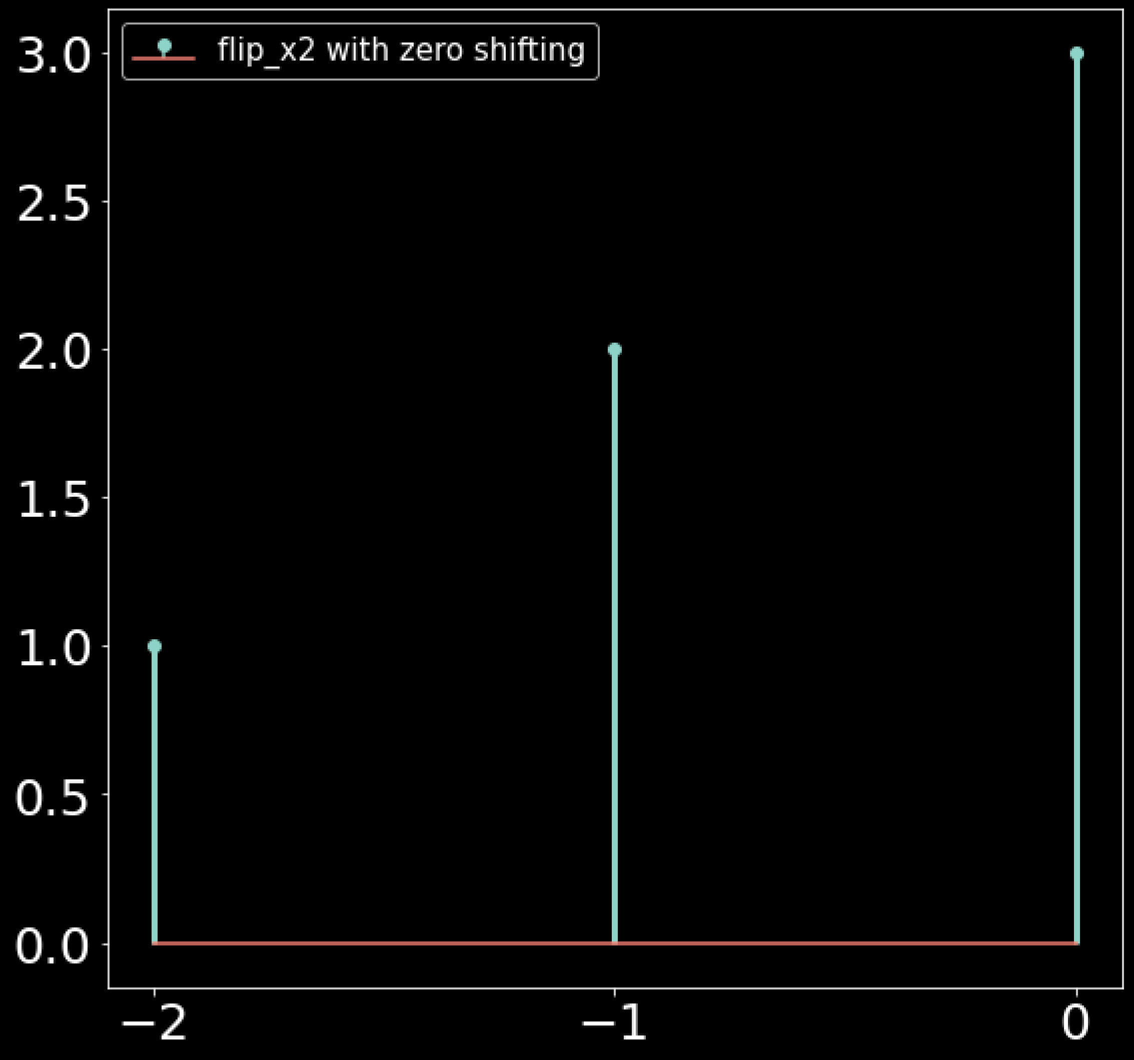
$$y(n) = \sum_{k=0}^{N-1} x_1(k)x_2(n-k)$$

Example



For k = 0 , means no shifting





For $k = 0$

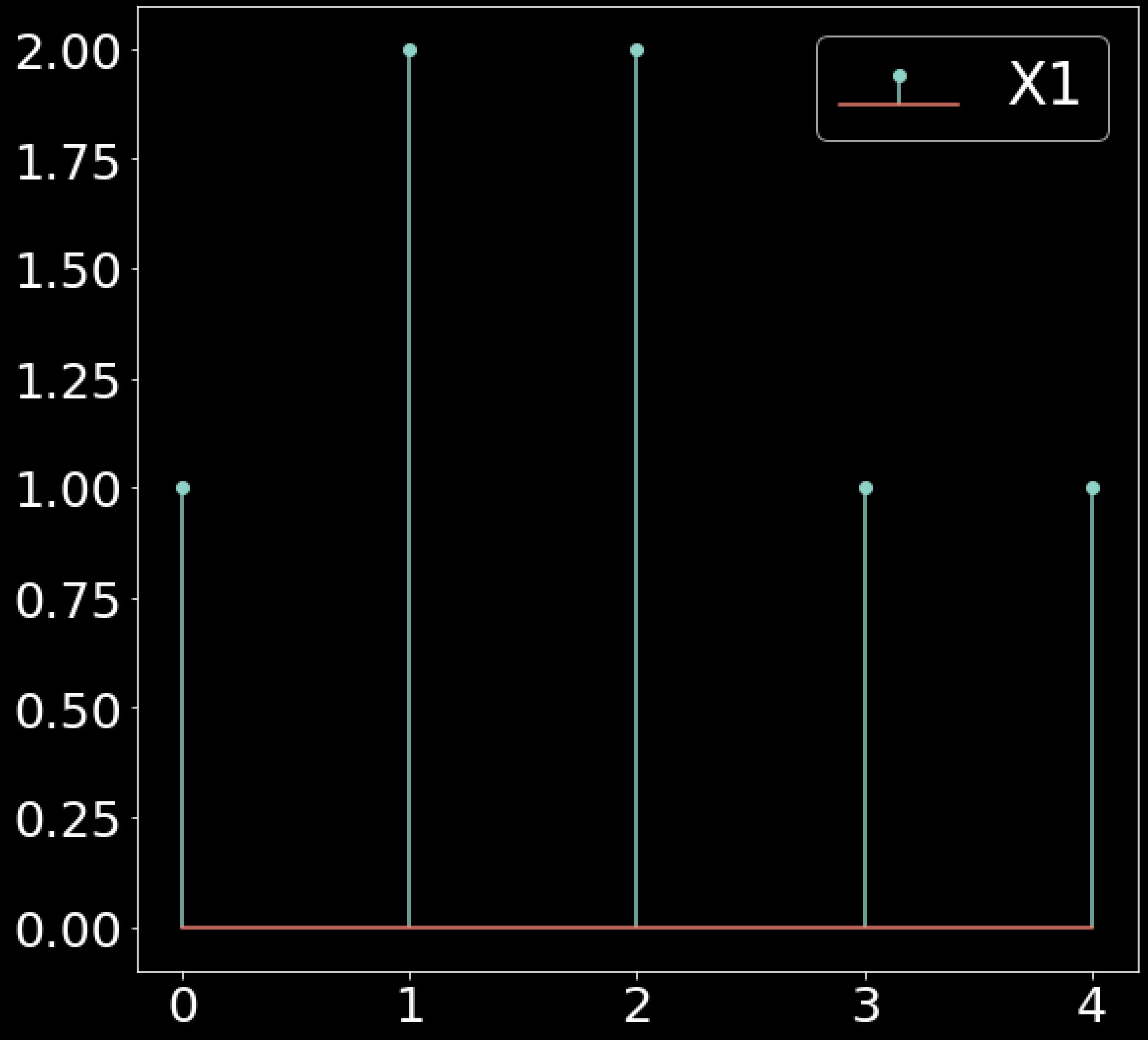
Now we have to perform the next step i.e multiplication. Multiplication of the discrete signals is always performed by sample-to-sample basis.

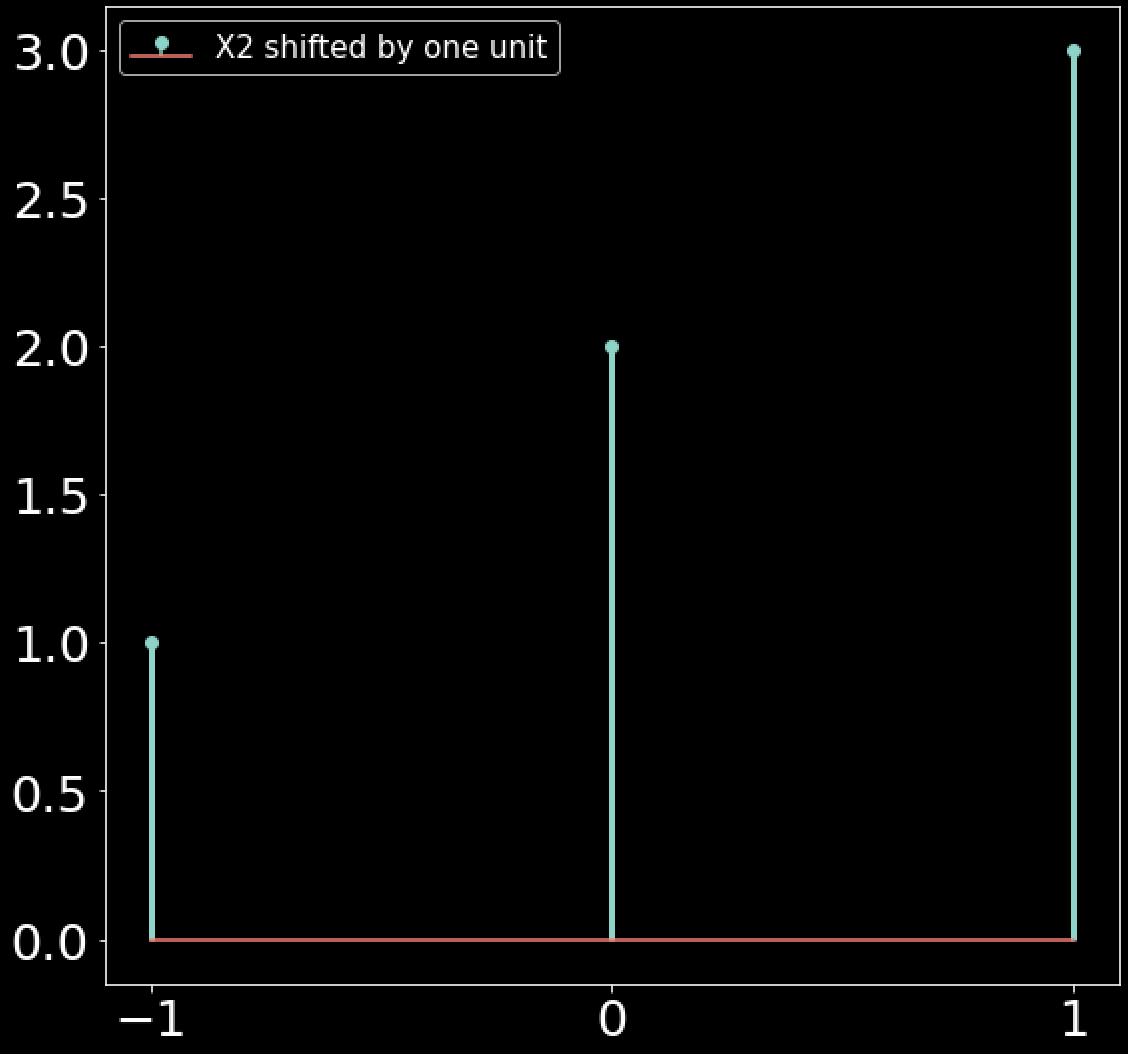
For x1 and flipped x2, they have a common sample only at time =0 i.e n=0

so the product sequence = [1 x 3 = 3]

Sum sequence = [3]

For k = 1 , means shift the flipped signal towards right by one unit





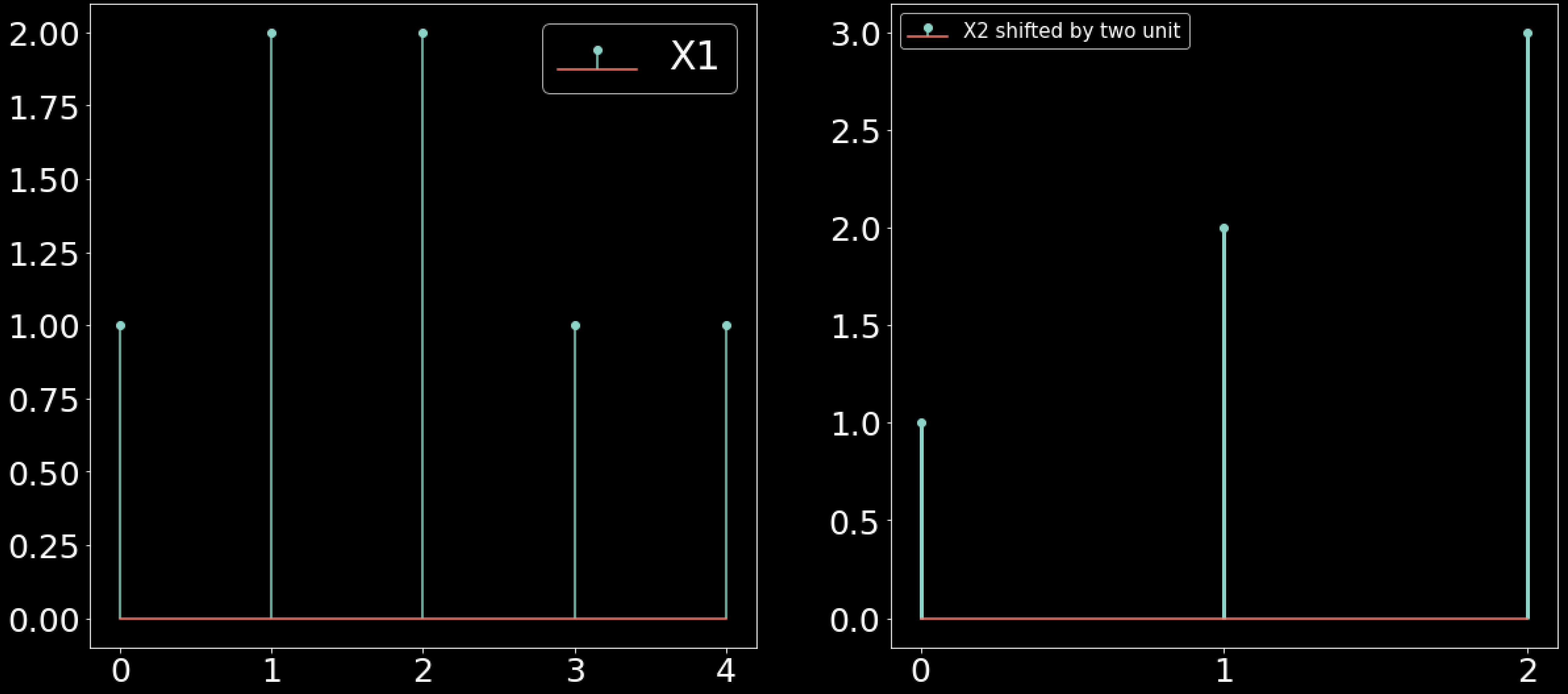
For $k = 1$

For x1 and shifted x2, they have a two common samples at time =0 and 1 i.e n=0 and 1

so the product sequence $= [1x2 = 2,\ 2x3 = 6]$

Sum sequence $= [2 + 6 = 8]$

For k = 2 , means shift the flipped signal towards right by two unit



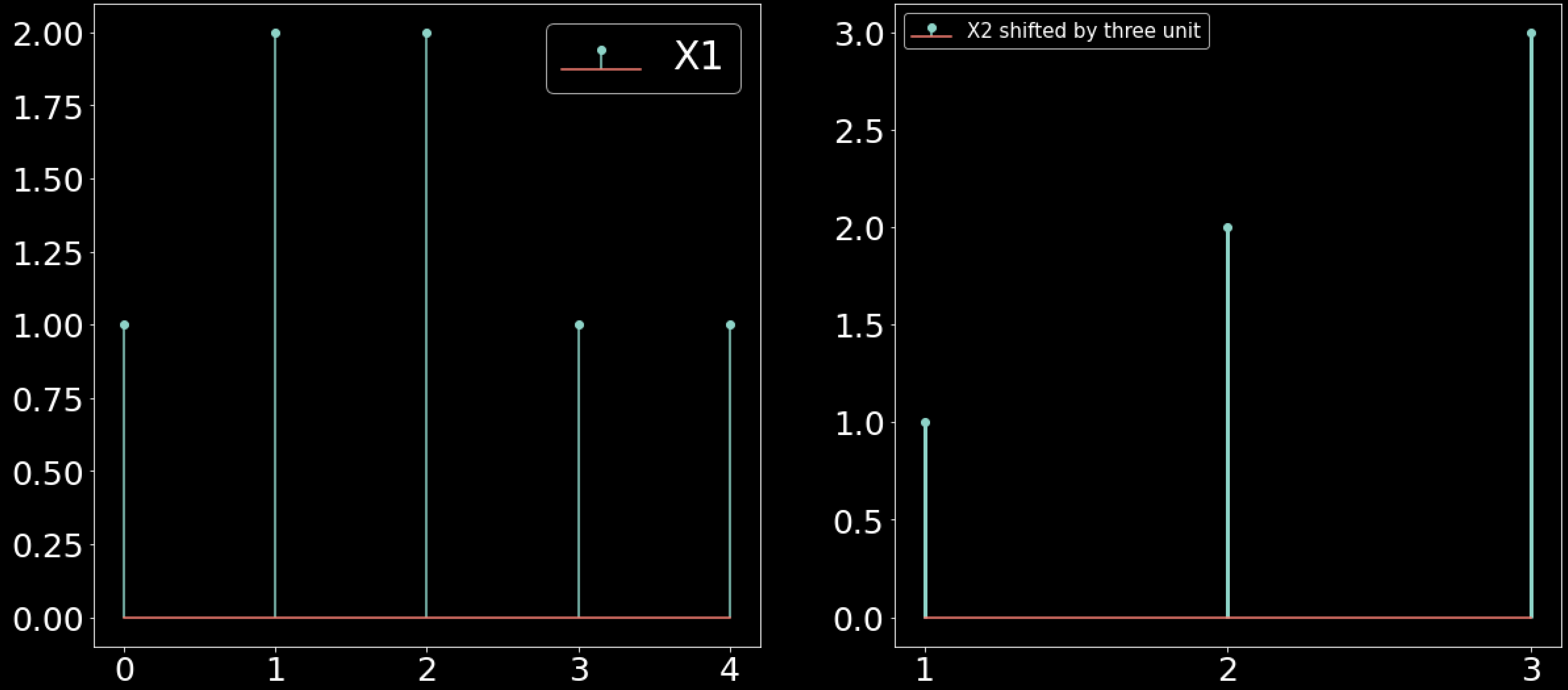
For $k = 2$

For x1 and shifted x2, they have a three common samples at time =0,1 and 2 i.e n=0,1 and 2.

so the product sequence $= [1x1 = 1,\ 2x2 = 4,\ 2x3 = 6]$

Sum sequence $= [1 + 4 + 6 = 11]$

For k = 3 , means shift the flipped signal towards right by three unit



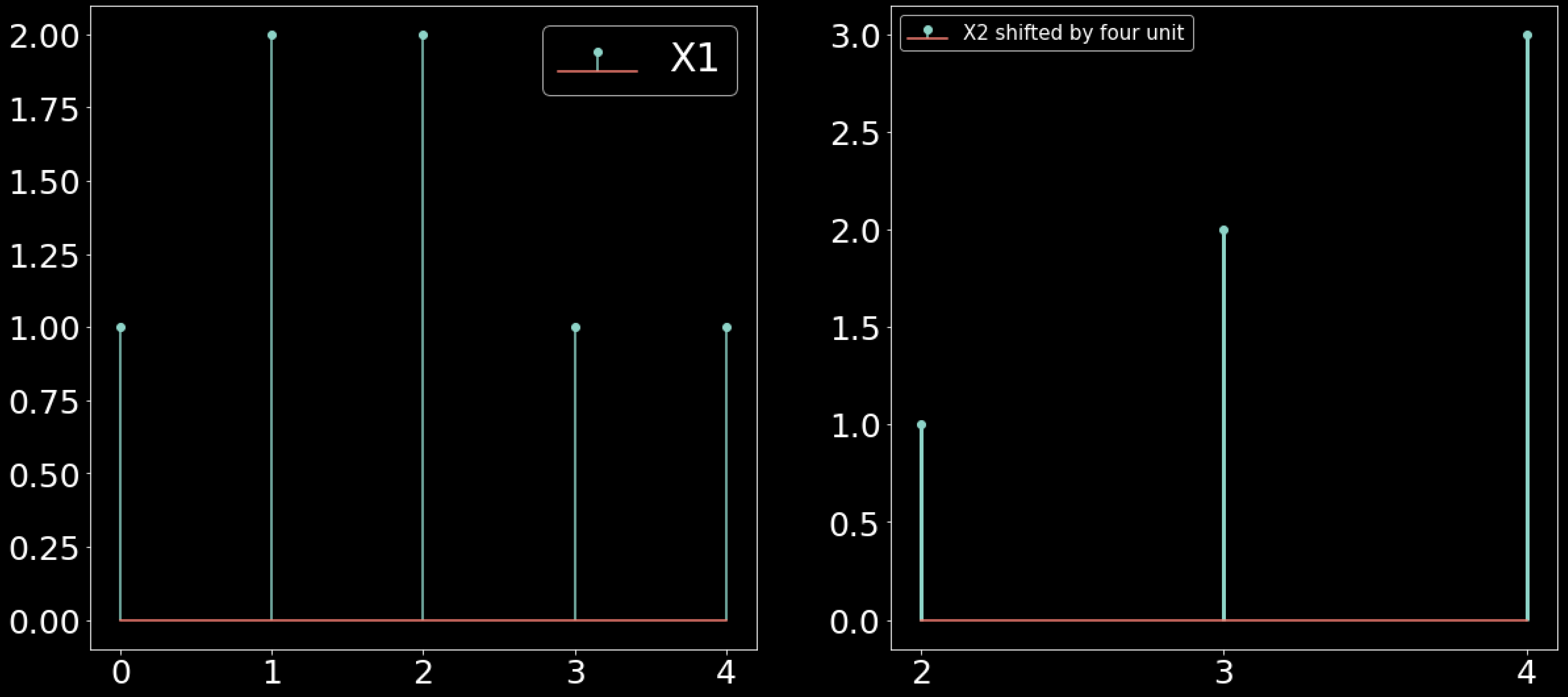
For $k = 3$

For x1 and shifted x2, they have a three common samples at time =1,2 and 3 i.e n=1,2 and 3.

so the product sequence $= [2x1 = 2,\ 2x2 = 4,\ 1x3 = 3]$

Sum sequence $= [2 + 4 + 3 = 9]$

For k = 4 , means shift the flipped signal towards right by four unit



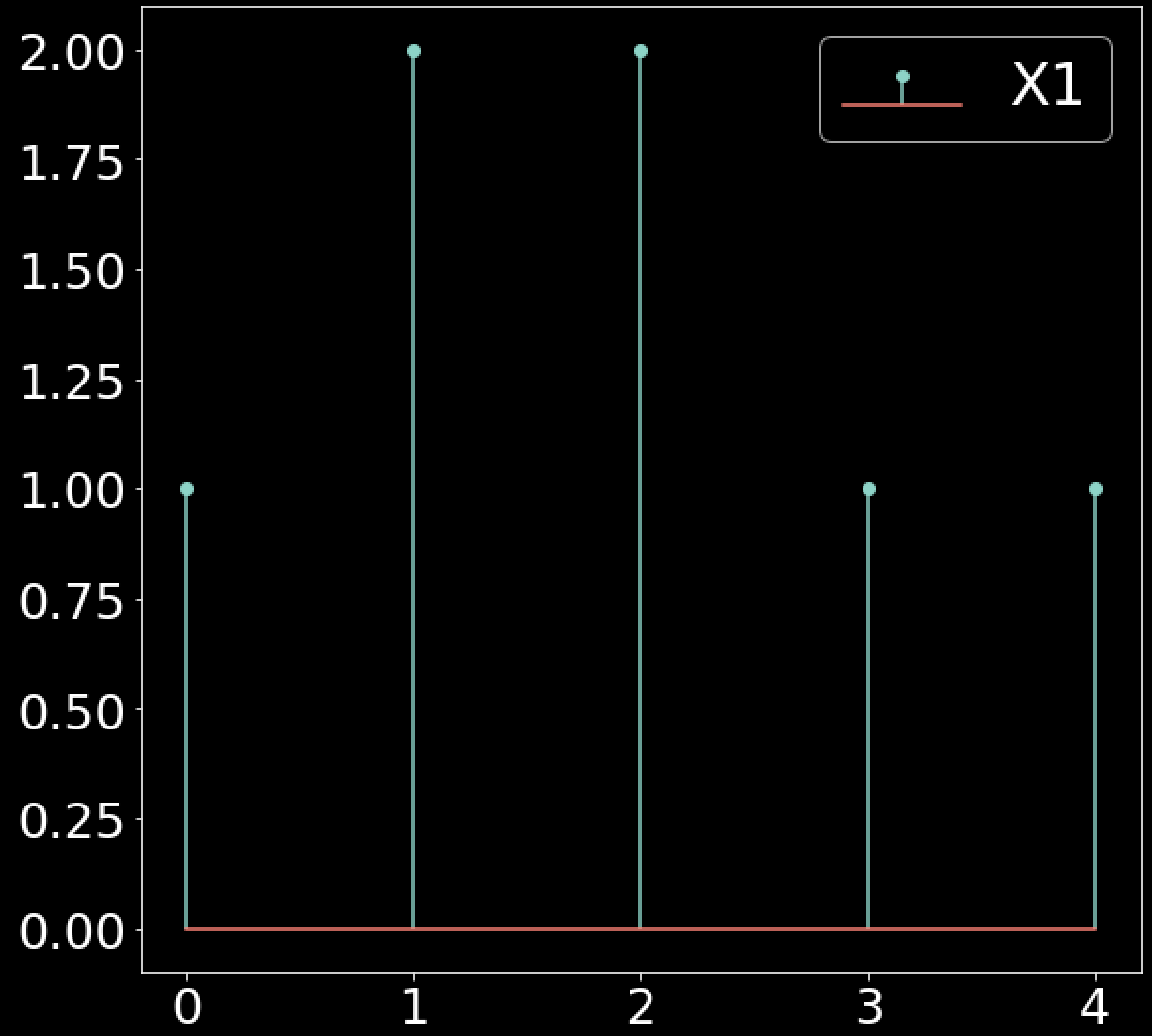
For $k = 4$

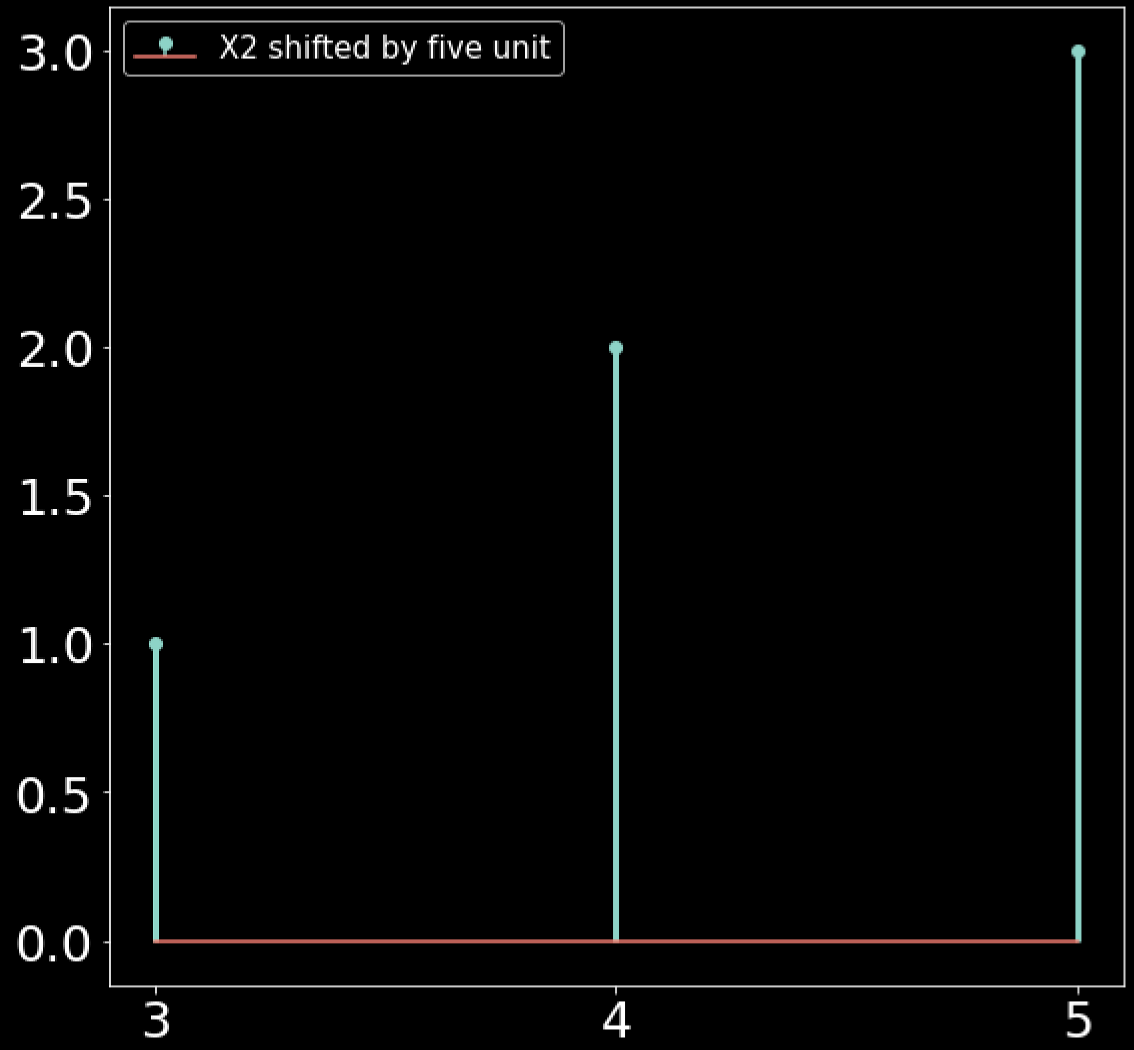
For x1 and shifted x2, they have a three common samples at time =2,3 and 4 i.e n=2,3 and 4.

so the product sequence = [2x1 = 2, 1x2 = 2, 1x3 = 3]

Sum sequence = [2 + 2 + 3 = 7]

For k = 5 , means shift the flipped signal towards right by five unit



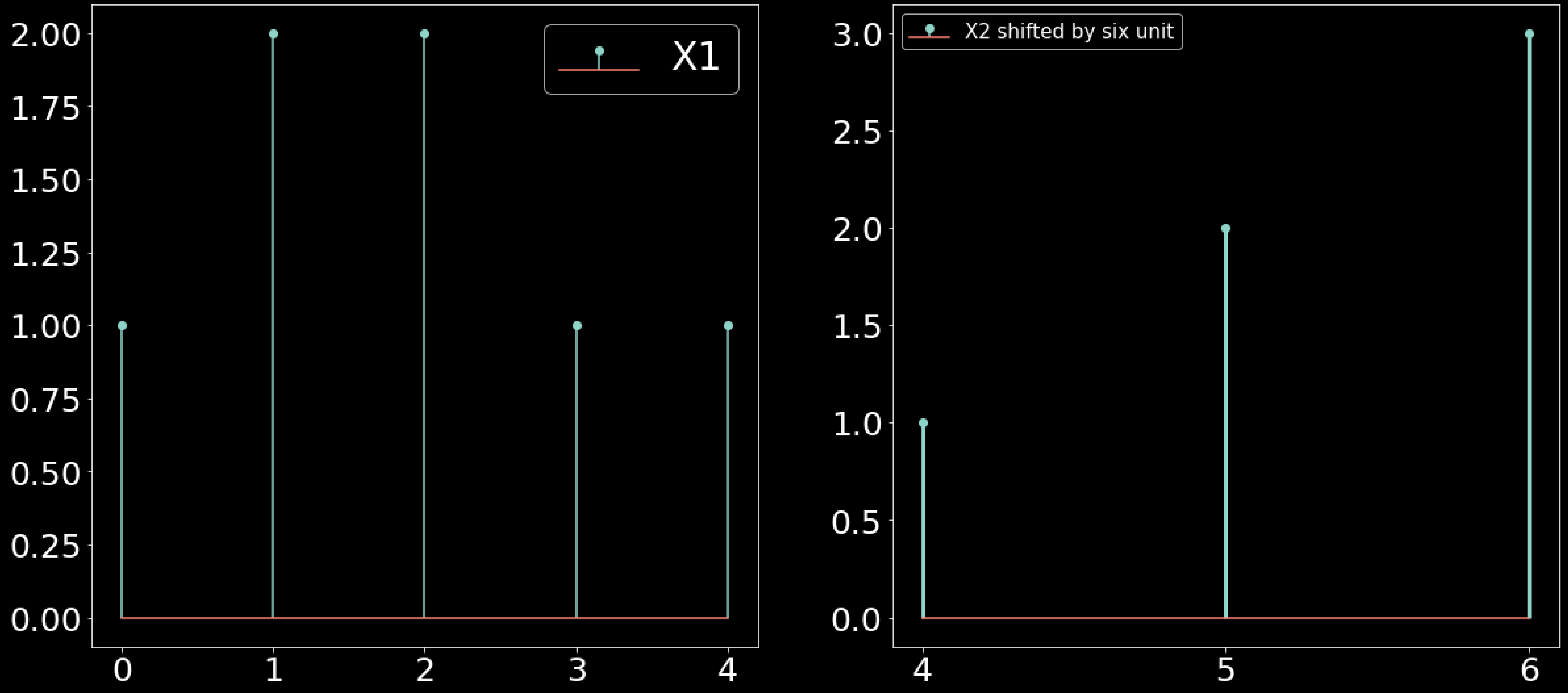


For k = 5

For x1 and shifted x2, they have a two common samples at time =3 and 4 i.e n=3 and 4.

so the product sequence = [1x1 = 1, 1x2 = 2]
Sum sequence = [1 + 2 = 3]

For k = 6 , means shift the flipped signal towards right by six unit

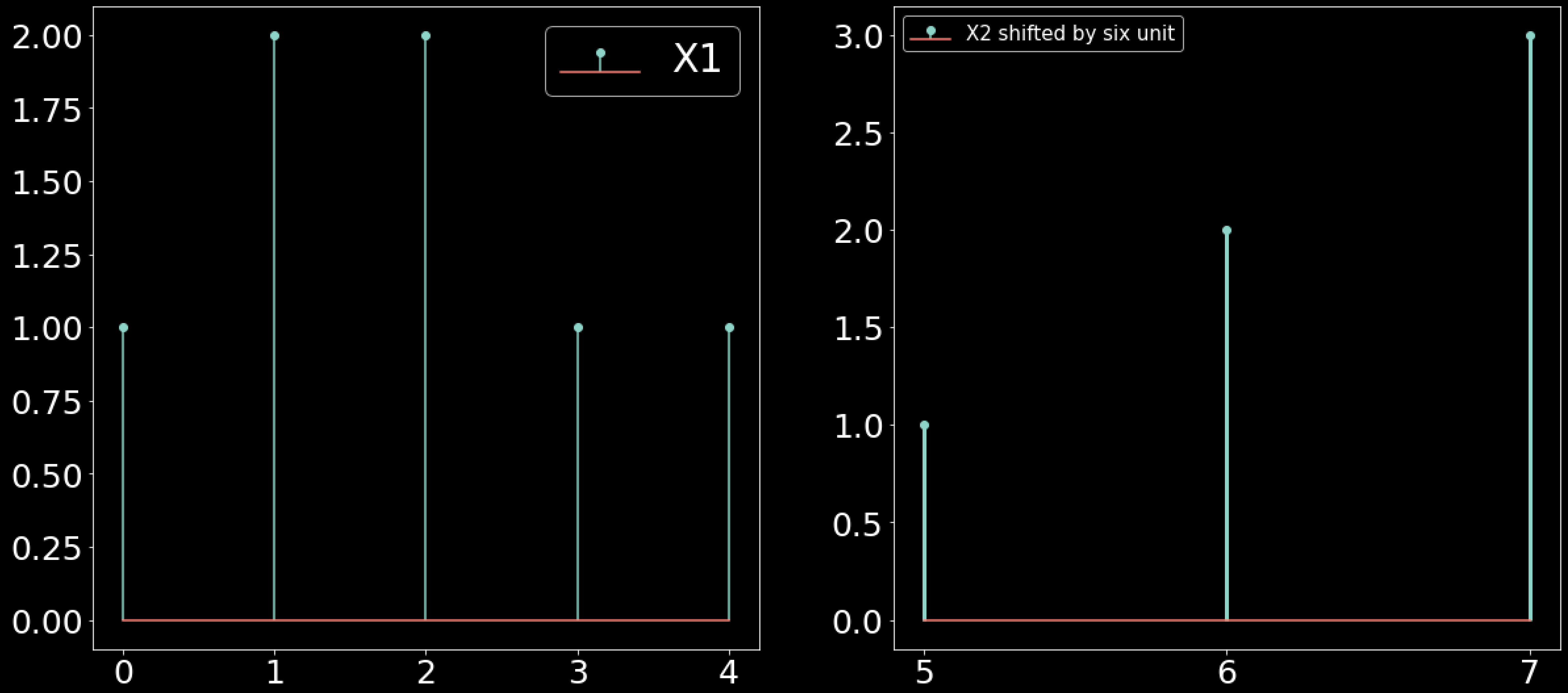


For $k = 6$

For x1 and shifted x2, they have only one common samples at time $= 4$ i.e $n = 4$.

so the product sequence $= [1x1 = 1]$
Sum sequence $= [1]$

For k = 7 , means shift the flipped signal towards right by seven unit



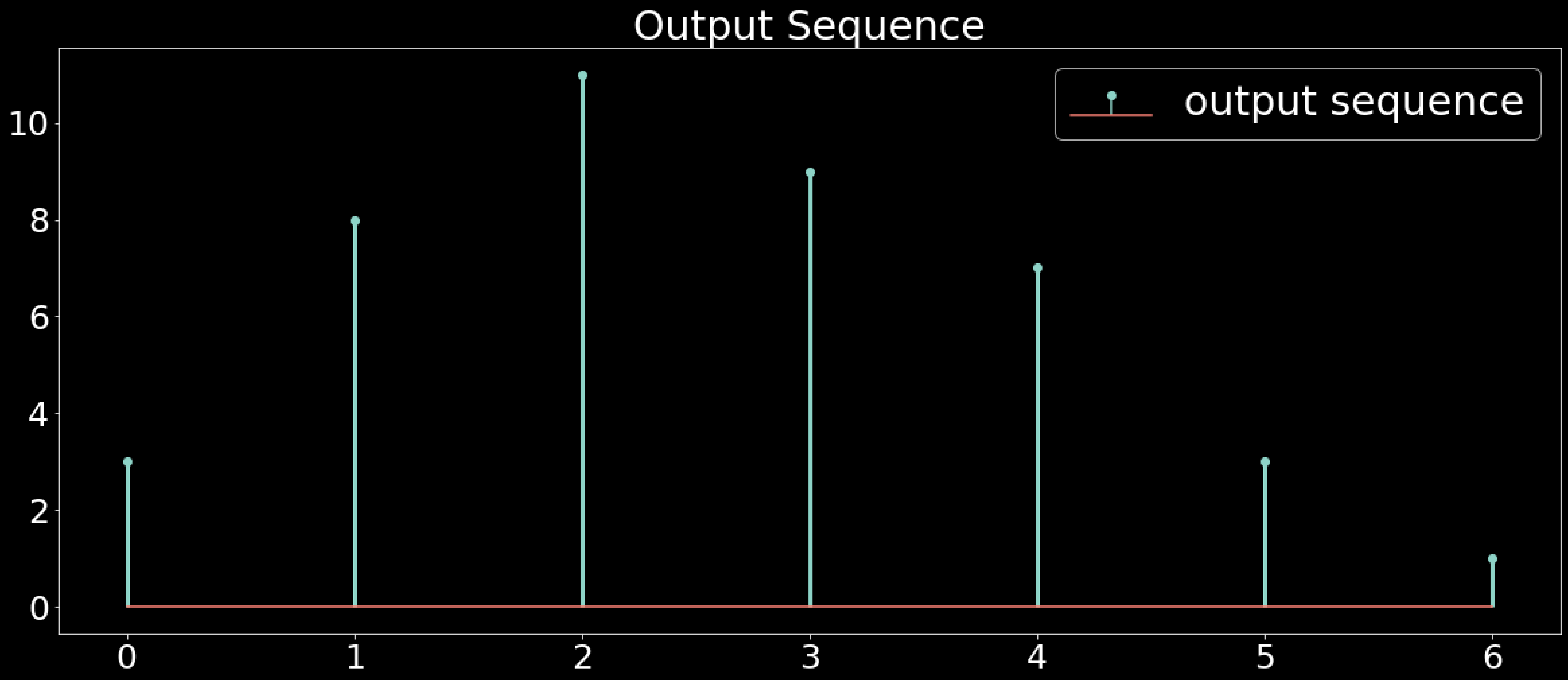
For $k = 7$

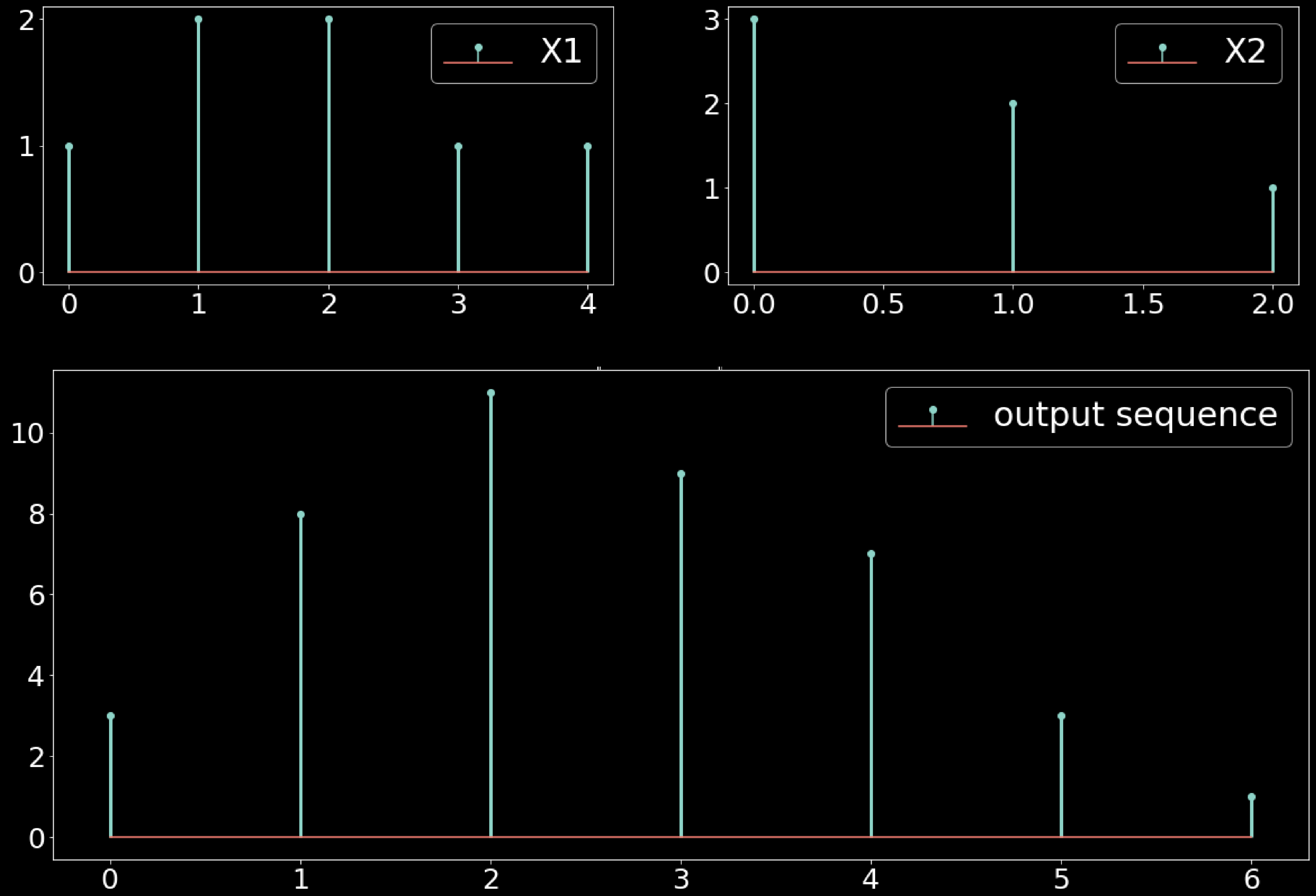
For x1 and shifted x2, they have no common samples so the product and sum sequences are zero.

The output sequence

$$y(n) = [\ 3, 8, 11, 9, 7, 3, 1]$$

The output sequence





X1
X2
output sequence

Convolution for mode = "full"

The number of samples in first signal $= nx_1 = 5$

The number of samples in the kernel $= nx_2 = 3$

$$\text{Number of samples in output sequence} = \text{nconv} = nx_1 + nx_2 - 1$$
$$= 5 + 3 - 1$$
$$= 7$$