Equation responsible for the existance of complex numbers.

$$x^2 + 1 = 0$$

$$x^2 = -1$$

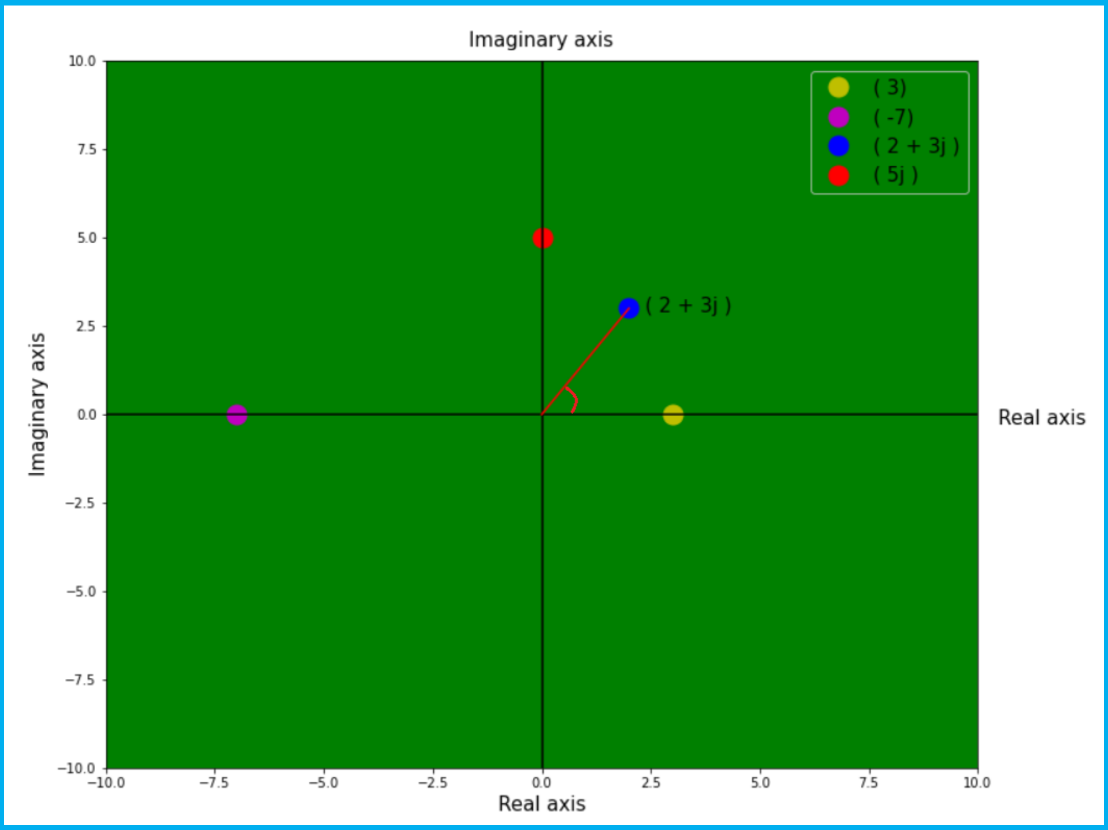
$$x = \sqrt{-1}$$

$$\boxed{j = \sqrt{-1}}$$

$$j^2 = -1$$

Complex Number is the sum of real and imaginary number.

$$z = x + yj$$


Imaginary axis
(3)
(-7)
(2 + 3j)
(5j)
(2 + 3j)
Real axis
Imaginary axis
Real axis
10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
−10.0
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0

j as an operator

suppose we have

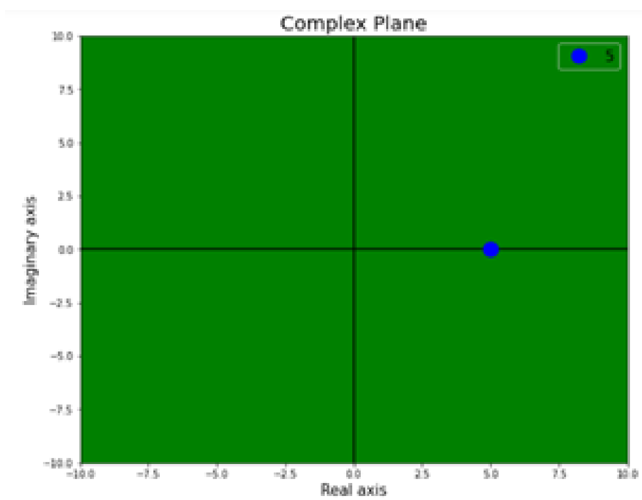
$$x = 5$$

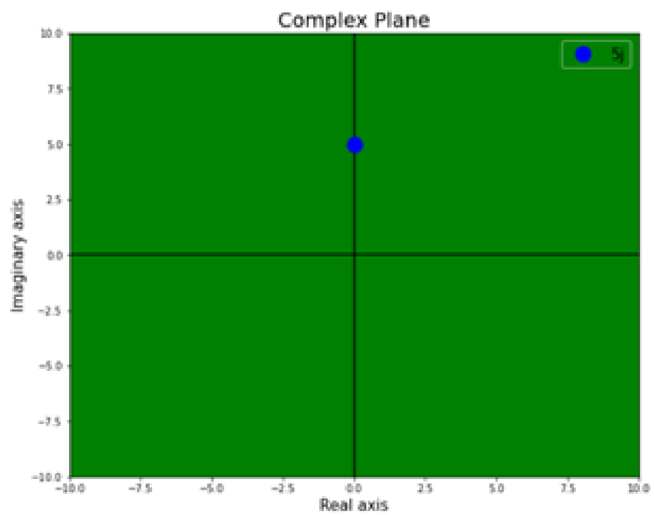
Multiply j with this real number

$$x = 5j$$

$$x = 5(1 < 90^0)$$

$$\boxed{j = 1 < 90^0}$$





Mathematical Operations on Complex Numbers

A. Addition and Subtraction

$$z_1 = 2 + 3j$$

$$z_2 = 7 + 5j$$

$$z = z_1 + z_2$$

$$z = (2 + 3j) + (7 + 5j)$$

$$z = (2 + 7) + (3j + 5j)$$

$$z = 9 + 8j$$

$$z = z_1 - z_2$$

$$z = (2 + 3j) - (7 + 5j)$$

$$z = 2 + 3j - 7 - 5j$$

$$z = 2 - 7 + 3j - 5j$$

$$z = -5 - 2j$$

B. Multiplication and Division

$$z_1 = 2 + 3j$$

$$z_2 = 7 + 5j$$

$$z = z_1 \times z_2$$

$$z = (2 + 3j)(7 + 5j)$$

$$z = 2(7 + 5j) + 3j(7 + 5j)$$

$$z = 14 + 10j + 21j + 15(j^2)$$

$$z = 14 + 31j + 15(-1)$$

$$z = 14 + 31j - 15$$

$$z = -1 + 31j$$

Division

Conjugate of a complex number

$$z = x + yj$$

The conjugate is

$$z = x - yj$$

if we have

$$z = 7 + 5j$$

The Conjugate of a number is

$$z = 7 - 5j$$

$$z_1 = 2 + 3j$$

$$z_2 = 7 + 5j$$

$$z = \frac{z_1}{z_2}$$

$$z = \frac{(2+3j)}{(7+5j)}$$

$$z = \frac{(2+3j)}{(7+5j)} \times \frac{(7-5j)}{(7-5j)}$$

$$z = \frac{2(7-5j) + 3j(7-5j)}{7^2 - (5j)^2} \qquad (a-b)(a+b) = a^2 - b^2$$

$$z = \frac{14 - 10j + 21j - 15(j^2)}{49 - 25(j^2)}$$

$$z = \frac{14 + 11j - 15(-1)}{49 - 25(-1)}$$

$$z = \frac{14 + 11j + 15}{(49 + 25)}$$

$$z = \frac{29 + 11j}{74}$$

$$z = \frac{29}{74} + \frac{11j}{74}$$

$$z = 0.392 + 0.148j$$

Magnitude and Phase Calculation for Complex Numbers

$z = 5 + 6j$

Magnitude of z

$$z = \sqrt{(real\,part)^2 + (img\,part)^2}$$

$$z = \sqrt{(5)^2 + (6)^2}$$

$$z = \sqrt{61}$$

$$z = 7.81$$

Angle of z with positive real axis.

$$z = tan^{-1}(\frac{img\,part}{real\,part})$$

$$z = tan^{-1}(\frac{6}{5})$$

$$z = 50.19^0$$

Generation of a complex Sine wave

Euler's Formula is given by

$$e^{j\theta} = cos\theta + jsin\theta$$

$$\theta = 2\pi ft + \phi$$

$$\boxed{e^{j(2\pi ft+\phi)} = cos(2\pi ft + \phi) + jsin(2\pi ft + \phi)}$$