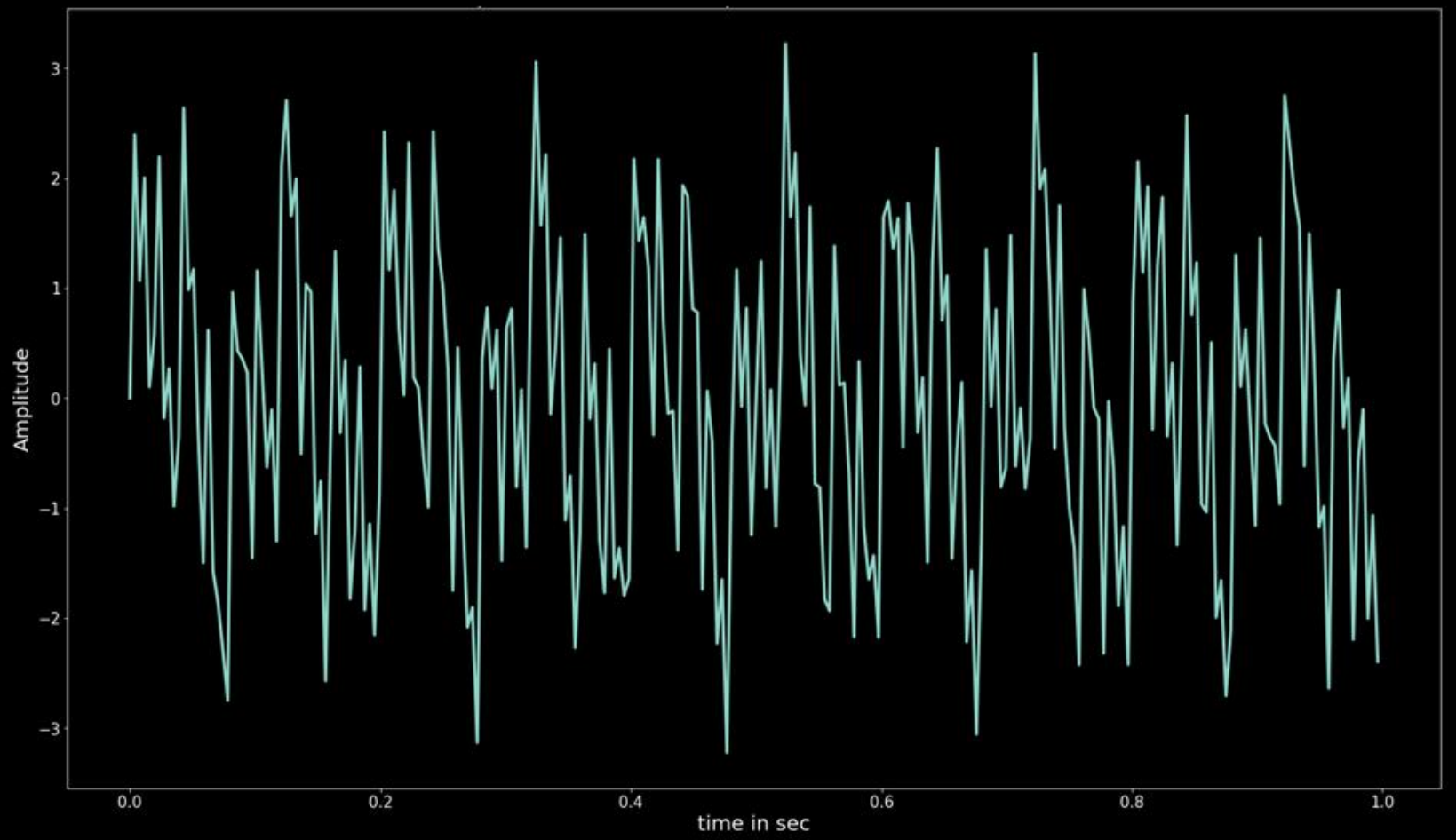
Frequency Domain Filtering

Topics to be covered

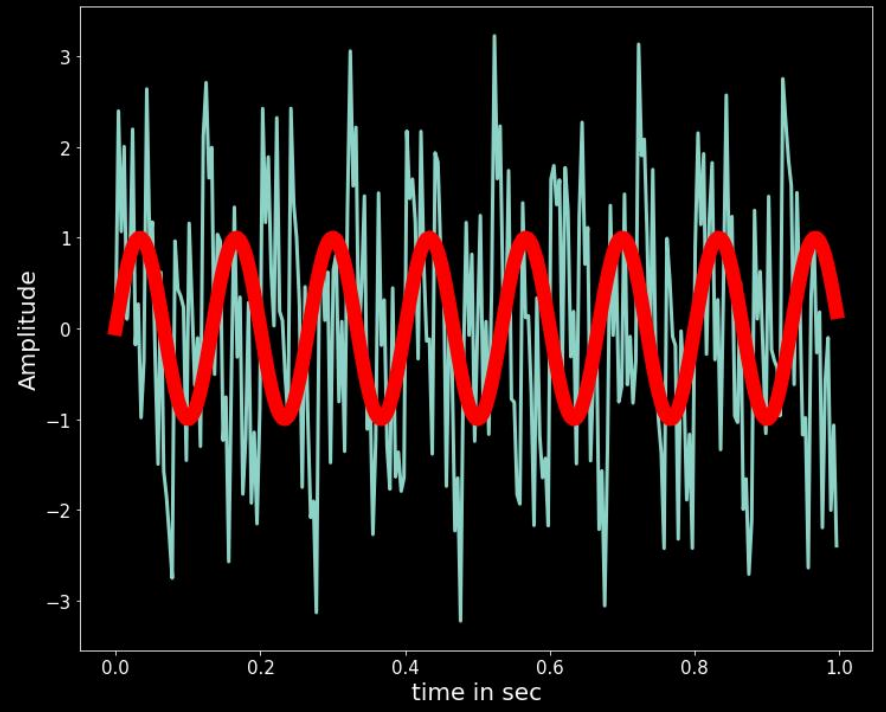
- 2D Fourier Transfom
- Frequency Domain Filtering
- Low Pass Filter
- High Pass Filter
- High Boost and Other Filters

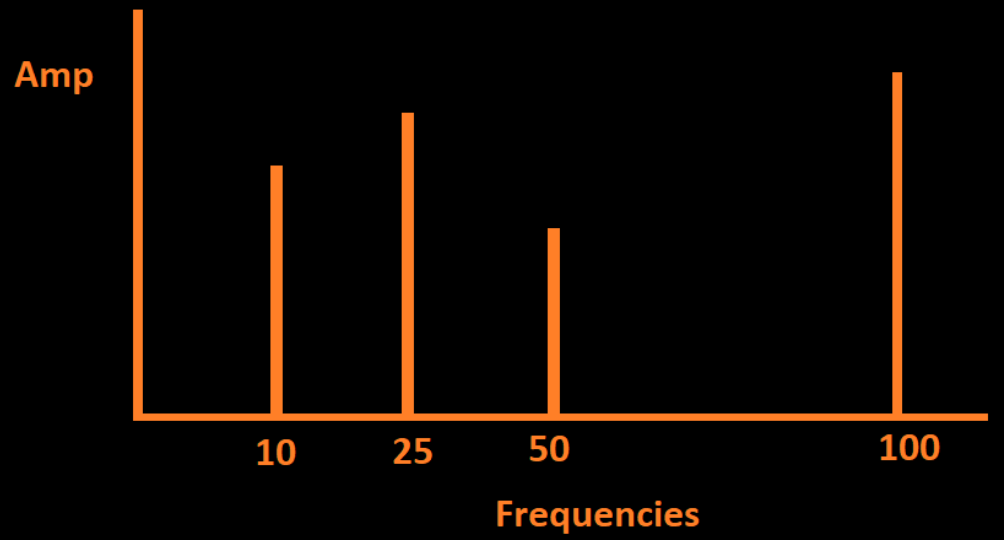
2D Fourier Transform

Signal with Frequencies 10, 25, 50 and 100 Hz



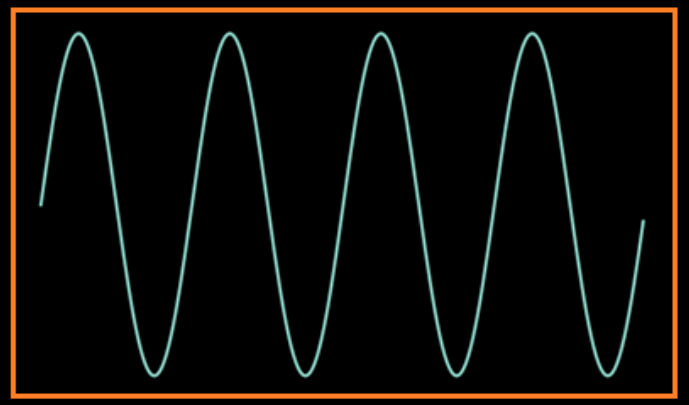
Signal with frequencies 10, 25, 50 and 100 Hz



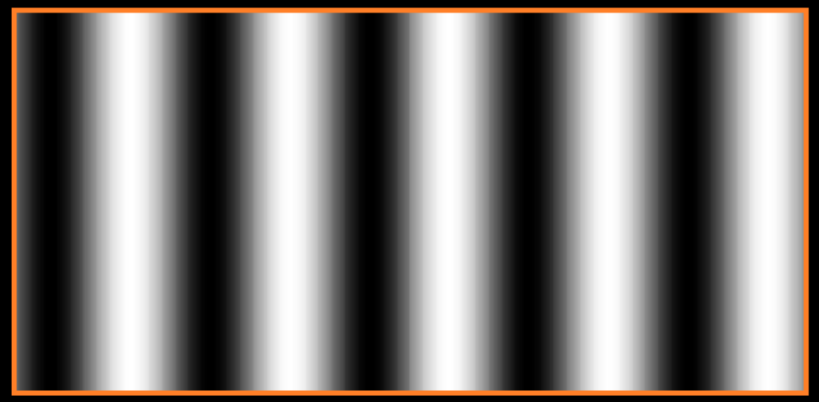


Sinusoidal gratings

A sinusoidal grating is a two-dimensional representation in which the amplitude varies sinusoidally along a certain direction.

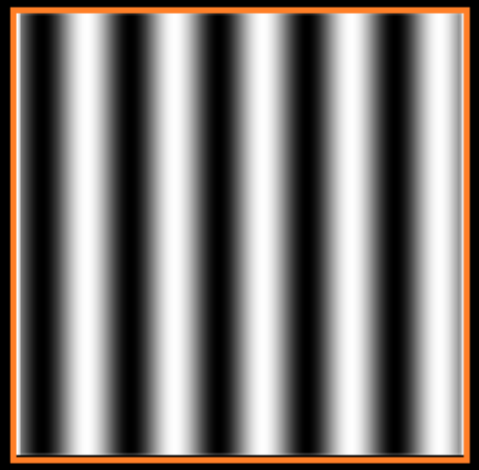


Sine Wave

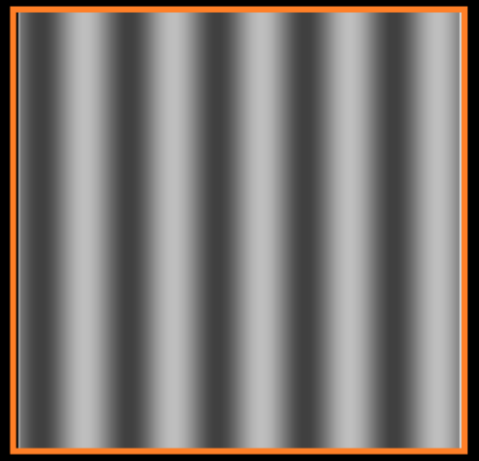


Grating of Sine Wave

Sinusoidal gratings

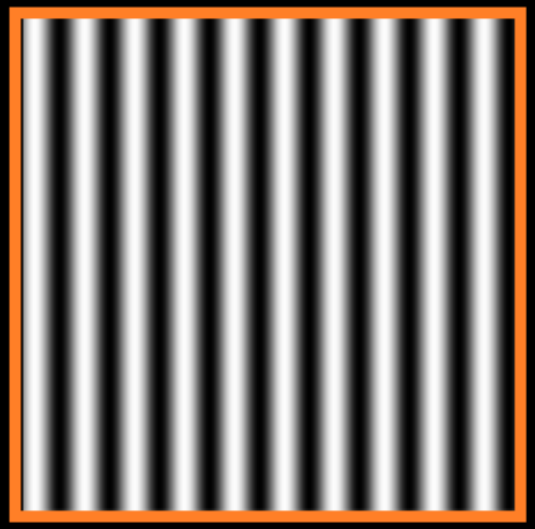


Large Amplitude

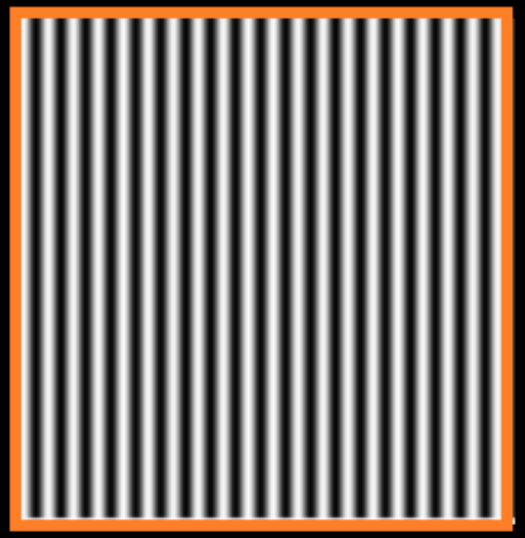


Small Amplitude

Sinusoidal gratings

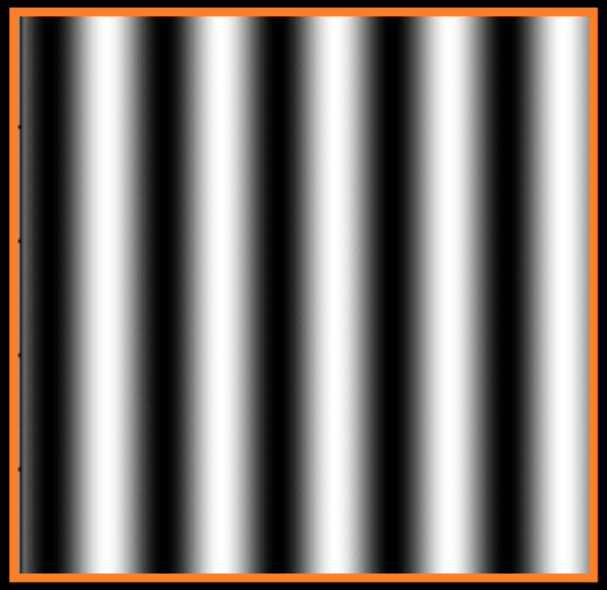


Low Frequency

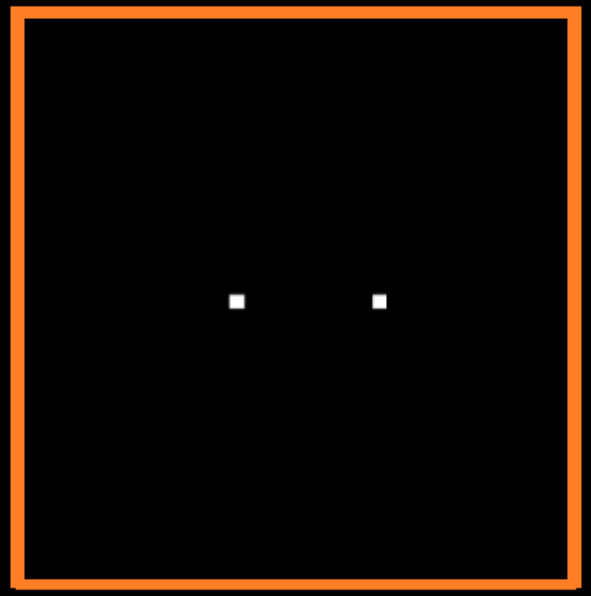


High Frequency

Fourier Transform of Grating

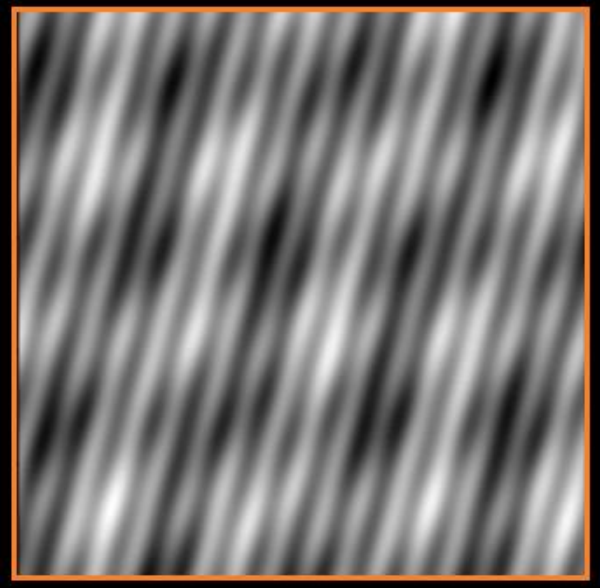


Grating

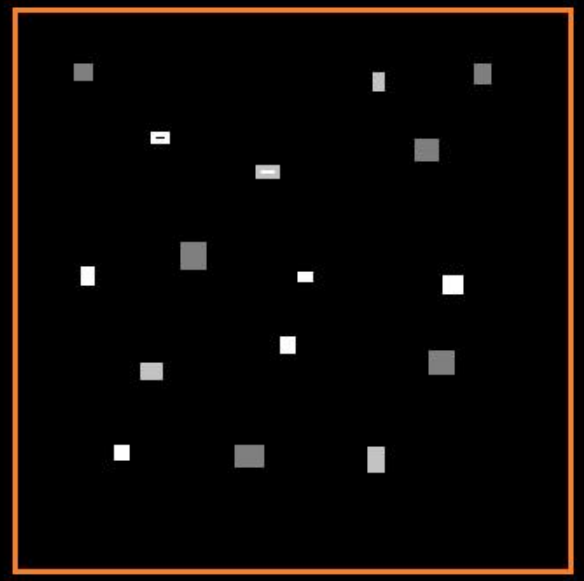


Fourier Transform

Fourier Transform of Grating



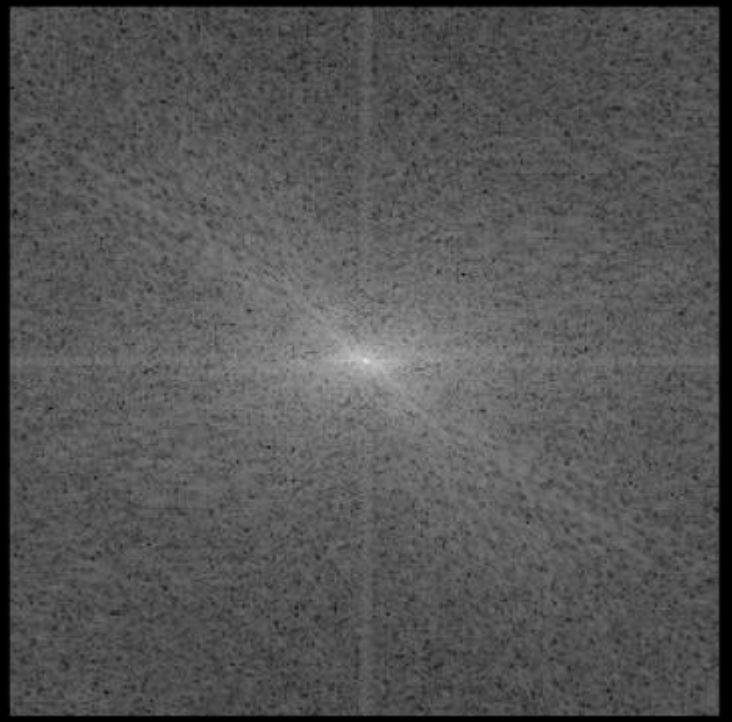
Grating



Fourier Transform

Fourier Transform of Image

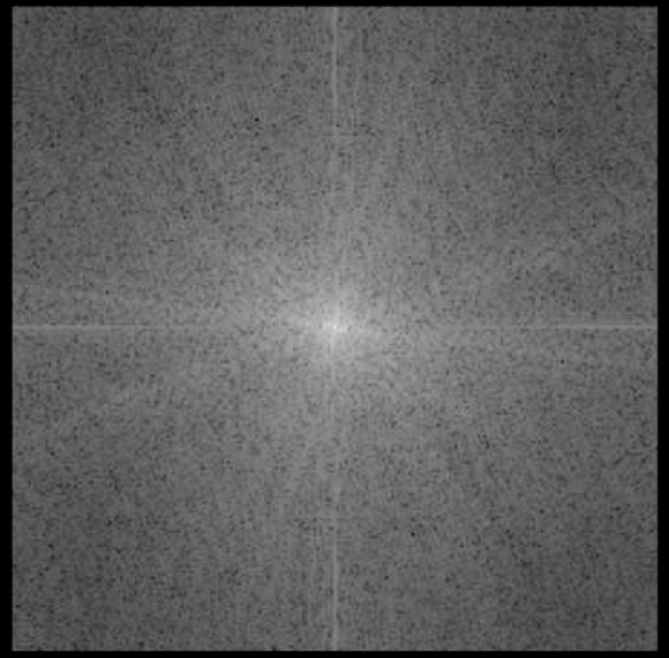
Image



Fourier Transform

Fourier Transform of Image

Image



Fourier Transform

Frequency Domain Filtering

The Convolution Theorem

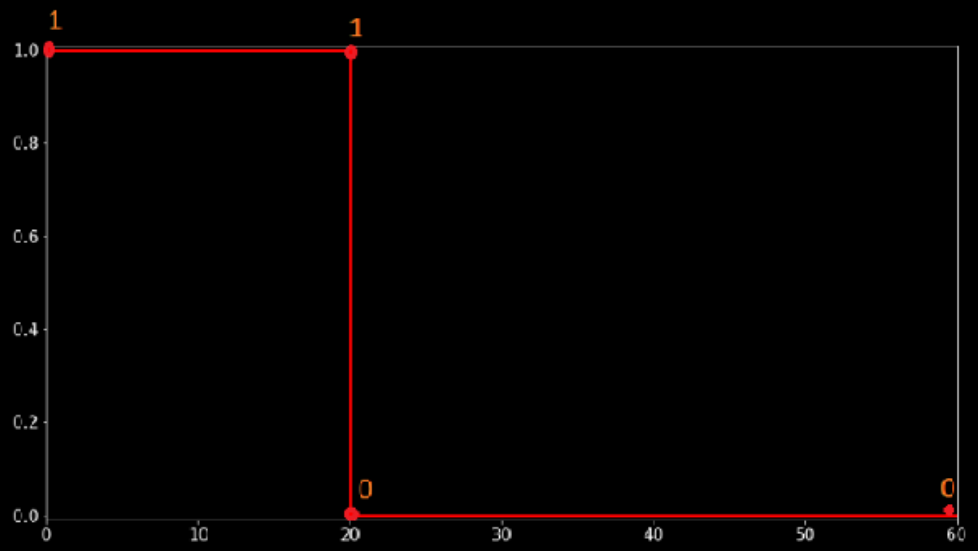
The convolution of two signals in time domain is equal to the point-wise multiplication of the signals in frequency domain.

Frequency Domain Filtering

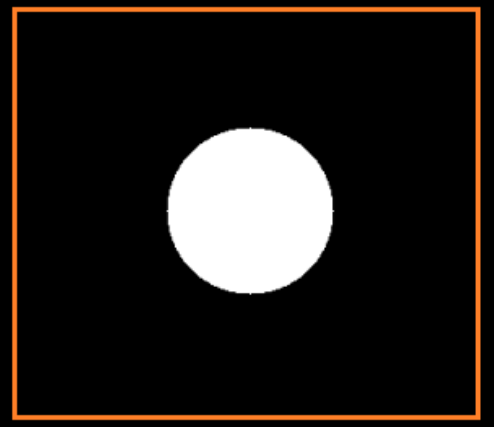
- Compute Fourier Transform of the image.
- Design a filter in frequency domain and define the cut-off frequency.
- Multiply the results of Fourier Transform of the image with the filter.
- Compute inverse Fourier Transform to get the filtered image.

Low Pass Filter

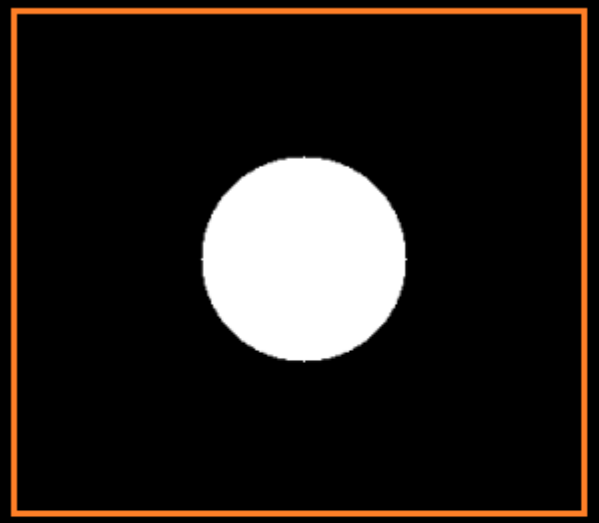
Low pass filter enhances all frequency components within a specified radius while attenuating all other frequencies.



fc = 20 Hz



2D Low Pass Filter



2D Low Pass Filter

$$H(u,v) = \begin{cases} 1, & \text{if } D(u,v) < D_0. \\ 0, & \text{if } D(u,v) > D_0. \end{cases}$$

where,

$D(u,v)$ is the distance between point (u,v) and the origin of $2D$ frequency

D_0 = Cut-off frequency

Frequency Domain Low Pass Filtering

Image

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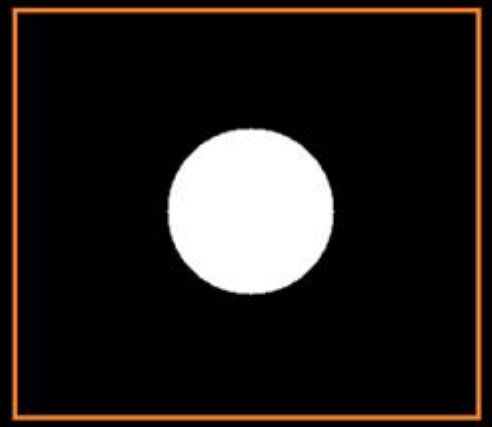

2D Low Pass Filter

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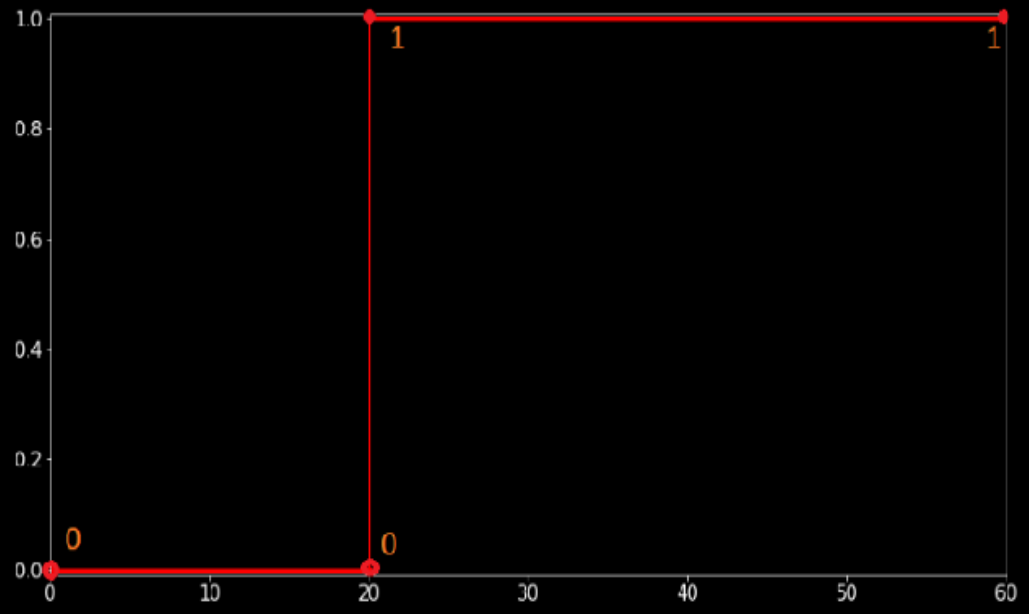
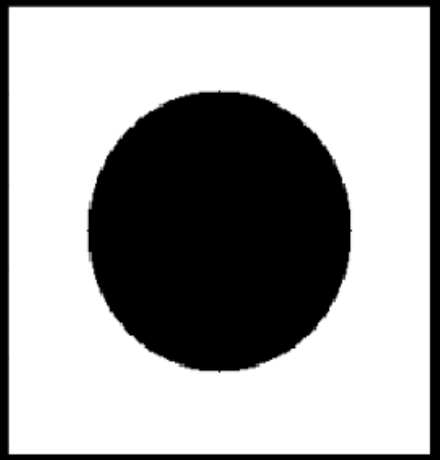
Low Pass Filtered

High Pass Filter

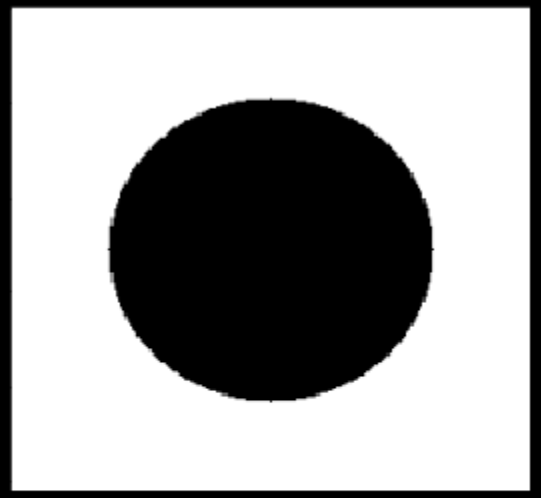
High pass filter enhances high frequency components while attenuating low frequencies. High pass filter controls the sharpening of the image.



fc = 20 Hz



2D High Pass Filter



2D High Pass Filter

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where,

$D(u,v)$ is the distance between point (u,v) and the origin of $2D$ frequency

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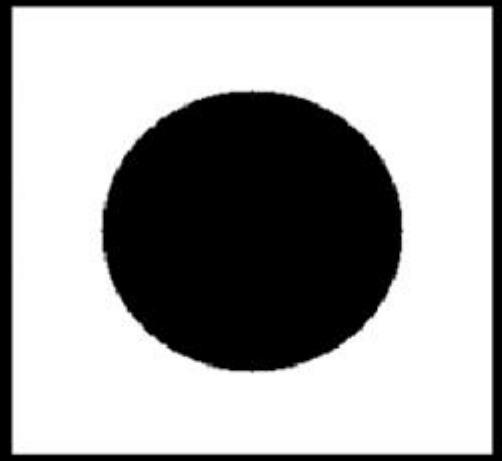
Frequency Domain High Pass Filtering

Image

*



2D High Pass Filter

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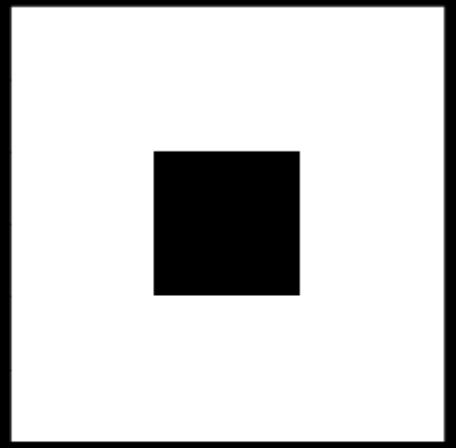
High Pass Filtered

High Boost Filter is High Pass Filter

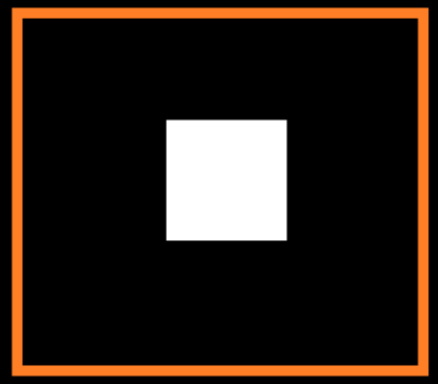
$$\begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

High Boost Filter

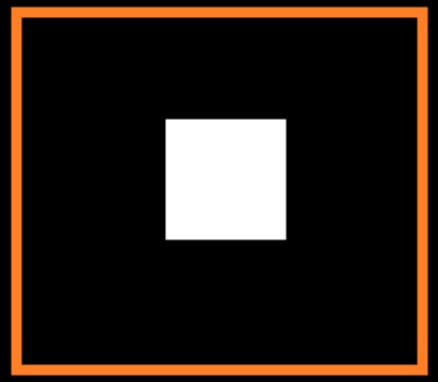


High Boost Filter
in frequency domain

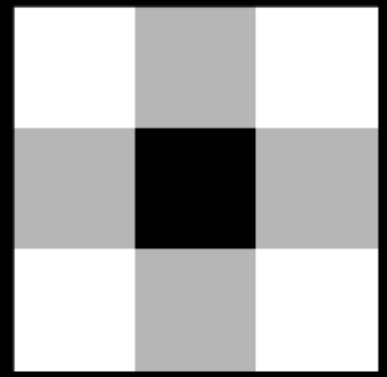
Other Filters



Mean Filter



Gaussian Filter



The Laplacian