Topics to be covered

- Moving Average or Mean Filter
- Gaussian Mean Filter
- Median Filter

Signal Denoising by Moving Average Filter

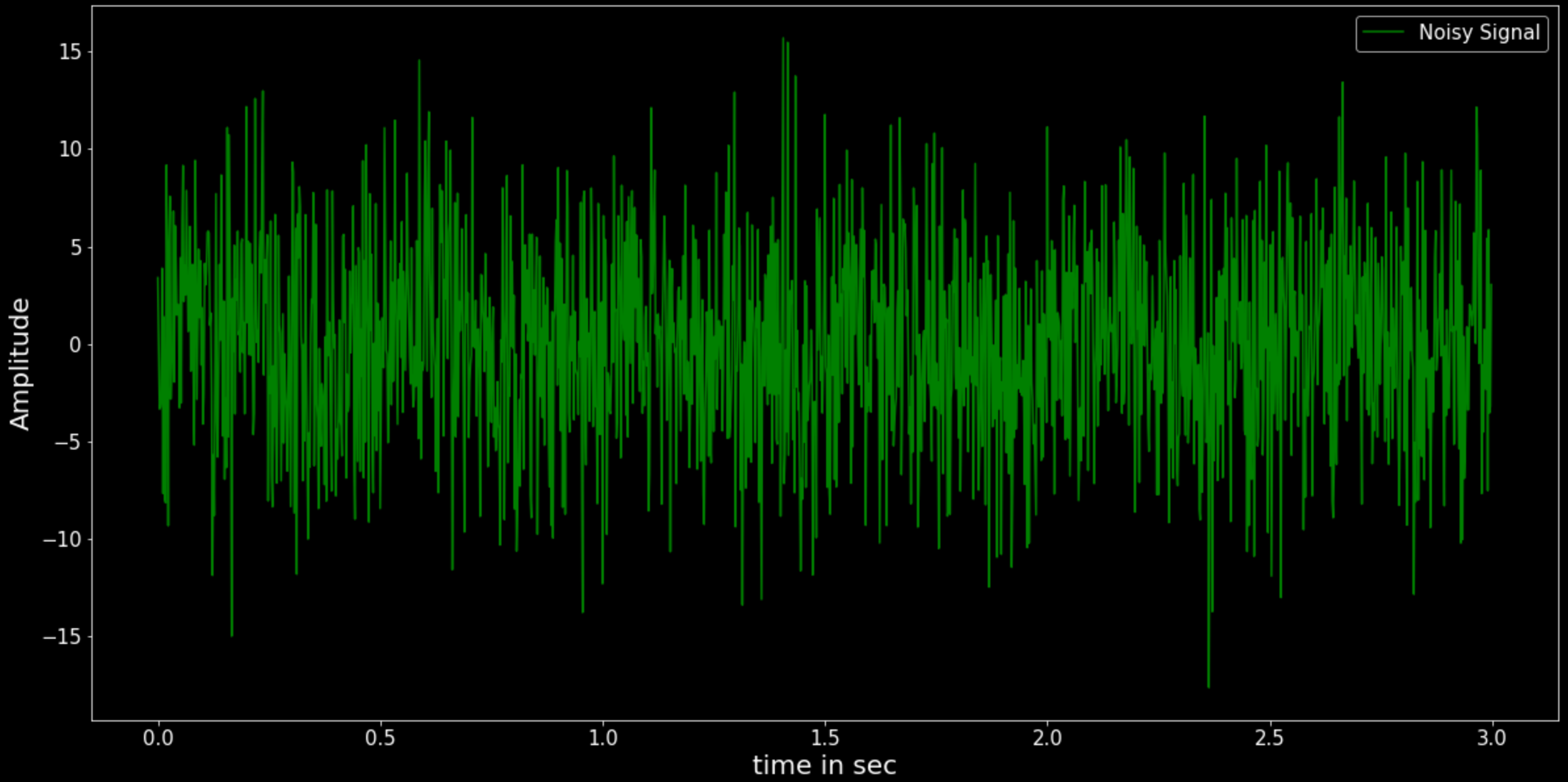
$$y(n) = \frac{1}{M} \sum_{i=0}^{M-1} x(n+i)$$

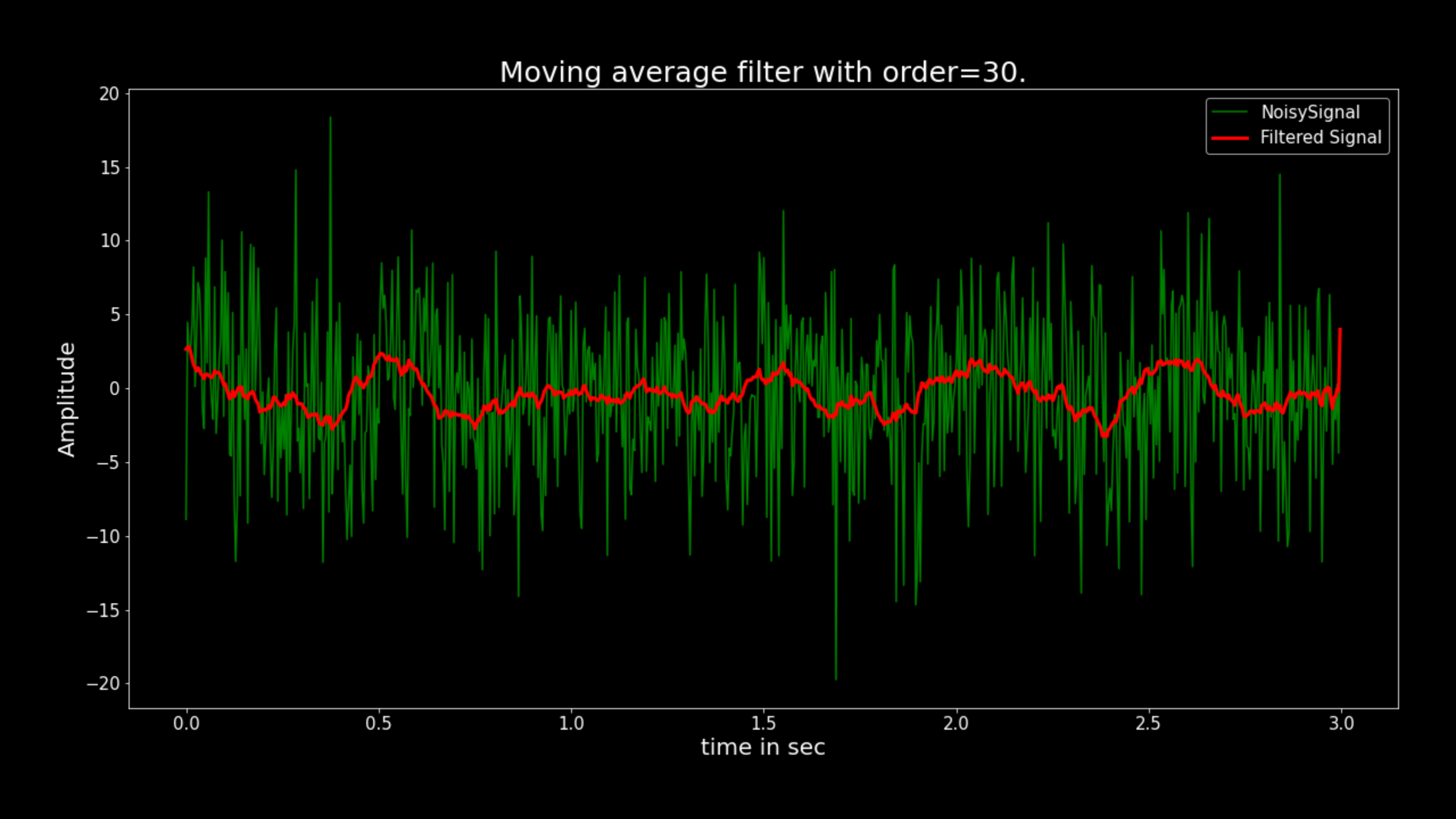
If we have a five point filter moving on the 10^{th} point then for symmetrically chosen points.

$$y[10] = \frac{y[8] + y[9] + y[10] + y[11] + y[12]}{5}$$

The other possibility is

$$y[10] = \frac{y[10] + y[11] + y[12] + y[13] + y[14]}{5}$$

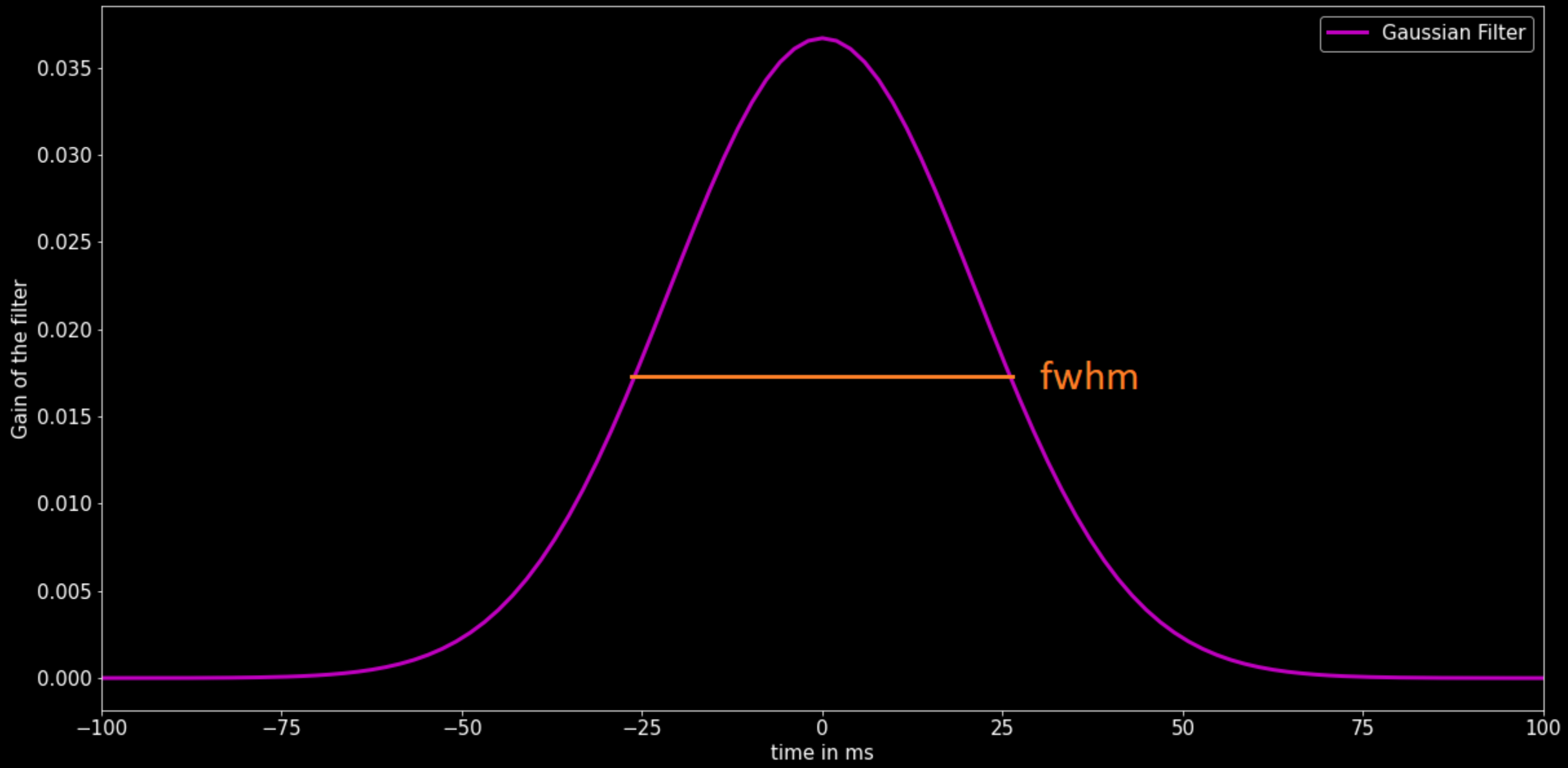

Noisy Signal
15
10
5
0
−5
−10
−15
Amplitude
0.0
0.5
1.0
1.5
2.0
2.5
3.0
time in sec

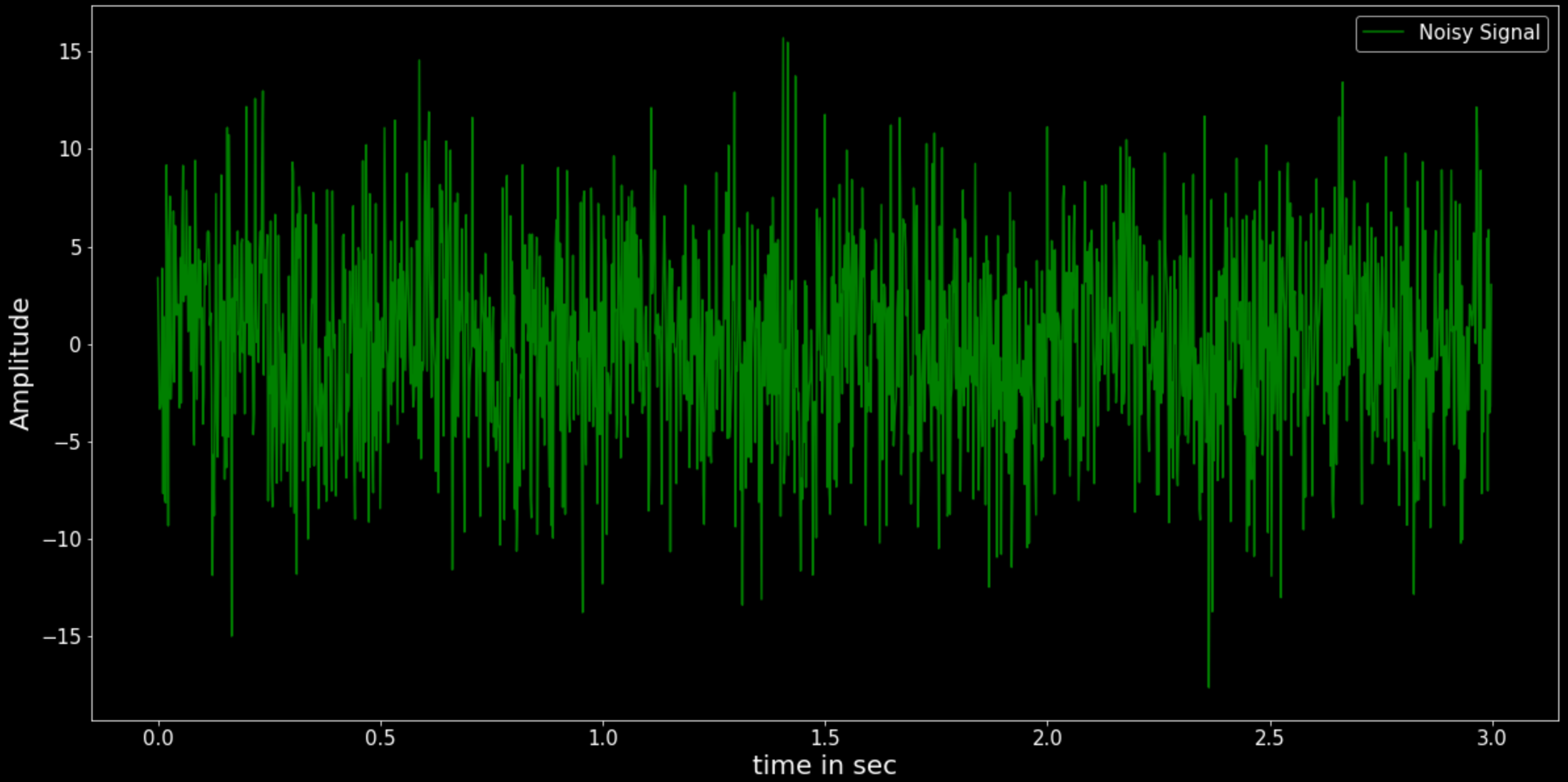

Moving average filter with order=30.
NoisySignal
Filtered Signal
Amplitude
time in sec
20
15
10
5
0
−5
−10
−15
−20
0.0
0.5
1.0
1.5
2.0
2.5
3.0

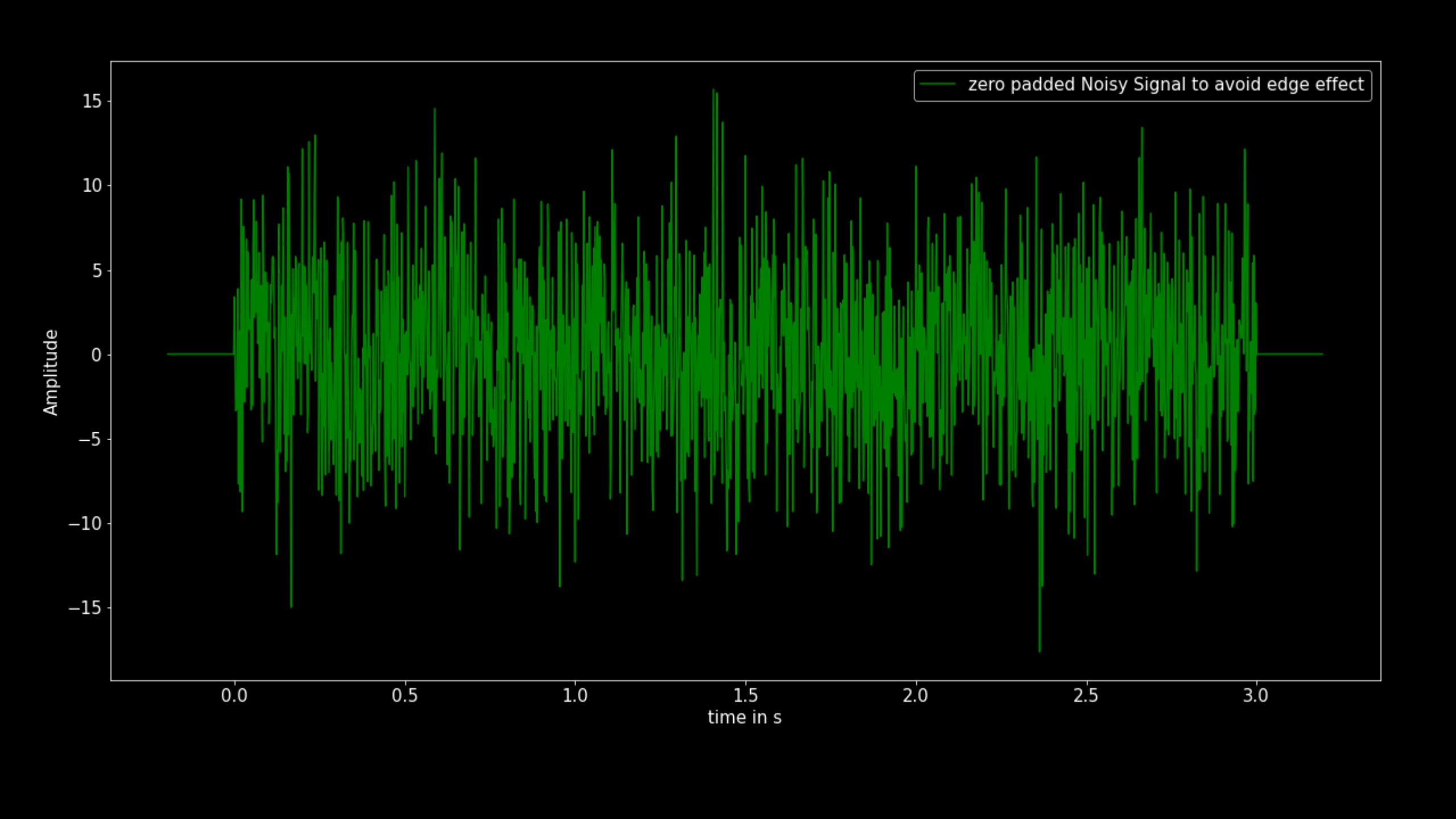
Gaussian Average or Gaussian Mean Filter

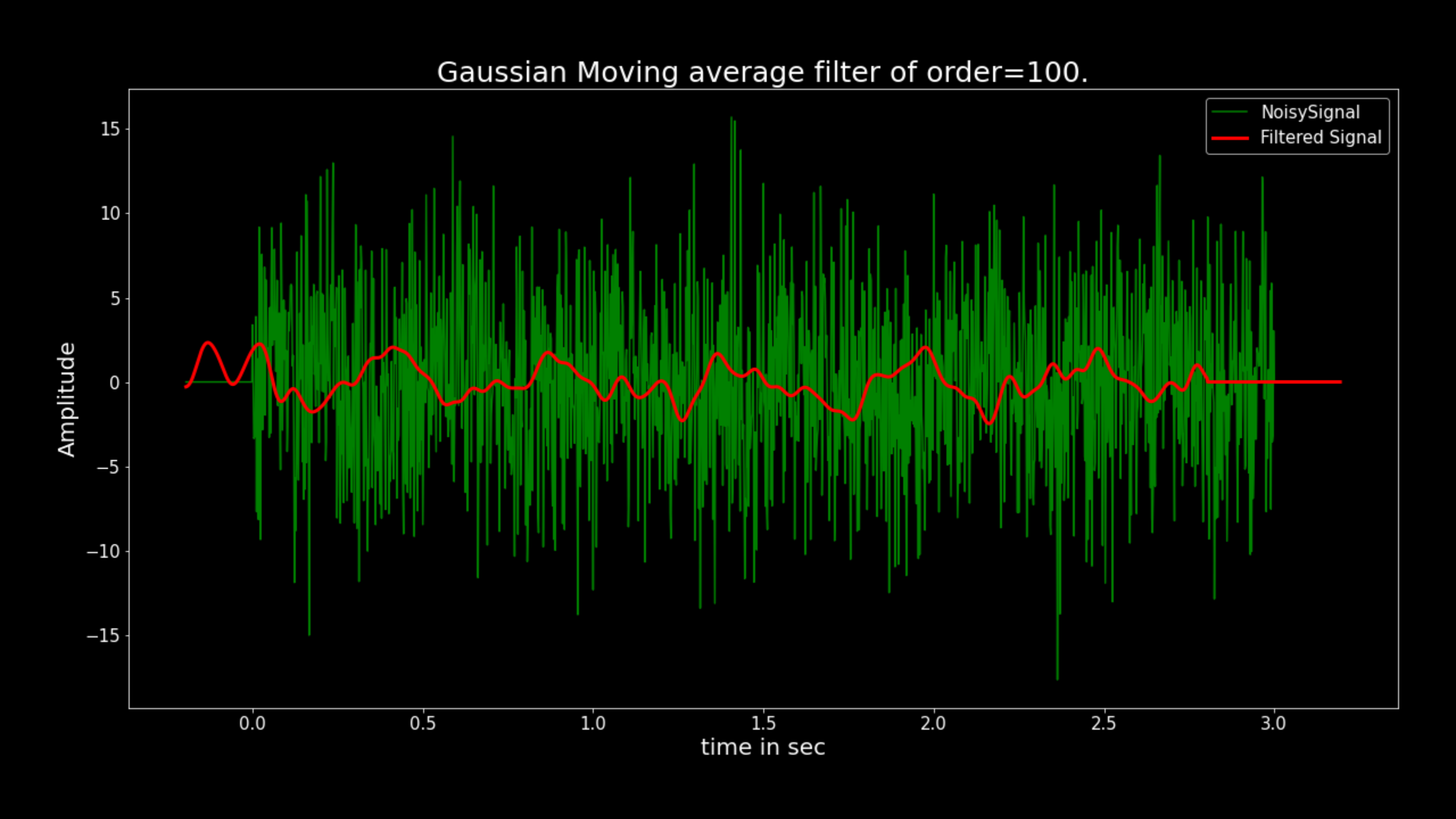
$$GaussianFilter = e^{\frac{-4ln(2)t^2}{(fwhm)^2}}$$

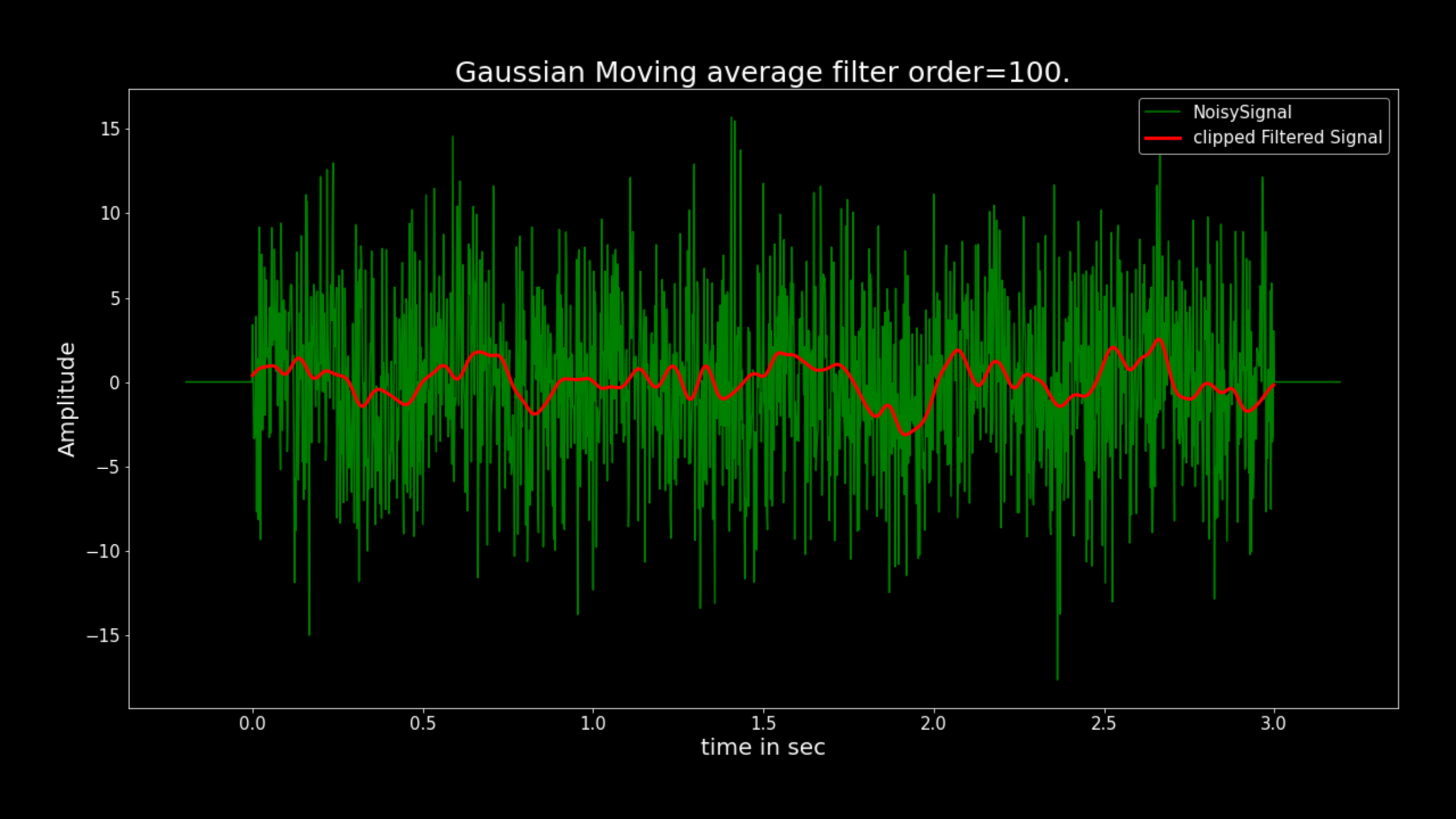
Where $fwhm$ is an important parameter of Gaussian and is called full width half maximum. The spread of the Gaussian depends upon this parameter. It is approximately 2.4 times the standard deviation.


Gaussian Filter
fwhm
Gain of the filter
time in ms
0.035
0.030
0.025
0.020
0.015
0.010
0.005
0.000
−100
−75
−50
−25
0
25
50
75
100


Noisy Signal
15
10
5
0
−5
−10
−15
Amplitude
0.0
0.5
1.0
1.5
2.0
2.5
3.0
time in sec


zero padded Noisy Signal to avoid edge effect
15
10
5
0
−5
−10
−15
Amplitude
0.0
0.5
1.0
1.5
2.0
2.5
3.0
time in s


Gaussian Moving average filter of order=100.
NoisySignal
Filtered Signal
Amplitude
time in sec
15
10
5
0
−5
−10
−15
0.0
0.5
1.0
1.5
2.0
2.5
3.0


Gaussian Moving average filter order=100.
NoisySignal
clipped Filtered Signal
Amplitude
time in sec
15
10
5
0
−5
−10
−15
0.0
0.5
1.0
1.5
2.0
2.5
3.0

Median Filter

Median of the Data

Consider the following data sequence

7, 9, 10, 14, 17

The median is the middle value of the data i.e 10.
median = 10.

If the data sequence has even number of elements i.e
7, 9, 10, 14, 17, 19

then the median is the mean value of the two middle values i.e
median = $\frac{10+14}{2}$

median = 12

If the data sequence is unsorted then we have to sort the data before finding the median.

Consider the following data.

12, 4, 3, 7, 11, 6, 13

After data sorting, we have

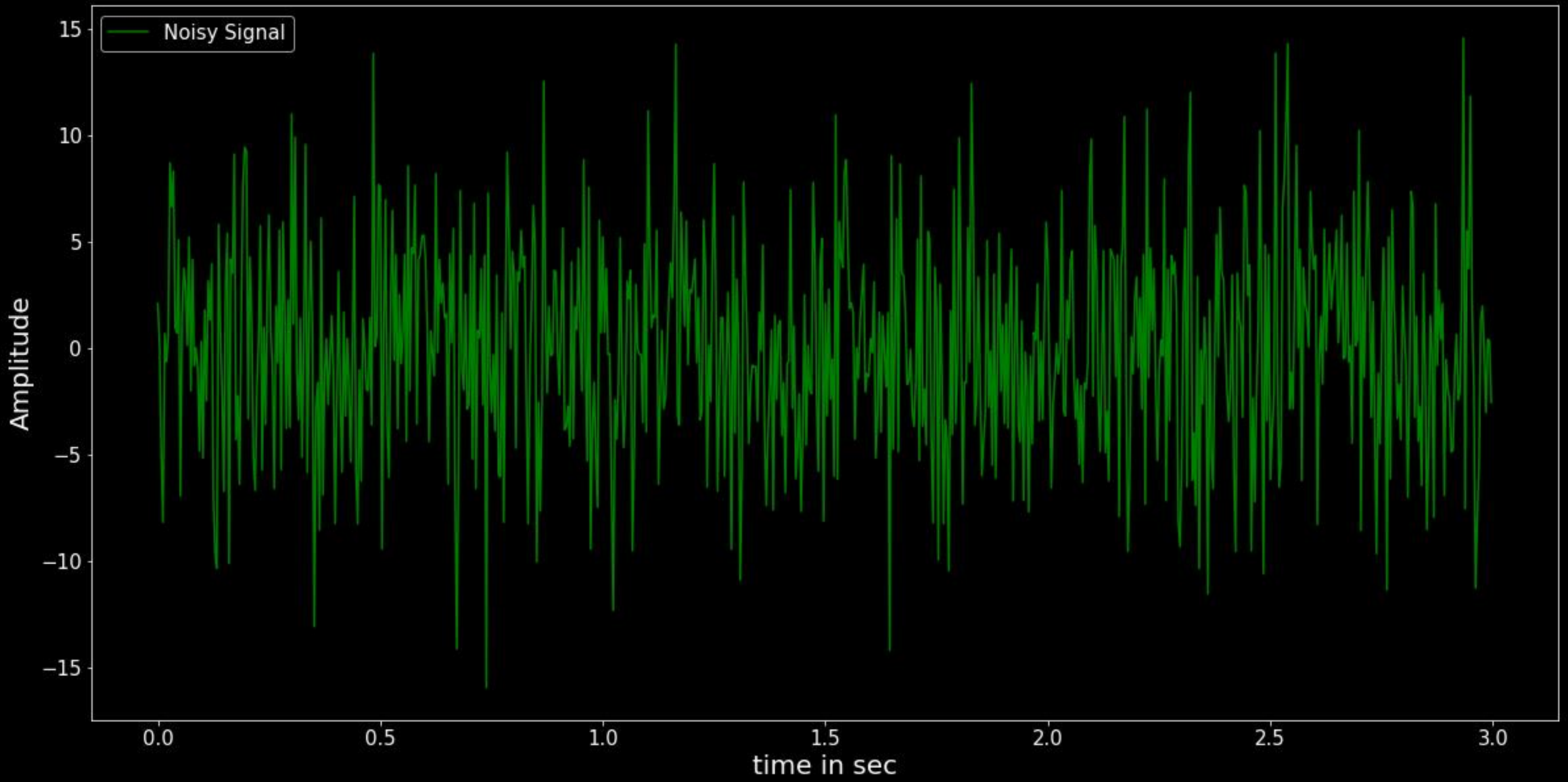
3, 4, 6, 7, 11, 12, 13

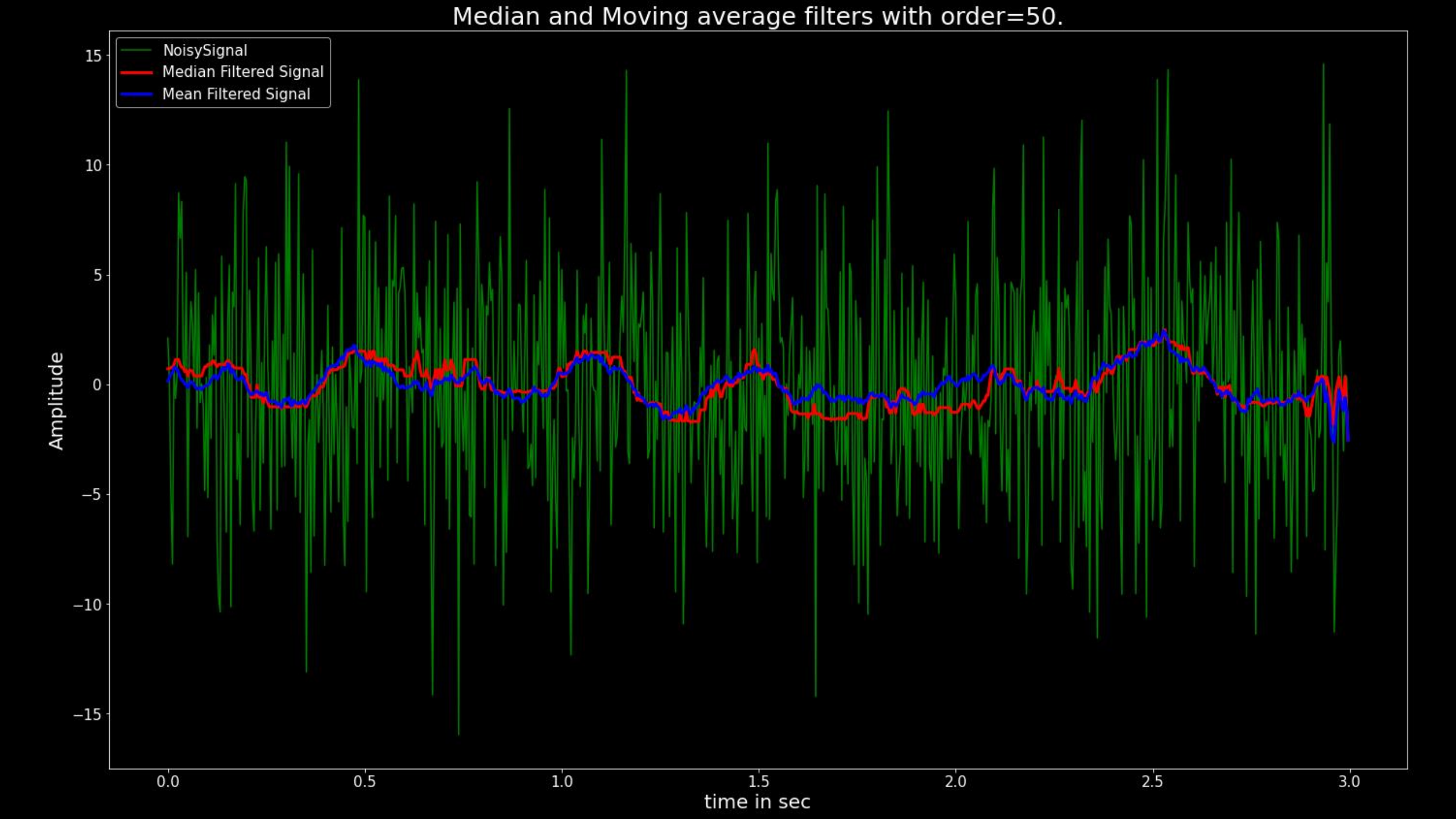
Now median is the middle value of the data i.e 7

median = 7

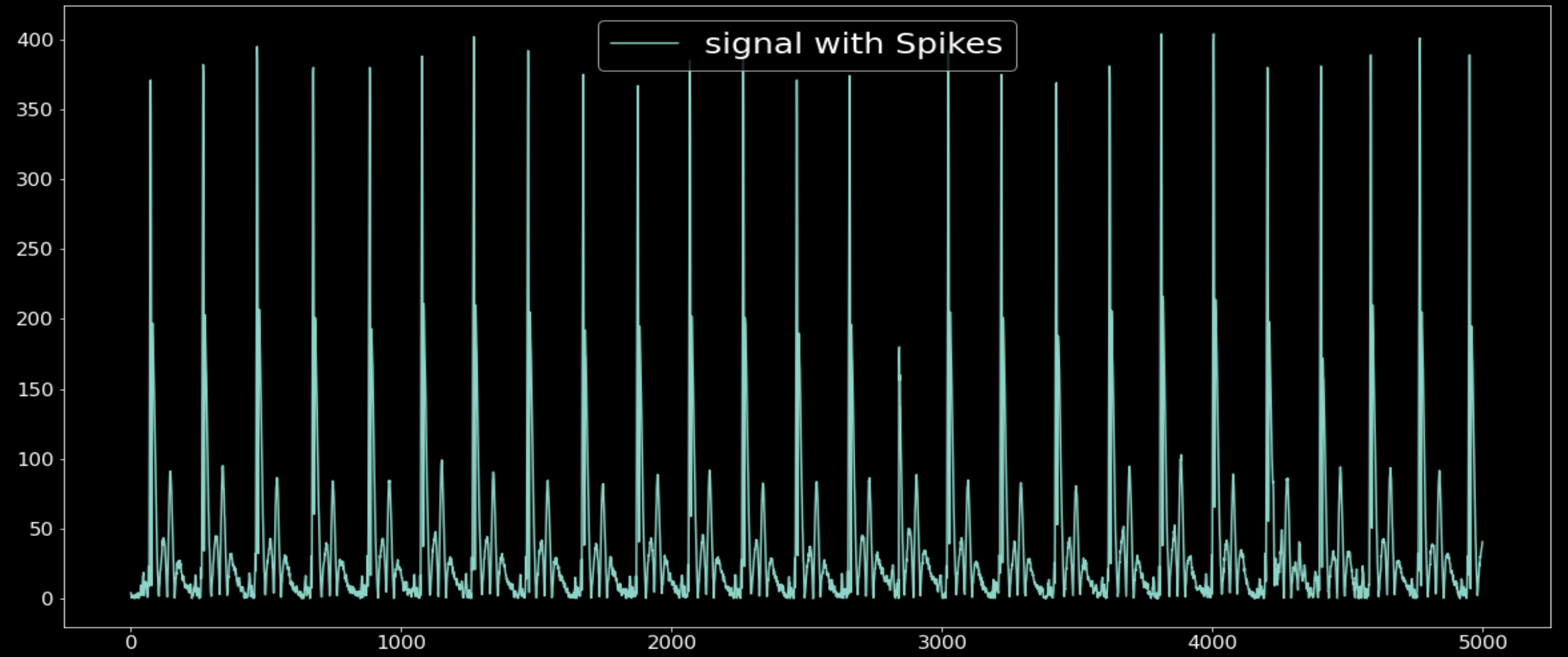
Median Filter

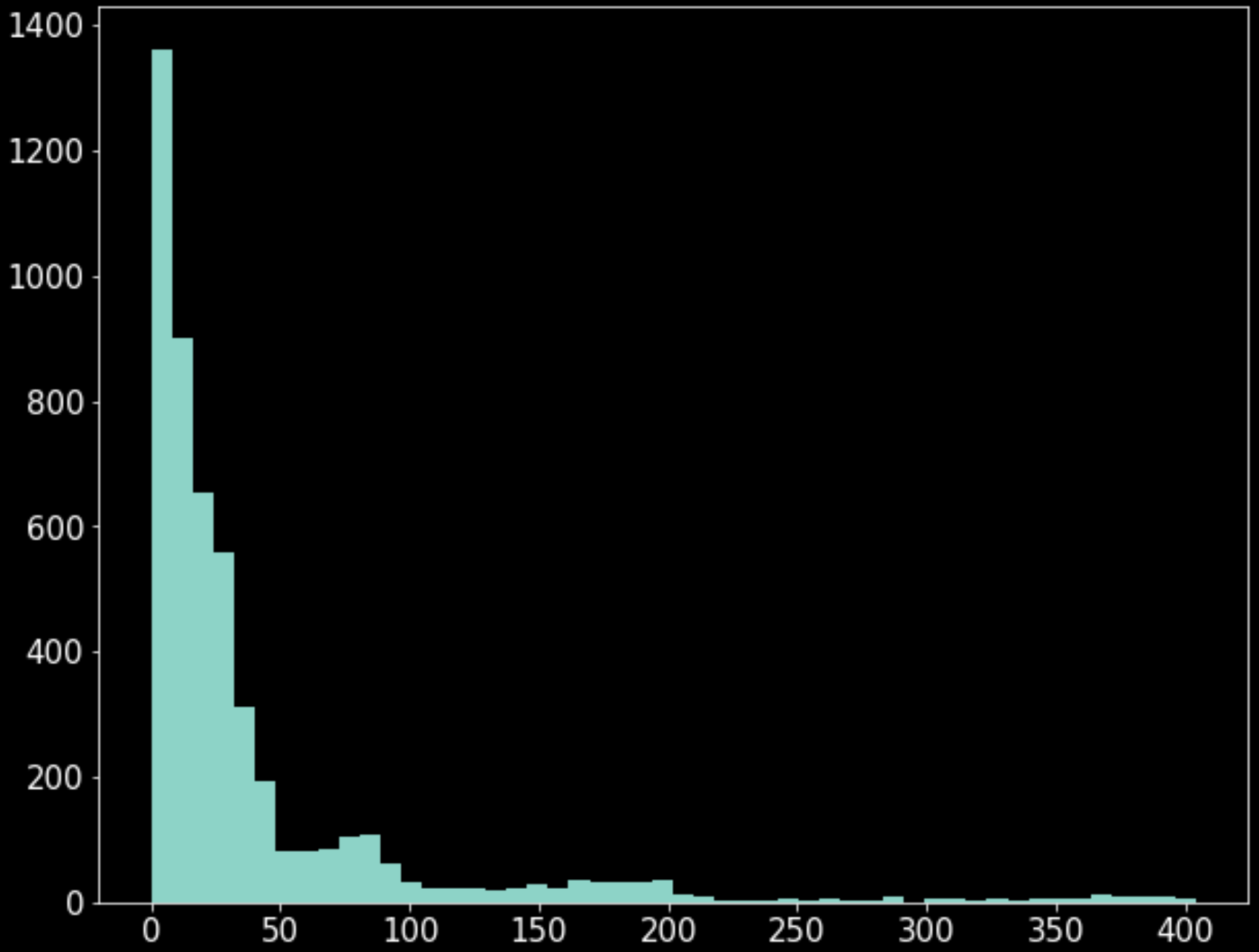
Median filter is a nonlinear filter. It works by moving through the noisy signal and replacing each data point of the noisy signal with median values of the neighbouring data points.

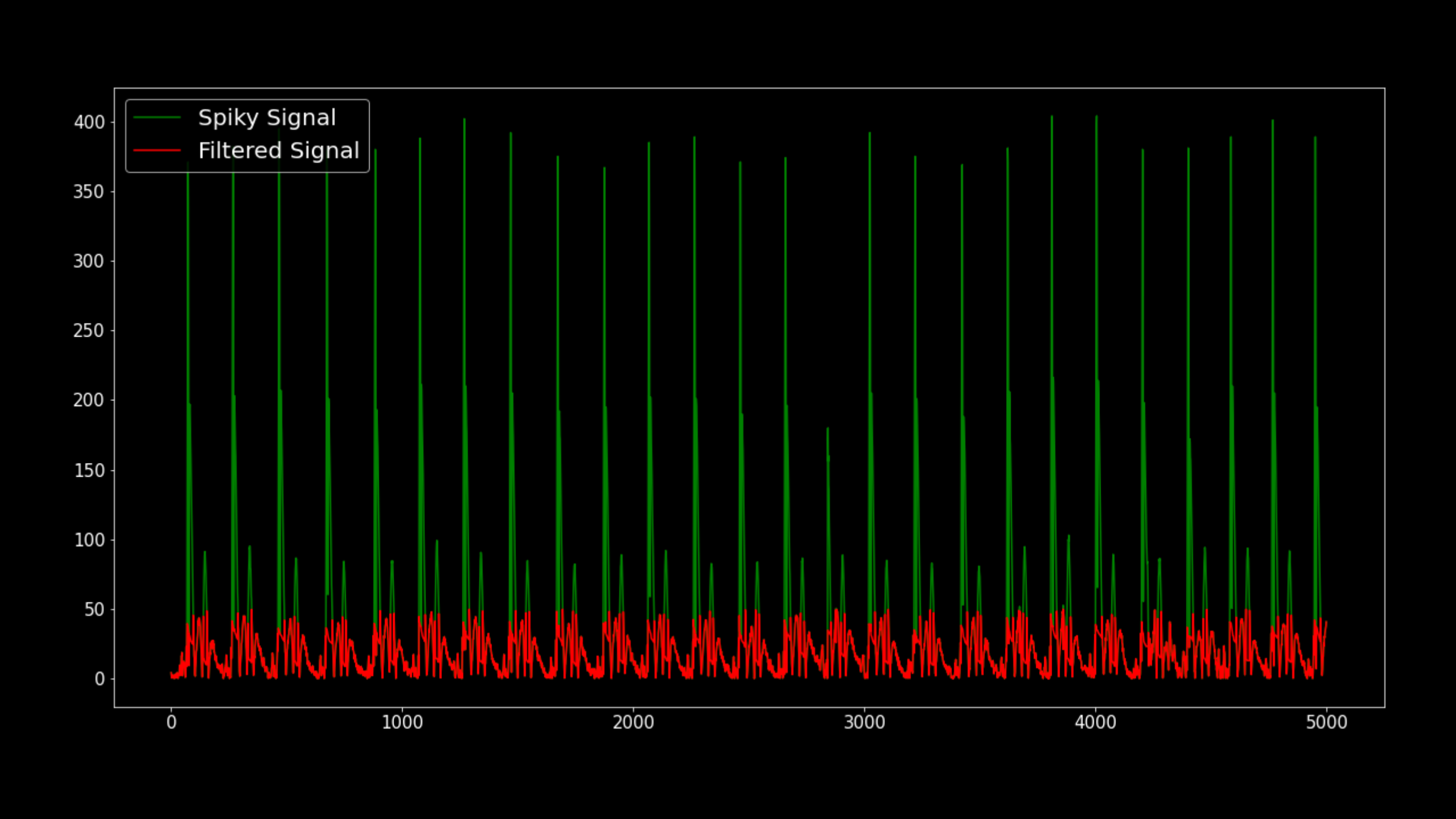

Noisy Signal
15
10
5
0
−5
−10
−15
Amplitude
0.0
0.5
1.0
1.5
2.0
2.5
3.0
time in sec


Median and Moving average filters with order=50.
NoisySignal
Median Filtered Signal
Mean Filtered Signal
Amplitude
time in sec
15
10
5
0
−5
−10
−15
0.0
0.5
1.0
1.5
2.0
2.5
3.0

Recovering signal from noisy spikes by Median Filter




1400
1200
1000
800
600
400
200
0
0
50
100
150
200
250
300
350
400


Spiky Signal
Filtered Signal
400
350
300
250
200
150
100
50
0
0
1000
2000
3000
4000
5000