Topics to be covered

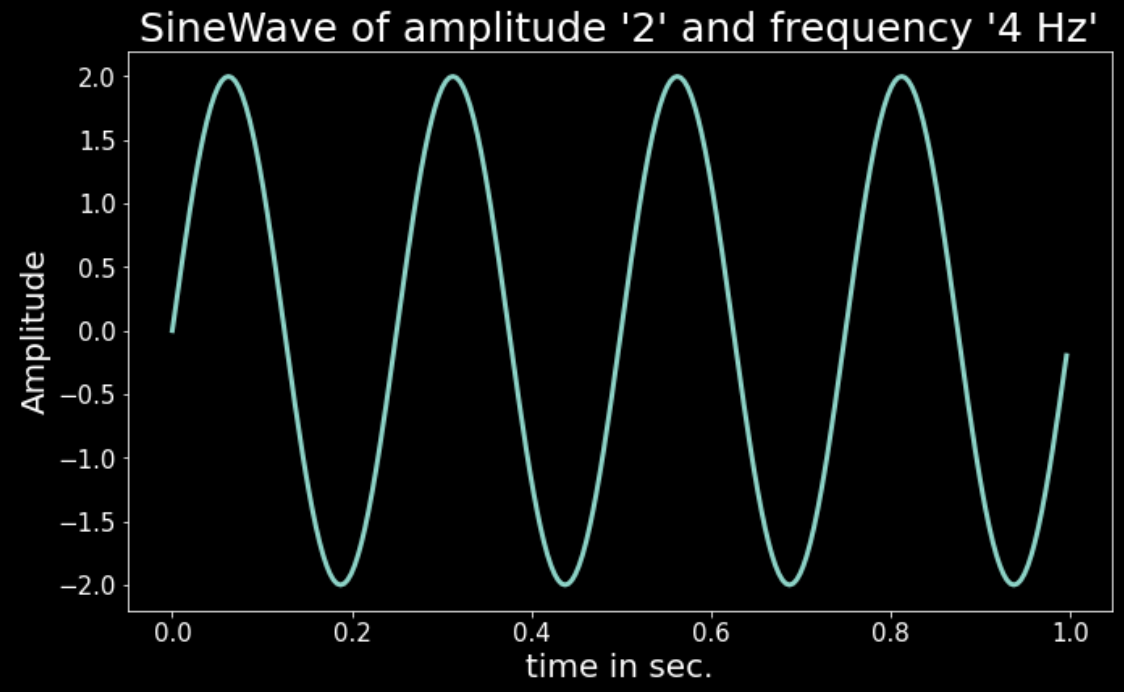
- Concept of Frequency
- Construction of a wave with sine, cosine and DC components
- Definition and Mechanism of Fourier Transform
- Step by Step calculation of Fourier Transform
- Fourier transform of signal with multiple frequencies
- Amplitude and Power Spectrum
- Inverse Fourier Transform
- Application of Fourier Transform

Fourier Transform

Amongst the several transformations that are used in Engineering and Sciences, the Fourier transform is probably the most popular one.

The purpose of the Fourier transform is to obtain further information from the signal that is not readily available in the raw form of the signal.

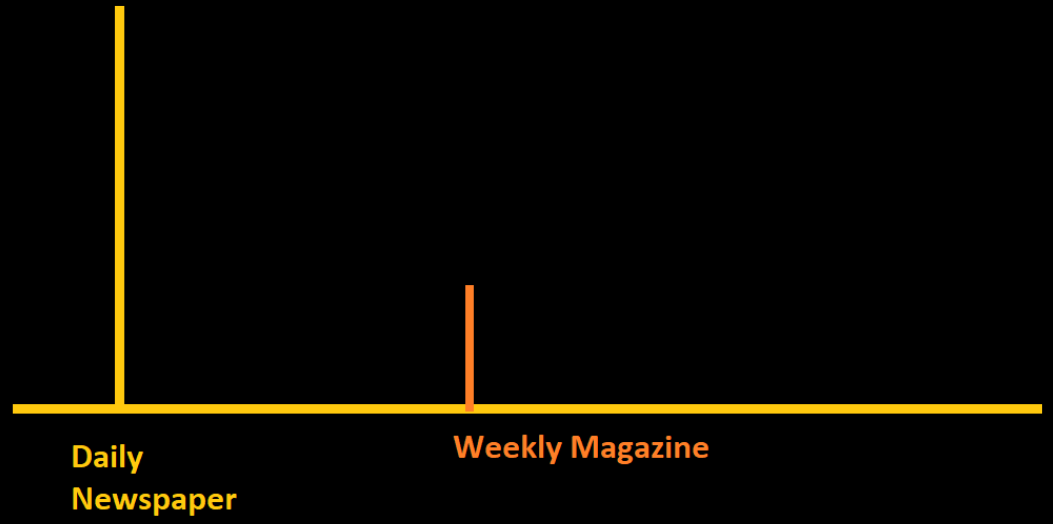
Most of the signals exists are in “time-domain” in their original or raw form. It means when we plot the signal then the “time”, which is independent variable, is on independent axis or X-axis and the other (dependent variable), which most of the cases is “amplitude” is on dependent axis or Y-axis. So, the plotting of the time-domain signal gives us the time-amplitude of the signal.



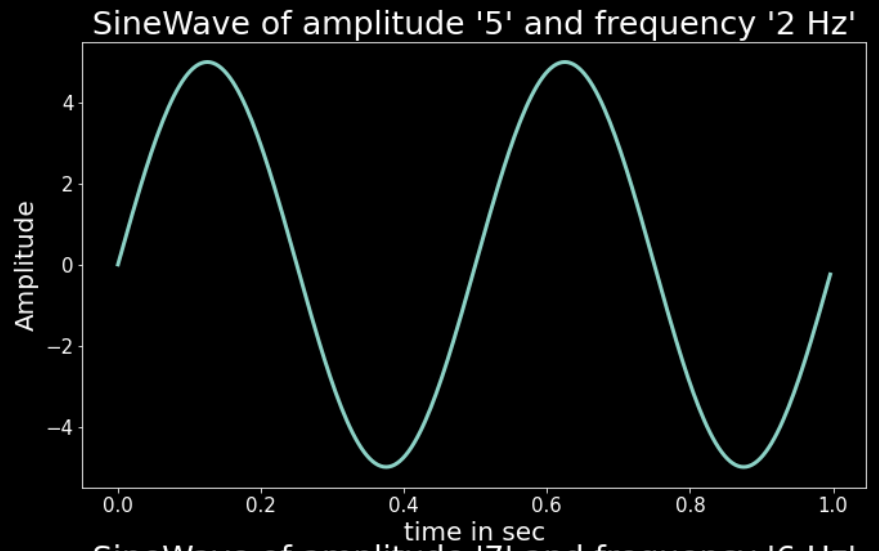
Concept of Frequency

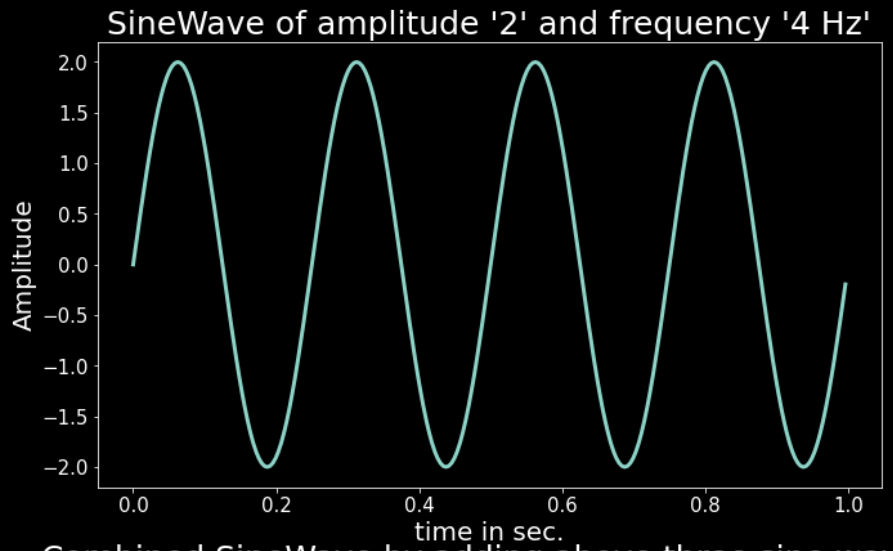
In various cases the most important information resides in the frequency contents or in frequency domain of the signal.

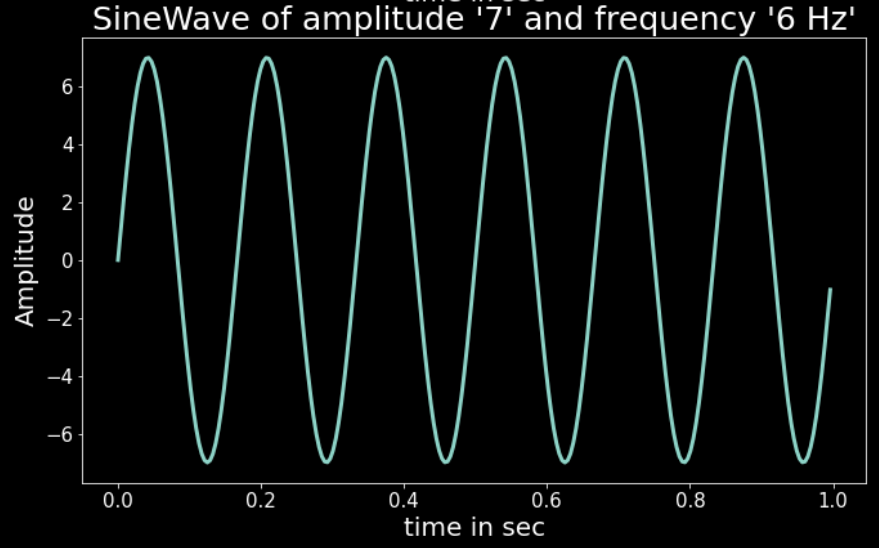
We know that if some physical quantity varies rapidly, we say that it has a high frequency as compare to those quantities which change slowly. For instance, the publication frequency of a daily published newspaper is higher than that of a weekly published magazine because the newspaper is published more frequently.

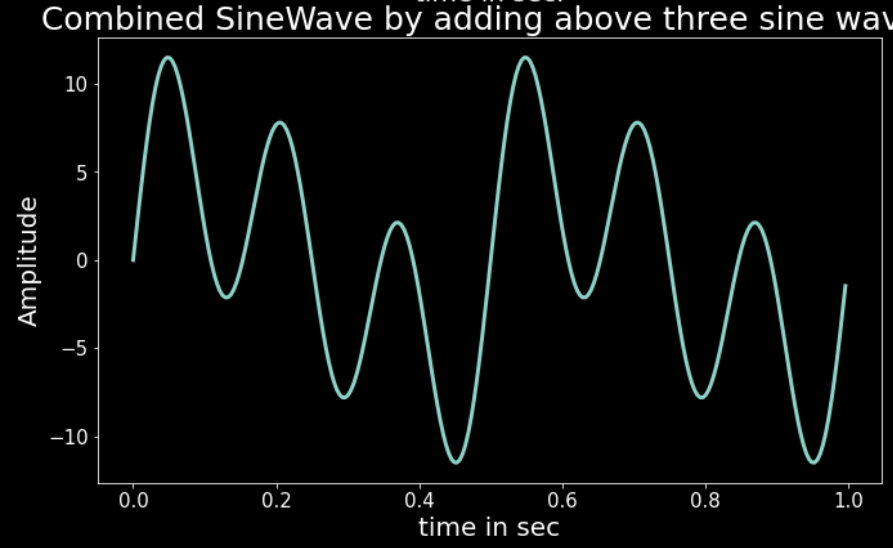


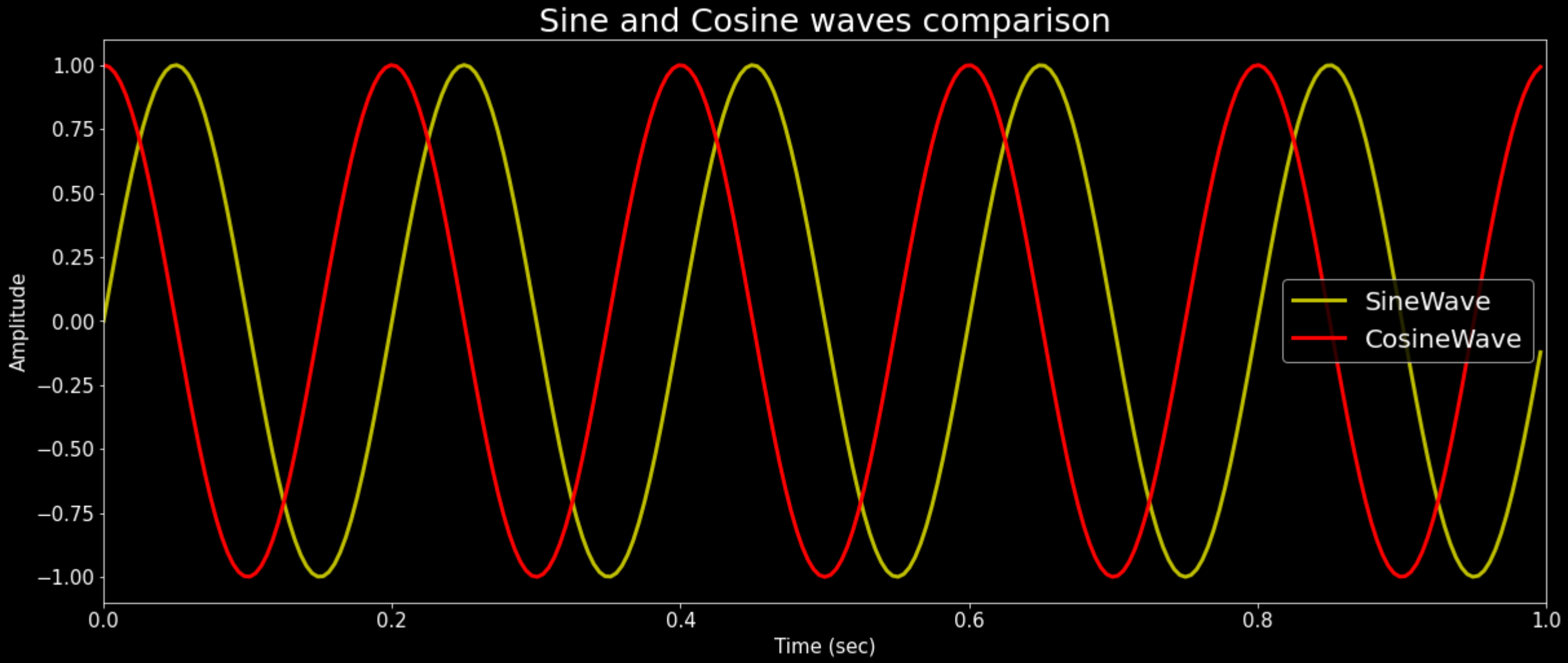
Constructing a Wave with different Sine Waves



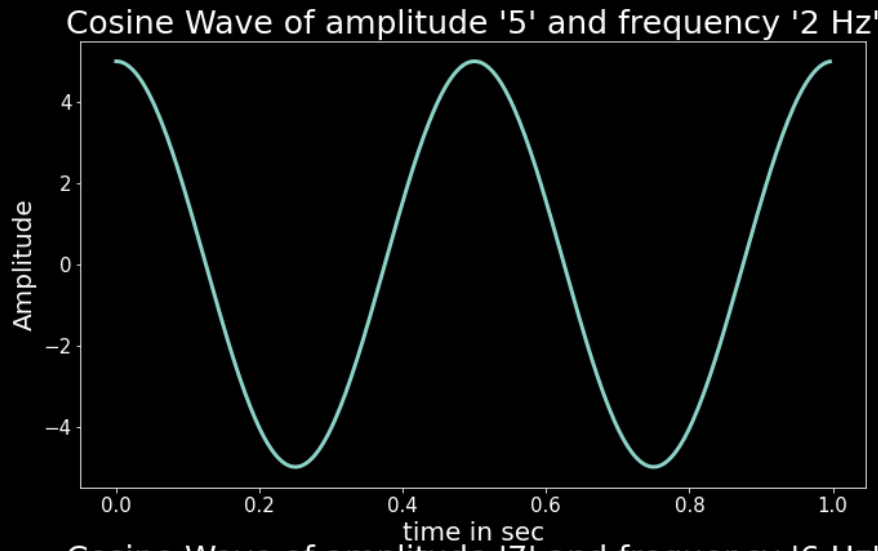


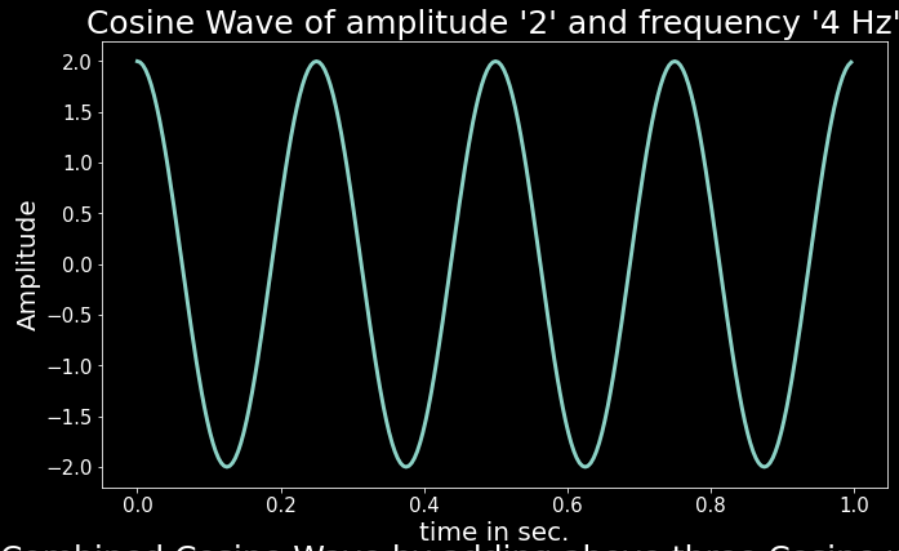


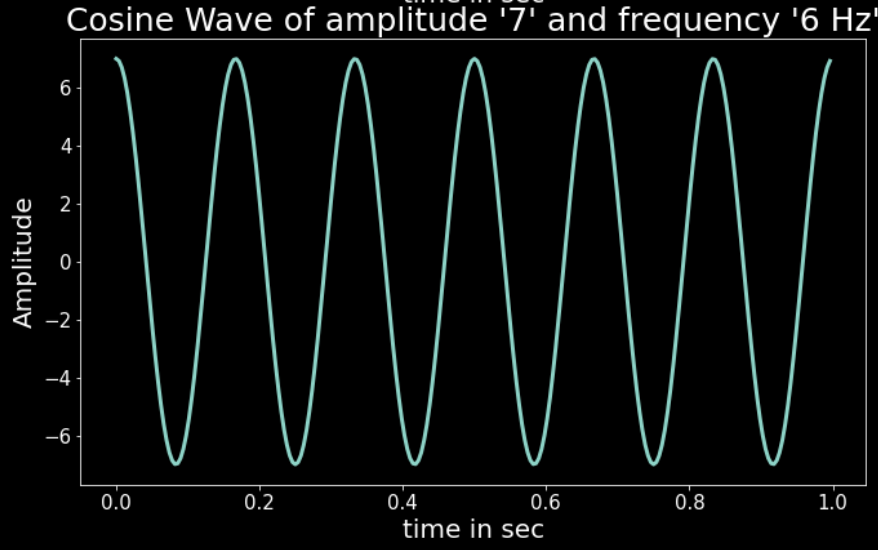


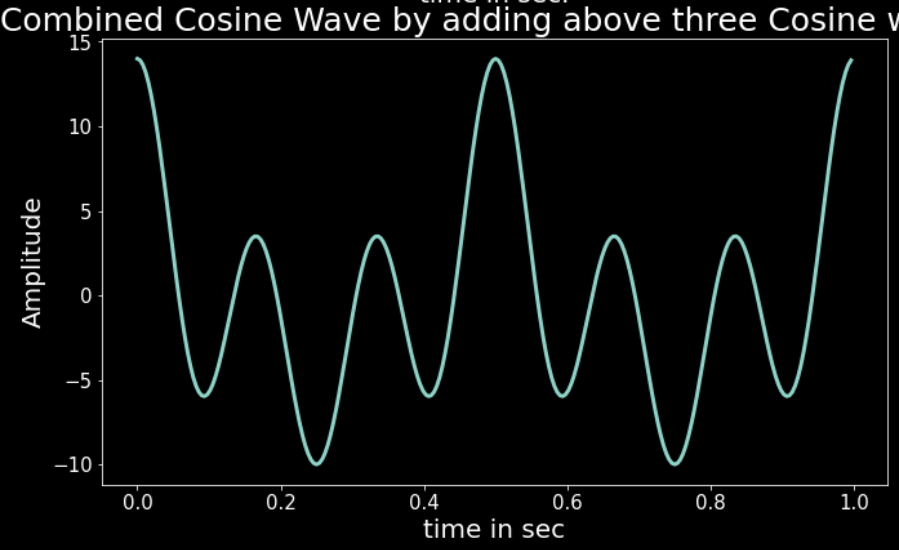

Sine and Cosine waves comparison
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
Amplitude
0.0
0.2
0.4
0.6
0.8
1.0
Time (sec)
SineWave
CosineWave

Constructing a Wave with different Cosine Waves

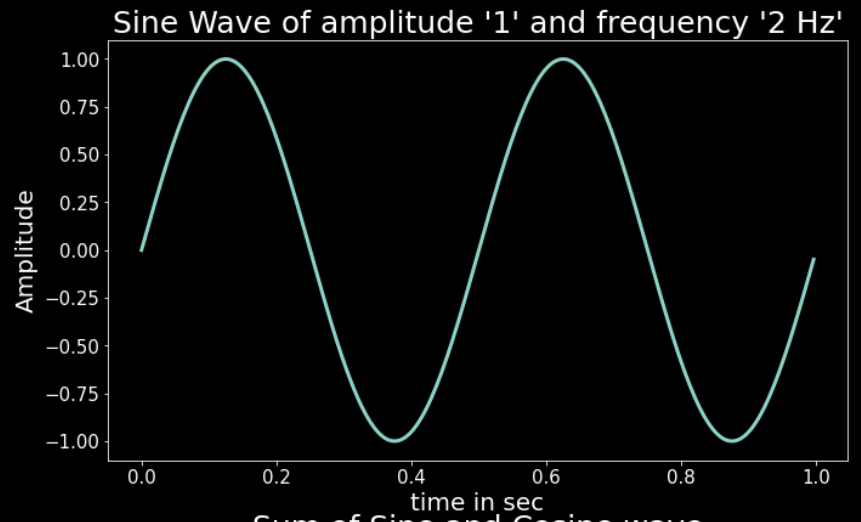


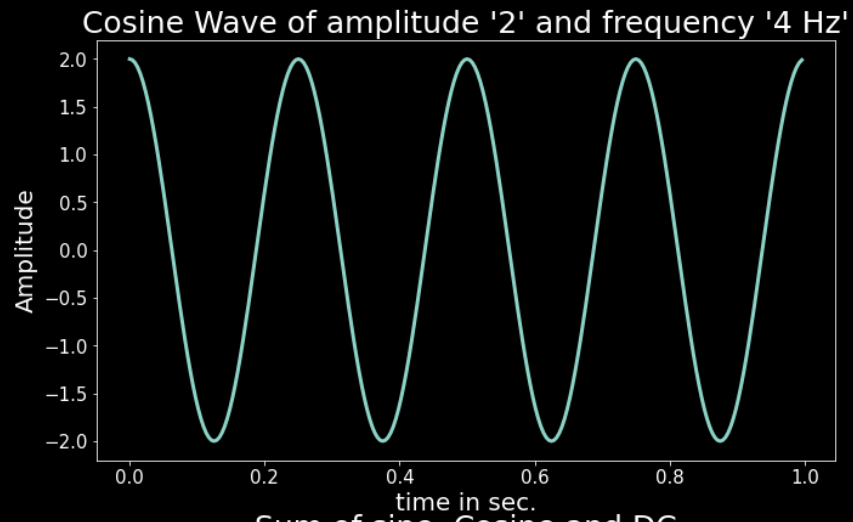


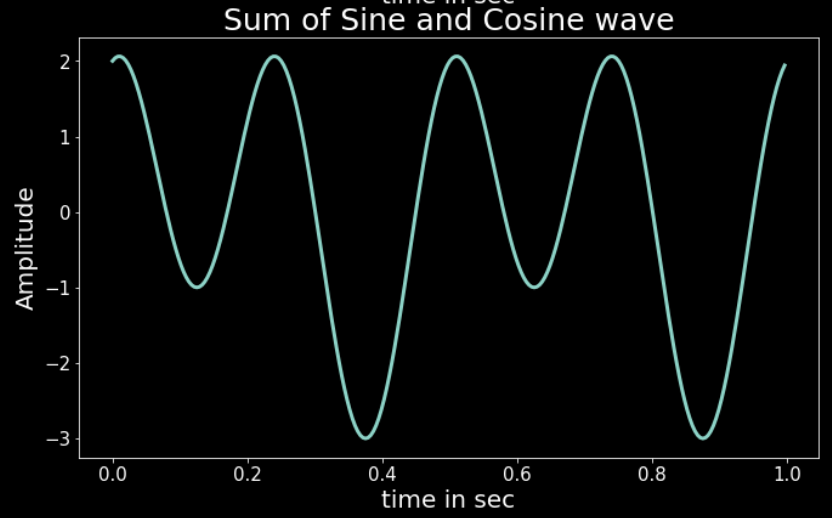


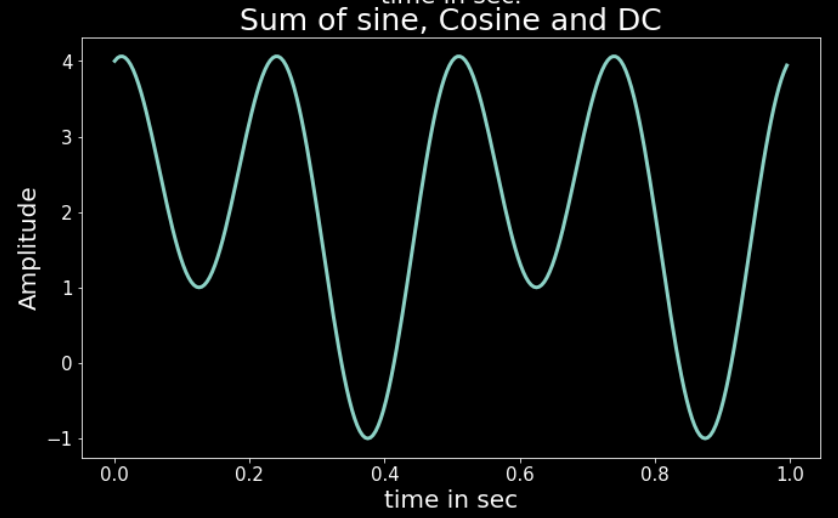


Constructing a Wave with SIne, cosine and DC

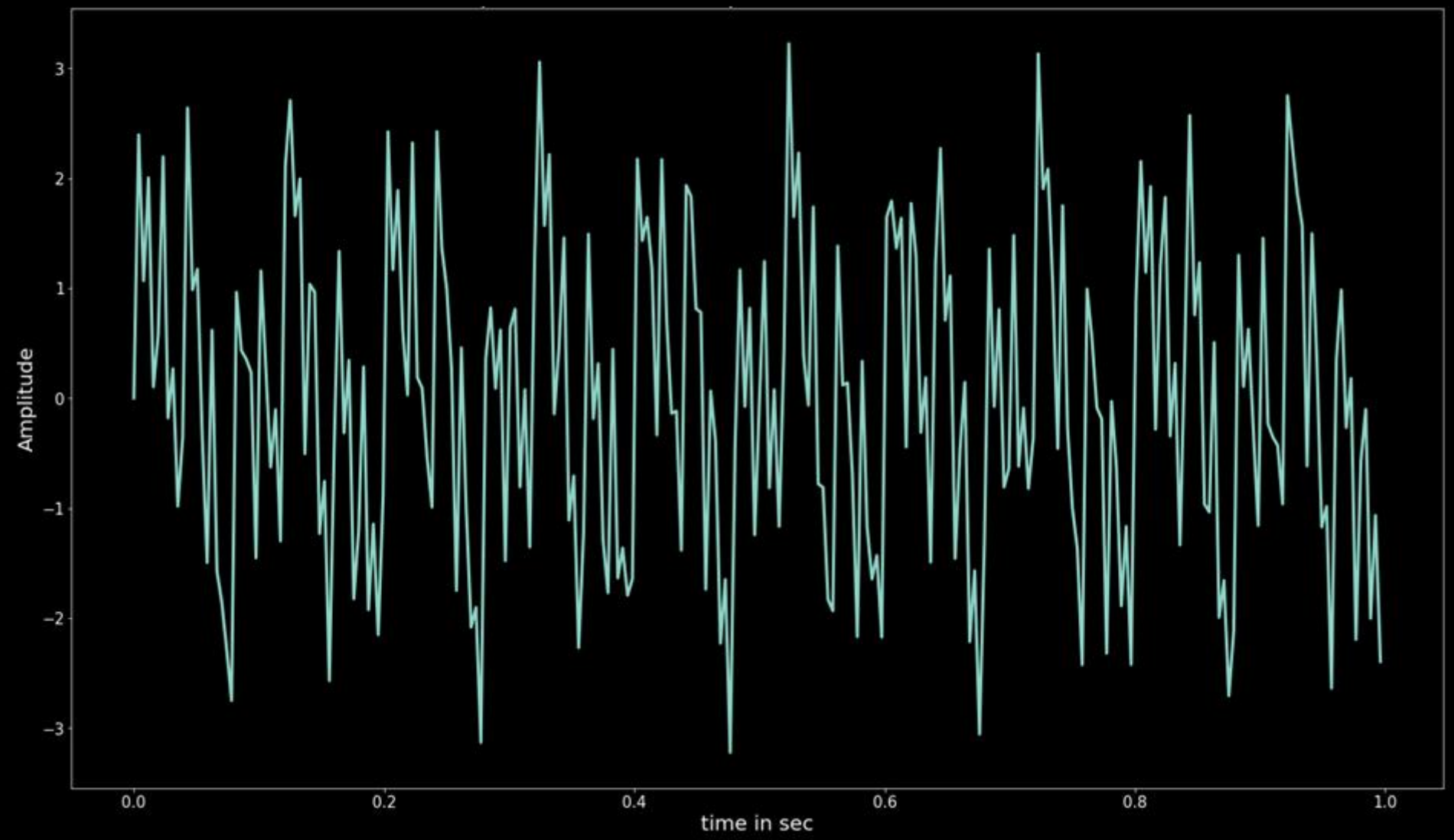








Signal with Frequencies 10, 25, 50 and 100 Hz

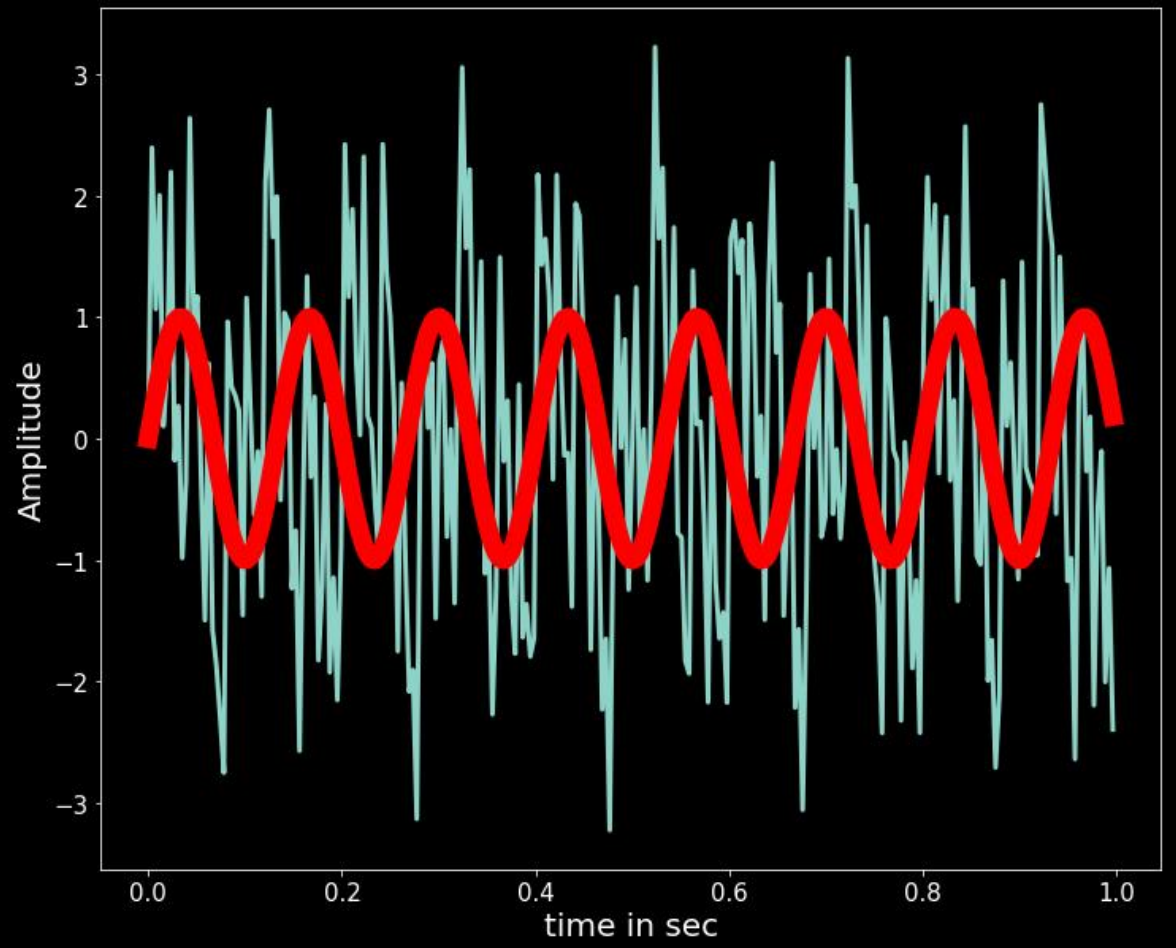


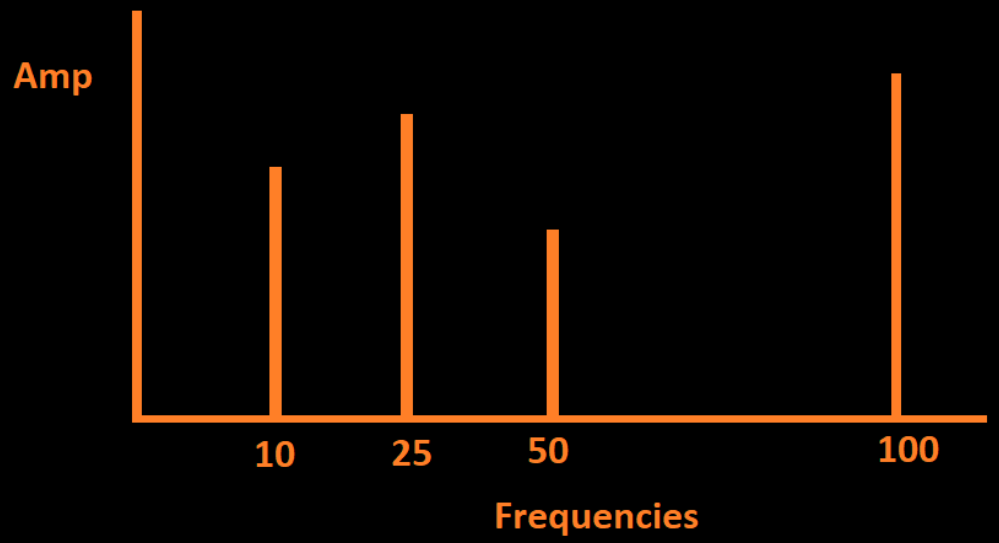
Procedure of Fourier Transform

Jean Baptiste Fourier , a French physicist, discovered that any periodic waveform can be decomposed into a sine wave and the combination of sine waves.

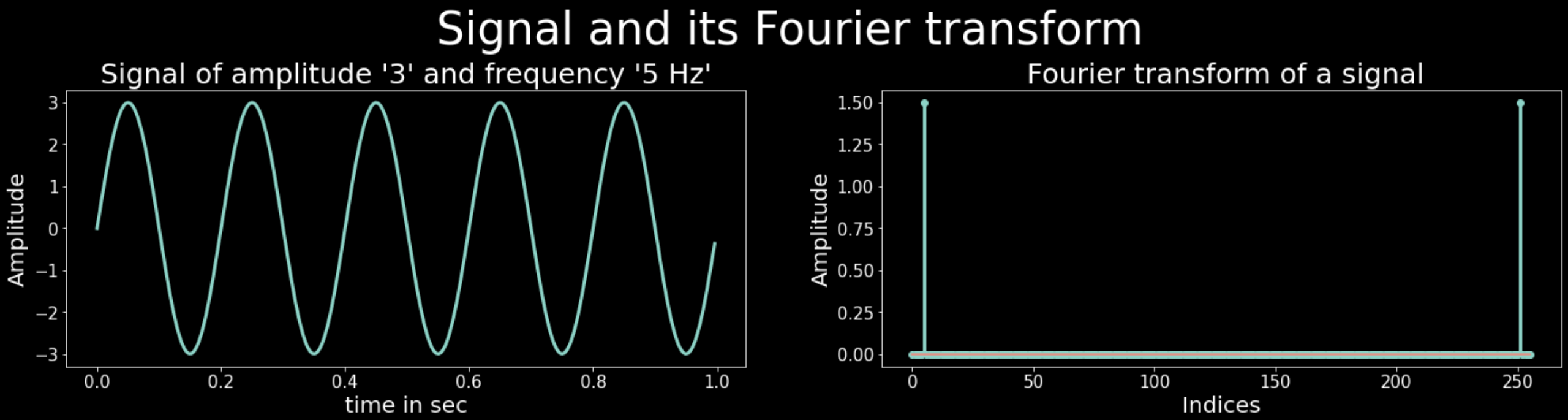
The mechanism of Fourier transform is to model the signal with the sine wave. If our signal has N data points, we select the the sine waves with atleast N different frequencies and then we find the similarities with the signal using dot product.

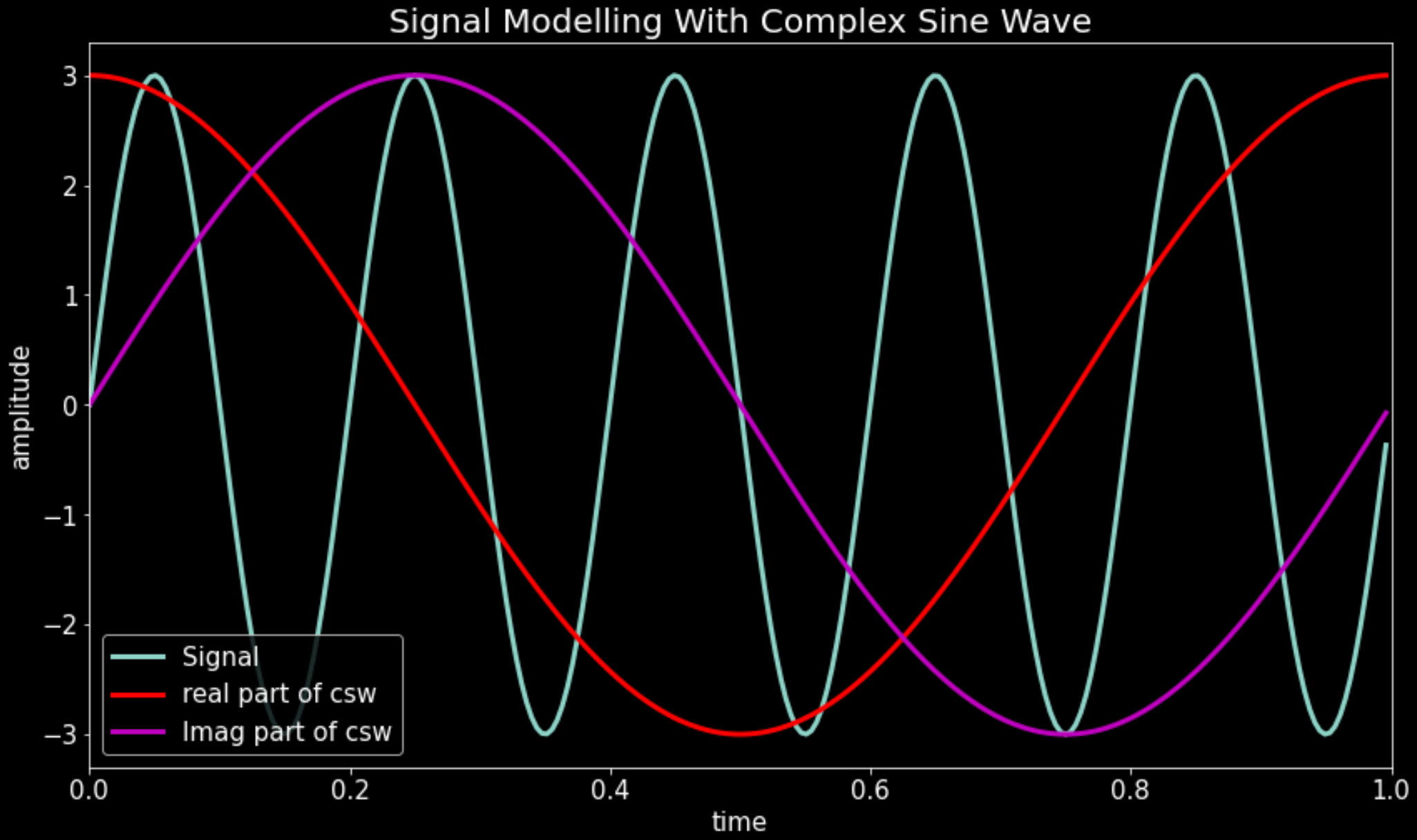
Signal with frequencies 10, 25, 50 and 100 Hz



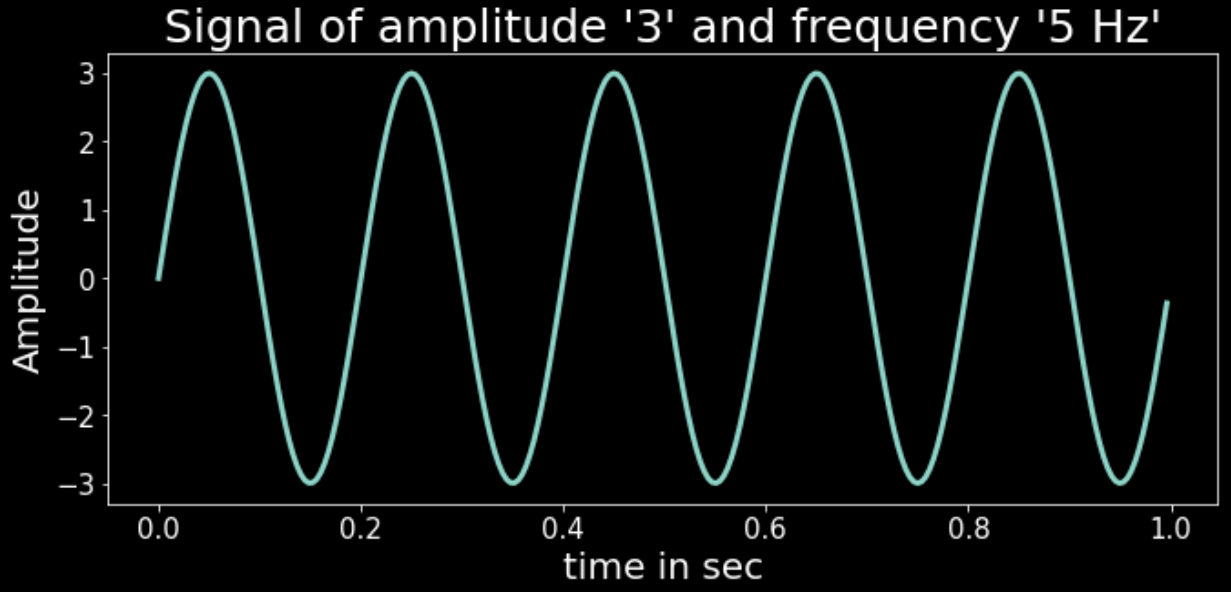


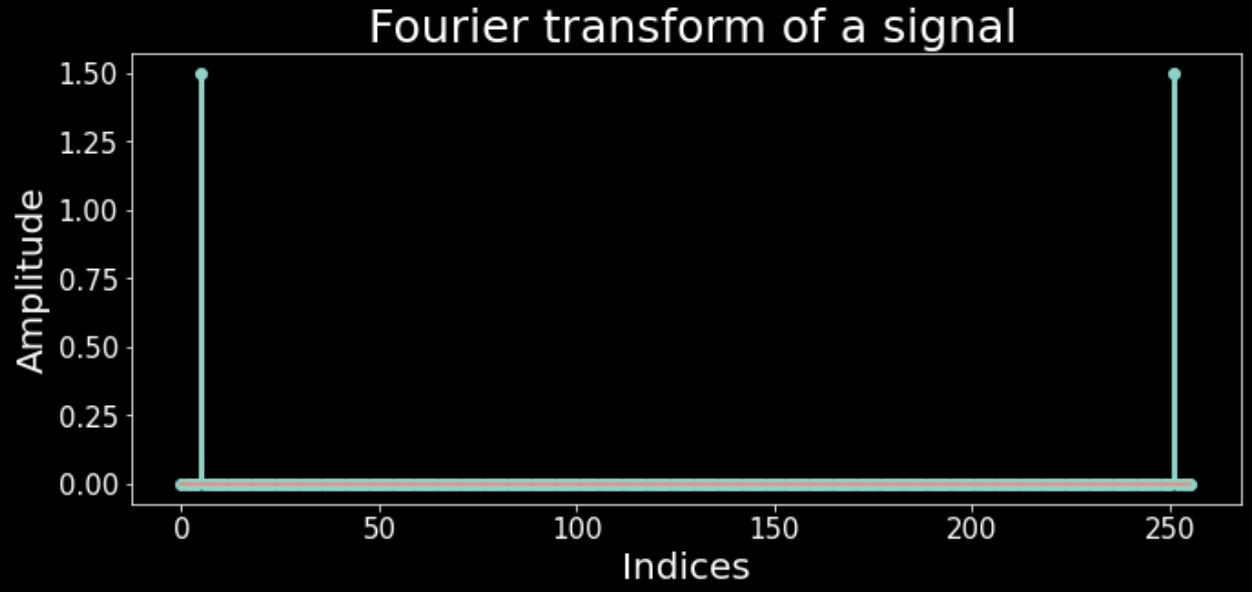
Fourier Transform of signal with amplitude ’3’ and frequency ’5’



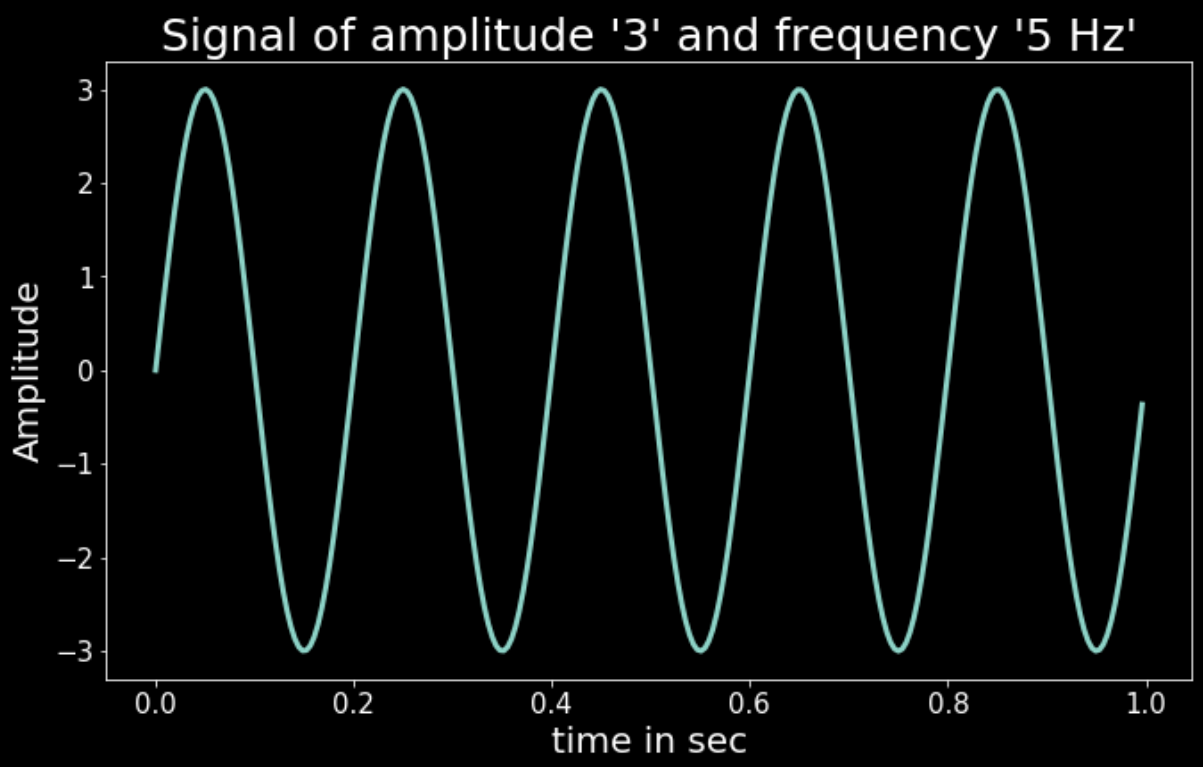

Signal Modelling With Complex Sine Wave
amplitude
time
Signal
real part of csw
Imag part of csw
3
2
1
0
−1
−2
−3
0.0
0.2
0.4
0.6
0.8
1.0

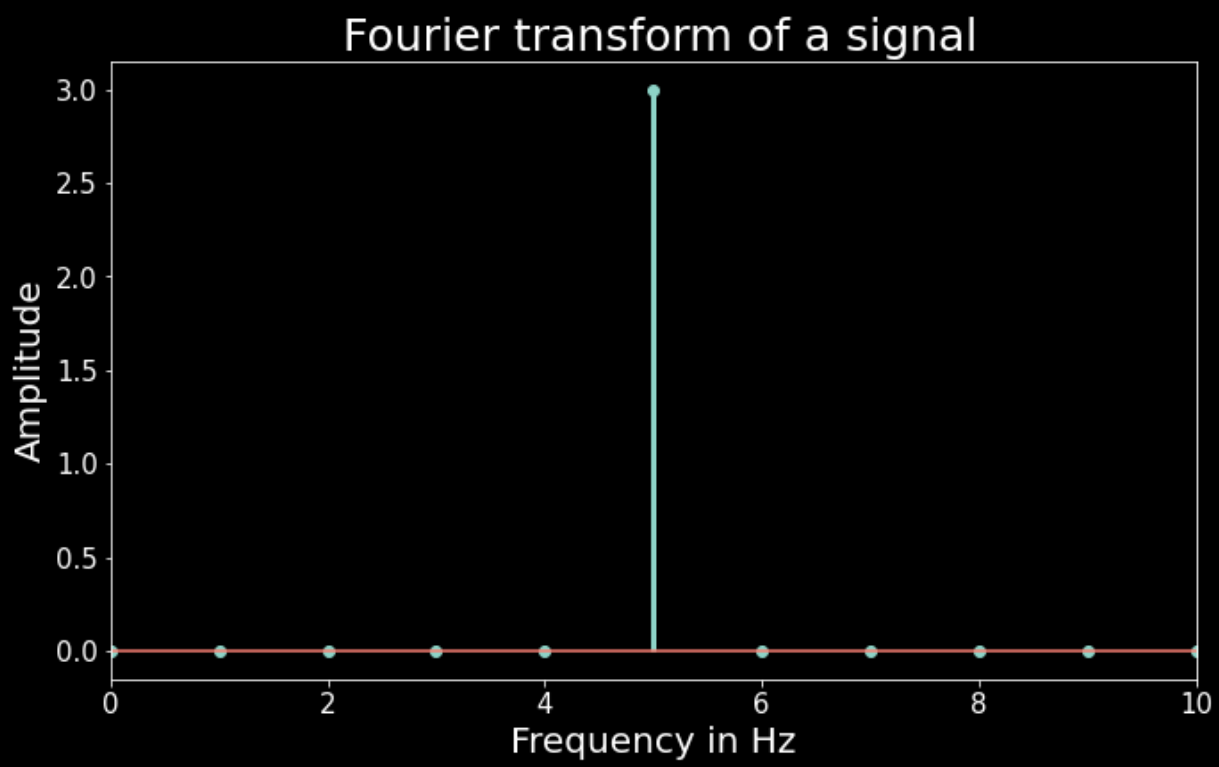
Signal and its Fourier transform



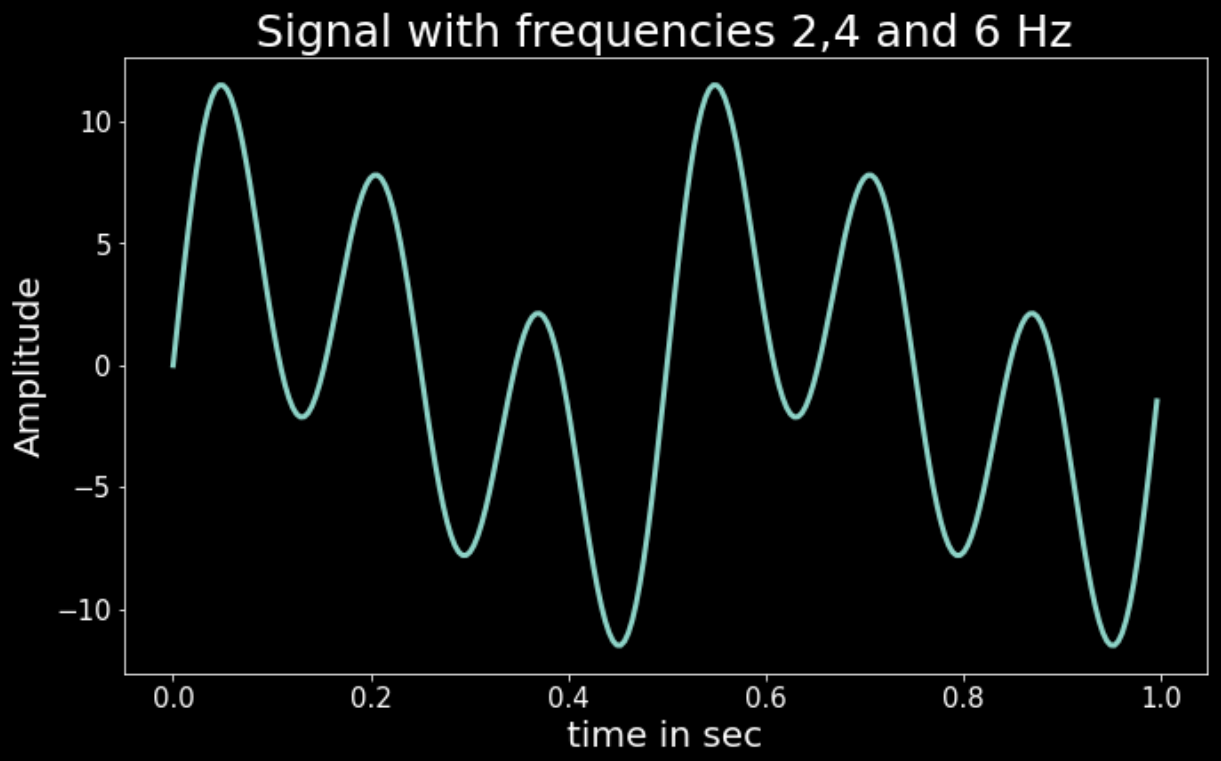


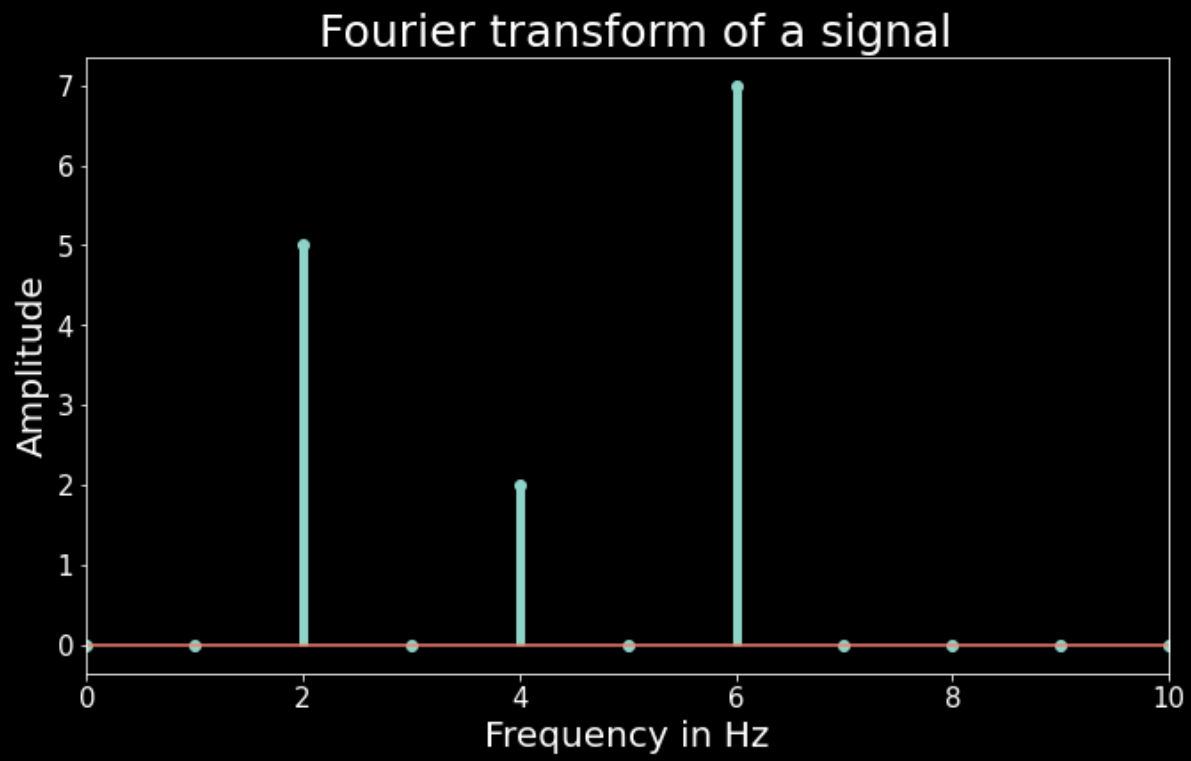
Fourier Transform of signal with amplitude ’3’ and frequency ’5’





Fourier Transform of signal with multiple frequencies

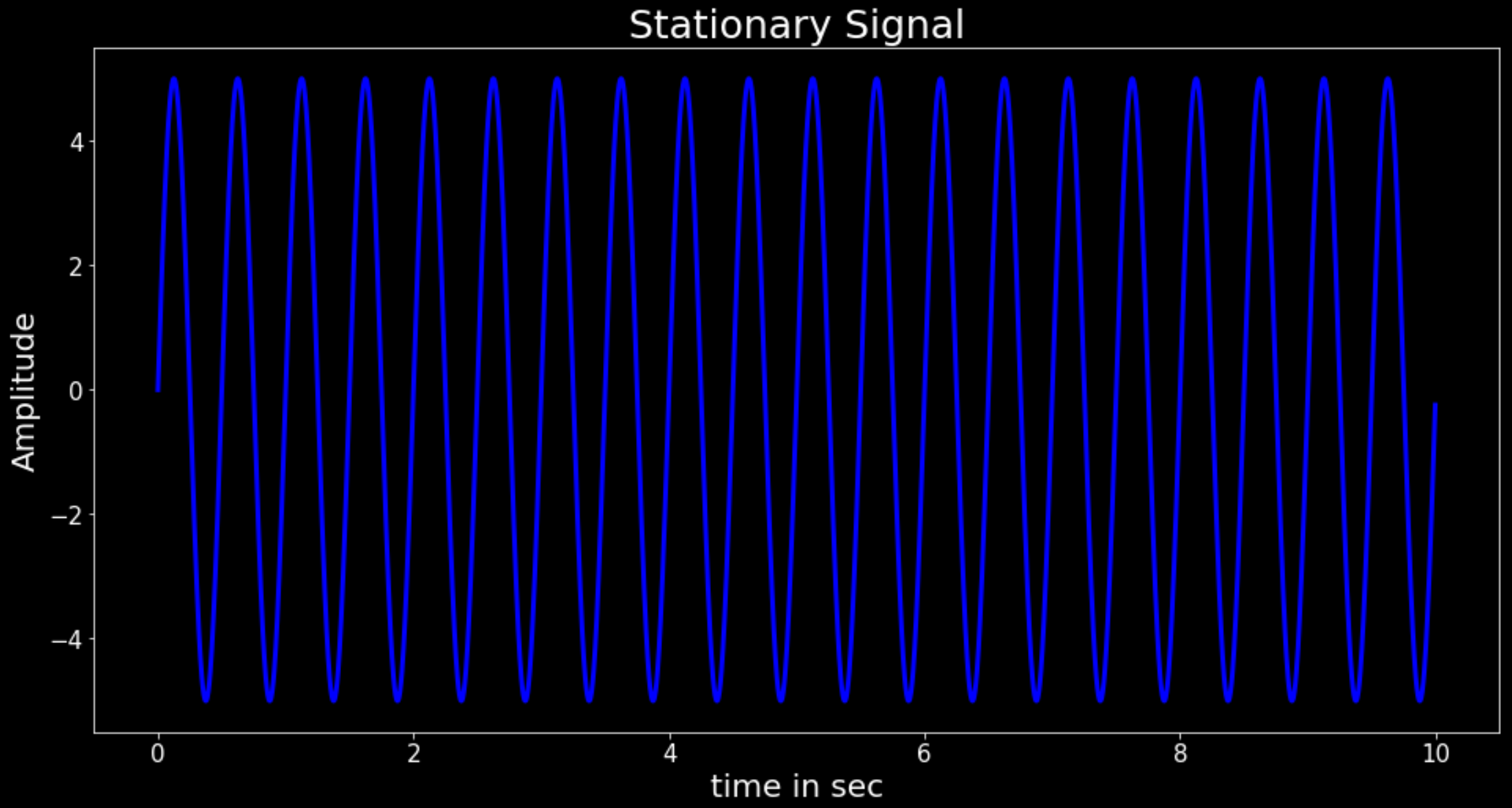


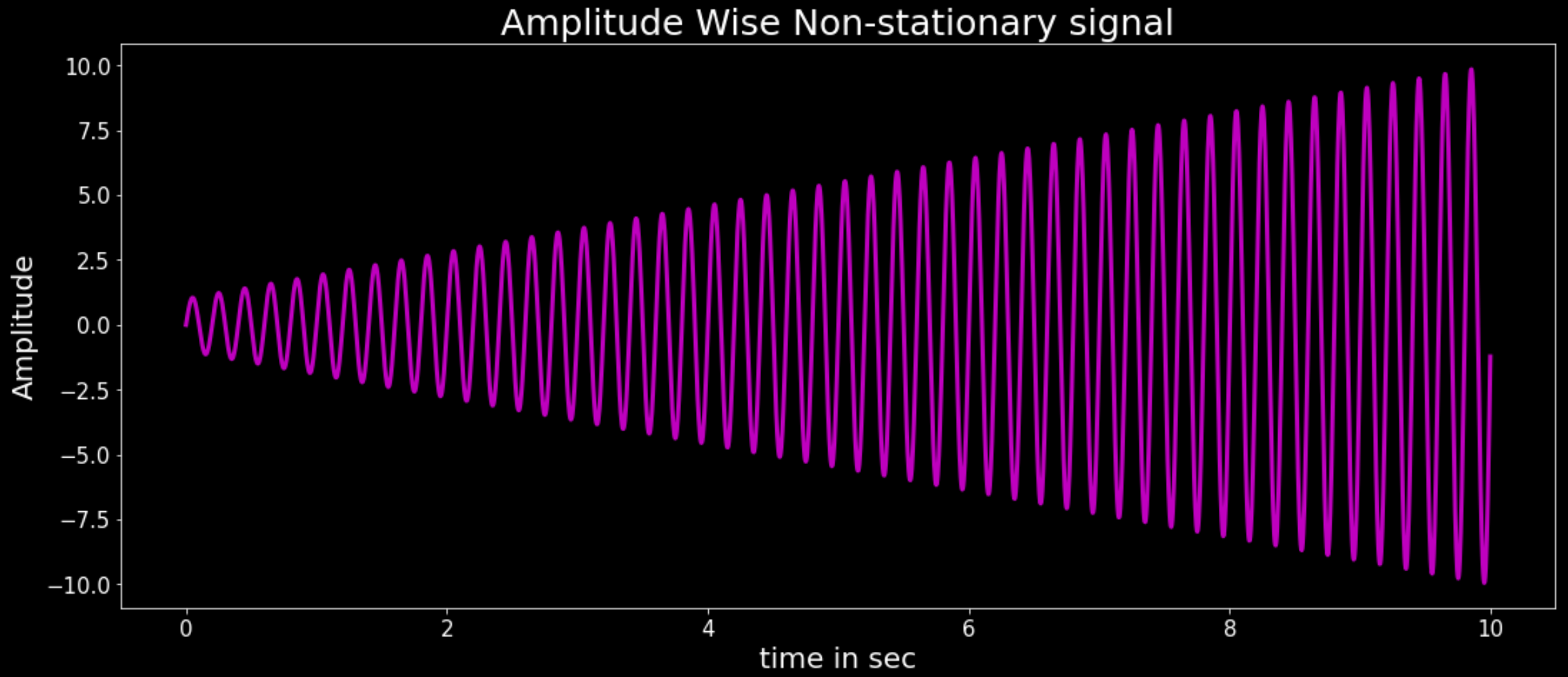


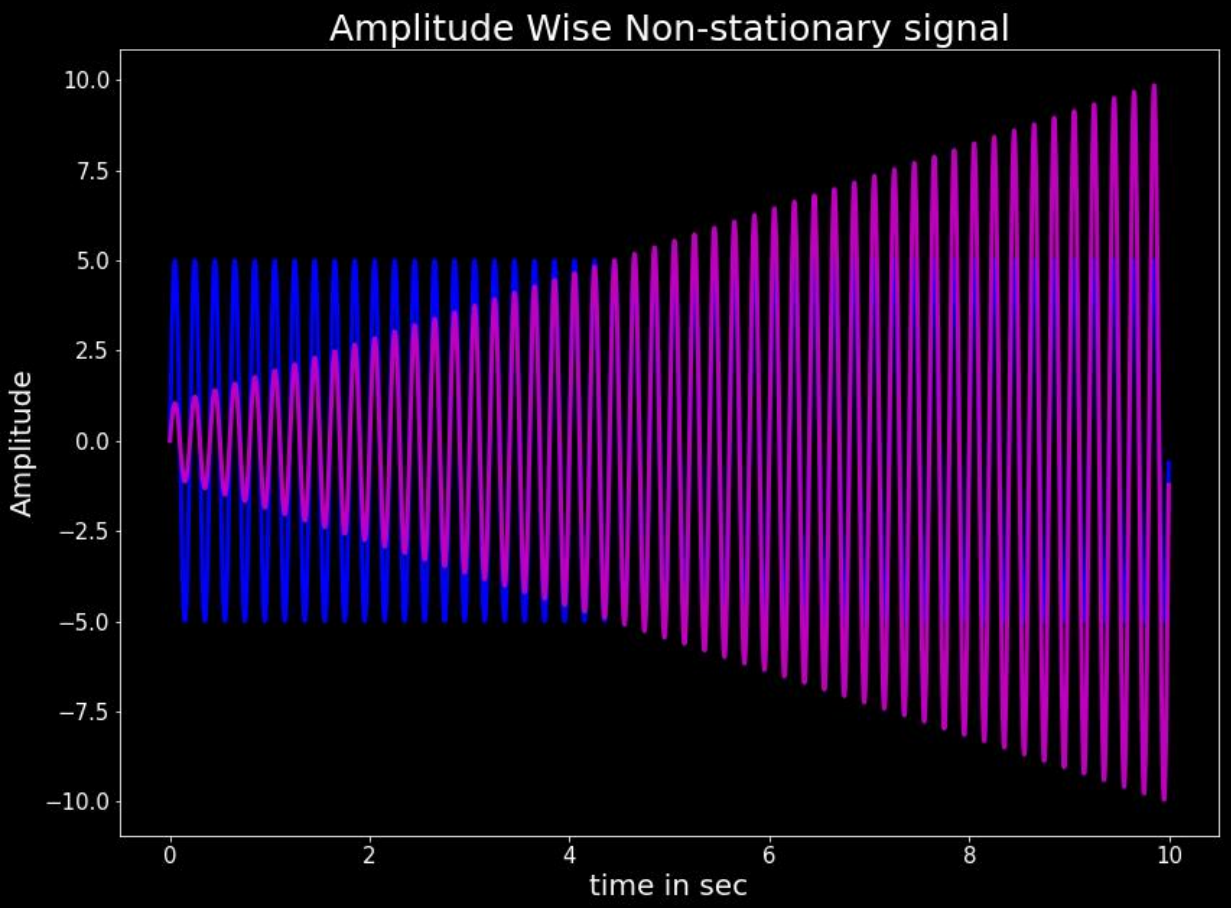
Applications of Fourier Transform

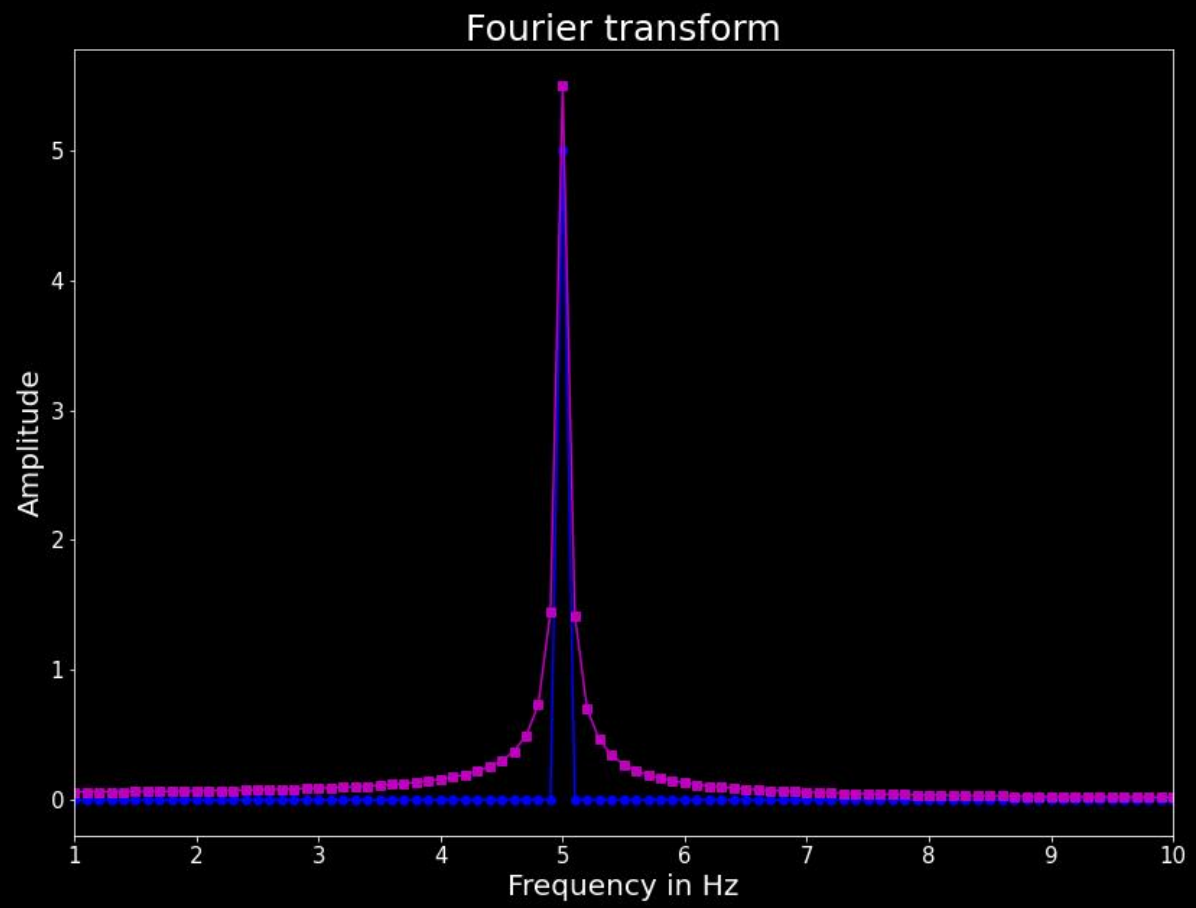
Stationarity of a signal

A signal is said to be stationary if its statistical properties do not change over time, otherwise, it is non-stationary.


Stationary Signal
Amplitude
4
2
0
−2
−4
0
2
4
6
8
10
time in sec


Amplitude Wise Non-stationary signal
Amplitude
10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
−10.0
0
2
4
6
8
10
time in sec


Amplitude Wise Non-stationary signal
10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
−10.0
Amplitude
0
2
4
6
8
10
time in sec


Fourier transform
5
4
3
2
1
0
Amplitude
1
2
3
4
5
6
7
8
9
10
Frequency in Hz