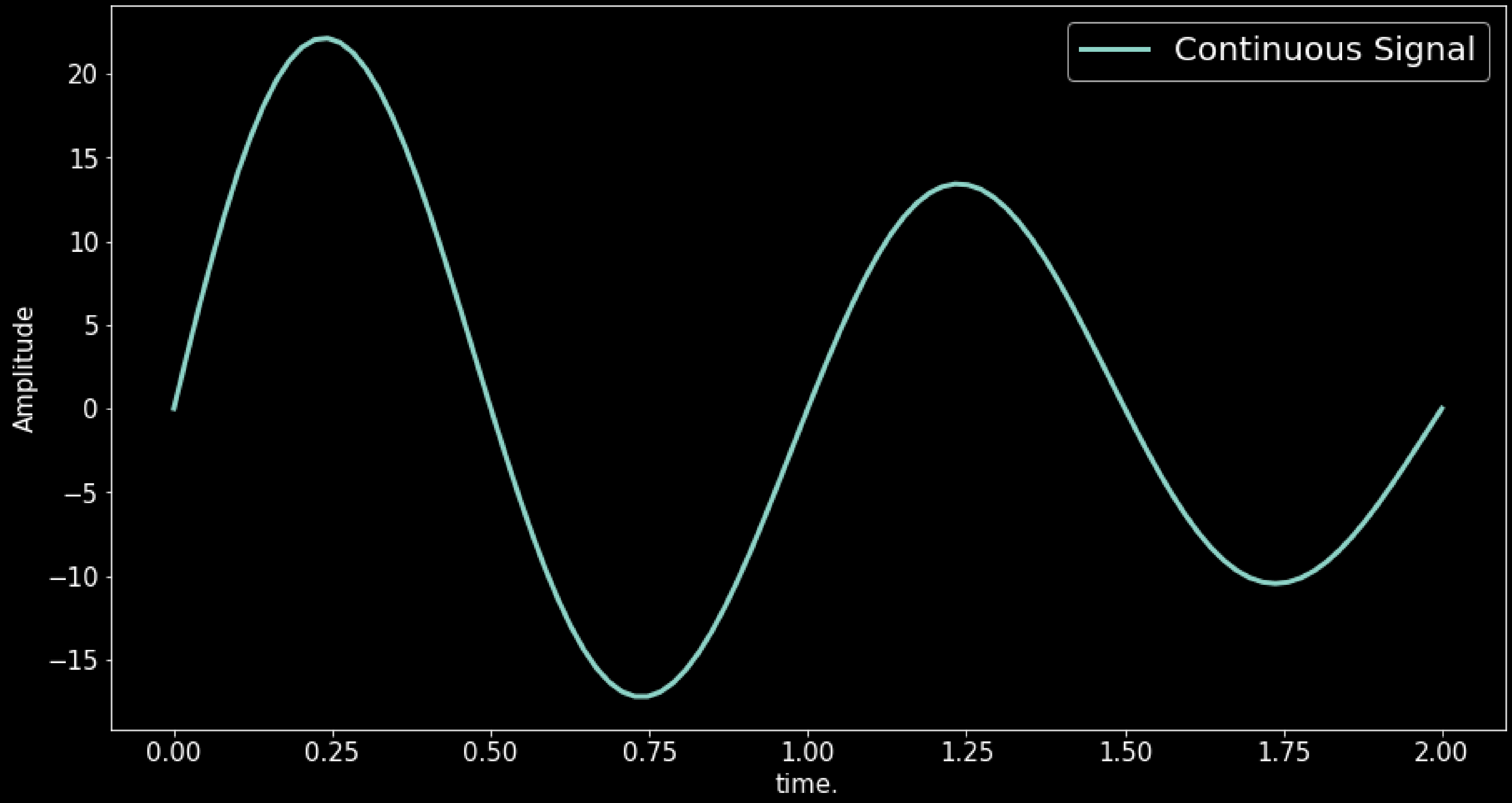
Topics to be covered

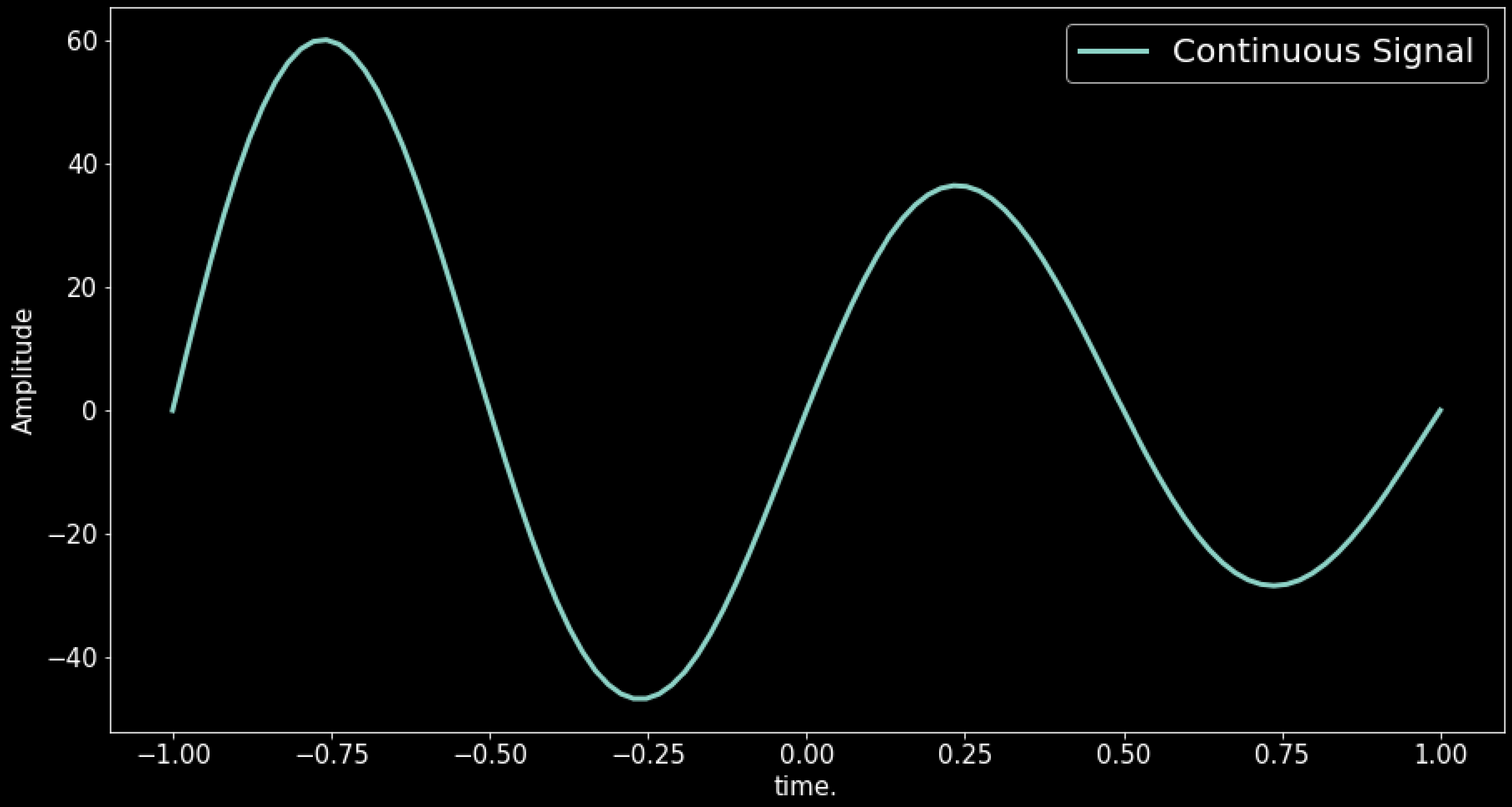
- Basic Elements of Signal Processing Unit
- Analog to Digital Conversion
- Fundamental Continuous time signals
- Fundamental Discrete time signals
- Sampling and Reconstruction of a Signal
- Nyquist Sampling Theorem

Signal Processing

Signal processing is the field of science which involves the manipulation of signal to get the desired shaping, transforming a signal from time domain to frequency and vice versa, smoothening the signal, separating the noise from signal i.e filtering, extracting information from the signal

Signals exist in nature are continuous signal. Continuous-time (or analog) signals exist for the continuous interval (t_1, t_2). This interval (t_1, t_2) can range from $-\infty$ to ∞.


Continuous Signal
Amplitude
time.
20
15
10
5
0
−5
−10
−15
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00


Continuous Signal
Amplitude
60
40
20
0
−20
−40
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00
time.

Basics of signal processing system

Since computer needs digital signals for processing, therefore, in order to use an analog signal on a computer it must be digitized with an analog-to-digital converter.Thus, there is a need for an interface between the analog signal and the digital signal processor.

Fig : Basic Elements of Signal Processing System

Analog to Digital Conversion

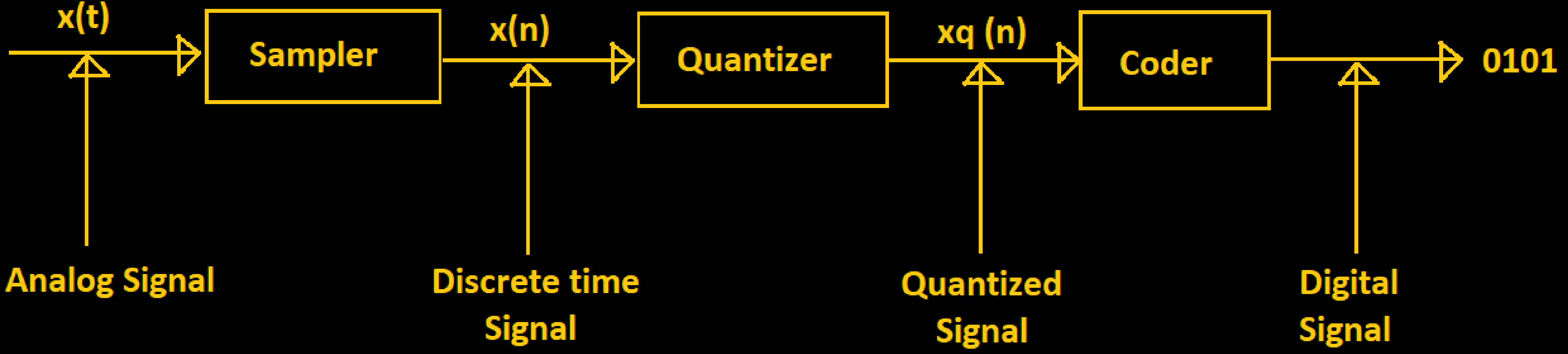
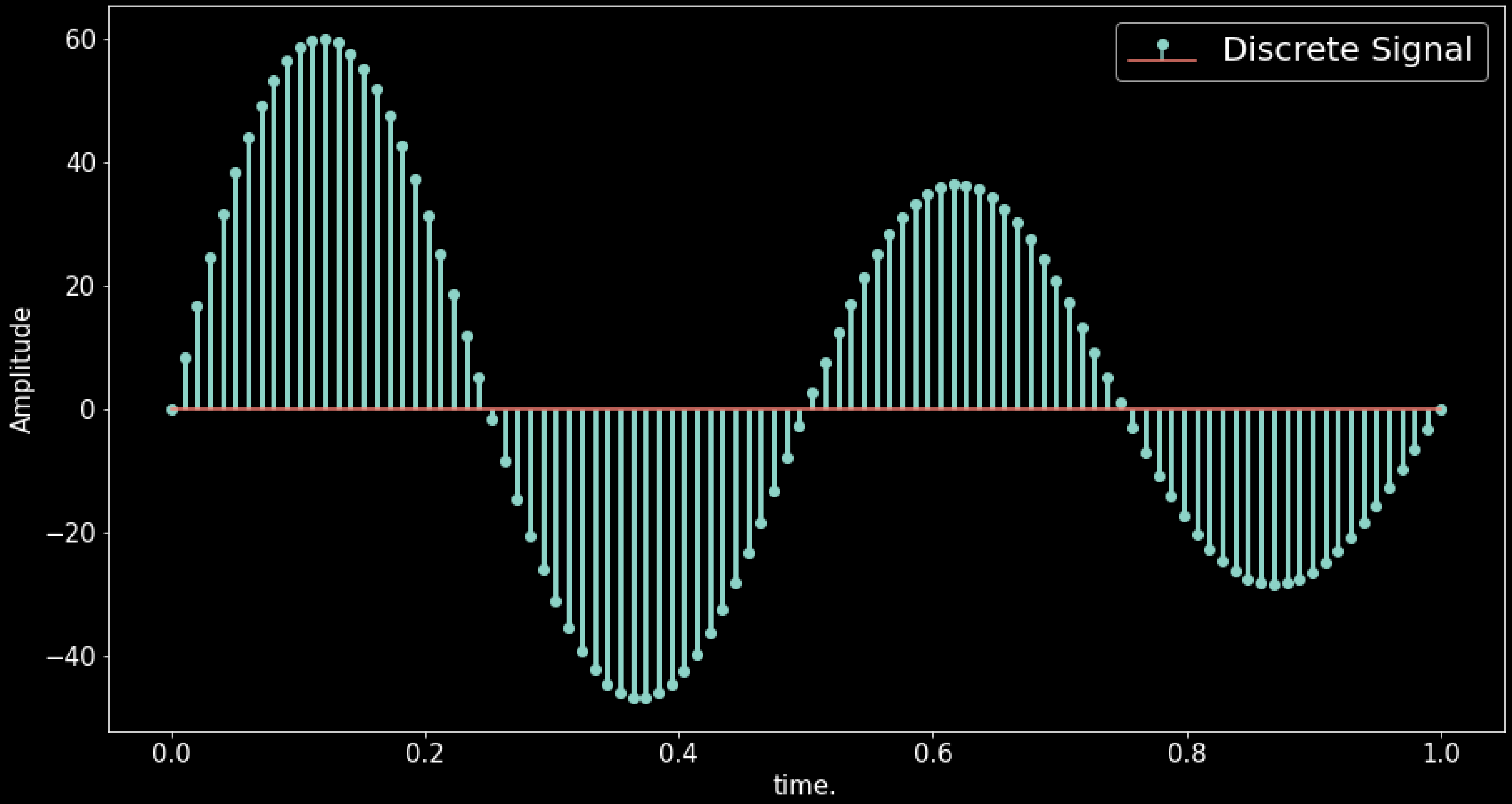
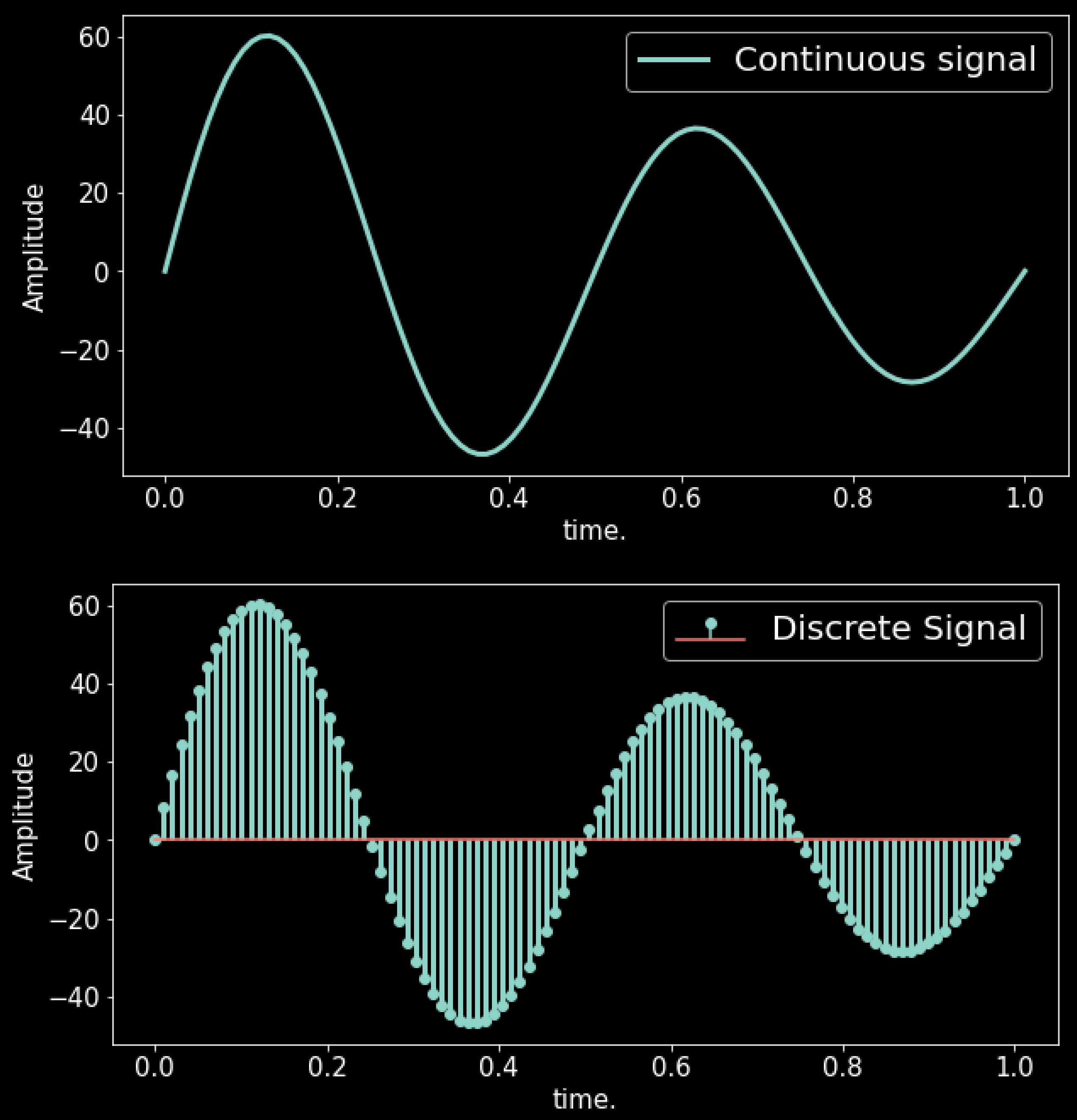


Fig : Elements of Analog to Digital Convertor

Discrete time Signal




Continuous signal
Amplitude
time.
Discrete Signal
Amplitude
time.

Analog to Digital Conversion

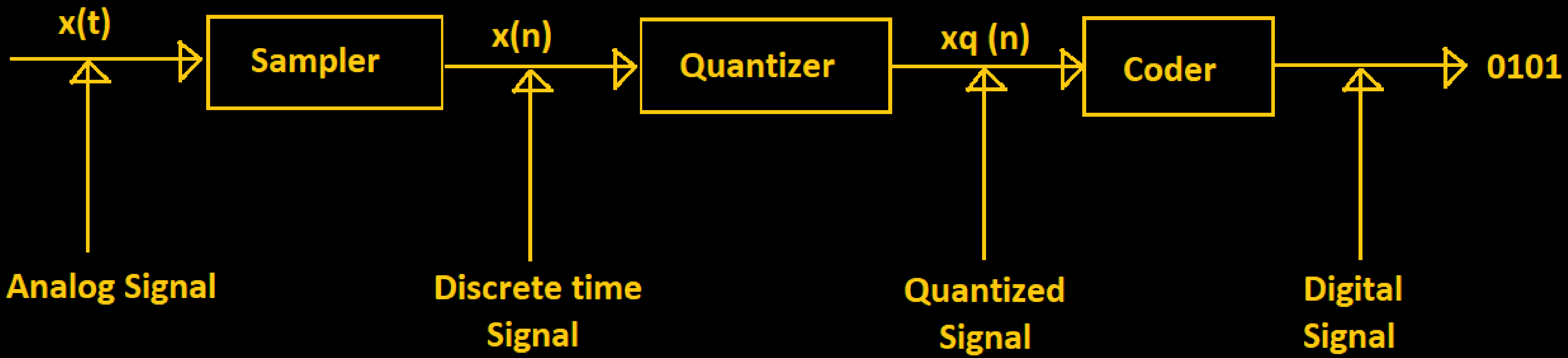
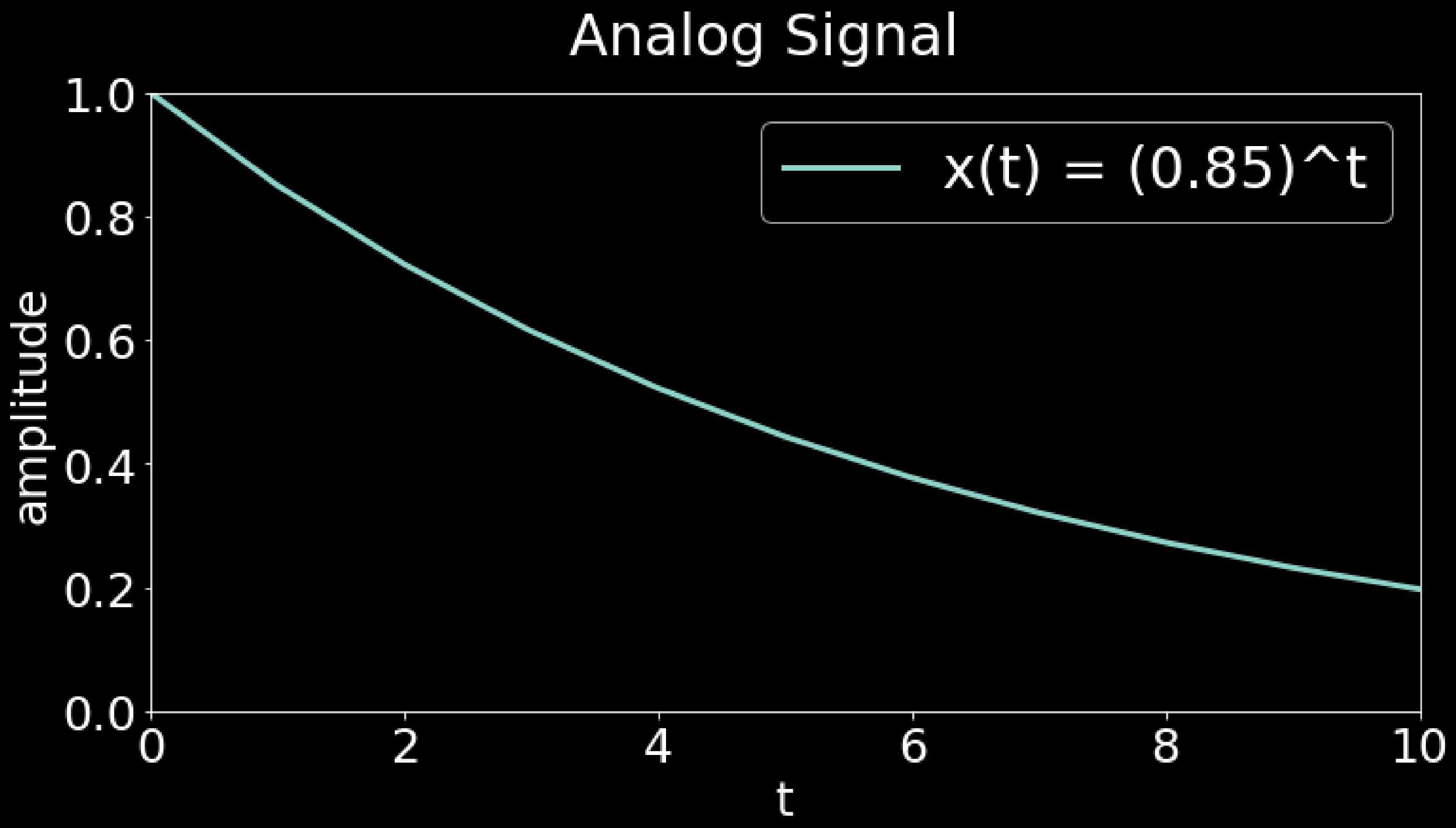


Fig : Elements of Analog to Digital Convertor

Example

Consider the following Analog signal



Sampling

Our analog signal is given by

$$x(t) = (0.85)^t$$

For sampling, we have to define a sampling interval, T. We define sampling interval by setting sampling frequency "f_s". For simplicity suppose

$$f_s = 1Hz.$$

$$T = \frac{1}{f_s}$$

$$T = 1s$$

For sampling replace $t = nT$.

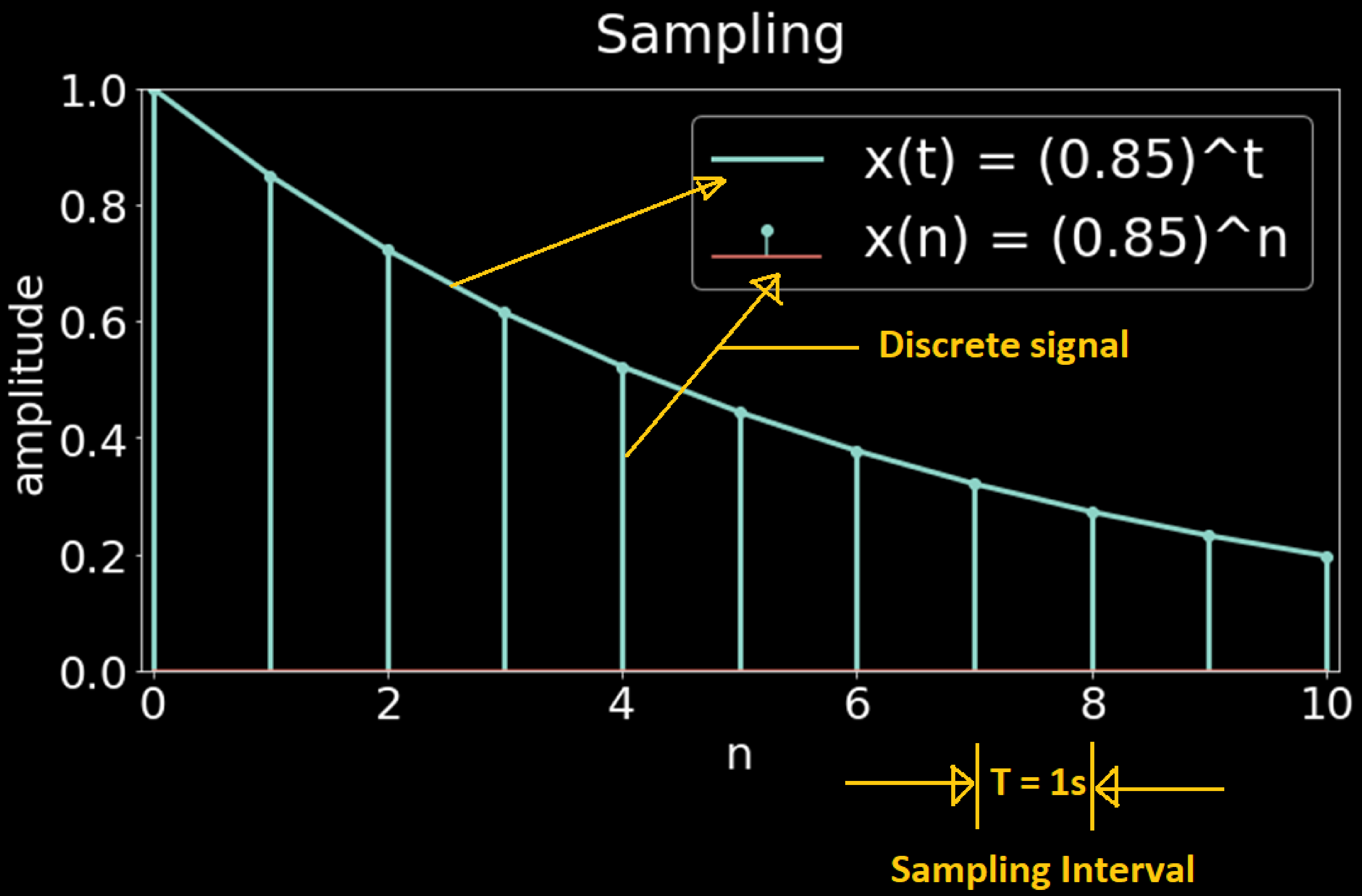
Thus,

$$x(nT) = (0.85)^{nT}$$

Since $T = 1s$, therefore,

$$x(n) = (0.85)^{n}$$

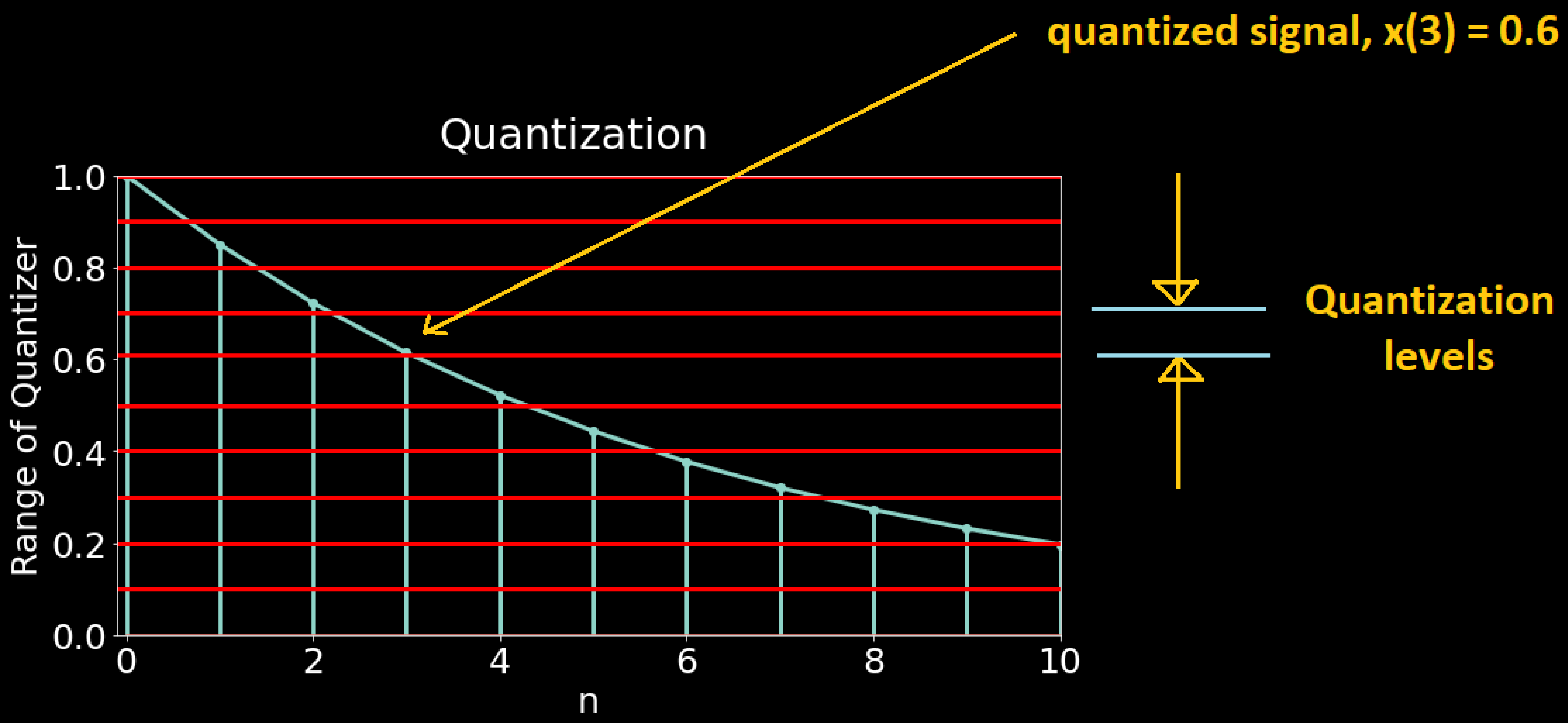
$x(n)$ is the discrete time signal with sampling interval of $1s$.


Sampling
1.0
0.8
0.6
0.4
0.2
0.0
amplitude
0
2
4
6
8
10
n
x(t) = (0.85)^t
x(n) = (0.85)^n
Discrete signal
T = 1s
Sampling Interval

Quantization

Quantization is the process of converting the amplitude of discrete signal into a digital signal by expressing each sample value as a finite (instead of infinite) number of digits.

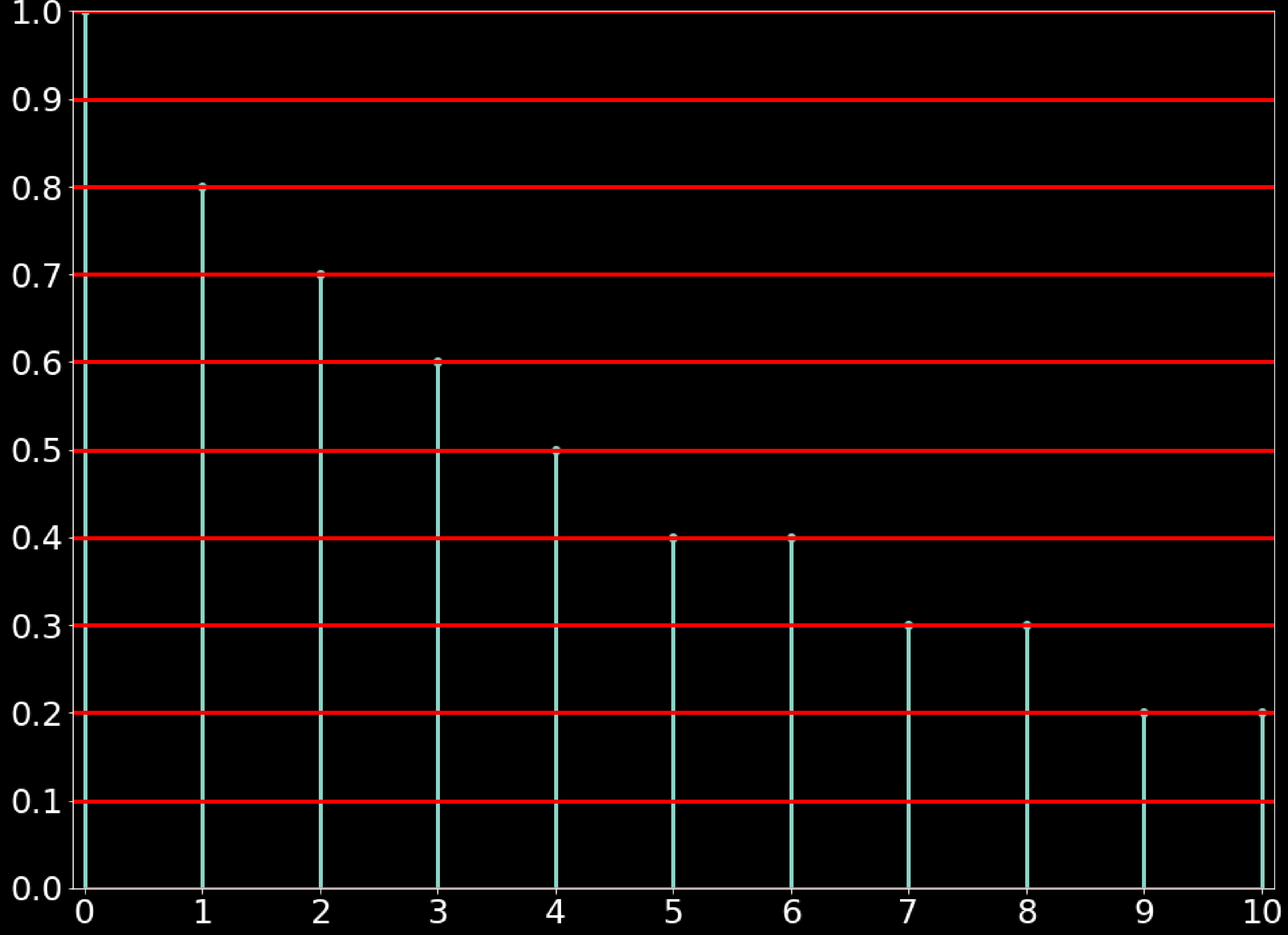
Accuracy of the signal representation is directly proportional to how many discrete levels are allowed to represent the magnitude of the signal.


quantized signal, x(3) = 0.6
Quantization
1.0
0.8
0.6
0.4
0.2
0.0
Range of Quantizer
0
2
4
6
8
10
n
Quantization
levels

Discrete time (n)	Discrete signal $(0.85)^n$	Quantized signal
0	1	1
1	0.85	0.9
2	0.7224999999999999	0.7
3	0.6141249999999999	0.6
4	0.5220062499999999	0.5
5	0.44370531249999995	0.4
6	0.37714951562499993	0.4
7	0.3205770882812499	0.3
8	0.27249052503906246	0.3
9	0.23161694628320306	0.2
10	0.1968744043407226	0.2

Quantized signal after rounding discrete time signal

Quantized Signal



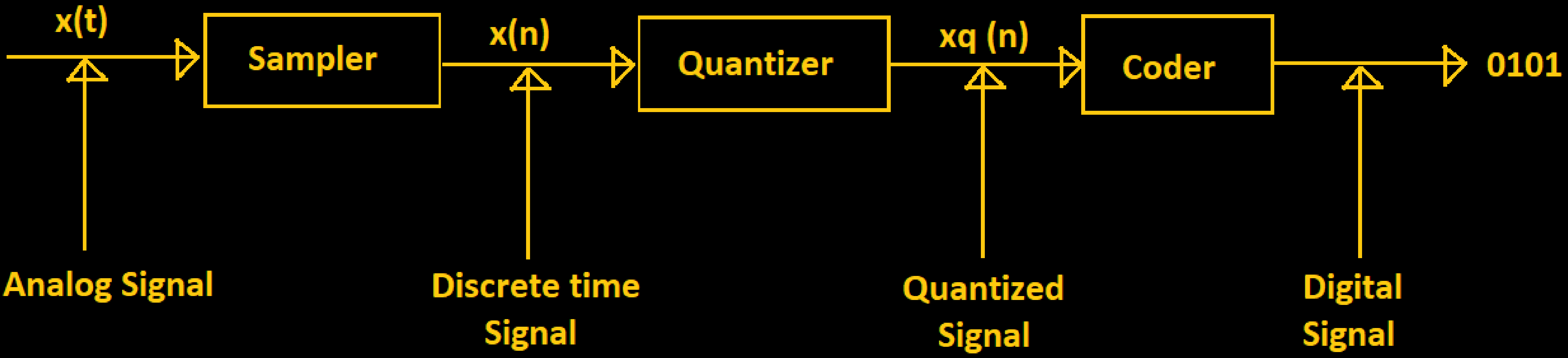


Fig : Elements of Analog to Digital Convertor

Coding

b_3	b_2	b_1	b_0	**Quantized Levels**
0	0	0	0	0.0
0	0	0	1	0.1
0	0	1	0	0.2
0	0	1	1	0.3
0	1	0	0	0.4
0	1	0	1	0.5
0	1	1	0	0.6
0	1	1	1	0.7
1	0	0	0	0.8
1	0	0	1	0.9
1	0	1	0	1.0

Some Fundamental Continuous time signal

Continuous time signal has infinite range of values over the time. It is denoted by $x(t)$. Now we will discuss some fundamental continuous time signals for positive value of the time $(t > 0)$.

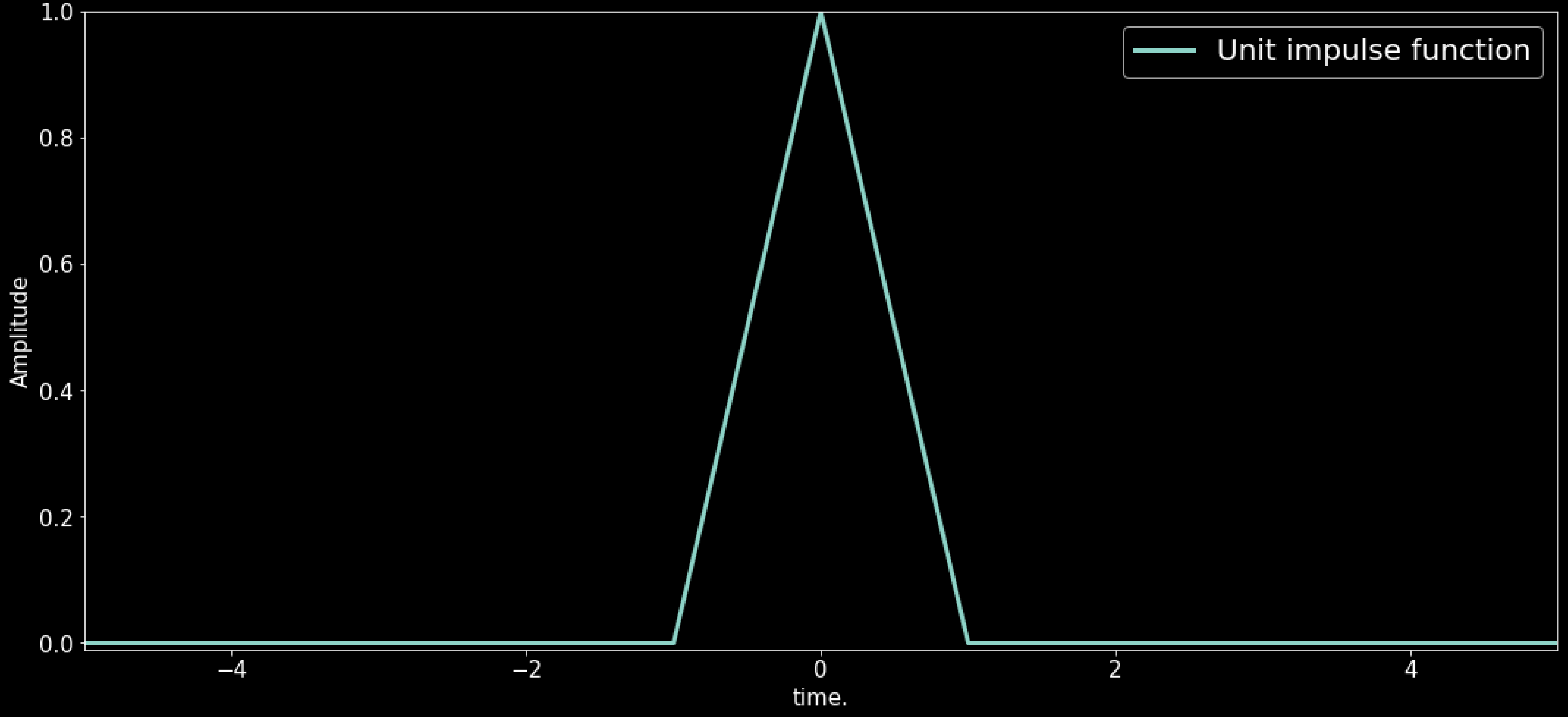
Unit Impulse signal

Unit impulse signal, denoted by $\delta(t)$, is given by

$$\delta(t) = 1 \quad for \quad t = 0$$

and

$$\delta(t) = 0 \quad for \quad t \neq 0$$



Unit impulse function
Amplitude
time.
1.0
0.8
0.6
0.4
0.2
0.0
−4
−2
0
2
4

Time shifted impulse

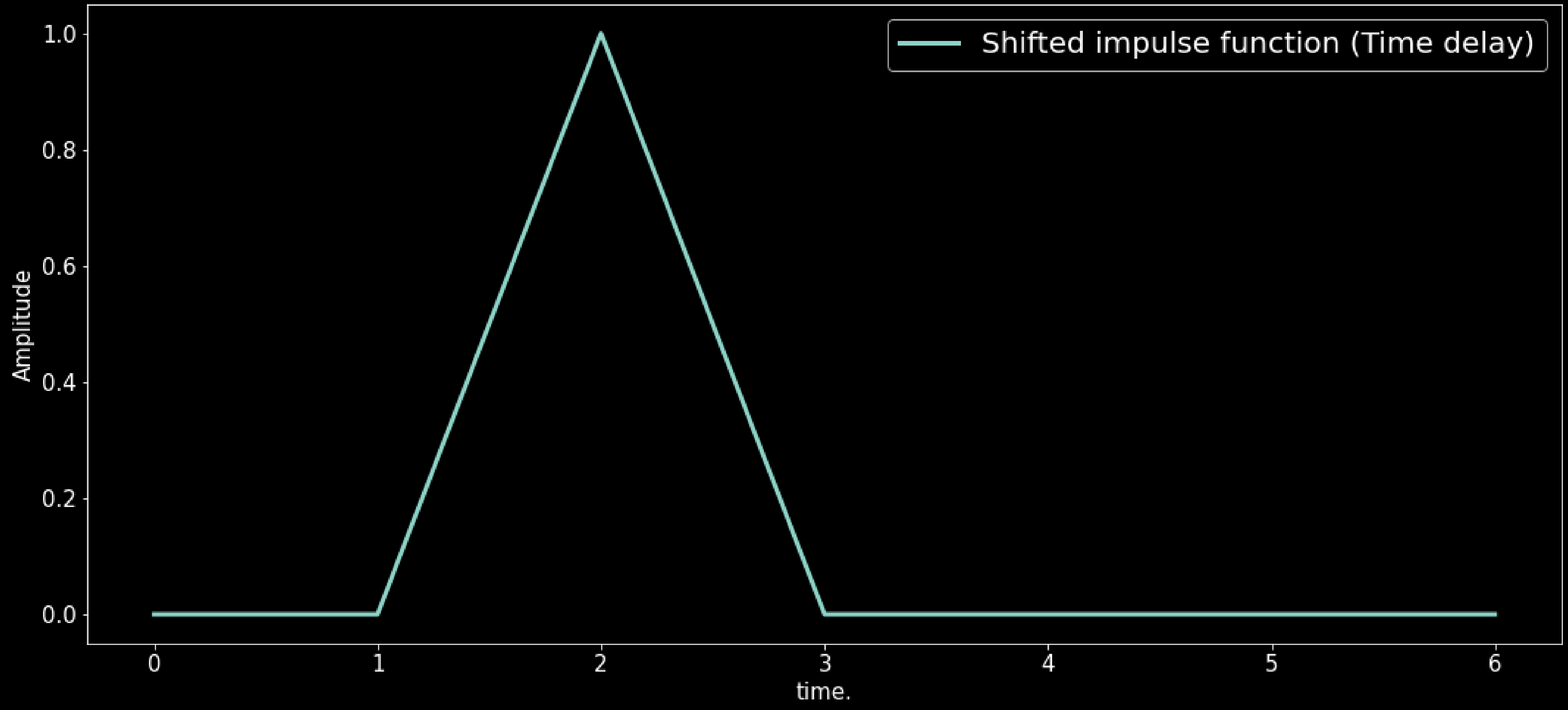
Impulse signal is one at $t = 0$, however, we can shift the signal on both sides of zero. There are two kinds of time shifting.

1. Time delay.
2. Time advance.

Time delay

In time delay, we move the signal towards right side of the zero i.e towards positive time axis. If we delay the signal towards right by two units then the equation of impulse becomes

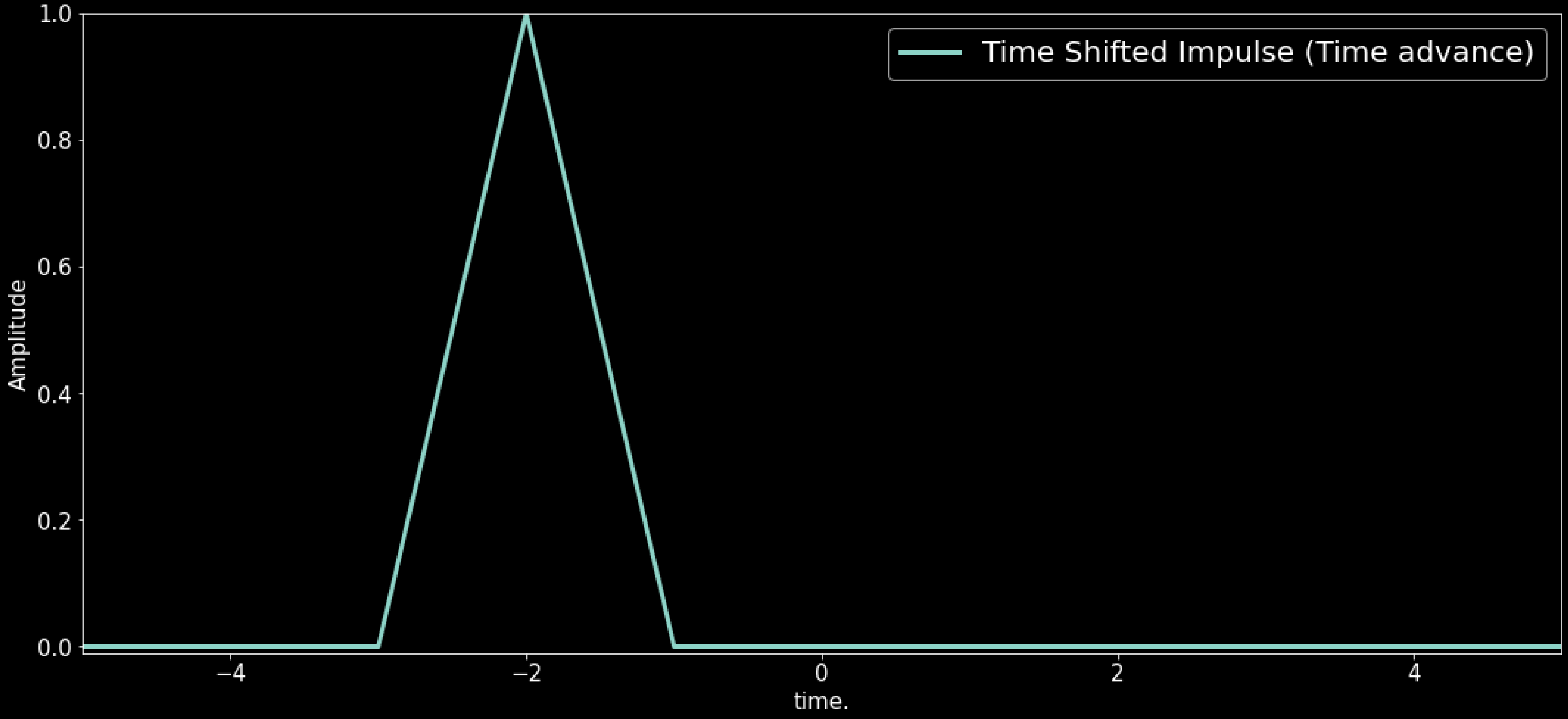
$$\delta(t-2) = 1$$



Time advance

In time advance, we move the signal towards left side of the zero i.e towards negative time axis. If we advance the signal towards left by two units then the equation of impulse becomes

$\delta(t+2) = 1$



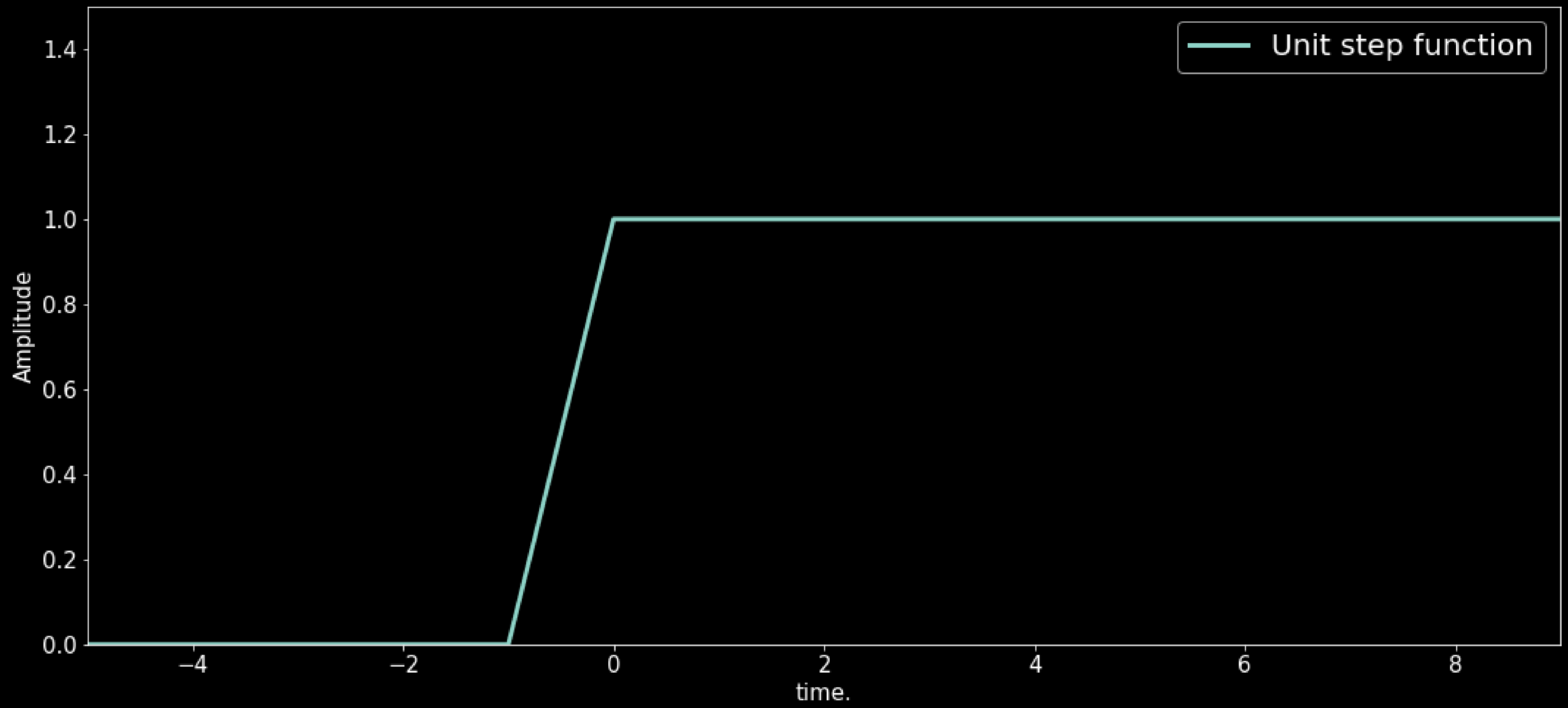
Unit Step signal

The unit step signal, denoted by $u(t)$, is described as a function having magnitude of 1 at time equal to and greater than zero.

$u(t) = 1$ for $t \geq 0$

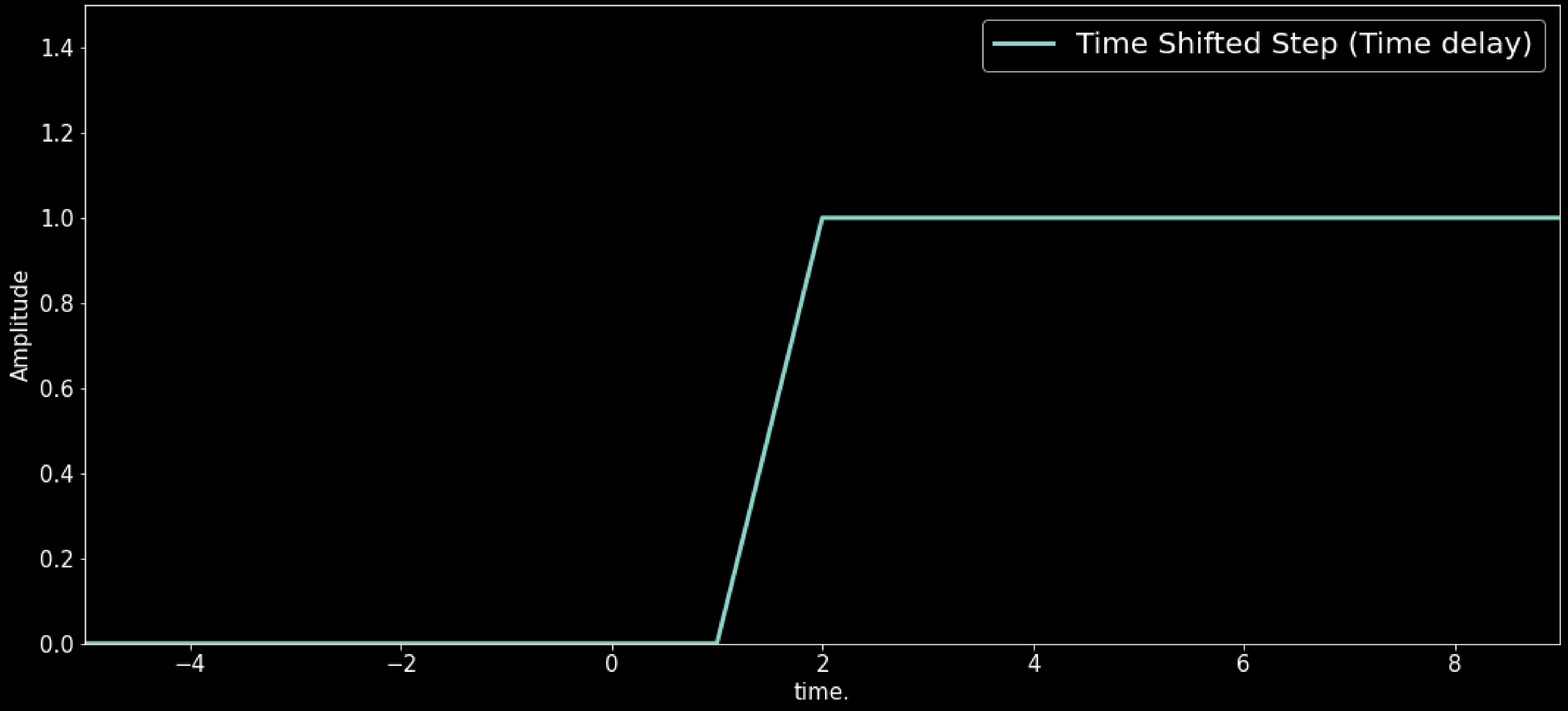
and

$u(t) = 0$ for $t < 0$

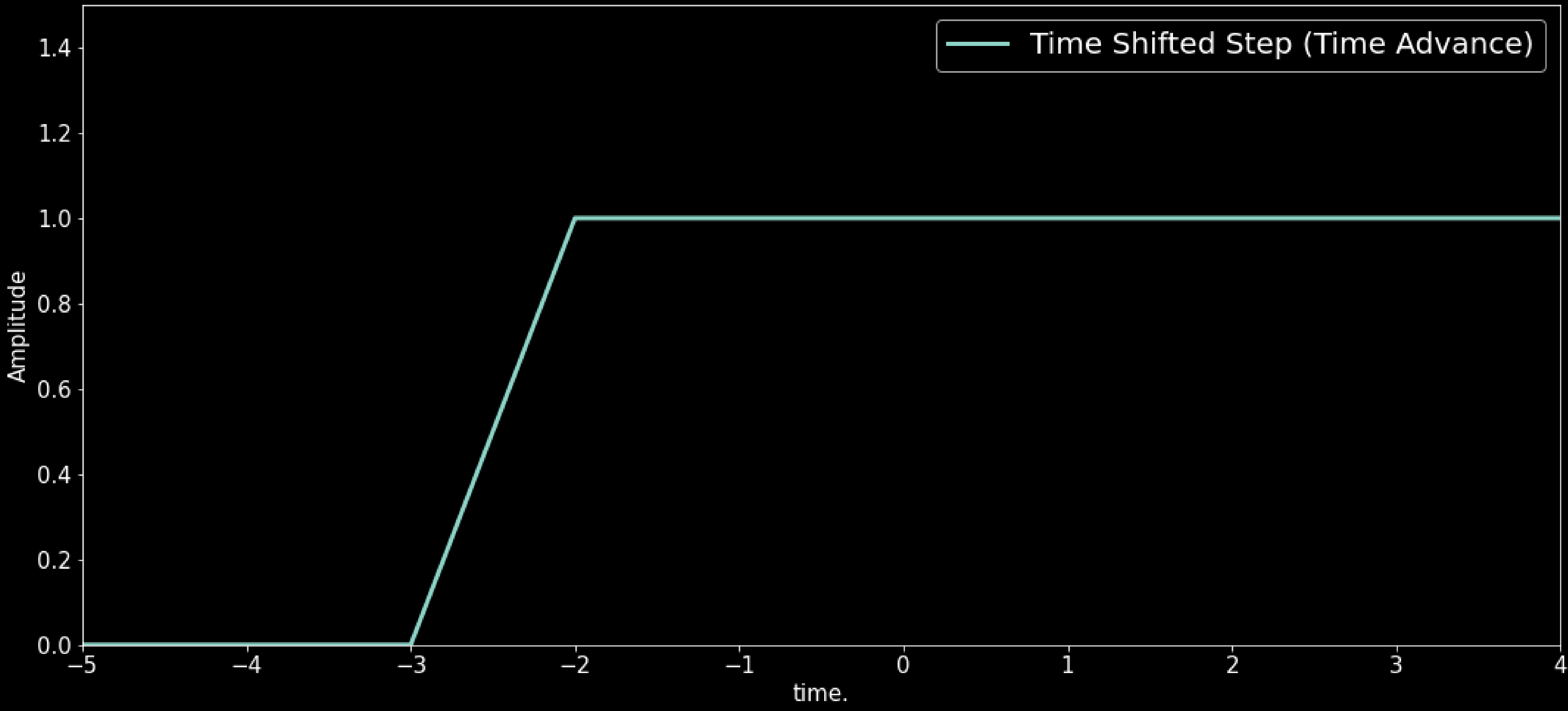


Unit step function
Amplitude
time.
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
−4
−2
0
2
4
6
8

Time shifted step



Time shifted step



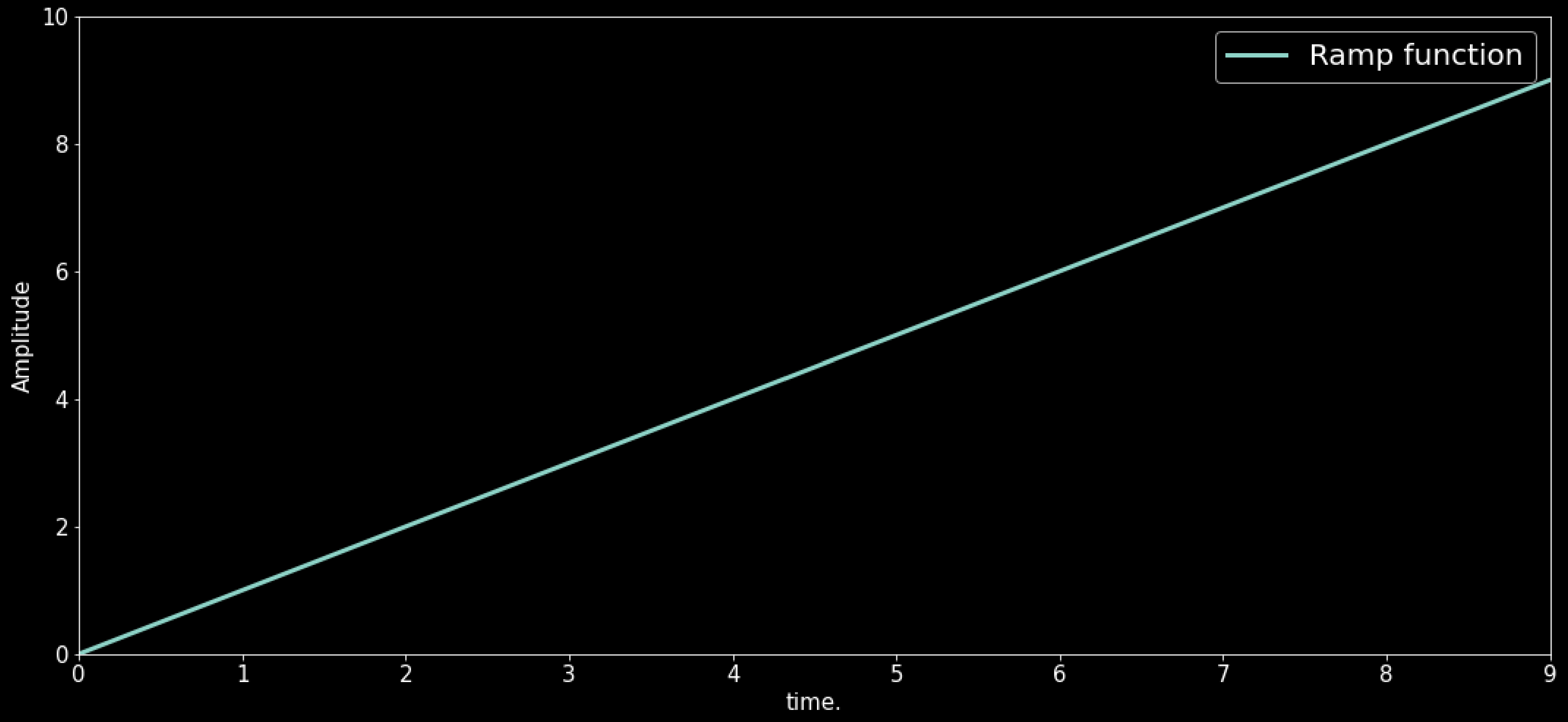
Unit Ramp signal

The ramp function is a uniformly increasing time domain signal of a constant slope. The ramp function is described as a function having a magnitude of t at $t \geq 0$.

$x(t) = t$ for $t \geq 0$.

and

$x(t) = 0$ for $t < 0$.



Ramp function
10
8
6
4
2
0
Amplitude
0
1
2
3
4
5
6
7
8
9
time.

Sinusoidal signal

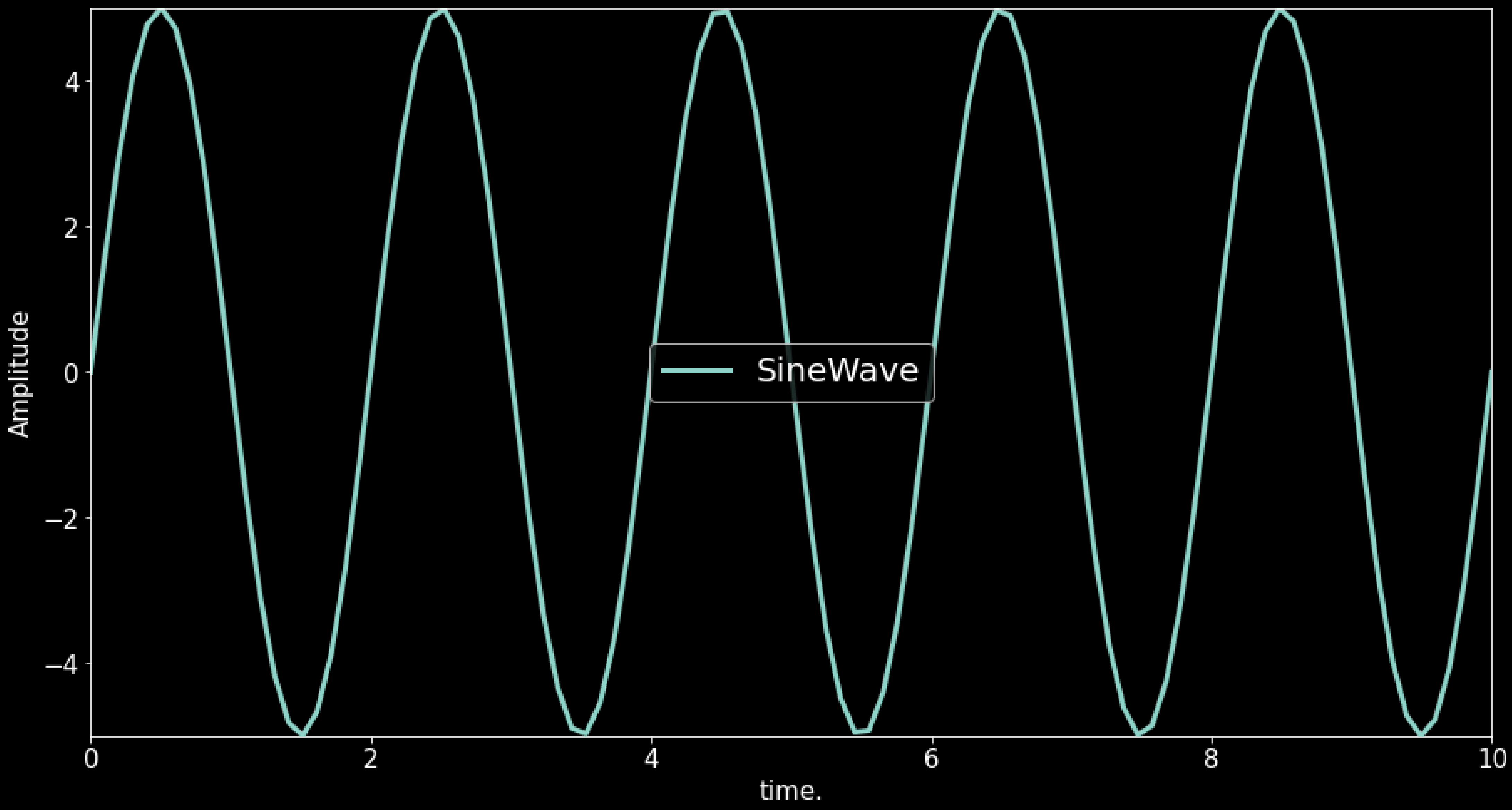
A sinusoidal signal consists of an oscillations that repeat over a fixed interval of time called time period of the signal. A sinusoidal signal for $t > 0$ is given by

$$x(t) = Asin(2\pi ft)$$

where,

$A =$ Amplitude of the signal.

$f =$ Frequency of the signal.



SineWave
Amplitude
time.
4
2
0
−2
−4
0
2
4
6
8
10

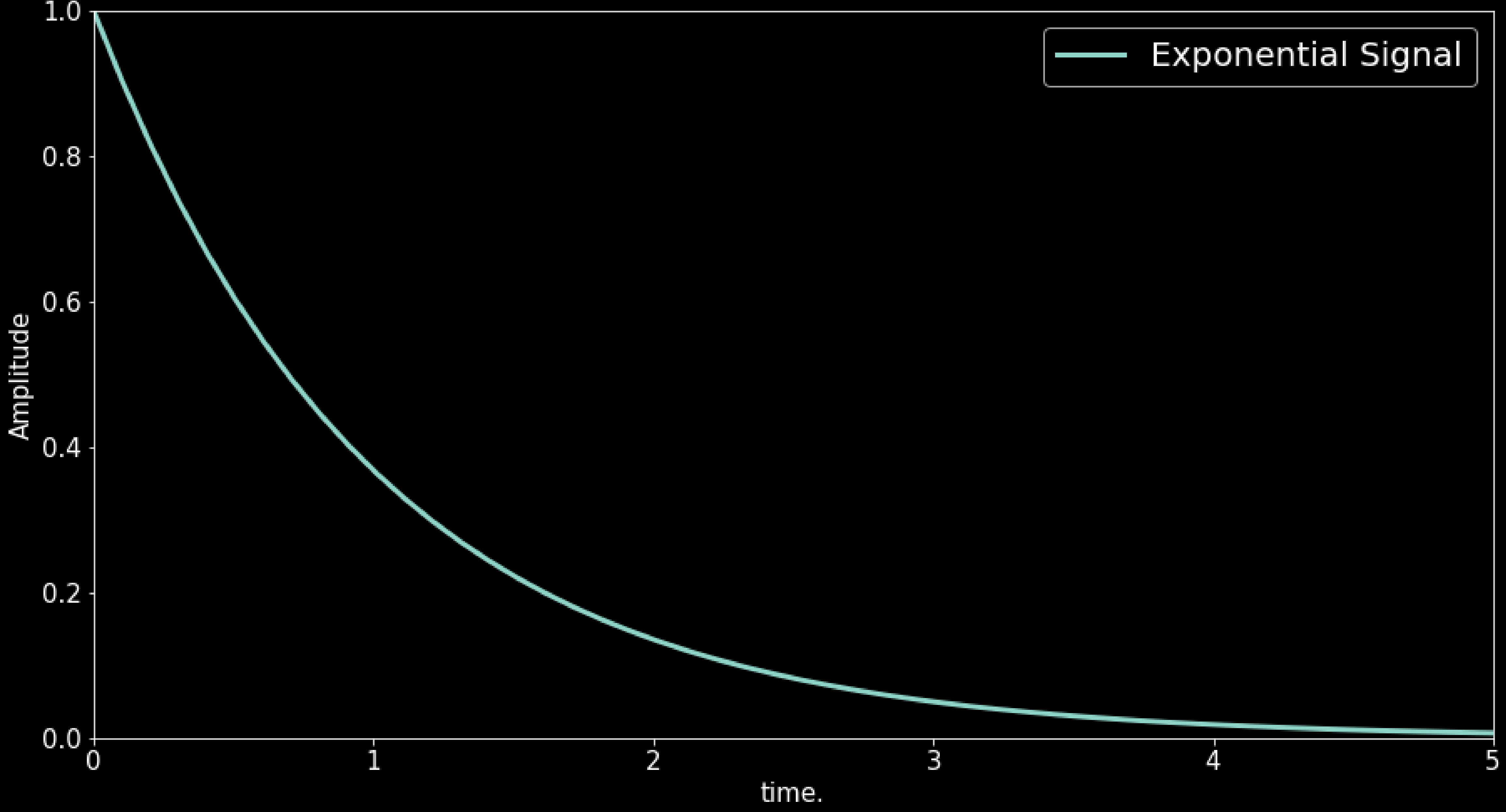
Unit Exponential signal

The unit exponential function is described as has a magnitude of 1 at time zero and exponentially decaying for time greater than zero. An exponential signal for $t > 0$ is given by

$x(t) = 1$ for $t = 0$.

and

$x(t) = e^{-t}$ for $t > 0$.


Exponential Signal
Amplitude
time.
1.0
0.8
0.6
0.4
0.2
0.0
0
1
2
3
4
5

Fundamental Discrete time signal

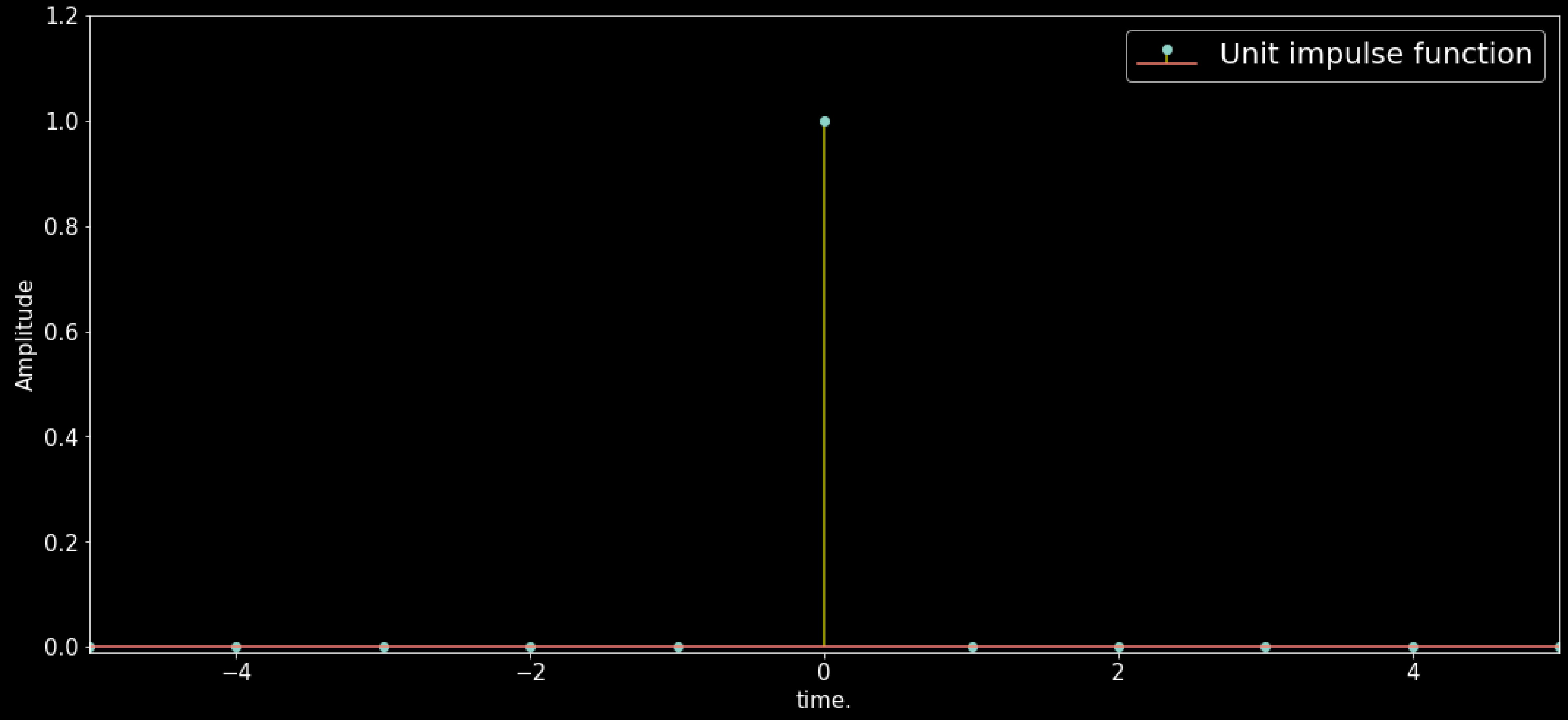
Unit Impulse signal

Unit impulse signal, denoted by $\delta(t)$, is given by

$$\delta(t) = 1 \quad for \quad t = 0$$

and

$$\delta(t) = 0 \quad for \quad t \neq 0$$


Unit impulse function
Amplitude
time.
1.2
1.0
0.8
0.6
0.4
0.2
0.0
−4
−2
0
2
4

Time shifted impulse

Impulse signal is one at $t = 0$, however, we can shift the signal on both sides of zero. There are two kinds of time shifting.

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Time delay

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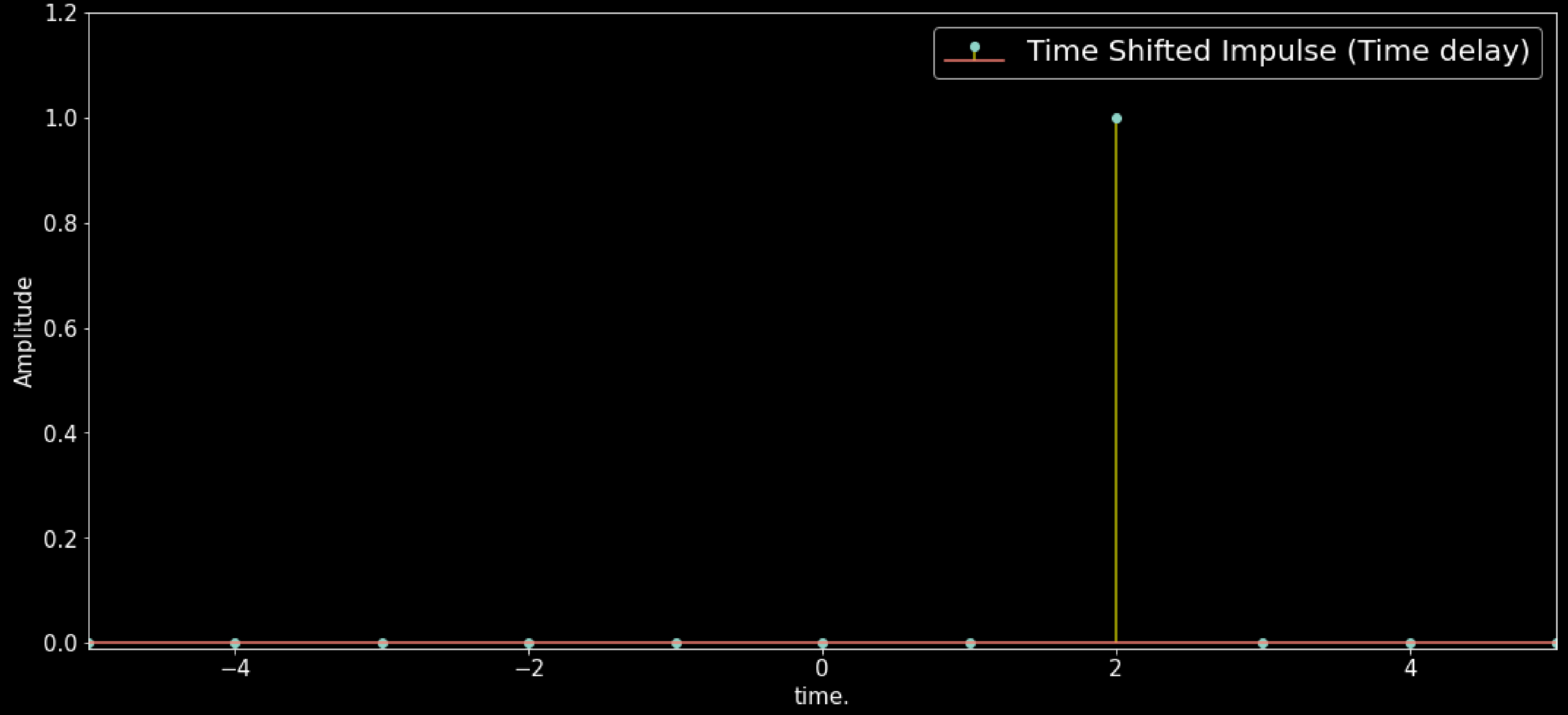
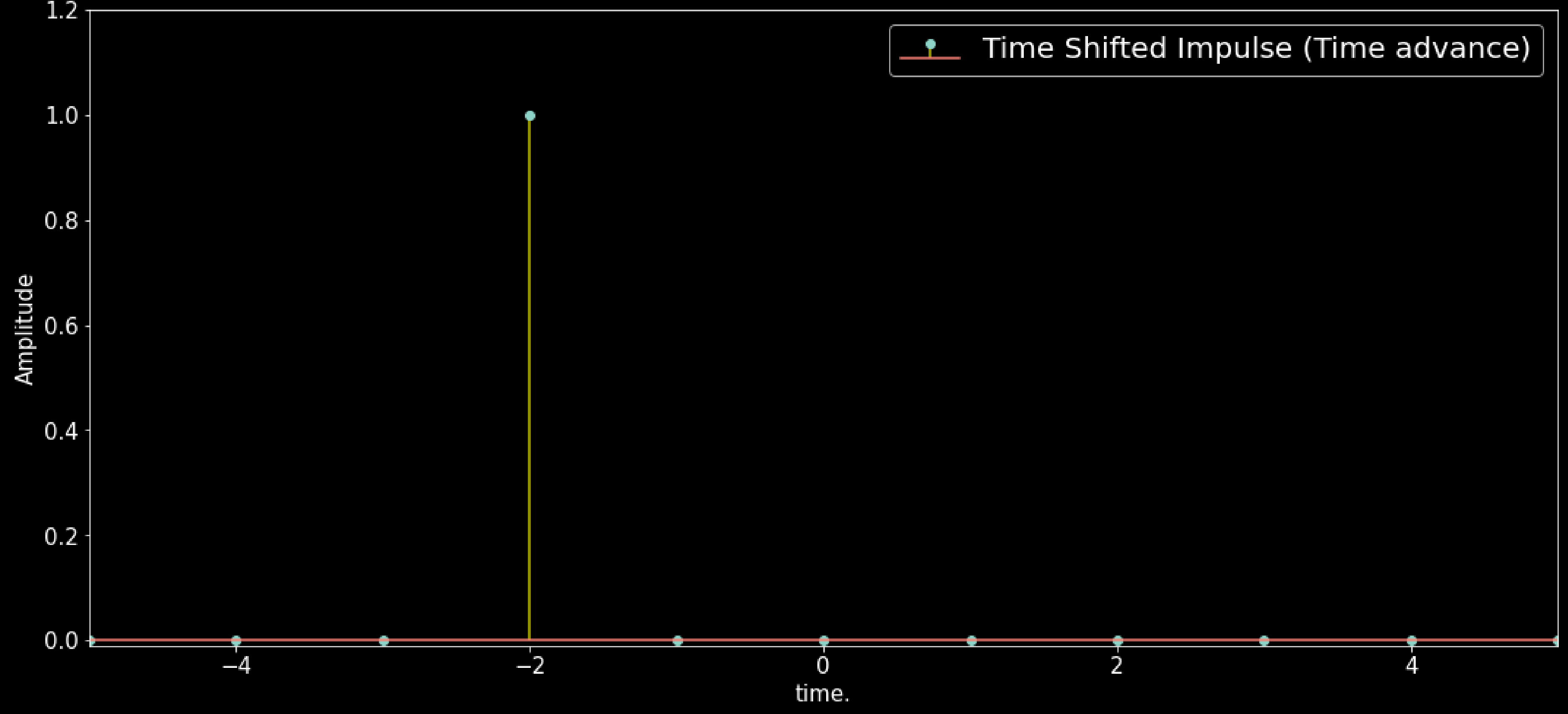
$$\delta(t-2) = 1$$



Time advance

In time advance, we move the signal towards left side of the zero i.e towards negative time axis. If we advance the signal towards left by two units then the equation of impulse becomes

$$\delta(t+2) = 1$$



Unit Step signal

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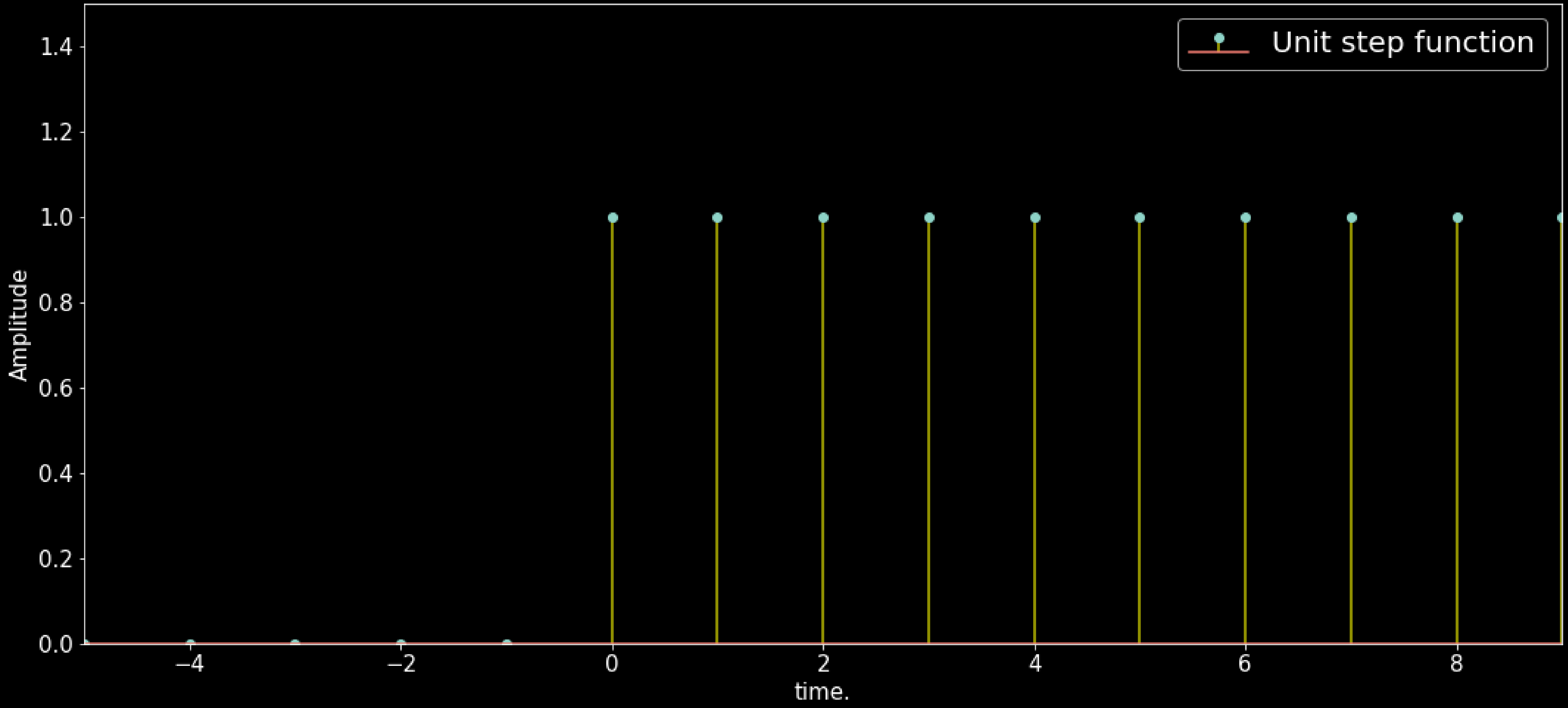
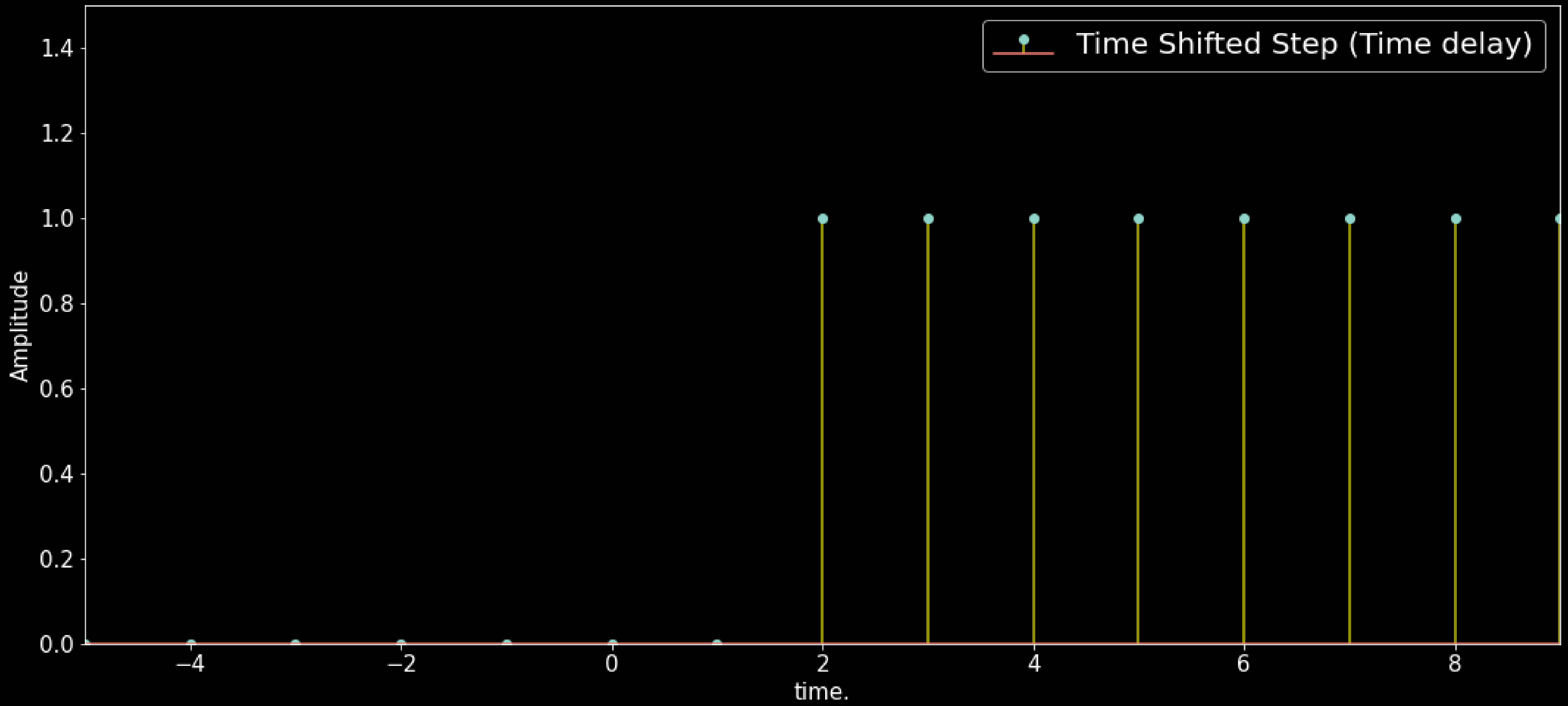
and

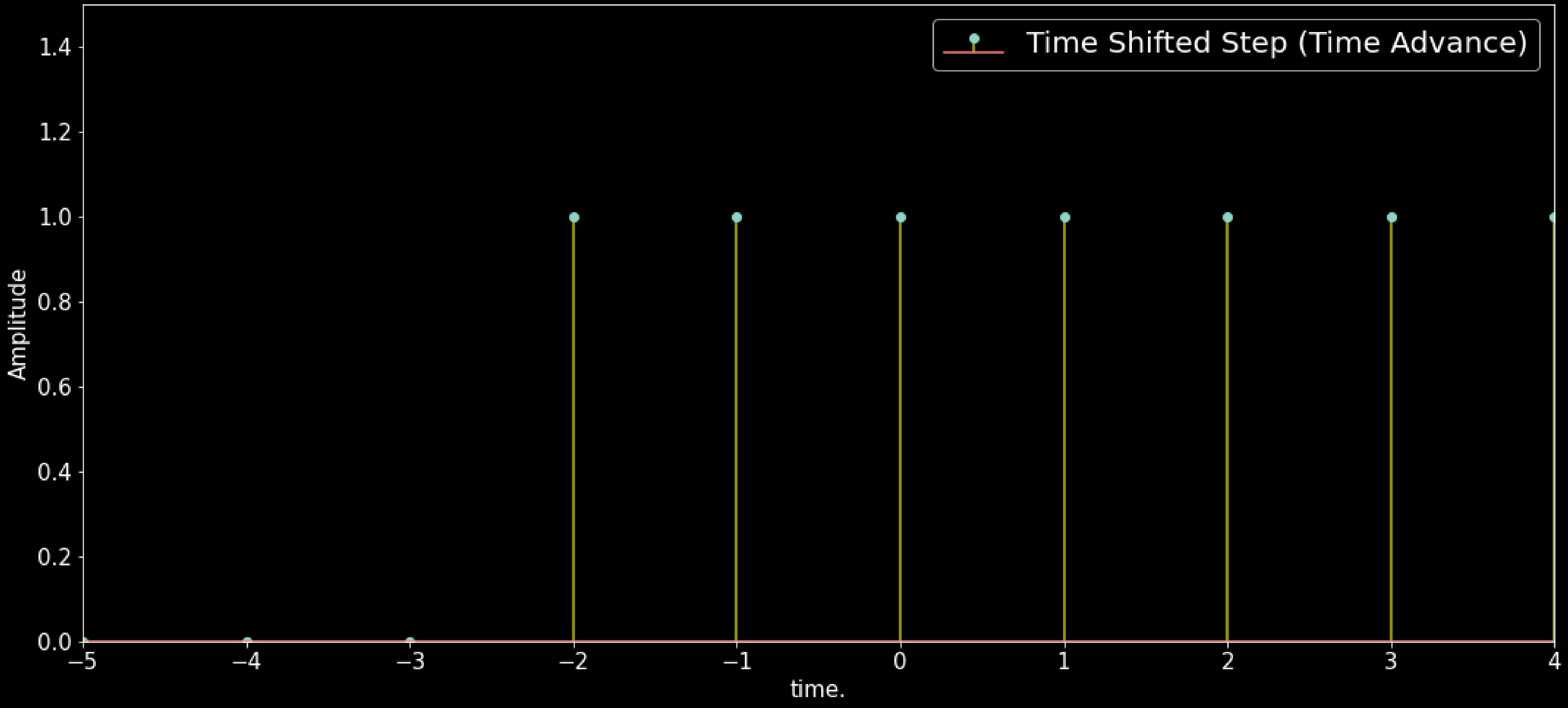
$u(t) = 0$ for $t < 0$


Unit step function
Amplitude
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
−4
−2
0
2
4
6
8
time.

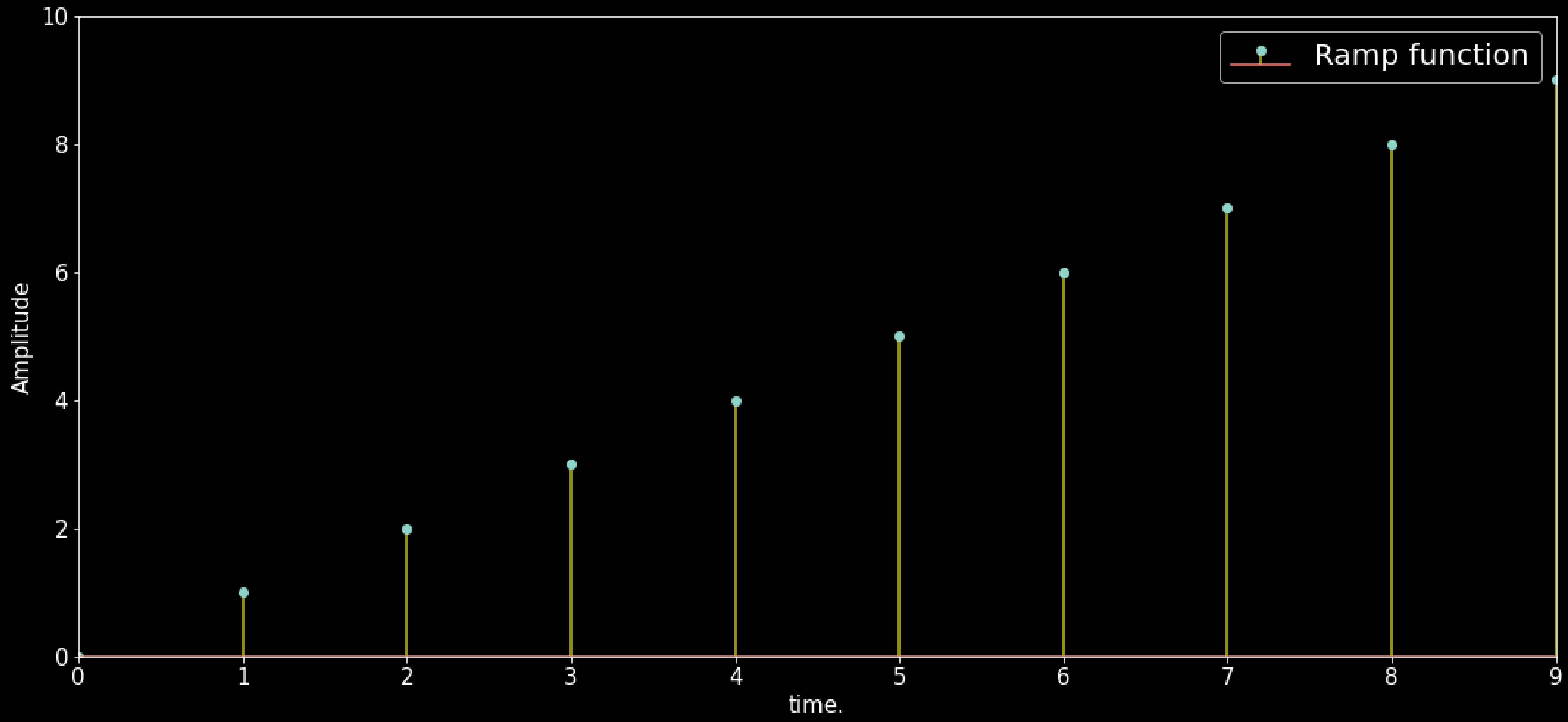
Time shifted step



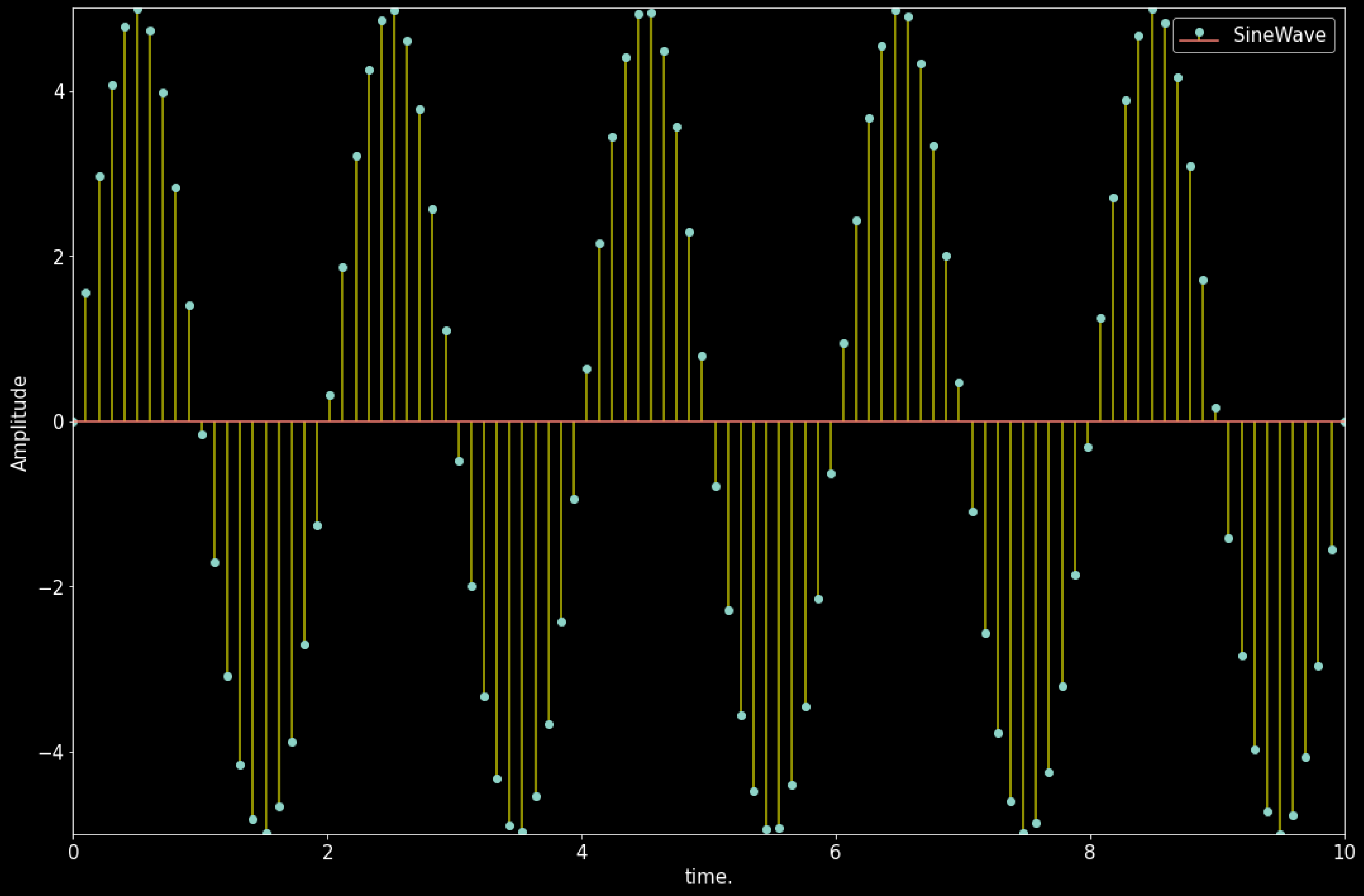
Time shifted step



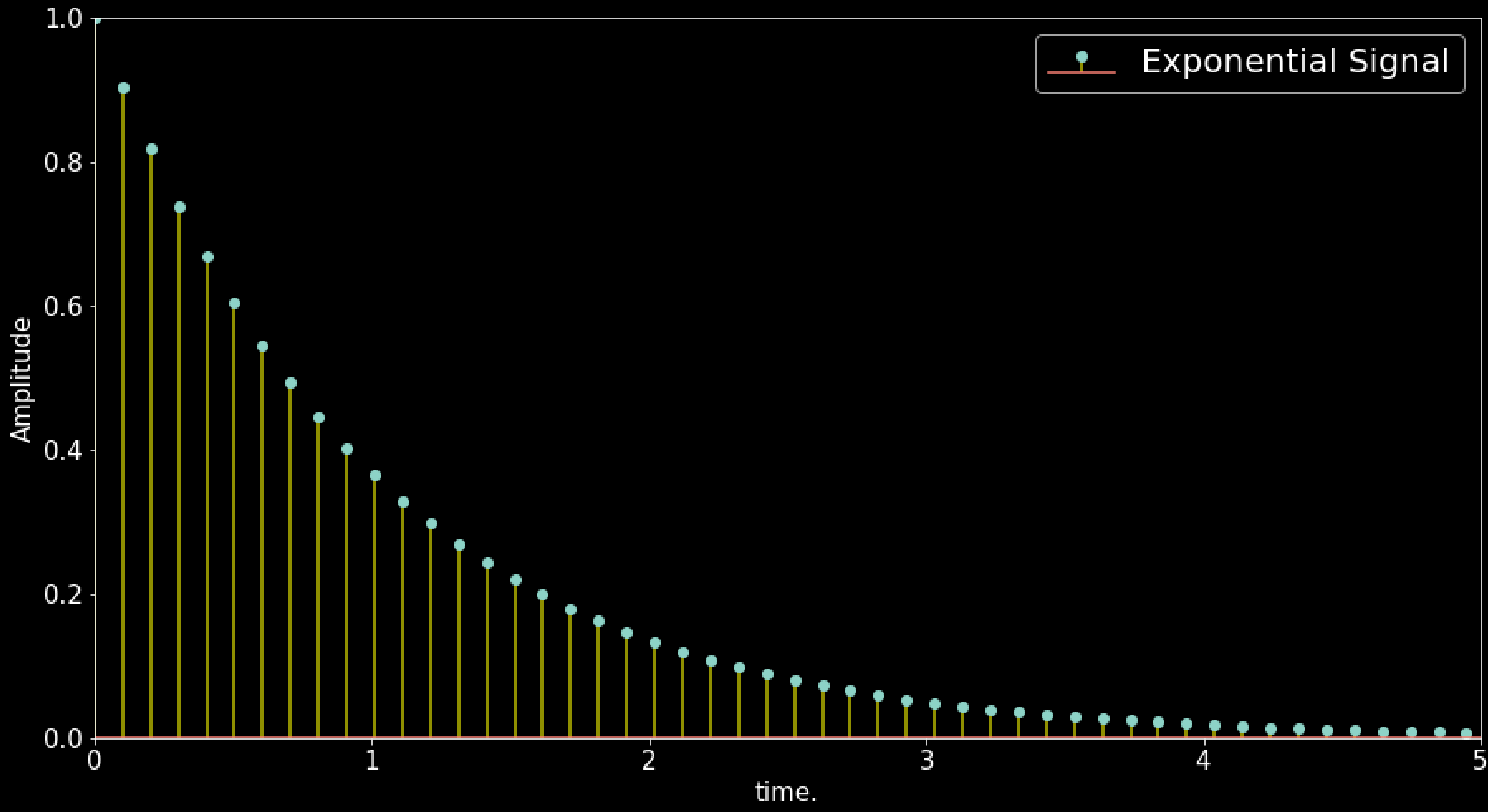
Unit Ramp signal



Sinusoidal signal



Unit Exponential signal



Sampling and Reconstruction

A continuous time signal $x(t)$ can be converted into discrete time signal $x(n)$ by replacing t with nT.

$$x(n) = x(nT)$$

where $x(n)$ is the discrete time signal obtained by taking samples of the continuoue time signal $x(t)$ every T seconds. The time interval T between successive samples is called the sampling period or sample interval and is given by

$$T = \frac{1}{f_s}$$

where f_s is called the sampling rate or the sampling frequency (Hertz).

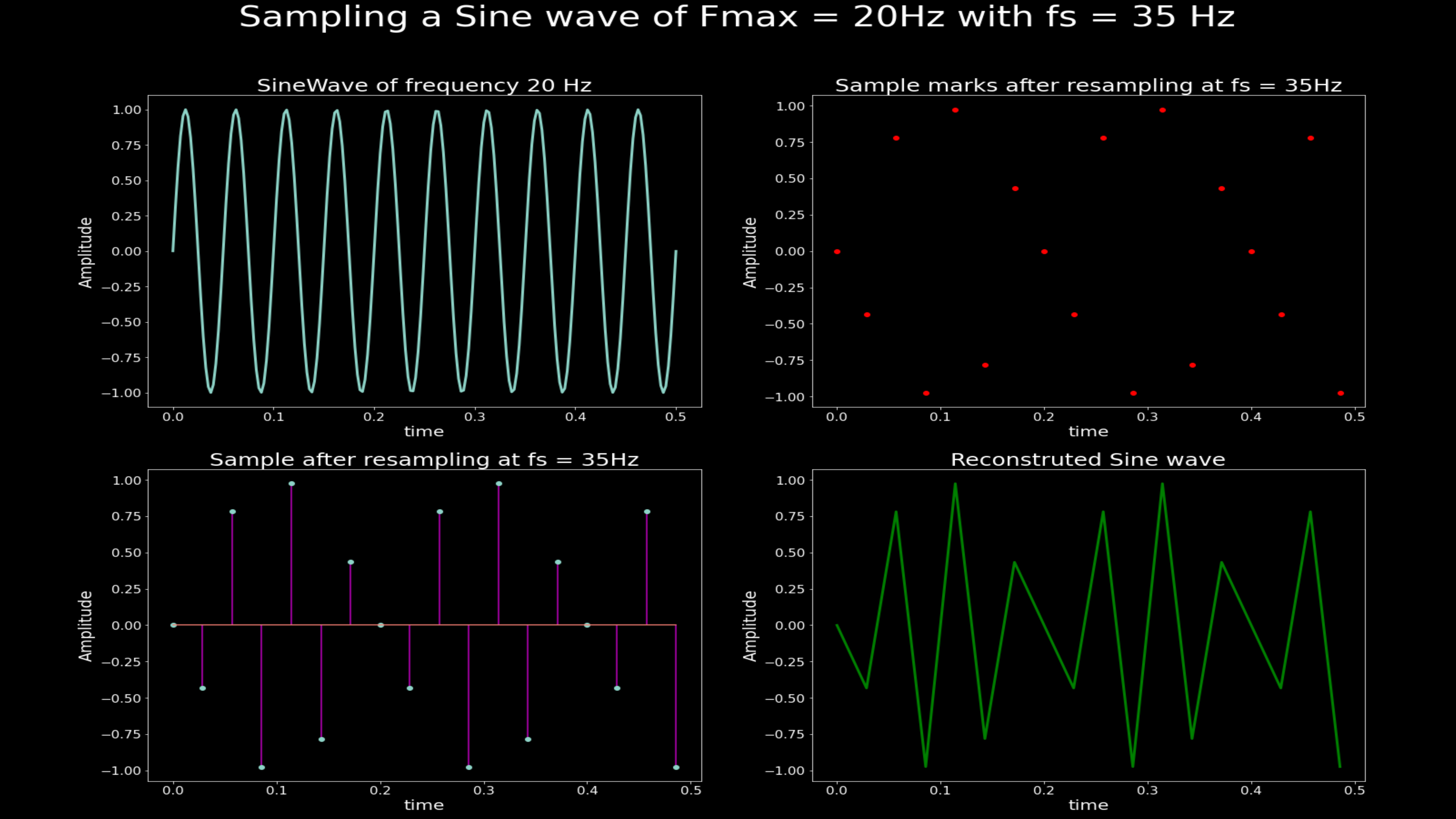
Nyquist sampling theorem

The Nyquist sampling theorem states that "the sampling frequency f_s should be greater or equal than twice the maximum frequency of the signal (continuous time signal) to be sampled."

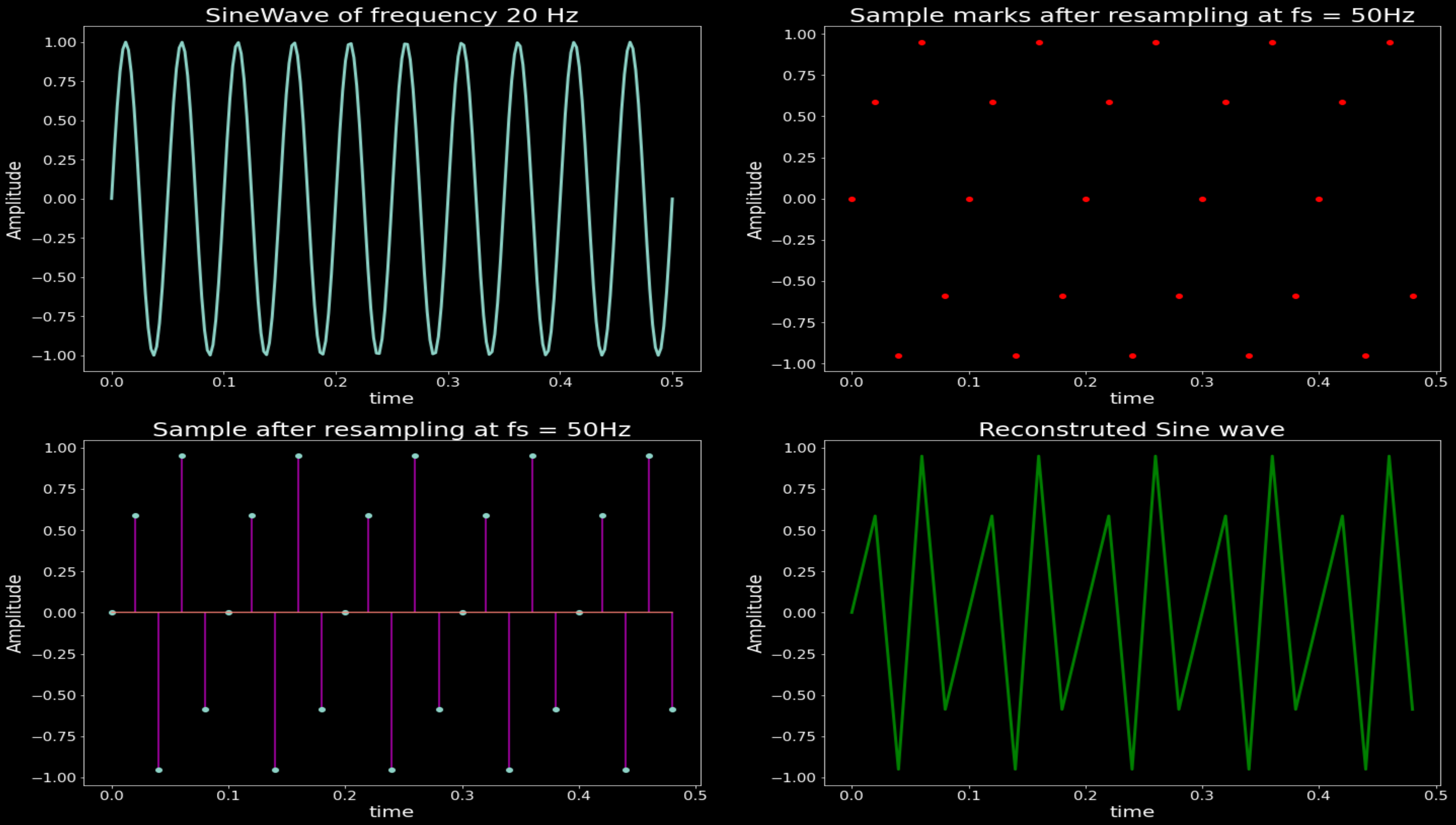
If F_{max} is the maximum frequency of the signal then according to sampling theorem

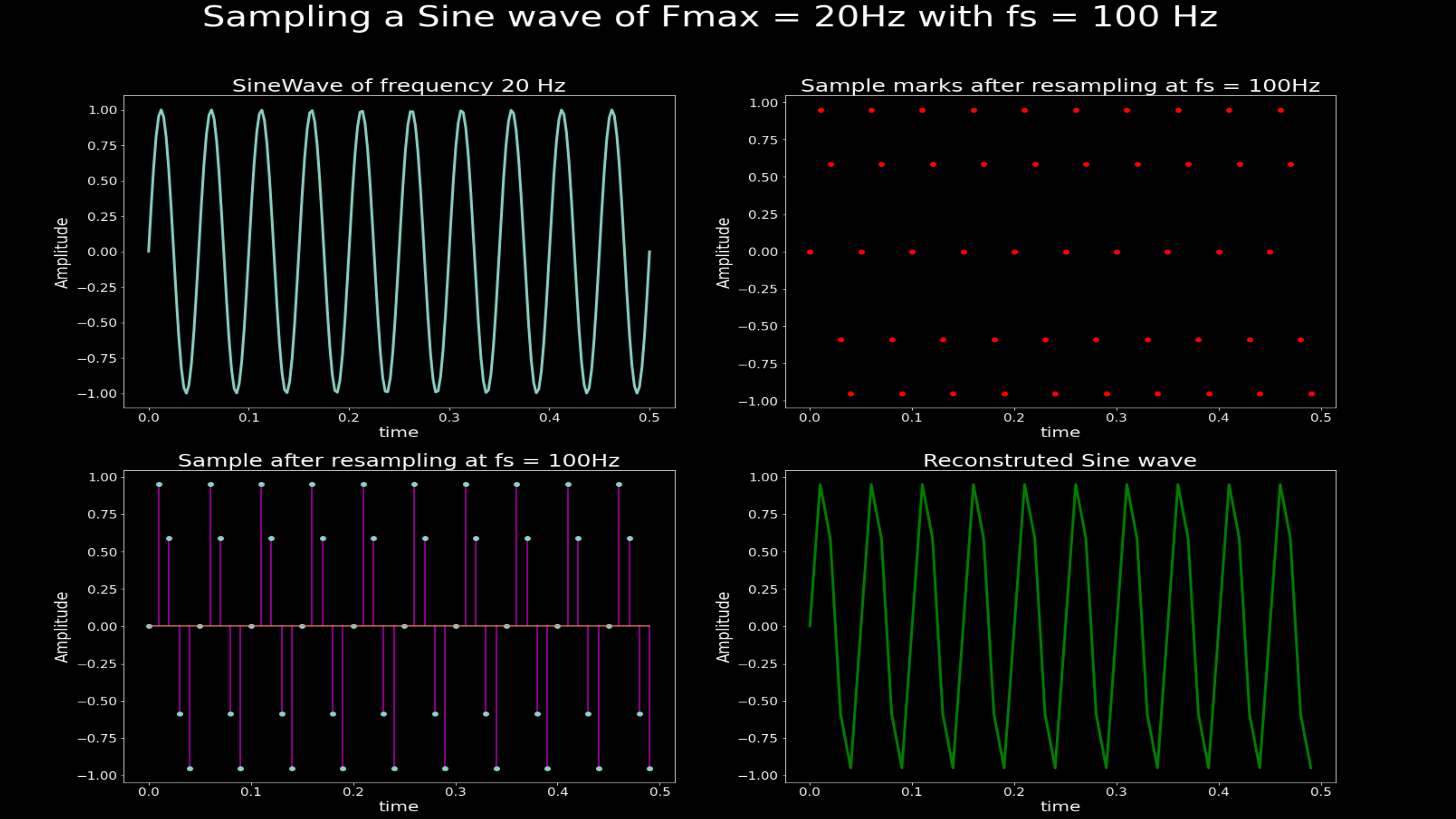
$$f_s \geq 2F_{max}$$

Sampling theorem is very important if we want to reconstruct the signal after sampling.


Sampling a Sine wave of Fmax = 20Hz with fs = 35 Hz
SineWave of frequency 20 Hz
Sample marks after resampling at fs = 35Hz
Sample after resampling at fs = 35Hz
Reconstruted Sine wave
Amplitude
time

Sampling a Sine wave of Fmax = 20Hz with fs = 50 Hz





Sampling a Sine wave of Fmax = 20Hz with fs = 100 Hz
SineWave of frequency 20 Hz
Sample marks after resampling at fs = 100Hz
Sample after resampling at fs = 100Hz
Reconstruted Sine wave
Amplitude
time

Sampling a wave of Fmax = 20Hz with fs = 100 Hz