Wavelet Transform

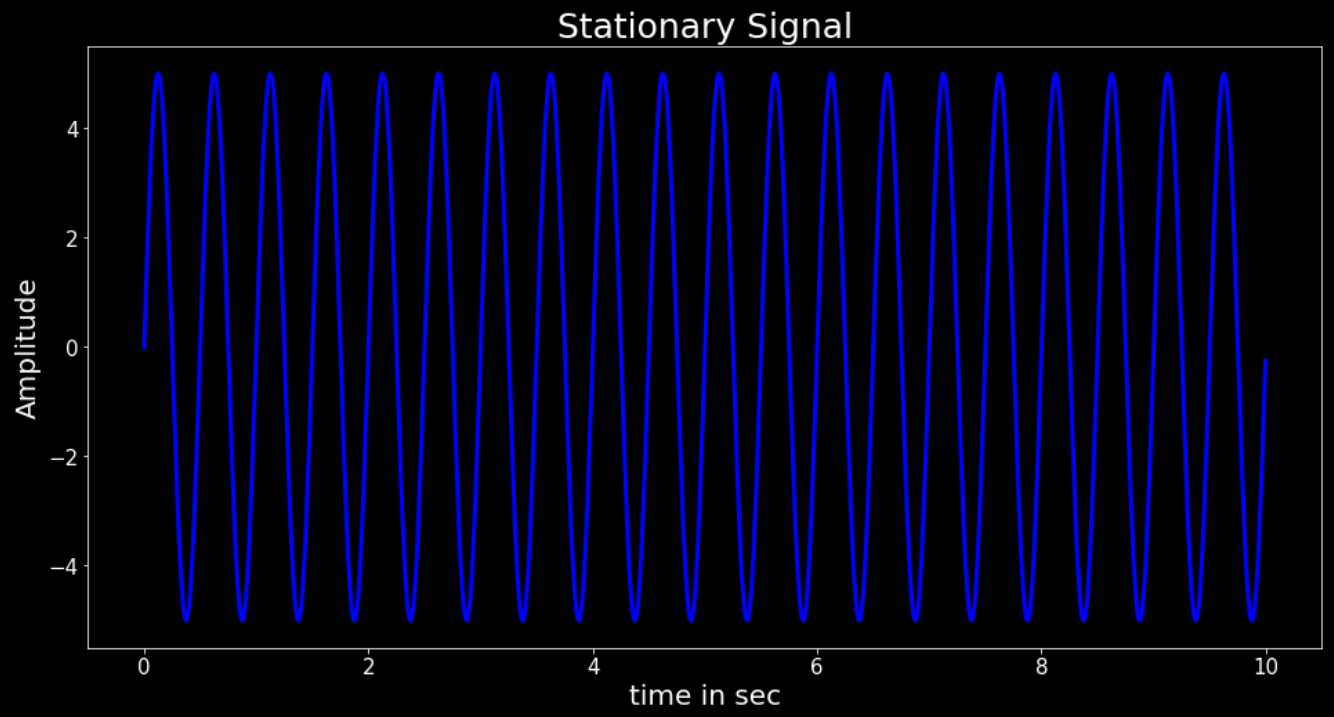
Topics to be covered

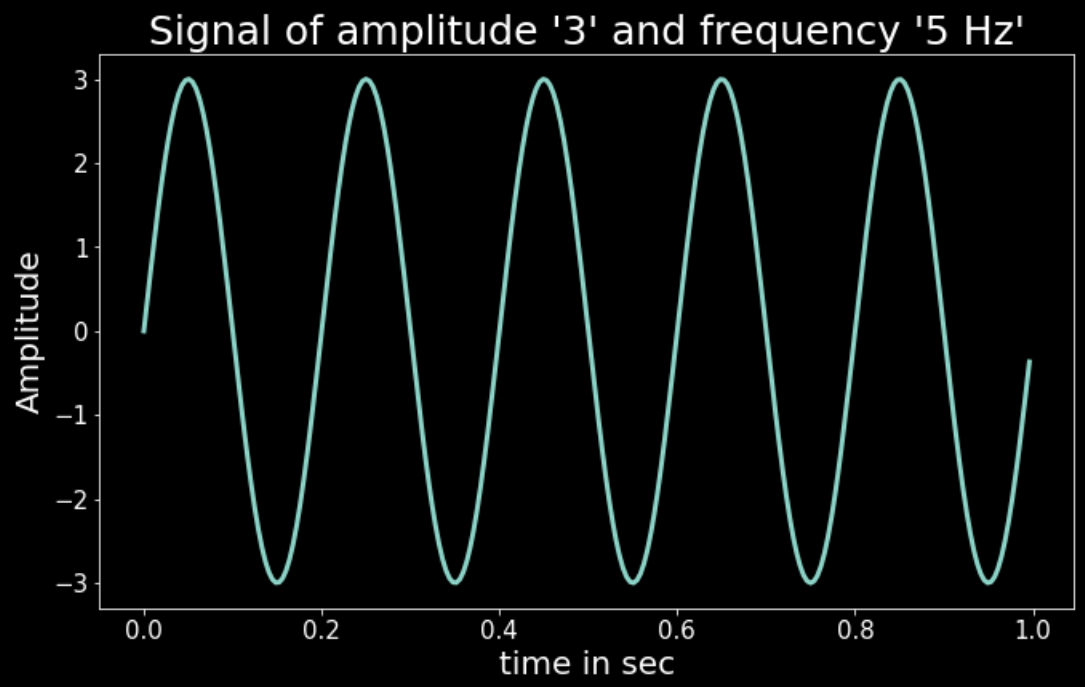
- Limitations of Fourier Transform.
- Why wavelet Transform ?
- Working of Wavelet Transform.
- Wavelet families for continuous and discrete wavelet transform.
- The discrete wavelet transform as a Filter bank.
- Single level decomposition and reconstruction.
- Multilevel decomposition.
- Continuous Wavelet Transform.

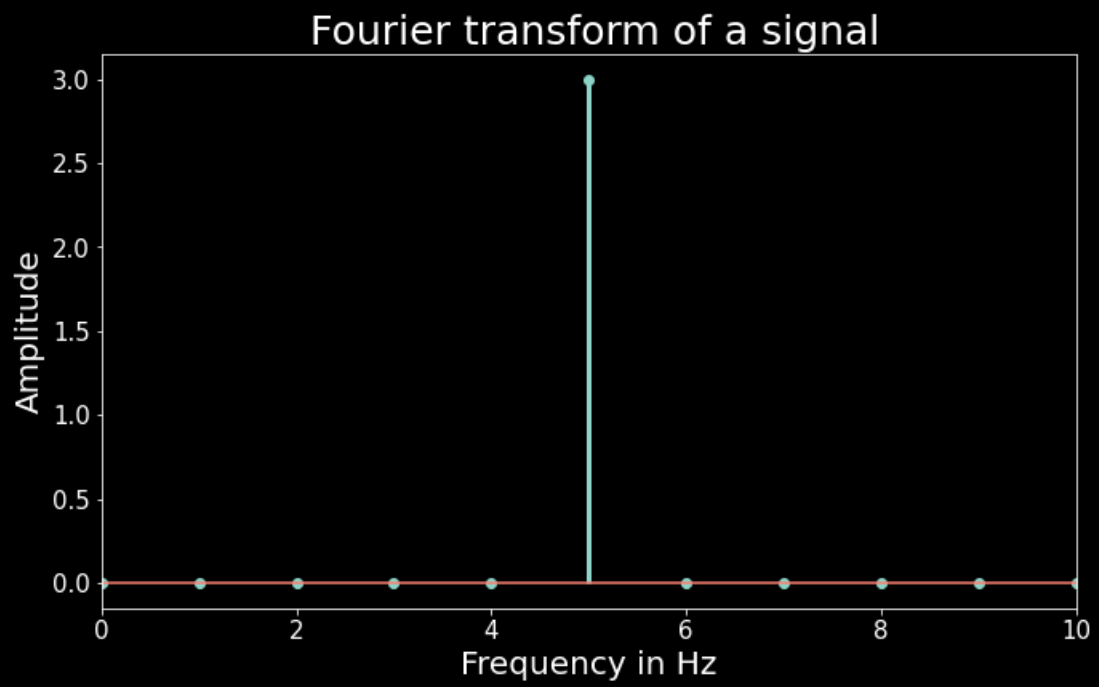
Limitations of Fourier Transform

Stationarity of a signal

A signal is said to be stationary if its statistical properties do not change over time, otherwise, it is non-stationary.

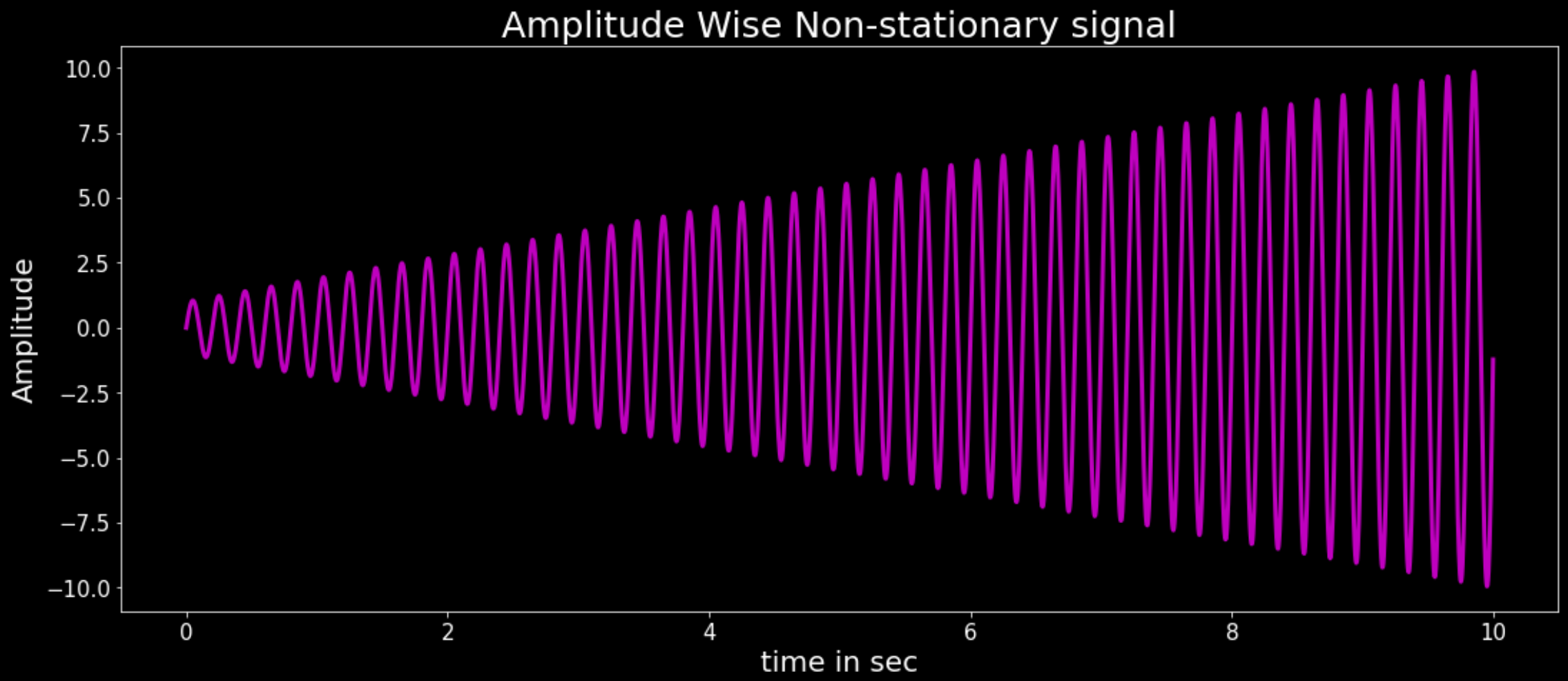


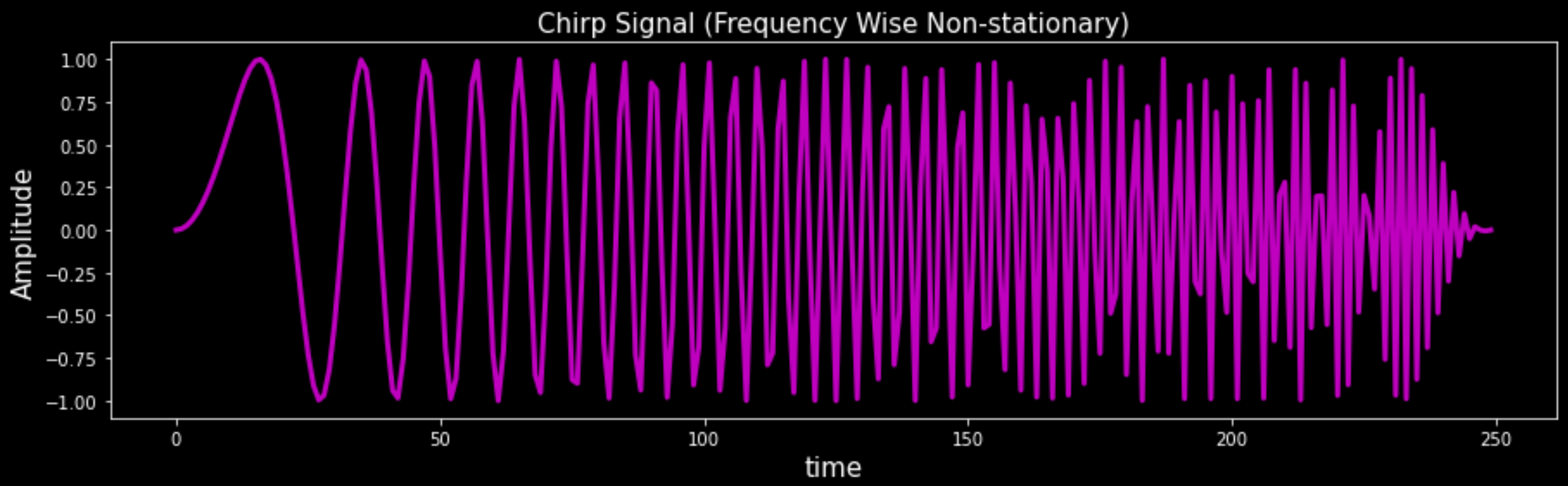

Signal of amplitude '3' and frequency '5 Hz'
Amplitude
time in sec
3
2
1
0
−1
−2
−3
0.0
0.2
0.4
0.6
0.8
1.0

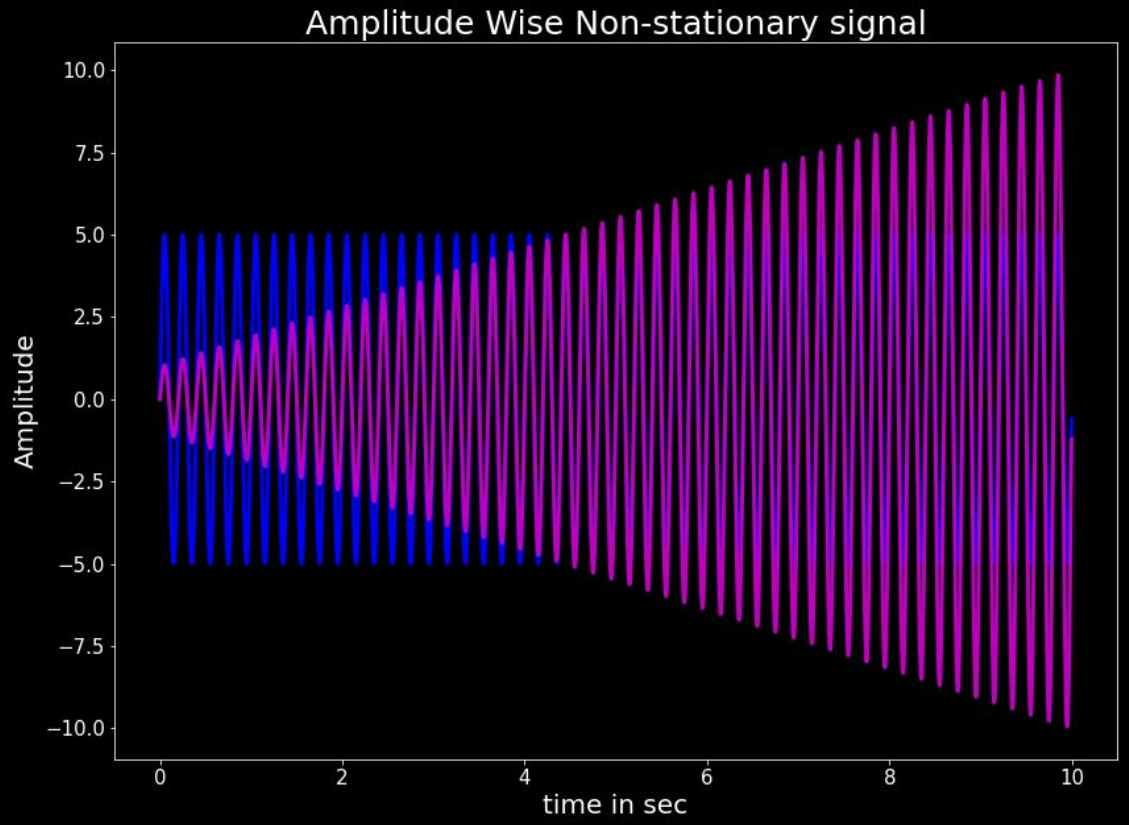

Fourier transform of a signal
Amplitude
Frequency in Hz
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0
2
4
6
8
10

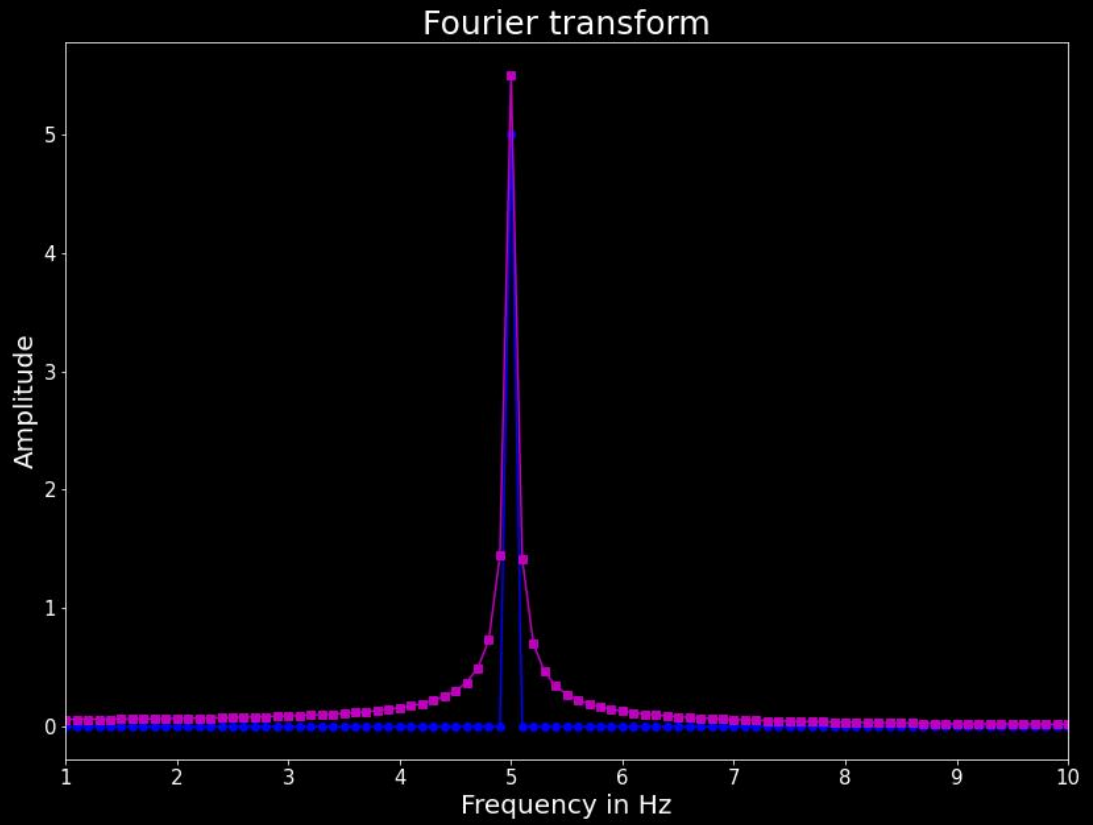
Limitations of Fourier Transform

- Fourier Transform (FT) works well when the signal is stationary, i.e the frequencies present in the signal are not time-dependent.
- FT becomes more and more uninterpretable as the non-stationarity of the signal increases.
- Most of the real life signals are non-stationary in nature. Thus, a better representation of a signal both in time and frequency domain is required.
- Some of the example of non-stationary signals include ECG signal and chirp signal.


Amplitude Wise Non-stationary signal
10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
−10.0
Amplitude
0
2
4
6
8
10
time in sec

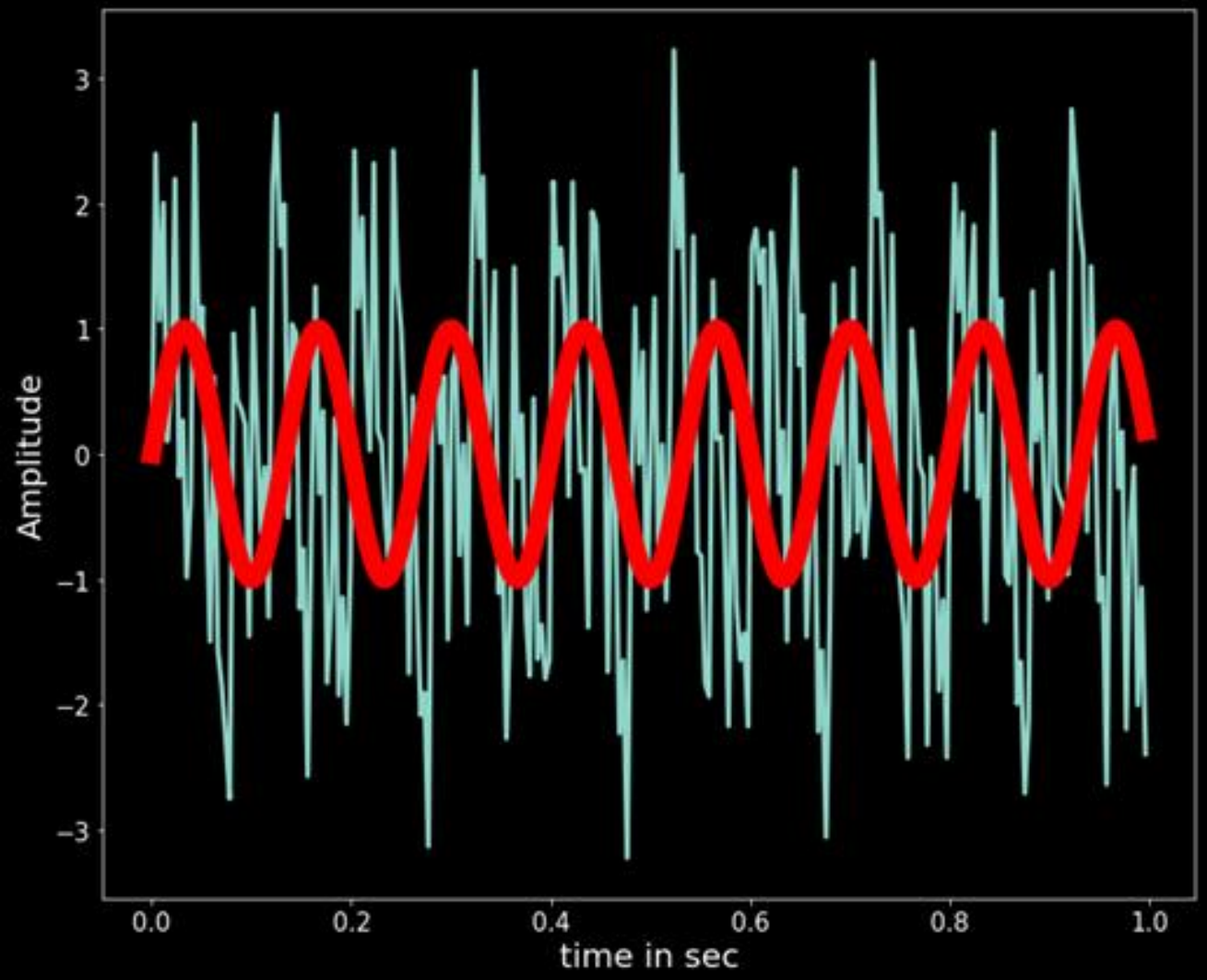

Chirp Signal (Frequency Wise Non-stationary)
Amplitude
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
0
50
100
150
200
250
time


Amplitude Wise Non-stationary signal
10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
−10.0
Amplitude
0
2
4
6
8
10
time in sec

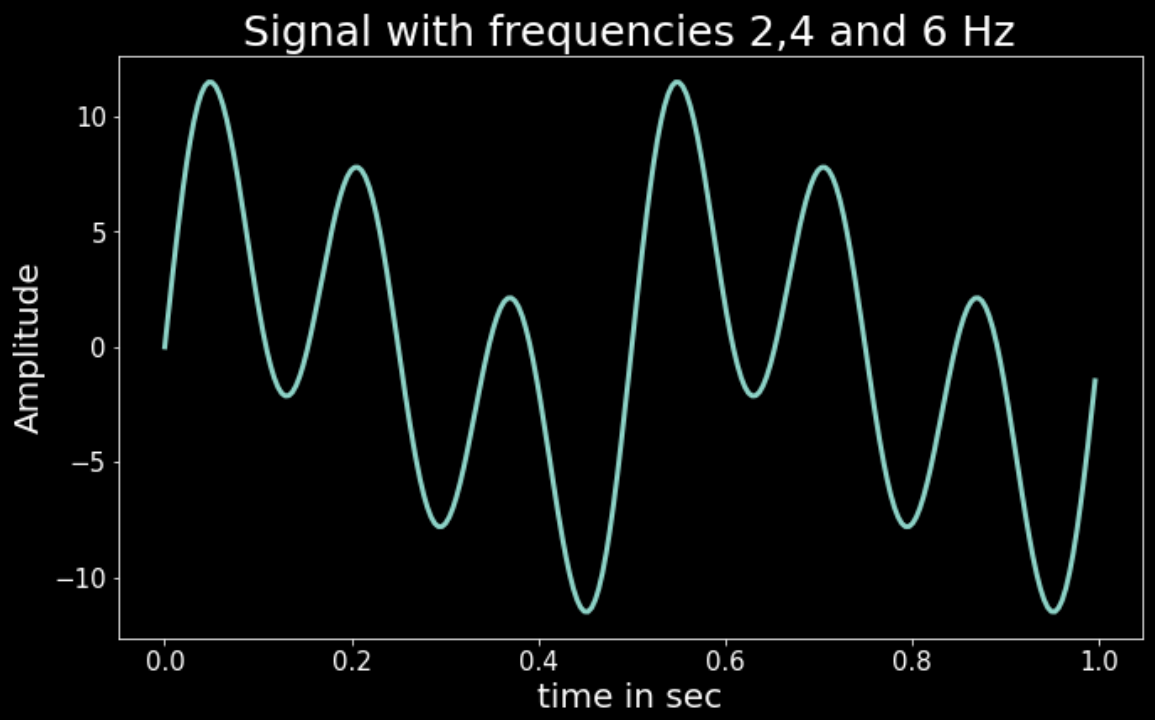

Fourier transform
5
4
3
2
1
0
Amplitude
1
2
3
4
5
6
7
8
9
10
Frequency in Hz

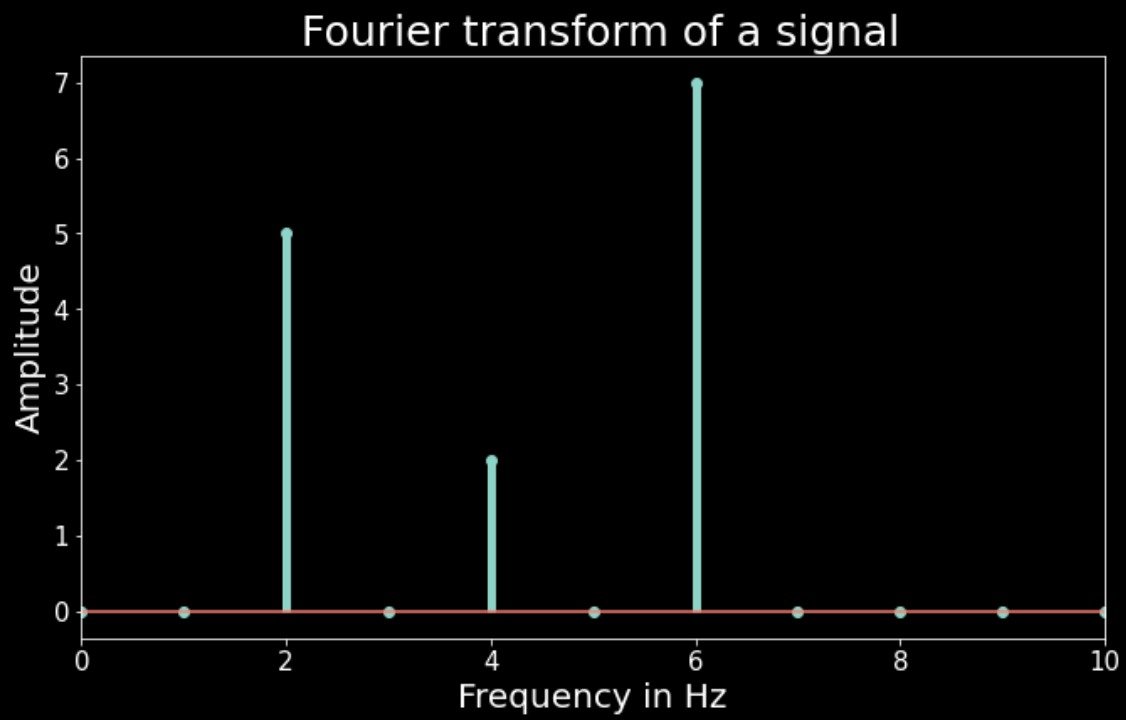
Why Wavelet Transform ?

How Fourier Transform Works ?



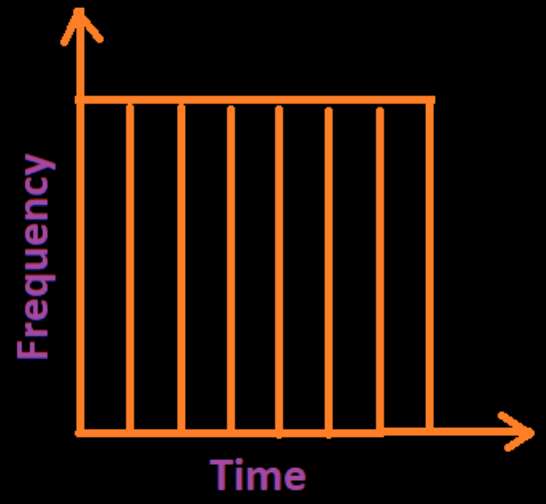
How Fourier Transform Works ?



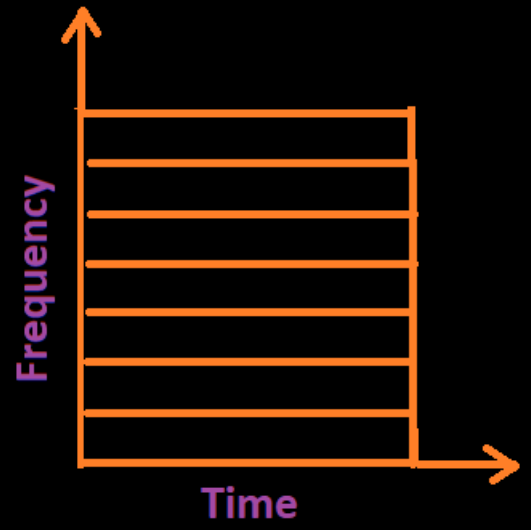


Why Wavelet Transform ?

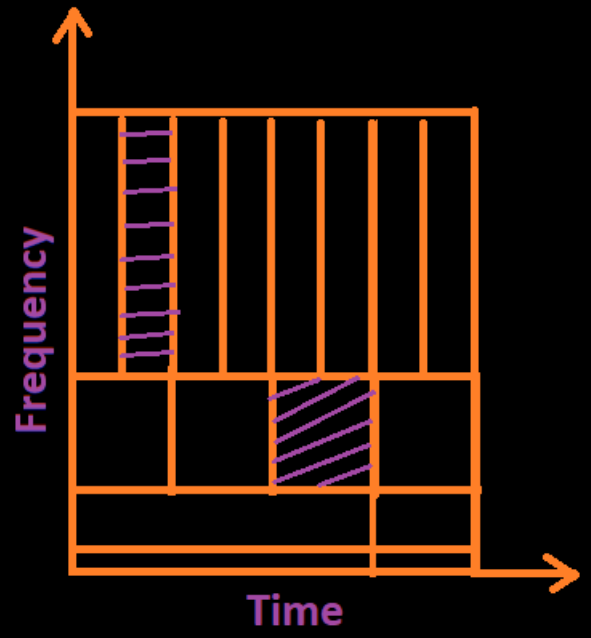
- Wavelet Transform (WT) has high resolution in both the time and frequency domain.
- It does not only tell us which frequencies are present in a signal, but also at which time intervals these frequencies are present.
- Wavelet Transform works with different scales. First, it uses large scale OR large window to look for large features and then uses small scales for looking into small features.



Tine domain signal



FT of a signal



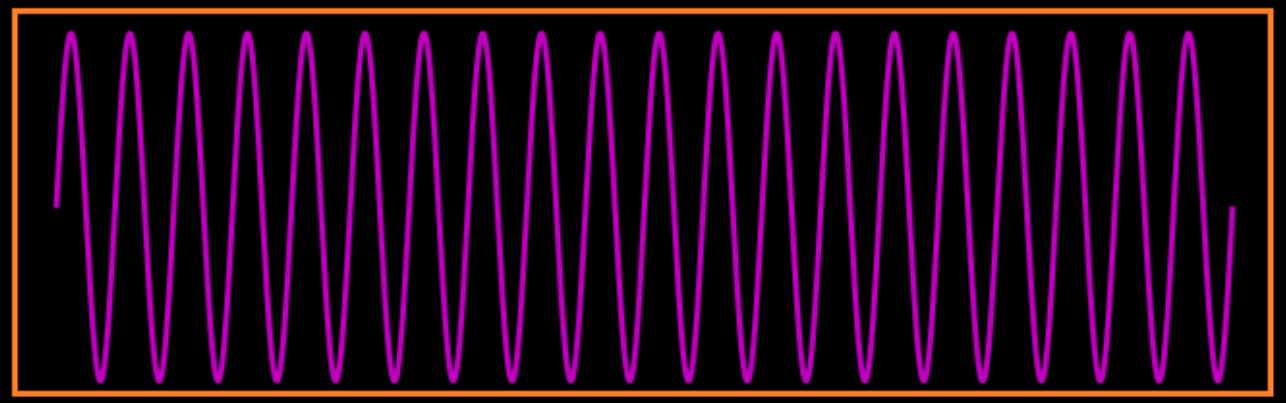
Wavelet Transform

Working of Wavelet Transform

Working of Wavelet Transform

- WT uses wavelet rather than a sine wave which was used for FT.
- Sine wave is not localized in time i.e it exists from negative infinity to positive infinity while a wavelet is localized in time. Localization allows the WT to obtain time and frequency information.
- We perform convolution between signal and wavelet (mother wavelet) and repeat convolution at different scales of wavelet.

Working of Wavelet Transform



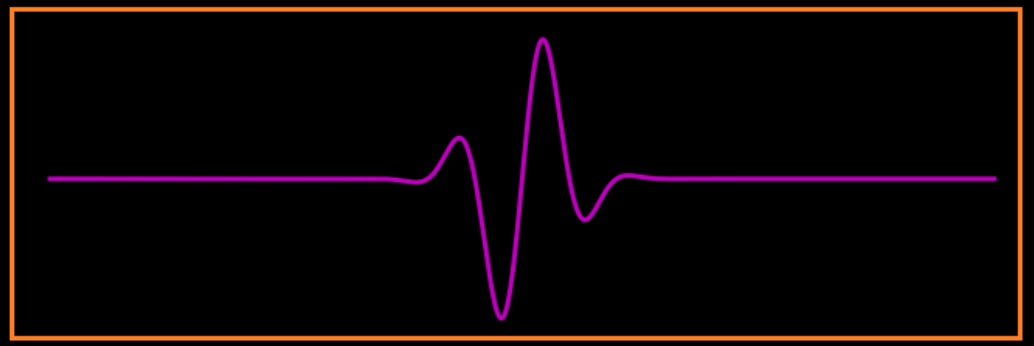
Sine Wave

Wavelet

Working of Wavelet Transform

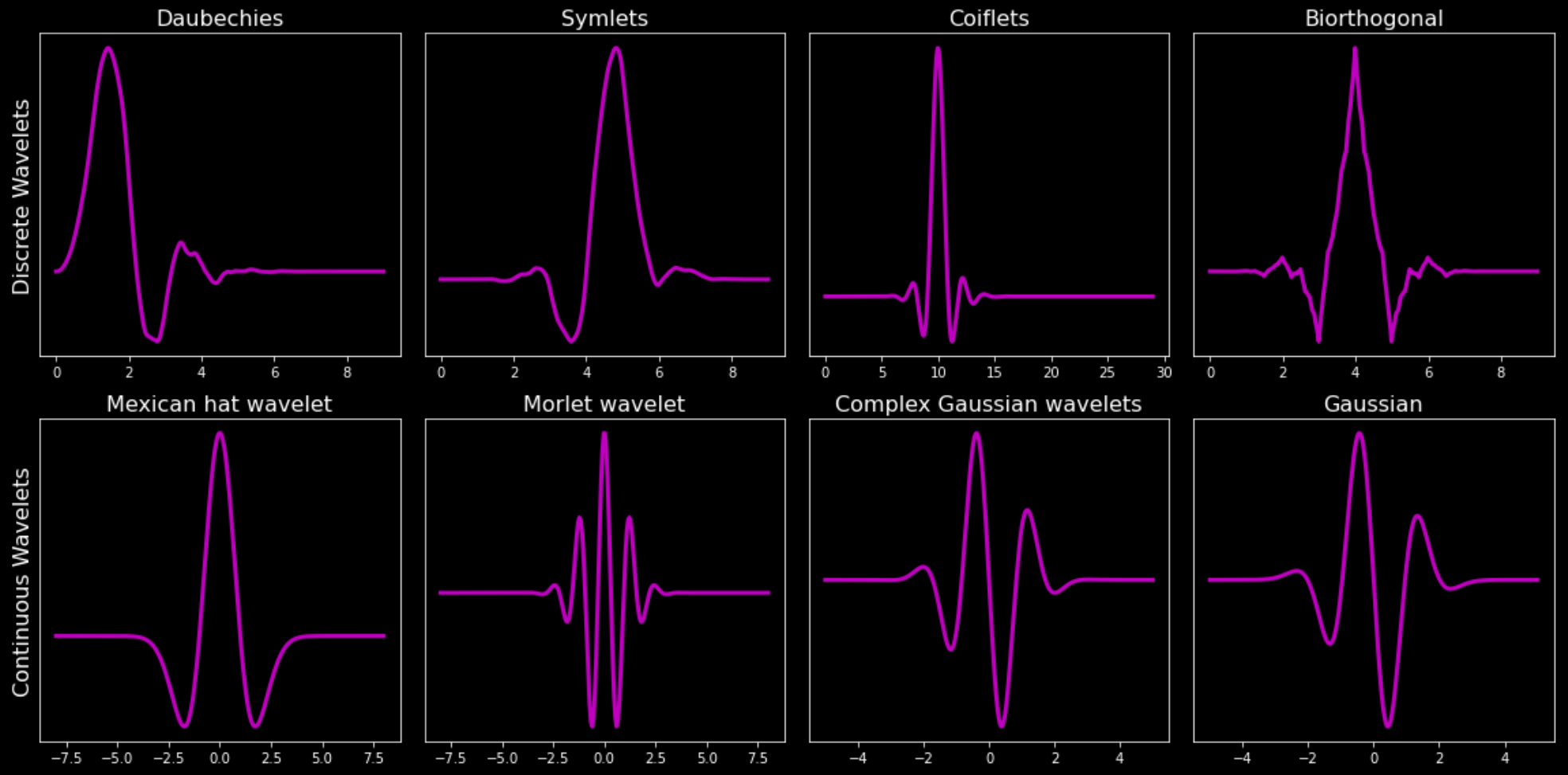
- Smaller the scale, more compressed will be a wavelet i.e wavelet changes rapidly and its frequency is high. Thus, scales are the inverse of the frequency.

Small scale
Large frequency



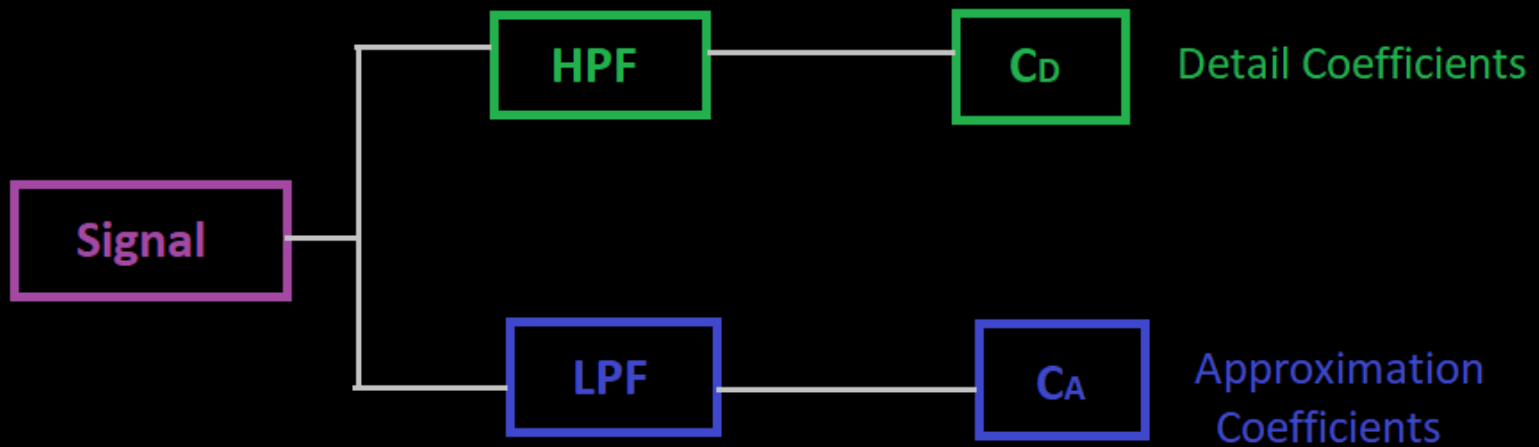
Large scale
Small frequency

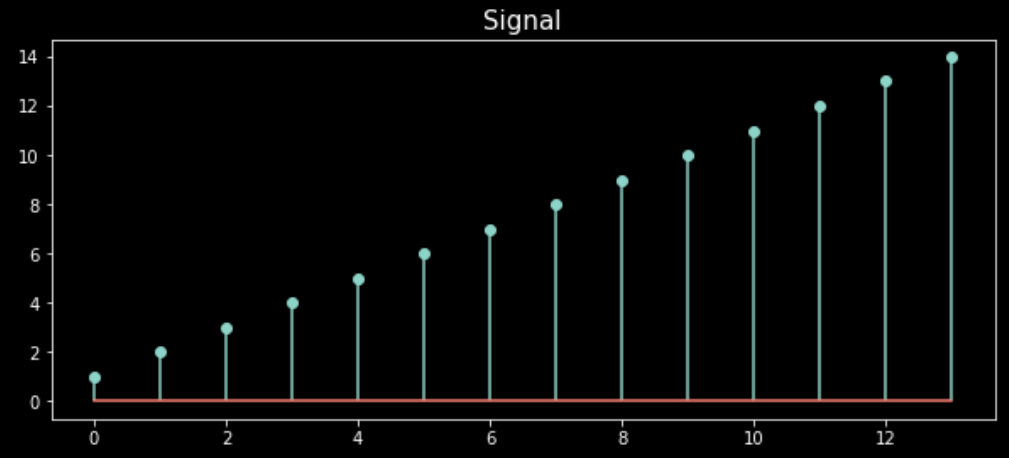
Wavelet Families

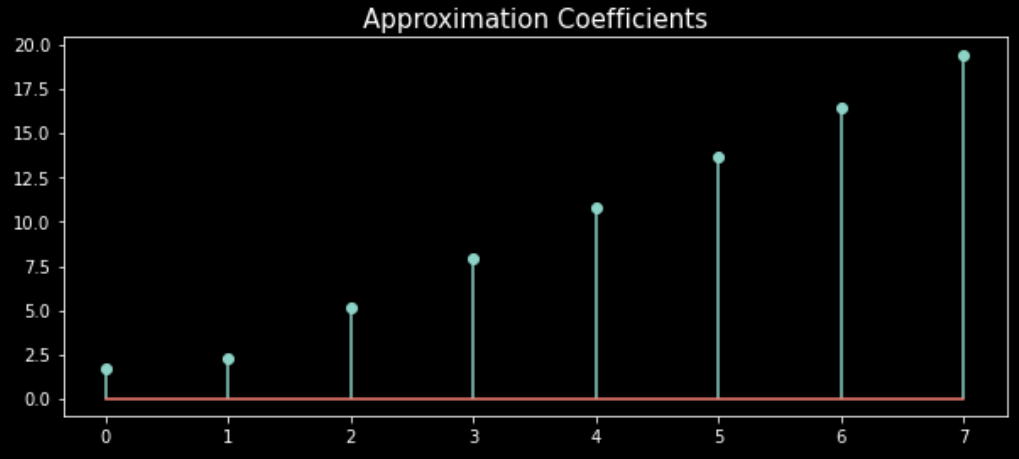

Daubechies
Symlets
Coiflets
Biorthogonal
Discrete Wavelets
Mexican hat wavelet
Morlet wavelet
Complex Gaussian wavelets
Gaussian
Continuous Wavelets

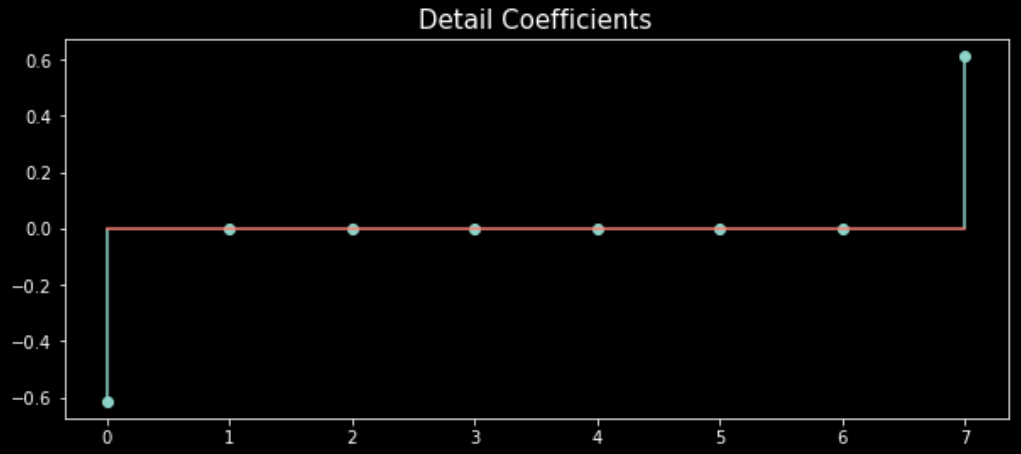
Single Level Decomposition

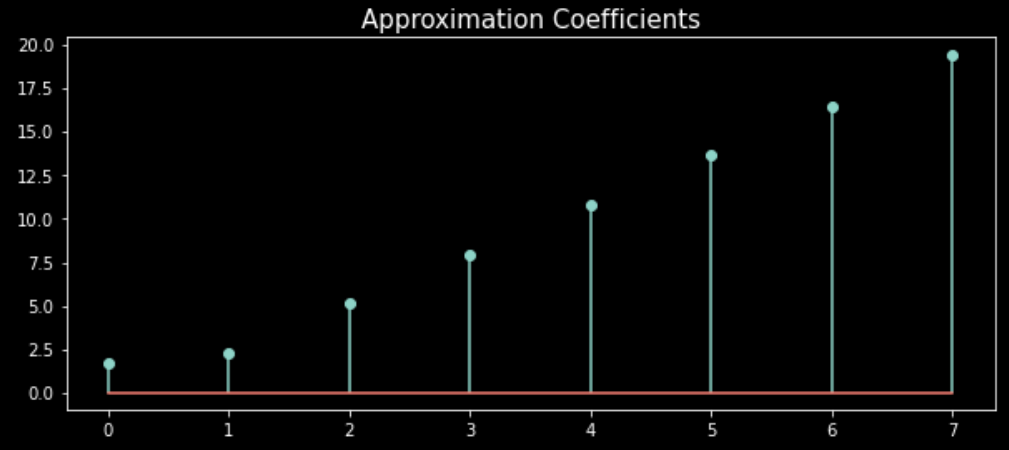
Single Level Decomposition

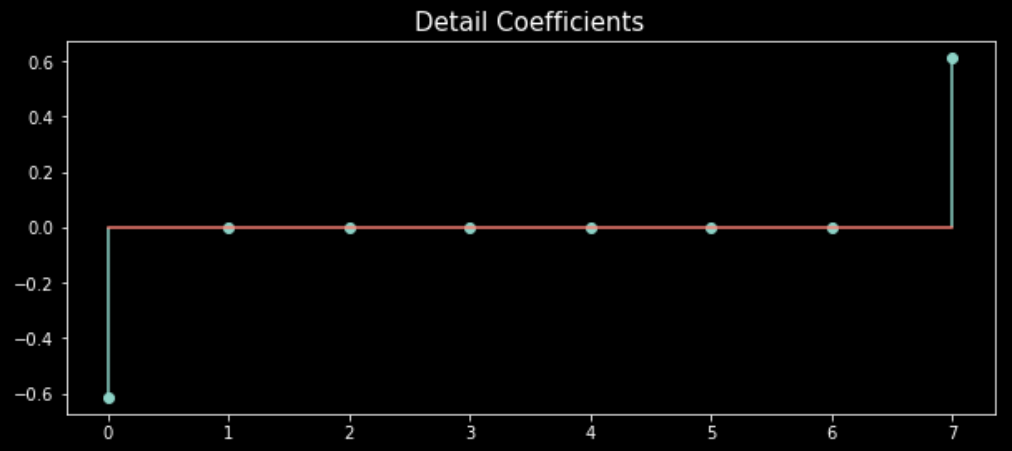


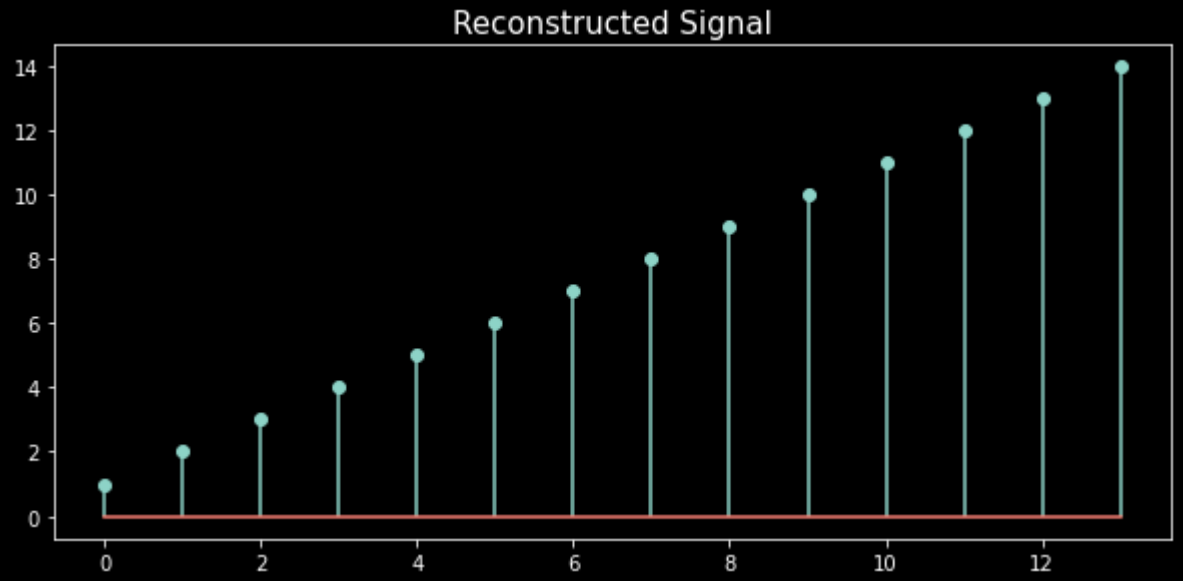

Signal
0
2
4
6
8
10
12
14

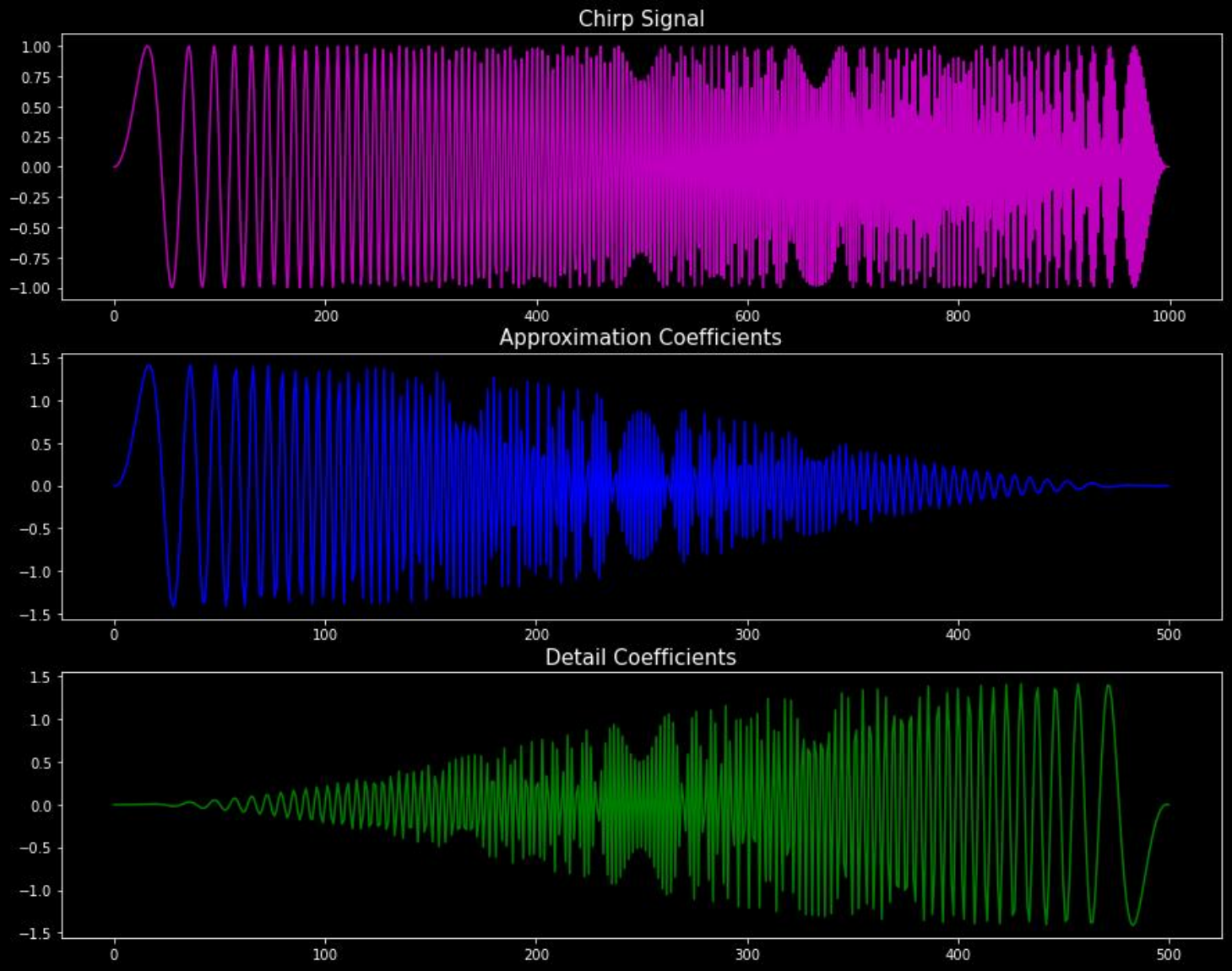

Approximation Coefficients
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
0
1
2
3
4
5
6
7

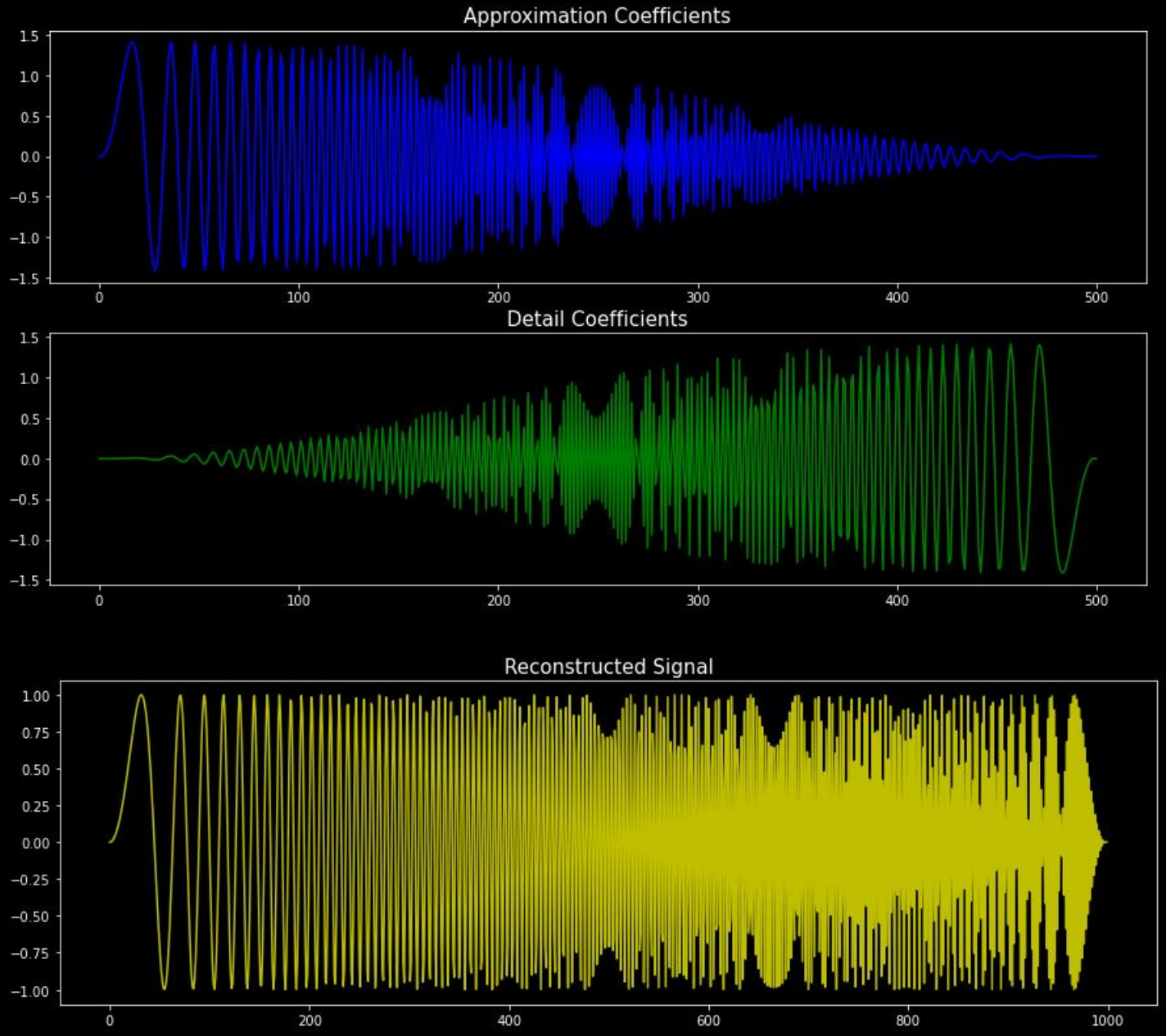

Detail Coefficients
−0.6
−0.4
−0.2
0.0
0.2
0.4
0.6
0
1
2
3
4
5
6
7


Approximation Coefficients
20.0
17.5
15.0
12.5
10.0
7.5
5.0
2.5
0.0
0
1
2
3
4
5
6
7

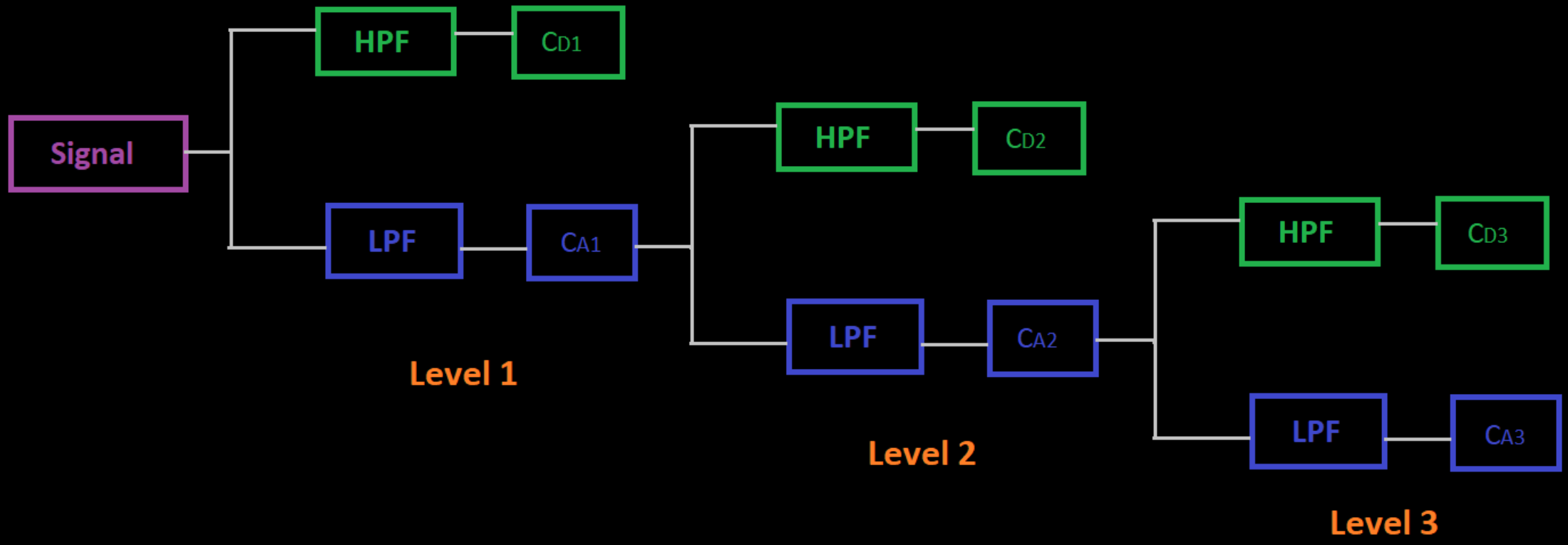

Detail Coefficients
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
0
1
2
3
4
5
6
7

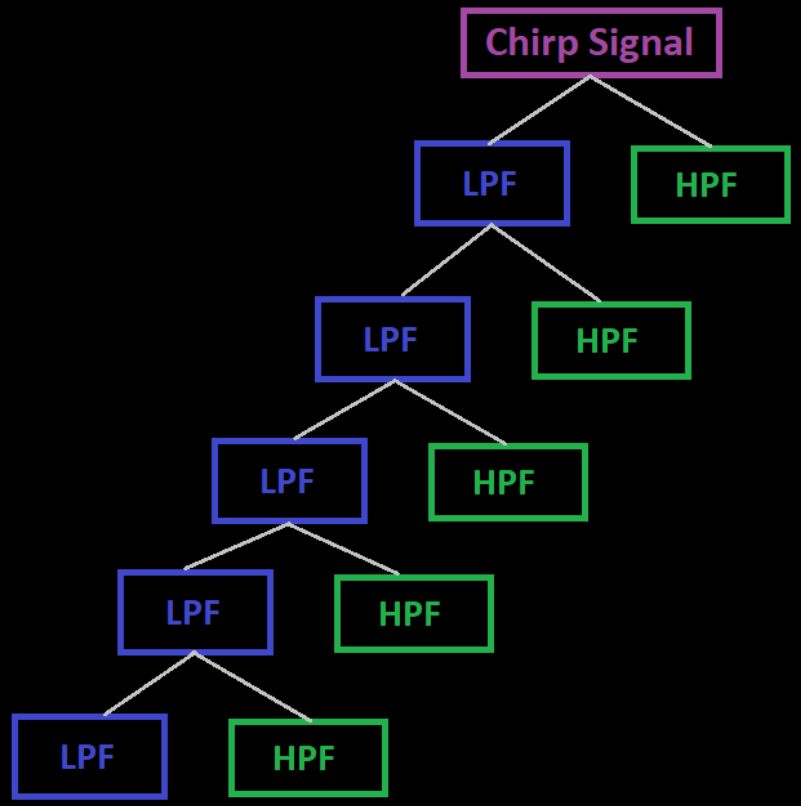

Reconstructed Signal
14
12
10
8
6
4
2
0
0
2
4
6
8
10
12

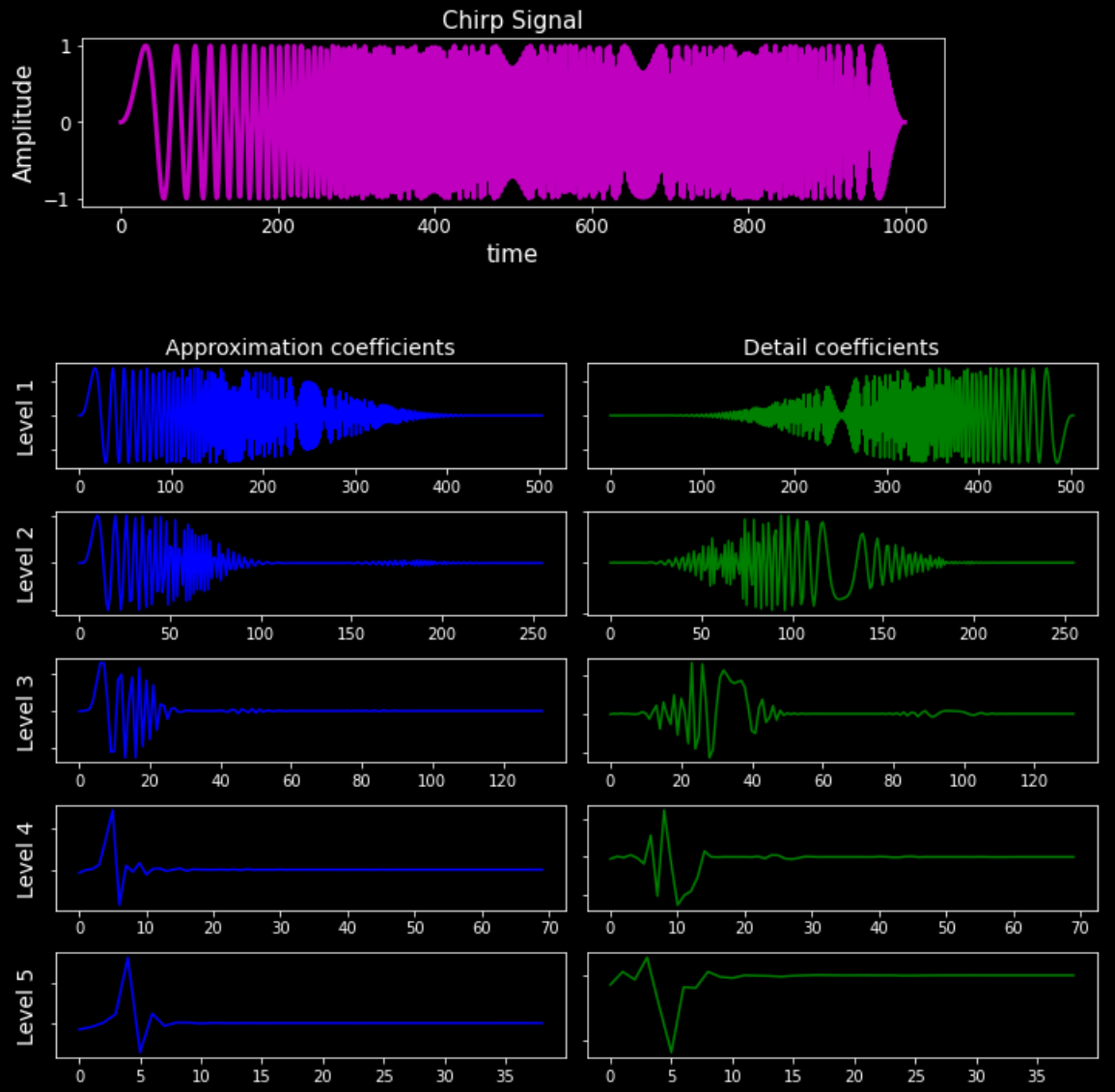

Chirp Signal
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
0
200
400
600
800
1000
Approximation Coefficients
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
0
100
200
300
400
500
Detail Coefficients
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
0
100
200
300
400
500


Approximation Coefficients
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
0
100
200
300
400
500
Detail Coefficients
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
0
100
200
300
400
500
Reconstructed Signal
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
0
200
400
600
800
1000

Multilevel Decomposition

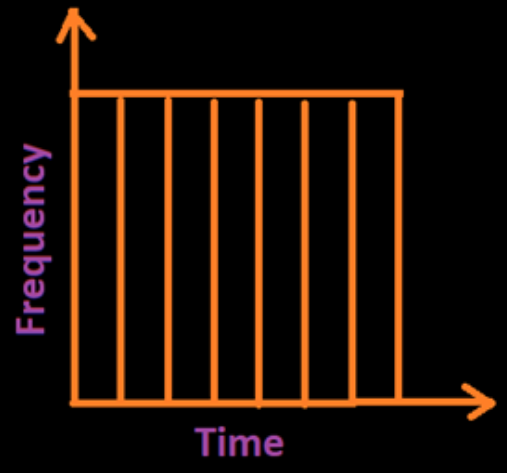

Signal
HPF
CD1
LPF
CA1
HPF
CD2
LPF
CA2
HPF
CD3
LPF
CA3
Level 1
Level 2
Level 3


Chirp Signal
LPF
HPF
LPF
HPF
LPF
HPF
LPF
HPF
LPF
HPF

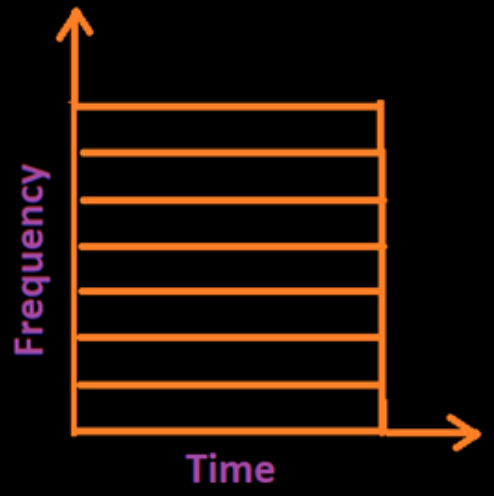

Chirp Signal
Amplitude
time
Approximation coefficients
Detail coefficients
Level 1
Level 2
Level 3
Level 4
Level 5

Continuous Wavelet Transform

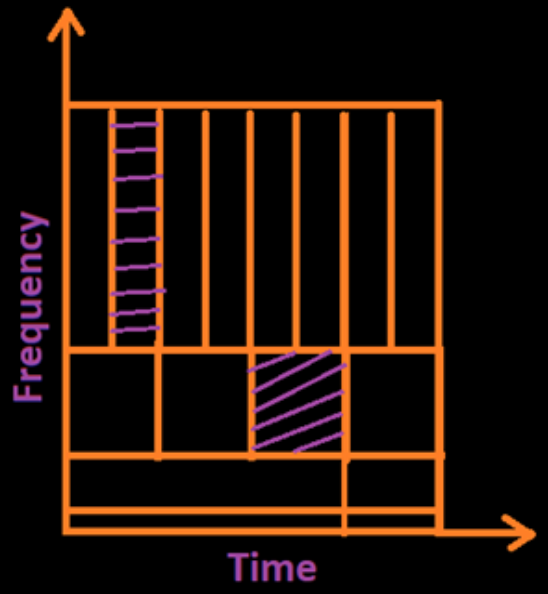
(Time Frequency Analysis)



Tine domain signal

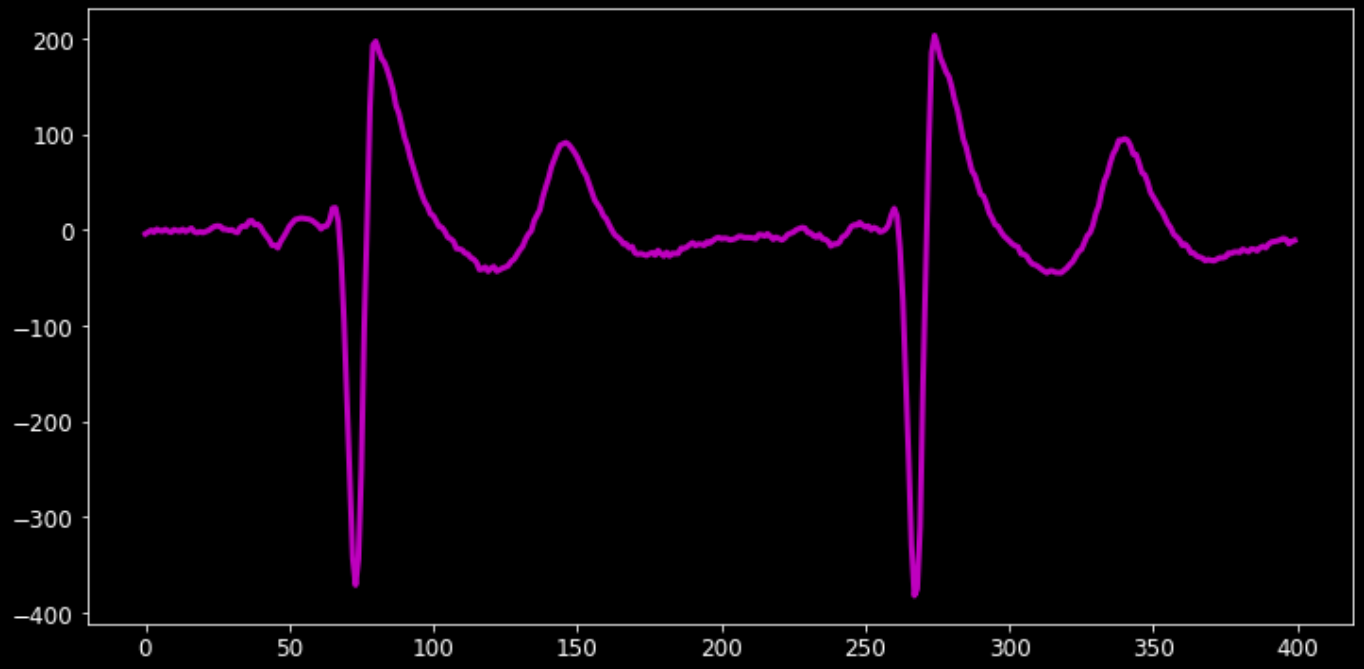


FT of a signal

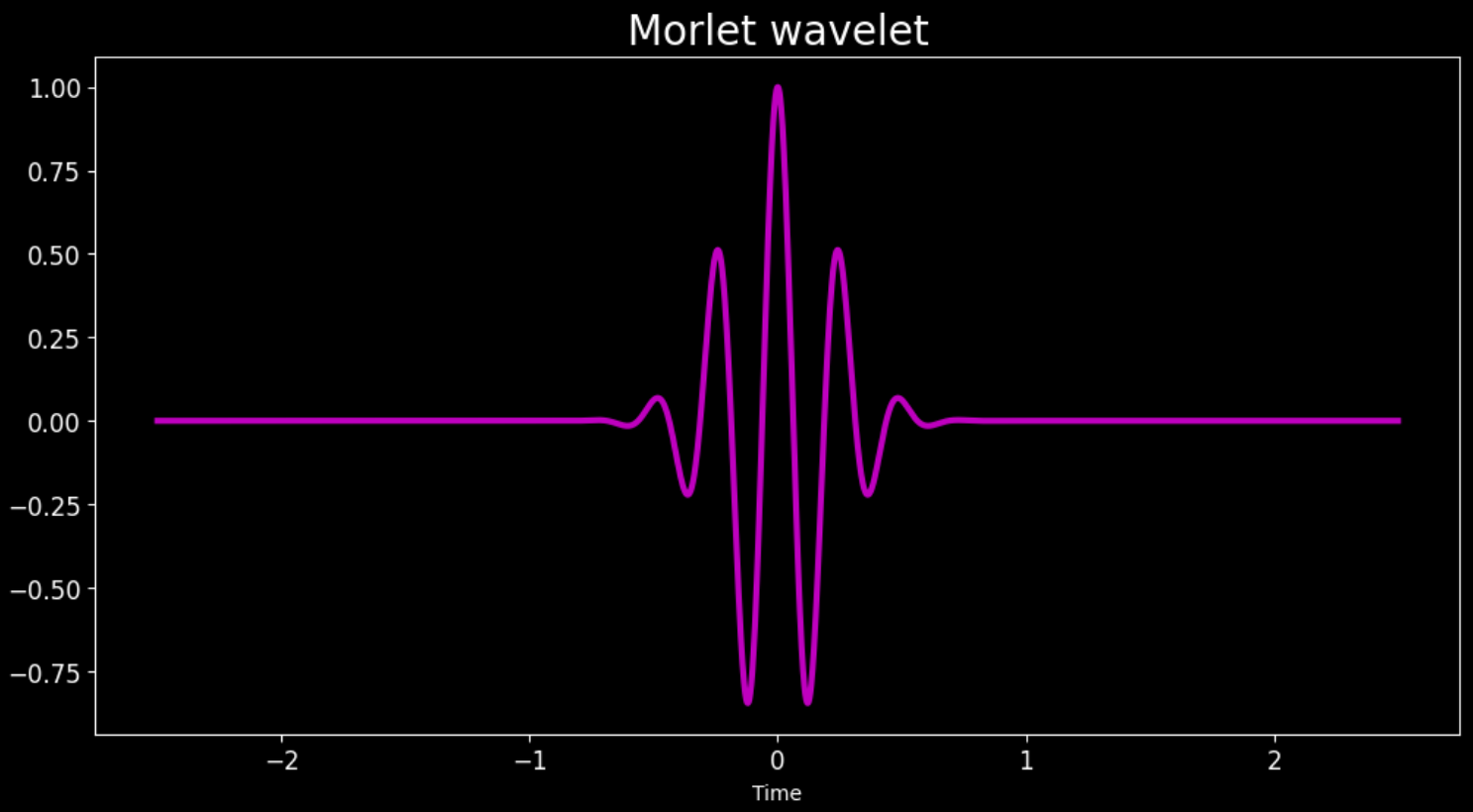


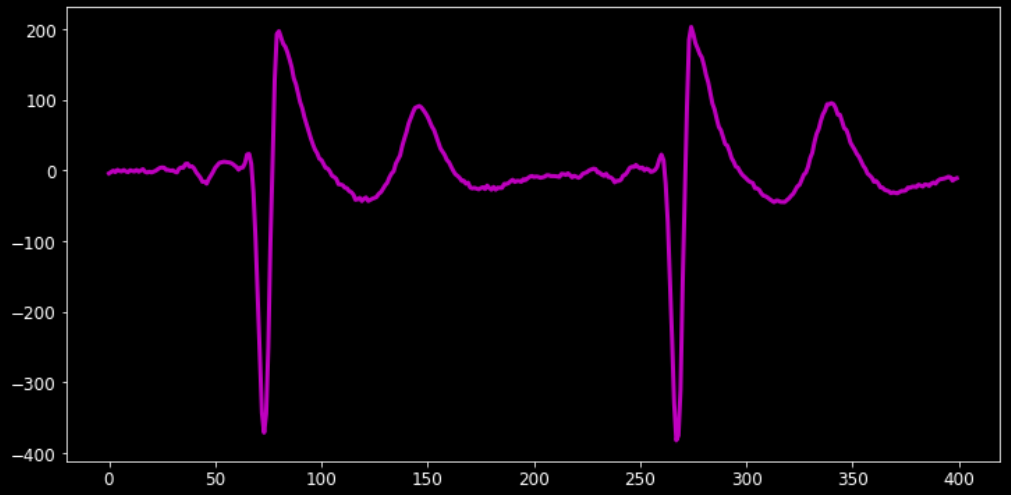
Wavelet Transform

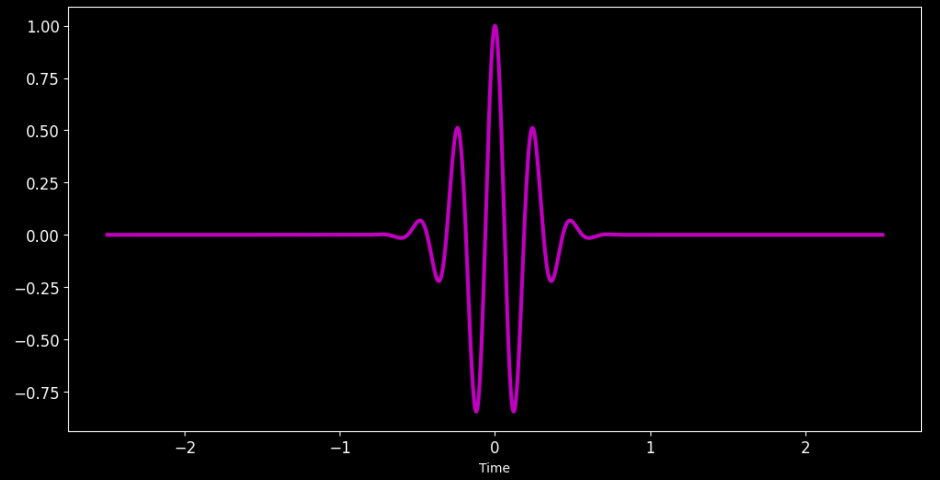
Signal

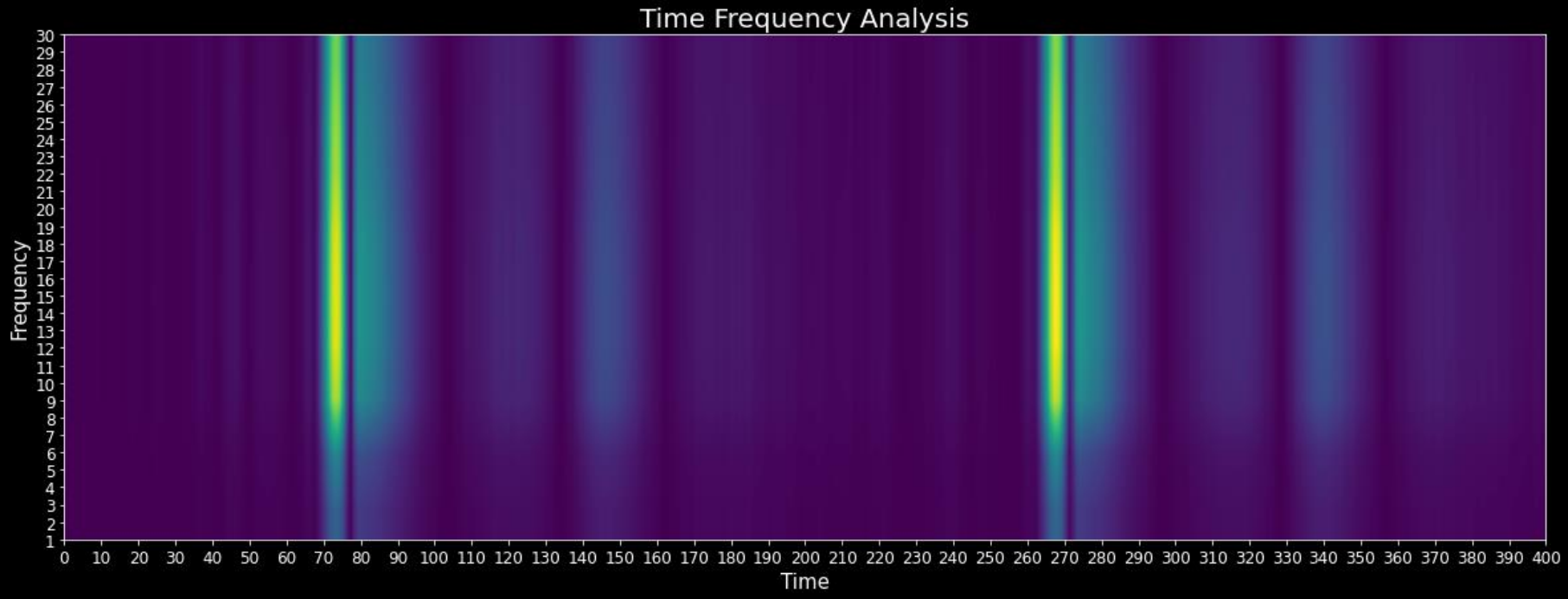


Morlet Wavelet




200
100
0
−100
−200
−300
−400
0
50
100
150
200
250
300
350
400


1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−2
−1
0
1
2
Time


Time Frequency Analysis
Frequency
Time
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
190
200
210
220
230
240
250
260
270
280
290
300
310
320
330
340
350
360
370
380
390
400