

## Midterm Examples

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## Announcements

Trees

## From Discussion 5 (Updated to be about the Tree class)

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For a Tree instance `t`:

- Its root label can be any value, and `t.label` evaluates to it.
- Its branches are trees, and `t.branches` evaluates to a list of branches.
- It is a leaf if it has no branches, and `t.is_leaf()` returns `True`.
- An identical tree can be constructed with `Tree(t.label, t.branches)`.
- You can call functions that take trees as arguments, such as `height(t)`.
- That's how you work with trees. No `t == x` or `t[0]` or `x in t` or `list(t)`, etc.
- To modify a Tree instance `t`, you can:
  - Change its label: `t.label = ...`
  - Change its branches: `t.branches = ...` or `t.branches.append(...)`
  - Modify one of its branches: `t.branches[0].label = ...`

Students received 49% of the points on average.

26% of students answered the question correctly.

## Fall 2017 CS 61A Midterm 2 Q5(a)

**Definition.** A pile (of leaves) for a tree  $t$  with no repeated leaf labels is a dictionary in which the label for each leaf of  $t$  is a key, and its value is the path from that leaf to the root. Each path from a node to the root is either an empty tuple, if the node is the root, or a two-element tuple containing the label of the node's parent and the rest of the path (i.e., the path to the root from the node's parent).

```
def pile(t):
    """Return a dict that contains every path from a leaf to the root of tree t.

>>> pile(Tree(5, [Tree(3, [Tree(1), Tree(2)]), Tree(6, [Tree(7)])]))
{1: (3, (5, ())), 2: (3, (5, ())), 7: (6, (5, ()))}
"""
p = {}
def gather(u, path):
    if u.is_leaf():
        p[u.label] = path
    for b in u.branches:
        gather(b, (u.label, path))
gather(t, ())
return p
```

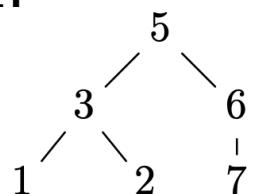
*Start at the top*

*all paths have ()*

*Base case:  
Put a leaf label in p*

*Recursive call:  
Build a longer path*

*u has a label that can be added to the path*



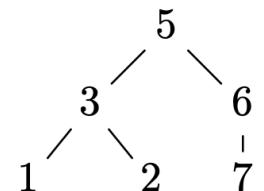
Students received 53% of the points on average.

24% of students answered the question correctly.

## Fall 2017 CS 61A Midterm 2 Q5(b)

Implement Path, a class whose `__init__` method takes a Tree `t` and a `leaf_label`. Assume all leaf labels of `t` are unique. When a Path is printed, labels in the path from the root to the leaf of `t` with label `leaf_label` are displayed, separated by dashes.

```
class Path:  
    """A path through a tree from the root to a leaf, identified by its leaf label.  
  
>>> a = Tree(5, [Tree(3, [Tree(1), Tree(2)]), Tree(6, [Tree(7)])])  
>>> print(Path(a, 7), Path(a, 2))  
5-6-7 5-3-2  
"""  
def __init__(self, t, leaf_label):  
    self.pile, self.end = pile(t), leaf_label  
    {...., 7: (6, (5, ())), 5: (), ()}  
    Build s  
    from this  
def __str__(self):  
    path, s = _____, _____  
    path is  
    a nested  
    tuple  
    while path:  
        path, s = _____, _____  
    return s  
    s is a string
```

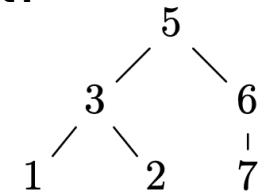


## Fall 2017 CS 61A Midterm 2 Q5(a) Revisited

**Definition.** A pile (of leaves) for a tree  $t$  with no repeated leaf labels is a dictionary in which the label for each leaf of  $t$  is a key, and its value is the path from that leaf to the root. Each path from a node to the root is either an empty tuple, if the node is the root, or a two-element tuple containing the label of the node's parent and the rest of the path (i.e., the path to the root from the node's parent). **Represent the path as a list of labels.**

```
def pile(t):
    """Return a dict that contains every path from a leaf to the root of tree t.

>>> pile(Tree(5, [Tree(3, [Tree(1), Tree(2)]), Tree(6, [Tree(7)])]))
{1: [5, 3, 1], 2: [5, 3, 2], 7: [5, 6, 7]}
"""
p = {}
def gather(u, path):
    if u.is_leaf():
        p[u.label] = path
    for b in u.branches:
        gather(b, (u.label, path))
    path + [u.label]
gather(t, ())
return p
```



OR

```
p[u.label] = path
gather(b, path + [b.label])
gather(t, [t.label])
```

# Recursion

## From Discussion 4 (With Some Extra Tips)

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Don't start trying to write code right away. Instead, start by describing the recursive case in words. Some examples:

- In `fib` from lecture, the recursive case is to add together the previous two Fibonacci numbers.
- In `count_partitions` from lecture, the recursive case is to partition `n-m` using parts up to size `m` and to partition `n` using parts up to size `m-1`.

### How to get the recursive description right?

**Use abstraction:** Pick an example, then figure out what a recursive call will do for you on that example, not by reading the code, but by reading the docstring.

**Implement a choice:** Most tree recursion problems involve making a sequence of choices (e.g., use a partition of size `m` or don't). The recursive case implements one of those choices; recursion implements the rest.

### How to get the base cases right?

Once you know what the recursive case is, find all the simple cases it leads to.

Students received 71% of the points on average.

41% of students answered the question correctly.

## Fall 2017 CS 61A Midterm 2 Q4(c)

Implement `ways`, which takes two values `start` and `end`, a non-negative integer `k`, and a list of one-argument functions `actions`. It returns the number of ways of choosing functions `f1, f2, ..., fj` from `actions` such that `f1(f2(...(fj(start))))` equals `end` and  $j \leq k$ . The same action function can be chosen multiple times. If a sequence of actions reaches `end`, then no further actions can be applied (see first example below).

```
def ways(start, end, k, actions):
    """Return the number of ways of reaching end from start by taking up to k actions.

    >>> ways(-1, 1, 5, [abs, lambda x: x+2])    # abs(-1) or -1+2, but not abs(abs(-1))
    2
    >>> ways(1, 10, 5, [lambda x: x+1, lambda x: x+4])  # 1+1+4+4, 1+4+4+1, or 1+4+1+4
    3
    >>> ways(1, 20, 5, [lambda x: x+1, lambda x: x+4])
    0
    >>> ways([3], [2, 3, 2, 3], 4, [lambda x: [2]+x, lambda x: 2*x, lambda x: x[:-1]])
    3
    .....
    if _____:
        start == end
        :
        return 1
    elif _____:
        k == 0
        :
        return 0
    return _____([_____ways(f(start), end, k - 1, actions) _____ for f in actions])
```

*# of ways starting with that action*

*Choose an action*

Students received 39% of the points on average.

4% of students answered the question correctly.

## Fall 2016 CS 61A Midterm 2 Q7(a)

Implement `sums`, which takes two positive integers  $n$  and  $k$ . It returns a list of lists containing all the ways that a list of  $k$  positive integers can sum to  $n$  (in any order).

```
def sums(n, k):
    """Return the ways in which K positive integers can sum to N.

>>> sums(2, 2)
[[1, 1]]
>>> sums(2, 3)
[]
>>> sums(4, 2)
[[3, 1], [2, 2], [1, 3]]
>>> sums(5, 3)
[[3, 1, 1], [2, 2, 1], [1, 3, 1], [2, 1, 2], [1, 2, 2], [1, 1, 3]]
"""
if _____:
    return _____
y = []
for x in _____:
    _____
    y.extend([_____ for s in sums(_____, _____)])
return y
```

*Build the list*   *Choose a way*   *Ways of completing the list*

*Choose the first number*

Students received 17% of the points on average.

1% of students answered the question correctly.

## Fall 2016 CS 61A Midterm 2 Q7(b)

**Note: Nowadays, this question would have been labeled an A+ question and worth 0 points.**

Implement `sums`, which takes two positive integers  $n$  and  $k$ . It returns a list of lists containing all the ways that a list of  $k$  positive integers can sum to  $n$  (in any order).

*x: a max number  
y: a list of lists of numbers  
Put each number up to x at the front of each list in y*

```
f = lambda x, y: (x and [_____ for z in y] + f(_____, _____)) or []
```

```
def sums(n, k):  
    """Return the ways in which K positive integers can sum to N."""
```

```
g = lambda w: (w and f(_____, _____)) or [[]]
```

```
return [v for v in g(k) if sum(v) == n]
```

*Lists of k positive integers*