

## Midterm Examples

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## Announcements

Trees

## From Discussion 5 (Updated to be about the Tree class)

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For a Tree instance `t`:

- Its root label can be any value, and `t.label` evaluates to it.
- Its branches are trees, and `t.branches` evaluates to a list of branches.
- It is a leaf if it has no branches, and `t.is_leaf()` returns `True`.
- An identical tree can be constructed with `Tree(t.label, t.branches)`.
- You can call functions that take trees as arguments, such as `height(t)`.
- That's how you work with trees. No `t == x` or `t[0]` or `x in t` or `list(t)`, etc.
- To modify a Tree instance `t`, you can:
  - Change its label: `t.label = ...`
  - Change its branches: `t.branches = ...` or `t.branches.append(...)`
  - Modify one of its branches: `t.branches[0].label = ...`

Students received 49% of the points on average.

26% of students answered the question correctly.

## Fall 2017 CS 61A Midterm 2 Q5(a)

**Definition.** A pile (of leaves) for a tree  $t$  with no repeated leaf labels is a dictionary in which the label for each leaf of  $t$  is a key, and its value is the path from that leaf to the root. Each path from a node to the root is either an empty tuple, if the node is the root, or a two-element tuple containing the label of the node's parent and the rest of the path (i.e., the path to the root from the node's parent).

```
def pile(t):  
    """Return a dict that contains every path from a leaf to the root of tree t.
```

```
>>> pile(Tree(5, [Tree(3, [Tree(1), Tree(2)]), Tree(6, [Tree(7)])]))  
{1: (3, (5, ())), 2: (3, (5, ())), 7: (6, (5, ()))}
```

```
    p = {}  
    def gather(u, path):  
        if u.is_leaf():
```

```
            p[u.label] = path
```

```
        for b in u.branches:
```

```
            gather(b, (u.label, path))
```

```
    gather(t, ())
```

```
    return p
```

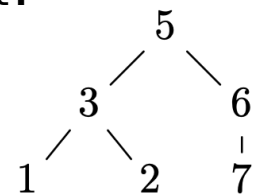
*all paths  
have ()*

*u has a label that can  
be added to the path*

*Base case:  
Put a leaf label in p*

*Recursive call:  
Build a longer path*

*Start at  
the top*



Students received 53% of the points on average.  
 24% of students answered the question correctly.

## Fall 2017 CS 61A Midterm 2 Q5(b)

Implement Path, a class whose `__init__` method takes a Tree `t` and a `leaf_label`. Assume all leaf labels of `t` are unique. When a Path is printed, labels in the path from the root to the leaf of `t` with label `leaf_label` are displayed, separated by dashes.

class Path:

"""A path through a tree from the root to a leaf, identified by its leaf label.

```
>>> a = Tree(5, [Tree(3, [Tree(1), Tree(2)]), Tree(6, [Tree(7)])])
```

```
>>> print(Path(a, 7), Path(a, 2))
```

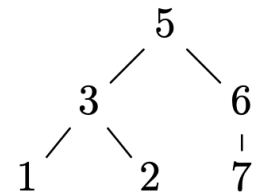
```
5-6-7 5-3-2
```

```
"""
```

```
def __init__(self, t, leaf_label):
    self.pile, self.end = pile(t), leaf_label
```

```
{..., 7: (6, (5, ()))}
(5, ())
()
```

Build *s*  
from this



```
def __str__(self):
```

path is  
a nested  
tuple

```
path, s = self.pile[self.end], str(self.end)
```

```
while path:
```

```
    path, s = path[1], str(path[0]) + '-' + s
```

```
    return s
```

*s is a string*

## Fall 2017 CS 61A Midterm 2 Q5(a) Revisited

**Definition.** A pile (of leaves) for a tree  $t$  with no repeated leaf labels is a dictionary in which the label for each leaf of  $t$  is a key, and its value is the path from that leaf to the root. Each path from a node to the root is either an empty tuple, if the node is the root, or a two-element tuple containing the label of the node's parent and the rest of the path (i.e., the path to the root from the node's parent). *Represent the path as a list of labels.*

```
def pile(t):  
    """Return a dict that contains every path from a leaf to the root of tree t.
```

```
>>> pile(Tree(5, [Tree(3, [Tree(1), Tree(2)]), Tree(6, [Tree(7)])]))  
{1: [5, 3, 1], 2: [5, 3, 2], 7: [5, 6, 7]}  
"""
```

```
p = {}
```

```
def gather(u, path):  
    if u.is_leaf():
```

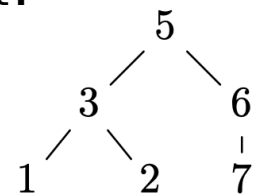
```
        p[u.label] = path + [u.label]
```

```
    for b in u.branches:
```

```
        gather(b, (u.label, path) path + [u.label])
```

```
    gather(t, (()) [])
```

```
    return p
```



```
p[u.label] = path
```

OR

```
gather(b, path + [b.label])
```

```
gather(t, [t.label])
```

# Recursion



## From Discussion 4 (With Some Extra Tips)

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Don't start trying to write code right away. Instead, start by describing the recursive case in words. Some examples:

- In `fib` from lecture, the recursive case is to add together the previous two Fibonacci numbers.
- In `count_partitions` from lecture, the recursive case is to partition `n-m` using parts up to size `m` and to partition `n` using parts up to size `m-1`.

### How to get the recursive description right?

**Use abstraction:** Pick an example, then figure out what a recursive call will do for you on that example, not by reading the code, but by reading the docstring.

**Implement a choice:** Most tree recursion problems involve making a sequence of choices (e.g., use a partition of size `m` or don't). The recursive case implements one of those choices; recursion implements the rest.

### How to get the base cases right?

Once you know what the recursive case is, find all the simple cases it leads to.

Students received 71% of the points on average.

41% of students answered the question correctly.

## Fall 2017 CS 61A Midterm 2 Q4(c)

Implement **ways**, which takes two values **start** and **end**, a non-negative integer **k**, and a list of one-argument functions **actions**. It returns the number of ways of choosing functions **f1, f2, ..., fj** from **actions** such that **f1(f2(...(fj(start))))** equals **end** and **j ≤ k**. The same action function can be chosen multiple times. If a sequence of actions reaches end, then no further actions can be applied (see first example below).

```
def ways(start, end, k, actions):
```

```
    """Return the number of ways of reaching end from start by taking up to k actions.
```

```
>>> ways(-1, 1, 5, [abs, lambda x: x+2])    # abs(-1) or -1+2, but not abs(abs(-1))
```

```
2
```

```
>>> ways(1, 10, 5, [lambda x: x+1, lambda x: x+4])    # 1+1+4+4, 1+4+4+1, or 1+4+1+4
```

```
3
```

```
>>> ways(1, 20, 5, [lambda x: x+1, lambda x: x+4])
```

```
0
```

```
>>> ways([3], [2, 3, 2, 3], 4, [lambda x: [2]+x, lambda x: 2*x, lambda x: x[:-1]])
```

```
3
```

```
"""
```

```
    if start == end:
```

```
        return 1
```

```
    elif k == 0:
```

```
        return 0
```

*# of ways starting with that action*

*Choose an action*

```
    return sum ([ ways(f(start), end, k - 1, actions) for f in actions])
```

4% of students answered the question correctly.

Implement `sums`, which takes two positive integers `n` and `k`. It returns a list of lists containing all the ways that a list of `k` positive integers can sum to `n` (in any order).

"""Return the ways in which K positive integers can sum to N.

[[1, 1]]

[ ]

$[[3, 1], [2, 2], [1, 3]]$

$$[[3, 1, 1], [2, 2, 1], [1, 3, 1], [2, 1, 2], [1, 2, 2], [1, 1, 3]]$$

■■■■■

\_\_\_\_\_

[illegible]

Figure 1. The effect of the number of trials on the number of correct responses. The number of correct responses was plotted against the number of trials for each condition. The error bars represent the standard error of the mean.

```
y.extend([s + [x] for s in sums(n - x, k - 1)])
```

```
return y
```

## Ways of completing the list

## Fall 2016 CS 61A Midterm 2 Q7(b)

Students received 17% of the points on average.  
1% of students answered the question correctly.

**Note: Nowadays, this question would have been labeled an A+ question and worth 0 points.**

Implement `sums`, which takes two positive integers `n` and `k`. It returns a list of lists containing all the ways that a list of `k` positive integers can sum to `n` (in any order).

*x: a max number  
y: a list of lists of numbers  
Put each number up to x at the front of each list in y*

```
f = lambda x, y: (x and [ [x] + z for z in y] + f(x - 1, y)) or []
```

```
def sums(n, k):  
    """Return the ways in which K positive integers can sum to N."""
```

```
    g = lambda w: (w and f(n, g(w-1))) or [[]]
```

```
    return [v for v in g(k) if sum(v) == n]
```

*Lists of k positive integers*