8. $A = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}, B = \begin{bmatrix} -2 & 1 \\ 3 & 4 \end{bmatrix}, C = \begin{bmatrix} 0 & -3 \\ 2 & 1 \end{bmatrix}$

a) $(A+B)+C = A+(B+C)$ (associatividade)

$(A+B) = \begin{bmatrix} -1 & 3 \\ 3 & 3 \end{bmatrix} + C = \begin{bmatrix} 0 & -3 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 5 & 4 \end{bmatrix}$ $(B+C) = \begin{bmatrix} -2 & -2 \\ 5 & 5 \end{bmatrix} + A = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 5 & 4 \end{bmatrix}$

b) $(A-B)-C = A-(B+C)$ (distributividade do sinal negativo)

$(A-B) = \begin{bmatrix} 3 & 1 \\ -3 & -5 \end{bmatrix} - C = \begin{bmatrix} 0 & -3 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ -5 & -6 \end{bmatrix}$ $A = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix} - (B+C) = \begin{bmatrix} -2 & -2 \\ 5 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ -5 & -6 \end{bmatrix}$

9. $N_1 = \begin{bmatrix} 7 & 8 & 6 \end{bmatrix}, N_2 = \begin{bmatrix} 6 & 9 & 5 \end{bmatrix}$ $N_2 - N_1 = \begin{bmatrix} -1 & 1 & -1 \end{bmatrix}$

10. a) $A \cdot B$ e $B \cdot A$ $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$ $AB = \begin{bmatrix} 1\cdot5+2\cdot7 & 1\cdot6+2\cdot8 \\ 3\cdot5+4\cdot7 & 3\cdot6+4\cdot8 \end{bmatrix} = \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}$

$BA = \begin{bmatrix} 5\cdot1+6\cdot3 & 5\cdot2+6\cdot4 \\ 7\cdot1+8\cdot3 & 7\cdot2+8\cdot4 \end{bmatrix} = \begin{bmatrix} 23 & 34 \\ 31 & 46 \end{bmatrix}$

b) CD $C = \begin{bmatrix} 2 & -1 & 0 \\ 1 & 3 & 4 \end{bmatrix}_{2\times3}, D = \begin{bmatrix} 1 & 2 \\ 0 & -3 \\ 4 & 5 \end{bmatrix}_{3\times2}$ $CD = \begin{bmatrix} 2\cdot1+(-1)\cdot0+0\cdot4 & 2\cdot2+(-1)(-3)+0\cdot5 \\ 1\cdot1+3\cdot0+4\cdot4 & 1\cdot2+3\cdot(-3)+4\cdot5 \end{bmatrix}$

$CD = \begin{bmatrix} 2 & 7 \\ 17 & 13 \end{bmatrix}$

c) EF e FE $E = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}_{1\times3}, F = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}_{3\times1}$ $EF = \begin{bmatrix} 1\cdot4+2\cdot5+3\cdot6 \end{bmatrix} = \begin{bmatrix} 32 \end{bmatrix}_{1\times1}$

$FE = \begin{bmatrix} 4\cdot1 & 4\cdot2 & [illegible] \\ 5\cdot1 & 5\cdot2 & 5\cdot3 \\ 6\cdot1 & 6\cdot2 & 6\cdot3 \end{bmatrix} \begin{bmatrix} 4 & 8 & 12 \\ 5 & 10 & 15 \\ 6 & 12 & 18 \end{bmatrix}$

11 a) $AB \neq BA$ $A = \begin{bmatrix} 1 & [illegible] \\ 3 & [illegible] \end{bmatrix}$ [illegible] $AB = \begin{bmatrix} 1\cdot2+2\cdot3 & 1\cdot3+2\cdot2 \\ 3\cdot2+4\cdot3 & 3\cdot3+4\cdot2 \end{bmatrix} = \begin{bmatrix} 8 & 7 \\ 18 & 17 \end{bmatrix}$

$BA = \begin{bmatrix} 2\cdot1+3\cdot3 & 2\cdot2+3\cdot4 \\ 3\cdot1+2\cdot3 & 3\cdot2+2\cdot4 \end{bmatrix} \begin{bmatrix} 11 & 16 \\ 9 & 14 \end{bmatrix}$ $\therefore AB \neq BA$

b) $\cancel{[illegible]}$ $(AB)C = A(BC)$ $AB = \begin{bmatrix} 8 & 7 \\ 18 & 17 \end{bmatrix} \cdot C = \begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 8\cdot2+7\cdot1 & 8\cdot2+7\cdot1 \\ 18\cdot2+17\cdot1 & 18\cdot2+17\cdot1 \end{bmatrix} = \begin{bmatrix} 23 & 23 \\ 53 & 53 \end{bmatrix}$

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot (BC) = \begin{bmatrix} 7 & 7 \\ 8 & 8 \end{bmatrix} = \begin{bmatrix} 23 & 23 \\ 53 & 53 \end{bmatrix}$$

c) $A(B+C) = AB + AC$

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot (B+C) = \begin{bmatrix} 4 & 5 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} 12 & 11 \\ 28 & 27 \end{bmatrix} \qquad AB = \begin{bmatrix} 8 & 7 \\ 18 & 17 \end{bmatrix} + AC = \begin{bmatrix} 4 & 4 \\ 10 & 10 \end{bmatrix} = \begin{bmatrix} 12 & 11 \\ 28 & 27 \end{bmatrix}$$

II $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ a) AI , IA $\quad A = \begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix} \quad AI = \begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix} \quad IA = \begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix}$

b) $AI = IA$

3 $O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ $\quad AO = OA = O \quad \forall A[a_{ij}]_{m\times m} \quad A = [a_{ij}]_{m\times m} = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix}_{m\times n}$

$$\begin{bmatrix} a_{11} & \cdots & a_{1m} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{mn} \end{bmatrix} \cdot \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{bmatrix} = C = [c_{ij}]_{n\times n} \;\Big|\; c_{ij} = \sum_{k=1}^{n} a_{ik} \cdot 0$$

$\therefore A \cdot O = O$

3. $\left[\begin{array}{cc|c} 2 & 1 & 5 \\ 3 & -2 & 4 \end{array}\right]$ a) $L_1 \leftrightarrow L_2$ $\left[\begin{array}{cc|c} 3 & -2 & 4 \\ 2 & 1 & 5 \end{array}\right]$ b) $L_2 \leftarrow L_2 \cdot (-1)$ $\left[\begin{array}{cc|c} 2 & 1 & 5 \\ -3 & 2 & -4 \end{array}\right]$

c) $L_2 \leftarrow 2L_1 + L_2$ $\left[\begin{array}{cc|c} 2 & 1 & 5 \\ 7 & 0 & 14 \end{array}\right]$

4. $\left[\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 2 & 4 & -1 & 3 \\ -1 & -2 & 2 & -1 \end{array}\right]$ $\begin{matrix} L_2 \leftarrow -2L_1 + L_2 \\ L_3 \leftarrow L_1 + L_3 \end{matrix}$ $\left[\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 0 & 0 & -3 & -5 \\ 0 & 0 & 3 & 3 \end{array}\right]$ $L_3 \leftarrow L_2 + L_3$ $\left[\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 0 & 0 & -3 & -5 \\ 0 & 0 & 0 & -2 \end{array}\right]$

Sistema impossível.

5. $\left[\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 2 & 4 & -1 & 3 \\ -1 & -2 & 2 & -1 \end{array}\right]$ $\begin{matrix} L_2 \leftarrow -2L_1 + L_2 \\ L_3 \leftarrow L_1 + L_3 \end{matrix}$ $\left[\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 0 & 0 & -3 & -5 \\ 0 & 0 & 3 & 3 \end{array}\right]$ $L_2 \leftarrow L_2 \cdot \left(-\frac{1}{3}\right)$ $\left[\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 0 & 0 & 1 & 5/3 \\ 0 & 0 & 3 & 3 \end{array}\right]$

$L_1 \leftarrow L_1 - L_2$
$L_3 \leftarrow -3L_2 + L_3$

$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 7/3 \\ 0 & 0 & 1 & 5/3 \\ 0 & 0 & 0 & -2 \end{array}\right]$ $0x + 0y + 0z = -2$

Sistema impossível.

6. a) $\begin{cases} x + y + z = 6 \\ 2x - y + z = 3 \\ x + 2y - z = 4 \end{cases}$ $\left[\begin{array}{ccc|c} 1 & 1 & 2 & 6 \\ 2 & -1 & 1 & 3 \\ 1 & 2 & -1 & 4 \end{array}\right]$ $\begin{matrix} L_2 = -2L_1 + L_2 \\ L_3 = -L_1 + L_3 \end{matrix}$ $\left[\begin{array}{ccc|c} 1 & 1 & 2 & 6 \\ 0 & -3 & -3 & -9 \\ 0 & 1 & -3 & -2 \end{array}\right]$ $L_2 \leftarrow -\frac{1}{3} L_2$

$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 6 \\ 0 & 1 & 1 & 3 \\ 0 & 1 & -3 & -2 \end{array}\right]$ $\begin{matrix} L_1 \leftarrow L_1 - L_2 \\ L_3 \leftarrow L_3 - L_2 \end{matrix}$ $\left[\begin{array}{ccc|c} 1 & 0 & 1 & 3 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & -2 & -5 \end{array}\right]$ $L_3 \leftarrow L_3 \cdot \left(-\frac{1}{2}\right)$ $\left[\begin{array}{ccc|c} 1 & 0 & 1 & 3 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 1 & 5/2 \end{array}\right]$

$\begin{matrix} L_2 \leftarrow L_2 - L_3 \\ L_1 \leftarrow L_1 - L_3 \end{matrix}$ $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1/2 \\ 0 & 1 & 0 & 1/2 \\ 0 & 0 & 1 & 5/2 \end{array}\right]$ $\begin{cases} x = 1/2 \\ y = 1/2 \\ z = 5/2 \end{cases}$

b) $\begin{cases} x + y + 2z = 8 \\ -x - 2y + 3z = 1 \\ 3x - 7y + 4z = 10 \end{cases}$ $\left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ -1 & -2 & 3 & 1 \\ 3 & -7 & 4 & 10 \end{array}\right]$ $\begin{matrix} L_2 \leftarrow L_2 + L_1 \\ L_3 \leftarrow L_3 - 3L_1 \end{matrix}$ $\left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ 0 & -1 & 5 & 9 \\ 0 & -10 & -2 & -14 \end{array}\right]$ $L_2 \leftarrow L_2(-1)$

$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ 0 & 1 & -5 & -9 \\ 0 & -10 & -2 & -14 \end{array}\right]$ $\begin{matrix} L_1 \leftarrow L_1 - L_2 \\ L_3 \leftarrow L_3 + 10L_2 \end{matrix}$ $\left[\begin{array}{ccc|c} 1 & 0 & 7 & 17 \\ 0 & 1 & -5 & -9 \\ 0 & 0 & -52 & -104 \end{array}\right]$ $L_3 \leftarrow L_3 - \left(\frac{1}{52}\right)$ $\left[\begin{array}{ccc|c} 1 & 0 & 7 & 17 \\ 0 & 1 & -5 & -9 \\ 0 & 0 & 1 & 2 \end{array}\right]$

$L_1 \leftarrow L_1 - 7L_3$
$L_2 \leftarrow L_2 + 5L_3$

$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right]$ $\begin{cases} x = 3 \\ y = 1 \\ z = 2 \end{cases}$

c) $\begin{cases} 2x+2y+2z=0 \\ -2x+5y+2z=1 \\ 8x+y+4z=-1 \end{cases}$ $\left[\begin{array}{ccc|c} 2 & 2 & 2 & 0 \\ -2 & 5 & 2 & 1 \\ 8 & 1 & 4 & -1 \end{array}\right]$ $L_1 \leftarrow L_1 \cdot \frac{1}{2}$ $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ -2 & 5 & 2 & 1 \\ 8 & 1 & 4 & -1 \end{array}\right]$ $\begin{array}{l} L_2 \leftarrow L_2 + 2L_1 \\ L_3 \leftarrow L_3 - 8L_1 \end{array}$

$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 7 & 4 & 1 \\ 0 & -7 & -4 & -1 \end{array}\right]$ $L_2 \leftarrow L_2 \cdot \frac{1}{7}$ $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 4/7 & 1/7 \\ 0 & -7 & -4 & -1 \end{array}\right]$ $\begin{array}{l} L_1 \leftarrow L_1 - L_2 \\ L_3 \leftarrow L_3 + 7L_2 \end{array}$ $\left[\begin{array}{ccc|c} 1 & 0 & 3/7 & -1/7 \\ 0 & 1 & 4/7 & 1/7 \\ 0 & 0 & 0 & 0 \end{array}\right]$

Infinitas soluções

d) $\left[\begin{array}{ccc|c} 0 & -2 & 3 & 1 \\ 3 & 6 & -3 & -2 \\ 6 & 8 & 3 & 5 \end{array}\right]$ $L_1 \leftrightarrow L_2$ $\left[\begin{array}{ccc|c} 3 & 6 & -3 & -2 \\ 0 & -2 & 3 & 1 \\ 6 & 8 & 3 & 5 \end{array}\right]$ $L_3 \leftarrow L_3 - 2L_1$ $\left[\begin{array}{ccc|c} 3 & 6 & -3 & -2 \\ 0 & -2 & 3 & 1 \\ 0 & -6 & 9 & 9 \end{array}\right]$

$L_2 \leftarrow -\frac{1}{2} L_2$

$\left[\begin{array}{ccc|c} 3 & 6 & -3 & -2 \\ 0 & 1 & -3/2 & -1/2 \\ 0 & -6 & 9 & 9 \end{array}\right]$ $\begin{array}{l} L_1 \leftarrow L_1 - 6L_2 \\ L_3 \leftarrow L_3 + 6L_2 \end{array}$ $\left[\begin{array}{ccc|c} 3 & 0 & 6 & 1 \\ 0 & 1 & -3/2 & -1/2 \\ 0 & 0 & 0 & 6 \end{array}\right]$

$0x+0y+0z=6$

Sistema impossível.

e) $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 2 & 3 & 1 & 5 \\ 3 & 4 & 2 & 7 \end{array}\right]$ $\begin{array}{l} L_2 \leftarrow L_2 - 2L_1 \\ L_3 \leftarrow L_3 - 3L_1 \end{array}$ $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & -1 & 1 \\ 0 & 1 & -1 & 1 \end{array}\right]$ $\begin{array}{l} L_1 \leftarrow L_1 - L_2 \\ L_3 \leftarrow L_3 - L_2 \end{array}$ $\left[\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$

Há infinitas soluções.

7. $\begin{cases} 2x+y+3z=1900 \\ 1x+3y+5z=2400 \\ 3x+2y+4z=2900 \end{cases}$ $\left[\begin{array}{ccc|c} 2 & 1 & 3 & 1900 \\ 1 & 3 & 5 & 2400 \\ 3 & 2 & 4 & 2900 \end{array}\right]$ $L_1 \leftrightarrow L_2$ $\left[\begin{array}{ccc|c} 1 & 3 & 5 & 2400 \\ 2 & 1 & 3 & 1900 \\ 3 & 2 & 4 & 2900 \end{array}\right]$

$\begin{array}{l} L_2 \leftarrow L_2 - 2L_1 \\ L_3 \leftarrow L_3 - 3L_1 \end{array}$

$\left[\begin{array}{ccc|c} 1 & 3 & 5 & 2400 \\ 0 & -5 & -7 & -2900 \\ 0 & -7 & -11 & -4300 \end{array}\right]$ $L_2 \leftarrow L_2 \cdot \left(-\frac{1}{5}\right)$ $\left[\begin{array}{ccc|c} 1 & 3 & 5 & 2400 \\ 0 & 1 & 7/5 & 580 \\ 0 & -7 & -11 & -4300 \end{array}\right]$

$\begin{array}{l} L_1 \leftarrow L_1 - 3L_2 \\ L_3 \leftarrow L_3 + 7L_2 \end{array}$

$\left[\begin{array}{ccc|c} 1 & 0 & 4/5 & 660 \\ 0 & 1 & 7/5 & 580 \\ 0 & 0 & -6/5 & -240 \end{array}\right]$ $L_3 \leftarrow L_3 \cdot \left(-\frac{6}{5}\right)$ $\left[\begin{array}{ccc|c} 1 & 0 & 4/5 & 660 \\ 0 & 1 & 7/5 & 580 \\ 0 & 0 & 1 & 200 \end{array}\right]$ $\begin{array}{l} L_1 \leftarrow L_1 - \frac{4}{5} L_3 \\ L_2 \leftarrow L_2 - \frac{7}{5} L_3 \end{array}$

$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 500 \\ 0 & 1 & 0 & 300 \\ 0 & 0 & 1 & 200 \end{array}\right]$ $\begin{cases} x=500 \\ y=300 \\ z=200 \end{cases}$

verdadeiro

2\. 11. (V) $(A+B)_{ij} = a_{ij} + b_{ij} = b_{ij} + a_{ij} = (B+A)_{ij}$

(F) $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$ $AB = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1(1)+1(1) & 1 \cdot 0+1 \cdot 1 \\ 0 \cdot 1+1(1) & 0 \cdot 0+1 \cdot 1 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$

$BA = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 \cdot 1+0 \cdot 0 & 1 \cdot 1+0 \cdot 1 \\ 1 \cdot 1+1 \cdot 0 & 1 \cdot 1+1 \cdot 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$ $\boxed{AB \neq BA}$

(V) $(A+0)_{ij} = a_{ij} + 0 = a_{ij}$ matriz nula = elemento neutro da adição.

(V) A matriz identidade é o elemento neutro da multiplicação $(AI_n)_{ij}$

$\sum_{k=1}^{n} a_{ik}(I_n)_{kj} = a_{ij}(I_n)_{jj} = a_{ij}(1) = a_{ij}$

(V) $(A^t)^t = A$

(V) $(A+A)^t = (2A)^t = 2A^t$ $A^t + A^t = 2A^t$ $(A+A)^t = A^t + A^t$

(V) $I_n^t = I_n \cdot I_n = \boxed{I_n}$

(V) $(A+A)^t = \boxed{A^t + A^t}$

(F) $A = \begin{pmatrix} 1 & 2 \\ 0 & 0 \end{pmatrix}$ $\boxed{\det(A) = 0 - 0 = 0}$ Não tem inversa.

(V) $(I_n + A^2)(I_n - 2A^2) = I_n \cdot I_n - 2I_n A^2 + A^2 I_n - 2A^2A^2$

$= I_n - 2A^2 - 2A^4$

$A^2 = -2A^4$

$= I_n - A^2 - 2A^2$

$= I_n - A^2 + A^2 = \boxed{I_n}$

(V) $A = P^t DP$ $A^t = (P^t DP)^t = P^t D^t P = P^t DP \Rightarrow A = A^t$

(F) $D = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}, A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ $DA = \begin{pmatrix} 2 & 2 \\ 3 & 3 \end{pmatrix}$ $AD = \begin{pmatrix} 2 & 3 \\ 2 & 3 \end{pmatrix}$ $\boxed{DA \neq AD}$

(V) $B = A \cdot A^t$ $B^t = (AA^t)^t = (A^t)^t \cdot A^t = A \cdot A^t \Rightarrow B^t = B$

(F) $A = A^t$, $B = B^t$ $C^t = (AB)^t = B^t \cdot A^t = BA$ $C^t = C$ para $BA = AB$

$A = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}, B = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$ $AB = \begin{pmatrix} 1 & 4 \\ 2 & 6 \end{pmatrix}$ $BA = \begin{pmatrix} 1 & 2 \\ 4 & 6 \end{pmatrix}$ $\boxed{AB \neq BA}$

3.4.1.

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 4 \\ 2 & 3 & 5 \end{bmatrix}, \quad x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 1000 \\ 2000 \\ 2500 \end{bmatrix} \quad x = A^{-1} B$$

$$\det(A) = 1 \cdot \begin{pmatrix} 1 & 4 \\ 3 & 5 \end{pmatrix} - 1 \cdot \begin{pmatrix} 2 & 4 \\ 2 & 5 \end{pmatrix} + 1 \cdot \begin{pmatrix} 2 & 1 \\ 2 & 3 \end{pmatrix} = -7 - 2 + 4 = -5$$

$$A^{-1} = \frac{1}{\det(A)} \quad A^{-1} = \begin{bmatrix} 7/5 & 1/5 & -3/5 \\ 2/5 & -3/5 & 2/5 \\ -4/5 & 1/5 & 1/5 \end{bmatrix} \quad X = A^{-1} \cdot B$$

$$x = \frac{7}{5} \cdot 1000 + \frac{1}{5} \cdot 2000 + \frac{3}{5} \cdot 2500 = 1400 + 800 - 1500 = 700$$

$$y = \frac{2}{5} \cdot 1000 - \frac{3}{5} \cdot 2000 + \frac{2}{5} \cdot 2500 = 400 - 1200 + 1000 = 200 \qquad X = \begin{bmatrix} 700 \\ 200 \\ 100 \end{bmatrix}$$

$$z = -\frac{4}{5} \cdot 1000 + \frac{1}{5} \cdot 2000 + \frac{1}{5} \cdot 2500 = -800 + 400 + 500 = 100$$

2. a) $\begin{bmatrix} 1 & 2 & 2 \\ 1 & 3 & 1 \\ 1 & 3 & 2 \end{bmatrix}$ $\det(A) = 1 \cdot \begin{pmatrix} 3 & 1 \\ 3 & 2 \end{pmatrix} - 2 \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} + 2 \begin{pmatrix} 1 & 3 \\ 1 & 3 \end{pmatrix} = 1 \cdot 3 - 2 \cdot 1 + 2 \cdot 0 = 1$

$C_{11} = \begin{pmatrix} 3 & 1 \\ 3 & 2 \end{pmatrix} = 3$ $C_{12} = -\begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} = -1$ $C_{13} = \begin{pmatrix} 1 & 3 \\ 1 & 3 \end{pmatrix} = 0$ $C_{21} = -\begin{pmatrix} 2 & 2 \\ 3 & 2 \end{pmatrix} = 2$

$C_{22} = \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix} = 0$ $C_{23} = -\begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix} = -1$ $C_{31} = \begin{pmatrix} 2 & 2 \\ 3 & 1 \end{pmatrix} = -4$ $C_{32} = -\begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix} = 1$

$C_{33} = \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix} = 1$ $\quad \operatorname{cof}(A) = \begin{bmatrix} 3 & 2 & -4 \\ -1 & 0 & 1 \\ 0 & -1 & 1 \end{bmatrix} \cdot 1 = \begin{bmatrix} 3 & 2 & -4 \\ -1 & 0 & 1 \\ 0 & -1 & 1 \end{bmatrix}$

d) $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 1 \end{bmatrix}$ $\det(A) = 1 \cdot \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix} - 2 \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} + 3 \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = -1 - 2 + 3 = 0$

Não é invertível.

e) $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & -1 & 2 \\ 1 & -1 & 2 & 1 \\ 1 & 3 & 3 & 2 \end{bmatrix}$ $\det(A) = 1 \cdot \begin{pmatrix} 2 & -1 & 2 \\ -1 & 2 & 1 \\ 3 & 3 & 2 \end{pmatrix} - 1 \begin{pmatrix} 1 & -1 & 2 \\ 1 & 2 & 1 \\ 1 & 3 & 2 \end{pmatrix} + 1 \begin{pmatrix} 1 & 2 & 2 \\ 1 & -1 & 1 \\ 1 & 3 & 2 \end{pmatrix} - 1 \begin{pmatrix} 1 & 2 & -1 \\ 1 & -1 & 2 \\ 1 & 3 & 3 \end{pmatrix}$

$L_4 = L_1 + L_2 + L_3$ logo A não é invertível.

f)

b) $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 3 & 1 & 2 \\ 1 & 2 & -1 & 1 \\ 5 & 9 & 1 & 6 \end{bmatrix}$ $\det(A) = 1 \cdot \underbrace{\begin{pmatrix} 3 & 1 & 2 \\ 2 & -1 & 1 \\ 9 & 1 & 6 \end{pmatrix}}_{M1} - 1 \underbrace{\begin{pmatrix} 1 & 1 & 2 \\ 1 & -1 & 1 \\ 5 & 1 & 6 \end{pmatrix}}_{M2} + 1 \underbrace{\begin{pmatrix} 1 & 3 & 2 \\ 1 & 2 & 1 \\ 5 & 9 & 6 \end{pmatrix}}_{M3} - 1 \underbrace{\begin{pmatrix} 1 & 3 & 1 \\ 1 & 2 & -1 \\ 5 & 9 & 1 \end{pmatrix}}_{M4}$

$$M_1 = 3\begin{pmatrix} -1 & 1 \\ 1 & 6 \end{pmatrix} - 1\begin{pmatrix} 2 & 1 \\ 9 & 6 \end{pmatrix} + 2\begin{pmatrix} 2 & -1 \\ 9 & 1 \end{pmatrix} = 3(-7) - 1(3) + 2(11) = -2$$

$$M2 = 1\begin{pmatrix} -1 & 1 \\ 1 & 6 \end{pmatrix} - 1\begin{pmatrix} 1 & 1 \\ 5 & 6 \end{pmatrix} + 2\begin{pmatrix} 1 & -1 \\ 5 & 1 \end{pmatrix} = 1(-7) - 1(1) + 2(6) = 4$$

$$M_3 = 1\begin{pmatrix} 2 & 1 \\ 9 & 6 \end{pmatrix} - 3\begin{pmatrix} 1 & 1 \\ 5 & 6 \end{pmatrix} + 2\begin{pmatrix} 1 & 2 \\ 5 & 9 \end{pmatrix} = 1(3) - 3(1) + 2(-1) = -2$$

$$M4 = 1\begin{pmatrix} 2 & -1 \\ 9 & 1 \end{pmatrix} - 3\begin{pmatrix} 1 & -1 \\ 5 & 1 \end{pmatrix} + 1\begin{pmatrix} 1 & 2 \\ 5 & 9 \end{pmatrix} = 1(11) - 3(6) + 1(-1) = -8$$

$\det(A) = 1(-2) - 1(4) + 1(-2) - 1(-8) = 0$ Não é inversível.

3. Os determinantes foram calculados anteriormente.

4. $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 2 & a \end{bmatrix}$ $\det(A) = 1\begin{pmatrix} 0 & 0 \\ 2 & a \end{pmatrix} - 1\begin{pmatrix} 1 & 0 \\ 1 & a \end{pmatrix} + 0\begin{pmatrix} 1 & 0 \\ 1 & 2 \end{pmatrix}$

$\det(A) = 1 \cdot 0 - 1 \cdot a + 0 = -a$ Como $\det(A) \neq 0$

$a \in \mathbb{R}$ é válido.

5. $\begin{cases} x + y + z = 6 \\ 2x + y + 3z = 14 \\ x + 2y + z = 10 \end{cases}$ $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 6 \\ 14 \\ 10 \end{bmatrix}$ $AX = B$

$\det(A) = 1\begin{pmatrix} 1 & 3 \\ 2 & 1 \end{pmatrix} - 1\begin{pmatrix} 2 & 3 \\ 1 & 1 \end{pmatrix} + 1\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$ $\det(A) = 1(-5) - 1(-1) + 1(3) = -1$

adj $C_{11} = \begin{pmatrix} 1 & 3 \\ 2 & 1 \end{pmatrix} = -5$ $C_{12} = -\begin{pmatrix} 2 & 3 \\ 1 & 1 \end{pmatrix} = 1$ $C_{13} = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} = 3$

$C_{21} = -\begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix} = 1$ $C_{22} = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} = 1$ $C_{23} = -\begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} = -1$

$C_{31} = \begin{pmatrix} 1 & 1 \\ 1 & 3 \end{pmatrix} = 2$ $C_{32} = -\begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix} = -1$ $C_{33} = \begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix} = -1$

$\operatorname{cof}(A) = \begin{bmatrix} -5 & 1 & 3 \\ 1 & 1 & -1 \\ 2 & -1 & -1 \end{bmatrix}$ $\operatorname{adj}(A) = \begin{bmatrix} -5 & 1 & 2 \\ 1 & 1 & -1 \\ 3 & -1 & -1 \end{bmatrix}$

$A^{-1} = -1 \cdot \begin{bmatrix} -5 & 1 & 2 \\ 1 & 1 & -1 \\ 3 & -1 & -1 \end{bmatrix} = \begin{bmatrix} 5 & -1 & -2 \\ -1 & -1 & 1 \\ -3 & 1 & 1 \end{bmatrix}$ $X = A^{-1} \cdot B = \begin{bmatrix} 5 & -1 & -2 \\ -1 & -1 & 1 \\ -3 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 6 \\ 14 \\ 10 \end{bmatrix} =$ $x = -4$, $y = -10$, $z = 6$

7. $A = \begin{bmatrix} 0 & 1 & 5 \\ 3 & -6 & 9 \\ 2 & 6 & 1 \end{bmatrix} \xrightarrow{L_1 \leftrightarrow L_2} \begin{bmatrix} 3 & -6 & 9 \\ 0 & 1 & 5 \\ 2 & 6 & 1 \end{bmatrix} \xrightarrow{L_3 \leftarrow L_3 - \frac{2}{3}L_1} \begin{bmatrix} 3 & -6 & 9 \\ 0 & 1 & 5 \\ 0 & 10 & -5 \end{bmatrix}$

$L_3 \leftarrow L_3 - 10L_2$

$\begin{bmatrix} 3 & -6 & 9 \\ 0 & 1 & 5 \\ 0 & 0 & -55 \end{bmatrix}$ $\det(U) = 3 \cdot 1 \cdot (-55) = -165$

$\det(A) = -(-165) = 165$

10. $A^2 = -2A^4$ então $(I + A^2)^{-1} = I - 2A^2$ $\quad A^2 = -2A^4 \quad A^4 = -\frac{1}{2}A^2$

$(I + A^2)(I - 2A^2) = I - 2A^2 + A^2 - 2A^4 = I - A^2 - 2A^4 \qquad A^4 = -\frac{1}{2}A^2$

$I - A^2 - 2\left(-\frac{1}{2}A^2\right) = I - A^2 + A^2 = I$ Verdadeiro.

$A^t = -A^2$ e invertível, então $\det(A) = -1$

$\det(A^t) = \det(-A^2) \qquad \det(A) = (-1)^n (\det(A))^2$

$\det(A) = (-1)^n \det(A^2) \qquad (\det(A))^2 - (-1)^n \det(A) = 0$

$\det(A)\det(A) - (-1)^n = 0$

Falso.

$B = A \cdot A^t \cdot A^{-1}$ então $\det(A) = \det(B) \qquad \det(B) = \det(A) \cdot \det(A^t) \cdot \det(A^{-1})$

$\det(A) \cdot \frac{1}{\det(A)} = \det(A)$

Verdadeiro.

$\det(A + B) = \det(A) + \det(B) \quad A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \det(A) + \det(B) = 1 + (-1) = 0$

$A + B = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \quad \det(A + B) = 0$

$C = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, D = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad \det(C) = -2 \quad \det(D) = 0 \quad \det(C) + \det(D) = -2$

logo é falso.

$C + D = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix} \quad \det(C + D) = -5$