Exercises Modern Robotics

3.2 - $p = (1/\sqrt{3}, -1/\sqrt{6}, 1/\sqrt{2})_{xyz}$

$$p' = Rot(\hat{z}, -120^\circ)\, Rot(\hat{y}, 135^\circ)\, Rot(\hat{x}, 30^\circ) \cdot p$$

$$p' = \begin{bmatrix} \cos(-120) & -\sin(-120) & 0 \\ \sin(120) & \cos(120) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(135) & 0 & \sin(135) \\ 0 & 1 & \\ -\sin(135) & 0 & \cos(135) \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(35^\circ) & -\sin(35^\circ) \\ 0 & \sin(35^\circ) & \cos(35^\circ) \end{bmatrix} \begin{bmatrix} 1/\sqrt{3} \\ -1/\sqrt{6} \\ 1/\sqrt{2} \end{bmatrix}$$

utilizando o python:

a) $p' = \begin{bmatrix} -0{,}559 \\ 0{,}512 \\ -0{,}652 \end{bmatrix}$

b) $R = \begin{bmatrix} 0{,}354 & 0{,}507 & -0{,}786 \\ 0{,}612 & -0{,}761 & -0{,}214 \\ -0{,}707 & -0{,}406 & -0{,}579 \end{bmatrix}$

3.3) $p_i' = Rp$ but R is unknown ...

$$\begin{bmatrix} 0 \\ 2 \\ \sqrt{2} \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \begin{bmatrix} \sqrt{2} \\ 0 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ -\sqrt{2} \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} -\sqrt{2} \\ \sqrt{2} \\ -2 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \begin{bmatrix} 0 \\ 2\sqrt{2} \\ 0 \end{bmatrix}$$

(1) $-\cancel{\sqrt{2}} = r_{12} \cdot 2\cancel{\sqrt{2}} \rightarrow r_{12} = -\frac{1}{2}$

(II) $\cancel{\sqrt{2}} = r_{22} \cdot 2\cancel{\sqrt{2}} \Rightarrow r_{22} = \frac{1}{2}$

(III) $-\cancel{2} = r_{32} \cdot \cancel{2}\sqrt{2} \Rightarrow r_{32} = -\frac{\sqrt{2}}{2}$

(IV) $\begin{cases} 0 = \sqrt{2} \cdot r_{11} + 2 \cdot r_{13} \\ \frac{1}{\sqrt{2}} = r_{11} + r_{12} - r_{23} = r_{11} - \frac{1}{2} - r_{13} \end{cases}$ $\longrightarrow r_{13} = -\frac{\sqrt{2}}{2} r_{11}$

$$\frac{1}{\sqrt{2}} + \frac{1}{2} = \frac{\sqrt{2}+1}{2} = r_{11} - r_{13} = r_{11} - \left(-\frac{\sqrt{2}}{2} r_{11}\right)$$

$$\frac{(\sqrt{2}+1)}{\cancel{2}} = \frac{(2+\sqrt{2})}{\cancel{2}} r_{11} \longrightarrow r_{11} = \frac{\sqrt{2}}{2}, \quad r_{13} = -\frac{1}{2}$$

(V) $\begin{cases} 2 = \sqrt{2} \cdot r_{21} + 2 r_{23} \\ 1/\sqrt{2} = r_{21} + r_{22} - r_{23} = r_{21} + \frac{1}{2} - r_{13} \end{cases}$ $\longrightarrow r_{23} = \frac{2 - \sqrt{2} r_{21}}{2}$

$$\frac{1}{\sqrt{2}} - \frac{1}{2} = \frac{\sqrt{2}}{2} - \frac{1}{2} = \frac{\sqrt{2}-1}{2} = r_{21} - r_{23} = r_{21} - \left(\frac{2-\sqrt{2}\,r_{21}}{2}\right)$$

$$\frac{\sqrt{2}-1}{\cancel{2}} = \frac{(2+\sqrt{2})\,r_{21} - 2}{\cancel{2}} \Rightarrow r_{21} = \frac{\sqrt{2}+1}{\sqrt{2}+2}\frac{(\sqrt{2}-2)}{(\sqrt{2}-2)}$$

$$r_{21} = \frac{\cancel{2} - 2\sqrt{2} + 1\sqrt{2} \cancel{-2}}{2-4} \Rightarrow r_{21} = \frac{\sqrt{2}}{2}$$

$$r_{23} = \frac{2 - \sqrt{2}.\, r_{21}}{2} = \frac{2 - \cancel{\sqrt{2}}\frac{\cancel{\sqrt{2}}}{\cancel{2}}}{2} \Rightarrow r_{23} = \frac{1}{2}$$

vi) $\begin{cases} \sqrt{2} = \sqrt{2}\, r_{31} + 2\, r_{33} \longrightarrow r_{33} = \frac{\sqrt{2}}{2}(1 - r_{31}) \\ -\sqrt{2} = r_{31} + r_{32} - r_{33} = r_{31} - \frac{\sqrt{2}}{2} - r_{33} \end{cases}$

$$-\sqrt{2} + \frac{\sqrt{2}}{2} = -\frac{\sqrt{2}}{2} = r_{31} - r_{33} = r_{31} - \frac{\sqrt{2}}{2}(1 - r_{31})$$

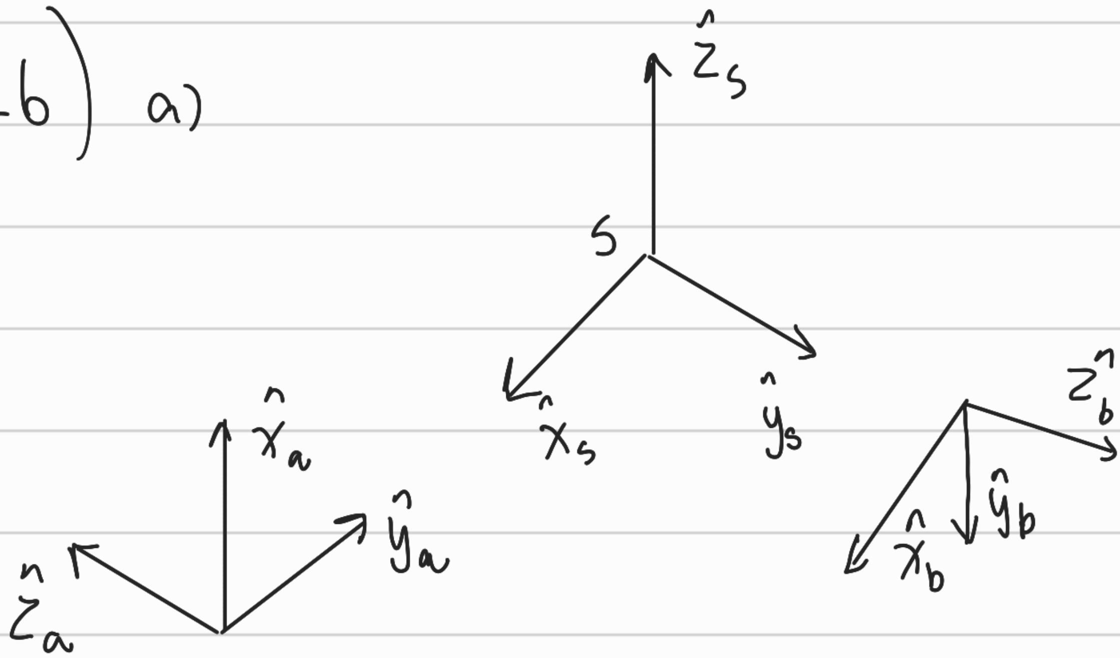
$$-\cancel{\frac{\sqrt{2}}{2}} + \cancel{\frac{\sqrt{2}}{2}} = r_{31} + \frac{\sqrt{2}}{2} r_{31} \Rightarrow r_{31} = 0$$

$$r_{33} = \frac{\sqrt{2}}{2}$$

Therefore, R is described as:

$$R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{2}}{2} & -1/2 & -1/2 \\ \frac{\sqrt{2}}{2} & 1/2 & 1/2 \\ 0 & -\sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix}$$

3.16) a)



b) $R_{sa} = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix}$

* write ${}^b x_s; {}^b y_s; {}^b z_s$ on the lines of the matrix

OR

* write ${}^s x_b; {}^s y_b; {}^s z_b$ on the columns of the matrix

$$R_{sb} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$$

And the transformations:

$$T_{sa} = \begin{bmatrix} 0 & -1 & 0 & 3 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{sb} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

c) In this case where the relationship between the axis is trivial, we can calculate the rotation matrix R_{bs} and analyse the initial point of s related to b.

$$R_{bs} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \quad \text{and } {}^{b}p_{s} = (0, 0, -2)$$

$$T_{bs} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Testing: $T_{sb} \cdot T_{sb}^{-1} = T_{sb} \cdot T_{bs} = I$ ✓

d) $T_{ab} = T_{as} \cdot T_{sb} = T_{sa}^{-1} \cdot T_{sb}$

$$T_{as} = \begin{bmatrix} R_{as} & {}^{a}p_s \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} R_{sa}^{T} & {}^{a}p_s \\ 0 & 1 \end{bmatrix}$$

$$R_{as} = \begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}$$

$${}^{s}p_{SA} = -{}^{s}p_{AS}$$

$${}^{A}p_{AS} = R_{AS} \, {}^{s}-p_{SA}$$

$${}^{A}p_{AS} = (0, 3, 0)$$

$$T_{as} = \begin{bmatrix} R_{as} & {}^{A}p_{AS} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

So, we can calculate T_{sb}

$$T_{ab} = T_{as} T_{sb}$$

$$= \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{ab} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 3 \\ 0 & 0 & -1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Checking the drawings at item a) we can see that the T_{ab} is correct.

e) $T = T_{sb} = Rot(\hat{x}, 90°) \cdot Trans(\hat{y}, 2)$

$T_1 = T_{sa} \cdot T = T_{sa} \cdot Rot(\hat{x}, 90°) \cdot Trans(\hat{y}, 2)$

↳ The rotation will be about x_a and y_a configuring a body - fixed transformation of T_{sa}

$T_2 = T\, T_{sa} = Rot(\hat{x}, 90) \cdot Trans(\hat{y}, 2)\, T_{sa}$

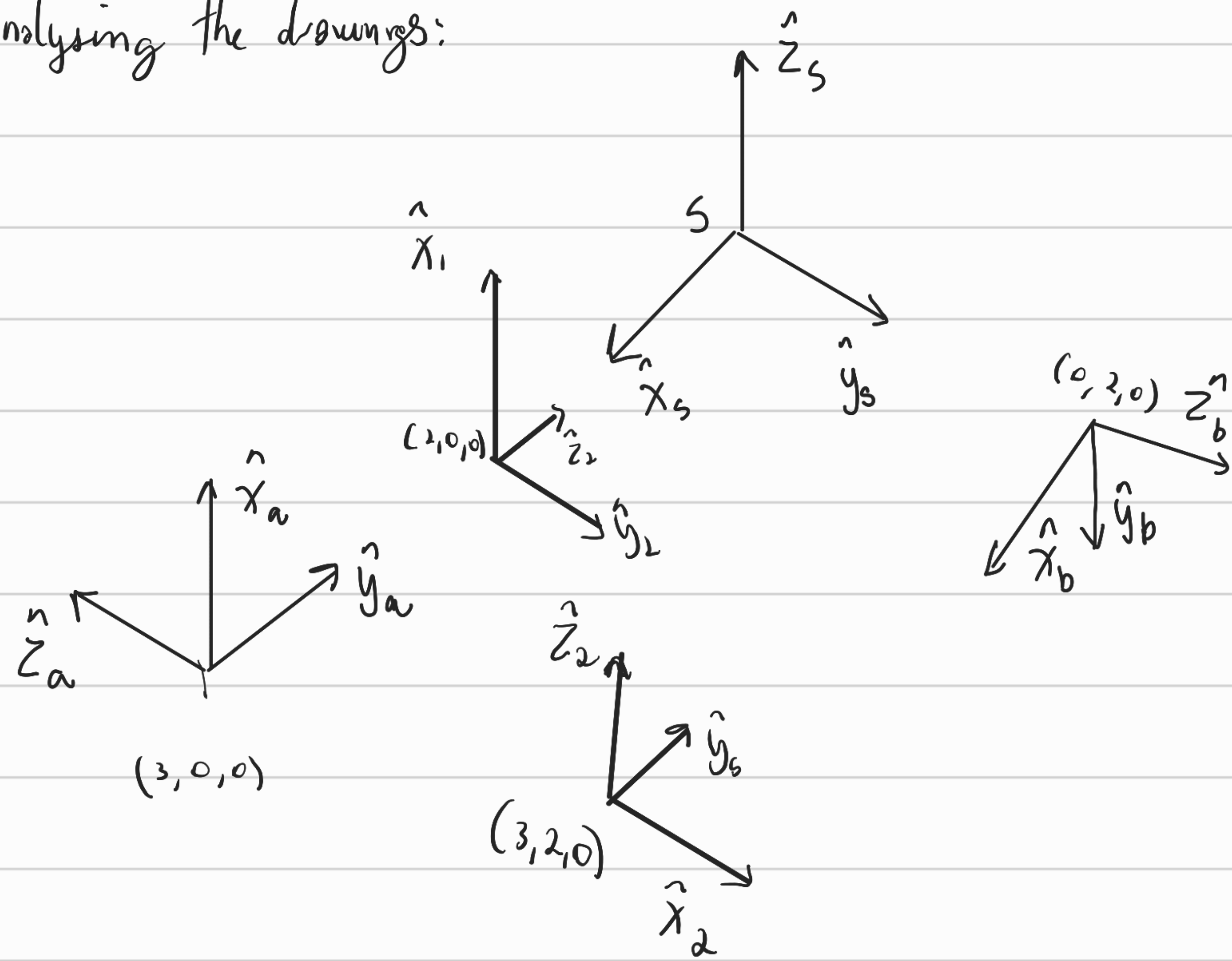
↳ This will transform the a frame by $90°$ on $\hat{x}_s$ and translate 2 units on y_s. So its a world - fixed transformation

We can see it by:

$$T_1 = T_{sa} \cdot T = \begin{bmatrix} 0 & -1 & 0 & 3 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 & +1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2 = T \cdot T_{sa} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 3 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 3 \\ 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Analysing the drawings:



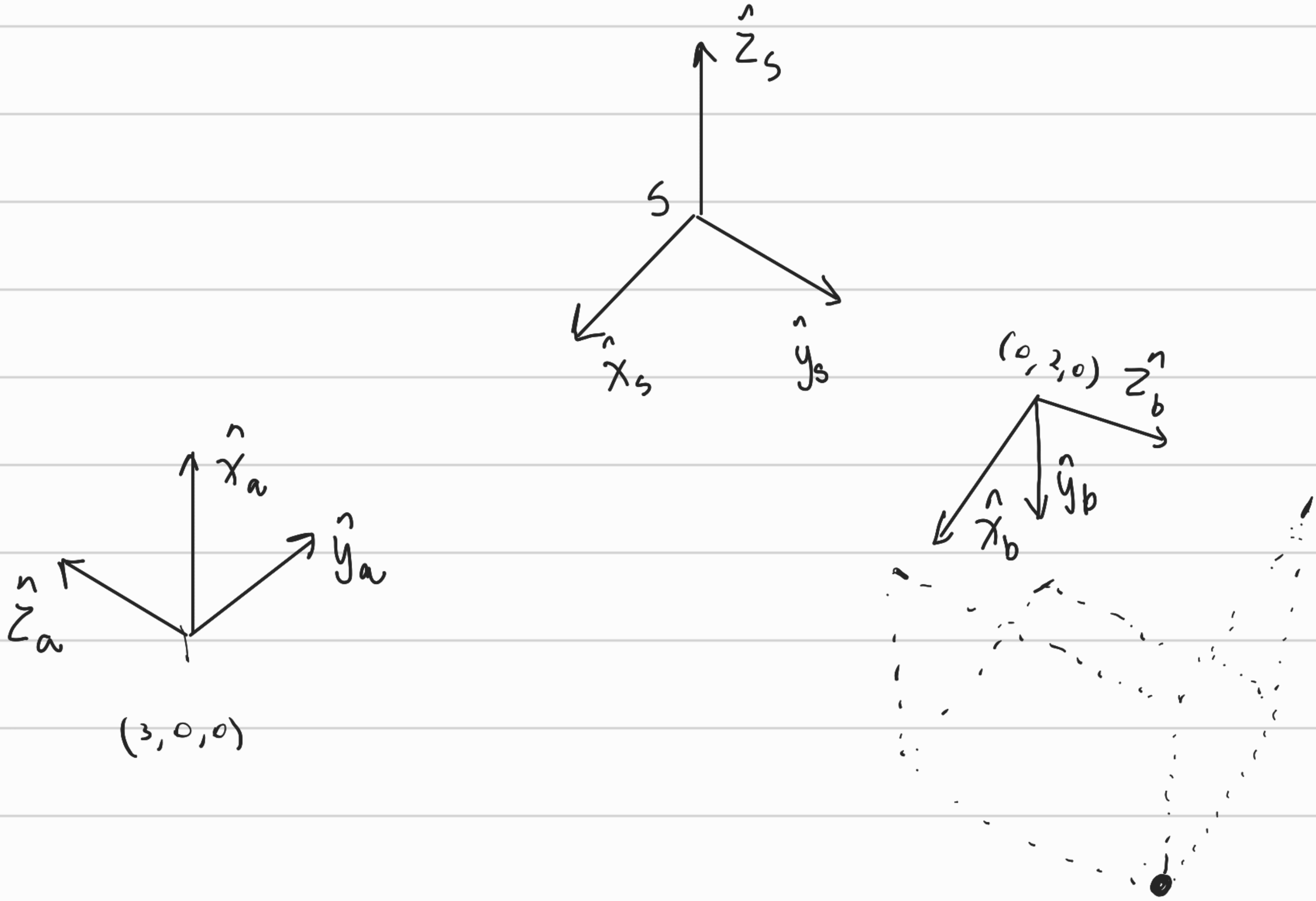
f) $p_b = (1, 2, 3)$

We can change the representation of p in b to s by:

$p_a = T_{sb} \cdot p_b$

$$P_a = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ -2 \\ 1 \end{bmatrix}$$

↳ homogeneous coordinates



$P_b = (1, 2, 3)$

$P_a = (1, 5, -2)$

g) Applying the transformation:

$$p' = T_{sb}\, p_s = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ -2 \\ 1 \end{bmatrix}$$

For this operation it can be interpreted as a rotation and a translation that define the b frame applied to the point p. So the point will rotate and translate the same as b frame was.

Applying the inverse:

$$p'' = T_{sb}^{-1}\, p_s = T_{bs} \cdot p_s = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \\ 0 \\ 1 \end{bmatrix}$$

For this operation, it can be interpreted as a change on the reference frame from s to b. So the point is not moved on the space, it only is written on a different representation.

3.17) a) $T_{ad} = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$$T_{cd} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

b) $T_{ab} = T_{ac} T_{cb} = T_{ad} T_{cd}^{-1} T_{bc}^{-1}$

$$T_{bc}^{-1} = \begin{bmatrix} R^T & -R^T p_b \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{cd}^{-1} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{ab} = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Using python for matrix multiplication:

$$T_{ab} = \begin{bmatrix} 0 & 1 & 0 & -1 \\ 1 & 0 & 0 & -3 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

We can check this by analyzing the drawings.

3.18) a) $T_{vs} = T_{va} T_{as} = T_{ar}^{-1} T_{ae} T_{es}$

$$= T_{ar}^{-1} T_{ea}^{-1} T_{es}$$

b) Let ${}^{e}p_{s} = (1,1,1)$

$\to$ reference frame

$\to$ origin of s.

$$ {}^{r}p_s = T_{re}\,{}^{e}p_s \qquad (1) $$

But we don't have T_{re}:

$$ T_{re} = T_{er}^{-1} = \begin{bmatrix} R^T & -R^T\,{}^{e}p_r \\ 0 & 1 \end{bmatrix} $$

$$ T_{re} = \begin{bmatrix} -1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Resolving (1):

$$ {}^{r}p_s = \begin{bmatrix} -1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} $$

3.20: The relationship between θ_H (angle of the high wheel) and the θ_L (low wheel angle) can be described as:

$$C_H = C_L \rightarrow \theta_H \cdot r_H = \theta_L r_L$$

$$\theta_H = \frac{\theta_L}{2} \quad \text{and} \quad w_H = \frac{w_L}{2}$$

writing the transformation from b to a:

$$T_{ab} = \underbrace{Rot(\hat{x}_a, w_{Rel} t)}_{\text{relative velocity}} \cdot \underbrace{Trans(\hat{y}, L)}_{\text{not constant}}$$

$$\theta_{rel} = \theta_H - \theta_L = -\frac{\theta_H}{2}$$

$$w_{rel} = -\frac{w_H}{2}$$

Therefore: $R_{ab} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\left(-\frac{w_H}{2} \cdot t\right) & -\sin\left(-\frac{w_H}{2} \cdot t\right) \\ 0 & \sin\left(-\frac{w_H}{2} \cdot t\right) & \cos\left(-\frac{w_H}{2} \cdot t\right) \end{bmatrix}$

Also, for the translation of b related to a:

$$(L\hat{y})_a = Rot(\hat{x}_a, \theta_L)(L\hat{y})_{world}$$

$$(L\hat{y})_a = Rot(\hat{x}_a, 2w_H t)(L\hat{y})_{world}$$

$$(L\hat{y})_a = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 2w_H t & -\sin 2w_H t \\ 0 & \sin 2w_H t & \cos 2w_H t \end{bmatrix} \begin{bmatrix} 0 \\ L \\ 0 \end{bmatrix}$$

$$(L\hat{y})_a = \begin{bmatrix} 0 \\ L\cos(2w_H t) \\ L\sin(2w_H t) \end{bmatrix}$$

With that we can write:

$$T_{ab} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\left(-\frac{w_H}{2}\cdot t\right) & -\sin\left(-\frac{w_H}{2}\cdot t\right) & L\cos(2w_H t) \\ 0 & \sin\left(-\frac{w_H}{2}\cdot t\right) & \cos\left(-\frac{w_H}{2}\cdot t\right) & L\sin(2w_H t) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Or related to θ_H

$$T_{ab} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\left(-\frac{\theta_H}{2}\right) & -\sin\left(-\frac{\theta_H}{2}\right) & L\cos(2\theta_H) \\ 0 & \sin\left(-\frac{\theta_H}{2}\right) & \cos\left(-\frac{\theta_H}{2}\right) & L\sin(2\theta_H) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

With $\cos(-\theta) = \cos(\theta)$ and $-\sin(\theta) = \sin(-\theta)$:

$$T_{ab} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\left(\frac{\theta_H}{2}\right) & \sin\left(\frac{\theta_H}{2}\right) & L\cos(2\theta_H) \\ 0 & -\sin\left(\frac{\theta_H}{2}\right) & \cos\left(\frac{\theta_H}{2}\right) & L\sin(2\theta_H) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The translation of b to c we can write:

$$T_{bc} = \begin{bmatrix} 1 & 0 & 0 & -D \\ 0 & 1 & 0 & r_H \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

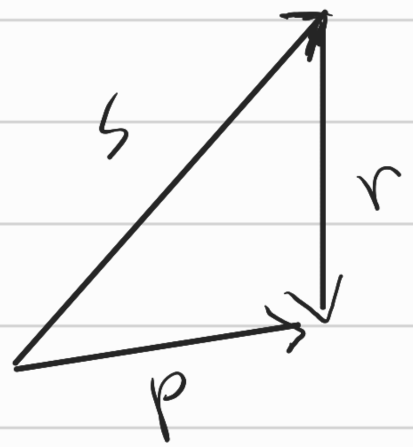
So:

$$T_{ac} = T_{ab} T_{bc}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\frac{\theta_H}{2}) & \sin(\frac{\theta_H}{2}) & L\cos(2\theta_H) \\ 0 & -\sin(\frac{\theta_H}{2}) & \cos(\frac{\theta_H}{2}) & L\sin(2\theta_H) \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -D \\ 0 & 1 & 0 & r_H \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{ac} = \begin{bmatrix} 1 & 0 & 0 & -D \\ 0 & \cos(\frac{\theta_H}{2}) & \sin(\frac{\theta_H}{2}) & r_H \cos(\frac{\theta_H}{2}) + L\cos(2\theta_H) \\ 0 & -\sin(\frac{\theta_H}{2}) & \cos(\frac{\theta_H}{2}) & -r_H \sin(\frac{\theta_H}{2}) + L\sin(2\theta_H) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3.21) a) Defining a new vector s:



$$s = (-100, 300, 500)_a$$

$$p = (0, 800, 0)_a$$

$$p - r = s \Rightarrow r = p - s$$

$$r = (100, 500, -500)_a$$

Changing the reference frame:

$$r_b = T_{ba}\, r_a = T_{ab}^{-1}\, r_a$$

But

R_{ba}

$$T_{ab}^{-1} = \begin{bmatrix} 0 & 1 & 0 & -300 \\ -1 & 0 & 0 & -200 \\ 0 & 0 & 1 & -500 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Calculating r_b:

$$r_b = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{R_{ba} =} \cdot \underbrace{\begin{bmatrix} 100 \\ 500 \\ -500 \end{bmatrix}}_{r_a} = \begin{bmatrix} 500 \\ -100 \\ -500 \end{bmatrix}$$

b) $\hat{z}$ and $\hat{y}$ axes are coplanar, $T_{bc} = ?$

By the figure, we can see that

$$T_{AC} = Trans(\hat{y}_a, p) \cdot Rot(\hat{x}_a, -60°)$$

Using python:

$$T_{AC} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0,5 & 0,866 & 800 \\ 0 & -0,866 & 0,5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

To know T_{bA}:

$$T_{bA} = T_{AB}^{-1} = \begin{bmatrix} 0 & 1 & 0 & -300 \\ -1 & 0 & 0 & -100 \\ 0 & 0 & 1 & -500 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Finally:

$$T_{bc} = T_{bA} \cdot T_{AC}$$

$$T_{bc} = \begin{bmatrix} 0 & 1 & 0 & -300 \\ -1 & 0 & 0 & -100 \\ 0 & 0 & 1 & -500 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0{,}5 & 0{,}866 & 800 \\ 0 & -0{,}866 & 0{,}5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{bc} = \begin{bmatrix} 0 & 0{,}5 & 0{,}866 & 500 \\ -1 & 0 & 0 & -100 \\ 0 & -0{,}866 & 0{,}5 & -500 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$