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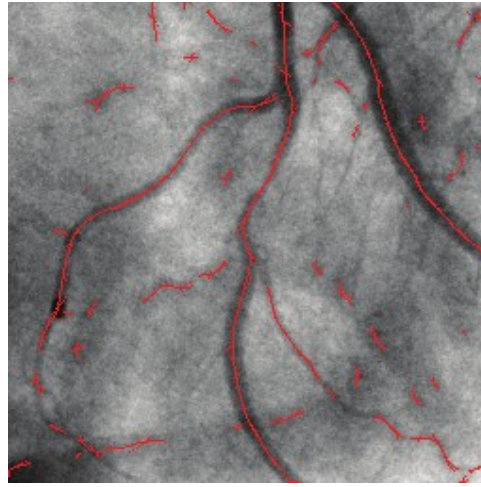
Student id: **3301311**

Educational Program: **EEMCS**

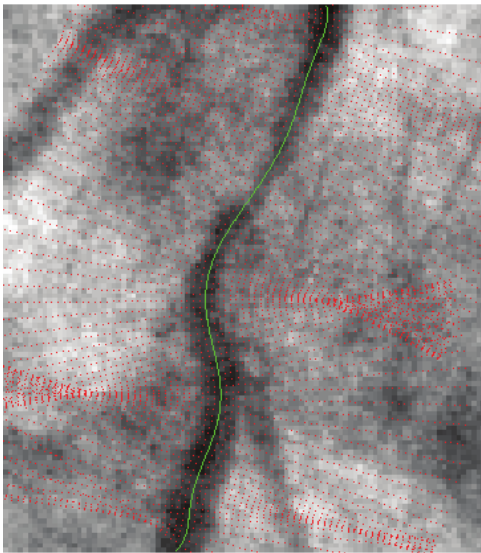
a)



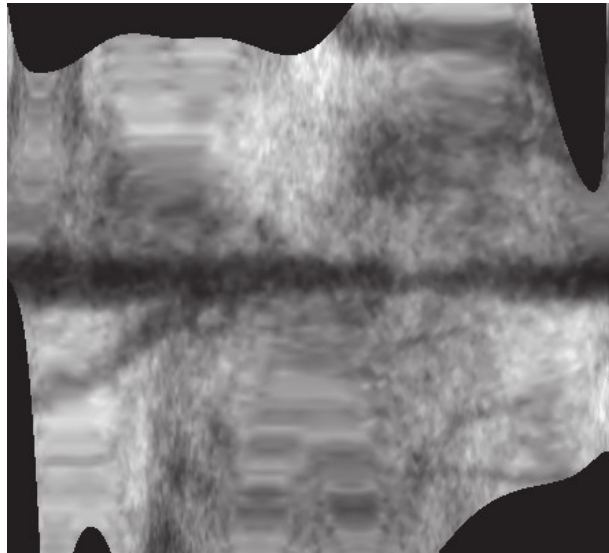
b)



c)



d)



**Figure 1 A coronary angiogram**

a) Original.

c) ROI with medial axis (green dots)  
and a grid that is aligned along  
and across the medial axis.

b) Original with lines detected.

d) ROI aligned along the medial axis.

## Exercise 3: edge detection and morphological operations

In the following two exercises, we develop and evaluate software for an image processing system that is able to measure the diameters of a blood vessel in coronary angiographic images such as the one shown in Figure 1a. These measurements are useful for the quantitative assessment of the degree of stenosis (narrowing) to diagnose the cardiac disease. The full image analysis system consists of the following steps:

1. Detection and localization of the blood vessels using a line detector (Figure 1b).
2. Interactive definition of a ROI (region-of-interest) containing the vessel with stenosis (Figure 1c).
3. Alignment of the image along the detected line, i.e. the medial axis of the blood vessel by application of a geometric transform is adapted to this medial axis (Figure 1d and e).
4. Edge detection along the blood vessel.
5. Measurement of the nominal diameter.
6. Measurement of the diameter at the stenosis.

### Part I: Edge based image segmentation

The purpose of this part is to develop an image processing system that is able to find the contours of the vessel.

1. The scale space

A 2D Gaussian point spread function can be created and visualized with:

```
sigma = 5; % set the width of the Gaussian
L = 2*ceil(sigma*.)+1; % fill in a constant to define the matrix size
xymax = (L-1)/2; % the maximum of the x and the y coordinate
xrange = -xymax:xymax; % the range of x values
yrange = xrange; % the range of y values
figure(1)
h = fspecial('gaussian', L, sigma); % create the PSF matrix
C = cat(3, ones(size(h)), ones(size(h)), zeros(size(h)));
% create a RGB matrix to define the colour of the surface plot
hd = surf(xrange, yrange, h, C, 'FaceColor', 'interp', 'Facelight', 'phong');
% create the surface plot of the gaussian
camlight right % add a light at the right side of the scene
% set appropriate axis limits
xlabel('x'); % add axis labels
ylabel('y');
zlabel('h(x,y)');
print -r150 -dpng ex3_1.png % print the result to file
```

Here, the function `fspecial` is used to create a Gaussian PSF matrix `h` with the deviation (width) of the Gaussian set to  $\sigma$  (matlab: `sigma`). The size of `h` should be such that the truncation error of the tails of the Gaussian is negligible. On the other hand, a too large size leads to a waste of computations. The function `surf` creates a surface plot of the PSF. The 3D array `C` defines the colour of the surface. The other functions, `camlight`, `xlabel`, etc. are needed to refine your graph<sup>1</sup>.

---

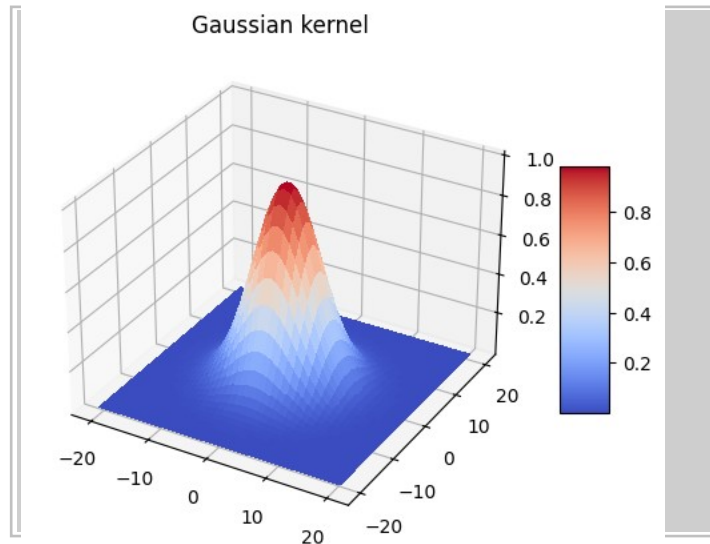
<sup>1</sup> **Note:** in the sequel, omission of axis labels in any graph (images excluded) in your report will be considered as a shortcoming of your report, and will have a negative impact on the grade.

- a) Initiate a new Matlab script. That starts with `%% name`, then clear the work space and close all figure windows, then add a line “ `%% question 1`”. Next copy and paste the code given above. Complete the highlighted lines.

Give the completed Matlab code of the 2nd line above:

```
L = 2*ceil(4*sigma)+1)
```

Insert the graph:



- b) Apply the filter to the test image stored in `ang2.png`. Apply this with four different values of  $\sigma$ . That is:  $\sigma = 1, 5, 10$  and  $20$ . You can use the following template for your code:

```
im = ... % read the image from file
sigma = [1 5 10 20]; % fill array with sigma values
for i=1:4 % do 4 times:
    h = fspecial(...; % create the PSF
    imfiltered = imfilter(...; % apply the filter
    subplot(2,2,i); % define a subplot
    imshow(imfiltered,[]); % show the image
    title(['\sigma = ' num2str(sigma(i))]); % include a plot title
end
```

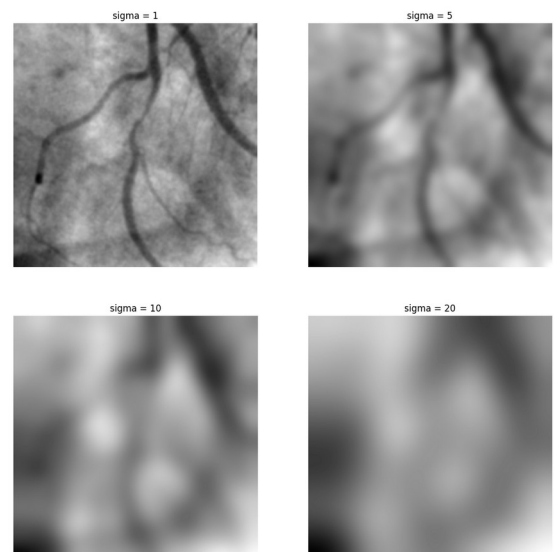
Select the options of `imfilter` such that the output image has the same size as the input image, and the boundary option is that boundary effects on the border of the image are visually minimized.

Give the completed Matlab code of the lines that contain the `fspecial` function and `imfilter` function:

```
def gkern(sigma=5, kernlen=None):
    kernlen = kernlen or (2*ceil(4*sigma)+1)
    gkern1d = gaussian(kernlen, std=sigma)
    .reshape(kernlen, 1)
    gkern2d = np.outer(gkern1d, gkern1d)
    return gkern2d

psf = gkern(s) # manually defined
img_blur = cv.filter2D(img, -1, psf)
```

Insert the resulting images:



What are the scales ( $\xi^2$ ) of these four images?

[1, 25, 100, 400]

## 2. Derivatives

Consider a continuous image  $f(x,y)$  of which only a sampled representation  $f(n\Delta, m\Delta)$  is available. Suppose that we are interested in the first derivative  $f_x(x,y) = \partial f(x,y) / \partial x$ . The purpose is to get a sampled version  $f_x(n\Delta, m\Delta)$  of this derivative by processing the digital image  $f(n\Delta, m\Delta)$ . However, because only a digital representation is available (and not the continuous one) such a derivative can only be approximated. In the continuous case, differentiation is defined within an infinitesimally small neighbourhood of a point. But in the discrete case, this neighbourhood is necessarily extended to a finite neighbourhood. The differentiation is necessarily approximated by a convolution that implements the differentiation combined with some sort of low-pass filtering.

In the continuous domain the combination of differentiation and low-pass filtering is nicely accomplished by:

$$\frac{\partial}{\partial x} f(x,y) \approx h(x,y) * f(x,y) \quad \text{with} \quad h(x,y) = \frac{\partial}{\partial x} \text{low-pass filter}$$

In other words, differentiation of a low-pass filtered image is equivalent to convolving the image with the derivative of the PSF of the low-pass filter. This opens the door to differentiation in the discrete (digital) domain: If  $h_x(x,y) = \partial h(x,y) / \partial x$ , then differentiation combined with low-pass filtering in the discrete domain is accomplished by a PSF matrix  $h_x(n\Delta, m\Delta)$ :

$$f_x(n\Delta, m\Delta) \approx f(n\Delta, m\Delta) * h_x(n\Delta, m\Delta)$$

In the scale space theory it has been argued that the best low-pass filter needed for differentiation is the Gaussian:

$$h(x,y) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right) \quad (1)$$

Sample code to create the PSF matrix  $h$  with a size of  $L \times L$  (as an alternative to using the function `fspecial`) is:

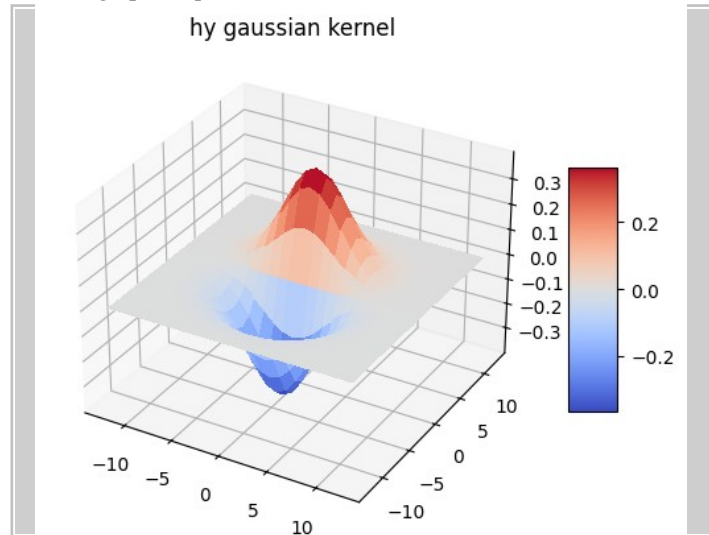
```
N = (L-1)/2; % get the size of half of the full range
[x,y]=meshgrid(-N:N,-N:N); % create the coordinates of a 2D orthogonal grid
h = exp(-(x.^2 + y.^2)/(2*sigma^2))/(2*pi*sigma^2);
```

- a) Give the analytical expression for  $h_y(x,y)$ . That is, differentiate expression (1) analytically with respect to  $y$ . Insert the found expression in Matlab syntax such as in the sample code for  $h(x,y)$ :

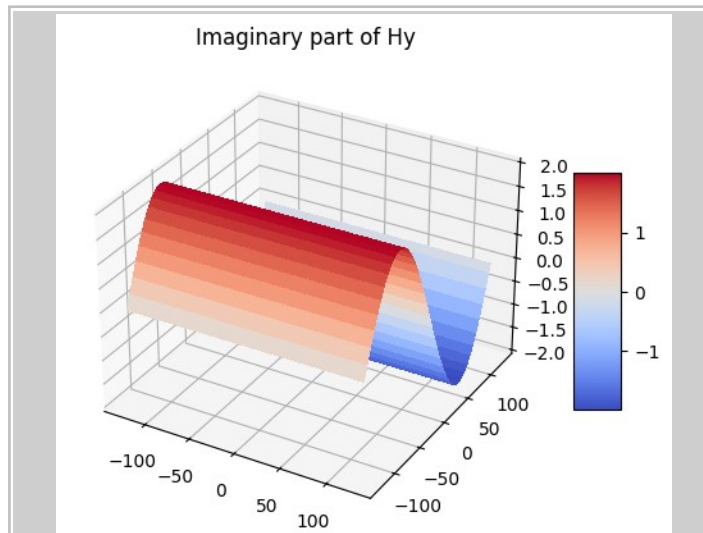
```
hy = -(exp(-(x^2 + y^2)/(2*sigma^2))*y)/(2*pi*sigma^4)
```

- b) Create a PSF matrix  $h_y$  that contains a sampled version  $h_y(n\Delta, m\Delta)$  of  $h_y(x,y)$ . Visualize the PSF as in question 1. Implement this with  $\xi = 3$  (which is equivalent to  $\xi = 3$  and  $\Delta = 1$ ).
- c) Use `psf2otf` to calculate the OTF matrix  $HY$  associated to  $h_y$ . You have to add an appropriate argument to this function such that the size of  $HY$  matches the size of the image `ang2.png`. Inspect manually (in the command window of Matlab) whether the real part of  $HY$  is zero, or that the imaginary part of  $HY$  is zero, or that none are zero. Hint use, for instance, `max(real(HY(:)))` and `max(imag(HY(:)))` to check the maximum of the real and imaginary part of the 2D array.
- d) Visualize the OTF as a surface plot. If in question 2d you have found that the real part of the OTF is zero, you visualize only the imaginary part. If the imaginary was zero, you visualize only the real part. If both were non-zero, you visualize both in two graphs. **Hints:**
- § Add the option `'edgecolor','none'` to the function `surf` to suppress the black mesh lines on the surface (the edges).
  - § For visualization, use `fftshift` so that the origin is in the centre of the graph.
  - § Use `xlim` and `ylim` to set appropriate limits to the x- and y axes.

Insert the graph of question 2b:



Insert the graph(s) of question 2d:



What is your findings in question 2c:

$\xi$  The real part of the OTF is zero:

$\xi$  The imaginary part of the OTF is zero:

Yes (yes/no)

No (yes/no)

- e) Explain the findings of question 2c. **Hint:** find out how the operator will responds to a harmonic function  $A \exp(2 \pi j (u x + v y))$ :

The  $H_y$  is an odd function in the  $y$ -axis, so is represented by a sum of cosines, with is represented by the imaginary component in the Fourier transformation

f) Describe and explain the observation of the plotted OTF by answering the following questions: Hints: - Consider the imaginary parts of  $H_y$ . - You can substitute by the values of  $u$  and  $v$  in the  $H_y$  equation to check the behaviour.

§ How does  $H_y$  behave at frequencies  $v = 0$ ?

As a sine wave

§ What is the explanation for that?

When you take the derivative of a Gaussian function, you introduce a linear term 'x' in the expression. This linear term 'x' is what leads to the sinusoidal component when you take the Fourier transform.

§ How does  $H_y$  behave at low frequencies of  $v$ , i.e.  $v \rightarrow 0$ ?

As a sine wave

§ What is the explanation for that?

The same as before

§ How does  $H_y$  behave at high frequencies of  $v$ , i.e.  $v \gg 1/\sigma$ ?

As a sine wave

§ What is the explanation for that?

The same as before

g) Apply the convolution with  $h_y$  to the test image.

Implement this convolution in the frequency domain using the OTF given in the matrix  $H_y$ . Insert the resulting image on the right side of this text (use the function `imwrite` and `mat2gray`, rather than a copy of a figure window):

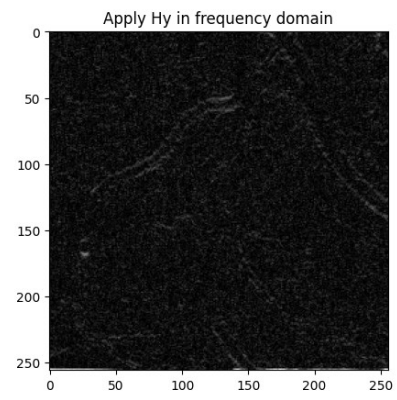
Explain the response of the operator to:

- § Areas without much contrast.
- § Positive step transitions such as at one of the boundary of the blood vessel.
- § Negative step transitions such as at the other boundary of the blood vessel
- § What is the influence of  $\sigma$  on these responses?

Areas without contrast are filtered out.

The negative and positive steps pass through the filter and can be seen in the image.

The image is not altered by changes in sigma





## INTERMEZZO

### The usage of the matlab function `mat2gray` :

Remember:

- § If the number representation of an image is of type `uint8`:
  - The range of pixel is: 0, 1, 2, ..., 255. So, the resolution of the grey scale is 1.
  - 0 is black and 255 is white
- § If the number representation is `double`:
  - The range of pixels is between 0 and 1, and the resolution of the grey scale is very fine ( $10^{-16}$ )
  - 0 is black and 1 is white

When writing an image `im` to file (jpg or tif), then:

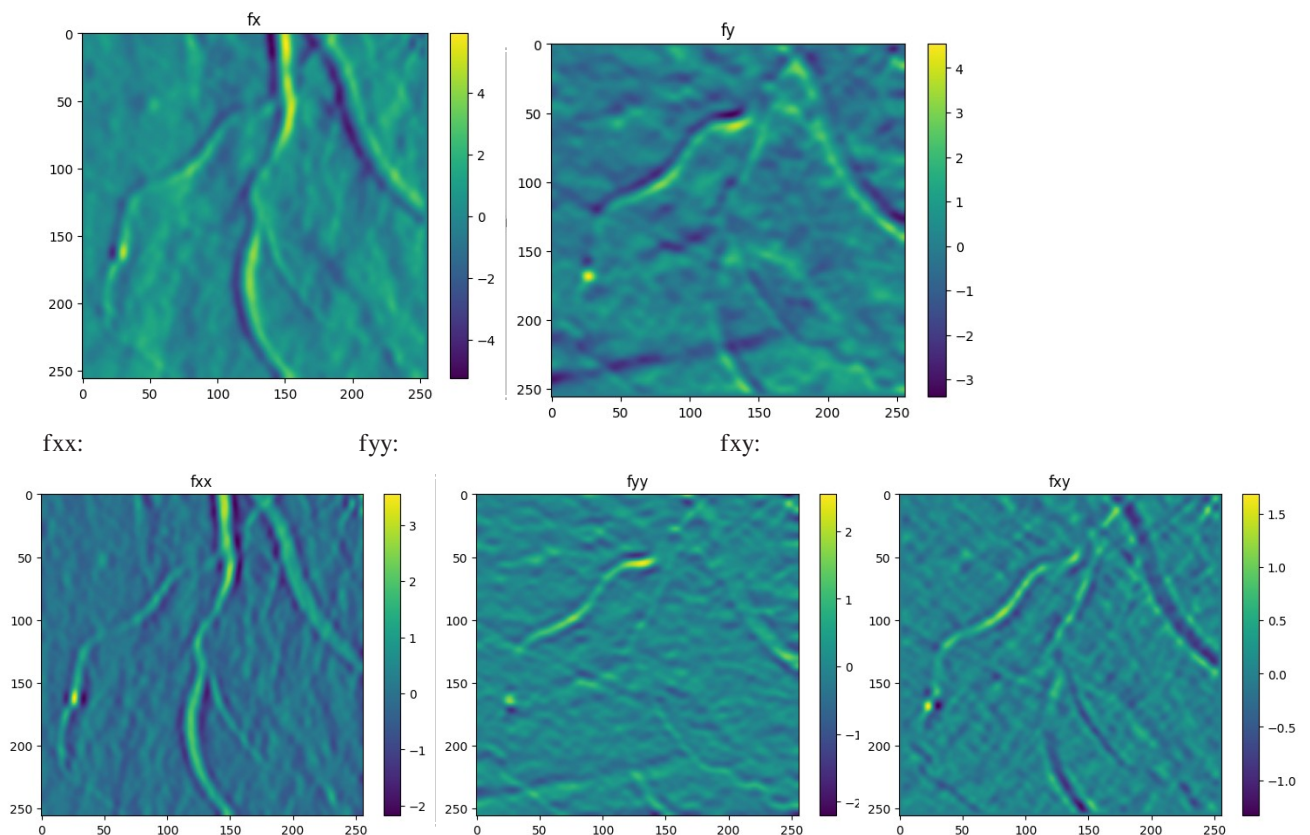
- § If the number representation of `im` is `uint8`, the pixels are written to file without processing. So, the bytes of the image are literally written to the file (apart from a possible compression).
- § If the number representation of `im` is `double`, then the pixels are rescaled by a factor of 255, that is: `im` becomes `im*255`, then converted to `uint8`, and finally written to file.

In the latter case, information may be lost. First of all, there will be round-off errors. Secondly, if the grey level of a pixel was larger than 1 (whiter than white), or smaller than 0 (blackier than black), then in the conversion from `double` to `uint8`, the pixel will be first truncated to 1 or to 0, respectively.

This truncation can be prevented by the function `mat2gray`. This function maps the greyscale of an image linearly (`im` becomes `a*im+b`) such that the maximum of `im` becomes 1 and the minimum becomes 0. This guarantees that no truncation takes place when writing to a file, but it also may enlarge the round-off error.

**FROM NOW IT IS UNDERSTOOD THAT YOU USE MAT2GRAY WHENEVER NEEDED !  
FAILING TO DO SO HAS A NEGATIVE IMPACT ON THE GRADE**

3. The function `ut_gauss` is an easy-to-use implementation <sup>2</sup> to calculate the derivatives  $f_x(n,m)$ ,  $f_y(n,m)$ ,  $f_{xx}(n,m)$ ,  $f_{yy}(n,m)$ , and  $f_{xy}(n,m)$ . Apply these operations to the test image. Use  $\varsigma = 4$



<sup>2</sup> We silently assume that  $\varsigma \geq 1$  from now on.

Knowing that the edge segments are vertically aligned, which of these images convey information about these segments, and what kind of information is that? Which of them do not convey information in this particular case?

The  $f_x$  contains information about how much the image changes in the horizontal direction. You can observe one dark blue region regarding the change from light to dark color, aside you can observe a yellow region regarding the change from dark to light color, which could characterize one edge.

The  $f_{xx}$  contains information about the change of the change, that is shows us the abrupt change with value of zero.

About that, the  $f_y$  and  $f_{yy}$  don't give to us any information about the vertical edge segments

#### 4. Gradient magnitude and Laplacian

The gradient magnitude of an image is defined as:

$$|\nabla f(n,m)| = \sqrt{f_x^2(n,m) + f_y^2(n,m)}$$

or, in short,  $\sqrt{f_x^2 + f_y^2}$ . The Laplacian is defined as:

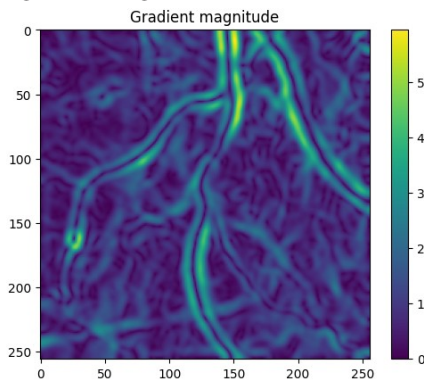
$$\nabla^2 f(n,m) = f_{xx}(n,m) + f_{yy}(n,m)$$

or shortly  $f_{xx} + f_{yy}$ .

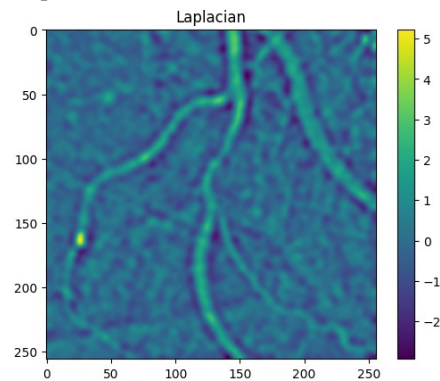
Using the results from question 3, calculate the gradient magnitude and the Laplacian of the test image.

**Don't use for-loops.** Use `scipy.ndimage`.

gradient magnitude



Laplacian:



How can the gradient magnitude on one hand and the Laplacian images on the other hand, be used to locate the edges?

It is possible to observe in the images that the edges are requested in both transformations. The gradient magnitude shows the biggest value for an abrupt change but is not very precise. While the Laplacian carries in the zeros of the image the center of the edge but is very sensitive to noise.



## 5. Marr-Hildreth's zero crossings

In the continuous domain the equation

$$f_{xx}(x,y) - f_{yy}(x,y) = 0$$

defines, under mild conditions, curves in the  $x,y$ -plane. These curves are called the *zero crossings* of the Laplacian.

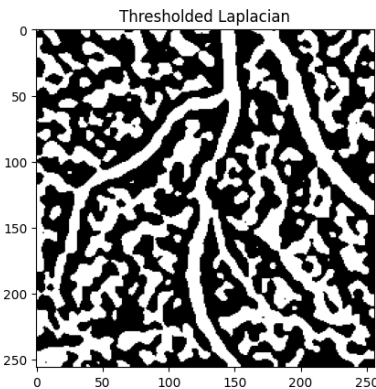
The goal of the Marr-Hildreth operation is to find an edge map. The edge map is a binary image in which all found edge elements are marked by 1 where all other pixels are 0. Marr and Hildreth mark each pixel as an edge element if the Laplacian exhibits a zero crossing near that pixel position.

A cheap zero crossing detection is accomplished by first finding areas with a positive Laplacian, and then to find the boundary of these areas. We apply this procedure to the Laplacian of the test image obtained in question 4:

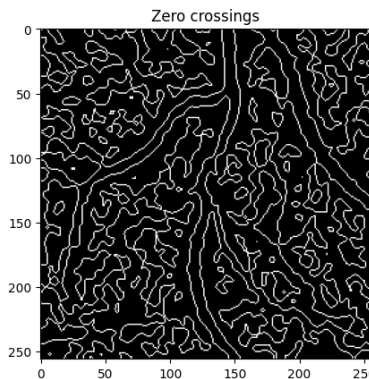
- Create a binary image in which a pixel is set to 1 if the corresponding Laplacian for that pixel is positive. (**Don't use for-loops!**)
- Determine the boundary pixels of the binary image, and display it on the screen. For that purpose:
  - Create a structuring element consisting of a 4-neighbourhood (use `strel` with option `diamond`.)
  - Erode the binary image by this structuring element, so that only the interior of the foreground is left.
  - Subtract this interior from the original binary image.

Note: `bwmorph` with option `'remove'` does almost the same job but is still less useful as it operates differently on the border of the image).

5a Thresholded Laplacian:



5b zero crossings Laplacian:



- This method for finding the zero crossings introduces a bias (a systematic error). Explain why.

The erosion and subtract reduce the real size of the object.

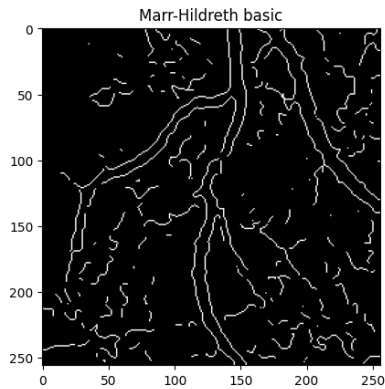
## 6. Hysteresis thresholding the gradient magnitude

The Marr-Hildreth operator produces many candidate edge elements, but many of them will be false. The reason for this is that the Laplacian may exhibit a zero crossing at positions for which the gradient magnitude is near zero. So, the condition for a zero crossing to be an acceptable edge element is that the gradient magnitude exceeds some suitable threshold.

- Determine the gradient magnitude as in question 4. Mask this image by the Marr-Hildreth zero crossings obtained in question 5. That is, all gradient magnitude pixels that are not a zero crossing should be made zero, while all other pixels are retained.
- Compare the masked gradient magnitude image against a suitable threshold  $T$ ; and show an edge map consisting of zero crossings for which the gradient magnitude is larger than  $T$ . Show the result on the screen. You can interactively select the threshold by visually evaluating the edge map. A useful tool is provided by `imcontrast`.

Found threshold: 20% (1.19)

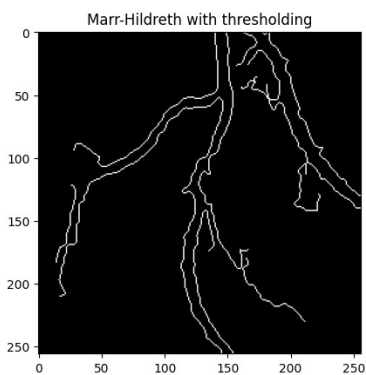
Insert found edge map:



Hysteresis thresholding is an improvement of this simple threshold operation. In hysteresis thresholding, the gradient magnitude is first masked by the zero crossings (so as to exclude all pixels which are not a zero crossing, as before). Next, the result is thresholded twice. In the first threshold operation, the threshold is larger than in the second threshold operation. The first map, called the `marker`, may miss many weak edges, but there will be no (or at least only a few) false edges. The second map, called the `mask`, contains both weak and strong edges, but also many false edges. The strategy is to accept an edge segment in the second image only if such an edge segment contains at least one edge element from the first edge map. Thus, the `marker` will be propagated into the `mask` to yield a more reliable edge map.

- c) Apply a threshold operation with a threshold that is somewhat larger than the threshold `T`. Name the resulting image `marker`.
- d) Apply a threshold operation with a threshold that is somewhat smaller than the threshold `T`. Name the resulting image `mask`.
- e) Propagate the `marker` image into the `mask` image, and interpret the result. **Hint:** use the function `imreconstruct`.

Resulting edge map:



Describe the (potential) advantages of hysteresis thresholding:

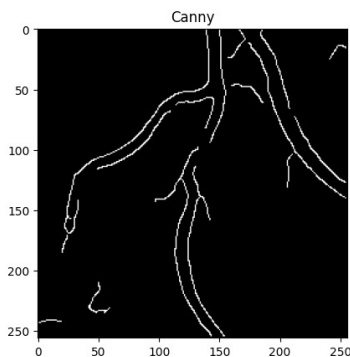
Hysteresis thresholding handles better noise, getting only the strong edges and connecting with weak edges.

## 7. Edge detection

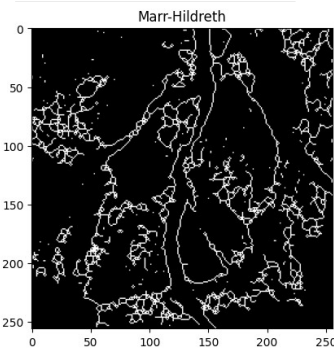
The Marr-Hildreth, including hysteresis thresholding is implemented in the function `ut_edge`. This function uses the function `ut_levelx` to find the zero crossings. `ut_levelx` implements a more accurate method to find the zero crossings than the one outlined in question 4. `ut_edge` also contains an implementation of the Canny edge detector.

- a) Apply `ut_edge` to the test image in order to get an edge map in which the edge elements form two non-fragmented edge segments. Select the options of `ut_edge` as follows:
- ☒ Set the option 'c' That is, apply Canny. Later we will also use 'm' (Marr-Hildreth)
  - ☒ Use option 's' to set the scale. For now use  $\varsigma = 3$ .
  - ☒ Use the option 'h' (hysteresis thresholding). The thresholds are specified as two additional parameters. Read the help of the function to understand how these parameters are defined. Select them interactively such that two non-broken edge segments arise. Perhaps, you need some iterations to establish the right thresholds.

Canny edge map:



Marr Hildreth edge map:



Describe the main difference between Marr-Hildreth and Canny edge detectors considering the following aspects: Localization criteria, sensitivity to noise and edge connectedness.

Marr-Hildreth has good edge localization, but is very sensitive to noise and can sometimes produce disconnected edges.

On the other hand, Canny provides more pos-processing steps, which provides better edge localization, is less noise-sensitive, and produces better edge connections

Insert m-code (note a copy-and- paste of this code to Matlab's native editor, and a single 'run' should suffice to run the whole code):

```
import numpy as np
import scipy
from skimage import morphology, feature
import cv2 as cv
from math import ceil
import matplotlib
import matplotlib.pyplot as plt

# From lecture notes
def psf2otf(psf, shape):
    M, N = shape
    # get size of psf
    sz = np.shape(psf)
    # calculate needed paddings
    n = N - sz[1]
    m = M - sz[0]
    n1 = int(np.floor(n / 2) + np.mod(n, 2))
    n2 = int(np.floor(n / 2))
    m1 = int(np.floor(m / 2) + np.mod(m, 2))
    m2 = int(np.floor(m / 2))
    # pad array with zeros
    psf = np.pad(psf, ((m1, m2), (n1, n2)))
    # shift origin
    psf = np.fft.ifftshift(psf)
    # apply DFT
    otf = np.fft.fft2(psf)
    return otf

# from https://stackoverflow.com/a/46892763
def gkern(sigma=5, kernlen=None):
    """Returns a 2D Gaussian kernel array."""
    kernlen = kernlen or (2*ceil(4*sigma)+1) # pdf = 0.95
    gkern1d = scipy.signal.windows.gaussian(kernlen, std=sigma).reshape(kernlen, 1)
    gkern2d = np.outer(gkern1d, gkern1d)
    return gkern2d

# Based on from https://docs.opencv.org/4.x/de/dbc/tutorial_py_fourier_transform.html
def fft2(img):
    dft = np.fft.fft2(img)
    dft_shift = np.fft.fftshift(dft)
    magnitude_spectrum = 20*np.log(np.abs(dft_shift))

    return dft_shift, magnitude_spectrum

def ifft2(dft_shift):
    dft = np.fft.ifftshift(dft_shift)
    idft = np.fft.ifft2(dft)
    img_back = np.abs(idft)

    return img_back

def plot_psf_3d(psf, title = None):
    kernlen = psf.shape[0]

    x = np.linspace(-kernlen/2, kernlen/2, kernlen)
    y = np.linspace(-kernlen/2, kernlen/2, kernlen)

    X, Y = np.meshgrid(x, y)

    fig, ax = plt.subplots(subplot_kw={"projection": "3d"})
    surf = ax.plot_surface(X, Y, psf, cmap=matplotlib.cm.coolwarm,
                          linewidth=0, antialiased=False)

    fig.colorbar(surf, shrink=0.5, aspect=5)

    if title is not None:
        ax.set_title(title)

    plt.show()
```

Insert m-code (note a copy-and- paste of this code to Matlab's native editor, and a single 'run' should suffice to run the whole code):

```
def main():
    img = cv.imread('ang2.png', cv.IMREAD_GRAYSCALE)

    # convert image to float
    img = np.float32(img) / 255.0

    plt.imshow(img, cmap='gray')
    plt.show()

    psf = gkern(5)
    plot_psf_3d(psf, 'Gaussian kernel')

    sigma = [1, 5, 10, 20]

    fig, axs = plt.subplots(2, 2, figsize=(13, 13))
    for i, s in enumerate(sigma):
        psf = gkern(s)
        img_blur = cv.filter2D(img, -1, psf)

        ax = axs[i//2, i%2]
        ax.imshow(img_blur, cmap='gray')
        ax.set_title(f'sigma = {s}')
        ax.axis('off')

    dx = np.array([[1, 0, -1]])
    dy = dx.T

    psf = gkern(3)
    otf = psf2otf(dy, img.shape)

    print(f"max real: {np.max(otf.real)}")
    print(f"max imag: {np.max(otf.imag)}")

    mtf = np.abs(otf)

    plot_psf_3d(otf.imag, 'Imaginary part of Hy')
    plot_psf_3d(otf.real, 'Real part of Hy')

    plt.show()

    psf = gkern(3)
    psf_dx = cv.filter2D(psf, -1, dx)
    psf_dy = cv.filter2D(psf, -1, dy)

    psf_dxx = cv.filter2D(psf_dx, -1, dx)
    psf_dyy = cv.filter2D(psf_dy, -1, dy)
    psf_dxy = cv.filter2D(psf_dx, -1, dy)

    plot_psf_3d(psf_dx, 'hx gaussian kernel')
    plot_psf_3d(psf_dy, 'hy gaussian kernel')
    plot_psf_3d(psf_dxx, 'hxx gaussian kernel')
    plot_psf_3d(psf_dyy, 'hyy gaussian kernel')
    plot_psf_3d(psf_dxy, 'hxy gaussian kernel')

    img_dx = cv.filter2D(img, -1, psf_dx)
    img_dy = cv.filter2D(img, -1, psf_dy)
    img_dxx = cv.filter2D(img, -1, psf_dxx)
    img_dyy = cv.filter2D(img, -1, psf_dyy)
    img_dxy = cv.filter2D(img, -1, psf_dxy)

    def plot_img(img, title):
        plt.imshow(img)
        plt.colorbar()
        plt.title(title)
        plt.show()

    plot_img(img_dx, 'fx')
    plot_img(img_dy, 'fy')
    plot_img(img_dxx, 'fxx')
    plot_img(img_dyy, 'fyy')
    plot_img(img_dxy, 'fxy')
```

Insert m-code (note a copy-and- paste of this code to Matlab's native editor, and a single 'run' should suffice to run the whole code):

```
# Gradient magnitude
img_grad = np.sqrt(img_dx**2 + img_dy**2)

plt.imshow(img_grad)

plt.title('Gradient magnitude')
plt.colorbar()
plt.show()

# Laplacian
img_lap = img_dxx + img_dyy

plt.imshow(img_lap)
plt.colorbar()
plt.title('Laplacian')

img_lap_thresholded = img_lap > 0

plt.imshow(img_lap_thresholded, cmap='gray')
plt.title('Thresholded Laplacian')

struct = morphology.diamond(1)
eroded = morphology.binary_erosion(img_lap_thresholded, struct)
img_lab_zero_crossings = img_lap_thresholded ^ eroded

plt.imshow(img_lab_zero_crossings, cmap='gray')
plt.title('Zero crossings')

# Marr-Hildreth

img_grad_zero_crossings = img_grad * img_lab_zero_crossings

plt.imshow(img_grad_zero_crossings)
plt.colorbar()
plt.title('Marr-Hildreth')

img_max = img_grad.max()
threshold_basic = img_max*0.2

print(threshold_basic)
img_marr_hildreth_basic = img_grad_zero_crossings > threshold_basic

plt.imshow(img_marr_hildreth_basic, cmap='gray')
plt.title('Marr-Hildreth basic')
plt.show()

low_threshold = img_max*0.1
high_threshold = img_max*0.5
print(low_threshold, high_threshold)

img_marr_hildreth_marker = img_grad_zero_crossings > high_threshold
img_marr_hildreth_mask = img_grad_zero_crossings > low_threshold

img_marr_hildreth_thresholded = morphology.reconstruction(img_marr_hildreth_marker, img_marr_hildreth_mask)

plt.imshow(img_marr_hildreth_thresholded, cmap='gray')
plt.title('Marr-Hildreth')
plt.show()

img_canny = feature.canny(img, sigma=3, low_threshold=low_threshold, high_threshold=high_threshold)

plt.imshow(img_canny, cmap='gray')
plt.title('Canny')
plt.show()

if __name__ == '__main__':
    main()
```



Insert m-code (note a copy-and- paste of this code to Matlab's native editor, and a single 'run' should suffice to run the whole code):