Commodity Exporters, Heterogeneous Importers, and the Terms of Trade

Joris Hoste* KU Leuven

Guilherme Tonsig Teijeiro† Stockholm School of Economics

November 22, 2022

[Preliminary version, please do not circulate]

Abstract

How important are terms-of-trade shocks relative to total-factor-productivity shocks as a source of consumption volatility in commodity-exporting economies? We develop a tractable version of Gopinath & Neiman (2014) with segmented financial markets and realistic real exchange rate determination and provide a bridge to the standard frictionless small-open economy (SOE) framework. We have two main results. First, we show how the differences between the models are captured by two partial elasticities for which we provide analytical expressions. Second, we show that a combination of these two partial elasticities determines the relative importance of shocks to the terms of trade independently of assumptions on market structure, returns to scale, selection into importing, and financial markets. We calibrate the economy to Chilean and Colombian customs firm-level data to show that the terms of trade are at least two times more important than in the standard SOE framework: thirty-four percent of this difference is accounted for by monopolistic competition, sixty-two by increasing returns to importing, and only four percent by firm heterogeneity and selection. However, we show that the latter are crucial in capturing moments of the microdata such as the slope of the sub-intensive margin of trade adjustment and the distribution of imports.

JEL codes: E13, E32, F32, F41 and F44

Keywords: Business cycles, Trade adjustment, Terms-of-trade and Commodity prices

*Electronic Adress: joris.hoste@kuleuven.be

†Electronic Adress: guilherme.tonsiggarciateijeiro@phdstudent.hhs.se

1 Introduction

An important subject in international business cycle research is parsing the importance of different fundamental shocks in generating economic fluctuations. The less our models rely on unobserved and exogenous shocks to explain the volatility in the data, the more useful they become (Abramovitz (1956); Cochrane (1994)). In the particular case of developing and commodity-exporting economies, this led to a large literature that evaluates how important are terms-of-trade shocks relative to productivity shocks. In this paper, we reevaluate this question in light of recent literature that emphasizes the importance of the firm-level structure of trade in explaining the mechanics of trade adjustment at business cycle frequency.

We analyze the sources of business cycle fluctuations in middle-income economies that are commodity exporters, that is, countries that both import and produce a diversified set of manufactured goods and services, yet depend on the export of a few primary products as a means of obtaining foreign currency to finance their imports. In particular, we develop a theoretical framework of imperfectly competitive and heterogeneous manufacturing firms that face per-variety fixed costs to import akin to Gopinath & Neiman (2014). It consists of an upstream manufacturing sector and a downstream services sector that both use labor and intermediate inputs to produce, and a commodities sector that is modeled as an endowment flow. It addresses the missing heterogeneity in firm-level trade adjustment to shocks relative to a frictionless baseline model and we use it to study how consumption differently responds to terms-of-trade shocks in the complete model.

This heterogeneity exists in multiple dimensions. Importers are on average larger firms (in sales volume), employ more workers, pay higher wages and generate more value-added per worker. In addition, firms that import more also import a more diversified set of goods, and from a larger set of foreign sources. These differences imply that firms adjust their imports in different margins. Larger firms are less likely to stop importing altogether (extensive margin) and less likely to stop importing particular goods altogether (sub-extensive margin). Taken together these facts imply that larger firms are more exposed to variations in the nominal exchange rate and the shocks that affect it, including shocks to the price of commodities. Consequently, it stands to reason that these shocks will lead to different responses of aggregate variables in models that accommodate these sources of heterogeneity. These are the issues with which we grapple in this paper.

Our first contribution is to develop a tractable general-equilibrium version of Gopinath & Neiman (2014) and to show that, up to a first-order approximation, the dynamics of the complete model and frictionless benchmark are represented by the same set of equations in which the models differ only in the quantitative expression of two partial-equilibrium elasticities that depend on deep parameters. The same is true for all specifications in between the two extreme cases, that is, we show that each modification of the benchmark model leading to the complete model (e.g. monopolistic competition, increasing returns to importing and selection) can be studied separately using the same structural equations.

Our second contribution is to provide a limiting result that shows that the importance of terms-of-trade shocks relative to productivity shocks for consumption volatility depends on one general-equilibrium elasticity that is a simple function of the two partial-equilibrium elasticities that make up the general structure. This limiting result does not depend on the particular asset market structure and holds for most of the popular incomplete asset market assumptions in the literature.[1] In addition, we show how the debt-elastic-interest-rate equation that is standard in the SOE literature and the segmented financial markets setup of Itskhoki & Mukhin (2021) lead to the same UIP

[1] More specifically, this result holds for financial autarky, segmented financial markets as in Itskhoki & Mukhin (2021), a noncontingent domestic currency bond and a noncontingent foreign currency bond. This result does not hold for complete financial markets, as perfect international insurance would eliminate all volatility in private consumption in an SOE.

equation, but under the segmented financial markets setup, this UIP equation is micro-founded and can be used to study the nominal exchange rate and the price index separately.

We tailor the analysis to commodity-exporting countries for two reasons. First, these countries are particularly exposed to shocks to their terms of trade (Kohn et al. (2021)) as fluctuations in the prices of a few commodities bare a significant impact on their trade balance, their nominal exchange rate, and the ability of domestic producers to import (Chen et al. (2010)). Second, we avoid the shortcomings in the measurement of the terms of trade voiced in Schmitt-Grohé & Uribe (2018). Fluctuations in export prices for a limited set of commodities are readily observed and our model of importing and production leads to theory-consistent aggregation of individual import prices.

The benchmark perfect competition model is composed of a manufacturing sector and a services sector with representative producers. The manufacturing firm produces according to a constant returns-to-scale technology that combines labor and an input bundle consisting of domestic and foreign inputs. Services are produced by combining labor and output from the manufacturing sector under constant returns-to-scale as well. To understand the contribution of each friction present in the complete model vis-a-vis the benchmark model, we move from the latter to the former in three steps. First, we consider the role of monopolistic competition in the manufacturing sector, which distorts the relative price of domestic and foreign intermediate inputs. Second, we add increasing returns to scale to importing by adding a love-for-variety aggregator on the imported intermediate input bundle, while requiring manufacturing producers to pay a constant fixed cost per imported variety in domestic labor[2]. Finally, we consider the role of heterogeneity and endogenous selection in importing.

Each friction has different effects on goods and labor market equilibrium relations in the model. However, as mentioned before, up to a first-order approximation all setups can be summarized in a general structure described by two partial-equilibrium elasticities. The first elasticity measures to what extent final consumption responds to changes in the openness of the economy[3]. The second elasticity captures the degree of expenditure switching in the economy following a change in the real exchange rate or, in other words, how the real exchange rate responds to the openness of the economy. Importantly, even though we impose constant fixed costs per imported variety and resort to a specific productivity distribution (which is not the case in Gopinath & Neiman (2014)), we show that the model still captures salient empirical firm-level trade-adjustment patterns, including the slope of the relative importance of the sub-intensive margin in the importer-size distribution and the Generalized Pareto Distribution of imports.

By embedding the production structure in a specific asset market structure, we close the model and use it to understand whether its different versions yield different results regarding the importance of terms-of-trade shocks in general equilibrium. We show that the contribution of different frictions depends on two competing effects. On the one hand, domestic distortions in the manufacturing sector lead to more labor being allocated to services in equilibrium, which reduces the exposure of final consumption to the terms of trade. On the other hand, domestic distortions increase the incentives for manufacturing producers to import, which increases exposure.

We show that when the economy is relatively open in equilibrium the second effect dominates and the share attributed to the terms of trade relative to productivity in consumption volatility rises. In the limiting case where terms-of-trade shocks and productivity shocks approach random walks the difference in the relative importance of terms-of-trade is summarized in terms of the two partial-equilibrium elasticities[4]. Different asset market structures only differ in the importance attributed to interest-rate shocks in the overall variance of consumption and do not alter the relative weight of productivity vis-a-vis the terms of trade. Put differently, while the assumptions on

[2]Gopinath & Neiman (2014) shows that this friction is essential to capture the rising importance of the sub-intensive versus the sub-extensive margin in the firm size distribution.

[3]Openness is defined as a variable proportional to the ratio of imports to total consumption.

[4]We think this is a relevant case in light of commodity price data and of the literature on emerging-market TFP processes (Aguiar & Gopinath (2007)).

financial markets change risk-sharing patterns and characterize the moments of the real exchange rate (e.g. excess volatility, disconnect, etc.), assumptions on technology and market structure determine how well the model matches facts of firm-level adjustments to external shocks.

Finally, we calibrate the model to macro and micro data from Chile and Colombia and we find that the terms of trade are two to five times more important than in standard small open economy (SOE) frameworks: thirty-four percent of this difference is accounted for by monopolistic competition, sixty-two by increasing returns to importing, and only four percent by firm heterogeneity and selection. While heterogeneity and selection are crucial to match cross-sectional patterns in trade adjustment, it is inconsequential for the relative importance of terms-of-trade shocks relative to productivity shocks. Accounting for monopolistic competition and increasing returns to importing substantially increase the importance of terms of trade shocks and are crucial to match the aggregate fifty-fifty split between the sub-intensive and sub-extensive trade adjustment margins.

Our paper is related to three strands of literature. The first broadly discusses business cycles in developing economies and tries to understand their main drivers. Stationary productivity shocks, terms-of-trade shocks, and interest rate shocks all seem to be contributing factors to output, consumption, investment, and real-exchange-rate volatility. Kydland & Zarazaga (2002) and Aguiar & Gopinath (2007) stress the importance of (non-)stationary productivity shocks in developing markets, while García-Cicco et al. (2010) point that these shocks have implausible implications for the dynamics of the trade balance and that calibration on a longer period attributes virtually no variation in output fluctuations to them.

Mendoza (1995) was the first to study the importance of terms-of-trade shocks through the lens of an IRBC model. While it attributed around 50% of output volatility to the terms of trade, other studies have found numbers ranging from less than 10% to over 80% (see Drechsel & Tenreyro (2018); Fernández et al. (2018); García-Cicco et al. (2010); Kohn et al. (2021); Kose (2002)). While all these papers discuss results for different countries and periods, they all use the same perfect competition benchmark with incomplete yet integrated financial markets. Schmitt-Grohé & Uribe (2018) point that in a cross-section of countries the correlation between the decomposed variance from the benchmark model and an SVAR model is low. We depart from this literature by integrating models that are successful in describing firm-level importing patterns and the dynamics of the real exchange rate.

The second branch of literature to which we contribute studies the micro-structure of trade. A growing number of papers document the role of heterogeneous trade adjustment to shocks and study when and to what extent these lead to aggregate effects which are not present in representative agent frameworks. Kehoe & Ruhl (2008) shows how shocks to the terms of trade cannot have first-order productivity effects in a neoclassical setting. Amiti & Konings (2007); Blaum et al. (2018); Goldberg et al. (2010); Gopinath & Neiman (2014); Halpern et al. (2015) answer this challenge by introducing increasing returns to importing and heterogeneity, thus creating an endogenous connection between the terms of trade and aggregate productivity.

This literature stresses how importing firms adjust to adverse terms-of-trade movements in two margins: by importing less of each previously imported product variety (sub-intensive margin) and by importing fewer varieties (sub-extensive margin). We contribute to this literature in two ways. First, we study well-identified commodity-price shocks which we show to be the dominant source of terms-of-trade movements in the data, instead of proxying for nominal-exchange-rate shocks using import prices. Second, we provide a tractable general equilibrium framework that allows researchers to decompose differences between frameworks friction-by-friction and to determine which are the most important channels through which models differ.

Finally, we are tangentially related to the literature on commodity currencies. Chen & Rogoff (2003) finds that commodity prices have a strong impact on the real exchange rate in commodity exporters and Chen et al. (2010)

shows that exchange rates can predict commodity prices through a forward-looking expectations channel. Fernández et al. (2018) and Känzig (2021) show how the real exchange rate tends to appreciate following a commodity price shock. Meese & Rogoff (1983), Engel & West (2005) and Rossi (2013) discuss the random walk behavior of the real exchange rate, while Itskhoki & Mukhin (2021) develop a model that is successful in matching this real exchange rate behavior. We provide tractable expressions for the general-equilibrium elasticities of the real exchange rate to productivity shocks, import-price shocks, commodity-price shocks and interest-rate shocks including in the Itskhoki & Mukhin (2021) framework.

The paper is structured as follows. In the next section, we provide the stylized facts that the model needs to match and motivate our modeling choices. Section 3 develops the theoretical model we use to analyze the contribution of different shocks to consumption volatility. Section 4 provides our two main theoretical results and 5 shows that the model developed in section 3 matches the stylized facts from 2. Section 6 and 7 contain the qualitative and quantitative comparison of the different models and section 8 concludes.

2 Data and Stylized Facts

Before describing the theoretical framework, we provide four stylized facts regarding trade adjustment and exchange rate determination in commodity-exporting countries and use these facts to motivate our modeling decisions.

2.1 Data Sources

We use firm-level manufacturing surveys and trade data from Chile and Colombia, which are good examples of economies we want to capture in our theoretical framework. In particular, they are middle-income countries with an imbalanced profile of imports and exports and high volatility in their terms of trade. The firm-level manufacturing data comes from the Colombian survey of manufacturing (Encuesta Anual Manufacturera, EAM) and the Chilean industrial survey (Encuesta Nacional Industrial Anual, ENIA) respectively. ENIA has firm-level data on sales, employees, wages and value-added for a sample of Chilean manufacturing producers for the years 2005 to 2019 which is collected and maintained by Instituto Nacional de Estadisticas[5]. EAM is Colombia's manufacturing survey[6] maintained by Departamento Administrativo Nacional de Estadística (DANE) and has data similar to that of ENIA for the years 2007 to 2019.

We also use data on firm-level trade data from both countries. Customs data in Chile is maintained by Servicio Nacional de Aduanas and has information on imports and exports by firms, HS code and country of destination. In Colombia, the customs data is maintained by DANE and has similar information. Finally, we complement the firm-level data with macroeconomic variables such as the terms of trade, real output, real consumption, the trade balance, the real exchange rate and the nominal exchange rate. This data comes from Banco de la Republica in Colombia[7] and from the Chilean Central Bank[8].

2.2 Stylized Facts

We show that Chile and Colombia (1) export a small set of goods and import a diversified set of goods; (2) their trade adjustment is dominated by the intensive margin, but that there is variation in the importance of the sub-intensive

[5]https://www.ine.cl/estadisticas/economia/industria-manufacturera/estructura-de-la-industria
[6]https://www.dane.gov.co/index.php/estadisticas-por-tema/industria/encuesta-anual-manufacturera-enam
[7]https://www.banrep.gov.co/es
[8]https://www.bcentral.cl/inicio

margin across the firm-size distribution (Gopinath & Neiman (2014), Blaum et al. (2018)); (3) their terms of trade are dominated by changes in commodity export prices; and (4) their terms-of-trade are more volatile than their real exchange rates.

2.2.1 Facts on trade adjustment

We start with the fact that Chile's and Colombia's exports tend to be concentrated in one or two types of commodities, while their imports tend to be much more diversified. We summarize this in the first stylized fact.

Fact 1 (Trade Imbalance)**.** *A few commodities account for the majority of the volume of exports of commodity exporters. Imports are much more diversified.*

Figure 1 illustrates the degree of this imbalance for Chile and Colombia. In particular, we plot their aggregate USD exports and imports over time once including all products and once excluding their main exporting commodity.[9] For both countries, we observe a substantial difference between the export series that includes all exported goods and the one that excludes their main exported commodity. Both countries' main export commodity accounts for 50% to 80% of total exports at most times. This pattern does not hold for imports. Importantly, these patterns are not confined to Chile and Colombia, as Kohn et al. (2021) shows that this trade imbalance towards commodity exports and differentiated goods imports holds for a wide set of developing countries.

While export adjustment is dominated by a few commodity exports, Gopinath & Neiman (2014) illustrates that import adjustment is heterogenous across importers.

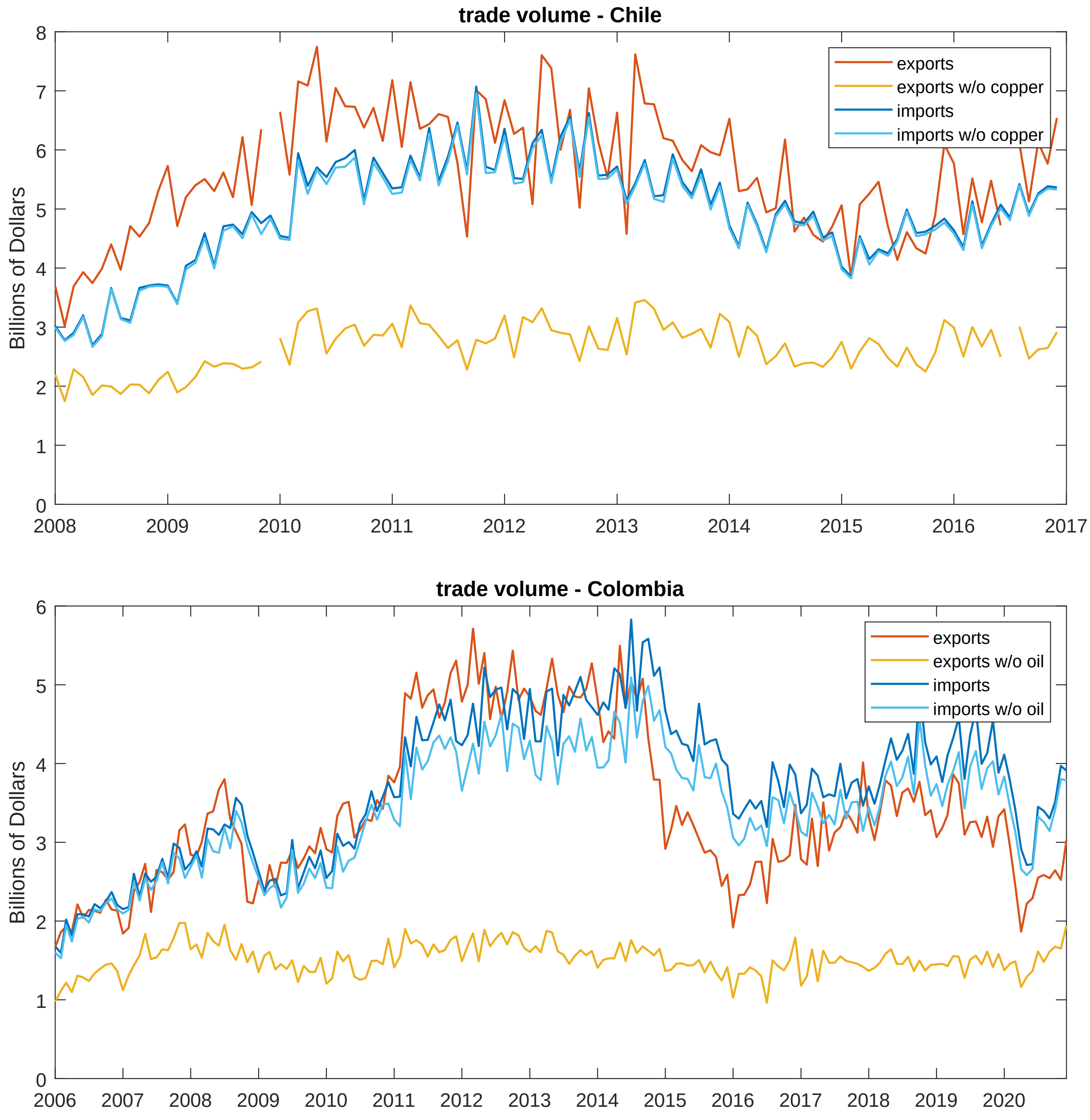
Fact 2 (Import Adjustment)**.** *Aggregate import movements are dominated by the intensive margin, and the sub-intensive margin dominates trade adjustment for large importers.*

It is well-known that there exists large cross-sectional heterogeneity in the extent to which firms engage in international trade. For instance, Bernard et al. (2012) shows that importers tend to be larger sales-wise, that they employ more workers, that they pay higher wages and that their workers generate more value-added on average. Figure (A.1 confirms these importer premia for Chilean and Colombian firms. In addition to being larger and more productive, Figure (A.2) shows that firms that import more in dollars also import a larger number of varieties from a larger set of source countries.

Figure 2 plots the month-over-month percent changes in imported volume and splits this change into a firm-level intensive margin (the extent to which import changes because continuing importers change their imports) and a firm-level extensive margin (the extent to which imports change because firms start or stop importing altogether). In line with Bernard et al. (2009), we find that for both countries the firm-level intensive margin dominates the extensive margin. Gopinath & Neiman (2014) shows how the intensive firm-level margin can be further decomposed into a sub-intensive and a sub-extensive margin as follows

[9] Chile's and Colombia's main exporting commodities are copper and oil respectively.

Figure 1: Aggregate Trade Flows



Notes: Trade volumes in current US dollars. The volumes net of oil excludes the following HS-4 codes: 2709-15, 3403, 3819, 3811 and 3911. The volumes net of copper exclude the following HS-4 codes: 2603, 2825, 2827 and all items under HS-2 74.

$$\frac{\Delta m_t}{m_{t-1}} = \underbrace{\sum_{i\in\Omega_t^f\setminus\Omega_{t-1}^f}\frac{m_{it}}{m_{t-1}} - \sum_{i\in\Omega_{t-1}\setminus\Omega_t^f}\frac{m_{it-1}}{m_{t-1}}}_{\text{extensive margin}}$$

$$+ \underbrace{\sum_{i\in\Omega_t^f\cap\Omega_{t-1}^f}\left[\underbrace{\sum_{j\in\Omega_{it}^p\setminus\Omega_{it-1}^p}\frac{m_{ijt}}{m_{it-1}} - \sum_{j\in\Omega_{it-1}^p\setminus\Omega_{it}^p}\frac{m_{ijt-1}}{m_{it-1}}}_{\text{subextensive margin}} + \underbrace{\sum_{j\in\Omega_{it}^p\cap\Omega_{it-1}^p}\frac{m_{ijt}-m_{ijt-1}}{m_{t-1}}}_{\text{subintensive margin}}\right]}_{\text{intensive margin}}$$

where Ω_t^f is the set of firms imporing in period t, Ω_{it}^p is the set of products imported by i at time t and m_{ijt} is the imported volume of product j by firm i. The sub-intensive margin captures the extent to which firms change their overall imports by importing different amounts of varieties they already import, while the sub-extensive margin measures the extent to which firms change their overall imports by changing the set of varieties being imported. In line with the results for Argentina from Gopinath & Neiman (2014), we find that the sub-intensive and sub-extensive margin explain around 50% each of the overall firm-level intensive margin.

The importance of the sub-intensive and sub-extensive margins differs greatly across the firm-size distribution. Figure 3 illustrates the importance of the extensive margin and the share of the intensive margin that can be attributed to the sub-intensive margin by firm-size percentile. We plot firms increasing and decreasing their imported volumes in separate charts. It is clear that the importance of the extensive margin falls with firm size and is close to zero for the upper tail of the importing distribution. The importance of the sub-extensive margin also falls with firm size, but it turns out that even the largest importers adjust both on the sub-intensive and sub-extensive margin, which explains their equal share in aggregate changes.

2.2.2 Facts about the terms of trade

We now turn to stylized facts that pertain to the sources of volatility in the terms of trade, and its volatility relative to that of the real exchange rate.

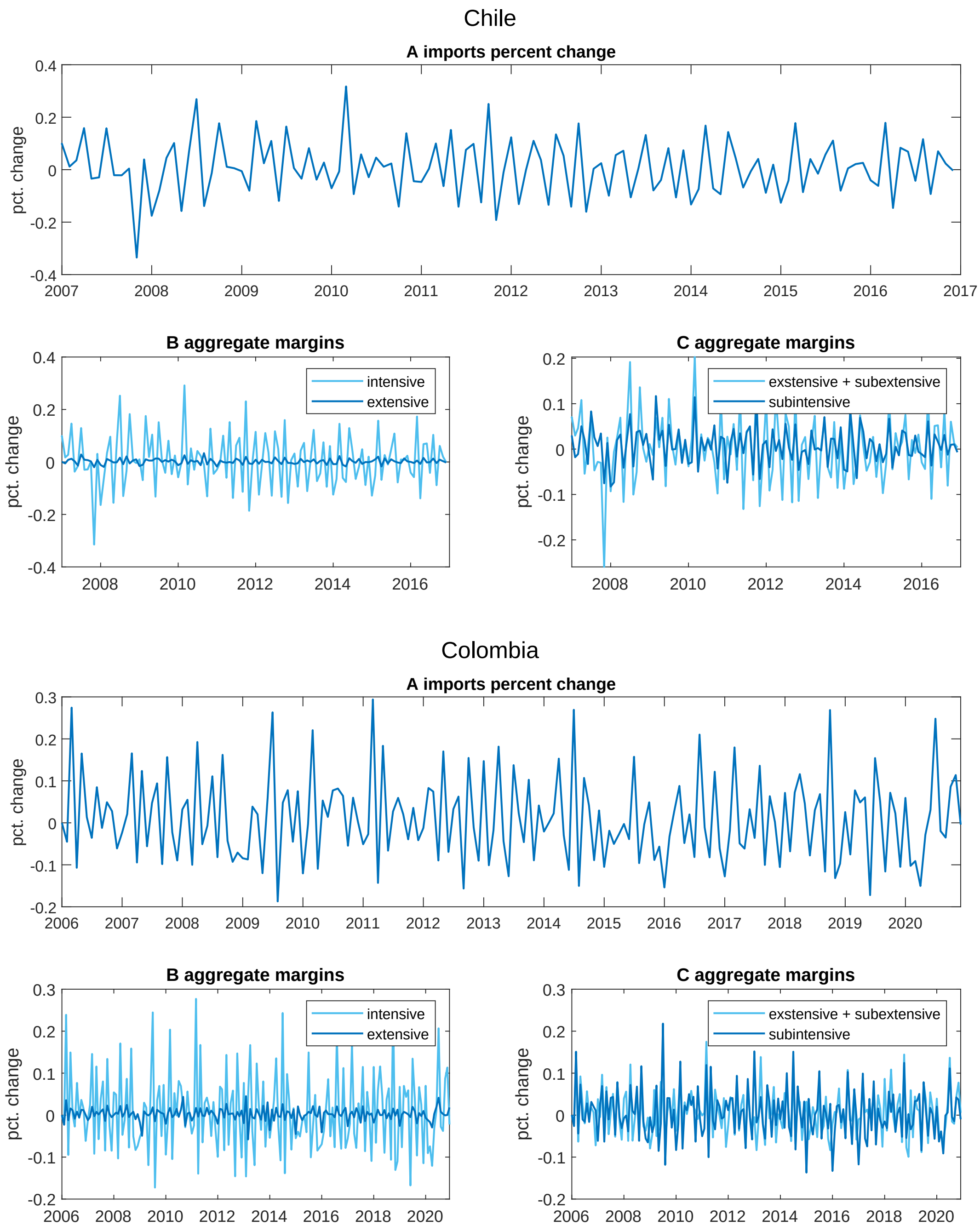
Fact 3 (Export-dominance of the terms of trade)**.** *The terms of trade are highly correlated with export prices and are uncorrelated with import prices. The export price index is dominated by a single commodity.*

Panel (a) of Figure 4 plots the correlation between changes in the terms of trade and changes in the import and export price indices for Chile and Colombia. Most of the variation in the terms of trade is accounted for by the price of exports. Panel (b) plots the correlation between changes in the export price index and the price of the main commodity exported by Chile and Colombia respectively. These figures show that most of the variation in the export price index is accounted for by the price of a single commodity[10]. Since export prices dominate the volatility of the terms of trade, Fact 3 leads us to interpret our model differently from Gopinath & Neiman (2014). Instead of focusing on the price of imports as a proxy for nominal exchange rate movements, we interpret most variation in the terms of trade as resulting from movements in the price of exports.

Fact 4 (Excess volatility of the terms of trade)**.** *The terms of trade are more volatile than the real exchange.*

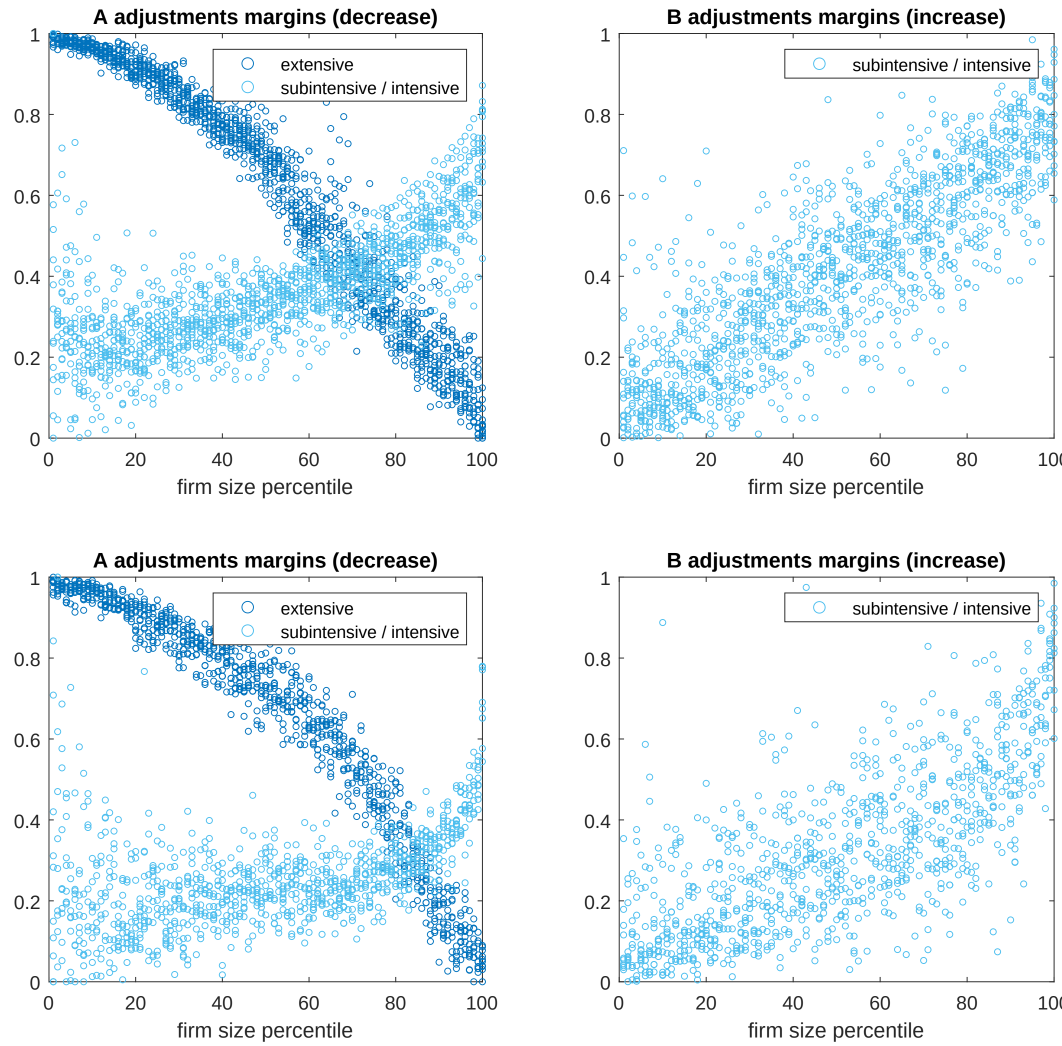
[10]Data for the terms of trade in Chile comes from Bennett et al. (2001).

Figure 2: Trade Adjustment Margins



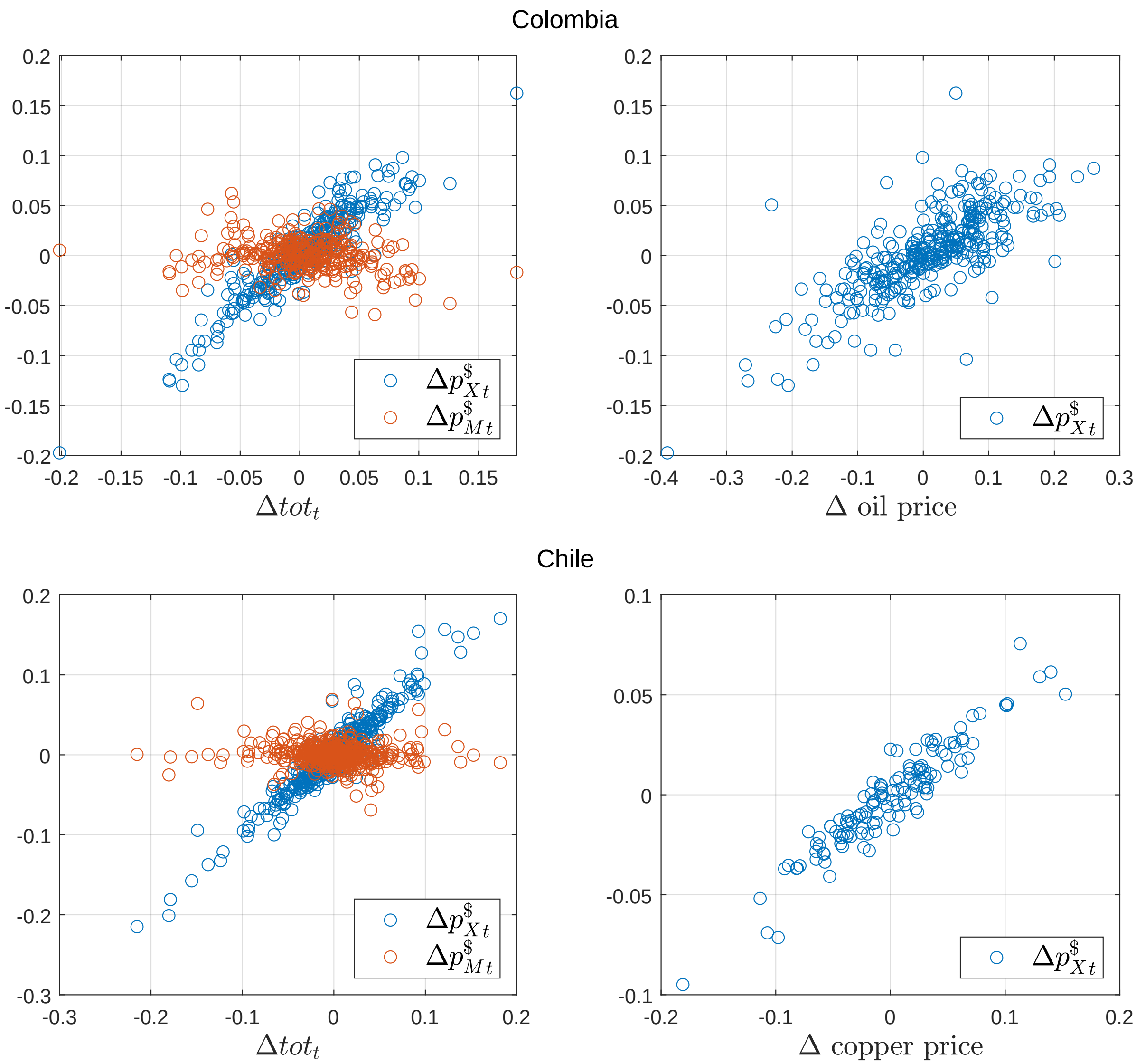
Notes: A) the percent change in import volumes month-by-month; B) the decomposition of the change in import volumes into intensive and extensive margins; and C) the decomposition of changes in imports by sub-extensive and sub-intensive margins.

Figure 3: Sub-intensive and sub-extensive margin in the cross-section



Notes: The sub-intensive margin teased out of the intensive margin. The upper plots illustrate Colombian data, while the lower plots illustrate Chilean data.

Figure 4: Terms-of-trade and Export and Import Prices



Notes: The right-hand panel plot the scatter of monthly changes in the terms of trade against the price indices of exports and imports. The right-hand panel plots the scatter of monthly changes in commodity prices against the export price index. Data cover 1995 to 2021 in Colombia and 1965 to 2000 in Chile.

We provide the time series properties and correlations of consumption, output, the trade balance, the real exchange rate and the terms-of-trade in Table A.1. As in many developing countries, consumption tends to be slightly more volatile than output, and the trade balance is more or less acyclical in Chile and Colombia. In contrast to many developed countries (e.g. Atkeson & Burstein (2008) and Itskhoki & Mukhin (2021)), the terms of trade are two times more volatile than the real exchange rate.

Taking stock, we stress that the facts presented are common across Chile and Colombia and that the ubiquitous nature of these patterns points to the importance of having theoretical frameworks that can account for them. To capture the cross-sectional variation in trade adjustment, a sufficiently rich structure of fixed costs to importing capturing the differing degrees to which importers of different sizes interact with foreign suppliers needs to be embedded in the standard SOE. Since Gopinath & Neiman (2014) shows that the aggregate manufacturing price index becomes much more responsive to changes in input prices, we test whether models that match the micro-moments have significantly different predictions for the macro-moments in general equilibrium.

3 Theoretical Framework

We study a small open economy in which the supply side is composed of three different sectors, namely a services sector, a manufacturing sector and a commodities sector. First, we describe how the final consumption good is produced by a services sector that uses labor and manufacturing inputs. Second, we describe a manufacturing sector that uses labor and an intermediate input bundle to produce. Third, we model the commodity sector as an endowment stream. Finally, we embed this production structure in a dynamic general equilibrium model which allows consumers to share risk with the rest of the world through segmented financial markets intermediated by risk-averse agents.

3.1 The supply side

3.1.1 The services sector

The services sector consists of a representative firm that combines intermediate inputs from the domestic manufacturing sector and labor and produces according to the following production function

$$Y_{St} = A_{St} {L_{St}}^{1-\mu} {X_{St}}^{\mu} \qquad \text{where} \qquad X_{St} = \left(\int_i {X_{Sit}}^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}$$

where the intermediate input bundle X_{St} is a CES bundle of domestic varieties produced by the manufacturing sector, μ is the parameter that determines the intermediate input share in the production of services and σ is the elasticity of substitution between varieties. The first-order conditions that determine labor and input demand are the following

$$L_{St} = (1-\mu)\frac{P_{St}}{W_t} Y_{St}, \qquad X_{St} = \mu \frac{P_{St}}{P_{Dt}} Y_{St} \qquad \text{and} \qquad X_{Sit} = \left(\frac{P_{it}}{P_{Dt}} \right)^{-\sigma} X_{St} \tag{3.1}$$

where P_{Dt} is the price index of domestic manufactured goods, P_{it} is the price of each individual variety and W_t are nominal wages paid to workers. The representative firm is assumed to operate in perfect competition, which together with the production function determines the price index of services[11].

$$P_{St} = \frac{1}{A_{St}} \frac{{W_t}^{1-\mu} {P_{Dt}}^{\mu}}{(1-\mu)^{1-\mu}\mu^{\mu}} \tag{3.2}$$

3.1.2 Manufacturing Sector

The domestic manufacturing sector combines labor and intermediate inputs which are either produced at home or imported. In transitioning from a frictionless model to the complete heterogeneous firm model, we consider four different setups of the manufacturing sector: (1) one with homogeneous firms in a perfectly competitive market; (2) one with homogeneous firms in monopolistic competition; (3) one with homogeneous firms in a monopolistic market with increasing returns to importing; finally, (4) one with heterogeneous firms which self-select into importing as in Gopinath & Neiman (2014).

Technology There is a continuous, unit measure of domestic manufacturing firms indexed by i. Domestic firms produce using the following Cobb-Douglas production function

$$Y_{it} = A_{Dt}\varphi_i {L_{Dit}}^{1-\gamma} {X_{Dit}}^{\gamma}$$

where firm i's productivity level is a combination of its time-invariant productivity φ_i and A_{Dt} which is a sector-level productivity shock. Manufacturing output Y_{it} is produced by combining productive labor L_{Dit} and intermediate inputs X_{Dit} where γ is the input elasticity in production. The intermediate input bundle is a CES aggregate of foreign and domestic intermediate input bundles

$$X_{Dit} = \left(\omega^{\frac{1}{\varepsilon}} {Q_{Dit}}^{\frac{\varepsilon-1}{\varepsilon}} + (1-\omega)^{\frac{1}{\varepsilon}} {Q_{Mit}}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

where Q_{Dit} and Q_{Mit} are firm i's domestic and imported inputs respectively. ε determines the degree of substitutability between the domestic and foreign input bundles and ω is a home-bias parameter that determines the degree of roundaboutness in the economy. Finally, domestic and imported input bundles are CES aggregates of individual domestic and foreign intermediate input varieties.

$$Q_{Dit} = \left(\int_j {q_{Dijt}}^{\frac{\sigma-1}{\sigma}} dj \right)^{\frac{\sigma}{\sigma-1}} \quad \text{and} \quad Q_{Mit} = \left(\int_{k \in \mathcal{L}_i} {q_{Mikt}}^{\frac{\theta-1}{\theta}} dk \right)^{\frac{\theta}{\theta-1}} \tag{3.3}$$

The domestic intermediate input bundle aggregates the varieties produced by the domestic manufacturing sector. The quantity used of the output of firm j by firm i is denoted by q_{Dijt} and substitution among these varieties depends on σ. Firms substitute across imported input varieties given an elasticity θ. Following Gopinath & Neiman (2014), we assume that individual imported varieties are indistinguishable from one another in their quality or source. This allows us to assume a common dollar price $P_M^{\$}$ for all imported varieties k, and such that the firm-specific imported intermediate input bundle price is the following.

$$P_{Mit} = E_t P_{Mt}^{\$} |\mathcal{L}_{it}|^{\frac{1}{1-\theta}}$$

[11] Modelling the relation between the services and manufacturing sector as vertical provides a parsimonious way to match the pattern that final consumer prices are much less responsive to nominal exchange rate movements compared to import prices or producer prices (see Burstein et al. (2005) and Burstein & Gopinath (2014)).

and that the firm-specific intermediate input price is

$$P_{Xit} = \left(\omega {P_{Dt}}^{1-\varepsilon} + (1-\omega){P_{Mt}}^{1-\varepsilon}|\mathcal{L}_{it}|^{\frac{1-\varepsilon}{1-\theta}}\right)^{\frac{1}{1-\varepsilon}}$$

In the setups without increasing returns to importing, we assume that $\mathcal{L}_{it} = 1 \; \forall t$, while in setups with increasing returns to importing the measure of imported varieties is chosen optimally.

Market Structure The manufacturing sector sells both to itself and to the services sector. Following a large literature in closed and open economy macroeconomics (Nakamura & Steinsson (2010), Gopinath & Neiman (2014) and Blaum et al. (2018)), we assume that both domestic manufacturing firms and service providers substitute between domestic varieties with the same elasticity σ. This implies that the final demand for manufacturing output is

$$Y_{it} = \left(\frac{P_{it}}{P_{Dt}}\right)^{-\sigma}(X_{St} + Q_{Dt})$$

where $Q_{Dt} \equiv \int_i Q_{Dit}di$ is the total demand for manufacturing output from domestic manufacturers. The domestic manufacturing price index is a CES aggregate of domestic variety prices $P_{Dt} = \left(\int_i {P_{it}}^{1-\sigma}di\right)^{\frac{1}{1-\sigma}}$. We assume that manufacturers compete under monopolistic competition which combined with CES demand for manufacturing output leads to a pricing rule that consists of a constant markup over marginal costs[12].

$$P_{it} = \frac{\sigma}{\sigma-1}\mathrm{MC}_{it}$$

By assuming that the manufacturing sector charges a constant markup over marginal costs, we deviate from the recent literature in international macroeconomics that accounts for pricing-to-market by allowing for more general forms of competition (e.g. Amiti et al. (2019) and Gopinath et al. (2020)). However, in Section 2 we showed that, in contrast to developed economies where the terms-of-trade is less volatile than the real exchange (Atkeson & Burstein (2008)), commodity exporters experience the opposite. For this reason and to obtain tractable expressions for the equilibrium processes, we abstract from pricing to market.

Optimal Input Allocation Conditional on choosing the measure of imported varieties $|\mathcal{L}_{it}|$, we derive the firm's marginal cost function by solving the firm's cost minimization problem. The first-order conditions for optimality in labor and intermediate input use are the following.

$$L_{Dit} = (1-\gamma)\frac{\mathrm{MC}_{it}}{W_t}Y_{it} \quad \text{and} \quad X_{Dit} = \gamma\frac{\mathrm{MC}_{it}}{P_{Xit}}Y_{it} \tag{3.4}$$

Optimal demand for domestic and imported bundles is governed by the first-order conditions of the second-tier problem of manufacturing producers and depends on the elasticity of substitution between input bundles

$$Q_{Dit} = \omega\left(\frac{P_{Xit}}{P_{Dt}}\right)^{\varepsilon}X_{Dit} \quad \text{and} \quad Q_{Mit} = (1-\omega)\left(\frac{P_{Xit}}{P_{Mit}}\right)^{\varepsilon}X_{Dit}$$

[12] In the setup under perfect competition, we evaluate the model in the limit where $\sigma/(\sigma-1) \to 1$ and manufacturing prices are equal to marginal costs.

Finally, the optimal demand for each type of variety is pinned down by the first-order conditions of the third tier of the manufacturing producer's problem.

$$q_{Dijt} = \left(\frac{P_{jt}}{P_{Dt}}\right)^{-\sigma} Q_{Dit} \quad \text{and} \quad q_{Mikt} = \left(\frac{P_{Mkt}}{P_{Mit}}\right)^{-\theta} Q_{Mit}$$

Combining these expressions with the production function, manufacturing firms' marginal cost function conditional on a sourcing strategy $|\mathcal{L}_{it}|$ is the following.

$$\mathrm{MC}_{it}(|\mathcal{L}_{it}|) = \frac{1}{A_{Dt}} \frac{1}{\varphi_i} \frac{{W_t}^{1-\gamma} {P_{Xit}}^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}$$

Optimal Sourcing Decision Without increasing returns to importing, the optimal sourcing strategy is $\mathcal{L}_{it} = 1$. Under increasing returns to importing, firms weigh the benefits of additional imported intermediate input varieties with the fixed costs of accessing them. This fixed cost is paid every period in domestic labor units, such that total fixed costs are $W_t f|\mathcal{L}_{it}|$ where f is the labor requirement per imported variety. Manufacturing firms maximize profits $[P_{it} - \mathrm{MC}_{it}(|\mathcal{L}_{it}|)]Y_{it}$ net of fixed costs. To find an explicit solution for the measure of imported varieties we need to assume that $\varepsilon = \theta$ and that the fixed costs to be paid are linear in the measure, leading to

$$|\mathcal{L}_{it}| = \frac{\omega}{1-\omega} \left(\frac{P_{Mt}}{P_{Dt}}\right)^{\varepsilon-1} \left[\left(\frac{\varphi_i}{\varphi_{Mt}}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}} - 1\right]$$

where φ_{Mt} is the cutoff productivity level defined by equating revenues to fixed costs and under which $|\mathcal{L}_{it}| = 0$[13].

As long as $\gamma(\sigma-1) < \varepsilon - 1$ the measure of imported varieties is increasing in productivity (Appendix B). We use the cutoff definition to re-express input prices as

$$P_{Xit} = {\gamma_{Dit}}^{\frac{1}{\varepsilon-1}} \omega^{-\frac{1}{\varepsilon-1}} P_{Dt} \quad \text{where} \quad \gamma_{Dit} = \begin{cases} \left(\frac{\varphi_{Mt}}{\varphi_i}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}} & \text{if } \varphi_i \geq \varphi_{Mt} \\ 1 & \text{otherwise} \end{cases}$$

where γ_{Dit} is the domestic intermediate input share which is decreasing in φ_i. As long as $\gamma(\sigma-1) < \varepsilon - 1$ holds, higher import prices can induce aggregate productivity effects by reducing the benefits from adding imported intermediate input varieties prices and forcing marginal importers to stop importing.

3.1.3 Commodity Sector

We follow Fernández et al. (2018) and model the commodity sector as an endowment process that is the only source of foreign currency for the economy. We make this assumption for two reasons. First, world commodity prices, e.g. oil prices for Colombia and copper prices for Chile, are plausibly exogenous to the domestic economies[14]. Second, adjusting physical commodity output is often hard to achieve in the short run due to significant time-to-build in

[13] The expression for φ_{Mt} is the following

$$\varphi_{Mt} = \left(\frac{\sigma}{\sigma-1}\right)^{\frac{\sigma}{\sigma-1}} \left[\frac{\gamma}{\varepsilon-1}(1-\omega)^{\gamma\frac{\sigma-1}{\varepsilon-1}} \frac{{P_{Dt}}^{\sigma}(X_{St}+Q_{Dt})}{fW_t}\right]^{-\frac{1}{\sigma-1}} \frac{1}{A_{Dt}} \frac{{W_t}^{1-\gamma}{P_{Mt}}^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}} \left[\frac{\omega}{1-\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{\varepsilon-1}\right]^{\frac{\varepsilon-1-\gamma(\sigma-1)}{(\sigma-1)(\varepsilon-1)}}$$

[14] This claim is supported by the fact that Colombia was only the 20^{th} largest oil producer in 2020 according to the US Energy Information Administration and that Colombia has never been a member of OPEC. Also, Chile account for a little under 10% of the world's raw copper production in 2015 according to the US Geological Survey 2017.

extraction capacity. For these reasons, income from commodity exports is well approximated by an endowment process that keeps physical output fixed in the short run, but accounts for income fluctuations stemming from changes in world commodity prices. In addition, by using these assumptions we discard the reallocation of labor in and out of the commodity sector[15].

3.2 Final Demand, Saving and Financial Markets

We study the SOE frameworks developed in the previous section in combination with three different assumptions for financial markets. First, we study the model in financial autarky. Even though not allowing for any risk-sharing through financial markets is an extreme assumption, it aids in intuition. Second, just as in the prior literature, we study the framework in a setup where the representative consumer has access to an internationally-traded dollar bond. Finally, we consider a segmented financial market structure and the implications of this setup on the contribution of different shocks to consumption volatility[16].

The economy is populated by a representative consumer that consumes services and has access to a one-period risk-free nominal bond. The consumer maximizes lifetime utility subject to a budget constraint and a transversality condition

$$\max_{\{C_{St}\}_{t=0}^{\infty}} U = \sum_{t=0}^{\infty} \beta^t \ln C_{St}$$

$$\text{s.t.} \qquad \frac{B_{t+1}}{R_t} - B_t = W_t L + \Pi_t + E_t P^{\$}_{Xt} X - P_{St} C_{St}$$

$$\lim_{j\to\infty} \mathbb{E}_t \left[\frac{B_{t+j+1}}{\prod_{s=0}^{\infty} R_{t+j}} \right] \geq 0$$

where B_t denotes the nominal international asset position, L is a fixed labor supply, E_t is the nominal exchange rate at time t, Π_t are profits paid out to consumers by firms in the manufacturing sector and $P_{St}C_{St}$ is the total expenditure on services in any given period t. Depending on the financial asset that is available to consumers, the equilibrium conditions will differ.

Financial Autarky In financial autarky the consumer is unable to share risk through financial markets, such that trade needs to be balanced period by period. In this case, the nominal exchange rate will adjust in a way that ensures that commodity exports equal imports of intermediate inputs.

$$TB_t = W_t L + \Pi_t + E_t P^{\$}_{Xt} X - P_{St} C_{St} = 0$$

Integrated Financial Markets Following the long literature that studies the contribution of different types of shocks to consumption volatility (e.g. Mendoza (1995),García-Cicco et al. (2010) and Drechsel & Tenreyro (2018)) we consider a case where the consumer shares risks with the rest of the world (ROW) using a dollar-denominated bond.

[15] For instance, Asker et al. (2019) model oil extraction through a Leontief production function in labor and extractive capital that is pre-determined in the short run. Hence, without additional investment in physical extraction capacity, there is no reallocation of productive labor to the commodities sector.

[16] To understand the contribution and transmission of both domestic and external shocks to consumption and output volatility, we need the model to match key properties of the real and nominal exchange rate processes. In particular, Brandt et al. (2006) shows that the nominal exchange rate is excessively volatile relative to macroeconomic variables, Meese & Rogoff (1983) and Engel & West (2005) show how the nominal exchange rate is largely disconnected from and barely predictable by macroeconomic variables and Backus & Smith (1993) establish that real exchange rates are excessively volatile relative to and not robustly correlated with relative consumption levels across countries.

The first-order conditions for optimal intertemporal substitution imply the following two equilibrium conditions

$$\beta \mathbb{E}_t \left[\frac{C_{St}}{C_{St+1}} \frac{P_{St}}{P_{St+1}} \frac{E_{t+1}}{E_t} R_t \right] = 1$$

$$\frac{E_{t+1} B_{t+1}^{\$}}{R_t} - E_t B_t^{\$} = W_t L + \Pi_t + E_t P_{Xt}^{\$} X - P_{St} C_{St}$$

where the Euler equation describes the optimal intertemporal allocation of consumption over time subject to the budget constraint. Without additional discipline on the interest rate R_t, this model is non-stationary (see Schmitt-Grohé & Uribe (2003)). In line with most papers in this literature, we ensure stationary by assuming a debt-elastic interest-rate equation.

$$R_t = R^{\$} + \chi_2 \left(e^{-(b_{t+1} - \bar{b})} - 1 \right) + \chi_1 \left(e^{\psi_t} - 1 \right)$$

where $R^{\$}$ is the nominal interest on dollar bonds received by foreign residents[17] and where $b_t = B_t/(P_S C_S)$. This setup ensures that returns on the foreign asset that domestic residents receive are inversely related to the size of the net foreign asset position. ψ_t is a risk premium shock that drives a wedge between the fundamental forces and the effective interest rate.

Segmented Financial markets The last possibility we consider is that the consumer can share risks with the ROW through financial intermediaries. We adapt the segmented-financial-markets framework developed in Jeanne & Rose (2002), Gabaix & Maggiori (2015) and Itskhoki & Mukhin (2021) to our small open economy setting. There are three types of agents: noise traders, risk-averse intermediaries and households. First, noise traders abroad have exogenous demand for local currency[18]. More specifically, there is a mass of n identical noise traders that pursue a zero-capital strategy by taking a long position $\frac{N_{t+1}}{R_t}$ in local currency bonds and financing this position by an offsetting short position $-E_t \frac{N_{t+1}^{\$}}{R_t^{\$}}$ in ROW currency bonds. As their demand is driven by exogenous ROW investment demand, their problem is fully described by

$$\frac{N_{t+1}}{R_t} = n \left(e^{\psi_t} - 1 \right), \qquad \psi_t = \rho_\psi \psi_{t-1} + \sigma_\psi \varepsilon_t^{\psi}, \qquad \text{and} \qquad \frac{N_{t+1}}{R_t} = -E_t \frac{N_{t+1}^{\$}}{R_t^{\$}}$$

where ψ_t is the AR(1) noise trader shock with persistency ρ_ψ, standard deviation σ_ψ. Second, the positions of households and noise traders are intermediated by a mass m of identical risk-averse intermediaries that reside abroad as well. In taking on currency risk, intermediaries choose a position d_{t+1} to maximize profits from carry-trade under constant absolute risk aversion (CARA) using a zero-capital strategy.

$$\max_{d_{t+1}} -\mathbb{E} \left[\frac{1}{\vartheta} e^{-\vartheta \left(R_t - \frac{E_{t+1}}{E_t} R_t^{\$} \right) \frac{d_{t+1}}{P_{t+1} R_t}} \right] \qquad \text{s.t.} \qquad \frac{D_{t+1}}{R_t} = -E_t \frac{D_{t+1}^{\$}}{R_t^{\$}}$$

[17] Without additional frictions or additional financial assets, one can view this interest rate as the one that the Federal Reserve would set when implementing their monetary policy activities.

[18] Given that they conduct their activities from abroad, we do not have to account for their profits.

ϑ is the risk-aversion parameter and $D_{t+1} \equiv md_{t+1}$ is the total intermediating position. In Appendix B.4, we derive the optimal portfolio choice of intermediaries

$$\frac{d_{t+1}}{P_{t+1}} = -\frac{i_t - i_t^{\$} - \mathbb{E}\left[\Delta e_{t+1}\right]}{\vartheta \sigma_e^2}$$

where $i_t \equiv \ln R_t$, $i_t^{\$} \equiv \ln R_t^{\$}$, $\Delta e_{t+1} \equiv \ln E_{t+1} - \ln E_t$ and σ_e^2 is the equilibrium variance of the nominal exchange rate. Risk-averse intermediaries invest in local currency bonds whenever the cost of financing through foreign currency bonds, given by $i_t^{\$}$, is lower than the returns from investing in local currency bonds which are given by the interest rate on local currency bonds and the expected appreciation of the ROW currency, which is given by $i_t^{\$} + \mathbb{E}[\Delta e_{t+1}]$. Risk-averse arbitrageurs will limit their positions whenever they are more risk-averse or when the bilateral nominal exchange rate displays large volatility. Finally, Households invest in local currency bonds B_{t+1} to smooth consumption in response to shocks which yields the following optimality conditions

$$\beta \mathbb{E}_t \left[\frac{C_{St}}{C_{St+1}} \frac{P_{St}}{P_{St+1}} R_t \right] = 1$$

$$\frac{B_{t+1}}{R_t} - B_t = W_t L + \Pi_t + E_t P_{Xt}^{\$} X - P_{St} C_{St}$$

where R_t is the nominal interest rate received on domestic currency bonds. Equilibrium in financial markets is attained when the demand for domestic financial assets is completely intermediated by financial intermediaries.

$$B_{t+1} + N_{t+1} + D_{t+1} = 0$$

Note that we do not augment the optimality conditions with an ad-hoc debt-elastic interest equation. Following Itskhoki & Mukhin (2021) the risk-sharing condition will endogenously depend on the international financial asset position and the noise trader shock which ensures stationarity.

3.3 Equilibrium

Definition 1 (Stable equilibrium). *Given the set of deep parameters* $\Theta = \Big\{ \gamma, \omega, \varepsilon, \sigma, \theta, \kappa, \underline{\varPhi}, \delta_1, \delta_2, R^{\$}, P_M^{\$}, P_X^{\$}, X, f \Big\}_{t=0}^{\infty}$ *and a set of exogenous processes* $\{P_{Xt}^{\$}, P_{Mt}^{\$}, A_{Dt}, A_{St}, \psi_t\}_{t=0}^{\infty}$*, a stable equilibrium is a set of price processes* $\{P_{Dt}, W_t, E_t\}_{t=0}^{\infty}$ *that ensures that the equilibrium processes for the endogenous variables* $\{B_{t+1}, C_{St}, Y_{St}, X_{St}, Q_{Dt}, L_{St}, L_{Dt},$ *satisfy (1) optimal intertemporal consumption allocation, (2) services and manufacturing producers maximize profits, (3) financial intermediaries maximize profits from carry trade, markets clear:*

Goods markets

$$Y_{St} = C_{St}, \qquad Y_{Dit} = \left(\frac{P_{it}}{P_{Dt}} \right)^{-\sigma} (X_{St} + Q_{Dt})$$

Labor markets

$$L = L_{St} + \int_i (L_{Dit} + L_{Mit})\, di$$

Financial markets

$$B_{t+1} + N_{t+1} + D_{t+1} = 0$$

Current account

$$\frac{B_{t+1}}{R_t} - B_t = W_t L + \Pi_t + E_t P^{\$}_{Xt} X - P_{St} C_{St}$$

and the transversality condition $\lim_{j\to\infty} \mathbb{E}_t \left[\frac{B_{t+j+1}}{\prod_{s=0}^{\infty} R_{t+j}}\right] \geq 0$ *holds.*

In all models we consider, we show that the equilibrium conditions can be written in terms of an auxiliary variable H_t, which we name the *endogenous openness* of the economy (see Appendix B). For instance, we can write the level of savings net of export revenue $W_t L + \Pi_t - P_{St} C_{St}$ as proportional to the product of total consumer spending $P_{St} C_{St}$ and the openness of the economy. In this way, we can rewrite the budget constraint or the current account equation as an expression that equates the change in the net foreign asset position to the trade balance which is the difference between commodity exports and imports of intermediate inputs[19].

$$\frac{B_{t+1}}{R_t} - B_t = E_t P^{\$}_{Xt} X - \mu\gamma \frac{\sigma - 1}{\sigma} H_t P_{St} C_{St}$$

In addition, in Appendix B we show that in all models the expression for the aggregate imported intermediate input share can be written as

$$S^M_t \equiv \frac{P_{Mt} Q_{Mt}}{P_{Xt} X_{Dt}} = \frac{\left(1 - \gamma \frac{\sigma-1}{\sigma}\right) H_t}{1 - \gamma \frac{\sigma-1}{\sigma} H_t}$$

which is increasing in H_t and ensures that $H_t \in (0, 1)$. While the exact expression of H_t depends on how the other equilibrium variables differ across the models we consider, in all of them H_t captures the degree of reliance of the domestic economy on imported inputs[20].

Rewriting the equilibria in terms of H_t is useful for two reasons. First, the steady state is implicitly defined as a fixed point that exists and is unique for all the models with homogeneous firms and for the heterogenous model at maximum heterogeneity[21].

Proposition 1 (Steady-state equilibrium)**.** *The steady-state equilibrium exists and is unique for the following models: (1) homogeneous firms in a perfectly competitive market; (2) homogeneous firms in monopolistic competition; (3) homogeneous firms in monopolistic competition with access to increasing returns to importing.*

Proof. See Appendix C. □

The argument behind the proofs[22] is that for each model we can construct an equation $F(H(\Theta); \Theta)$ such that $\lim_{H\to 0} F(H(\Theta); \Theta) = -1$ and $\lim_{H\to 1} F(H(\Theta); \Theta) = \infty$, where Θ is the set of deep parameters. By Bolzano's

[19]Given that commodity exports are completely exogenously determined, we equate the endogenous response of imports with the endogenous response of overall openness of the economy.

[20]In the model with homogenous manufacturers that compete under monopolistic competition without increasing returns is given by,

$$H_t = \frac{1}{1 + \left(1 - \gamma \frac{\sigma-1}{\sigma}\right) \frac{\omega}{1-\omega} \left(\frac{E_t P^{\$}_{Mt}}{P_{Dt}}\right)^{\varepsilon - 1}}$$

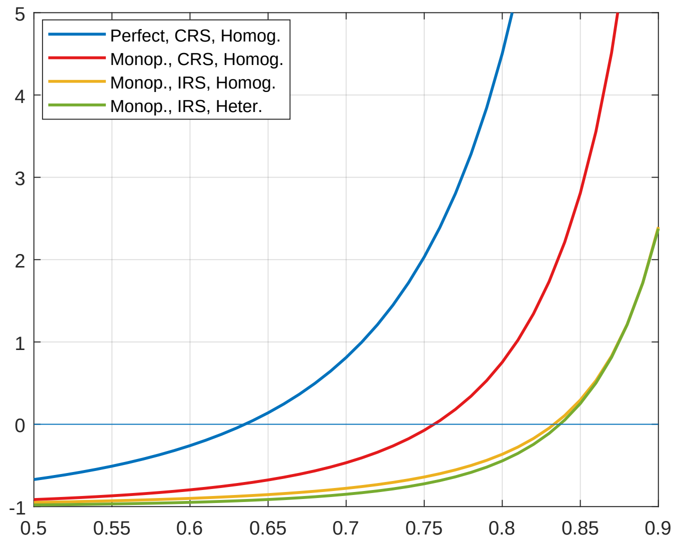
and intuitively depends on the relative input price of foreign and domestic intermediate inputs and the home-bias parameter ω. For small values of ω manufacturing producers are more dependent on imported inputs and H_t is closer to one. The same is true when the price of domestic inputs in domestic currency is high relative to that of imported inputs.

[21]We follow the convention in the literature to approximate the equilibrium around a steady state in which the net foreign asset position is zero (see Drechsel & Tenreyro (2018),Schmitt-Grohé & Uribe (2018) and Itskhoki & Mukhin (2021)).

[22]The proof uses the implicit function theorem to show that

$$\frac{\partial H}{\partial \frac{\sigma-1}{\sigma}} = -\frac{\partial F / \partial H}{\partial F / \partial \frac{\sigma-1}{\sigma}} < 0 \Rightarrow \frac{\partial H}{\partial \frac{\sigma}{\sigma-1}} > 0$$

Figure 5: Steady state H equation for different models



Theorem, there exists at least one root $H \in (0,1)$. THe uniqueness of the steady state follows from the fact that $F(H(\Theta),\Theta)$ is monotonically increasing in H and that $\partial F/\partial H > 0$ for $H \in (0,1)$[23].

The same argument can not be used in the heterogeneous model with arbitrary levels of heterogeneity. Nevertheless, when heterogeneity approaches the upper limit, $\kappa \to \frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}$ at which the moments of the model remain finite, the same argument can be used and the steady state exists and is unique. In addition, when the model converges to an economy with a degenerate productivity distribution, $\kappa \to \infty$, the model collapses to the homogenous firm model with increasing returns to scale on the importing bundle for which proposition 1 ensures existence and uniqueness.

We conjecture that the steady state also exists and is unique in the intermediate heterogeneity cases. To support this claim, we plot the implicit function that defines the fixed point equation for different values of steady-state openness for each of the models in Figure 5. This Figure shows that the implicit functions of the homogeneous and heterogeneous models with increasing returns to importing behave very similarly and therefore we believe conjecturing existence and uniqueness in the model with arbitrary levels of heterogeneity is sensible.

The second reason for casting the steady-state equilibrium in terms of the openness of the economy is that we can obtain comparative statics regarding equilibrium openness and labor allocations relative to changes in the deep parameters.

Corollary 1 (Openness and markups)**.** *Steady-state openness $H(\Theta)$ is increasing in the markup $\frac{\sigma}{\sigma-1}$ of the manufacturing sector.*

$$\frac{\partial H(\Theta)}{\partial \frac{\sigma}{\sigma-1}} > 0.$$

[23]For example, in the cases of perfect and monopolistic competition, it follows that

$$F(H(\Theta),\Theta) = \frac{\Lambda_1^j(\Theta)(1-H)^{-\frac{1}{\varepsilon-1}\frac{1}{1-\gamma}} H^{\frac{\varepsilon}{\varepsilon-1}} \left(1-\gamma\frac{\sigma-1}{\sigma}H\right)^{\frac{\gamma}{1-\gamma}\frac{1}{\varepsilon-1}}}{1+\Lambda_2^j(\Theta)\left(1-\gamma\frac{\sigma-1}{\sigma}H\right)} - 1$$

where $\Lambda_i^j(\Theta)$ are real and positive auxiliary variables where i indexes the models.

Proof. See Appendix C. □

The intuition behind this result is that when the manufacturing sector operates under monopolistic competition, domestic intermediate input prices are above marginal costs of production which leads to a positive wedge between the value of domestic intermediate inputs in production and its cost in its factor use. As a result, there is a departure from the efficient allocation where the manufacturing sector is producing less than optimal and domestic producers source inputs disproportionally from abroad thus increasing the equilibrium openness of the economy.

The second reason why writing the equilibrium in terms of H is useful, and that is that labor allocations can then be written in terms of H and a subset of structural parameters $\tilde{\Theta} = \{\mu, \sigma, \gamma, \varepsilon\}$ as follows.

$$L_{Dt} = \frac{\mu}{1-\mu} \frac{1-\gamma\frac{\sigma-1}{\sigma}H_t}{1-\gamma\frac{\sigma-1}{\sigma}} (1-\gamma)\frac{\sigma-1}{\sigma} L_{St} \quad \text{and} \quad L_{Mt} = \frac{1}{\varepsilon-1}\frac{\mu}{1-\mu}\gamma\frac{\sigma-1}{\sigma} H_t L_{St}$$

In all models, the ratio of labor allocated to manufacturing relative to services is high when the input share in the service sector μ is high, the input elasticity in the manufacturing sector γ is low, the economy is relatively closed (H is low) and when markups are low. In the model with increasing returns to importing, a second relation between labor allocated to services and labor allocated to paying the fixed cost of importing holds.

Intuitively, labor used for importing is relatively higher compared to labor allocated to services the less substitutable intermediate inputs are, the higher the intermediate input share in manufacturing and services is and the more open the economy is. Another interesting implication of the latter two expressions is that if H_t is constant, for instance when $\omega = 0$ such that there is no roundabout production, then labor allocation is static and changes in the production of the final good purely come from additional intermediate input usage[24]. Importantly, combining these relationships the steady-state labor allocation $L_S(H(\Theta), \tilde{\Theta})$ can also be written solely as a function of the equilibrium openness and a subset of deep parameters.

$$L_S\left(H(\Theta), \tilde{\Theta}\right) = \left[1 + \frac{\mu}{1-\mu}\gamma\frac{\sigma-1}{\sigma}H\left(\frac{1-\gamma\frac{\sigma-1}{\sigma}H}{\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H}\frac{1-\gamma}{\gamma} + \frac{1}{\varepsilon-1}\right)\right]^{-1}$$

Corollary 2 (Labor allocations and markups)**.** *Steady-state labor allocated to the services sector* $L_S\left(H(\Theta), \tilde{\Theta}\right)$ *is increasing in the markup* $\frac{\sigma}{\sigma-1}$ *of the manufacturing sector.*

$$\frac{\partial L_S\left(H(\Theta), \tilde{\Theta}\right)}{\partial \frac{\sigma}{\sigma-1}} > 0.$$

Proof. See Appendix C. □

The idea behind this result is that the inefficiencies that lead the economy to be more open also lead to less productive labor being allocated to manufacturing and more to services. This is because while the manufacturing sector avoids double marginalization by switching from domestic to imported inputs, the services sector switches from inputs to labor. Finally, the model with increasing returns to importing yields a more open economy compared to the model without increasing returns when the fixed cost of sourcing additional product varieties is not too large:

Corollary 3 (Increasing returns to importing)**.** *As the fixed cost of importing input varieties approaches zero, openness in the setup with increasing returns importing is higher than in the setup without increasing returns to*

[24] See for example Antràs et al. (2022) for a similar result.

importing.

Proof. See Appendix C. □

The introduction of increasing returns to importing changes the sourcing problem in two ways. On the one hand, the love-for-variety aggregator provides incentives to add more intermediate input varieties and lower marginal costs further. On the other hand, using more intermediate input varieties requires a fixed cost to be paid for each variety. Corollary 3 states that when this cost approaches zero[25], the benefits of adding intermediate inputs varieties increasingly outweigh the costs of accessing them making the economy more open.

In conjunction, these corollaries illustrate the two competing forces that change the exposure of the economy to terms-of-trade shocks. On the one hand, the inefficiencies introduced by monopolistic competition lead the manufacturing sector to be more exposed, and this is compounded by increasing returns to importing when the fixed cost associated with sourcing imported intermediate input varieties is low. On the other hand, the services sector becomes less exposed as more labor is allocated directly to services in response to a high markup in the manufacturing sector. In the section that follows, we show how the same forces that change the model's behavior in levels also determine the model's dynamics.

4 First-order solutions

In this section, we restrict attention to stable first-order solutions around the zero-debt steady state. We assume that the exogenous stochastic processes $\{a_{St}, a_{Dt}, p^{\$}_{Xt}, p^{\$}_{Mt}, \delta_t\}$ follow a shock-specific AR(1)-process, which are not model-specific. That is, they are the same regardless of assumptions on market structure and firm heterogeneity.[26] Up to a first-order, the goods and labor market clearing block of the models admits a general structure in which the comovement between consumption and the real exchange rate is mediated by the movement of endogenous openness. We show that the models only differ in their prediction for the two elasticities that determine how consumption and the real exchange rate comove with endogenous openness. Using the general structure, we show that the relative contribution of terms-of-trade shocks to productivity shocks to consumption volatility is pinned down by these elasticities and discuss why the different models provide different predictions for each of the elasticities.

4.1 General Structure of Goods and Labor Markets

To a first-order approximation, the equations that describe the optimal decisions on the supply side of the economy and the goods and labor market clearing conditions can be recombined into two equations.

Theorem 1 (General Structure)**.** *Regardless of the assumptions on the degree of financial market integration, market structure, returns to importing and selection, the equations of labor and goods markets lead to two equations that express consumption and the real exchange rate as a function of η_t and exogenous shocks as follows*

$$c_{St} = \frac{\mu}{1-\gamma} a_{Dt} + a_{St} + \nu_{cH}\eta_t \tag{4.1}$$

[25]Values in Gopinath & Neiman (2014) and Halpern et al. (2015) are not far from this case.

[26]In particular, we assume that the exogenous shock processes take the following form:

$$x_{t+1} = \rho_x x_t + \varepsilon_{xt}$$

where x_t is one of the exogenous processes where ρ_x is the persistence of the shock and the innovation is a white noise process $\varepsilon_{xt} \sim \mathcal{N}(0, \sigma_x^2)$ with finite variance σ_x^2.

$$\eta_t = \frac{1}{\nu_{qH}} \left(\frac{1-\mu}{1-\gamma} a_{Dt} - a_{St} + p^{\$}_{Mt} + q_t \right) \tag{4.2}$$

where $q_t \equiv e_t - p_{St}$ is the real exchange rate and η_t are percent changes deviations from steady state openness. It also follows from the assumptions on labor and goods markets that $\nu_{cH} > 0$ and $\nu_{qH} < 0$.

Proof. See Appendix B and D. □

The general structure is composed of two equations of which elasticities ν_{cH} and ν_{qH} depend on the specific structure of the model. The first equation provides the relation between consumption, exogenous shocks and endogenous openness. We obtain this relation by combining the linearized expressions for the services price index, the manufacturing price index and the model-specific definition of endogenous openness. These substitutions yield equilibrium relations between real manufacturing prices, real wages and openness.

$$w_t - p_{St} = \frac{1}{1-\mu} a_{St} - \frac{\mu}{1-\mu} \left(p_{Dt} - p_{St} \right), \qquad p_{Dt} - p_{St} = a_{St} - \frac{1-\mu}{1-\gamma} a_{Dt} - \nu_{pH} \eta_t$$

Percent changes in real wages are positively related to percent changes in productivity in services reflecting higher labor productivity, and negatively related to percent changes in the relative price of manufacturing goods as this erodes the purchasing power of nominal wages. Relative manufacturing prices are negatively related to manufacturing productivity, positively associated with services productivity (through the equilibrium effect on real wages), and positively associated with openness[27]. The extent to which relative manufacturing prices respond to changes in openness, captured by ν_{pH}, differ across the models. To arrive at the consumption equation, we combine these two expressions with labor market clearing.

$$c_{St} = w_t - p_{St} + \nu_{lH} \eta_t = \frac{\mu}{1-\gamma} a_{Dt} + a_{St} + \nu_{cH} \eta_t$$

Labor market clearing implies that consumption is determined by real wages and openness. In appendix D we show that this term reflects how labor allocated to services reacts to changes in openness. It implies that consumption depends on the purchasing power of labor in terms of the final good and the extent to which more of it can be produced through the reallocation of labor to that sector. Combining the labor market clearing condition with the expression for real wages yields the consumption equation.

The second equation in the system is the expenditure switching equation which describes how openness responds to shocks and the real exchange rate. This expression is the result of combining the linearized versions of the model-specific definition of endogenous openness, the relation between the productivity cut-off and endogenous variables and relative manufacturing prices. It is written as a combination of the equation that defines openness in each model and the productivity cutoff equation[28].

$$\underbrace{\left(\underbrace{\frac{1-\gamma}{\gamma} \frac{\varepsilon - 1}{1-\mu} \left(\frac{\left(1 - \gamma^{\frac{\sigma-1}{\sigma}}\right) H}{1 - \gamma^{\frac{\sigma-1}{\sigma}} H} \right) \nu_{pH}}_{\text{Profits}} - \underbrace{\nu_{lH}}_{\text{Fixed costs}} \right)}_{\text{IRS channel}} (1 - H) \eta_t = - \underbrace{(\varepsilon - 1)(1 - H) \left(p^{\$}_{Mt} + q_t - (p_{Dt} - p_{St}) \right)}_{\text{Substitution channel}}$$

[27]This could occur because of a negative $p^{\$}_{Mt}$ or a positive δ_t shock that induces an appreciation of the local currency.

[28]Given that the expressions for H_t are model specific, we illustrate the steps with the homogeneous firms model as it captures the substitution and increasing returns to importing channel of expenditure switching well. The heterogeneous firms model has a similar, albeit more convoluted, expression.

This expression captures the two channels that determine expenditure switching in this economy: a substitution channel and an increasing returns to scale channel. In the absence of increasing returns to importing, manufacturing firms choose the optimal intermediate input bundle by minimizing costs. In this scenario, the only thing that matters is the relative price of foreign inputs to domestic inputs and the elasticity of substitution between the bundles[29]. With increasing returns to importing, manufacturing firms also solve a profit maximization problem in which they decide on how many intermediate input varieties to source from abroad. In doing so, they weigh the additional profits (from lowering their marginal costs) with the additional fixed costs of importing more varieties. The pass-through from changes in aggregate relative manufacturing prices into aggregate manufacturing profits is captured by the coefficient on ν_{pH}. In addition, given that fixed costs are paid in domestic labor, the extent to which fixed costs change is captured by ν_{lH}.

The heterogeneous firm model admits the same structure, but the difference lies in the coefficient on ν_{pH} which in addition to the pass-through coefficient into profits also captures the fact that not all firms in the economy will access the IRS technology. The final expenditure switching relation is obtained by combining this expression with the expression for relative manufacturing prices. In contrast to a large literature (e.g. Obstfeld & Rogoff (1995),Galì & Monacelli (2005) and Itskhoki & Mukhin (2021)), our expenditure switching channel does not originate from substitution between domestic and foreign product varieties for final demand, but stems from optimal input allocation on the supply side (see Obstfeld (2001) for an early reference).

4.2 Relative Importance of Terms-of-trade Shocks

The previous result indicates that any differences between the simple setup with perfect competition and the complete structure with heterogeneous firms, increasing returns to importing and selection, are captured in different expressions for ν_{cH} and ν_{qH}. Therefore, we can use this structure to derive the equilibrium processes for consumption and the real exchange rate under different financial market structures for each of the models. We start with financial autarky as the simple equilibrium processes convey most of the intuition. We then show that this intuition carries over to more complex and realistic financial asset market structures.

4.2.1 Financial autarky

In the absence of financial risk-sharing possibilities, trade must balance every period. Therefore, there exists an equality between how much consumption and imports/openness change and how much commodity prices and the real exchange rate changes.

$$c_{St} + \eta_t = p^{\$}_{Xt} + q_t$$

Combined with the general structure of goods and labor markets, this leads to a three-equations three-variables system.

Corollary 4 (Equilibrium processes - Financial Autarky)**.** *Regardless of assumptions on returns to scale, firm heterogeneity, competition in the manufacturing sector or selection into importing, the equations that determine the*

[29]The $1-H$-term reflects a baseline effect: the lower the initial level of openness, the larger the percentage change in the openness for the same change in the relative input prices.

responses of consumption and the real exchange rate to structural shocks in financial autarky are the following.

$$c_{St} = a_{St} + \frac{1}{1-\gamma}(\mu - \nu_c)a_{Dt} + \nu_c(p^{\$}_{Xt} - p^{\$}_{Mt})$$
$$q_t = a_{St} - \frac{1}{1-\gamma}\big((1-\mu) - \nu_q\big)a_{Dt} - \nu_q p^{\$}_{Xt} - (1-\nu_q)p^{\$}_{Mt}$$

where

$$\nu_c \equiv \frac{\nu_{cH}}{1+\nu_{cH}-\nu_{qH}} \quad \text{and} \quad \nu_q \equiv -\frac{\nu_{qH}}{1+\nu_{cH}-\nu_{qH}}$$

and where $\nu_c > 0$ and $\nu_q > 0$ following Theorem 1.

Proof. See Appendix E. □

Following Corollary 4, any differences between frameworks can be thought of as differences in ν_c and ν_q. To the extent that $\nu_c \in [0, \mu)$, models that have a higher ν_c put more weight on terms-of-trade shocks as a source for consumption movements and less weight on exogenous manufacturing TFP shocks.

4.2.2 Financial Markets

International asset markets provide risk-sharing possibilities such that trade does not need to balance in every period. In this case, the current account equation implies trade imbalances are financed through changes in the net foreign asset position.

$$\beta b_{t+1} - b_t = \mu\gamma\frac{\sigma-1}{\sigma}\eta\big(q_t - c_{St} + p_{Xt} - \eta_t\big)$$

The extent to which consumers want to borrow or lend is governed by the Euler equation. Under integrated financial markets, the Euler equation has to be complemented with a debt-elastic interest rate equation that induces the stationarity of the system by forcing interest rates to rise in the amount borrowed[30]. This is because in the SOE framework the interest rate is not determined by market clearing. Under segmented financial markets, the role of the debt-elastic-interest-rate equation is fulfilled by the asset-market-clearing equation that balances the demand for the domestic currency of the consumer, the noise traders and financial intermediaries. After combining the Euler equation with either the debt-elastic interest rate or asset-market-clearing equation, we arrive at the same linearized risk-sharing condition[31]

$$\mathbb{E}_t\big[\Delta c_{St+1} - \Delta q_{t+1}\big] = -\chi_1\psi_t + \chi_2 b_{t+1}$$

[30] Previous papers have imposed stationarity by assuming other debt-elastic interest rate equations, and here's how our equation would look if we were to follow their assumptions. García-Cicco et al. (2010) assumed international financial markets with a one-period-ahead, non-contingent bond that pays in the numeraire good and leads to the following Euler equation

$$\mathbb{E}_t\big[\Delta c_{St+1} - \Delta q_{t+1}\big] = -\mu_t + \delta b_{t+1}$$

where μ_t is an interest premium shock. In Drechsel & Tenreyro (2018) the economy borrows from abroad in units of its own final good and the Euler equation would be

$$\mathbb{E}_t\big[\Delta c_{St+1} - \delta q_{t+1}\big] = -\mu_t + \delta_1 p_{Xt} + \delta_2 b_{t+1}$$

such that the interest rate also responds directly to commodity price by assumption. Kohn et al. (2021) assume international financial markets with a non-contingent bond that pays in the tradable good and which Euler is

$$\mathbb{E}_t\big[\Delta c_{St+1} - \Delta q_{t+1}\big] = \delta_1 y_{St} + \delta_2 b_{t+1}$$

such that the interest premium responds to changes in GDP.

[31] Even if the nominal exchange rate does not enter the Euler equation in the segmented financial markets framework given that domestic households save in domestic currency bonds, the nominal exchange rate enter the linearized risk-sharing condition by inserting the optimal decision of the financial intermediaries.

where b_t is the percentage change in the nominal bond position and ψ_t is the risk premium shock. The relationship between the nominal interest rate, the nominal bonds position and the risk premium shock is governed by χ_1 and χ_2 respectively. They are either exogenously specified parameters under integrated financial markets or a combination of deep parameters and the conditional variance of the nominal exchange ($\sigma^2_{\Delta e}$) under segmented financial markets. In any case, the general structure of goods and labor markets remains the same under both financial market structures and delivers the following equilibrium process for consumption and the real exchange rate.

Corollary 5 (Equilibrium processes - Financial Markets)**.** *Regardless of assumptions on returns to scale, firm heterogeneity, market structure or selection into importing*

1. *The equilibrium processes that determine the responses of consumption and the real exchange rate to structural shocks are stationary.*
2. *The equilibrium processes are given by*

$$c_{St} = a_{St} + \frac{\mu - \nu_c(1+\alpha_D)}{1-\gamma} + \nu_c\left((1-\alpha_X(\nu_{cH}-\nu_{qH}))\,p_{Xt} - (1+\alpha_M)p_{Mt}\right) - \frac{\alpha_\delta \nu_{cH}\chi_1}{1-\rho_\delta}\delta_t$$
$$q_t = a_{St} - \frac{(1-\mu)-\nu_q(1+\alpha_D)}{1-\gamma}a_{Dt} - \nu_q\left(1-\alpha_X(\nu_{cH}-\nu_{qH})\right)p^{\$}_{Xt} - \left(1-\nu_q(1+\alpha_M)\right)p^{\$}_{Mt} - \frac{\alpha_\delta \nu_{qH}\chi_1}{1-\rho_\delta}\delta_t$$

where

$$\alpha_y = (1-\lambda_1 L)^{-1}\left(1-(1/\beta)L\right)\frac{1-\rho_y}{\lambda_2-\rho_y}\frac{1}{\nu_{cH}-\nu_{qH}}$$

where $y = \{D, X, M, \delta\}$, L is the lag operator and $\lambda_1(\nu_{cH}, \nu_{qH})$ and $\lambda_2(\nu_{cH}, \nu_{qH})$ are the non-explosive and explosive eigenvalues of the system, respectively.

Proof. See Appendix B.4. □

While the equilibrium processes for consumption and the real exchange rate become ARMA$(1,1)$ processes and are more complicated compared to the AR(1) processes under financial autarky, the same reallocation of weights across shocks as under financial autarky applies. This becomes apparent if we consider the limiting case where all shocks approach a random walk, $\rho_y \to \infty \forall y$[32]. Even if the share of the variance of consumption explained by terms of trade fluctuations now also depends on the relative importance of interest-rate shocks, which in turn depends on goods and labor market structure, the share explained by terms of trade *relative* to the share explained by productivity shocks is still captured by ν_c only.

Theorem 2 (Terms-of-trade relative to TFP)**.** *Under (1) financial autarky and (2) either integrated and segmented financial markets with $\rho_y \to \infty$ with $y = \{D, X, M\}$, the importance of ToT shocks relative to exogenous TFP shocks, as captured by the share of the total variance of consumption explained by ToT shocks over the share of the total variance explained by TFP shocks is*

$$\frac{s_X}{s_A} = \frac{\sigma_X^2}{\sigma_A^2}\frac{(\nu_c)^2}{\frac{\sigma_S^2}{\sigma_D^2} + \left(\frac{\mu-\nu_c}{1-\gamma}\right)^2}$$

where σ^2 are the variances of the shock processes. In addition, the relative importance of terms-of-trade shocks is rising in ν_c, that is $\partial\frac{s_x}{s_a}/\partial\nu_c > 0$.

[32]This is not the case for the financial shock. Therefore, we need in addition that $\sigma_\delta = 0$ in this limiting case.

Proof. Follows directly from the ratio of variances $\mathrm{var}(c_t)$ in Corollary 4. $\square$

Theorem 1 and 2 collectively make up the central results of the paper. Whereas Theorem 1 provides a unifying framework for the frictionless benchmark and the full model with IRS and heterogeneous producers, Theorem 2 illustrates that when the shocks approach a random walk, all we need to know to understand different predictions regarding the relative importance of terms-of-trade shocks are the different predictions regarding ν_c[33].

5 Aggregate TFP and adjustment margins

To achieve tractability we assumed that fixed costs per variety increase linearly with the number of imported varieties and that the elasticity of substitution across imported varieties θ is the same as the elasticity of substitution between the imported and domestic intermediate input bundles ε. In this section, we show that these simplifying assumptions are relatively innocuous, as the model can still replication the following micro-moments: (1) the firm-level import distribution has a heavier-than-Pareto tail; (2) the importance of the sub-intensive margin increases with firm size; and (3) terms-of-trade shocks generate endogenous TFP movements.

5.1 Import Distribution

We define aggregate imports in domestic currency M_t as follows.

$$M_t \equiv \int_{\varphi_{Mt}}^{\infty} P_{Mt}(\varphi)\, Q_{Mt}(\varphi)\, g(\varphi)\, d\varphi$$

where φ_{Mt} is the cutoff productivity level required to import, $P_{Mt}(\varphi)\, Q_{Mt}(\varphi)$ are the firm-level imports in terms of domestic currency and $g(\varphi)$ is the distribution of firm-level productivity. To derive the distribution of total imports across firms, we rely on the following lemma.

Lemma 1. *Following the assumptions of the model with increasing returns to scale and selection, the dollar amount imported by firm i, $M_{it}^{\$}$, can be written as the product of the fixed costs of importing and the firm-specific import measure.*

$$E_t M_t^{\$}(\varphi) = (\varepsilon - 1) W_t f \mathcal{L}_t(\varphi)$$

Proof. See Appendix B.6. $\square$

Lemma 1 states that firm-level imports in domestic currency can be written as a combination of a term that is common for all firms times the number of intermediate input varieties sourced by the firm. The expression is rather intuitive as it equates total spending on imports in pesos to the cost of importing each variety $(W_t f)$ times the measure of varieties imported $\mathcal{L}_t(\varphi)$ adjusted by the elasticity of substitution between the domestic and imported input bundles. Combining this intermediate result with the assumption that firm-level productivity follows a Pareto distribution, we obtain an expression for the distribution of imports across firms conditional on importing.

Proposition 2. *The distribution of firm imports conditional on importing is Generalized Pareto as follows.*

$$\Pr\left(M_{it}^{\$} < M | M > 0\right) = 1 - \left[1 + \frac{1}{\varepsilon - 1}\frac{E_t}{W_t f}\frac{1-\omega}{\omega}\left(\frac{P_{Dt}}{E_t P_{Mit}^{\$}}\right)^{\varepsilon - 1} M\right]^{-\kappa \frac{\varepsilon - 1 - \gamma(\sigma - 1)}{(\sigma - 1)(\varepsilon - 1)}}$$

[33]Given the wild behavior of commodity prices and the evidence of a non-stationary component in manufacturing TFP (e.g. Aguiar & Gopinath (2007) and Drechsel & Tenreyro (2018)), we believe this is representative limiting case.

Figure 6: Power Law for Imports

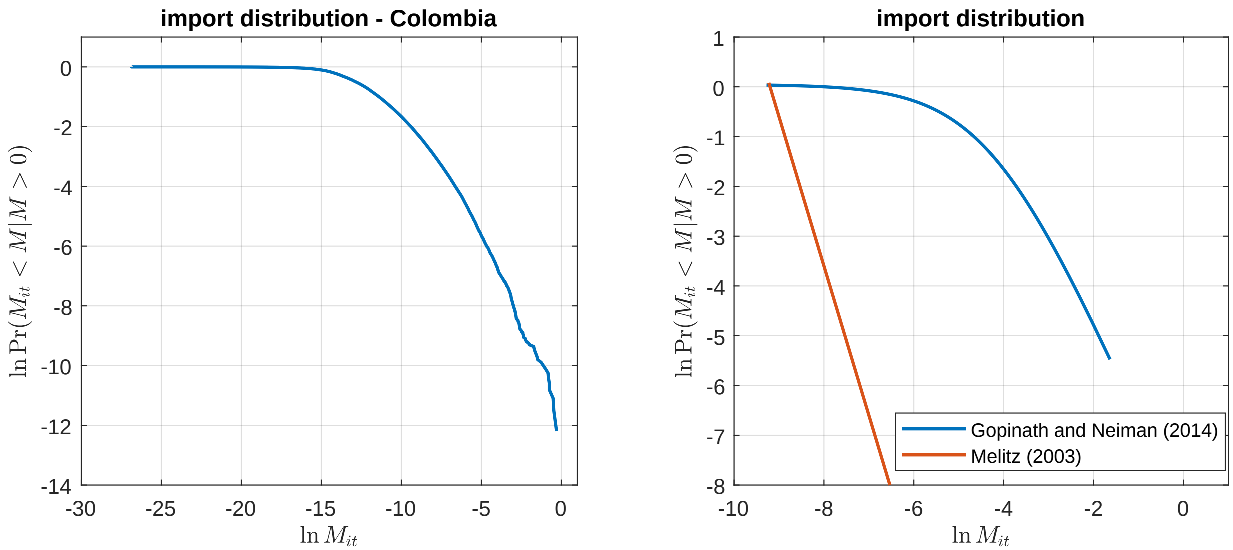


Figure 7: The left-hand panel plots the log-log Pareto plot of the distribution of firm imports in Colombian data for the years 2006-2020. The right-hand panel plots the same log-log plot but of the model equilibrium following the expression in Proposition 2.

Proof. The proposition follows directly from Lemma 1 and the assumption of a Pareto productivity distribution.

$$\begin{aligned}
\Pr\left(M_{it}^{\$} < M|M > 0\right) &= \Pr\left(\varphi_i < \left(\frac{1}{\varepsilon - 1}\frac{E_t}{W_t f}\frac{1-\omega}{\omega}\left(\frac{P_{Dt}}{E_t P_{Mit}^{\$}}\right)^{\varepsilon-1} + 1\right)^{\frac{\varepsilon-1-\gamma(\sigma-1)}{(\sigma-1)(\varepsilon-1)}} \varphi_{Mt}|\varphi_i > \varphi_{Mt}\right) \\
&= F\left(\left(\frac{1}{\varepsilon - 1}\frac{E_t}{W_t f}\frac{1-\omega}{\omega}\left(\frac{P_{Dt}}{E_t P_{Mit}^{\$}}\right)^{\varepsilon-1} + 1\right)^{\frac{\varepsilon-1-\gamma(\sigma-1)}{(\sigma-1)(\varepsilon-1)}} \varphi_{Mt}\right)\left(1 - F(\varphi_{Mt})\right)^{-1}
\end{aligned}$$

□

In Figure 6, we illustrate the importance of assuming that manufacturing firms pay a fixed cost per imported variety instead of assuming that firms pay simply one fixed costs to import, as in Melitz (2003). The left panel Figure 6 plots the relationship between log imports and the log of the cumulative distribution of imports and illustrates the presence of many small importers and a few large importers. In panel (b) of Figure 6 we plot the same relationship for the two types of models.

In a model where firms pay only one fixed cost to access imported intermediate input, the import distribution would follow a Simple Pareto distribution and the relationship between the log import level and the log cumulative density of imports would be linear with slope $-\frac{\kappa}{\sigma-1}$, which is not the case in the data. However, when manufacturing firms have to incur a fixed cost per imported variety, the import distribution is Generalized Pareto with a much heavier tail. The model predicts a much more important role for a few large importers and can generate the presence of many more small importers.[34]

[34]It turns out the number of small importers in the data is even higher than the complete can generate. As discussed in Arkolakis (2010) modeling fixed costs as market penetration costs could potentially generate more small importers.

5.2 Adjustment margins

In section 2, we showed that the intensive margin accounts for all of the changes in aggregate imports over time in the data. In addition, for continuing importers, the importance of the sub-intensive margin relative to the sub-extensive margin rises in the distribution of imports. Using Lemma 1 and Proposition 2, we show that our version of Gopinath & Neiman (2014) fits these patterns.

Proposition 3 (Trade adjustment)**.** *Following an aggregate shock, changes in aggregate imports can be summarized as follows.*

1. *For any shock, all changes in aggregate imports are accounted for by the intensive margin of trade.*

2. *Conditional on a commodity price shock $p^{\$}_{Xt}$, the model with heterogeneous firms predicts that the share of the sub-intensive margin relative to the overall change to total dollar-imports per firm is given by*

$$\frac{m_{intt}^{\$}(\varphi)}{m_{intt}^{\$}(\varphi)+m_{extt}^{\$}(\varphi)}=\frac{\frac{\mu}{1-\mu}\nu_{p\eta}-\nu_{qH}}{\frac{\mu}{1-\mu}\nu_{p\eta}-\nu_{qH}+(\varepsilon-1)\left(\nu_{p\eta}+\nu_{qH}+\frac{1}{1-\gamma_{Di}}\frac{1}{\varepsilon-1-\gamma(\sigma-1)}\frac{\sigma-1}{\kappa-(\sigma-1)}\frac{1}{1-\eta+\left(1-\gamma\frac{\sigma-1}{\sigma}\right)(1-\tilde{\kappa})\eta}\right)}$$

and is decreasing in the domestic input share γ_{Di}.

Proof. (1) is true by construction in the models with a representative producer, while in the model with selection and heterogeneous firms it follows from applying Leibniz's rule to the total amount imported per firm. Following Lemma 1, total imports can be expressed as a combination of firm-specific terms and an aggregate term as follows where $\tilde{M}_t=(\varepsilon-1)W_tf/E_t$ and such that

$$-\frac{\partial\ln M_t}{\partial\ln x_t}=-\frac{x_t}{M_t}\left[\underbrace{\int_{\varphi_{Mt}}^{\infty}\frac{\partial}{\partial x_t}\tilde{M}_t\mathcal{L}_t(\varphi)dG(\varphi)}_{\text{Intensive}}-\underbrace{\tilde{M}_t\mathcal{L}_t(\varphi_{Mt})\frac{\partial}{\partial x_t}\varphi_{Mt}}_{\text{Extensive}}\right]$$

and the extensive margin part is zero since $\mathcal{L}_t(\varphi_{Mt})=0$, that is, the measure evaluated at the cutoff is nil. This is true for any shock.
(2) follows from approximating the equation in Lemma (1) as follows

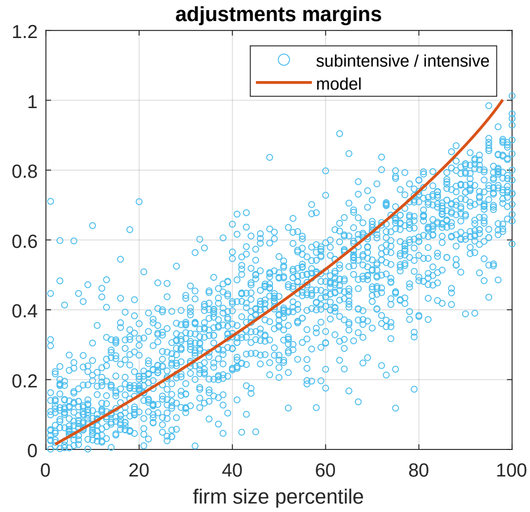
$$m_{it}^{\$}=\underbrace{w_t-e_t}_{\text{Subintensive}}+\underbrace{(\varepsilon-1)(e_t+p_{Mt}^{\$}-p_{Dt})+\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}\frac{1}{1-\gamma_{Di}}\frac{(\kappa-(\sigma-1))^{-1}}{1-H+(1-\tilde{\kappa})\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H}\eta_t}_{\text{Subextensive}}$$

of which details are found in Appendix B.6. ☐

Proposition 3 guarantees that all of the adjustments in aggregate imports happen at the intensive margin, which is the case in the data. With no heterogeneity, this is true by construction. However, in the model with heterogeneity and selection, the same is true because the contribution of the extensive margin depends on the measure of imported intermediate inputs which is zero when evaluated at the cut-off productivity level.

This simplified version of Gopinath & Neiman (2014) also generates the positive relationship between firm size and the importance of the sub-intensive margin. This is because larger firms import more and have smaller domestic input shares. We focus on commodity price shocks for two reasons. First, section 2 indicates that these

Figure 8: Sub-Intensive vs. Sub-Extensive Margin



Notes: The figure plots the relationship between the level of imports and the share attributed to the sub-intensive margin observed in the data and predicted by the model in the baseline calibration. The theoretical relationship is obtained by noting that Proposition 2 allows us to solve for any percentile of the distribution and its associated level of imports. Consequently, we can map any percentile to a productivity level $\varphi_p = (1-p)^{-\frac{1}{\kappa}}\varphi_{Mt}$ and each productivity level in turn to its domestic input share γ_{Dp}, which, are finally used to map import size percentiles to their associated sub-intensive margin shares.

shocks account for most movements in the terms of trade. Second, the change in the sub-intensive and sub-extensive margins has the same sign in response to this specific shock, which makes the ratio of margins easier to interpret.[35] Even though this moment is not targeted by the baseline calibration, Figure 8 shows that the model closely matches the slope in the data.

5.3 Manufacturing TFP

A second feature of the reality that only a model with heterogeneous importers and selection can match is endogenous movements in manufacturing TFP that arise from productivity and interest rate shocks. We use the structure of the model to obtain an exact expression for the aggregate and manufacturing production functions. Aggregate manufacturing output is rewritten as follows [36].

$$Y_{Dt} = \underbrace{A_{Dt}}_{\text{Technology}} \underbrace{{L_{Dt}}^{1-\gamma}{X_{Dt}}^{1-\gamma}}_{\text{Input and factor use}} \underbrace{\left[\int_{\underline{\varphi}}^{\infty}\left(\varphi_i\left(\frac{L_{Dt}(\varphi)}{L_{Dt}}\right)^{1-\gamma}\left(\frac{X_{Dt}(\varphi)}{X_{Dt}}\right)^{\gamma}\right)^{\frac{\sigma-1}{\sigma}} d(\varphi)\right]^{\frac{\sigma}{\sigma-1}}}_{\text{Allocative efficiency}}$$

There are three elements in the production function that map into the framework of Baqaee & Farhi (2020). The first term is composed of the exogenous productivity shock in the manufacturing sector, A_{Dt}, which we refer to as the *technology* term of the production function. The second term captures the contribution of *input and factor use*

[35]In response to productivity shocks and import price shocks, the margins move in opposite directions. This makes defining the share attributed to a particular margin non-trivial.

[36]We provide the details in Appendix H. We do not have to resort to a first-order approximation to the production function as Gopinath & Neiman (2014). However, when we turn to the quantitative exercise, we will consider a first-order solution to the model and work with a first-order approximation to this exact expression.

to output. L_{Dt} accounts for the direct effect of increased use of productive labor in manufacturing on output[37]. X_{Dt} is an intermediate input aggregator that accounts for total input use. Note that all variables are scaled by their respective elasticities in production[38]. Finally, the *allocative efficiency* term represents the reallocation of productive labor and inputs between firms. Whenever very productive firms (high φ) are allocated more labor and inputs, output increases above and beyond the increase in aggregate labor and inputs allocated to the manufacturing sector.

In the absence of changes in the productivity cut-off for importing the model collapses back into a representative-producer framework in which terms-of-trade shocks do not have first-order effects on aggregate productivity as in Kehoe & Ruhl (2008). We formalize this result in the following proposition.

Proposition 4 (The need for selection)**.** *In the absence of selection, the heterogeneous manufacturing sector can be replaced by a representative producer with the following productivity level.*

$$\varphi_D = \underline{\varphi} \left(\frac{\kappa}{\kappa - \frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}} \right)^{\frac{\varepsilon-1-\gamma(\sigma-1)}{(\sigma-1)(\varepsilon-1)}}$$

Proof. Appendix I. □

It follows that heterogeneity, fixed costs and roundabout production are necessary, but not sufficient conditions for terms-of-trade shocks to induce aggregate productivity effects. Instead, in our model selection into importing is the ingredient that makes this set of assumptions sufficient. An intuition for this result is presented in Blaum et al. (2018), which shows that the percentage change in the domestic input share is a sufficient statistic to measure the aggregate gains from input trade. Due to the log-linearity of our model, the percentage change in the domestic share is the same across heterogeneous producers. Hence, without selection, there is no difference in the change in the domestic input share between a representative-firm model with roundabout production and a heterogeneous-firms model with fixed costs of importing and roundabout production.

6 Comparing the models

Section 4 showed that different models have different predictions for consumption volatility because they generate different values for ν_c. In this section, we provide a general expression for ν_c and show that, while we cannot unambiguously rank the models, we can unpack the mechanisms that determine these values. We show that the models have different predictions for the elasticity of private consumption to openness, which is captured by ν_{cH}. Given that the latter depends on the relationship between openness and aggregate real manufacturing prices, and on how labor reallocates to the services sector in response to changes in openness, the models differ in their predictions for ν_c as well.

6.1 Understanding ν_c

The following Proposition establishes that we can always write ν_c as a combination of two distinct elements.

[37]Even though the model has inelastic total labor supply, an increase in productive labor can happen at the expense of a reduction in labor used in importing.

[38]Note that this is different from Gopinath & Neiman (2014) where inputs are scaled with cost shares and the inverse of the markup in the manufacturing sector. Baqaee & Farhi (2020) show that the differences arise because Gopinath & Neiman (2014) uses the Basu & Fernald (2002) decomposition which does not properly separate changes in output due to changes in technology and allocative efficiency.

Proposition 5 (Consumption elasticity ν_c). *The general equilibrium elasticity ν_c has the following common structure across frameworks.*

$$\nu_c = \underbrace{\mu\gamma H_i(\Theta)}_{Hulten} \times \underbrace{\Lambda_i\left(H_i(\Theta), \tilde{\Theta}\right)}_{Distortion}$$

where i index the different models $i = \{Perfect, Monopolistic, IRS, Selection\}$ and where $\tilde{\Theta} \equiv \{\mu, \gamma, \sigma, \varepsilon, \}$ is a subset of deep parameters.

Proof. See Appendix E. □

The first part of ν_c is the product of the input elasticity in services μ, the input elasticity in manufacturing γ and the steady-state openness level H. For H close to zero, that is, for closed economies, the part of the variance of consumption that can be explained by movements in the terms of trade is small, which is intuitive[39]. We name this the Hulten term because it states that, up to a first-order, the impact of terms-of-trade shocks on consumption depends on the input shares in production and the reliance on foreign intermediate inputs.

The second part of the expression is a correction term that accounts for the domestic distortions, such that $\Lambda = 1$ only in the frictionless perfect competition benchmark. It is always the case that when all inputs are imported ($H = 1$ in steady state) $\Lambda = 1$ and the dynamics of all setups are the same. In between economies that are completely closed and completely open it is ambiguous whether or not $\Lambda > 1$, but we know when this is more likely to happen. In particular, the lower the elasticity of substitution between imported and domestic inputs ε, the more likely it is that $\Lambda > 1$ (In the Cobb-Douglas case $\varepsilon = 1$ this is certain). It is also more likely to happen when the input elasticities (both γ and μ) are high. These results are rather intuitive: the less substitutable domestic and imported inputs and the more the services sector and the manufacturing sector itself depend on inputs to produce the more likely it is that frictions matter and that the economy is more exposed to commodity-price shocks through the manufacturing-prices channel.

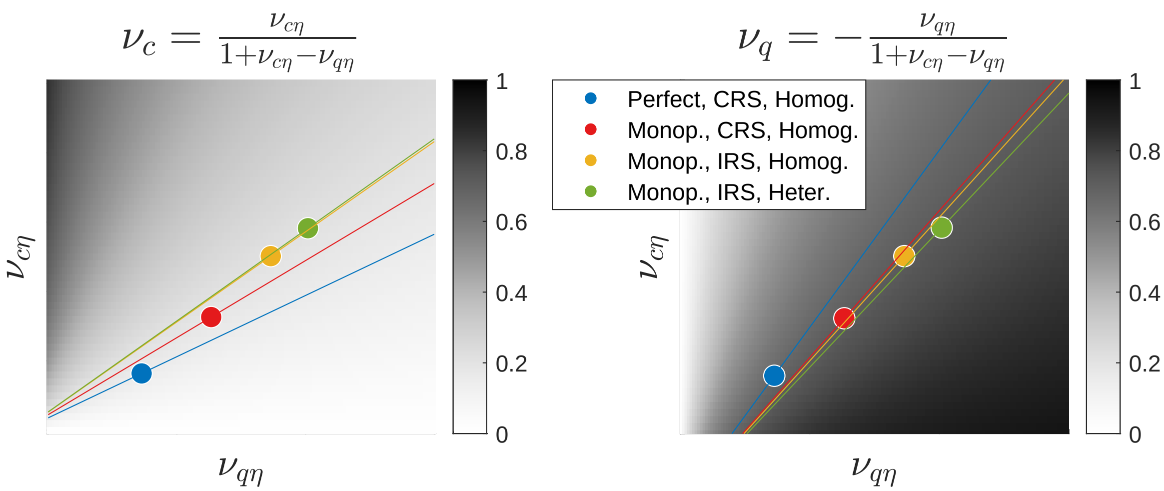
6.2 Amplification mechanism

Since ν_c and ν_q are functions of the partial elasticities ν_{cH} and ν_{qH}, we can represent the general-equilibrium elasticities in a (ν_{cH}, ν_{qH})-plane. Figure 9 represents the corresponding values of ν_c and ν_q in greyscale for different values of ν_{cH} and ν_{qH}. Any model that is represented in Theorem 1 can be thought of as a point in this space. For instance, Broda (2004) presents a model where the monetary authority is pegging to the dollar and forcing most adjustments through real variables. Such a model can be thought of as being in the upper-left corner as it is a model in which consumption responds much more strongly to terms-of-trade shocks than the real exchange rate. Models in which most of the adjustment happens through the real exchange rate with little movement in consumption, known as exchange-rate-disconnect models (e.g. Itskhoki & Mukhin (2021)), would be in the lower-right corner.

Figure 9 also plots the values of ν_c and ν_q for each of the setups we consider together with their isoquants. For instance, the red dots represent ν_c and ν_q in monopolistic competition with no heterogeneity or selection. According to the figure, the monopolistic competition model generates a higher ν_c compared to the perfect competition case. Following Theorem 2, we expect the monopolistic competition model to attribute a larger share of the variance of consumption to terms-of-trade shocks relative to productivity shocks. Figure 9 also shows that the relative importance of terms-of-trade shocks further rises in the homogeneous and heterogeneous firm models with increasing returns to importing.

[39]A small steady-state H means the economy relies very little on imports for production which can be because steady-state import prices are high or home-bias in production is substantial, etc.

Figure 9: Heatmap of the general elasticities in the ν_{cH} and $-\nu_{qH}$ space



Notes: Each model is signaled with a dot and lines of the same color indicate coordinate combinations that would lead to the same general equilibrium elasticities.

To understand how ν_c changes qualitatively across models, note that it is sufficient to understand how ν_{cH} changes[40]. In Appendix D, we prove that ν_{cH} is the following

$$\nu_{cH} = \frac{\mu}{1-\mu}\nu_{pH} + \nu_{lH}$$

where ν_{pH} captures how aggregate real manufacturing prices change when the openness of the economy changes, while ν_{lH} in turn captures how the labor allocation to the services sector or the supply of the final good changes when openness changes in response to shocks. While the responsiveness of manufacturing prices to changes in openness increases with the additional frictions, the response of labor allocated to services to changes in openness is ambiguous.

6.2.1 Manufacturing Prices

As shown in Section 4.1, the response of aggregate manufacturing prices can be written as a function of productivity shocks and the change in equilibrium openness of the economy.

$$p_{Dt} - p_{St} = a_{St} - \frac{1-\mu}{1-\gamma}a_{Dt} - \nu_{pH}\eta_t,$$

where $$\nu_{pH} \equiv \underbrace{\underbrace{(1-\mu)\frac{\gamma}{1-\gamma}}_{\text{Input shares}}\underbrace{\frac{\left(1-\gamma^{\frac{\sigma-1}{\sigma}}\right)H}{1-\gamma^{\frac{\sigma-1}{\sigma}}H}}_{\text{Import share}}\underbrace{\frac{1}{(1-H)(\varepsilon-1)}}_{\text{Import subst.}}}_{\text{no selection}}\underbrace{\frac{\varepsilon-1}{\gamma(\sigma-1)}\frac{\tilde{\kappa}(1-H)}{(1-\tilde{\kappa}H)\left(1-\gamma^{\frac{\sigma-1}{\sigma}}\right)+(1-H)\gamma^{\frac{\sigma-1}{\sigma}}}}_{\text{selection}}$$

It can be separated into a part that is shared among the models and a part that is specific to the model with heterogeneous producers. The "no selection" term depends on input elasticities, the fraction of imported inputs, and the degree of input substitutability. First, the higher the input elasticity in manufacturing γ the higher the exposure

[40]To compare the models quantitatively, one needs to understand how ν_{qH} changes as well.

of the manufacturing sector to η_t. As manufacturing output becomes increasingly important in the production of services, the response of manufacturing prices relative to the price of the final good falls, that is, the input share in services μ dampens the exposure.

Second, the partial elasticity also depends on the share of imported inputs in total input spending. If steady-state openness is high (e.g. due to a low home-bias parameter) the imported input share is also high and the exposure of aggregate real manufacturing prices to changes in openness increases. Finally, the import substitution term depends on the steady-state openness of the economy and the elasticity of substitution across intermediate inputs ε. When ε is high, domestic inputs are better substitutes for imported inputs, such that manufacturing firms can more easily substitute away from more expensive foreign inputs and insulate prices from foreign shocks.

Through ν_{pH}, monopolistic competition leads to higher responsiveness of consumption to openness because steady-state openness is higher since as $H \to 1$ the import substitution term increases exponentially. This is reflected in a higher imported intermediate input share and a higher import substitutability term. The increasing-returns-to-importing model leads to a further increase in the responsiveness of manufacturing prices, as manufacturing firms can further decrease the level of marginal costs by adding imported intermediate input varieties.

Selection adds an extensive margin on the firm level to the already existing intensive margin. This is reflected in the term $\frac{\varepsilon-1}{\gamma(\sigma-1)}$ which is always greater than one (a similar argument as in Chaney (2008))[41]. The second term is smaller than one and captures the fact that as not all firms select into importing. Nevertheless, the first effect always dominates in our case. Note that when heterogeneity is at its lowest, the model with homogeneous firms and the model with heterogeneous firms, both with increasing returns to importing, coincide as there is no selection and the productivity distribution is degenerate. Any deviation from this lower limit implies a stronger response. While selection always leads to amplification from a qualitative point of view, Figure 9 shows that quantitatively this effect seems to be rather small.

6.2.2 Labor Markets

In Appendix D we show that the labor market clearing condition can be combined with goods market clearing for the final consumption good and the first order condition for labor use in services to arrive at an expression for the labor allocation to services as a function of equilibrium openness. Therefore, the change in the labor allocation to services can be written solely as a function of changes in openness:

$$l_{St} = \nu_{lH}\eta_t, \qquad \text{where} \qquad \nu_{lH} \equiv \frac{\mu\gamma H}{\chi_i(\Theta) - \mu\gamma H}$$

which is increasing in the steady-state openness and where $\chi_i(\Theta)$ is a combination of deep parameters which is different across the models. In the perfect competition model, the combination of parameters simplifies to $\chi_{\text{perfect}} = 1$. When the economy is completely closed ($H = 0$) labor allocated to services does not respond to foreign shocks. When the economy is completely open ($H = 1$) it follows that $\chi_{\text{perfect}} = \mu\gamma/(1-\mu\gamma)$.

The effect of monopolistic competition on the partial-equilibrium elasticity of l_{St} to η_t is ambiguous. On the one hand, higher markups in the manufacturing sector lead to a higher H (corollary 1), which means l_{St} is more responsive to openness. On the other hand, higher markups imply that more labor is allocated to services in steady-state such that $\chi_{\text{Monopolistic}} > 1$. This is more likely to be true when μ is low and γ is high (conditional on H). The relative importance of these two forces determines whether ν_{lH} increases under monopolistic competition.

The comparison between the model with monopolistic competition without increasing returns to importing to

[41] Recall this is necessary for the model to produce finite moments.

the model with increasing returns to importing reveals a similar ambiguity. On the one hand, the economy is likely to be more open which increases responsiveness. On the other hand, part of the labor endowment is used to pay for imports and the percentage of labor allocated to services falls. Finally, the comparison between the models with homogeneous and heterogeneous firms with increasing returns to importing is not ambiguous as it boils down to comparing openness.

7 Quantitative Exercise

In this section, we complement the qualitative comparison of the different models with two distinct quantitative exercises. First, we calibrate the parameters in the model to standard values in the literature and describe the resulting impulse responses split into the four possible setups. In a second exercise, we leverage the rule of thumb described in Theorem 2 to compute the relative importance of TOT to TFP across the different models.

7.1 Calibration

Table 1 describes the calibrated parameters and their sources. There are three sets of parameters that need to be calibrated, namely the ones related to the manufacturing sector, the ones related to services, and the ones related to the intertemporal problem. The input elasticity parameters γ and μ are calibrated to match the cost shares of the manufacturing and services sectors, respectively. For the manufacturing input share, Chilean data has values closer to 0.60, while Colombian data has values closer to 0.70, so we pick a value in between to study a representative economy. There are no such cross-country differences when it comes to μ, so we set it to 0.40. A third parameter that influences cost shares in the model is ω, but this parameter cannot be easily matched to an observable moment in the data. In addition, we cannot separately identify the home-bias parameter from the relative price of domestic and imported intermediate input prices in steady state. Therefore, we follow Gopinath & Neiman (2014) and Blaum et al. (2018) and set ω to 0.50. The elasticity of substitution across final product varieties σ varies in the literature. Gopinath & Neiman (2014) uses a value of 4.00, while Blaum et al. (2018) uses the ratio of firms' revenues to total cost to back out the elasticity at the sectoral level. They find values in the range of 1.87 to 7.39, with most values in the 3.00 to 3.50 range. Kasahara & Rodrigue (2008) uses a version of Olley & Pakes (1996) and finds values in the range of 3.14 to 4.44. We set σ to 3.00, which is in the range of estimates.

The elasticity of substitution between imported and domestic inputs ε is also set to 4.00 in Gopinath & Neiman (2014), however, Blaum et al. (2018) finds it to be lower and in the 2.38 range, so we set it to the intermediate value of 3.00. The elasticity of substitution between imported varieties θ is restricted to the same value as ε such that the model has an analytical solution as described in the manufacturing sector subsection. The entry fixed cost is calibrated to 0.0075 in Gopinath & Neiman (2014), while it is calibrated to 0.0472 in Blaum et al. (2018), which is a substantial difference. We set values between 0.005 and 0.05, but it does not significantly change quantitative results.

Parameters χ_1 and χ_2 are standard in the literature and determine the debt-elastic interest rate equation quantitatively. Kohn et al. (2021) and Fernández et al. (2018) set χ_1 to 0.001, while García-Cicco et al. (2010) sets it to 2.80 in a setup with additional exogenous elements in the equation, which is a calibration followed by Drechsel & Tenreyro (2018), which is what they call a setup with financial frictions and is meant to capture a higher sensitivity of interest rates to debt levels in developing economies. We set our baseline calibration to 0.001 in line with the literature with no financial frictions.

Table 1: Calibration of main parameters

Parameter	Value	Reference
Manufacturing sector		
Parameter	Value	Reference
γ	0.65	Country IO-tables
ω	0.50	Gopinath & Neiman (2014), Blaum et al. (2018)
ε	3.00	Gopinath & Neiman (2014), Blaum et al. (2018)
θ	3.00	Restriction
$\underline{\varphi}$	1.00	Melitz & Redding (2015)
κ	6.50	Restriction
f	0.05	Blaum et al. (2018)
Services sector		
Parameter	Value	Reference
μ	0.40	Country IO-tables
σ	3.00	Gopinath & Neiman (2014),Kasahara & Rodrigue (2008), Blaum et al. (2018)
Intertemporal		
Parameter	Value	Reference
β	0.98	Kohn et al. (2021)
χ_1	1	-
χ_2	0.001	-
$\bar{b}$	0	Itskhoki & Mukhin (2021)

Targeting H We use Proposition 1 and notice that several steady-state variables jointly determine the equilibrium H, but do not enter meaningfully into any of the relevant elasticities that make up the dynamic system. For example, in the perfect and monopolistic competition cases, it follows that $(LP_M^{\$})/(P_X^{\$}X)$ jointly determine H, so we don't need to take a stand on the particular values of foreign prices, the export quantity and the labor force in levels. We use Colombian and Chilean national accounts to calibrate H. In particular, we use the ratio of total imports to total household consumption together with the calibrated input shares such that $\hat{H} = T^{-1}\sum_t^T(1/\mu\gamma)(P_M M)/(PC)$. In Colombia, this average is 0.87 and is calculated using quarterly data covering the years 2006-2020 while in Chile this average is 0.93 for the year 2008-2018, which we use to target an H of 0.90 in our calibration.

Targeting κ We use Proposition 2 to calibrate κ by leveraging the fact that we have an exact solution for what the tail exponent of the import distribution is. We combined the calibrated elasticities together with the piecewise maximum-likelihood estimate of the tail exponent of the import distribution of the Colombian data. Following Amand & Pelgrin (2016), we use data above the 95th percentile adding to a total of roughly 75 thousand observations over 12 years.

7.2 Impulse Responses

Figure 10 illustrates the responses of the system to three shocks, namely a commodity price shock, a manufacturing productivity shock and an interest rate premium shock. When commodity prices increase, consumption increases, the real exchange rate strengthens, and the trade balance improves. Consumption rises as the same commodity endowment yields a bigger return increasing income that can be spent on the final good. However, as the commodity price shock is stationary, not all additional income is spent on more consumption today. Therefore, imports do not increase as much as exports and the trade balance improves. As the net foreign asset position improves, the real exchange rate strengthens. In line with the qualitative discussion described in the previous section, consumption responds more strongly to the same commodity price shock. Also in line with the qualitative predictions, consumption responds less to the same productivity shock.

A second important difference is the response of the trade balance to productivity shocks. Since the direct impact of productivity shocks to import intensity η_t is mediated by $(1+\nu_{cH}-\nu_{qH})^{-1}$, the impact of productivity shocks on the trade balance is muted when all channels are included. This points to a model that, in addition to all that was previously considered, provides a better fit to the weak procyclical nature of the trade balance in the data.

A third difference is between the case of selection and all the other cases. When there is self-selection into and out of importing, both commodity-price shocks and interest-rate-premium shocks lead to first-order productivity responses. However, we find that these responses are small: for a one-percent positive shock to commodity prices, manufacturing TFP goes up by 10^{-3} percent. The same is true for interest-rate-premium shocks. A positive premium shock leads the representative consumer to save more and reduces consumption and improves the trade balance. Conversely, the increased demand for foreign currency as a means of savings weakens the real exchange rate.

7.3 The rule-of-thumb decomposition

Theorem 2 allows us to approximate the relative relevance of the terms of trade to total factor productivity for any ratio of their respective variances. For example, in the literature we have values ranging from nine in Mendoza (1995), four in Kose (2002), a wide range in Fernández et al. (2018) from less than one in Chile, twelve in Brazil and fourthy-four in Colombia, to thirty-six in Drechsel & Tenreyro (2018)[42]. Regardless of which, we know the relative importance of terms of trade shocks relative to productivity shocks in our calibration.

$$\frac{s_X}{s_A} \propto \frac{(\nu_c)^2}{\frac{\sigma_S^2}{\sigma_D^2}+\left(\frac{\mu-\nu_c}{1-\gamma}\right)^2} = \begin{cases} 0.0201-0.0662 & \text{Perfect competition} \\ 0.0300-0.1213 & \text{Monopolistic competition} \\ 0.0477-0.2761 & \text{Increasing returns} \\ 0.0489-0.2901 & \text{Selection} \end{cases}$$

The upper and lower bounds correspond to the cases where σ_S^2/σ_D^2 are one and zero, respectively. A model that assumes perfect competition might understate the importance of TOT by a factor of two when compared to a complete model with increasing returns to scale and selection into importing. However, thirty-four percent of the gap is closed by monopolistic competition alone, and an additional sixty-two is explained by increasing returns to scale. The inclusion of a selection mechanism into the model makes up for the remaining four percent.

7.4 Filtering and Quantitative Decomposition

[To be completed]

[42] Mendoza (1995) uses $\sigma_X = 4$ and $\sigma_A = 11.77$. Kose (2002) sets a variance of traded TFP to $\sigma_A = 0.0029$ following a variance of 0.0025 for TFP innovations with a persistence parameter of 0.4, and uses a variance of $\sigma_X = 0.012$ for TOT following a percent standard deviation of 10.09 in the data. Data in Fernández et al. (2018) comes from Table 3 page 110. Percent standard deviations in Drechsel & Tenreyro (2018) are $\sigma_X = 0.1765$ and $\sigma_A = 0.0295$.

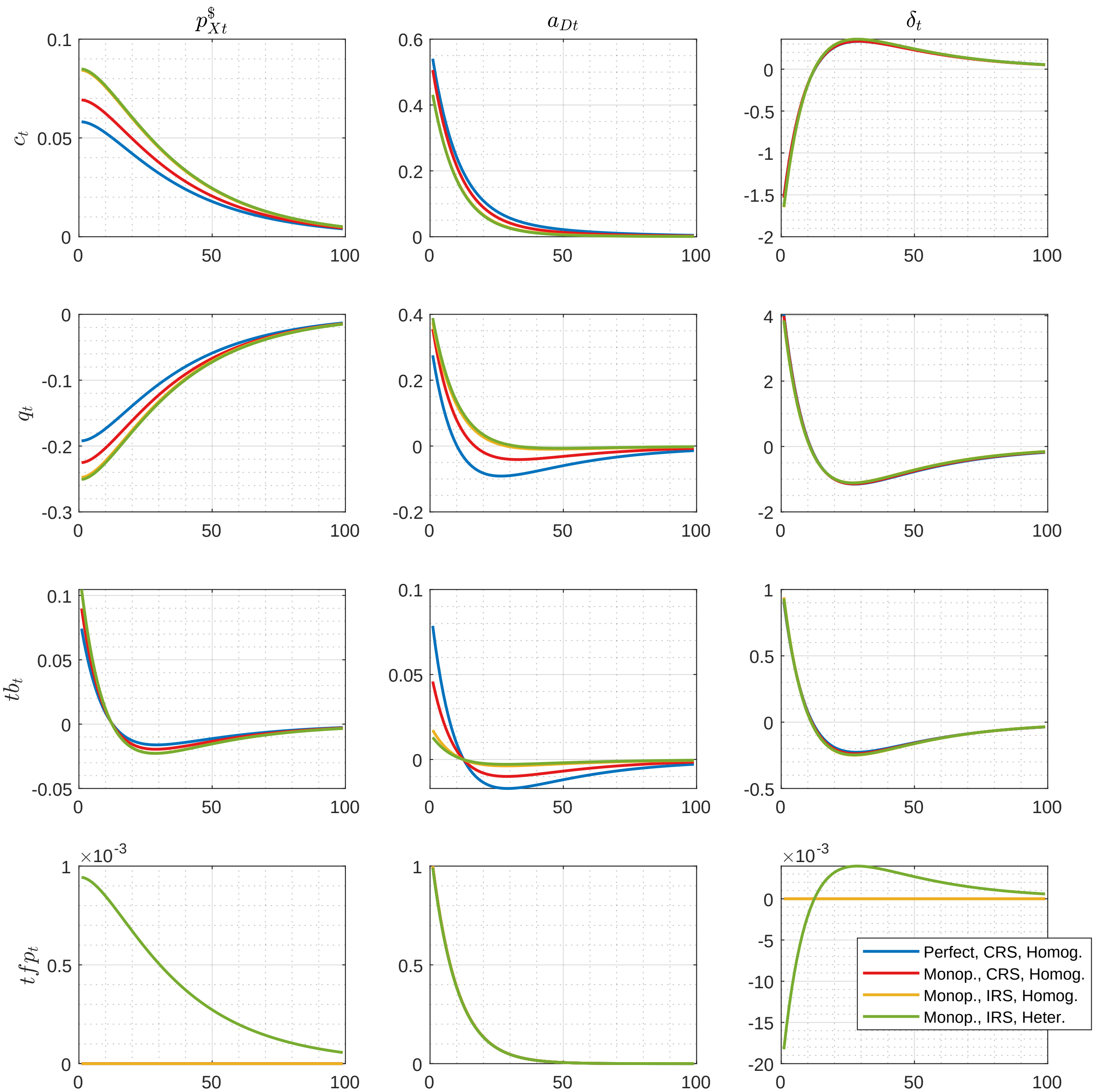


Figure 10: Impulse responses of every setup of the framework. Shocks are columns and responses are rows.

8 Conclusion

In this paper, we investigated whether matching salient micro-level patterns of trade adjustment changes the relative importance of terms-of-trade shocks relative to productivity shocks in commodity-exporting countries. To match these patterns, we developed a model of a small open economy that exports commodities and imports manufactured inputs and produces a final good in a downstream services sector. Domestic manufacturing producers are heterogeneous in their productivity and self-select into importing but have to pay a fixed cost per imported intermediate input variety. This leads to an equilibrium in which more productive domestic manufacturing producers are more exposed to nominal exchange rate fluctuations and adjust both on the sub-intensive and sub-extensive margin.

We show how this model nests simpler cases that are more common in the literature, including homogeneous firm models with perfect competition and monopolistic competition without increasing returns to importing and a homogeneous firm model with increasing returns to importing. In equilibrium, the added frictions lead to an economy that is more open to importing, as manufacturing producers seek to avoid double marginalization at home and import more.

We show how the first-order dynamics can be summarized in a common structure that depends on the partial-equilibrium elasticity of consumption to openness and openness to the real exchange rate. In addition, these partial-equilibrium elasticities determine the relative importance of terms-of-trade shocks relative to productivity shocks. We show how accounting for these micro-moments increases the importance of terms-of-trade shocks by a factor between two and five.

The model with heterogeneity in firm-level productivity and selection into importing captures untargeted moments of the microdata with relative ease. These include the slope of the relative importance of the sub-intensive margin in the cross-sectional distribution of firm size and the Generalized Pareto distribution that is so characteristic of the import distribution in the data. However, the ability to match these patterns does not seem to alter the importance of terms-of-trade shocks relative to productivity shocks in explaining consumption volatility.

References

Abramovitz, M. (1956). Resource and output trends in the united states since 1870. *The American Economic Review*, *46*, 5-23.

Aguiar, M., & Gopinath, G. (2007). Emerging market business cycles: The cycle is the trend. *Journal of Political Economy*, *115*.

Amand, M., & Pelgrin, F. (2016). Pareto distributions in international trade: hard to identify, easy to estimate. *Easy to Estimate (November 13, 2016).*

Amiti, M., Itskhoki, O., & Konings, J. (2019). International shocks, variable markups, and domestic prices. *The Review of Economic Studies*, *86*, 2356-2402.

Amiti, M., & Konings, J. (2007). Trade liberalization, intermediate inputs, and productivity: Evidence from indonesia. *American Economic Review*, *97*, 1611-1638.

Antràs, P., Gutiérrez, A., Fort, T. C., & Tintelnot, F. (2022). *Trade policy and global sourcing: A rationale for tariff escalation.*

Arkolakis, C. (2010). Market penetration costs and the new consumers margin in international trade. *Journal of Political Economy*, *118*, 1151-1199.

Asker, J., Collard-Wexler, A., & Loecker, J. D. (2019, 4). (mis)allocation, market power, and global oil extraction. *American Economic Review*, *109*, 1568-1615.

Atkeson, A., & Burstein, A. (2008). Pricing-to-market, trade costs, and international relative prices. *The American Economic Review*, *98*, 1998-2031.

Backus, D. K., & Smith, G. W. (1993). Consumption and real exchange rates in dynamic economies with non-traded goods. *Journal of International Economics*, *29*, 297-316.

Baqaee, D. R., & Farhi, E. (2020). Productivity and misallocation in general equilibrium. *The Quarterly Journal of Economics*, *135*, 105-163.

Basu, S., & Fernald, J. G. (2002). Aggregate productivity and aggregate technology. *European Economic Review*, *46*, 963-991.

Bennett, H., Valdés, R. O., et al. (2001). *Series de términos de intercambio de frecuencia mensual para la economía chilena: 1965-1999* (No. 98). Banco Central de Chile May.

Bernard, A. B., Jensen, J. B., Redding, S. J., & Schott, P. K. (2009, 5). The margins of us trade. *American Economic Review Papers & Proceedings*, *99*, 487-493.

Bernard, A. B., Jensen, J. B., Redding, S. J., & Schott, P. K. (2012). The empirics of firm heterogeneity and international trade. *Annual Review of Economics*, *4*, 283-313.

Blaum, J., Lelarge, C., & Peters, M. (2018). The gains from input trade with heterogeneous importers. *American Economic Journal: Macroeconomics*, *10*, 77-127.

Brandt, M. W., Cochrane, J. H., & Santa-Clara, P. (2006, 5). International risk sharing is better than you think, or exchange rates are too smooth. *Journal of Monetary Economics*, *53*, 671-698.

Broda, C. (2004, 5). Terms-of-trade and exchange rate regimes in developing countries. *Journal of International Economics*, *63*, 31-58.

Burstein, A., Eichenbaum, M., & Rebelo, S. (2005). Large devaluations and the real exchange rate. *Journal of Political Economy*, *113*, 742-784.

Burstein, A., & Gopinath, G. (2014). International prices and exchange rates. *Handbook of International Economics*, *4*, 391-451.

Chaney, T. (2008, 9). Distorted gravity: The intensive and extensive margins of international trade. *American Economic Review*, *98*, 1707-1721.

Chen, Y. C., & Rogoff, K. (2003). Commodity currencies. *Journal of International Economics*, *60*, 133-160.

Chen, Y.-C., Rogoff, K. S., & Rossi, B. (2010). Can exchange rates forecast commodity prices? *The Quarterly Journal of Economics*, *125*, 1145-1194.

Cochrane, J. H. (1994). Shocks. In (Vol. 41, p. 295-364).

Drechsel, T., & Tenreyro, S. (2018, 5). Commodity booms and busts in emerging economies. *Journal of International Economics*, *112*, 200-218.

Engel, C., & West, K. D. (2005). Exchange rates and fundamentals. *Journal of Political Economy*, *113*, 485-517.

Fernández, A., González, A., & Rodríguez, D. (2018, 3). Sharing a ride on the commodities roller coaster: Common factors in business cycles of emerging economies. *Journal of International Economics*, *111*, 99-121.

Gabaix, X., & Maggiori, M. (2015, 8). International liquidity and exchange rate dynamics. *The Quarterly Journal of Economics*, *130*, 1369-1420.

Galì, J., & Monacelli, T. (2005). Monetary policy and exchange rate interactions in a small open economy. *The Review of Economic Studies*, *72*, 707-734.

García-Cicco, J., Pancrazi, R., & Uribe, M. (2010). Real business cycles in emerging countries? *American Economic Review*, *100*, 2510-2531.

Goldberg, P. K., Khandelwal, A. K., Pavcnik, N., & Topolova, P. (2010). Imported intermediate inputs and domestic product variety: Evidence from india. *The Quarterly Journal of Economics*, *125*, 1727-1767.

Gopinath, G., Boz, E., Casas, C., Díez, F. J., Gourinchas, P. O., & Plagborg-Møller, M. (2020). Dominant currency paradigm. *The American Economic Review*, *110*, 677-719.

Gopinath, G., & Neiman, B. (2014). Trade adjustment and productivity in large crises. *The American Economic Review*, *104*, 793-831.

Halpern, L., Koren, M., & Szeidl, A. (2015). Imported inputs and productivity. *American Economic Review*, *105*, 3660-3703.

Itskhoki, O., & Mukhin, D. (2021). Exchange rate disconnect in general equilibrium. *Journal of Political Economy*, *129*, 2183-2232.

Itskhoki, O., & Mukhin, D. (2022). Mussa puzzle redux. *Revise & Resubmit at Econometrica*, 1-59.

Jeanne, O., & Rose, A. K. (2002). Noise trading and exchange rate regimes. *The Quarterly Journal of Economics*, *117*, 537-569.

Kasahara, H., & Rodrigue, J. (2008, 8). Does the use of imported intermediates increase productivity? plant-level evidence. *Journal of Development Economics*, *87*, 106-118.

Kehoe, T. J., & Ruhl, K. J. (2008). Are shocks to the terms of trade shocks to productivity? *Review of Economic Dynamics*, *11*, 804-819.

Kohn, D., Leibovici, F., & Tretvoll, H. (2021, 7). Trade in commodities and business cycle volatility. *American Economic Journal: Macroeconomics*, *13*, 173-208. doi: 10.1257/mac.20180131

Kose, A. M. (2002). Explaining business cycles in small open economies 'how much do world prices matter?'. *Journal of International Economics*, *56*, 299-327.

Kydland, F. E., & Zarazaga, C. E. (2002). Argentina's lost decade. *Review of Economic Dynamics*, *5*, 152-165.

Känzig, D. R. (2021, 4). The macroeconomic effects of oil supply news: Evidence from opec announcements. *American Economic Review*, *11*, 1092-1125.

Meese, R. A., & Rogoff, K. (1983). Empirical exchange rate models of the seventies: Do they fit out of sample? *Journal of International Economics*, *14*, 3-24.

Melitz, M. J. (2003). The impact of trade on intra-industry reallocations and aggregate industry productivity. *Econometrica*, *71*, 1695-1725.

Melitz, M. J., & Redding, S. J. (2015). New trade models, new welfare implications. *American Economic Review*, *105*, 1105-1146.

Mendoza, E. G. (1995). The terms of trade, the real exchange rate, and economic fluctuations. *International Economic Review*, *36*, 101-137.

Nakamura, E., & Steinsson, J. (2010). Monetary non-neutrality in a multisector menu cost model. *The Quarterly Journal of Economics*, *125*, 961-1013.

Obstfeld, M. (2001). International macroeconomics: Beyond the mundell-fleming model. *IMF Staff Papers*, *47*, 1-39.

Obstfeld, M., & Rogoff, K. (1995). Exchange rate dynamics redux. *Source: Journal of Political Economy*, *103*, 624-660.

Rossi, B. (2013). Exchange rate predictability. *Journal of Economic Literature*, *51*, 1063-1119.

Schmitt-Grohé, S., & Uribe, M. (2003). Closing small open economy models. *Journal of International Economics*, *61*, 163-185.

Schmitt-Grohé, S., & Uribe, M. (2018). How important are terms-of-trade shocks? *International Economic Review*, *59*, 85-111.

A Descriptive statistics

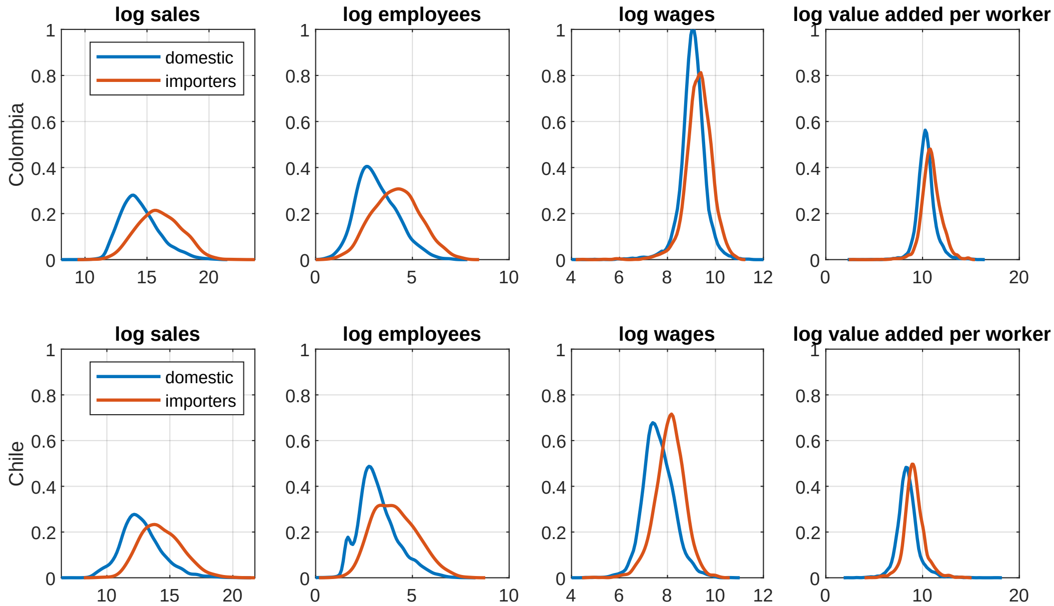


Figure A.1: Kernel densities of log sales, number of employees, wages and value added per worker. Includes only firms that either are exclusively participating in the domestic market, that is, are no importing or exporting, and firms that are importers only. Firms that export *and* import tend to be even larger on all metrics presented.

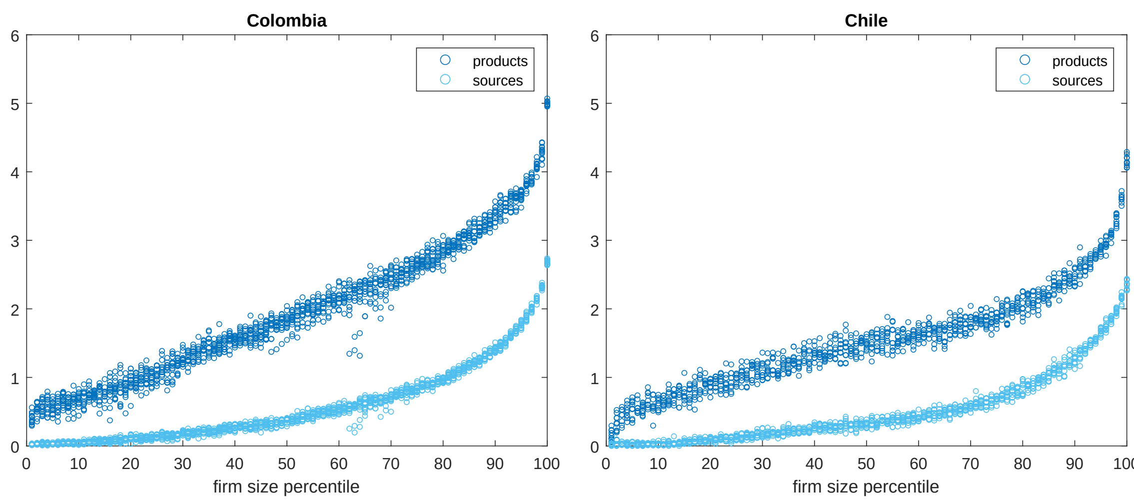


Figure A.2: Log number of products and sources by import size percentile.

Table A.1: Time series properties of aggregates

	Chile		Colombia		Chile			Colombia		
	σ_{x_{t-1},x_t}	σ_x	σ_{x_{t-1},x_t}	σ_x		σ_{xy}			σ_{xy}	
y_t	0.619	2.160	0.556	2.679	1.000			1.000		
c_t	0.507	3.913	0.578	3.010	0.884	1.000		0.914	1.000	
tb_t					-0.197	-0.156	1.000	0.039	0.050	1.000
q_t		0.020		0.025						
s_t		0.044		0.042						

Notes: Relative standard deviations, AR(1) persistence and correlations between output, consumption and the trade balance. Data is quarterly and covers the year 2005-2022 for Colombia and 1996-2021 for Chile. The trade balance is computed as exports minus imports over GDP.

B Non-linear solutions

B.1 Preferences

Households maximize an infinite stream of utility derived from the consumption of the final consumption goods, subject to the country's budget constraint. Their utility maximization problem is given by:

$$\begin{aligned} \max_{\{C_{St}\}_{t=0}^{\infty}} \; & U \sum_{t=0}^{\infty} \ln\left(C_{St}\right) \\ \text{s.t.} \quad & \frac{B_{t+1}}{R_t} - B_t = E_t P_{Xt}^{\$} X + W_t L - P_{St} C_{St} \\ & \lim_{j\to\infty} \mathbb{E}_t \left[\frac{B_{t+j+1}}{\Pi_{s=0}^{\infty} R_{t+j}}\right] \geq 0 \end{aligned}$$

Households allocate their consumption over time by solving the following maximization problem:

$$\max_{\{C_{St}\}_{t=0}^{\infty}} \mathcal{L} = \sum_{t=0}^{\infty} \left(\ln\left(C_{St}\right) + \lambda_t \left(\frac{B_{t+1}}{R_t} - B_t - E_t P_{Xt}^{\$} X - W_t L + P_{St} C_{St}\right)\right)$$

Combining first-order conditions with respect to consumption and the financial asset holding yields the following Euler equation:

$$\beta \mathbb{E}_t \left[\frac{P_{St} C_{St}}{P_{St+1} C_{St+1}} R_t\right] = 1$$

Therefore, the equilibrium conditions are given by

$$\begin{aligned} \beta \mathbb{E}_t \left[\frac{P_{St} C_{St}}{P_{St+1} C_{St+1}} R_t\right] &= 1 \\ \frac{B_{t+1}}{R_t} - B_t &= E_t P_{Xt}^{\$} X + W_t L - P_{St} C_{St} \\ \lim_{j\to\infty} \mathbb{E}_t \left[\frac{B_{t+j+1}}{\Pi_{s=0}^{\infty} R_{t+j}}\right] &\geq 0 \end{aligned} \tag{B.1.0.1}$$

B.2 Services sector

The services sector is made up from Homogeneous producers that combine labor (L_{St}) with the final manufacturing output (Y_{St}) to produce the final consumption good Y_{St}. They have access to the following technology:

$$Y_{St} = A_{St} L_{St}^{1-\mu} X_{St}^{\mu}$$

Services producers solve the following cost minimization problem:

$$\min_{L_{St}, X_{St}} W_t L_{St} + P_{Dt} X_{St}$$
$$\text{s.t.} \qquad Y_{St} = A_{St} L_{St}^{1-\mu} X_{St}^{\mu}$$

This yields the following first order conditions

$$W_t L_{St} = (1-\mu)\mathrm{MC}_{St} Y_{St} \quad \text{and} \quad P_{Dt} X_{St} = \mu \mathrm{MC}_{St} Y_{St}$$

and the following marginal cost function:

$$\mathrm{MC}_{St} = \frac{1}{A_{St}} \frac{{W_t}^{1-\mu} {P_{Dt}}^{\mu}}{(1-\mu)^{1-\mu} \mu^{\mu}}$$

Because services producers compete in a perfectly competitive manner, they price to marginal cost. Therefore the price of services is given by:

$$\mathrm{P}_{St} = \frac{1}{A_{St}} \frac{{W_t}^{1-\mu} {P_{Dt}}^{\mu}}{(1-\mu)^{1-\mu} \mu^{\mu}} \tag{B.2.0.1}$$

B.3 Manufacturing sector

The equilibrium manufacturing price index depends on the assumed production structure.We consider four options: (1) Homogeneous firms that compete under perfect competition and do not have access to an increasing returns to scale importing technology, (2) Homogeneous firms that compete under monopolistic competition and do not have access to an increasing returns to scale importing technology, (3) Homogeneous firms that compete under monopolistic competition and have access to an increasing returns to scale importing technology, (4) heterogeneous firms that compete under monopolistic competition and that can self-select into an increasing returns to scale importing technology.

B.3.1 Homogeneous Firms under Perfect Competition

In this section we provide the derivations for the model where domestic manufacturers are Homogeneous in their productivity and where the importing technology is not subject to economies of scale. Manfacturers compete under monopolistic competition and have access to the following technology:

$$Y_{Dit} = \varphi_D A_{Dt} L_{Dit}^{1-\gamma} X_{Dit}^{\gamma} \quad \text{where} \quad X_{Dit} = \left(\omega^{\frac{1}{\varepsilon}} Q_{Dit}^{\frac{\varepsilon-1}{\varepsilon}} + (1-\omega)^{\frac{1}{\varepsilon}} Q_{Mit}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

Optimal input allocation They solve a two-tiered cost minimization problem:

$$\begin{aligned}
\min_{L_{Dit}, X_{Dit}} \quad & W_t L_{Dit} + P_{Xt} X_{Dit} \\
\text{s.t.} \quad & Y_{Dit} = \varphi_D A_{Dt} L_{Dit}^{1-\gamma} X_{Dit}^{\gamma} \\
& X_{Dit} = \left(\omega^{\frac{1}{\varepsilon}} Q_{Dit}^{\frac{\varepsilon-1}{\varepsilon}} + (1-\omega)^{\frac{1}{\varepsilon}} Q_{Mit}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}
\end{aligned}$$

The first-order conditions for the cost minizimation problem are the following. In the upper tier manufacturing choose the optimal labor-intermediate inputs bundle (L_{Dit}, X_{Dit}) subject to input prices W_t and P_{Xit}. The first-order conditions are given:

$$W_t L_{Dit} = (1-\gamma)\mathrm{MC}_{Dit} Y_{Dit} \quad \text{and} \quad P_{Xit} X_{Dit} = \gamma \mathrm{MC}_{Dit} Y_{Dit}$$

In the lower tier, manufacturers decide on the optimal mix of domestic and imported intermediate inputs (Q_{Dit}, Q_{Mit}) given inputs prices P_{Dt} and $E_t P_{Mt}^{\$}$, both denominated in domestic currency. The first-order conditions from the second tier problem are given by:

$$P_{Dt} Q_{Dt} = \omega \left(\frac{P_{Xit}}{P_{Dt}} \right)^{\varepsilon-1} P_{Xt} X_{Dt} \quad \text{and} \quad E_t p_{Mt}^{\$} Q_{Mit} = (1-\omega) \left(\frac{P_{Xit}}{E_t p_{Mt}^{\$}} \right)^{\varepsilon-1} P_{Xit} X_{Dit}$$

Given that these prices do not depend on the identity of the firm, we can drop the i subscript and combine them to write the marginal cost function as:

$$\mathrm{MC}_{Dt} = \frac{1}{\varphi_D} \frac{1}{A_{Dt}} \frac{{W_t}^{1-\gamma} {P_{Xt}}^{\gamma}}{(1-\gamma)^{1-\gamma} \gamma^{\gamma}} \quad \text{where} \quad P_{Xt} = \left(\omega {P_{Dt}}^{1-\varepsilon} + (1-\omega) E_t {p_{Mt}^{\$}}^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}$$

Manufacturing price index Combining the fact that $P_{Dit} = \mathrm{MC}_{Dit}$ the expression for the marginal cost function and the fact that manufacturers are assumed to be identical, we obtain the aggregate price index for manufacturing goods.

$$\begin{aligned}
P_{Dt} &\equiv \left(\int_i P_{Dit}^{1-\sigma} \right)^{\frac{1}{1-\sigma}} di \\
&= \left(\int_i (\mathrm{MC}_{Dit})^{1-\sigma} \, di \right)^{\frac{1}{1-\sigma}} \\
&= \left(\int_i \left(\frac{1}{\varphi_D} \frac{1}{A_{Dt}} \frac{{W_t}^{1-\gamma} \left(\omega {P_{Dt}}^{1-\varepsilon} + (1-\omega) {P_{Mit}}^{1-\varepsilon} \right)^{\frac{\gamma}{1-\varepsilon}}}{(1-\gamma)^{1-\gamma} \gamma^{\gamma}} \right)^{1-\sigma} di \right)^{\frac{1}{1-\sigma}} \qquad \text{(B.3.1.1)} \\
&= \frac{1}{\varphi_D} \frac{1}{A_{Dt}} \frac{{W_t}^{1-\gamma} \left(\omega {P_{Dt}}^{1-\varepsilon} + (1-\omega) {P_{Mt}}^{1-\varepsilon} \right)^{\frac{\gamma}{1-\varepsilon}}}{(1-\gamma)^{1-\gamma} \gamma^{\gamma}}
\end{aligned}$$

B.3.2 Homogeneous Firms under Monopolistic Competition

In this section we provide the derivations for the model where domestic manufacturers are homogeneous in their productivity and where the importing technology is not subject to economies of scale. Manfacturers compete under monopolistic competition and have access to the following technology:

$$Y_{Dit} = \varphi_D A_{Dt} L_{Dit}^{1-\gamma} X_{Dit}^{\gamma} \quad \text{where} \quad X_{Dit} = \left(\omega^{\frac{1}{\varepsilon}} Q_{Dit}^{\frac{\varepsilon-1}{\varepsilon}} + (1-\omega)^{\frac{1}{\varepsilon}} Q_{Mit}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}$$

Optimal input allocation They solve a two-tiered cost minimization problem:

$$\begin{aligned} \min_{L_{Dit}, X_{Dit}} \quad & W_t L_{Dit} + P_{Xt} X_{Dit} \\ \text{s.t.} \quad & Y_{Dit} = \varphi_D A_{Dt} L_{Dit}^{1-\gamma} X_{Dit}^{\gamma} \\ & X_{Dit} = \left(\omega^{\frac{1}{\varepsilon}} Q_{Dit}^{\frac{\varepsilon-1}{\varepsilon}} + (1-\omega)^{\frac{1}{\varepsilon}} Q_{Mit}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}} \end{aligned}$$

The first-order conditions for the cost minizimation problem are the following. In the upper tier manufacturing choose the optimal labor-intermediate inputs bundle (L_{Dit}, X_{Dit}) subject to input prices W_t and P_{Xit}. The first-order conditions are given:

$$W_t L_{Dit} = (1-\gamma)\text{MC}_{Dit} Y_{Dit} \quad \text{and} \quad P_{Xit} X_{Dit} = \gamma \text{MC}_{Dit} Y_{Dit}$$

In the lower tier, manufacturers decide on the optimal mix of domestic and imported intermediate inputs (Q_{Dit}, Q_{Mit}) given inputs prices P_{Dt} and $E_t P_{Mt}^{\$}$, both denominated in domestic currency. The first-order conditions from the second tier problem are given by:

$$P_{Dt} Q_{Dt} = \omega \left(\frac{P_{Xit}}{P_{Dt}}\right)^{\varepsilon-1} P_{Xt} X_{Dt} \quad \text{and} \quad E_t p_{Mt}^{\$} Q_{Mit} = (1-\omega) \left(\frac{P_{Xit}}{E_t p_{Mt}^{\$}}\right)^{\varepsilon-1} P_{Xit} X_{Dit}$$

Given that these prices do not depend on the identity of the firm, we can drop the i subscript and combine them to write the marginal cost function as:

$$\text{MC}_{Dt} = \frac{1}{\varphi_D} \frac{1}{A_{Dt}} \frac{{W_t}^{1-\gamma} {P_{Xt}}^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}} \quad \text{where} \quad P_{Xt} = \left(\omega {P_{Dt}}^{1-\varepsilon} + (1-\omega) E_t {p_{Mt}^{\$}}^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}$$

Manufacturing price index Combining the fact that $P_{Dt} = \frac{\sigma}{\sigma-1}\text{MC}_{Dt}$, the expression for the marginal cost function and the fact that manufacturers are assumed to be identical, we obtain the aggregate price index for manufacturing goods.

$$\begin{aligned} P_{Dt} &\equiv \left(\int_i P_{Dit}^{1-\sigma}\right)^{\frac{1}{1-\sigma}} di \\ &= \left(\int_i \left(\frac{\sigma}{\sigma-1}\text{MC}_{Dit}\right)^{1-\sigma} di\right)^{\frac{1}{1-\sigma}} \\ &= \frac{\sigma}{\sigma-1}\left(\int_i \left(\frac{1}{\varphi_D}\frac{1}{A_{Dt}} \frac{{W_t}^{1-\gamma}\left(\omega {P_{Dt}}^{1-\varepsilon} + (1-\omega){P_{Mit}}^{1-\varepsilon}\right)^{\frac{\gamma}{1-\varepsilon}}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right)^{1-\sigma} di\right)^{\frac{1}{1-\sigma}} \qquad \text{(B.3.2.1)} \\ &= \frac{\sigma}{\sigma-1}\frac{1}{\varphi_D}\frac{1}{A_{Dt}} \frac{{W_t}^{1-\gamma}\left(\omega {P_{Dt}}^{1-\varepsilon} + (1-\omega){P_{Mt}}^{1-\varepsilon}\right)^{\frac{\gamma}{1-\varepsilon}}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}} \end{aligned}$$

B.3.3 Homogeneous Firms under Monopolistic Competition and IRS Importing

In this section we provide the derivations for the model where domestic manufacturers are Homogeneous in their productivity and where the importing technology is subject to economies of scale. Manfacturers have access to the following technology:

$$Y_{Dit} = \varphi_D A_{Dt} L_{Dt}^{1-\gamma} X_{Dit}^{\gamma}$$

$$\text{where} \quad X_{Dit} = \left(\omega^{\frac{1}{\varepsilon}} Q_{Dit}^{\frac{\varepsilon-1}{\varepsilon}} + (1-\omega)^{\frac{1}{\varepsilon}} Q_{Mit}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}} \text{ and } \qquad Q_{Mit} = \left(\int_{k\in|\mathcal{L}_{it}|} q_{Mkit}{}^{\frac{\theta-1}{\theta}} dk\right)^{\frac{\theta}{\theta-1}}$$

The optimal production strategy is determined in two steps. First, conditional on the sourcing strategy $|\mathcal{L}_{it}|$, manufacturers choose the cost minimizing bundle of labor and intermediate inputs and the cost-minizing bundle of domestic and foreign intemediate inputs for each level of output. Second, given this production structure manufacturers determine the optimal measure $|\mathcal{L}_{it}|$ of imported intermediate input varieties subject to the fixed costs of importing.

Conditional optimal input allocation They solve a two-tiered cost minimization problem:

$$\begin{aligned} \min_{L_{Dit}, X_{Dt}\big||\mathcal{L}_{it}|} \quad & W_t L_{Dit} + P_{Xt} X_{Dit} \\ \text{s.t.} \quad & Y_{Dit} = \varphi_D A_{Dt} L_{Dit}^{1-\gamma} X_{Dit}^{\gamma} \\ & X_{Dit} = \left(\omega^{\frac{1}{\varepsilon}} Q_{Dit}^{\frac{\varepsilon-1}{\varepsilon}} + (1-\omega)^{\frac{1}{\varepsilon}} Q_{Mit}\left(|\mathcal{L}_{it}|\right)^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}} \end{aligned}$$

The first-order conditions for the cost minizimation problem are the following. In the upper tier manufacturing choose the optimal labor-intermediate inputs bundle (L_{Dit}, X_{Dit}) subject to input prices W_t and P_{Xt}. The first-order conditions are given:

$$W_t L_{Dit} = (1-\gamma)\text{MC}_{Dit} Y_{Dit} \quad \text{and} \quad P_{Xit} X_{Dt} = \gamma \text{MC}_{Dit} Y_{Dit}$$

In the lower tier, manufacturers decide on the optimal mix of domestic and imported intermediate inputs $(Q_{Dit}, Q_{Mit}(|\mathcal{L}_{it}|))$ given inputs prices P_{Dt} and $P_{Mit}(|\mathcal{L}_{it}|)$, both denominated in domestic currency. The first-order conditions from the second tier problem are given by:

$$P_{Dt} Q_{Dit} = \omega \left(\frac{P_{Xit}}{P_{Dt}}\right)^{\varepsilon-1} P_{Xit} X_{iDt} \quad \text{and} \quad P_{Mit} Q_{Mit} = (1-\omega)\left(\frac{P_{Xit}}{P_{Mit}(|\mathcal{L}_{it}|)}\right)^{\varepsilon-1} P_{Xit} X_{Dit}$$

These first-order conditions can be combined to write the marginal cost function as:

$$\text{MC}_{Dit} = \frac{1}{\varphi_D} \frac{1}{A_{Dt}} \frac{W_t{}^{1-\gamma} P_{Xit}{}^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}} \quad \text{where} \quad P_{Xit} = \left(\omega P_{Dt}{}^{1-\varepsilon} + (1-\omega) P_{Mit}(|\mathcal{L}_{it}|)^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}$$

Sourcing strategy Given the optimal production structure conditional on the sourcing strategy, we now solve for the optimal sourcing strategy assuming that firms choose the sourcing strategy that maximizes their profits:

$$\begin{aligned}
&\max_{|\mathcal{L}_{it}|} \left(P_{Dit} - MC_{Dit}\right) Y_{it} - W_t f |\mathcal{L}_{it}| \\
\text{s.t.} \qquad & MC_{Dit} = \frac{1}{A_{Dt}} \frac{1}{\varphi_D} \frac{{W_t}^{1-\gamma} {P_{Xit}}^{\gamma}}{\gamma^\gamma (1-\gamma)^{1-\gamma}} \\
& P_{Xit} = \left[\omega {P_{Dt}}^{1-\varepsilon} + (1-\omega)\left(E_t P^{\$}_{Mt}\right)^{1-\varepsilon} |\mathcal{L}_{it}|\right]^{\frac{1}{1-\varepsilon}} \\
& Y_{Dt} = X_{St} + Q_{Dt} \\
& P_{Dit} = \frac{\sigma}{\sigma-1} MC_{Dit}
\end{aligned}$$

where we have used the assumption that $\theta = \epsilon$ such that $P_{Mt} = E_t p^{\$}_{Mt} |\mathcal{L}_{it}|^{\frac{1}{\varepsilon-1}}$ or when all constraints are substituted in

$$\max_{|\mathcal{L}_{it}|} \frac{1}{\sigma-1}\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma} P^{\sigma}_{Dt}(X_{St}+Q_{Dt}) \left[\frac{1}{A_{Dt}} \frac{1}{\varphi_D} \frac{{W_t}^{1-\gamma}}{(1-\gamma)^{1-\gamma}\gamma^\gamma} \left(\omega {P_{Dt}}^{1-\varepsilon} + (1-\omega)\left(E_t P^{\$}_{Mt}\right)^{1-\varepsilon} |\mathcal{L}_{it}|\right)^{\frac{\gamma}{1-\varepsilon}}\right]^{1-\sigma} - W_t f |\mathcal{L}_{it}|$$

Now we propose a change of variables in the maximization problem. Let

$$Z_t = \left(\omega {P_{Dt}}^{1-\varepsilon} + (1-\omega) {P_{Mt}}^{1-\varepsilon} |\mathcal{L}_{it}|\right)^{\gamma\frac{\sigma-1}{\varepsilon-1}} \Rightarrow |\mathcal{L}_{it}| = \frac{{Z_t}^{\frac{\varepsilon-1}{\gamma(\sigma-1)}} - \omega {P_{Dt}}^{1-\varepsilon}}{(1-\omega)\left(E_t P^{\$}_{Mt}\right)^{1-\varepsilon}}$$

such that the maximization problem becomes

$$\max_{|\mathcal{L}_{it}|} \frac{1}{\sigma-1}\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma} {P_{Dt}}^{\sigma}(X_{St}+Q_{Dt}) \left[\frac{1}{A_{Dt}} \frac{1}{\varphi_D} \frac{{W_t}^{1-\gamma}}{(1-\gamma)^{1-\gamma}\gamma^\gamma}\right]^{1-\sigma} Z_t - W_t f \frac{{Z_t}^{\frac{\varepsilon-1}{\gamma(\sigma-1)}} - \omega {P_{Dt}}^{1-\varepsilon}}{(1-\omega)\left(E_t P^{\$}_{Mt}\right)^{1-\varepsilon}}$$

The first order condition of this problem is the following.

$$\frac{1}{\sigma-1}\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma} {P_{Dt}}^{\sigma}(X_{St}+Q_{Dt}) \left[\frac{1}{A_{Dt}} \frac{1}{\varphi_D} \frac{{W_t}^{1-\gamma}}{(1-\gamma)^{1-\gamma}\gamma^\gamma}\right]^{1-\sigma} - W_t f \frac{\varepsilon-1}{\gamma(\sigma-1)} \frac{{Z_t}^{\frac{\varepsilon-1}{\gamma(\sigma-1)}-1}}{(1-\omega)\left(E_t P^{\$}_{Mt}\right)^{1-\varepsilon}} = 0$$

Hence we have an expression for Z_t:

$${Z_t}^{\frac{\varepsilon-1-\gamma(\sigma-1)}{\gamma(\sigma-1)}} = \frac{1}{\sigma-1}\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma} \frac{\gamma(\sigma-1)}{\varepsilon-1} \frac{{P_{Dt}}^{\sigma}(X_{St}+Q_{Dt})}{f W_t} (1-\omega)\left(E_t P^{\$}_{Mt}\right)^{1-\varepsilon} \left[\frac{1}{A_{Dt}} \frac{1}{\varphi_D} \frac{{W_t}^{1-\gamma}}{(1-\gamma)^{1-\gamma}\gamma^\gamma}\right]^{1-\sigma}$$

and consequently

$$\begin{aligned}
&\left(\omega {P_{Dt}}^{1-\varepsilon} + (1-\omega)\left(E_t P^{\$}_{Mt}\right)^{1-\varepsilon} |\mathcal{L}_{it}|\right)^{\frac{\varepsilon-1-\gamma(\sigma-1)}{\varepsilon-1}} \\
&\qquad = \left(\frac{\sigma}{\sigma-1}\right)^{-\sigma} \frac{\gamma}{\varepsilon-1} \frac{{P_{Dt}}^{\sigma}(X_{St}+Q_{Dt})}{f W_t} (1-\omega)\left(E_t P^{\$}_{Mt}\right)^{1-\varepsilon} \left[\frac{1}{A_{Dt}} \frac{1}{\varphi_D} \frac{{W_t}^{1-\gamma}}{(1-\gamma)^{1-\gamma}\gamma^\gamma}\right]^{1-\sigma}
\end{aligned}$$

We can then solve for the measure of imported varieties.

$$|\mathcal{L}_{it}| = \left[\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma} \frac{\gamma(1-\omega)^{\frac{\gamma(\sigma-1)}{\varepsilon-1}}}{\varepsilon-1} \frac{P_{Dt}{}^{\sigma}(X_{St}+Q_{Dt})}{fW_t} \left(\frac{1}{A_{Dt}}\frac{1}{\varphi_D}\frac{W_t{}^{1-\gamma}P_{Mt}^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right)^{1-\sigma}\right]^{\frac{\varepsilon-1}{\varepsilon-1-\gamma(\sigma-1)}} - \frac{\omega}{1-\omega}\left(\frac{P_{Dt}}{E_t P_{Mt}^{\$}}\right)^{1-\varepsilon}$$

This expression does not depend on the idendity of the firm and therefore all firms have the same sourcing strategy. At the same time, this expression defines the minimal level of productivity φ_D necessary for firms to import and to cover the fixed costs. This is found at $|\mathcal{L}_{it}|(\varphi_{Mt}) = 0$:

$$\varphi_{Mt} = \left(\frac{\sigma}{\sigma-1}\right)^{\frac{\sigma}{\sigma-1}} \left(\frac{\gamma(1-\omega)^{\frac{\gamma(\sigma-1)}{\varepsilon-1}}}{\varepsilon-1} \frac{P_{Dt}{}^{\sigma}(X_{St}+Q_{Dt})}{fW_t}\right)^{-\frac{1}{\sigma-1}} \frac{1}{A_{Dt}} \frac{W_t{}^{1-\gamma}E_t P_{Mt}^{\$}{}^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}} \left[\frac{\omega}{1-\omega}\left(\frac{P_{Dt}}{E_t P_{Mt}^{\$}}\right)^{1-\varepsilon}\right]^{\frac{\varepsilon-1-\gamma(\sigma-1)}{(\sigma-1)(\varepsilon-1)}}$$

We can use this expression to write the measure of imported inputs more succintly as a function of the importing cutoff where we drop the subscript i:

$$|\mathcal{L}_t| = \frac{\omega}{1-\omega}\left(\frac{P_{Dt}}{E_t P_{Mt}^{\$}}\right)^{1-\varepsilon} \left[\left(\frac{\varphi_D}{\varphi_{Mt}}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}} - 1\right]$$

We can then use this result to solve for firm-specific input prices and unit costs, respectively. We have that

$$P_{Xt} = \left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{\sigma-1}{\varepsilon-1-\gamma(\sigma-1)}} \omega^{\frac{1}{1-\varepsilon}} P_{Dt}$$

Manufacturing price index Combining the fact that $P_{Dit} = \frac{\sigma}{\sigma-1}\mathrm{MC}_{Dit}$, the expression for the marginal cost function and the fact that manufacturers are assumed to be identical, we obtain the aggregate price index for manufacturing goods.

$$\begin{aligned}
P_{Dit} &\equiv \left(\int_i P_{Dit}^{1-\sigma}\right)^{\frac{1}{1-\sigma}} di \\
&= \left(\int_i \left(\frac{\sigma}{\sigma-1}\mathrm{MC}_{Dit}\right)^{1-\sigma} di\right)^{\frac{1}{1-\sigma}} \\
&= \frac{\sigma}{\sigma-1}\left(\int_i \left(\frac{1}{\varphi_D}\frac{1}{A_{Dt}} \frac{W_t{}^{1-\gamma}\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{\gamma(\sigma-1)}{\varepsilon-1-\gamma(\sigma-1)}}\omega^{\frac{\gamma}{1-\varepsilon}}P_{Dt}^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right)^{1-\sigma} di\right)^{\frac{1}{1-\sigma}} \qquad \text{(B.3.3.1)} \\
P_{Dt} &= \frac{\sigma}{\sigma-1}\frac{1}{\varphi_D}\frac{1}{A_{Dt}} \frac{W_t{}^{1-\gamma}\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{\gamma(\sigma-1)}{\varepsilon-1-\gamma(\sigma-1)}}\omega^{\frac{\gamma}{1-\varepsilon}}P_{Dt}^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}
\end{aligned}$$

B.3.4 heterogeneous Firms under Monopolistic Competition and IRS Importing

In this section we provide the derivations for the model where domestic manufacturers are heterogeneous in their productivity and where they can self-select into an importing technology that is subject to economies of scale.

Manfacturers have access to the following technology:

$$Y_{Dit} = \varphi_D A_{Dt} L_{Dt}^{1-\gamma} X_{Dit}^{\gamma}$$

$$\text{where} \quad X_{Dit} = \left(\omega^{\frac{1}{\varepsilon}} Q_{Dit}^{\frac{\varepsilon-1}{\varepsilon}} + (1-\omega)^{\frac{1}{\varepsilon}} Q_{Mit}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}} \text{ and } \qquad Q_{Mit} = \left(\int_{k\in|\mathcal{L}_{it}|} q_{Mkit}{}^{\frac{\theta-1}{\theta}} dk\right)^{\frac{\theta}{\theta-1}}$$

The optimal production strategy is determined in two steps. First, conditional on the sourcing strategy $|\mathcal{L}_{it}|$, manufacturers choose the cost minimizing bundle of labor and intermediate inputs and the cost-minizing bundle of domestic and foreign intemediate inputs for each level of output. Second, given this production structure manufacturers determine the optimal measure $|\mathcal{L}_{it}|$ of imported intermediate input varieties subject to the fixed costs of importing.

Conditional optimal input allocation They solve a two-tiered cost minimization problem:

$$\begin{aligned}
\min_{L_{Dit}, X_{Dt}\big| |\mathcal{L}_{it}|} \quad & W_t L_{Dit} + P_{Xt} X_{Dit} \\
\text{s.t.} \quad & Y_{Dit} = \varphi_D A_{Dt} L_{Dit}^{1-\gamma} X_{Dit}^{\gamma} \\
& X_{Dit} = \left(\omega^{\frac{1}{\varepsilon}} Q_{Dit}^{\frac{\varepsilon-1}{\varepsilon}} + (1-\omega)^{\frac{1}{\varepsilon}} Q_{Mit}\left(|\mathcal{L}_{it}|\right)^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}
\end{aligned}$$

The first-order conditions for the cost minizimation problem are the following. In the upper tier manufacturing choose the optimal labor-intermediate inputs bundle (L_{Dit}, X_{Dit}) subject to input prices W_t and P_{Xt}. The first-order conditions are given:

$$W_t L_{Dit} = (1-\gamma)\text{MC}_{Dit} Y_{Dit} \quad \text{and} \quad P_{Xit} X_{Dt} = \gamma \text{MC}_{Dit} Y_{Dit}$$

In the lower tier, manufacturers decide on the optimal mix of domestic and imported intermediate inputs $(Q_{Dit}, Q_{Mit}\left(|\mathcal{L}_{it}|\right))$ given inputs prices P_{Dt} and $P_{Mit}\left(|\mathcal{L}_{it}|\right)$, both denominated in domestic currency. The first-order conditions from the second tier problem are given by:

$$P_{Dt} Q_{Dit} = \omega \left(\frac{P_{Xit}}{P_{Dt}}\right)^{\varepsilon-1} P_{Xit} X_{iDt} \quad \text{and} \quad P_{Mit} Q_{Mit} = (1-\omega)\left(\frac{P_{Xit}}{P_{Mit}\left(|\mathcal{L}_{it}|\right)}\right)^{\varepsilon-1} P_{Xit} X_{Dit}$$

These first-order conditions can be combined to write the marginal cost function as:

$$\text{MC}_{Dit} = \frac{1}{\varphi_D} \frac{1}{A_{Dt}} \frac{W_t{}^{1-\gamma} P_{Xit}{}^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}} \quad \text{where} \quad P_{Xit} = \left(\omega P_{Dt}{}^{1-\varepsilon} + (1-\omega) P_{Mit}(|\mathcal{L}_{it}|)^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}$$

Sourcing strategy The end problem to be solved by the manufacturing producer after solving for optimal prices and input use is to chose a measure of imported varieties. The problem is structured as follows

$$\begin{aligned}
&\max_{|\mathcal{L}_{it}|}(p_{it}-c_{it})Y_{it}-W_t f|\mathcal{L}_{it}| \\
\text{s.t.}\qquad & c_{it}=\frac{1}{A_{Dt}}\frac{{W_t}^{1-\gamma}{P_{Xit}}^{\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}}\frac{1}{\varphi_i} \\
& P_{Xit}=\left[\omega{P_{Dt}}^{1-\varepsilon}+(1-\omega){P_{Mt}}^{1-\varepsilon}|\mathcal{L}_{it}|\right]^{\frac{1}{1-\varepsilon}} \\
& Y_{it}=\left(\frac{p_{it}}{P_{Dt}}\right)^{-\sigma}(X_{St}+Q_{Dt}) \\
& p_{it}=\frac{\sigma}{\sigma-1}c_{it}
\end{aligned}$$

or when all constraints are substituted in

$$\max_{|\mathcal{L}_{it}|}\frac{1}{\sigma-1}\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma}P_{Dt}^{\sigma}(X_{St}+Q_{Dt})\left[\frac{1}{A_{Dt}}\frac{{W_t}^{1-\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\frac{1}{\varphi_i}\left(\omega{P_{Dt}}^{1-\varepsilon}+(1-\omega){P_{Mt}}^{1-\varepsilon}|\mathcal{L}_{it}|\right)^{\frac{\gamma}{1-\varepsilon}}\right]^{1-\sigma}-W_t f|\mathcal{L}_{it}|$$

Now we propose a change of variables in the maximization problem. Let

$$Z_t=\left(\omega{P_{Dt}}^{1-\varepsilon}+(1-\omega){P_{Mt}}^{1-\varepsilon}|\mathcal{L}_{it}|\right)^{\gamma\frac{\sigma-1}{\varepsilon-1}}\Rightarrow|\mathcal{L}_{it}|=\frac{{Z_t}^{\frac{\varepsilon-1}{\gamma(\sigma-1)}}-\omega{P_{Dt}}^{1-\varepsilon}}{(1-\omega){P_{Mt}}^{1-\varepsilon}}$$

such that the maximization problem becomes

$$\max_{|\mathcal{L}_{it}|}\frac{1}{\sigma-1}\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma}{P_{Dt}}^{\sigma}(X_{St}+Q_{Dt})\left[\frac{1}{A_{Dt}}\frac{{W_t}^{1-\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\frac{1}{\varphi_i}\right]^{1-\sigma}Z_t-W_t f\frac{{Z_t}^{\frac{\varepsilon-1}{\gamma(\sigma-1)}}-\omega{P_{Dt}}^{1-\varepsilon}}{(1-\omega){P_{Mt}}^{1-\varepsilon}}$$

The first order condition of this problem is the following.

$$\frac{1}{\sigma-1}\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma}{P_{Dt}}^{\sigma}(X_{St}+Q_{Dt})\left[\frac{1}{A_{Dt}}\frac{{W_t}^{1-\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\frac{1}{\varphi_i}\right]^{1-\sigma}-W_t f\frac{\varepsilon-1}{\gamma(\sigma-1)}\frac{{Z_t}^{\frac{\varepsilon-1}{\gamma(\sigma-1)}-1}}{(1-\omega)P_{Mt}^{1-\varepsilon}}=0$$

Hence we have an expression for Z_t:

$${Z_t}^{\frac{\varepsilon-1-\gamma(\sigma-1)}{\gamma(\sigma-1)}}=\frac{1}{\sigma-1}\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma}\frac{\gamma(\sigma-1)}{\varepsilon-1}\frac{{P_{Dt}}^{\sigma}(X_{St}+Q_{Dt})}{fW_t}(1-\omega){P_{Mt}}^{1-\varepsilon}\left[\frac{1}{A_{Dt}}\frac{{W_t}^{1-\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\frac{1}{\varphi_i}\right]^{1-\sigma}$$

and consequently

$$\begin{aligned}
&\left(\omega{P_{Dt}}^{1-\varepsilon}+(1-\omega){P_{Mt}}^{1-\varepsilon}|\mathcal{L}_{it}|\right)^{\frac{\varepsilon-1-\gamma(\sigma-1)}{\varepsilon-1}} \\
&\qquad=\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma}\frac{\gamma}{\varepsilon-1}\frac{{P_{Dt}}^{\sigma}(X_{St}+Q_{Dt})}{fW_t}(1-\omega)P_{Mt}^{1-\varepsilon}\left[\frac{1}{A_{Dt}}\frac{{W_t}^{1-\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\frac{1}{\varphi_i}\right]^{1-\sigma}
\end{aligned}$$

We can then solve for the measure of imported varieties.

$$|\mathcal{L}_{it}| = \left[\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma} \frac{\gamma(1-\omega)^{\frac{\gamma(\sigma-1)}{\varepsilon-1}}}{\varepsilon-1} \frac{P_{Dt}{}^{\sigma}(X_{St}+Q_{Dt})}{fW_t} \left(\frac{1}{A_{Dt}} \frac{W_t{}^{1-\gamma} P_{Mt}^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right)^{1-\sigma} \varphi_i{}^{\sigma-1} \right]^{\frac{\varepsilon-1}{\varepsilon-1-\gamma(\sigma-1)}} - \frac{\omega}{1-\omega}\left(\frac{P_{Dt}}{P_{Mt}}\right)^{1-\varepsilon}$$

We can use this expression to determine the condition under which the measure of imported varieties is increasing in productivity

$$\frac{\partial|\mathcal{L}_i|}{\partial\varphi_i} > 0 \Leftrightarrow \frac{(\sigma-1)(\varepsilon-1)}{(\varepsilon-1)-\gamma(\sigma-1)} > 0 \Rightarrow \gamma < \frac{\varepsilon-1}{\sigma-1}$$

and to solve for the cutoff productivity value that leads a firm to import inputs.

$$\varphi_{Mt} = \left(\frac{\sigma}{\sigma-1}\right)^{\frac{\sigma}{\sigma-1}} \left(\frac{\gamma(1-\omega)^{\frac{\gamma(\sigma-1)}{\varepsilon-1}}}{\varepsilon-1} \frac{P_{Dt}{}^{\sigma}(X_{St}+Q_{Dt})}{fW_t}\right)^{-\frac{1}{\sigma-1}} \frac{1}{A_{Dt}} \frac{W_t{}^{1-\gamma} P_{Mt}{}^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}} \left[\frac{\omega}{1-\omega}\left(\frac{P_{Dt}}{P_{Mt}}\right)^{1-\varepsilon}\right]^{\frac{\varepsilon-1-\gamma(\sigma-1)}{(\sigma-1)(\varepsilon-1)}}$$

We can use this expression to solve for the measure of imported inputs as a function of the importing cutoff.

$$|\mathcal{L}_{it}| = \frac{\omega}{1-\omega}\left(\frac{P_{Dt}}{P_{Mt}}\right)^{1-\varepsilon} \left[\left(\frac{\varphi_i}{\varphi_{Mt}}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}} - 1\right]$$

if $\varphi_i > \varphi_{Mt}$ and zero otherwise. We can then use this result to solve for firm-specific input prices and unit costs, respectively. We have that

$$P_{Xit} = \left(\frac{\varphi_{Mt}}{\varphi_i}\right)^{\frac{\sigma-1}{\varepsilon-1-\gamma(\sigma-1)}} \omega^{\frac{1}{1-\varepsilon}} P_{Dt}$$

if $\varphi_i \geq \varphi_{Mt}$ and $P_{Xit} = \omega^{\frac{1}{1-\varepsilon}} P_{Dt}$ when $\varphi_i < \varphi_{Mt}$.

Manufacturing price index We combine the expression for $P_{Dt}{}^{1-\sigma}$, P_{X_it} and aggregate across the firm size distribution:

$$\begin{aligned}
P_{Dt}^{1-\sigma} &= \int_i p_{it}^{1-\sigma} di = \int_i \left(\frac{\sigma}{\sigma-1} c_{it}\right)^{1-\sigma} di \\
&= \int_i \left[\frac{\sigma}{\sigma-1} \frac{1}{A_{Dt}} \frac{W_t{}^{1-\gamma} P_{Xit}{}^{\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}} \frac{1}{\varphi_i}\right]^{1-\sigma} di \\
&= \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} \left(\frac{1}{A_{Dt}} \frac{W_t{}^{1-\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right)^{1-\sigma} \int_i \left[P_{Xit}{}^{\gamma} \frac{1}{\varphi_i}\right]^{1-\sigma} di \\
&= \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} \left(\frac{1}{A_{Dt}} \frac{W_t{}^{1-\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right)^{1-\sigma} \left\{ \int_{\underline{\varphi}}^{\varphi_{Mt}} \left[\omega^{\frac{1}{1-\varepsilon}} P_{Dt}\right]^{\gamma(1-\sigma)} \varphi_i^{\sigma-1} g(\varphi) d\varphi \right. \\
&\quad \left. + \int_{\varphi_{Mt}}^{\infty} \left[\left(\frac{\varphi_{Mt}}{\varphi_i}\right)^{\frac{\sigma-1}{\varepsilon-1-\gamma(\sigma-1)}} \omega^{\frac{1}{1-\varepsilon}} P_{Dt}\right]^{\gamma(1-\sigma)} \varphi_i^{\sigma-1} g(\varphi) d\varphi \right\} \\
&= \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} \left(\frac{1}{A_{Dt}} \frac{W_t{}^{1-\gamma} P_{Dt}^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right)^{1-\sigma} \omega^{\frac{\gamma(\sigma-1)}{\varepsilon-1}} \left\{ \int_{\underline{\varphi}}^{\varphi_{Mt}} \varphi_i^{\sigma-1} g(\varphi) d\varphi \right. \\
&\quad \left. + \int_{\varphi_{Mt}}^{\infty} \left(\frac{\varphi_{Mt}}{\varphi_i}\right)^{\frac{\sigma-1}{\varepsilon-1-\gamma(\sigma-1)}\gamma(1-\sigma)} \varphi_i^{\sigma-1} g(\varphi) d\varphi \right\}
\end{aligned}$$

Now we impose that the distribution of productivities is Pareto:

$$g(\varphi) = \kappa \underline{\varphi}^{\kappa} \varphi^{-\kappa-1}$$

The first integral becomes

$$\int_{\underline{\varphi}}^{\varphi_{Mt}} \varphi^{\sigma-1} \kappa \underline{\varphi}^{\kappa} \varphi^{-\kappa-1} d\varphi = \frac{\kappa \underline{\varphi}^{\kappa}}{\sigma-1-\kappa} \varphi^{\sigma-1-\kappa} |_{\underline{\varphi}}^{\varphi_{Mt}} = \frac{\kappa \underline{\varphi}^{\kappa}}{\sigma-1-\kappa} \left({\varphi_{Mt}}^{\sigma-1-\kappa} - \underline{\varphi}^{\sigma-1-\kappa} \right)$$

while the second one becomes

$$\begin{aligned}
\int_{\varphi_{Mt}}^{\infty} \left(\frac{\varphi_{Mt}}{\varphi_i} \right)^{\frac{\sigma-1}{\varepsilon-1-\gamma(\sigma-1)}\gamma(1-\sigma)} & \varphi^{\sigma-1} \kappa \underline{\varphi}^{\kappa} \varphi^{-\kappa-1} d\varphi \\
&= \varphi_{Mt}^{\frac{\sigma-1}{\varepsilon-1-\gamma(\sigma-1)}\gamma(1-\sigma)} \frac{\kappa \underline{\varphi}^{\kappa}}{\kappa - \frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}} \varphi_{Mt}^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}-\kappa} \\
&= \frac{\kappa \underline{\varphi}^{\kappa}}{\kappa - \frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}} \varphi_{Mt}^{\sigma-1-\kappa}
\end{aligned}$$

so that prices are

$$P_{Dt}^{1-\sigma} = \left(\frac{\sigma}{\sigma-1} \omega^{\frac{\gamma}{1-\varepsilon}} \frac{1}{A_{Dt}} \frac{{W_t}^{1-\gamma} {P_{Dt}}^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}} \right)^{1-\sigma} \left[\varphi_{Mt}^{\sigma-1-\kappa} \left(\frac{\kappa \underline{\varphi}^{\kappa}}{\sigma-1-\kappa} + \frac{\kappa \underline{\varphi}^{\kappa}}{\kappa - \frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}} \right) - \underline{\varphi}^{\sigma-1-\kappa} \frac{\kappa \underline{\varphi}^{\kappa}}{\sigma-1-\kappa} \right] \tag{B.3.4.1}$$

B.4 Financial markets

In this appendix, we explain the optimal solution to the risk-averse intermediaries' portfolio choice. This derivation is taken from Itskhoki & Mukhin (2021) and Itskhoki & Mukhin (2022). First, we explain the derivation of the objective function. Second, we describe how discrete time problem can be reformulated into a continuous time problem as long as time periods are short. Finally, we solve the portfolio choice problem in continuous time and characterize the optimal portfolio choice.

Objective function Risk-averse intermediaries want to maximize the profits of the carry trade between the home and ROW currency. Real profits are given by

$$\begin{aligned}
\Pi_{t+1}^{I*} &= \frac{1}{P_{t+1}^{*}} \left(\underbrace{R_t^{*}}_{\text{ROI on ROW bonds}} \underbrace{d_{t+1}^{*}\theta_t^{*}}_{\text{Investment in ROW bonds}} - \underbrace{\frac{R_t}{\mathcal{E}_{t+1}}}_{\text{Financing cost}} \underbrace{\mathcal{E}_t d_{t+1}^{*}\theta_t^{*}}_{\text{Financing in local bonds}} \right) \\
&= \frac{1}{P_{t+1}^{*}} \left(R_t^{*} \frac{d_{t+1}^{*}}{R_t^{*}} - \frac{R_t}{\mathcal{E}_{t+1}} \mathcal{E}_t \frac{d_{t+1}^{*}}{R_t^{*}} \right) \\
&= \frac{1}{P_{t+1}^{*}} \left(R_t^{*} - \frac{R_t}{\mathcal{E}_{t+1}} \mathcal{E}_t \right) \frac{d_{t+1}^{*}}{R_t^{*}} \\
&= \frac{\tilde{R}_t^{*}}{P_{t+1}^{*}} \frac{d_{t+1}^{*}}{R_t^{*}}
\end{aligned}$$

where θ_t^* is the price of a ROW foreign currency bond at time t that pays R_t^* at time $t+1$, such that $\theta_t^* = \frac{1}{R_t^*}$ and where we deflate nominal profits by the ROW price level P_{t+1}^* as the intermediaries originate from the ROW. Now, lets re-rewrite the objective function:

$$\begin{aligned}
\mathbb{E}\left[-\frac{1}{\vartheta}e^{-\vartheta\frac{\tilde{R}_{t+1}^*}{P_{t+1}^*}\frac{d_{t+1}^*}{R_t^*}}\right] &= \mathbb{E}\left[-\frac{1}{\vartheta}e^{-\vartheta\frac{\tilde{R}_{t+1}^*}{R_t^*}\frac{P_t^*}{P_{t+1}^*}\frac{d_{t+1}^*}{P_t^*}}\right] \\
&= \mathbb{E}\left[-\frac{1}{\vartheta}e^{-\vartheta\left(1-\frac{\mathcal{E}_t R_t}{\mathcal{E}_{t+1}R_t^*}\right)\frac{P_t^*}{P_{t+1}^*}\frac{d_{t+1}^*}{P_t^*}}\right] \\
&= \mathbb{E}\left[-\frac{1}{\vartheta}e^{-\vartheta\left(1-e^{\ln\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}+\ln\frac{\mathcal{R}_t}{\mathcal{R}_t^*}}\right)\frac{P_t^*}{P_{t+1}^*}\frac{d_{t+1}^*}{P_t^*}}\right] \\
&= \mathbb{E}\left[-\frac{1}{\vartheta}e^{-\vartheta\left(1-e^{i_t-i_t^*-\Delta e_{t+1}}\right)e^{-\pi_{t+1}}\frac{d_{t+1}^*}{P_t^*}}\right]
\end{aligned}$$

Continuous time limit In Appendix **??**, we show that all financial market models can be represented by a two-by-two system of linear difference equations in deviations of the real exchange rate and the net foreign asset position from their steady state. More importantly, we show that each of the models gives rise to stationary ARMA-processes for the real exchange rate and the net foreign asset position, and thus for all other general equilibrium variables. Therefore, we have that in equilibrium

$$(x_{t+1}, \pi_{t+1})|\mathcal{I}_t \sim \mathcal{N}(\boldsymbol{\mu}_t, \boldsymbol{\sigma^2})$$

where $\mathcal{I}_t$ is the information set at time t that consists of the exogenous processes whose values have been realized at time t, $\boldsymbol{\mu}_t$ is given by

$$\boldsymbol{\mu}_t \equiv \mathbb{E}_t\begin{pmatrix} x_{t+1} \\ \pi_{t+1}^* \end{pmatrix} = \begin{pmatrix} i_t - i_t^* - \mathbb{E}[\Delta e_{t+1}] \\ \mathbb{E}[\pi_{t+1}^*] \end{pmatrix}$$

and $\boldsymbol{\sigma}^2$ is given by:

$$\boldsymbol{\sigma} \equiv \mathbb{V}_t\begin{pmatrix} x_{t+1} \\ \pi_{t+1}^* \end{pmatrix} = \begin{pmatrix} \sigma_{e,t}^2 & -\sigma_{e\pi^*,t} \\ -\sigma_{e\pi^*,t} & \sigma_{\pi^*,t}^2 \end{pmatrix}$$

Now, define $x_{t+1} \equiv= i_t - i_t^* - \Delta e_{t+1}$, then (x_{t+1}, π_{t+1}^*) can be treated as the discrete time interval difference of the associated continuous time process $(d\mathcal{X}, d\mathcal{P}_t^*)$ given by:

$$\begin{pmatrix} d\mathcal{X} \\ d\mathcal{P}_t^* \end{pmatrix} = \boldsymbol{\mu}_t dt + \boldsymbol{\sigma} d\mathcal{Z}_t$$

where $\boldsymbol{\mu}_t$ and $\boldsymbol{\sigma}$ are defined as before and where $d\mathcal{Z}_t$ is a standard two-dimensional Brownian motion given by

$$d\mathcal{Z}_t = \begin{pmatrix} 0 \\ 0 \end{pmatrix} dt + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} d\varepsilon_{1,t} \\ d\varepsilon_{2,t} \end{pmatrix}$$

and $d\varepsilon_{1,t}$ and $d\varepsilon_{1,t}$ are white noise processes. This allows us to re-write the objective function as by applying Ito's

Lemma to approximate the exponential functions and using the defintion of $(d\mathcal{X}, d\mathcal{P}_t^*)$:

$$
\begin{aligned}
&\mathbb{E}\left[-\frac{1}{\vartheta}e^{-\vartheta\left(1-e^{d\mathcal{X}_{t+1}}\right)e^{-d\mathcal{P}_{t+1}^*}\frac{d_{t+1}^*}{P_t^*}}\right] \\
&= \mathbb{E}\left[-\frac{1}{\vartheta}e^{-\vartheta\left(-d\mathcal{X}_{t+1}-\frac{1}{2}d\mathcal{X}_{t+1}^2\right)\left(1-d\mathcal{P}_{t+1}^*-\frac{1}{2}\left(d\mathcal{P}_{t+1}^*\right)^2\right)\frac{d_{t+1}^*}{P_t^*}}\right] \\
&= \mathbb{E}\left[-\frac{1}{\vartheta}e^{-\vartheta\left(-d\mathcal{X}_{t+1}+d\mathcal{X}_{t+1}d\mathcal{P}_{t+1}^*-\frac{1}{2}d\mathcal{X}_{t+1}\left(d\mathcal{P}_{t+1}^*\right)^2-\frac{1}{2}d\mathcal{X}_{t+1}^2+\frac{1}{2}d\mathcal{X}_{t+1}^2d\mathcal{P}_{t+1}^*-\frac{1}{4}d\mathcal{X}_{t+1}^2\left(d\mathcal{P}_{t+1}^*\right)^2\right)\frac{d_{t+1}^*}{P_t^*}}\right] \\
&= \mathbb{E}\left[-\frac{1}{\vartheta}e^{-\vartheta\left(-d\mathcal{X}_{t+1}+d\mathcal{X}_{t+1}d\mathcal{P}_{t+1}^*-\frac{1}{2}d\mathcal{X}_{t+1}^2\right)\frac{d_{t+1}^*}{P_t^*}}\right] \\
&= \mathbb{E}\left[-\frac{1}{\vartheta}e^{\vartheta\left(\mu_{1,t}dt+\sigma_e d\mathcal{W}_{1,t}+\sigma_{e,\pi^*}dt+\frac{1}{2}\sigma_e^2dt\right)\frac{d_{t+1}^*}{P_t^*}}\right] \\
&= \mathbb{E}\left[-\frac{1}{\vartheta}e^{\vartheta\left(\left(\mu_{1,t}+\sigma_{e,\pi^*}+\frac{1}{2}\sigma_e^2\right)\frac{d_{t+1}^*}{P_t^*}dt+\sigma_e\frac{d_{t+1}^*}{P_t^*}d\mathcal{W}_{1,t}\right)}\right]
\end{aligned}
$$

Now, we use the property of the lognormal distribution to transform the objective function:

$$
\begin{aligned}
&\mathbb{E}\left[-\frac{1}{\vartheta}e^{\vartheta\left(\left(\mu_{1,t}+\sigma_{e,\pi^*}+\frac{1}{2}\sigma_e^2\right)\frac{d_{t+1}^*}{P_t^*}dt+\sigma_e\frac{d_{t+1}^*}{P_t^*}d\mathcal{W}_{1,t}\right)}\right] \\
&= -\frac{1}{\vartheta}e^{\mathbb{E}_t\left[\vartheta\left(\left(\mu_{1,t}+\sigma_{e,\pi^*}+\frac{1}{2}\sigma_e^2\right)\frac{d_{t+1}^*}{P_t^*}dt+\sigma_e\frac{d_{t+1}^*}{P_t^*}d\mathcal{W}_{1,t}\right)\right]+\frac{1}{2}\mathrm{var}_t\left[\vartheta\left(\left(\mu_{1,t}+\sigma_{e,\pi^*}+\frac{1}{2}\sigma_e^2\right)\frac{d_{t+1}^*}{P_t^*}dt+\sigma_e\frac{d_{t+1}^*}{P_t^*}d\mathcal{W}_{1,t}\right)\right]} \\
&= -\frac{1}{\vartheta}e^{\vartheta\left(\mu_{1,t}+\sigma_{e,\pi^*}+\frac{1}{2}\sigma_e^2\right)\frac{d_{t+1}^*}{P_t^*}dt+\frac{\vartheta^2\sigma_e^2}{2}\left(\frac{d_{t+1}^*}{P_t^*}\right)^2dt}
\end{aligned}
$$

Therefore, the maximization problem is equivalent to:

$$
\max_{d_{t+1}^*}\mathbb{E}_t\left[-\vartheta\left(\mu_{1,t}+\sigma_{e,\pi^*}+\frac{1}{2}\sigma_e^2\right)\frac{d_{t+1}^*}{P_t^*}+\frac{\vartheta^2\sigma_e^2}{2}\left(\frac{d_{t+1}^*}{P_t^*}\right)^2\right]
$$

where we have applied the log-transformation which globally increasing.

Optimal portfolio choice Given the previous equivalence result, the problem reduces to a standard quadratic equation with a single root. Therefore, the optimal portfolio choice is characterized by:

$$
\frac{d_{t+1}^*}{P_{t+1}^*} = \frac{\mu_{1,t}+\sigma_{e,\pi^*}+\frac{1}{2}\sigma_e^2}{\vartheta\sigma_e^2} = \frac{i_t-i_t^*-\mathbb{E}[\Delta e_{t+1}]\sigma_{e,\pi^*}+\frac{1}{2}\sigma_e^2}{\vartheta\sigma_e^2}
$$

B.5 Trade balance and Labor market clearing

Combining market clearing conditions on goods markets and labor markets leads to intuitive expressions for the amount of saving and the labor market clearing. These expressions depend on the assumed production and market structure in the manufacturing sector.

B.5.1 Homogeneous firms under Perfect Competition

Goods market clearing Goods market clearing implies that the demand for manufacturing output by services producers and by other manufacturing producers equals final output in the manufacturing sector and that total consumption equals output in services

$$Y_{Dit} = X_{St} + \int_j Q_{Djt} dj, \qquad Y_{St} = C_{St}$$

Plugging in the residual demand schedules, we have

$$\begin{aligned}
Y_{Dit} &= X_{Sit} + \int_j Q_{Dijt} dj \\
&= \left(\frac{P_{it}}{P_{Dt}}\right)^{-\sigma} X_{St} + \int_j \left(\frac{P_{it}}{P_{Dt}}\right)^{-\sigma} Q_{Djt} dj \\
&= \left(\frac{P_{it}}{P_{Dt}}\right)^{-\sigma} \left(X_{St} + \int_j Q_{Djt} dj\right) \\
&= \left(\frac{P_{it}}{P_{Dt}}\right)^{-\sigma} (X_{St} + Q_{Dt})
\end{aligned}$$

where $Q_{Dt} \equiv \int_j Q_{Djt} dj$. We can also write this in aggregate form by using the corresponding the aggregation for manufacturing output as dictated by the demand system:

$$\begin{aligned}
Y_{Dt} &\equiv \left(\int_i (Y_{Dit})^{\frac{\sigma-1}{\sigma}} di\right)^{\frac{\sigma}{\sigma-1}} \\
&= \left(\int_i \left(\left(\frac{P_{it}}{P_{Dt}}\right)^{-\sigma} (X_{St} + Q_{Dt})\right)^{\frac{\sigma-1}{\sigma}} di\right)^{\frac{\sigma}{\sigma-1}} \\
&= \left(\int_i P_{it}^{1-\sigma} di\right)^{\frac{\sigma}{\sigma-1}} P_{Dt}^{\sigma} (X_{St} + Q_{Dt}) \\
&= X_{St} + Q_{Dt}
\end{aligned}$$

where we have used the definition of the price index.

Labor market clearing Labor market clearing requires that all labor demanded by the manufacturing and services sectors equals the supply of labor which we assume is perfectly inelastic. Because manufacturing producers are Homogeneous, we can write the labor market clearing in terms of aggregate variables.

$$\begin{aligned}
L_t &= L_{St} + \int_i L_{Dit} di \\
&= L_{St} + \int_i (1-\gamma) \frac{Y_{Dit} MC_{Dit}}{W_t} di \\
&= L_{St} + \int_i (1-\gamma) \frac{Y_{Dt} MC_{Dt}}{W_t} di \\
&= L_{St} + L_{Dt}
\end{aligned}$$

Trade balance The trade balance represents fundamental demand or supply of international foreign assets and depends on the assumed product structure. We re-write this in turn:

$$\begin{aligned}
TB_t &= E_t P^{\$}_{Xt} X + W_t L_t - P_{St} C_{St} \\
&= E_t P^{\$}_{Xt} X + W_t (L_{St} + L_{Dt}) - P_{St} C_{St} \\
&= E_t P^{\$}_{Xt} X + (1-\mu) P_{St} Y_{St} + (1-\gamma) P_{Dt} Y_{Dt} - P_{St} C_{St} \\
&= E_t P^{\$}_{Xt} X - \mu P_{St} C_{St} + (1-\gamma) P_{Dt} (Q_{Dt} + X_{St})
\end{aligned}$$

Now, we can re-write $(Q_{Dt} + X_{St})$ by combining the first-order condition for domestic intermediate inputs:

$$\begin{aligned}
Q_{Dt} &\equiv \int_i Q_{Dit} di \\
&= \int_i \omega \left(\frac{P_{Xit}}{P_{Dt}}\right)^{\varepsilon} \frac{P_{Xit}}{P_{Xit}} X_{Dit} di \\
&= \int_i \omega \left(\frac{P_{Xit}}{P_{Dt}}\right)^{\varepsilon} \gamma \frac{MC_{Dit}}{P_{Xit}} Y_{Dit} di \\
&= \int_i \omega \left(\frac{P_{Xit}}{P_{Dt}}\right)^{\varepsilon} \gamma \frac{MC_{Dit}}{P_{Xit}} \left(\frac{P_{Dit}}{P_{Dt}}\right)^{-\sigma} (Q_{Dt} + X_{St}) di \\
&= \int_i \omega \left(\frac{P_{Xit}}{P_{Dt}}\right)^{\varepsilon} \gamma \frac{P_{Dit}}{P_{Xit}} \left(\frac{P_{Dit}}{P_{Dt}}\right)^{-\sigma} (Q_{Dt} + X_{St}) di \\
&= \gamma \omega \left(\frac{P_{Xt}}{P_{Dt}}\right)^{\varepsilon} \frac{P_{Dt}^{\sigma}}{P_{Xit}} (Q_{Dt} + X_{St}) \int_i (P_{Dit})^{1-\sigma} di \\
&= \gamma \omega \left(\frac{P_{Xt}}{P_{Dt}}\right)^{\varepsilon - 1} (Q_{Dt} + X_{St}) \\
\left[1 - \gamma\omega \left(\frac{P_{Xt}}{P_{Dt}}\right)^{\varepsilon-1}\right] Q_{Dt} &= \gamma \omega \left(\frac{P_{Xt}}{P_{Dt}}\right)^{\varepsilon - 1} X_{St} \\
Q_{Dt} &= \frac{\gamma\omega \left(\frac{P_{Xt}}{P_{Dt}}\right)^{\varepsilon-1}}{1 - \gamma\omega \left(\frac{P_{Xt}}{P_{Dt}}\right)^{\varepsilon-1}} X_{St} \\
Q_{Dt} + X_{St} &= \frac{1}{1 - \gamma\omega \left(\frac{P_{Xt}}{P_{Dt}}\right)^{\varepsilon-1}} X_{St}
\end{aligned}$$

Plugging this is into the budget constraint yields

$$\begin{aligned}
E_t P^{\$}_{Xt} X + W_t L_t - P_{St} C_{St} &= E_t P^{\$}_{Xt} X - \mu P_{St} C_{St} + (1-\gamma) \frac{1}{1-\gamma\omega\left(\frac{P_{Xt}}{P_D t}\right)^{\varepsilon-1}} P_{Dt} X_{St} \\
&= E_t P^{\$}_{Xt} X - \mu P_{St} C_{St} + (1-\gamma)\mu \frac{1}{1-\gamma\omega\left(\frac{P_{Xt}}{P_D t}\right)^{\varepsilon-1}} P_{St} C_{St} \\
&= E_t P^{\$}_{Xt} X - \mu \left[1 - (1-\gamma) \frac{1}{1-\gamma\omega\left(\frac{P_{Xt}}{P_D t}\right)^{\varepsilon-1}} \right] P_{St} C_{St} \\
&= E_t P^{\$}_{Xt} X - \mu\gamma \frac{1-\omega\left(\frac{P_{Xt}}{P_D t}\right)^{\varepsilon-1}}{1-\gamma\omega\left(\frac{P_{Xt}}{P_D t}\right)^{\varepsilon-1}} P_{St} C_{St}
\end{aligned}$$

Now, we can conviently re-write $\frac{1-\omega\left(\frac{P_{Xt}}{P_D t}\right)^{\varepsilon-1}}{1-\gamma\omega\left(\frac{P_{Xt}}{P_D t}\right)^{\varepsilon-1}}$ using the intermediate input price index:

$$\begin{aligned}
P_{Xt}^{1-\varepsilon} &= \omega P_{Dt}^{1-\varepsilon} + (1-\omega) P_{Mt}^{1-\varepsilon} \\
\frac{1}{\omega}\left(\frac{P_{Xt}}{P_{Dt}}\right)^{1-\varepsilon} &= 1 + \frac{1-\omega}{\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{1-\varepsilon} \\
\omega\left(\frac{P_{Dt}}{P_{Xt}}\right)^{1-\varepsilon} &= \frac{1}{1+\frac{1-\omega}{\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{1-\varepsilon}}
\end{aligned}$$

Then we have that:

$$\begin{aligned}
\frac{1-\omega\left(\frac{P_{Xt}}{P_D t}\right)^{\varepsilon-1}}{1-\gamma\omega\left(\frac{P_{Xt}}{P_D t}\right)^{\varepsilon-1}} &= \frac{1-\frac{1}{1+\frac{1-\omega}{\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{1-\varepsilon}}}{1-\gamma\frac{1}{1+\frac{1-\omega}{\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{1-\varepsilon}}} \\
&= \frac{\frac{1-\omega}{\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{1-\varepsilon}}{1+\frac{1-\omega}{\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{1-\varepsilon}-\gamma} \\
&= \frac{1}{1+(1-\gamma)\frac{\omega}{1-\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{\varepsilon-1}}
\end{aligned}$$

Therefore the trade balance can be written as

$$TB_t = E_t P^{\$}_{Xt} X - \mu\gamma H_t P_{St} C_{St} \quad \text{where} \quad H_t \equiv \frac{1}{1+(1-\gamma)\frac{\omega}{1-\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{\varepsilon-1}} \tag{B.5.1.1}$$

Note that the problem for the consumer boils down to satifsying the trade balance condition in financial autarky. In addition, note that we can write the foreign intermediate input share as:

$$S_t^M \equiv \frac{P_{Mt} Q_{Mt}}{P_{Xt} X_{Dt}} = 1 - \frac{P_{Dt} Q_{Dt}}{P_{Xt} X_{Dt}} = 1 - \omega\left(\frac{P_{Dt}}{P_{Xt}}\right)^{1-\varepsilon} = \frac{\frac{1-\omega}{\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{1-\varepsilon}}{1+\frac{1-\omega}{\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{1-\varepsilon}}$$

which in terms of H_t becomes:

$$
\begin{aligned}
H_t &= \frac{1}{1+(1-\gamma)\frac{\omega}{1-\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{\varepsilon-1}} \\
\frac{\omega}{1-\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{\varepsilon-1} &= \frac{1-H_t}{(1-\gamma)H_t} \\
\frac{1-\omega}{\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{1-\varepsilon} &= \frac{(1-\gamma)H_t}{1-H_t} \\
1+\frac{1-\omega}{\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{1-\varepsilon} &= \frac{1-\gamma H_t}{1-H_t}
\end{aligned}
$$

Therefore, we have that the imported intermediate input share is given by:

$$
S_t^M = \frac{\frac{1-\omega}{\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{1-\varepsilon}}{1+\frac{1-\omega}{\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{1-\varepsilon}} = \frac{\frac{(1-\gamma)H_t}{1-H_t}}{\frac{(1-\gamma\eta)H_t}{1-H_t}} = \frac{(1-\gamma)H_t}{1-\gamma H_t}
$$

Labor market clearing - revisited We have:

$$
\begin{aligned}
w_t L_t &= w_t L_{St} + w_t L_{Dt} \\
&= (1-\mu)PStY_{St} + (1-\gamma)P_{Dt}Y_{Dt} \\
&= (1-\mu)PStY_{St} + (1-\gamma)P_{Dt}(Q_{Dt}+X_{St})
\end{aligned}
$$

Now, use the fact that $Q_{Dt}+X_{St} = \frac{1}{1-\gamma\omega\left(\frac{P_{Xt}}{P_{Dt}}\right)^{\varepsilon-1}}X_{St}$ which we can re-write in terms of H_t:

$$
\begin{aligned}
H_t &= \frac{1-\omega\left(\frac{P_{Xt}}{P_Dt}\right)^{\varepsilon-1}}{1-\gamma\omega\left(\frac{P_{Xt}}{P_Dt}\right)^{\varepsilon-1}} \\
\omega\left(\frac{P_{Xt}}{P_Dt}\right)^{\varepsilon-1} - \gamma\omega\left(\frac{P_{Xt}}{P_Dt}\right)^{\varepsilon-1} H_t &= 1-H_t \\
(1-\gamma H_t)\omega\left(\frac{P_{Xt}}{P_Dt}\right)^{\varepsilon-1} &= 1-H_t \\
\gamma\omega\left(\frac{P_{Xt}}{P_Dt}\right)^{\varepsilon-1} &= \frac{\gamma(1-H_t)}{1-\gamma H_t} \\
1-\gamma\omega\left(\frac{P_{Xt}}{P_Dt}\right)^{\varepsilon-1} &= \frac{1-\gamma}{1-\gamma H_t} \\
\frac{1}{1-\gamma\omega\left(\frac{P_{Xt}}{P_{Dt}}\right)^{\varepsilon-1}} &= \frac{1-\gamma H_t}{1-\gamma}
\end{aligned}
$$

Inserting this expression, we arrive at the labor market clearing condition

$$
\begin{aligned}
W_t L_t &= (1-\mu) PStY_{St} + (1-\gamma) P_{Dt}(Q_{Dt} + X_{St}) \\
&= (1-\mu) PStY_{St} + (1-\gamma)\frac{1-\gamma H_t}{1-\gamma} P_{Dt} X_{St} \\
&= (1-\mu) PStY_{St} + (1-\gamma)\frac{1-\gamma H_t}{1-\gamma} \mu P_{St} Y_{St} \\
&= (1-\mu+\mu-\mu\gamma H_t)\, P_{St} Y_{St}
\end{aligned}
$$

Using goods market clearing for final goods $Y_{St} = C_{St}$, we arrive at the labor market clearing condition:

$$
W_t L_t = X_1 (\chi_1 - \mu\gamma H_t)\, P_{St} C_{St} \qquad \text{where} \qquad X_1 = 1, \quad \chi_1 = 1 \tag{B.5.1.2}
$$

In addition, note that we can write labor allocated to the service sector solely as a fuction of H_t as well:

$$
\begin{aligned}
w_t L_t &= X_1 (\chi_1 - \mu\gamma H_t)\, P_{St} Y_{St} \\
&= X_1 (\chi_1 - \mu\gamma H_t)\, \frac{W_t L_{st}}{1-\mu} \\
L_{St} &= \frac{1-\mu}{\chi_1 - \mu\gamma H_t} \frac{L_t}{X_1}
\end{aligned}
$$

B.5.2 Homogeneous firms under Monopolistic Competition

Goods market clearing Goods market clearing implies that the demand for manufacturing output by services producers and by other manufacturing producers equals final output in the manufacturing sector and that total consumption equals output in services

$$
Y_{Dit} = X_{St} + \int_j Q_{Djt} dj, \qquad Y_{St} = C_{St}
$$

Plugging in the residual demand schedules, we have

$$
\begin{aligned}
Y_{Dit} &= X_{Sit} + \int_j Q_{Dijt} dj \\
&= \left(\frac{P_{it}}{P_{Dt}}\right)^{-\sigma} X_{St} + \int_j \left(\frac{P_{it}}{P_{Dt}}\right)^{-\sigma} Q_{Djt} dj \\
&= \left(\frac{P_{it}}{P_{Dt}}\right)^{-\sigma} \left(X_{St} + \int_j Q_{Djt} dj\right) \\
&= \left(\frac{P_{it}}{P_{Dt}}\right)^{-\sigma} (X_{St} + Q_{Dt})
\end{aligned}
$$

where $Q_{Dt} \equiv \int_j Q_{Djt} dj$. We can also write this in aggregate form by using the corresponding the aggregation for manufacturing output as dictated by the demand system:

$$
\begin{aligned}
Y_{Dt} &\equiv \left(\int_i (Y_{Dit})^{\frac{\sigma-1}{\sigma}}\, di\right)^{\frac{\sigma}{\sigma-1}} \\
&= \left(\int_i \left(\left(\frac{P_{it}}{P_{Dt}}\right)^{-\sigma} (X_{St}+Q_{Dt})\right)^{\frac{\sigma-1}{\sigma}} di\right)^{\frac{\sigma}{\sigma-1}} \\
&= \left(\int_i P_{it}^{1-\sigma} di\right)^{\frac{\sigma}{\sigma-1}} P_{Dt}^{\sigma}\left(X_{St}+Q_{Dt}\right) \\
&= X_{St}+Q_{Dt}
\end{aligned}
$$

where we have used the definition of the price index.

Labor market clearing Labor market clearing requires that all labor demanded by the manufacturing and services sectors equals the supply of labor which we assume is perfectly inelastic. Because manufacturing producers are Homogeneous, we can write the labor market clearing in terms of aggregate variables.

$$
\begin{aligned}
L_t &= L_{St} + \int_i L_{Dit} di \\
&= L_{St} + \int_i (1-\gamma)\frac{Y_{Dit}MC_{Dit}}{W_t} di \\
&= L_{St} + \int_i (1-\gamma)\frac{Y_{Dt}MC_{Dt}}{W_t} di \\
&= L_{St} + L_{Dt}
\end{aligned}
$$

Trade balance The trade balance represents fundamental demand or supply of international foreign assets and depends on the assumed product structure. We re-write this in turn:

$$
\begin{aligned}
TB_t &= E_t P_{Xt}^{\$} X + W_t L_t - P_{St}C_{St} \\
&= E_t P_{Xt}^{\$} X + W_t L + \Pi_t - P_{St}C_{St} \\
&= E_t P_{Xt}^{\$} X + W_t(L_{St}+L_{Dt}) + \frac{1}{\sigma}P_{Dt}Y_{Dt} - P_{St}C_{St} \\
&= E_t P_{Xt}^{\$} X + (1-\mu)P_{St}Y_{St} + (1-\gamma)MC_{Dt}Y_{Dt} + \frac{1}{\sigma}P_{Dt}Y_{Dt} - P_{St}C_{St} \\
&= E_t P_{Xt}^{\$} X + (1-\mu)P_{St}Y_{St} + (1-\gamma)\frac{\sigma-1}{\sigma}P_{Dt}Y_{Dt} + \frac{1}{\sigma}P_{Dt}Y_{Dt} - P_{St}C_{St} \\
&= E_t P_{Xt}^{\$} X - \mu P_{St}C_{St} + \left(\frac{1}{\sigma} + (1-\gamma)\frac{\sigma-1}{\sigma}\right) P_{Dt}(Q_{Dt}+X_{St})
\end{aligned}
$$

Now, we can re-write $(Q_{Dt} + X_{St})$ by combining the first-order condition for domestic intermediate inputs:

$$
\begin{aligned}
Q_{Dt} &\equiv \int_i Q_{Dit} di \\
&= \int_i \omega \left(\frac{P_{Xit}}{P_{Dt}}\right)^{\varepsilon} \frac{P_{Xit}}{P_{Xit}} X_{Dit} di \\
&= \int_i \omega \left(\frac{P_{Xit}}{P_{Dt}}\right)^{\varepsilon} \gamma \frac{MC_{Dit}}{P_{Xit}} Y_{Dit} di \\
&= \int_i \omega \left(\frac{P_{Xit}}{P_{Dt}}\right)^{\varepsilon} \gamma \frac{MC_{Dit}}{P_{Xit}} \left(\frac{P_{Dit}}{P_{Dt}}\right)^{-\sigma} (Q_{Dt} + X_{St}) di \\
&= \int_i \omega \left(\frac{P_{Xit}}{P_{Dt}}\right)^{\varepsilon} \gamma \frac{\frac{\sigma-1}{\sigma} P_{Dit}}{P_{Xit}} \left(\frac{P_{Dit}}{P_{Dt}}\right)^{-\sigma} (Q_{Dt} + X_{St}) di \\
&= \gamma \frac{\sigma-1}{\sigma} \omega \left(\frac{P_{Xt}}{P_{Dt}}\right)^{\varepsilon} \frac{P_{Dt}^{\sigma}}{P_{Xit}} (Q_{Dt} + X_{St}) \int_i (P_{Dit})^{1-\sigma} di \\
&= \gamma \frac{\sigma-1}{\sigma} \omega \left(\frac{P_{Xt}}{P_{Dt}}\right)^{\varepsilon-1} (Q_{Dt} + X_{St}) \\
\left[1 - \gamma \frac{\sigma-1}{\sigma} \omega \left(\frac{P_{Xt}}{P_{Dt}}\right)^{\varepsilon-1}\right] Q_{Dt} &= \gamma \frac{\sigma-1}{\sigma} \omega \left(\frac{P_{Xt}}{P_{Dt}}\right)^{\varepsilon-1} X_{St} \\
Q_{Dt} &= \frac{\gamma \frac{\sigma-1}{\sigma} \omega \left(\frac{P_{Xt}}{P_{Dt}}\right)^{\varepsilon-1}}{1 - \gamma \frac{\sigma-1}{\sigma} \omega \left(\frac{P_{Xt}}{P_{Dt}}\right)^{\varepsilon-1}} X_{St} \\
Q_{Dt} + X_{St} &= \frac{1}{1 - \gamma \frac{\sigma-1}{\sigma} \omega \left(\frac{P_{Xt}}{P_{Dt}}\right)^{\varepsilon-1}} X_{St}
\end{aligned}
$$

Plugging this is in

$$
\begin{aligned}
E_t P_{Xt}^{\$} X + W_t L_t - P_{St} C_{St} &= E_t P_{Xt}^{\$} X - \mu P_{St} C_{St} + \left(\frac{1}{\sigma} + (1-\gamma)\frac{\sigma-1}{\sigma}\right) \frac{1}{1 - \gamma \frac{\sigma-1}{\sigma} \omega \left(\frac{P_{Xt}}{P_D t}\right)^{\varepsilon-1}} P_{Dt} X_{St} \\
&= E_t P_{Xt}^{\$} X - \mu P_{St} C_{St} + \mu \left(\frac{1}{\sigma} + (1-\gamma)\frac{\sigma-1}{\sigma}\right) \frac{1}{1 - \gamma \frac{\sigma-1}{\sigma} \omega \left(\frac{P_{Xt}}{P_D t}\right)^{\varepsilon-1}} P_{St} C_{St} \\
&= E_t P_{Xt}^{\$} X - \mu \left[1 - \left(\frac{1}{\sigma} + (1-\gamma)\frac{\sigma-1}{\sigma}\right) \frac{1}{1 - \gamma \frac{\sigma-1}{\sigma} \omega \left(\frac{P_{Xt}}{P_D t}\right)^{\varepsilon-1}}\right] P_{St} C_{St} \\
&= E_t P_{Xt}^{\$} X - \mu \gamma \frac{\sigma-1}{\sigma} \frac{1 - \omega \left(\frac{P_{Xt}}{P_D t}\right)^{\varepsilon-1}}{1 - \gamma \frac{\sigma-1}{\sigma} \omega \left(\frac{P_{Xt}}{P_D t}\right)^{\varepsilon-1}} P_{St} C_{St}
\end{aligned}
$$

Now, we can conviently re-write $\frac{1-\omega\left(\frac{P_{Xt}}{P_{Dt}}\right)^{\varepsilon-1}}{1-\gamma\frac{\sigma-1}{\sigma}\omega\left(\frac{P_{Xt}}{P_{Dt}}\right)^{\varepsilon-1}}$ using the intermediate input price index:

$$
\begin{aligned}
P_{Xt}^{1-\varepsilon} &= \omega P_{Dt}^{1-\varepsilon} + (1-\omega)P_{Mt}^{1-\varepsilon} \\
\frac{1}{\omega}\left(\frac{P_{Xt}}{P_{Dt}}\right)^{1-\varepsilon} &= 1 + \frac{1-\omega}{\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{1-\varepsilon} \\
\omega\left(\frac{P_{Dt}}{P_{Xt}}\right)^{1-\varepsilon} &= \frac{1}{1+\frac{1-\omega}{\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{1-\varepsilon}}
\end{aligned}
$$

Then we have that:

$$
\begin{aligned}
\frac{1-\omega\left(\frac{P_{Xt}}{P_{Dt}}\right)^{\varepsilon-1}}{1-\gamma\frac{\sigma-1}{\sigma}\omega\left(\frac{P_{Xt}}{P_{Dt}}\right)^{\varepsilon-1}} &= \frac{1-\frac{1}{1+\frac{1-\omega}{\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{1-\varepsilon}}}{1-\gamma\frac{\sigma-1}{\sigma}\frac{1}{1+\frac{1-\omega}{\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{1-\varepsilon}}} \\
&= \frac{\frac{1-\omega}{\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{1-\varepsilon}}{1+\frac{1-\omega}{\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{1-\varepsilon}-\frac{\sigma-1}{\sigma}\gamma} \\
&= \frac{1}{1+(1-\gamma\frac{\sigma-1}{\sigma})\frac{\omega}{1-\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{\varepsilon-1}}
\end{aligned}
$$

Therefore the trade balance can be written as

$$
TB_t = E_t P_{Xt}^{\$} X - \mu\gamma\frac{\sigma-1}{\sigma}H_t P_{St} C_{St} \quad \text{where} \quad H_t \equiv \frac{1}{1+(1-\gamma\frac{\sigma-1}{\sigma})\frac{\omega}{1-\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{\varepsilon-1}} \tag{B.5.2.1}
$$

Note that the problem for the consumer boils down to satifsying the trade balance condition in financial autarky. In addition, we can write the foreign intermediate input share as:

$$
S_t^M \equiv \frac{P_{Mt}Q_{Mt}}{P_{Xt}X_{Dt}} = 1 - \frac{P_{Dt}Q_{Dt}}{P_{Xt}X_{Dt}} = 1-\omega\left(\frac{P_{Dt}}{P_{Xt}}\right)^{1-\varepsilon} = \frac{\frac{1-\omega}{\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{1-\varepsilon}}{1+\frac{1-\omega}{\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{1-\varepsilon}}
$$

which in terms of H_t becomes:

$$
\begin{aligned}
H_t &= \frac{1}{1+(1-\gamma\frac{\sigma-1}{\sigma})\frac{\omega}{1-\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{\varepsilon-1}} \\
\frac{\omega}{1-\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{\varepsilon-1} &= \frac{1-H_t}{(1-\gamma\frac{\sigma-1}{\sigma})H_t} \\
\frac{1-\omega}{\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{1-\varepsilon} &= \frac{(1-\gamma\frac{\sigma-1}{\sigma})H_t}{1-H_t} \\
1+\frac{1-\omega}{\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{1-\varepsilon} &= \frac{1-\gamma\frac{\sigma-1}{\sigma}H_t}{1-H_t}
\end{aligned}
$$

Therefore, we have that the imported intermediate input share is given by:

$$S_t^M = \frac{\frac{1-\omega}{\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{1-\varepsilon}}{1+\frac{1-\omega}{\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{1-\varepsilon}} = \frac{\frac{(1-\gamma\frac{\sigma-1}{\sigma})H_t}{1-H_t}}{\frac{(1-\gamma\frac{\sigma-1}{\sigma}\eta)H_t}{1-H_t}} = \frac{(1-\gamma\frac{\sigma-1}{\sigma})H_t}{1-\gamma\frac{\sigma-1}{\sigma}H_t}$$

Labor market clearing - revisited Labor market clearing requires that all labor demanded by the manufacturing and services sectors equals the supply of labor which we assume is perfectly inelastic. We have:

$$\begin{aligned}
w_t L &= w_t L_{St} + w_t L_{Dt} \\
&= (1-\mu)PStY_{St} + (1-\gamma)MC_{Dt}Y_{Dt} \\
&= (1-\mu)PStY_{St} + (1-\gamma)\frac{\sigma-1}{\sigma}P_{Dt}(Q_{Dt}+X_{St})
\end{aligned}$$

Now, use the fact that $Q_{Dt}+X_{St} = \frac{1}{1-\gamma\frac{\sigma-1}{\sigma}\omega\left(\frac{P_{Xt}}{P_{Dt}}\right)^{\varepsilon-1}}X_{St}$ which we can re-write in terms of H_t:

$$\begin{aligned}
H_t &= \frac{1-\omega\left(\frac{P_{Xt}}{P_Dt}\right)^{\varepsilon-1}}{1-\gamma\frac{\sigma-1}{\sigma}\omega\left(\frac{P_{Xt}}{P_Dt}\right)^{\varepsilon-1}} \\
\omega\left(\frac{P_{Xt}}{P_Dt}\right)^{\varepsilon-1} - \gamma\frac{\sigma-1}{\sigma}\omega\left(\frac{P_{Xt}}{P_Dt}\right)^{\varepsilon-1} H_t &= 1-H_t \\
(1-\gamma\frac{\sigma-1}{\sigma}H_t)\omega\left(\frac{P_{Xt}}{P_Dt}\right)^{\varepsilon-1} &= 1-H_t \\
\gamma\frac{\sigma-1}{\sigma}\omega\left(\frac{P_{Xt}}{P_Dt}\right)^{\varepsilon-1} &= \frac{\gamma\frac{\sigma-1}{\sigma}(1-H_t)}{1-\gamma\frac{\sigma-1}{\sigma}H_t} \\
1-\gamma\omega\left(\frac{P_{Xt}}{P_Dt}\right)^{\varepsilon-1} &= \frac{1-\gamma\frac{\sigma-1}{\sigma}}{1-\gamma\frac{\sigma-1}{\sigma}H_t} \\
\frac{1}{1-\gamma\omega\left(\frac{P_{Xt}}{P_{Dt}}\right)^{\varepsilon-1}} &= \frac{1-\gamma\frac{\sigma-1}{\sigma}H_t}{1-\gamma\frac{\sigma-1}{\sigma}}
\end{aligned}$$

Inserting this expression, we arrive at the labor market clearing condition

$$\begin{aligned}
W_t L &= (1-\mu)PStY_{St} + (1-\gamma)\frac{\sigma-1}{\sigma}P_{Dt}(Q_{Dt}+X_{St}) \\
&= (1-\mu)PStY_{St} + (1-\gamma)\frac{\sigma-1}{\sigma}\frac{1-\gamma\frac{\sigma-1}{\sigma}H_t}{1-\gamma\frac{\sigma-1}{\sigma}}P_{Dt}X_{St} \\
&= (1-\mu)PStY_{St} + (1-\gamma)\frac{\sigma-1}{\sigma}\frac{1-\gamma\frac{\sigma-1}{\sigma}H_t}{1-\gamma\frac{\sigma-1}{\sigma}}\mu P_{St}Y_{St} \\
&= \left[1-\mu+\mu(1-\gamma)\frac{\sigma-1}{\sigma}\frac{1-\gamma\frac{\sigma-1}{\sigma}H_t}{1-\gamma\frac{\sigma-1}{\sigma}}\right]P_{St}Y_{St} \\
&= \frac{1}{1-\gamma\frac{\sigma-1}{\sigma}}\left[(1-\mu)\left(1-\gamma\frac{\sigma-1}{\sigma}\right)+\mu(1-\gamma)\frac{\sigma-1}{\sigma}-\mu\gamma(1-\gamma)\frac{\sigma-1}{\sigma}H_t\right]P_{St}Y_{St} \\
&= \frac{(1-\gamma)\left(\frac{\sigma-1}{\sigma}\right)^2}{1-\gamma\frac{\sigma-1}{\sigma}}\left[(1-\mu)\frac{1-\gamma\frac{\sigma-1}{\sigma}}{1-\gamma}\left(\frac{\sigma}{\sigma-1}\right)^2+\mu\frac{\sigma}{\sigma-1}-\mu\gamma H_t\right]P_{St}Y_{St}
\end{aligned}$$

Using goods market clearing for final goods $Y_{St} = C_{St}$, we arrive at the labor market clearing condition:

$$W_t L = X_2 \left[\chi_2 - \mu\gamma H_t\right] P_{St} C_{St}$$

$$\text{where} \qquad X_2 \equiv \frac{(1-\gamma)\left(\frac{\sigma-1}{\sigma}\right)^2}{1-\gamma^{\frac{\sigma-1}{\sigma}}}, \qquad \chi_2 \equiv (1-\mu)\frac{1-\gamma^{\frac{\sigma-1}{\sigma}}}{1-\gamma}\left(\frac{\sigma}{\sigma-1}\right)^2 + \mu\frac{\sigma}{\sigma-1} \tag{B.5.2.2}$$

In addition, note that we can write labor allocated to the service sector solely as a fuction of H_t as well:

$$\begin{aligned} w_t L_t &= X_2 \left[\chi_2 - \mu\gamma H_t\right] P_{St} C_{St} \\ &= X_2 \left[\chi_2 - \mu\gamma H_t\right] \frac{W_t L_{st}}{1-\mu} \\ L_{St} &= \frac{(1-\mu)}{\chi_2 - \mu\gamma H_t} \frac{L_t}{X_2} \end{aligned}$$

B.5.3 Homogeneous Firms under Monopolistic Competition and IRS Importing

Goods market clearing Goods market clearing implies that the demand for manufacturing output by services producers and by other manufacturing producers equals final output in the manufacturing sector and that total consumption equals output in services

$$Y_{Dit} = X_{St} + \int_j Q_{Djt} dj, \qquad Y_{St} = C_{St}$$

Plugging in the residual demand schedules, we have

$$\begin{aligned} Y_{Dit} &= X_{Sit} + \int_j Q_{Dijt} dj \\ &= \left(\frac{P_{it}}{P_{Dt}}\right)^{-\sigma} X_{St} + \int_j \left(\frac{P_{it}}{P_{Dt}}\right)^{-\sigma} Q_{Djt} dj \\ &= \left(\frac{P_{it}}{P_{Dt}}\right)^{-\sigma} \left(X_{St} + \int_j Q_{Djt} dj\right) \\ &= \left(\frac{P_{it}}{P_{Dt}}\right)^{-\sigma} \left(X_{St} + Q_{Dt}\right) \end{aligned}$$

where $Q_{Dt} \equiv \int_j Q_{Djt} dj$. We can also write this in aggregate form by using the corresponding the aggregation for manufacturing output as dictated by the demand system:

$$\begin{aligned} Y_{Dt} &\equiv \left(\int_i \left(Y_{Dit}\right)^{\frac{\sigma-1}{\sigma}} di\right)^{\frac{\sigma}{\sigma-1}} \\ &= \left(\int_i \left(\left(\frac{P_{it}}{P_{Dt}}\right)^{-\sigma} \left(X_{St} + Q_{Dt}\right)\right)^{\frac{\sigma-1}{\sigma}} di\right)^{\frac{\sigma}{\sigma-1}} \\ &= \left(\int_i P_{it}^{1-\sigma} di\right)^{\frac{\sigma}{\sigma-1}} P_{Dt}^{\sigma} \left(X_{St} + Q_{Dt}\right) \\ &= X_{St} + Q_{Dt} \end{aligned}$$

where we have used the definition of the price index.

Labor market clearing Labor market clearing requires that all labor demanded by the manufacturing and services sectors equals the supply of labor which we assume is perfectly inelastic.

$$L = L_{St} + \int_i (L_{Dit} + L_{Mit}) di$$

Trade balance The trade balance represents fundamental demand or supply of international foreign assets and depends on the assumed product structure. We re-write this in turn:

$$\begin{aligned}
TB_t &= E_t P^{\$}_{Xt} X + W_t L_t - P_{St} C_{St} \\
&= E_t P^{\$}_{Xt} X + W_t L + \int_i \Pi_{it} di - P_{St} C_{St} \\
&= E_t P^{\$}_{Xt} X + W_t \left(L_{St} + \int_i (L_{Dit} + L_{Mit}) di \right) + \int_i \left(\frac{1}{\sigma} P_{Dit} Y_{Dit} - W_t L_{Mit} \right) di - P_{St} C_{St} \\
&= E_t P^{\$}_{Xt} X + W_t L_{St} + W_t L_{Dt} + \frac{1}{\sigma} P_{Dt} Y_{Dt} - P_{St} C_{St} \\
&= E_t P^{\$}_{Xt} X + (1-\mu) P_{St} Y_{St} + (1-\gamma) MC_{Dt} Y_{Dt} + \frac{1}{\sigma} P_{Dt} Y_{Dt} - P_{St} C_{St} \\
&= E_t P^{\$}_{Xt} X + (1-\mu) P_{St} Y_{St} + (1-\gamma) \frac{\sigma - 1}{\sigma} P_{Dt} Y_{Dt} + \frac{1}{\sigma} P_{Dt} Y_{Dt} - P_{St} C_{St} \\
&= E_t P^{\$}_{Xt} X - \mu P_{St} C_{St} + \left(\frac{1}{\sigma} + (1-\gamma) \frac{\sigma - 1}{\sigma} \right) P_{Dt} (Q_{Dt} + X_{St})
\end{aligned}$$

Now, we can re-write $(Q_{Dt} + X_{St})$ by combining the first-order condition for domestic intermediate inputs:

$$
\begin{aligned}
Q_{Dt} &= \int_j Q_{Djt} dj \\
&= \int_j \omega \left(\frac{P_{Dt}}{P_{Xjt}} \right)^{-\varepsilon} \frac{P_{Xjt} X_{Djt}}{P_{Xjt}} dj \\
&= \int_j \omega\gamma \left(\frac{P_{Dt}}{P_{Xjt}} \right)^{-\varepsilon} \frac{MC_{Djt} Y_{Djt}}{P_{Xjt}} dj \\
&= \int_j \omega\gamma \left(\frac{P_{Dt}}{P_{Xjt}} \right)^{-\varepsilon} \frac{MC_{Djt}}{P_{Xjt}} \left(\frac{P_{jt}}{P_{Dt}} \right)^{-\sigma} \left(X_{St} + Q_{Dt} \right) dj \\
&= \int_j \omega\gamma \left(\frac{P_{Dt}}{P_{Xjt}} \right)^{-\varepsilon} \frac{MC_{Djt}}{P_{Xjt}} \left(\frac{\frac{\sigma}{\sigma-1} MC_{jt}}{P_{Dt}} \right)^{-\sigma} \left(X_{St} + Q_{Dt} \right) dj \\
&= \omega\gamma \left(\frac{\sigma}{\sigma-1} \right)^{-\sigma} P_{Dt}^{\sigma-\varepsilon} \left(X_{St} + Q_{Dt} \right) \int_j \left(\frac{1}{\varphi_D A_{Dt}} \frac{W_t^{1-\gamma} P_{Xjt}^{\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}} \right)^{1-\sigma} P_{Xjt}^{\varepsilon-1} dj \\
&= \omega\gamma \left(\frac{\sigma}{\sigma-1} \right)^{-\sigma} P_{Dt}^{\sigma-\varepsilon} \left(X_{St} + Q_{Dt} \right) \left(\frac{1}{\varphi_D A_{Dt}} \frac{W_t^{1-\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}} \right)^{1-\sigma} \int_j P_{Xjt}^{\varepsilon-1-\gamma(\sigma-1)} dj \\
&= \omega\gamma \left(\frac{\sigma}{\sigma-1} \right)^{-\sigma} P_{Dt}^{\sigma-\varepsilon} \left(X_{St} + Q_{Dt} \right) \left(\frac{1}{\varphi_D A_{Dt}} \frac{W_t^{1-\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}} \right)^{1-\sigma} \left(\frac{\varphi_{Mt}}{\varphi_D} \right)^{\sigma-1} \omega^{-\frac{\varepsilon-1-\gamma(\sigma-1)}{\varepsilon-1}} P_{Dt}^{\varepsilon-1-\gamma(\sigma-1)} \\
&= \gamma \left(\frac{\sigma}{\sigma-1} \right)^{-\sigma} \omega^{\frac{\gamma(\sigma-1)}{\varepsilon-1}} P_{Dt}^{\sigma-1} \left(X_{St} + Q_{Dt} \right) \left(\frac{1}{\varphi_D A_{Dt}} \frac{W_t^{1-\gamma} P_{Dt}^{\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}} \right)^{1-\sigma} \left(\frac{\varphi_{Mt}}{\varphi_D} \right)^{\sigma-1} \\
&= \gamma \left(\frac{\sigma}{\sigma-1} \right)^{-\sigma} \omega^{\frac{\gamma(\sigma-1)}{\varepsilon-1}} \left(X_{St} + Q_{Dt} \right) \left(\frac{1}{\varphi_D A_{Dt}} \frac{W_t^{1-\gamma} P_{Dt}^{\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}} \right)^{1-\sigma} \left(\frac{\varphi_{Mt}}{\varphi_D} \right)^{\sigma-1} \left[\frac{\sigma}{\sigma-1} \frac{1}{\varphi_D A_{Dt}} \frac{W_t^{1-\gamma} P_{Dt}^{\gamma} \omega^{\frac{\gamma}{\varepsilon-1}}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}} \left(\frac{\varphi_{Mt}}{\varphi_D} \right)^{\frac{\gamma(\sigma-}{\varepsilon-1-\gamma}} \right. \\
&= \frac{\sigma-1}{\sigma} \gamma \left(\frac{\varphi_{Mt}}{\varphi_D} \right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}} \left(X_{St} + Q_{Dt} \right)
\end{aligned}
$$

Then, we can write

$$
\begin{aligned}
Q_{Dt} &= \frac{\frac{\sigma-1}{\sigma} \gamma \left(\frac{\varphi_{Mt}}{\varphi_D} \right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}}{1 - \frac{\sigma-1}{\sigma} \gamma \left(\frac{\varphi_{Mt}}{\varphi_D} \right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}} X_{St} \\
Q_{Dt} + X_{St} &= \frac{1}{1 - \frac{\sigma-1}{\sigma} \gamma \left(\frac{\varphi_{Mt}}{\varphi_D} \right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}} X_{St}
\end{aligned}
$$

Plug this back into the trade balance equation:

$$
\begin{aligned}
TB_t &= E_t P^{\$}_{Xt} X - \mu P_{St} C_{St} + \left(\frac{1}{\sigma} + (1-\gamma)\frac{\sigma-1}{\sigma}\right) P_{Dt}(Q_{Dt} + X_{St}) \\
&= E_t P^{\$}_{Xt} X - \mu P_{St} C_{St} + \left(\frac{1}{\sigma} + (1-\gamma)\frac{\sigma-1}{\sigma}\right) \frac{1}{1 - \frac{\sigma-1}{\sigma}\gamma \left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}} P_{Dt} X_{St} \\
&= E_t P^{\$}_{Xt} X - \mu P_{St} C_{St} + \left(\frac{1}{\sigma} + (1-\gamma)\frac{\sigma-1}{\sigma}\right) \frac{1}{1 - \frac{\sigma-1}{\sigma}\gamma \left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}} \mu P_{St} Y_{St} \\
&= E_t P^{\$}_{Xt} X \mu \left[\left(\frac{1}{\sigma} + (1-\gamma)\frac{\sigma-1}{\sigma}\right) \frac{1}{1 - \frac{\sigma-1}{\sigma}\gamma \left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}} - 1\right] P_{St} C_{St} \\
&= E_t P^{\$}_{Xt} X - \frac{\sigma-1}{\sigma}\mu\gamma \frac{1 - \left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}}{1 - \frac{\sigma-1}{\sigma}\gamma \left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}} P_{St} C_{St}
\end{aligned}
$$

which yields the expression for the saving:

$$
TB_t = E_t P^{\$}_{Xt} X - \frac{\sigma-1}{\sigma}\mu\gamma H_t P_{St} C_{St}, \qquad H_t \equiv \frac{1 - \left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}}{1 - \frac{\sigma-1}{\sigma}\gamma \left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}} \tag{B.5.3.1}
$$

In addition, we can write the foreign intermediate input share as:

$$
S^M_t \equiv \frac{P_{Mt} Q_{Mt}}{P_{Xt} X_{Dt}} = 1 - \frac{P_{Dt} Q_{Dt}}{P_{Xt} X_{Dt}} = 1 - \omega \left(\frac{P_{Dt}}{P_{Xt}}\right)^{1-\varepsilon} = 1 - \omega \left(\frac{P_{Dt}}{P_{Dt}\omega^{-\frac{1}{\varepsilon-1}} \left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)}{\varepsilon-1-\gamma(\sigma-1)}}}\right)^{1-\varepsilon} = 1 - \left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}
$$

which in terms of H_t becomes:

$$
\begin{aligned}
H_t &= \frac{1 - \left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}}{1 - \frac{\sigma-1}{\sigma}\gamma \left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}} \\
\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}} &= \frac{1 - H_t}{1 - \gamma\frac{\sigma-1}{\sigma} H_t} \\
1 - \left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}} &= \frac{\left(1 - \gamma\frac{\sigma-1}{\sigma}\right) H_t}{1 - \gamma\frac{\sigma-1}{\sigma} H_t}
\end{aligned}
$$

Therefore, we have that the imported intermediate input share is given by:

$$
S^M_t = \frac{\left(1 - \gamma\frac{\sigma-1}{\sigma}\right) H_t}{1 - \gamma\frac{\sigma-1}{\sigma} H_t}
$$

Labor market clearing - revisited We start by re-writing demand for labor being used in the importing of intermediate input varieties. To this end, we re-write profits and go back to the first order condition for the optimal number of imported varieties. Profits can be written as:

$$\begin{aligned}
\Pi_{it} &= \frac{1}{\sigma} P_{Dit} Y_{Dit} - W_t f |\lambda_{it}| \\
&= \frac{1}{\sigma} P_{Dit} \left(\frac{P_{it}}{P_{Dt}} \right)^{-\sigma} \left[X_{St} + \int_j Q_{Djt} dj \right] - W_t f |\lambda_{it}| \\
&= \frac{1}{\sigma} P_{Dit}^{1-\sigma} \left[P_{Dt}^{\sigma} X_{St} + \int_j P_{Dt}^{\sigma} \left(\frac{P_{Dt}}{P_{Xjt}} \right)^{-\varepsilon} dj \right] - W_t f |\lambda_{it}| \\
&= \frac{1}{\sigma} P_{Dit}^{1-\sigma} \widetilde{Y_{Dt}} - W_t f |\lambda_{it}|
\end{aligned}$$

where we have defined $\widetilde{Y_{Dt}} \equiv P_{Dt}^{\sigma} X_{St} + \int_j P_{Dt}^{\sigma} \left(\frac{P_{Dt}}{P_{Xjt}} \right)^{-\varepsilon} dj$. The first-order condition for the optimal number of imported varieties is given by:

$$\begin{aligned}
\frac{\partial \Pi_{it}}{\partial |\lambda_{it}|} - W_t f = 0 \\
\frac{\partial \ln \Pi_{it}}{\partial |\lambda_{it}|} \Pi_{it} = 0
\end{aligned}$$

Now,

$$\begin{aligned}
\frac{\partial \ln \Pi_{it}}{\partial |\lambda_{it}|} &= (1-\sigma) \frac{\partial \ln P_{Dit}}{\partial |\lambda_{it}|} \\
&= (1-\sigma)\gamma \frac{\partial \ln P_{Xit}}{\partial |\lambda_{it}|} \\
&= (1-\sigma)\gamma \frac{\partial P_{Xit}}{\partial |\lambda_{it}|} \frac{1}{P_{Xit}} \\
&= (1-\sigma)\gamma \frac{1}{1-\varepsilon} P_{Xit}^{\frac{\varepsilon}{\varepsilon-1}} (1-\omega) P_{Mt}^{1-\varepsilon} \frac{1}{P_{Xit}} \\
&= \gamma \frac{1-\sigma}{1-\varepsilon} (1-\omega) \left(\frac{P_{Mt}}{P_{Xit}} \right)^{1-\varepsilon} \\
&= \gamma \frac{1-\sigma}{1-\varepsilon} (1-\omega) \left(\frac{P_{Mt} |\lambda_{it}|^{\frac{1}{1-\varepsilon}}}{P_{Xit}} \right)^{1-\varepsilon} \frac{1}{|\lambda_{it}|} \\
&= \gamma \frac{1-\sigma}{1-\varepsilon} (1-\gamma_{it}) \frac{1}{|\lambda_{it}|} \\
\frac{\partial \Pi_{it}}{\partial |\lambda_{it}|} &= \gamma \frac{1-\sigma}{1-\varepsilon} (1-\gamma_{it}) \frac{\Pi_{it}}{|\lambda_{it}|}
\end{aligned}$$

where $1 - \gamma_{it} \equiv \left(\frac{P_{Mt} |\lambda_{it}|^{\frac{1}{1-\varepsilon}}}{P_{Xit}} \right)^{1-\varepsilon}$ is the domestic intermediate input share. Going back to the first-order condition, we have:

$$W_t f = \gamma \frac{1-\sigma}{1-\varepsilon}(1-\gamma_{it})\frac{\Pi_{it}}{|\lambda_{it}|}$$
$$|\lambda_{it}| W_t f = \gamma \frac{1-\sigma}{1-\varepsilon}(1-\gamma_{it})\Pi_{it}$$
$$L_{Mit} W_t = \gamma \frac{1-\sigma}{1-\varepsilon}(1-\gamma_{it})\Pi_{it}$$
$$L_{Mit} W_t = \frac{\gamma}{1-\varepsilon}\frac{1-\sigma}{\sigma}(1-\gamma_{it})P_{Dit}Y_{Dit}$$
$$L_{Mit} W_t = \frac{\gamma}{\varepsilon-1}(1-\gamma_{it})MC_{Dit}Y_{Dit}$$
$$L_{Mit} W_t = \frac{\gamma}{1-\gamma}\frac{1}{\varepsilon-1}(1-\gamma_{it})W_t L_{Dit}$$
$$L_{Mit} = \frac{\gamma}{1-\gamma}\frac{1}{\varepsilon-1}(1-\gamma_{it})L_{Dit}$$
$$L_{Mit} = \frac{\gamma}{1-\gamma}\frac{1}{\varepsilon-1}\left(1-\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{(\varepsilon-1)-\gamma(\sigma-1)}}\right)L_{Dit}$$

where we have used the alternative expression for the domestic intermediate input share. The labor market condition becomes:

$$\begin{aligned}
W_t L_t &= L_{St} + \int_i (L_{Dit} + L_{Mit})di \\
&= L_{St} + \int_i \left[1 + \frac{\gamma}{1-\gamma}\frac{1}{\varepsilon-1}\left(1-\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{(\varepsilon-1)-\gamma(\sigma-1)}}\right)\right] L_{Dit}di \\
&= L_{St} + (1-\gamma)\frac{\sigma-1}{\sigma}\left[1 + \frac{\gamma}{1-\gamma}\frac{1}{\varepsilon-1}\left(1-\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{(\varepsilon-1)-\gamma(\sigma-1)}}\right)\right] P_{Dt}Y_{Dt} \\
&= (1-\mu)P_{St}C_{St} + (1-\gamma)\frac{\sigma-1}{\sigma}\left[1 + \frac{\gamma}{1-\gamma}\frac{1}{\varepsilon-1}\left(1-\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{(\varepsilon-1)-\gamma(\sigma-1)}}\right)\right] P_{Dt}(X_S + Q_{Dt}) \\
&= (1-\mu)P_{St}C_{St} + (1-\gamma)\frac{\sigma-1}{\sigma}\left[1 + \frac{\gamma}{1-\gamma}\frac{1}{\varepsilon-1}\left(1-\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{(\varepsilon-1)-\gamma(\sigma-1)}}\right)\right] \frac{1}{1-\frac{\sigma-1}{\sigma}\gamma\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}} P_{Dt}X_{St} \\
&= (1-\mu)P_{St}Y_{St} + (1-\gamma)\frac{\sigma-1}{\sigma}\left[1 + \frac{\gamma}{1-\gamma}\frac{1}{\varepsilon-1}\left(1-\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{(\varepsilon-1)-\gamma(\sigma-1)}}\right)\right] \frac{1}{1-\frac{\sigma-1}{\sigma}\gamma\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}} \mu P_{St}Y_{St} \\
&= (1-\mu)P_{St}Y_{St} + \frac{\mu(1-\gamma)\frac{\sigma-1}{\sigma}}{1-\frac{\sigma-1}{\sigma}\gamma\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}} P_{St}Y_{St} + \mu(1-\gamma)\frac{\sigma-1}{\sigma}\frac{\gamma}{1-\gamma}\frac{1}{\varepsilon-1}\frac{1-\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{(\varepsilon-1)-\gamma(\sigma-1)}}}{1-\frac{\sigma-1}{\sigma}\gamma\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}}
\end{aligned}$$

Now re-write $\frac{1}{1-\frac{\sigma-1}{\sigma}\gamma\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}}$ as a function of H_t:

$$H_t = \frac{1-\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}}{1-\frac{\sigma-1}{\sigma}\gamma\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}}$$

$$(1-\frac{\sigma-1}{\sigma}\gamma H_t)\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}} = 1-H_t$$

$$\frac{\sigma-1}{\sigma}\gamma\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}} = \frac{\frac{\sigma-1}{\sigma}\gamma(1-H_t)}{1-\frac{\sigma-1}{\sigma}\gamma H_t} 1-\frac{\sigma-1}{\sigma}\gamma\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}} = \frac{1-\frac{\sigma-1}{\sigma}\gamma}{1-\frac{\sigma-1}{\sigma}\gamma H_t}$$

$$\frac{1}{1-\frac{\sigma-1}{\sigma}\gamma\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}} = \frac{1-\frac{\sigma-1}{\sigma}\gamma H_t}{1-\frac{\sigma-1}{\sigma}\gamma}$$

Pluggin this back into the labor market clearing condition

$$\begin{aligned}
W_tL_t &= (1-\mu)P_{St}Y_{St} + \mu(1-\gamma)\frac{\sigma-1}{\sigma}\frac{1-\frac{\sigma-1}{\sigma}\gamma H_t}{1-\frac{\sigma-1}{\sigma}\gamma}P_{St}Y_{St} + \mu(1-\gamma)\frac{\sigma-1}{\sigma}\frac{\gamma}{1-\gamma}\frac{1}{\varepsilon-1}H_tP_{St}Y_{St} \\
&= \left[(1-\mu) + \mu(1-\gamma)\frac{\sigma-1}{\sigma}\frac{1-\frac{\sigma-1}{\sigma}\gamma H_t}{1-\frac{\sigma-1}{\sigma}\gamma} + \mu(1-\gamma)\frac{\sigma-1}{\sigma}\frac{\gamma}{1-\gamma}\frac{1}{\varepsilon-1}H_t\right]P_{St}Y_{St} \\
&= \frac{1}{1-\gamma\frac{\sigma-1}{\sigma}}\left[(1-\mu)\left(1-\gamma\frac{\sigma-1}{\sigma}\right) + \mu(1-\gamma)\frac{\sigma-1}{\sigma} - \left((1-\gamma)\left(\frac{\sigma-1}{\sigma}\right)^2 - \frac{\frac{\sigma-1}{\sigma}}{\varepsilon-1}\left(1-\frac{\sigma-1}{\sigma}\gamma\right)\right)\mu\gamma H_t\right]P_{St}Y_{St} \\
&= \frac{(1-\gamma)\left(\frac{\sigma-1}{\sigma}\right)^2 - \frac{\frac{\sigma-1}{\sigma}}{\varepsilon-1}\left(1-\frac{\sigma-1}{\sigma}\gamma\right)}{1-\gamma\frac{\sigma-1}{\sigma}}\left[\frac{(1-\mu)\left(1-\gamma\frac{\sigma-1}{\sigma}\right)+\mu(1-\gamma)\frac{\sigma-1}{\sigma}}{(1-\gamma)\left(\frac{\sigma-1}{\sigma}\right)^2 - \frac{\frac{\sigma-1}{\sigma}}{\varepsilon-1}\left(1-\frac{\sigma-1}{\sigma}\gamma\right)} - \mu\gamma H_t\right]P_{St}Y_{St} \\
&= \frac{(1-\gamma)\left(\frac{\sigma-1}{\sigma}\right)^2 - \frac{\frac{\sigma-1}{\sigma}}{\varepsilon-1}\left(1-\frac{\sigma-1}{\sigma}\gamma\right)}{1-\gamma\frac{\sigma-1}{\sigma}}\left[\frac{\left((1-\mu)\frac{1-\gamma\frac{\sigma-1}{\sigma}}{1-\gamma}\frac{\sigma}{\sigma-1}+\mu\right)\frac{\sigma}{\sigma-1}}{1-\frac{1}{\varepsilon-1}\left(\frac{\frac{\sigma}{\sigma-1}-\gamma}{1-\gamma}\right)} - \mu\gamma H_t\right]P_{St}Y_{St}
\end{aligned}$$

Therefore, using goods market clearing in the services sector, we can write the labor market clearing condition:

$$W_tL_t = X_3\left[\chi_3 - \mu\gamma H_t\right]P_{St}C_{St}$$
$$\text{where} \qquad X_3 \equiv \frac{(1-\gamma)\left(\frac{\sigma-1}{\sigma}\right)^2 - \frac{\frac{\sigma-1}{\sigma}}{\varepsilon-1}\left(1-\frac{\sigma-1}{\sigma}\gamma\right)}{1-\gamma\frac{\sigma-1}{\sigma}}, \qquad \chi_3 \equiv \frac{\left((1-\mu)\frac{1-\gamma\frac{\sigma-1}{\sigma}}{1-\gamma}\frac{\sigma}{\sigma-1}+\mu\right)\frac{\sigma}{\sigma-1}}{1-\frac{1}{\varepsilon-1}\left(\frac{\frac{\sigma}{\sigma-1}-\gamma}{1-\gamma}\right)} \tag{B.5.3.2}$$

In addition, note that we can write labor allocated to the service sector solely as a fuction of H_t as well:

$$\begin{aligned}
w_tL_t &= X_3\left[\chi_3 - \mu\gamma H_t\right]P_{St}C_{St} \\
&= X_3\left[\chi_3 - \mu\gamma H_t\right]\frac{W_tL_{st}}{1-\mu} \\
L_{St} &= \frac{(1-\mu)}{\chi_3-\mu\gamma H_t}\frac{L_t}{X_3}
\end{aligned}$$

B.5.4 Heterogeneous Firms under Monopolistic Competition and IRS Importing

Goods market clearing Goods market clearing implies that the demand for manufacturing output by services producers and by other manufacturing producers equals final output in the manufacturing sector and that total consumption equals output in services

$$Y_{Dit} = X_{St} + \int_j Q_{Djt} dj, \qquad Y_{St} = C_{St}$$

Plugging in the residual demand schedules, we have

$$\begin{aligned}
Y_{Dit} &= X_{Sit} + \int_j Q_{Dijt} dj \\
&= \left(\frac{P_{it}}{P_{Dt}}\right)^{-\sigma} X_{St} + \int_j \left(\frac{P_{it}}{P_{Dt}}\right)^{-\sigma} Q_{Djt} dj \\
&= \left(\frac{P_{it}}{P_{Dt}}\right)^{-\sigma} \left(X_{St} + \int_j Q_{Djt} dj\right) \\
&= \left(\frac{P_{it}}{P_{Dt}}\right)^{-\sigma} (X_{St} + Q_{Dt})
\end{aligned}$$

where $Q_{Dt} \equiv \int_j Q_{Djt} dj$. We can also write this in aggregate form by using the corresponding the aggregation for manufacturing output as dictated by the demand system:

$$\begin{aligned}
Y_{Dt} &\equiv \left(\int_i (Y_{Dit})^{\frac{\sigma-1}{\sigma}} di\right)^{\frac{\sigma}{\sigma-1}} \\
&= \left(\int_i \left(\left(\frac{P_{it}}{P_{Dt}}\right)^{-\sigma} (X_{St} + Q_{Dt})\right)^{\frac{\sigma-1}{\sigma}} di\right)^{\frac{\sigma}{\sigma-1}} \\
&= \left(\int_i P_{it}^{1-\sigma} di\right)^{\frac{\sigma}{\sigma-1}} P_{Dt}^{\sigma} (X_{St} + Q_{Dt}) \\
&= X_{St} + Q_{Dt}
\end{aligned}$$

where we have used the definition of the price index.

Labor market clearing Labor market clearing requires that all labor demanded by the manufacturing and services sectors equals the supply of labor which we assume is perfectly inelastic.

$$L = L_{St} + \int_i (L_{Dit} + L_{Mit}) di$$

Trade balance The trade balance represents fundamental demand or supply of international foreign assets and depends on the assumed product structure. We re-write this in turn:

$$
\begin{aligned}
TB_t &= E_t P_{Xt}^{\$} X + W_t L_t - P_{St} C_{St} \\
&= E_t P_{Xt}^{\$} X + W_t L + \int_i \Pi_{it} di - P_{St} C_{St} \\
&= E_t P_{Xt}^{\$} X + W_t \left(L_{St} + \int_i (L_{Dit} + L_{Mit}) di \right) + \int_i \left(\frac{1}{\sigma} P_{Dit} Y_{Dit} - W_t L_{Mit} \right) di - P_{St} C_{St} \\
&= E_t P_{Xt}^{\$} X + W_t L_{St} + W_t L_{Dt} + \frac{1}{\sigma} P_{Dt} Y_{Dt} - P_{St} C_{St} \\
&= E_t P_{Xt}^{\$} X + (1-\mu) P_{St} Y_{St} + (1-\gamma) MC_{Dt} Y_{Dt} + \frac{1}{\sigma} P_{Dt} Y_{Dt} - P_{St} C_{St} \\
&= E_t P_{Xt}^{\$} X + (1-\mu) P_{St} Y_{St} + (1-\gamma) \frac{\sigma-1}{\sigma} P_{Dt} Y_{Dt} + \frac{1}{\sigma} P_{Dt} Y_{Dt} - P_{St} C_{St} \\
&= E_t P_{Xt}^{\$} X - \mu P_{St} C_{St} + \left(\frac{1}{\sigma} + (1-\gamma) \frac{\sigma-1}{\sigma} \right) P_{Dt} (Q_{Dt} + X_{St})
\end{aligned}
$$

Now, we can re-write $(Q_{Dt} + X_{St})$ by combining the first-order condition for domestic intermediate inputs:

$$
\begin{aligned}
Q_{Dt} &= \int_j Q_{Djt} dj \\
&= \int_j \omega \left(\frac{P_{Dt}}{P_{Xjt}} \right)^{-\varepsilon} \frac{P_{Xjt} X_{Djt}}{P_{Xjt}} dj \\
&= \int_j \omega\gamma \left(\frac{P_{Dt}}{P_{Xjt}} \right)^{-\varepsilon} \frac{MC_{Djt} Y_{Djt}}{P_{Xjt}} dj \\
&= \int_j \omega\gamma \left(\frac{P_{Dt}}{P_{Xjt}} \right)^{-\varepsilon} \frac{MC_{Djt}}{P_{Xjt}} \left(\frac{P_{jt}}{P_{Dt}} \right)^{-\sigma} (X_{St} + Q_{Dt}) \, dj \\
&= \int_j \omega\gamma \left(\frac{P_{Dt}}{P_{Xjt}} \right)^{-\varepsilon} \frac{MC_{Djt}}{P_{Xjt}} \left(\frac{\frac{\sigma}{\sigma-1} MC_{jt}}{P_{Dt}} \right)^{-\sigma} (X_{St} + Q_{Dt}) \, dj \\
&= \omega\gamma \left(\frac{\sigma}{\sigma-1} \right)^{-\sigma} P_{Dt}^{\sigma-\varepsilon} (X_{St} + Q_{Dt}) \int_{\underline{\varphi}}^{\infty} \left(\frac{1}{A_{Dt}} \frac{1}{\varphi} \frac{W_t^{1-\gamma} P_{Xt}(\varphi)^{\gamma}}{\gamma^{\gamma} (1-\gamma)^{1-\gamma}} \right)^{1-\sigma} P_{Xjt}^{\varepsilon-1} g(\varphi) d\varphi \\
&= \omega\gamma \left(\frac{\sigma}{\sigma-1} \right)^{-\sigma} P_{Dt}^{\sigma-\varepsilon} (X_{St} + Q_{Dt}) \left(\frac{1}{A_{Dt}} \frac{W_t^{1-\gamma}}{\gamma^{\gamma} (1-\gamma)^{1-\gamma}} \right)^{1-\sigma} \int_{\underline{\varphi}}^{\infty} P_{Xt}(\varphi)^{\varepsilon-1-\gamma(\sigma-1)} \varphi^{\sigma-1} g(\varphi) d\varphi \\
&= \omega\gamma \left(\frac{\sigma}{\sigma-1} \right)^{-\sigma} P_{Dt}^{\sigma-\varepsilon} (X_{St} + Q_{Dt}) \left(\frac{1}{A_{Dt}} \frac{W_t^{1-\gamma}}{\gamma^{\gamma} (1-\gamma)^{1-\gamma}} \right)^{1-\sigma} \left(\omega^{-\frac{1}{\varepsilon-1}} P_{Dt} \right)^{\varepsilon-1-\gamma(\sigma-1)} \\
&\qquad \left[\int_{\underline{\varphi}}^{\varphi_{Mt}} \varphi^{\sigma-1} g(\varphi) d\varphi + + \int_{\varphi_{Mt}}^{\infty} \varphi^{\sigma-1} \left(\frac{\varphi_{Mt}}{\varphi} \right)^{\sigma-1} g(\varphi) d\varphi \right] \\
&= \omega^{\frac{\gamma(\sigma-1)}{\varepsilon-1}} \gamma \left(\frac{\sigma}{\sigma-1} \right)^{-\sigma} P_{Dt}^{\sigma-1} (X_{St} + Q_{Dt}) \left(\frac{1}{A_{Dt}} \frac{W_t^{1-\gamma} P_{Dt}^{\gamma}}{\gamma^{\gamma} (1-\gamma)^{1-\gamma}} \right)^{1-\sigma} \\
&\qquad \left[\int_{\underline{\varphi}}^{\varphi_{Mt}} \varphi^{\sigma-1} g(\varphi) d\varphi + \int_{\varphi_{Mt}}^{\infty} \varphi_{Mt}^{\sigma-1} g(\varphi) d\varphi \right]
\end{aligned}
$$

Now, we use the assumption that productivity is distributed according to a pareto distribution: $g(\varphi) = \kappa \left(\underline{\varphi}\right)^{\kappa} \varphi^{-\kappa-1}$, then we have that:

$$
\begin{aligned}
Q_{Dt} &= \omega^{\frac{\gamma(\sigma-1)}{\varepsilon-1}}\gamma\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma}P_{Dt}^{\sigma-1}\left(X_{St}+Q_{Dt}\right)\left(\frac{1}{A_{Dt}}\frac{W_t^{1-\gamma}P_{Dt}^{\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}}\right)^{1-\sigma} \\
&\qquad \kappa\left(\underline{\varphi}\right)^{\kappa}\left[\int_{\underline{\varphi}}^{\varphi_{Mt}}\varphi^{\sigma-\kappa-2}d\varphi+\varphi_{Mt}^{\sigma-1}\int_{\varphi_{Mt}}^{\infty}\varphi^{-\kappa-1}d\varphi\right] \\
&= \omega^{\frac{\gamma(\sigma-1)}{\varepsilon-1}}\gamma\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma}P_{Dt}^{\sigma-1}\left(X_{St}+Q_{Dt}\right)\left(\frac{1}{A_{Dt}}\frac{W_t^{1-\gamma}P_{Dt}^{\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}}\right)^{1-\sigma} \\
&\qquad \kappa\left(\underline{\varphi}\right)^{\kappa}\left[\frac{1}{\sigma-1-\kappa}\left(\varphi_{Mt}^{\sigma-1-\kappa}-\underline{\varphi}^{\sigma-1-\kappa}\right)+\frac{1}{\kappa}\varphi_{Mt}^{\sigma-1-\kappa}\right] \\
&= \omega^{\frac{\gamma(\sigma-1)}{\varepsilon-1}}\gamma\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma}P_{Dt}^{\sigma-1}\left(X_{St}+Q_{Dt}\right)\left(\frac{1}{A_{Dt}}\frac{W_t^{1-\gamma}P_{Dt}^{\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}}\right)^{1-\sigma} \\
&\qquad \kappa\left(\underline{\varphi}\right)^{\kappa}\left[\varphi_{Mt}^{\sigma-1-\kappa}\left(\frac{1}{\kappa}-\frac{1}{\kappa-(\sigma-1)}\right)+\frac{\underline{\varphi}^{\sigma-1-\kappa}}{\sigma-1-\kappa}\right]
\end{aligned}
$$

Now use $P_{Dt}^{1-\sigma}=\left(\frac{\sigma}{\sigma-1}\omega^{\frac{\gamma}{1-\varepsilon}}\frac{1}{A_{Dt}}\frac{{W_t}^{1-\gamma}{P_{Dt}}^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right)^{1-\sigma}\left[\varphi_{Mt}^{\sigma-1-\kappa}\left(\frac{\kappa\underline{\varphi}^{\kappa}}{\sigma-1-\kappa}+\frac{\kappa\underline{\varphi}^{\kappa}}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}\right)-\underline{\varphi}^{\sigma-1-\kappa}\frac{\kappa\underline{\varphi}^{\kappa}}{\sigma-1-\kappa}\right]$ and write:

$$
\begin{aligned}
Q_{Dt} &= \gamma\frac{\sigma-1}{\sigma}\left(X_{St}+Q_{Dt}\right)\frac{\varphi_{Mt}^{\sigma-1-\kappa}\left(\frac{1}{\kappa}-\frac{1}{\kappa-(\sigma-1)}\right)+\frac{\underline{\varphi}^{\sigma-1-\kappa}}{\sigma-1-\kappa}}{\varphi_{Mt}^{\sigma-1-\kappa}\left(\frac{1}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}-\frac{1}{\kappa-(\sigma-1)}\right)+\frac{\underline{\varphi}^{\sigma-1-\kappa}}{\kappa-(\sigma-1)}} \\
&= \gamma\frac{\sigma-1}{\sigma}\left(X_{St}+Q_{Dt}\right)\frac{\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa}\left(\frac{1}{\kappa}-\frac{1}{\kappa-(\sigma-1)}\right)+\frac{1}{\sigma-1-\kappa}}{\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa}\left(\frac{1}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}-\frac{1}{\kappa-(\sigma-1)}\right)+\frac{1}{\kappa-(\sigma-1)}} \\
Q_{Dt} &= \frac{\gamma\frac{\sigma-1}{\sigma}\frac{\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa}\left(\frac{1}{\kappa}-\frac{1}{\kappa-(\sigma-1)}\right)+\frac{1}{\sigma-1-\kappa}}{\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa}\left(\frac{1}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}-\frac{1}{\kappa-(\sigma-1)}\right)+\frac{1}{\kappa-(\sigma-1)}}}{1-\gamma\frac{\sigma-1}{\sigma}\frac{\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa}\left(\frac{1}{\kappa}-\frac{1}{\kappa-(\sigma-1)}\right)+\frac{1}{\sigma-1-\kappa}}{\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa}\left(\frac{1}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}-\frac{1}{\kappa-(\sigma-1)}\right)+\frac{1}{\kappa-(\sigma-1)}}}X_{St} \\
Q_{Dt}+X_{St} &= \frac{1}{1-\gamma\frac{\sigma-1}{\sigma}\frac{\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa}\left(\frac{1}{\kappa}-\frac{1}{\kappa-(\sigma-1)}\right)+\frac{1}{\sigma-1-\kappa}}{\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa}\left(\frac{1}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}-\frac{1}{\kappa-(\sigma-1)}\right)+\frac{1}{\kappa-(\sigma-1)}}}X_{St}
\end{aligned}
$$

Plugging this back into the trade balance equation:

$$
\begin{aligned}
TB_t &= E_t P^{\$}_{Xt} X - \mu P_{St} C_{St} + \left(\frac{1}{\sigma} + (1-\gamma)\frac{\sigma-1}{\sigma}\right) P_{Dt}(Q_{Dt} + X_{St}) \\
&= E_t P^{\$}_{Xt} X - \mu P_{St} C_{St} + \left(\frac{1}{\sigma} + (1-\gamma)\frac{\sigma-1}{\sigma}\right) \frac{1}{1-\gamma\frac{\sigma-1}{\sigma}\frac{\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa}\left(\frac{1}{\kappa}-\frac{1}{\kappa-(\sigma-1)}\right)+\frac{1}{\sigma-1-\kappa}}{\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa}\left(\frac{1}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}-\frac{1}{\kappa-(\sigma-1)}\right)+\frac{1}{\kappa-(\sigma-1)}}} P_{Dt} X_{St} \\
&= E_t P^{\$}_{Xt} X - \mu\gamma\frac{\sigma-1}{\sigma}\left[\frac{1-\frac{\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa}\left(\frac{1}{\kappa}-\frac{1}{\kappa-(\sigma-1)}\right)+\frac{1}{\sigma-1-\kappa}}{\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa}\left(\frac{1}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}-\frac{1}{\kappa-(\sigma-1)}\right)+\frac{1}{\kappa-(\sigma-1)}}}{1-\gamma\frac{\sigma-1}{\sigma}\frac{\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa}\left(\frac{1}{\kappa}-\frac{1}{\kappa-(\sigma-1)}\right)+\frac{1}{\sigma-1-\kappa}}{\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa}\left(\frac{1}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}-\frac{1}{\kappa-(\sigma-1)}\right)+\frac{1}{\kappa-(\sigma-1)}}}\right] P_{St} C_{St}
\end{aligned}
$$

Therefore, we can write the trade balance equation as:

$$
TB_t = E_t P^{\$}_{Xt} X - \mu\gamma\frac{\sigma-1}{\sigma} H_t P_{St} C_{St},
$$

$$
\text{where} \quad H_t \equiv \frac{\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa}\frac{1}{1-\gamma\frac{\sigma-1}{\sigma}}\left(\frac{1}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}-\frac{1}{\kappa}\right)}{\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa}\frac{1}{1-\gamma\frac{\sigma-1}{\sigma}}\left[\left(\frac{1}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}-\frac{1}{\kappa-(\sigma-1)}\right)+\frac{\gamma\frac{\sigma-1}{\sigma}}{1-\gamma\frac{\sigma-1}{\sigma}}\left(\frac{1}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}-\frac{1}{\kappa}\right)\right]+\frac{1}{\kappa-(\sigma-1)}} \tag{B.5.4.1}
$$

Labor market clearing - revisited We start by re-writing demand for labor being used in the importing of intermediate input varieties. To this end, we re-write profits and go back to the first order condition for the optimal number of imported varieties. Profits can be written as:

$$
\begin{aligned}
\Pi_{it} &= \frac{1}{\sigma} P_{Dit} Y_{Dit} - W_t f |\lambda_{it}| \\
&= \frac{1}{\sigma} P_{Dit}\left(\frac{P_{it}}{P_{Dt}}\right)^{-\sigma}\left[X_{St} + \int_j Q_{Djt} dj\right] - W_t f |\lambda_{it}| \\
&= \frac{1}{\sigma} P_{Dit}^{1-\sigma}\left[P_{Dt}^{\sigma} X_{St} + \int_j P_{Dt}^{\sigma}\left(\frac{P_{Dt}}{P_{Xjt}}\right)^{-\varepsilon} dj\right] - W_t f |\lambda_{it}| \\
&= \frac{1}{\sigma} P_{Dit}^{1-\sigma}\widetilde{Y_{Dt}} - W_t f |\lambda_{it}|
\end{aligned}
$$

where we have defined $\widetilde{Y_{Dt}} \equiv P_{Dt}^{\sigma} X_{St} + \int_j P_{Dt}^{\sigma}\left(\frac{P_{Dt}}{P_{Xjt}}\right)^{-\varepsilon} dj$. The first-order condition for the optimal number of imported varieties is given by:

$$
\begin{gathered}
\frac{\partial \Pi_{it}}{\partial |\lambda_{it}|} - W_t f = 0 \\
\frac{\partial \ln \Pi_{it}}{\partial |\lambda_{it}|}\Pi_{it} = 0
\end{gathered}
$$

Now,

$$
\begin{aligned}
\frac{\partial \ln \Pi_{it}}{\partial |\lambda_{it}|} &= (1-\sigma)\frac{\partial \ln P_{Dit}}{\partial |\lambda_{it}|} \\
&= (1-\sigma)\gamma \frac{\partial \ln P_{Xit}}{\partial |\lambda_{it}|} \\
&= (1-\sigma)\gamma \frac{\partial P_{Xit}}{\partial |\lambda_{it}|}\frac{1}{P_{Xit}} \\
&= (1-\sigma)\gamma \frac{1}{1-\varepsilon} P_{Xit}^{\frac{\varepsilon}{\varepsilon-1}}(1-\omega)P_{Mt}^{1-\varepsilon}\frac{1}{P_{Xit}} \\
&= \gamma\frac{1-\sigma}{1-\varepsilon}(1-\omega)\left(\frac{P_{Mt}}{P_{Xit}}\right)^{1-\varepsilon} \\
&= \gamma\frac{1-\sigma}{1-\varepsilon}(1-\omega)\left(\frac{P_{Mt}|\lambda_{it}|^{\frac{1}{1-\varepsilon}}}{P_{Xit}}\right)^{1-\varepsilon}\frac{1}{|\lambda_{it}|} \\
&= \gamma\frac{1-\sigma}{1-\varepsilon}(1-\gamma_{it})\frac{1}{|\lambda_{it}|} \\
\frac{\partial \Pi_{it}}{\partial |\lambda_{it}|} &= \gamma\frac{1-\sigma}{1-\varepsilon}(1-\gamma_{it})\frac{\Pi_{it}}{|\lambda_{it}|}
\end{aligned}
$$

where $1-\gamma_{it} \equiv \left(\frac{P_{Mt}|\lambda_{it}|^{\frac{1}{1-\varepsilon}}}{P_{Xit}}\right)^{1-\varepsilon}$ is the domestic intermediate input share. Going back to the first-order condition, we have:

$$
\begin{aligned}
W_t f &= \gamma\frac{1-\sigma}{1-\varepsilon}(1-\gamma_{it})\frac{\Pi_{it}}{|\lambda_{it}|} \\
|\lambda_{it}|W_t f &= \gamma\frac{1-\sigma}{1-\varepsilon}(1-\gamma_{it})\Pi_{it} \\
L_{Mit}W_t &= \gamma\frac{1-\sigma}{1-\varepsilon}(1-\gamma_{it})\Pi_{it} \\
L_{Mit}W_t &= \frac{\gamma}{1-\varepsilon}\frac{1-\sigma}{\sigma}(1-\gamma_{it})P_{Dit}Y_{Dit} \\
L_{Mit}W_t &= \frac{\gamma}{\varepsilon-1}(1-\gamma_{it})MC_{Dit}Y_{Dit} \\
L_{Mit}W_t &= \frac{\gamma}{1-\gamma}\frac{1}{\varepsilon-1}(1-\gamma_{it})W_t L_{Dit} \\
L_{Mit} &= \frac{\gamma}{1-\gamma}\frac{1}{\varepsilon-1}(1-\gamma_{it})L_{Dit} \\
L_{Mit} &= \frac{\gamma}{1-\gamma}\frac{1}{\varepsilon-1}\left(1-\left(\frac{\varphi_{Mt}}{\varphi}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{(\varepsilon-1)-\gamma(\sigma-1)}}\right)L_{Dit}
\end{aligned}
$$

Now,

$$
\begin{aligned}
W_t L_{Mt} &\equiv \int_i W_t L_{Mit} di = \int_{\underline{\varphi}}^{\infty} W_t L_{Mt}(\varphi) g(\varphi) d\varphi \\
&= \int_{\varphi_{Mt}}^{\infty} W_t L_{Mt}(\varphi) g(\varphi) d\varphi \\
&= \int_{\varphi_{Mt}}^{\infty} \frac{\gamma}{1-\gamma} \frac{1}{\varepsilon - 1} \left(1 - \left(\frac{\varphi_{Mt}}{\varphi} \right)^{\frac{(\sigma-1)(\varepsilon-1)}{(\varepsilon-1)-\gamma(\sigma-1)}} \right) L_{Dt}(\varphi) g(\varphi) d\varphi \\
&= \frac{\gamma}{\varepsilon-1} \frac{\sigma-1}{\sigma} \int_{\varphi_{Mt}}^{\infty} \left(1 - \left(\frac{\varphi_{Mt}}{\varphi} \right)^{\frac{(\sigma-1)(\varepsilon-1)}{(\varepsilon-1)-\gamma(\sigma-1)}} \right) P_{Dt}(\varphi) Y_{Dt}(\varphi) g(\varphi) d\varphi \\
&= \frac{\gamma}{\varepsilon-1} \frac{\sigma-1}{\sigma} P_{Dt}^{\sigma} \left(X_{St} + Q_{Dt} \right) \int_{\varphi_{Mt}}^{\infty} \left(1 - \left(\frac{\varphi_{Mt}}{\varphi} \right)^{\frac{(\sigma-1)(\varepsilon-1)}{(\varepsilon-1)-\gamma(\sigma-1)}} \right) P_{Dt}(\varphi)^{1-\sigma} g(\varphi) d\varphi \\
&= \frac{\gamma}{\varepsilon-1} \left(\frac{\sigma-1}{\sigma} \right)^{\sigma} P_{Dt}^{\sigma} \left(X_{St} + Q_{Dt} \right) \left(\frac{1}{A_{Dt}} \frac{1}{\varphi} \frac{W_t^{1-\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}} \right)^{1-\sigma} \\
&\qquad \int_{\varphi_{Mt}}^{\infty} \left(1 - \left(\frac{\varphi_{Mt}}{\varphi} \right)^{\frac{(\sigma-1)(\varepsilon-1)}{(\varepsilon-1)-\gamma(\sigma-1)}} \right) \varphi^{\sigma-1} P_{Xt}(\varphi)^{\gamma(1-\sigma)} g(\varphi) d\varphi \\
&= \frac{\gamma}{\varepsilon-1} \left(\frac{\sigma-1}{\sigma} \right)^{\sigma} P_{Dt}^{\sigma} \left(X_{St} + Q_{Dt} \right) \left(\frac{1}{A_{Dt}} \frac{1}{\varphi} \frac{W_t^{1-\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}} \right)^{1-\sigma} \omega^{\frac{\gamma(\sigma-1)}{\varepsilon-1}} P_{Dt}^{\gamma(1-\sigma)} \\
&\qquad \int_{\varphi_{Mt}}^{\infty} \left(1 - \left(\frac{\varphi_{Mt}}{\varphi} \right)^{\frac{(\sigma-1)(\varepsilon-1)}{(\varepsilon-1)-\gamma(\sigma-1)}} \right) \varphi^{\sigma-1} \left(\frac{\varphi_{Mt}}{\varphi} \right)^{\frac{(\sigma-1)\gamma(1-\sigma)}{(\varepsilon-1)-\gamma(\sigma-1)}} g(\varphi) d\varphi \\
&= \frac{\gamma}{\varepsilon-1} \left(\frac{\sigma-1}{\sigma} \right)^{\sigma} P_{Dt}^{\sigma} \left(X_{St} + Q_{Dt} \right) \left(\frac{1}{A_{Dt}} \frac{1}{\varphi} \frac{W_t^{1-\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}} \right)^{1-\sigma} \omega^{\frac{\gamma(\sigma-1)}{\varepsilon-1}} P_{Dt}^{\gamma(1-\sigma)} \\
&\qquad \int_{\varphi_{Mt}}^{\infty} \left[\left(\frac{\varphi_{Mt}}{\varphi} \right)^{\frac{(\sigma-1)\gamma(1-\sigma)}{(\varepsilon-1)-\gamma(\sigma-1)}} - \left(\frac{\varphi_{Mt}}{\varphi} \right)^{\sigma-1} \right] \varphi^{\sigma-1} g(\varphi) d\varphi
\end{aligned}
$$

Use the assumption that productivity is distributed according to a pareto distribution: $g(\varphi) = \kappa \left(\underline{\varphi} \right)^{\kappa} \varphi^{-\kappa-1}$, then we have that:

$$
\begin{aligned}
W_t L_{Mt} &= \frac{\gamma}{\varepsilon-1} \left(\frac{\sigma-1}{\sigma} \right)^{\sigma} P_{Dt}^{\sigma} \left(X_{St} + Q_{Dt} \right) \left(\frac{1}{A_{Dt}} \frac{1}{\varphi} \frac{W_t^{1-\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}} \right)^{1-\sigma} \omega^{\frac{\gamma(\sigma-1)}{\varepsilon-1}} P_{Dt}^{\gamma(1-\sigma)} \kappa \left(\underline{\varphi} \right)^{\kappa} \\
&\qquad \int_{\varphi_{Mt}}^{\infty} \left[\left(\frac{\varphi_{Mt}}{\varphi} \right)^{\frac{(\sigma-1)\gamma(1-\sigma)}{(\varepsilon-1)-\gamma(\sigma-1)}} - \left(\frac{\varphi_{Mt}}{\varphi} \right)^{\sigma-1} \right] \varphi^{\sigma-\kappa-2} d\varphi \\
&= \frac{\gamma}{\varepsilon-1} \left(\frac{\sigma-1}{\sigma} \right)^{\sigma} P_{Dt}^{\sigma-1} P_{Dt} \left(X_{St} + Q_{Dt} \right) \left(\frac{1}{A_{Dt}} \frac{1}{\varphi} \frac{W_t^{1-\gamma} P_{Dt}^{\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}} \right)^{1-\sigma} \omega^{\frac{\gamma(\sigma-1)}{\varepsilon-1}} \\
&\qquad \kappa \left(\underline{\varphi} \right)^{\kappa} \varphi_{Mt}^{\sigma-1-\kappa} \left[\frac{1}{\kappa - \frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}} - \frac{1}{\kappa - (\sigma-1)} \right]
\end{aligned}
$$

Use again $P_{Dt}^{1-\sigma} = \left(\frac{\sigma}{\sigma-1}\omega^{\frac{\gamma}{1-\varepsilon}}\frac{1}{A_{Dt}}\frac{{W_t}^{1-\gamma}{P_{Dt}}^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right)^{1-\sigma}\left[\varphi_{Mt}^{\sigma-1-\kappa}\left(\frac{\kappa\underline{\varphi}^{\kappa}}{\sigma-1-\kappa}+\frac{\kappa\underline{\varphi}^{\kappa}}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}\right)-\underline{\varphi}^{\sigma-1-\kappa}\frac{\kappa\underline{\varphi}^{\kappa}}{\sigma-1-\kappa}\right]$ and write:

$$\begin{aligned}
W_tL_{Mt} &= \frac{\gamma}{\varepsilon-1}\frac{\sigma-1}{\sigma}P_{Dt}\left(X_{St}+Q_{Dt}\right)\frac{\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa}\left(\frac{1}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}-\frac{1}{\kappa}\right)}{\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa}\left(\frac{1}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}-\frac{1}{\kappa-(\sigma-1)}\right)+\frac{1}{\kappa-(\sigma-1)}} \\
&= \frac{\gamma}{\varepsilon-1}\frac{\sigma-1}{\sigma}H_tP_{Dt}X_{St}
\end{aligned}$$

where we have used the expression for $(X_{St}+Q_{Dt})$. Now, obtain an expression for $\int_i W_tL_{Dit}di$

$$\begin{aligned}
W_tL_{Dt} &\equiv \int_i W_tL_{Dit}di \\
&= \int_{\underline{\varphi}}^{\infty} W_tL_{Dt}(\varphi)g(\varphi)d\varphi \\
&= \int_{\underline{\varphi}}^{\infty} (1-\gamma)\frac{\sigma-1}{\sigma}P_{Dt}(\varphi)Y_{Dt}(\varphi)g(\varphi)d\varphi \\
&= (1-\gamma)\frac{\sigma-1}{\sigma}P_{Dt}^{\sigma}\left(X_{St}+Q_{Dt}\right)\int_{\underline{\varphi}}^{\infty} P_{Dt}(\varphi)^{1-\sigma}g(\varphi)d\varphi \\
&= (1-\gamma)\frac{\sigma-1}{\sigma}P_{Dt}\left(X_{St}+Q_{Dt}\right)
\end{aligned}$$

Lets re-write $X_{St}+Q_{Dt}$ as a function of H_t. From the definition of H_t:

$$\left(1-\gamma\frac{\sigma-1}{\sigma}\frac{\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa}\left(\frac{1}{\kappa}-\frac{1}{\kappa-(\sigma-1)}\right)+\frac{1}{\sigma-1-\kappa}}{\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa}\left(\frac{1}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}-\frac{1}{\kappa-(\sigma-1)}\right)+\frac{1}{\kappa-(\sigma-1)}}\right)H_t = 1-\frac{\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa}\left(\frac{1}{\kappa}-\frac{1}{\kappa-(\sigma-1)}\right)+\frac{1}{\sigma-1-\kappa}}{\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa}\left(\frac{1}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}-\frac{1}{\kappa-(\sigma-1)}\right)+\frac{1}{\kappa-(\sigma-1)}}$$

$$\left(1-\gamma\frac{\sigma-1}{\sigma}H_t\right)\frac{\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa}\left(\frac{1}{\kappa}-\frac{1}{\kappa-(\sigma-1)}\right)+\frac{1}{\sigma-1-\kappa}}{\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa}\left(\frac{1}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}-\frac{1}{\kappa-(\sigma-1)}\right)+\frac{1}{\kappa-(\sigma-1)}} = 1-H_t$$

$$1-\frac{\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa}\left(\frac{1}{\kappa}-\frac{1}{\kappa-(\sigma-1)}\right)+\frac{1}{\sigma-1-\kappa}}{\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa}\left(\frac{1}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}-\frac{1}{\kappa-(\sigma-1)}\right)+\frac{1}{\kappa-(\sigma-1)}} = \frac{1-\gamma\frac{\sigma-1}{\sigma}}{1-\gamma\frac{\sigma-1}{\sigma}H_t}$$

$$\frac{1}{\frac{\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa}\left(\frac{1}{\kappa}-\frac{1}{\kappa-(\sigma-1)}\right)+\frac{1}{\sigma-1-\kappa}}{\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa}\left(\frac{1}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}-\frac{1}{\kappa-(\sigma-1)}\right)+\frac{1}{\kappa-(\sigma-1)}}} = \frac{1-\gamma\frac{\sigma-1}{\sigma}H_t}{1-\gamma\frac{\sigma-1}{\sigma}}$$

$$Q_{Dt}+X_{St} = \frac{1-\gamma\frac{\sigma-1}{\sigma}H_t}{1-\gamma\frac{\sigma-1}{\sigma}}X_{St}$$

Now, return to the labor market clearing condition.

$$
\begin{aligned}
W_tL_t &= W_tL_{St} + \int_i (W_tL_{Dit} + W_tL_{Mit})di \\
&= W_tL_{St} + W_tL_{Dt} + W_tL_{Mt} \\
&= (1-\mu)P_{St}Y_{St} + (1-\gamma)\frac{\sigma-1}{\sigma}\frac{1-\gamma\frac{\sigma-1}{\sigma}H_t}{1-\gamma\frac{\sigma-1}{\sigma}}P_{Dt}X_{St} + \frac{\gamma}{\varepsilon-1}\frac{\sigma-1}{\sigma}H_tP_{Dt}X_{St} \\
&= (1-\mu)P_{St}Y_{St} + \mu(1-\gamma)\frac{\sigma-1}{\sigma}\frac{1-\frac{\sigma-1}{\sigma}\gamma H_t}{1-\frac{\sigma-1}{\sigma}\gamma}P_{St}Y_{St} + \mu(1-\gamma)\frac{\sigma-1}{\sigma}\frac{\gamma}{1-\gamma}\frac{1}{\varepsilon-1}H_tP_{St}Y_{St} \\
&= \left[(1-\mu) + \mu(1-\gamma)\frac{\sigma-1}{\sigma}\frac{1-\frac{\sigma-1}{\sigma}\gamma H_t}{1-\frac{\sigma-1}{\sigma}\gamma} + \mu(1-\gamma)\frac{\sigma-1}{\sigma}\frac{\gamma}{1-\gamma}\frac{1}{\varepsilon-1}H_t\right]P_{St}Y_{St} \\
&= \frac{1}{1-\gamma\frac{\sigma-1}{\sigma}}\left[(1-\mu)\left(1-\gamma\frac{\sigma-1}{\sigma}\right) + \mu(1-\gamma)\frac{\sigma-1}{\sigma} - \left((1-\gamma)\left(\frac{\sigma-1}{\sigma}\right)^2 - \frac{\frac{\sigma-1}{\sigma}}{\varepsilon-1}\left(1-\frac{\sigma-1}{\sigma}\gamma\right)\right)\mu\gamma H_t\right]P_{St}Y_{St} \\
&= \frac{(1-\gamma)\left(\frac{\sigma-1}{\sigma}\right)^2 - \frac{\frac{\sigma-1}{\sigma}}{\varepsilon-1}\left(1-\frac{\sigma-1}{\sigma}\gamma\right)}{1-\gamma\frac{\sigma-1}{\sigma}}\left[\frac{(1-\mu)\left(1-\gamma\frac{\sigma-1}{\sigma}\right)+\mu(1-\gamma)\frac{\sigma-1}{\sigma}}{(1-\gamma)\left(\frac{\sigma-1}{\sigma}\right)^2 - \frac{\frac{\sigma-1}{\sigma}}{\varepsilon-1}\left(1-\frac{\sigma-1}{\sigma}\gamma\right)} - \mu\gamma H_t\right]P_{St}Y_{St} \\
&= \frac{(1-\gamma)\left(\frac{\sigma-1}{\sigma}\right)^2 - \frac{\frac{\sigma-1}{\sigma}}{\varepsilon-1}\left(1-\frac{\sigma-1}{\sigma}\gamma\right)}{1-\gamma\frac{\sigma-1}{\sigma}}\left[\frac{\left((1-\mu)\frac{1-\gamma\frac{\sigma-1}{\sigma}}{1-\gamma}\frac{\sigma}{\sigma-1}+\mu\right)\frac{\sigma}{\sigma-1}}{1-\frac{1}{\varepsilon-1}\left(\frac{\frac{\sigma}{\sigma-1}-\gamma}{1-\gamma}\right)} - \mu\gamma H_t\right]P_{St}Y_{St}
\end{aligned}
$$

Therefore, using goods market clearing in the services sector, we can write the labor market clearing condition:

$$
W_tL_t = X_4\left[\chi_4 - \mu\gamma H_t\right]P_{St}C_{St}
$$

$$
\text{where} \qquad X_4 \equiv \frac{(1-\gamma)\left(\frac{\sigma-1}{\sigma}\right)^2 - \frac{\frac{\sigma-1}{\sigma}}{\varepsilon-1}\left(1-\frac{\sigma-1}{\sigma}\gamma\right)}{1-\gamma\frac{\sigma-1}{\sigma}}, \qquad \chi_4 \equiv \frac{\left((1-\mu)\frac{1-\gamma\frac{\sigma-1}{\sigma}}{1-\gamma}\frac{\sigma}{\sigma-1}+\mu\right)\frac{\sigma}{\sigma-1}}{1-\frac{1}{\varepsilon-1}\left(\frac{\frac{\sigma}{\sigma-1}-\gamma}{1-\gamma}\right)} \tag{B.5.4.2}
$$

In addition, note that we can write labor allocated to the service sector solely as a fuction of H_t as well:

$$
\begin{aligned}
w_tL_t &= X_4\left[\chi_4 - \mu\gamma H_t\right]P_{St}C_{St} \\
&= X_4\left[\chi_4 - \mu\gamma H_t\right]\frac{W_tL_{st}}{1-\mu} \\
L_{St} &= \frac{(1-\mu)}{\chi_4 - \mu\gamma H_t}\frac{L_t}{X_4}
\end{aligned}
$$

B.6 Adjustment Margins

We start by proving that firm-specific variety-level imports q_{Mikt} are not k specific or i specific, that is, they are the same for every importing firm.

$$
\begin{aligned}
q_{Mikt} &= \left(\frac{P_{Mt}}{P_{Mit}}\right)^{-\theta} Q_{Mit} = \left(\frac{P_{Mt}}{P_{Mit}}\right)^{-\theta} (1-\omega)\left(\frac{P_{Mt}}{P_{Mit}}\right)^{\varepsilon} X_{Dit} = \left(\frac{P_{Mt}}{P_{Mit}}\right)^{-\theta} (1-\omega)\left(\frac{P_{Mt}}{P_{Mit}}\right)^{\varepsilon} \gamma \frac{MC_{it}}{P_{Xit}} Y_{it} \\
&= \left(\frac{P_{Mt}}{P_{Mit}}\right)^{-\theta} (1-\omega)\left(\frac{P_{Mt}}{P_{Mit}}\right)^{\varepsilon} \gamma \frac{MC_{it}}{P_{Xit}} \left(\frac{P_{it}}{P_{Dt}}\right)^{-\sigma} (X_{St}+Q_{Dt}) \\
&\underbrace{=}_{P_{it}\to MC_{it}} \gamma(1-\omega)\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma} (P_{Mt})^{-\theta} \underbrace{(P_{Mit})^{\theta-\varepsilon}}_{=1} (P_{Xit})^{\varepsilon-1} (MC_{it})^{1-\sigma} (P_{Dt})^{\sigma} (X_{St}+Q_{Dt}) \\
&= \gamma(1-\omega)\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma} (P_{Mt})^{-\theta} (P_{Xit})^{\varepsilon-1} \left(\frac{1}{A_{Dt}}\frac{1}{\varphi_i}\frac{W_t^{1-\gamma}P_{Xit}^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right)^{1-\sigma} (P_{Dt})^{\sigma} (X_{St}+Q_{Dt}) \\
&\underbrace{=}_{P_{Xit}} \gamma(1-\omega)\left(\frac{\sigma-1}{\sigma}P_{Dt}\right)^{\sigma} (P_{Mt})^{-\varepsilon} \left(\frac{1}{\varphi_i}\frac{W_t^{1-\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right)^{1-\sigma} (X_{St}+Q_{Dt}) \left[\left(\frac{\varphi_{Mt}}{\varphi_i}\right)^{\frac{\sigma-1}{\varepsilon-1-\gamma(\sigma-1)}} \omega^{-\frac{1}{\varepsilon-1}} P_{Dt}\right]^{\varepsilon-1-\gamma(\sigma-1)} \\
&= \gamma(1-\omega)\left(\frac{\sigma-1}{\sigma}P_{Dt}\right)^{\sigma} (P_{Mt})^{-\varepsilon} \left(\frac{1}{\varphi_i}\frac{W_t^{1-\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right)^{1-\sigma} (X_{St}+Q_{Dt}) \left[\left(\frac{\varphi_{Mt}}{\varphi_i}\right)^{\frac{\sigma-1}{\varepsilon-1-\gamma(\sigma-1)}} \omega^{-\frac{1}{\varepsilon-1}} P_{Dt}\right]^{\varepsilon-1-\gamma(\sigma-1)} \\
&= \gamma(1-\omega)\left(\frac{\sigma-1}{\sigma}P_{Dt}\right)^{\sigma} (P_{Mt})^{-\varepsilon+\gamma(\sigma-1)} \left(\frac{1}{\varphi_i}\frac{W_t^{1-\gamma}P_{Mt}^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right)^{1-\sigma} (X_{St}+Q_{Dt}) \left(\omega^{-\frac{1}{\varepsilon-1}} P_{Dt}\right)^{\varepsilon-1-\gamma(\sigma-1)} \left(\frac{\varphi_{Mt}}{\varphi_i}\right)^{\sigma-1}
\end{aligned}
$$

Notice how both elements that depend on firm-level productivity cancel out, leading to

$$
q_{Mikt} = \left(\frac{\sigma-1}{\sigma}\right)^{\sigma} \gamma(1-\omega)(P_{Dt})^{\sigma}(X_{St}+Q_{Dt}) \left(\frac{1}{A_{Dt}}\frac{W_t^{1-\gamma}P_{Mt}^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right)^{1-\sigma} (P_{Mt})^{-1} \left(\omega^{-\frac{1}{\varepsilon-1}}\frac{P_{Dt}}{P_{Mt}}\right)^{\varepsilon-1-\gamma(\sigma-1)} \varphi_{Mt}^{\sigma-1}
$$

Now recall the expression for the cutoff

$$
\varphi_{Mt}^{\sigma-1} = \left(\frac{\sigma}{\sigma-1}\right)^{\sigma} \left(\frac{\gamma}{\varepsilon-1}(1-\omega)^{\gamma\frac{\sigma-1}{\varepsilon-1}} \frac{(P_{Dt})^{\sigma}(X_{St}+Q_{Dt})}{f w_t}\right)^{-1} \left(\frac{1}{A_{Dt}}\frac{W_t^{1-\gamma}P_{Mt}^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right)^{\sigma-1} \left(\frac{\omega}{1-\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{\varepsilon-1}\right)^{\frac{\varepsilon-1-\gamma(\sigma-1)}{\varepsilon-1}}
$$

Notice that there are many common elements in the last two equations, leading to significant simplification

$$
q_{Mikt} = (\varepsilon-1)\frac{W_t f}{P_{Mt}}
$$

The total amount imported per fir in peso is then $M_{it} = (\varepsilon-1)W_t f \mathcal{L}_{it}$.

B.6.1 Linear margin adjustment

We start with total imports by firm in dollars

$$
M_{it}^{\$} = (\varepsilon-1)\frac{W_t f}{E_t}\mathcal{L}_{it} = \underbrace{(\varepsilon-1)\frac{W_t f}{E_t}}_{sub-intensive margin} \underbrace{\frac{\omega}{1-\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{\varepsilon-1}\left[\left(\frac{\varphi_i}{\varphi_{Mt}}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}} - 1\right]}_{sub-extensive margin}
$$

Now we approximate it to the first order.

$$m_{it}^{\$} = w_t - e_t + (\varepsilon - 1)\left(e_t + p_{Mt}^{\$} - p_{Dt}\right) - \frac{(\sigma-1)(\varepsilon-1)}{\varepsilon - 1 - \gamma(\sigma-1)} \frac{\left(\frac{\varphi_i}{\varphi_M}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}}{\left(\frac{\varphi_i}{\varphi_M}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}} - 1} \varphi_{Mt}$$

Now we use the definition of the domestic input share

$$\gamma_{Dit} \equiv \left(\frac{\varphi_i}{\varphi_{Mt}}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}$$

leading to

$$m_{it}^{\$} = w_t - e_t + (\varepsilon - 1)\left(e_t + p_{Mt}^{\$} - p_{Dt}\right) - \frac{(\sigma-1)(\varepsilon-1)}{\varepsilon - 1 - \gamma(\sigma-1)} \frac{\frac{1}{\gamma_{Di}}}{\frac{1}{\gamma_{Di}} - 1} \varphi_{Mt}$$

Recall the linear equation for openness in the model with selection

$$\varphi_{Mt} = -\frac{1}{\kappa - (\sigma-1)} \frac{1}{1 - \left[\left(1 - \gamma\frac{\sigma-1}{\sigma}\right)\tilde{\kappa} + \gamma\frac{\sigma-1}{\sigma}\right] H} \eta_t$$

We split the margins, starting with the subintensive

$$\begin{aligned}
w_t - e_t &= \frac{1}{1-\mu}\left(a_{St} + p_{St} - \mu p_{Dt}\right) - e_t \\
&= \frac{1}{1-\mu}\left[a_{St} + p_{St} - \mu\left(a_{St} + p_{St} - \frac{1-\mu}{1-\gamma} a_{Dt} - \nu_{p\eta}\eta_t\right)\right] - e_t \\
&= a_{St} + p_{St} + \frac{\mu}{1-\gamma} a_{Dt} + \frac{\mu}{1-\mu}\nu_{p\eta}\eta_t - e_t \\
&= a_{St} + \frac{\mu}{1-\gamma} a_{Dt} + \frac{\mu}{1-\mu}\nu_{p\eta}\eta_t - q_t
\end{aligned}$$

Now recall the equation for η_t in autarky

$$\eta_t = \frac{1}{1 + \nu_{cH} - \nu_{qH}}\left(-\frac{1}{1-\gamma} a_{Dt} + p_{Xt}^{\$} - p_{Mt}^{\$}\right)$$

and the equation for the real exchange rate

$$q_t = a_{St} - \frac{1-\mu}{1-\gamma} a_{Dt} - p_{Mt}^{\$} + \nu_{qH}\eta_t$$

which we plug into the equation of the subintensive margin

$$
\begin{aligned}
m_t^{int} = w_t - e_t &= a_{St} + \frac{\mu}{1-\gamma} a_{Dt} + \frac{\mu}{1-\mu}\nu_{p\eta}\eta_t - \left(a_{St} - \frac{1-\mu}{1-\gamma} a_{Dt} - p_{Mt}^{\$} + \nu_{qH}\eta_t \right) \\
&= \frac{1}{1-\gamma} a_{Dt} + p_{Mt}^{\$} + \left(\frac{\mu}{1-\mu}\nu_{p\eta} - \nu_{qH} \right) \frac{1}{1+\nu_{cH}-\nu_{qH}} \left(-\frac{1}{1-\gamma} a_{Dt} + p_{Xt}^{\$} - p_{Mt}^{\$} \right) \\
&= \frac{1+\nu_{cH}-\nu_{qH} - \frac{\mu}{1-\mu}\nu_{p\eta} + \nu_{qH}}{1+\nu_{cH}-\nu_{qH}} \left(\frac{1}{1-\gamma} a_{Dt} + p_{Mt}^{\$} \right) + \frac{\frac{\mu}{1-\mu}\nu_{p\eta} - \nu_{qH}}{1+\nu_{cH}-\nu_{qH}} p_{Xt}^{\$} \\
&= \frac{1+\nu_{l\eta}}{1+\nu_{cH}-\nu_{qH}} \left(\frac{1}{1-\gamma} a_{Dt} + p_{Mt}^{\$} \right) + \frac{\frac{\mu}{1-\mu}\nu_{p\eta} - \nu_{qH}}{1+\nu_{cH}-\nu_{qH}} p_{Xt}^{\$}
\end{aligned}
$$

and now we solve the sub-extensive margin

$$
\begin{aligned}
m_t^{ext} &= (\varepsilon-1)\left(e_t + p_{Mt}^{\$} - p_{Dt} \right) + \frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)} \frac{1}{1-\gamma_{Di}} \frac{1}{\kappa-(\sigma-1)} \frac{1}{1-\left[\left(1-\gamma^{\frac{\sigma-1}{\sigma}}\right)\tilde{\kappa} + \gamma^{\frac{\sigma-1}{\sigma}}\right] H}\eta_t \\
&= (\varepsilon-1)\left(e_t + p_{Mt}^{\$} - \left(a_{St} + p_{St} - \frac{1-\mu}{1-\gamma} a_{Dt} - \nu_{p\eta}\eta_t \right) \right) \\
&+ \frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)} \frac{1}{1-\gamma_{Di}} \frac{1}{\kappa-(\sigma-1)} \frac{1}{1-\left[\left(1-\gamma^{\frac{\sigma-1}{\sigma}}\right)\tilde{\kappa} + \gamma^{\frac{\sigma-1}{\sigma}}\right] H}\eta_t \\
&= (\varepsilon-1)\left(q_t + p_{Mt}^{\$} - a_{St} + \frac{1-\mu}{1-\gamma} a_{Dt} + \nu_{p\eta}\eta_t \right) \\
&+ \frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)} \frac{1}{1-\gamma_{Di}} \frac{1}{\kappa-(\sigma-1)} \frac{1}{1-\left[\left(1-\gamma^{\frac{\sigma-1}{\sigma}}\right)\tilde{\kappa} + \gamma^{\frac{\sigma-1}{\sigma}}\right] H}\eta_t \\
&= (\varepsilon-1)\left(a_{St} - \frac{1-\mu}{1-\gamma} a_{Dt} - p_{Mt}^{\$} + \nu_{qH}\eta_t + p_{Mt}^{\$} - a_{St} + \frac{1-\mu}{1-\gamma} a_{Dt} + \nu_{p\eta}\eta_t \right) \\
&+ \frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)} \frac{1}{1-\gamma_{Di}} \frac{1}{\kappa-(\sigma-1)} \frac{1}{1-\left[\left(1-\gamma^{\frac{\sigma-1}{\sigma}}\right)\tilde{\kappa} + \gamma^{\frac{\sigma-1}{\sigma}}\right] H}\eta_t \\
&= (\varepsilon-1)\left(\nu_{qH} + \nu_{p\eta}\right)\eta_t + \frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)} \frac{1}{1-\gamma_{Di}} \frac{1}{\kappa-(\sigma-1)} \frac{1}{1-\left[\left(1-\gamma^{\frac{\sigma-1}{\sigma}}\right)\tilde{\kappa} + \gamma^{\frac{\sigma-1}{\sigma}}\right] H}\eta_t \\
&= (\varepsilon-1)\left(\nu_{qH} + \nu_{p\eta} + \frac{\sigma-1}{\varepsilon-1-\gamma(\sigma-1)} \frac{\frac{1}{1-\gamma_{Di}}\frac{1}{\kappa-(\sigma-1)}}{1-\left[\left(1-\gamma^{\frac{\sigma-1}{\sigma}}\right)\tilde{\kappa} + \gamma^{\frac{\sigma-1}{\sigma}}\right] H} \right)\eta_t \\
&= (\varepsilon-1)\left(\nu_{qH} + \nu_{p\eta} + \frac{\sigma-1}{\varepsilon-1-\gamma(\sigma-1)} \frac{\frac{1}{1-\gamma_{Di}}\frac{1}{\kappa-(\sigma-1)}}{1-\left[\left(1-\gamma^{\frac{\sigma-1}{\sigma}}\right)\tilde{\kappa} + \gamma^{\frac{\sigma-1}{\sigma}}\right] H} \right)\left[-\left(\frac{1}{1-\gamma} a_{Dt} + p_{Mt}^{\$} \right) + p_{Xt}^{\$} \right]
\end{aligned}
$$

C Equilibrium

In this appendix, we prove that the equilibrium exists and is unique in all variations of the model studied in the paper. We combine the five main equations of the model, i.e., the manufacturing and service prices equations, the trade balance equation, the market clearing equation and the endogenous openness equation, into a unique implicit equation in H only.

C.1 Perfect competition

The set of equations that determine the equilibrium is the following.

$$\begin{aligned}
P_D &= \frac{W^{1-\gamma}P_D^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\omega^{-\frac{\gamma}{\varepsilon-1}}\left[1+\frac{1-\omega}{\omega}\left(\frac{P_D}{EP_M^{\$}}\right)^{\varepsilon-1}\right]^{-\frac{\gamma}{\varepsilon-1}} \\
P_S &= \frac{W^{1-\mu}P_D{}^{\mu}}{(1-\mu)^{1-\mu}\mu^{\mu}} \\
EP_X^{\$}X &= \mu\gamma H P_S C_S \\
WL &= X_1\left[\chi_1-\mu\gamma H\right]P_S C_S \\
H &= \frac{1}{1+(1-\gamma)\frac{\omega}{1-\omega}\left(\frac{EP_M^{\$}}{P_D}\right)^{\varepsilon-1}}
\end{aligned}$$

We start by using the H equation and the services price equation and substitute them into the manufacturing price equation and to solve for P_D.

$$\begin{aligned}
P_D &= \frac{1}{\varphi_D}\left(\left((1-\mu)^{1-\mu}\mu^{\mu}\right)P_S P_D^{-\mu}\right)^{\frac{1-\gamma}{1-\mu}}\frac{P_D^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\omega^{-\frac{\gamma}{\varepsilon-1}}\left(\frac{1-\gamma H}{1-H}\right)^{\frac{\gamma}{1-\varepsilon}} \\
P_D^{\frac{1}{1-\mu}} &= \left(\left((1-\mu)^{1-\mu}\mu^{\mu}\right)P_S\right)^{\frac{1}{1-\mu}}\left(\frac{1}{\varphi_D}\frac{1}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right)^{\frac{1}{1-\gamma}}\omega^{-\frac{\gamma}{1-\gamma}\frac{1}{\varepsilon-1}}\left(\frac{1-H}{1-\gamma H}\right)^{\frac{\gamma}{1-\gamma}\frac{1}{\varepsilon-1}}
\end{aligned}$$

Second, we use trade balance, market clearing and service prices and then use the H equation again

$$\begin{aligned}
EP_M^{\$} &= \frac{\mu\gamma H\left(\left((1-\mu)^{1-\mu}\mu^{\mu}\right)P_S P_D^{-\mu}\right)^{\frac{1}{1-\mu}}}{X_1\left[\chi_1-\mu\gamma H\right]}\frac{LP_M^{\$}}{P_X^{\$}X} \\
\left(\frac{1-H}{(1-\gamma)\frac{\omega}{1-\omega}H}\right)^{\frac{1}{\varepsilon-1}}P_D^{\frac{1}{1-\mu}} &= \frac{\mu\gamma H}{1-\mu\gamma H}\left(\left((1-\mu)^{1-\mu}\mu^{\mu}\right)P_S\right)^{\frac{1}{1-\mu}}\frac{LP_M^{\$}}{P_X^{\$}X}
\end{aligned}$$

Finally, we plug the expression for P_D in to find an equation in H only as follows:

$$\left(\frac{1}{\varphi_D}\frac{\omega^{-\frac{\gamma}{\varepsilon-1}}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right)^{-\frac{1}{1-\gamma}}\left((1-\gamma)\frac{\omega}{1-\omega}\right)^{-\frac{1}{\varepsilon-1}}\frac{LP_M^{\$}}{P_X^{\$}X}\frac{\mu\gamma H^{\frac{\varepsilon-1}{\varepsilon}}(1-H)^{-\frac{1}{1-\gamma}\frac{1}{\varepsilon-1}}(1-\gamma H)^{\frac{\gamma}{1-\gamma}\frac{1}{\varepsilon-1}}}{X_1\left[\chi_1-\mu\gamma H\right]}=1$$

which can be written in Proposition 1 as

$$\begin{aligned}
F^1\left(H(\Theta),\Theta\right) &= \frac{\Lambda_1^1(\Theta)(1-H)^{-\frac{1}{\varepsilon-1}\frac{1}{1-\gamma}}H^{\frac{\varepsilon}{\varepsilon-1}}(1-\gamma H)^{\frac{\gamma}{1-\gamma}\frac{1}{\varepsilon-1}}}{X_1\left[\chi_1-\mu\gamma H\right]}-1 \\
\text{where}\qquad \Lambda_1^1(\Theta) &= \mu\gamma\left(\frac{1}{\varphi_D}\frac{\omega^{-\frac{\gamma}{\varepsilon-1}}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right)^{-\frac{1}{1-\gamma}}\left((1-\gamma)\frac{\omega}{1-\omega}\right)^{-\frac{1}{\varepsilon-1}}\frac{LP_M^{\$}}{P_X^{\$}X}
\end{aligned}$$

Let $F^1\left(H(\Theta),\Theta\right):[0,1]\to\mathbb{R}$ such that for any $x\in[0,1]$ and $\Lambda>0$it follows that

$$\lim_{H\to x}F^1\left(H(\Theta),\Theta\right)=\Lambda\frac{x^{\frac{\varepsilon}{\varepsilon-1}}(1-\gamma x)^{\frac{\gamma}{1-\gamma}\frac{1}{\varepsilon-1}}}{(1-x)^{\frac{1}{\varepsilon-1}\frac{1}{1-\gamma}}(1-\mu\gamma x)}-1=F^1(x,\Theta)$$

and such that

$$\lim_{H\to 0} F^1(H(\Theta),\Theta) = -1 \quad \text{and} \quad \lim_{H\to 1} F^1(H(\Theta),\Theta) = \infty$$

then $F^1(H(\Theta),\Theta)$ is continuous and has a root in $[0,1]$ by Bolzano's Theorem. The latter two limits follow from the $(1-H)^{-\frac{1}{\varepsilon-1}\frac{1}{1-\gamma}}$ and $H^{\frac{\varepsilon}{\varepsilon-1}}$ parts of $F^1(H(\Theta),\Theta)$ and guarantee the existence of an equilibrium $H\in[0,1]$.

C.2 Monopolistic competition

The set of equations that determine equilibrium in the economy with monopolistic competition is the following

$$\begin{aligned}
P_D &= \frac{\sigma}{\sigma-1}\frac{W^{1-\gamma}P_D^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\omega^{-\frac{\gamma}{\varepsilon-1}}\left[1+\frac{1-\omega}{\omega}\left(\frac{P_D}{EP_M^{\$}}\right)^{\varepsilon-1}\right]^{-\frac{\gamma}{\varepsilon-1}}\\
P_S &= \frac{W^{1-\mu}{P_D}^{\mu}}{(1-\mu)^{1-\mu}\mu^{\mu}}\\
EP_X^{\$}X &= \mu\gamma\frac{\sigma-1}{\sigma}HP_SC_S\\
WL &= X_2\left[\chi_2-\mu\gamma H\right]P_SC_S\\
H &= \frac{1}{1+\left(1-\gamma\frac{\sigma-1}{\sigma}\right)\frac{\omega}{1-\omega}\left(\frac{EP_M^{\$}}{P_D}\right)^{\varepsilon-1}}
\end{aligned}$$

We start by using the H equation and the services price equation and substitute them into the manufacturing price equation and solving it for P_D.

$$\begin{aligned}
P_D &= \frac{\sigma}{\sigma-1}\frac{1}{\varphi_D}\left(\left((1-\mu)^{1-\mu}\mu^{\mu}\right)P_S{P_D}^{-\mu}\right)^{\frac{1-\gamma}{1-\mu}}\frac{P_D^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\omega^{-\frac{\gamma}{\varepsilon-1}}\left(\frac{1-H}{1-\gamma\frac{\sigma-1}{\sigma}H}\right)^{\frac{\gamma}{\varepsilon-1}}\\
P_D^{1-\mu} &= \left(\frac{\sigma-1}{\sigma}\varphi_D\right)^{-\frac{1}{1-\gamma}}\left(\left((1-\mu)^{1-\mu}\mu^{\mu}\right)P_S\right)^{\frac{1}{1-\mu}}\left(\frac{1}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right)^{\frac{1}{1-\gamma}}\omega^{-\frac{\gamma}{1-\gamma}\frac{1}{\varepsilon-1}}\left(\frac{1-H}{1-\gamma\frac{\sigma-1}{\sigma}H}\right)^{\frac{\gamma}{1-\gamma}\frac{1}{\varepsilon-1}}
\end{aligned}$$

Second, we use trade balance, market clearing and service prices and then use the H equation again

$$\begin{aligned}
EP_M^{\$} &= \frac{\mu\gamma\frac{\sigma-1}{\sigma}H}{X_2\left[\chi_2-\mu\gamma H\right]}\frac{LP_M^{\$}}{P_X^{\$}X}\left(\left((1-\mu)^{1-\mu}\mu^{\mu}\right)P_S{P_D}^{-\mu}\right)^{\frac{1}{1-\mu}}\\
\left[\frac{1-H}{\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H}\frac{1-\omega}{\omega}\right]^{\frac{1}{\varepsilon-1}}P_D^{\frac{1}{1-\mu}} &= \frac{\mu\gamma\frac{\sigma-1}{\sigma}H}{X_2\left[\chi_2-\mu\gamma H\right]}\frac{LP_M^{\$}}{P_X^{\$}X}\left(\left((1-\mu)^{1-\mu}\mu^{\mu}\right)P_S\right)^{\frac{1}{1-\mu}}
\end{aligned}$$

Finally, we solve for P_D to find an equation in H only as follows

$$\left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\varepsilon-1}}\left(\frac{\sigma}{\sigma-1}\frac{1}{\varphi_D}\frac{\omega^{-\frac{\gamma}{\varepsilon-1}}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right)^{\frac{1}{1-\gamma}}\left[\frac{1-H}{\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H}\left(\frac{1-H}{1-\gamma\frac{\sigma-1}{\sigma}H}\right)^{\frac{\gamma}{1-\gamma}}\right]^{\frac{1}{\varepsilon-1}} = \frac{\mu\gamma\frac{\sigma-1}{\sigma}H\frac{LP_M^{\$}}{P_X^{\$}X}}{X_2\left[\chi_2-\mu\gamma H\right]}$$

which when collecting terms becomes

$$\left(\frac{\sigma}{\sigma-1}\frac{1}{\varphi_D}\frac{\omega^{-\frac{\gamma}{\varepsilon-1}}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right)^{-\frac{1}{1-\gamma}}\left(\left(1-\gamma\frac{\sigma-1}{\sigma}\right)\frac{\omega}{1-\omega}\right)^{-\frac{1}{\varepsilon-1}}\frac{LP_M^{\$}}{P_X^{\$}X}\frac{\mu\gamma\frac{\sigma-1}{\sigma}H^{\frac{\varepsilon-1}{\varepsilon}}(1-H)^{-\frac{1}{1-\gamma}\frac{1}{\varepsilon-1}}\left(1-\gamma\frac{\sigma-1}{\sigma}H\right)^{\frac{\gamma}{1-\gamma}\frac{1}{\varepsilon-1}}}{X_2\left[\chi_2-\mu\gamma H\right]} = 1$$

which can be written in Proposition (1) as

$$F^2\left(H\left(\Theta\right),\Theta\right)=\frac{\Lambda_1^2\left(\Theta\right)\left(1-H\right)^{-\frac{1}{\varepsilon-1}\frac{1}{1-\gamma}}H^{\frac{\varepsilon}{\varepsilon-1}}\left(1-\gamma\frac{\sigma-1}{\sigma}H\right)^{\frac{\gamma}{1-\gamma}\frac{1}{\varepsilon-1}}}{X_2\left[\chi_2-\mu\gamma H\right]}-1$$

$$\text{where}\qquad \Lambda_1^2\left(\Theta\right)=\left(\frac{\sigma}{\sigma-1}\frac{1}{\varphi_D}\frac{\omega^{-\frac{\gamma}{\varepsilon-1}}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right)^{-\frac{1}{1-\gamma}}\left(\left(1-\gamma\frac{\sigma-1}{\sigma}\right)\frac{\omega}{1-\omega}\right)^{-\frac{1}{\varepsilon-1}}\frac{LP_M^{\$}}{P_X^{\$}X}\mu\gamma\frac{\sigma-1}{\sigma}$$

The same arguments made in perfect competition also guarantee the existence of the equilibrium under monopolistic competition.

C.3 Increasing returns to scale

The set of equations that determine equilibrium in the economy with increasing returns to scale in importing is the following.

$$P_D=\frac{\sigma}{\sigma-1}\frac{W^{1-\gamma}P_D^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\omega^{-\frac{\gamma}{\varepsilon-1}}\frac{1}{\varphi_D}\left(\frac{\varphi_M}{\varphi_D}\right)^{\frac{\gamma(\sigma-1)}{\varepsilon-1-\gamma(\sigma-1)}}$$

$$P_S=\frac{W^{1-\mu}{P_D}^{\mu}}{(1-\mu)^{1-\mu}\mu^{\mu}}$$

$$EP_X^{\$}X=\mu\gamma\frac{\sigma-1}{\sigma}HP_SC_S$$

$$WL=X_3\left[\chi_3-\mu\gamma H\right]P_SC_S$$

$$H=\frac{1-\left(\frac{\varphi_M}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}}{1-\gamma\frac{\sigma-1}{\sigma}\left(\frac{\varphi_M}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}}$$

$$\frac{\varphi_M}{\varphi_D}=\frac{1}{\varphi_D}\left(\frac{\sigma}{\sigma-1}\right)^{\frac{\sigma}{\sigma-1}}\left(\frac{\gamma}{\varepsilon-1}(1-\omega)^{\gamma\frac{\sigma-1}{\varepsilon-1}}\frac{P_D^{\sigma-1}}{Wf}P_D(X_S+Q_D)\right)^{-\frac{1}{\sigma-1}}\left(\frac{1}{A_D\Phi_D}\frac{W^{1-\gamma}P_M^{\gamma}}{(1-\gamma)^{(}1-\gamma)\gamma^{\gamma}}\right)\left[\frac{\omega}{1-\omega}\left(\frac{EP_M}{P_D}\right)^{\varepsilon-1}\right]^{\frac{\varepsilon-1-}{(\sigma-}}$$

We use the last equation to solve for the productivity ratio as a function of H.

$$H-\gamma\frac{\sigma-1}{\sigma}H\left(\frac{\varphi_M}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}=1-\left(\frac{\varphi_M}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}\Rightarrow\left(\frac{\varphi_M}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}=\frac{1-H}{1-\gamma\frac{\sigma-1}{\sigma}H}$$

such that the price equation can be written as follows

$$P_D=\frac{\sigma}{\sigma-1}\frac{W^{1-\gamma}P_D^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\omega^{-\frac{\gamma}{\varepsilon-1}}\frac{1}{\varphi_D}\left(\frac{1-H}{1-\gamma\frac{\sigma-1}{\sigma}H}\right)^{\frac{\gamma}{\varepsilon-1}}$$

leading to a similar equation as before

$$P_D=\frac{\sigma}{\sigma-1}\frac{1}{\varphi_D}\left(\left((1-\mu)^{1-\mu}\mu^{\mu}\right)P_S{P_D}^{-\mu}\right)^{\frac{1-\gamma}{1-\mu}}\frac{P_D^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\omega^{-\frac{\gamma}{\varepsilon-1}}\left(\frac{1-H}{1-\gamma\frac{\sigma-1}{\sigma}H}\right)^{\frac{\gamma}{\varepsilon-1}}$$

$$P_D^{\frac{1}{1-\mu}}=\left(\frac{\sigma-1}{\sigma}\varphi_D\right)^{-\frac{1}{1-\gamma}}\left(\left((1-\mu)^{1-\mu}\mu^{\mu}\right)P_S\right)^{\frac{1}{1-\mu}}\left(\frac{1}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right)^{\frac{1}{1-\gamma}}\omega^{-\frac{\gamma}{1-\gamma}\frac{1}{\varepsilon-1}}\left(\frac{1-H}{1-\gamma\frac{\sigma-1}{\sigma}H}\right)^{\frac{\gamma}{1-\gamma}\frac{1}{\varepsilon-1}}$$

In addition, plug the first-order condition for labor use in services and the services price index into the trade balance condition:

$$EP_M^{\$} = \mu\gamma H \frac{\left((1-\mu)^{1-\mu}\mu^{\mu} P_S P_D^{-\mu}\right)^{\frac{1}{1-\mu}}}{X_3\left[\chi_3 - \mu\gamma H\right]} \frac{LP_M^{\$}}{P_X^{\$} X}$$

Use $P_D\left(X_s + Q_D\right) = \frac{1-\gamma\frac{\sigma-1}{\sigma}H}{1-\frac{\sigma-1}{\sigma}}\mu P_S C_S$ and to write the cut-off equation as and then use the first-order condition for labor use in services and the services price index:

$$\begin{aligned}
\left(\frac{\Phi_M}{\Phi_D}\right) &= \left(\frac{\sigma}{\sigma-1}\right)^{\frac{\sigma}{\sigma-1}} \left(\frac{\gamma(1-\omega)^{\frac{\gamma(\sigma-1)}{\varepsilon-1}}}{\varepsilon-1} \frac{P_D^{\sigma-1}}{Wf} \frac{1-\gamma\frac{\sigma-1}{\sigma}H}{1-\frac{\sigma-1}{\sigma}} \mu P_S C_S\right)^{-\frac{1}{\sigma-1}} \left(\frac{1}{A_D\Phi_D} \frac{W^{1-\gamma}\left(EP_M^{\$}\right)^{\gamma}}{(1-\gamma)^{(}1-\gamma)\gamma^{\gamma}}\right) \left[\frac{\omega}{1-\omega}\left(\frac{EP_M}{P_D}\right)^{\varepsilon-1}\right]^{\frac{\varepsilon-1-\gamma}{(\sigma-1)}} \\
&= \left(\frac{\sigma}{\sigma-1}\right)^{\frac{\sigma}{\sigma-1}} \left(\frac{\gamma(1-\omega)^{\frac{\gamma(\sigma-1)}{\varepsilon-1}}}{\varepsilon-1} \frac{P_D^{\sigma-1}}{f} \frac{1-\gamma\frac{\sigma-1}{\sigma}H}{1-\frac{\sigma-1}{\sigma}} \frac{L}{X_3\left[\chi_3-\mu\gamma H\right]}\right)^{-\frac{1}{\sigma-1}} \left(\frac{\left((1-\mu)^{1-\mu}\mu^{\mu}P_s\right)^{\frac{1-\mu}{1-\gamma}}}{A_D\Phi_D(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right) \\
&\quad \left(\frac{\omega}{1-\omega}\right)^{\frac{\varepsilon-1-\gamma(\sigma-1)}{(\varepsilon-1)(\sigma-1)}} P_D^{-\left(\frac{\varepsilon-1}{\sigma-1}+\frac{1-\gamma}{1-\mu}\right)} \left(EP_M^{\$}\right)^{\frac{\varepsilon-1}{\sigma-1}}
\end{aligned}$$

Plug in the expression for manufacturing prices and the cut-off as a function of H and collect terms to obtain an expression solely as a function of H:

$$\begin{aligned}
&\frac{\mu}{1-\mu}\gamma\frac{\sigma-1}{\sigma}\left(\frac{\omega}{1-\omega}(1-\gamma\frac{\sigma-1}{\sigma})\right)^{\frac{1}{\varepsilon-1}} \left(\frac{\sigma}{\sigma-1}\frac{1}{A_D\Phi_D}\frac{\omega^{-\frac{\gamma}{\varepsilon-1}}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right)\left(\frac{LP_M^{\$}}{P_X^{\$}X}\right)\left(\frac{\varepsilon-1}{\gamma}\frac{1-\mu}{\mu}f\right)^{\frac{1}{\varepsilon-1}}(1-\mu)^{\frac{\varepsilon-2}{\varepsilon-1}} L^{-\frac{1}{\varepsilon-1}} \\
&\qquad \frac{(1-H)^{-\frac{1}{1-\gamma}\frac{1}{\varepsilon}} H\left(1-\gamma\frac{\sigma-1}{\sigma}H\right)^{\frac{\gamma}{1-\gamma}\frac{1}{\varepsilon-1}}}{\left(X_3\left[\chi_3-\mu\gamma H\right]\right)^{\frac{\varepsilon-2}{\varepsilon-1}}} = 1
\end{aligned}$$

which can be written in Proposition (1) as

$$F^3\left(H\left(\Theta\right),\Theta\right) = \frac{\Lambda_3^1\left(\Theta\right)\left(1-H\right)^{-\frac{1}{1-\gamma}\frac{1}{\varepsilon}} H\left(1-\gamma\frac{\sigma-1}{\sigma}H\right)^{\frac{\gamma}{1-\gamma}\frac{1}{\varepsilon-1}}}{\left(X_3\left[\chi_3-\mu\gamma H\right]\right)^{\frac{\varepsilon-2}{\varepsilon-1}}} - 1$$

$$\begin{aligned}
\text{where} \qquad \Lambda_1^3\left(\Theta\right) &= \left(\frac{\sigma}{\sigma-1}\frac{1}{\varphi_D}\frac{\omega^{-\frac{\gamma}{\varepsilon-1}}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right)^{-\frac{1}{1-\gamma}} \left(\left(1-\gamma\frac{\sigma-1}{\sigma}\right)\frac{\omega}{1-\omega}\right)^{-\frac{1}{\varepsilon-1}} \frac{LP_M^{\$}}{P_X^{\$}X}\mu\gamma\frac{\sigma-1}{\sigma}L^{-\frac{1}{\varepsilon-1}} \\
&\quad \left(\frac{\varepsilon-1}{\gamma}\frac{1-\mu}{\mu}f\right)^{\frac{1}{\varepsilon-1}}(1-\mu)^{\frac{\varepsilon-2}{\varepsilon-1}}
\end{aligned}$$

This allows us to proof corrolary 3:

$$\lim_{f\to 0} F^2\left(\Theta\right) - F^3\left(\Theta\right) = F^2\left(\Theta\right) - \lim_{f\to 0} F^3\left(\Theta\right) = F^2\left(\Theta\right) > 0$$

Together with the fact that $\frac{\partial F^2(\Theta)}{\partial H} > 0$ and $\frac{\partial F^3(\Theta)}{\partial H} > 0$, this implies that $\lim_{f\to 0} H^3 - H^2 > 0$.

C.4 Selection

$$F(H(\Theta),\Theta) = \frac{\Lambda_1^j(\Theta)\Big((1-H)+\left(1-\gamma\frac{\sigma-1}{\sigma}\right)(1-\tilde{\kappa})H\Big)^{-\frac{1}{\varepsilon-1}\frac{1}{1-\gamma}} H^{1+\frac{1}{\varepsilon-1}\frac{\kappa}{\kappa-(\sigma-1)}}\left(1-\gamma\frac{\sigma-1}{\sigma}H\right)^{\frac{1}{(1-\gamma)(\sigma-1)}}}{\Big(1+\Lambda_2(\Theta)\Big(1-\gamma\frac{\sigma-1}{\sigma}H\Big)+\Lambda_3(\Theta)\gamma\frac{\sigma-1}{\sigma}H\Big)^{\frac{\varepsilon-2}{\varepsilon-1}}} - 1$$

D General structure

In this section we provide the first order linearized solutions to the non-linear equilibrium systems. We consider a first-order Taylor approximation around the steady state which we know exists and is unique in the homogeneous firm models under perfect competition, monopolistic competition and monopolistic competition under increasing returns to scale. In addition, we know the steady state exists and is unique in the limiting cases for $\kappa \to \infty$ and $\kappa \to \frac{\varepsilon-1}{\varepsilon-1-\gamma(\sigma-1)}$ or the heterogeneous firm model with selection and we conjecture that this remains true away from these limits.

D.1 Homogeneous Firms under Perfect Competition

In this section we derive the equilibrium system for the model with homogeneous producers that compete under perfect competition.

Rewriting in terms of H_t The non-linear equilibrium goods and labor markets block can be fully re-written in terms of H_t. In this case, only the manufacturing price index needs re-writing:

$$\begin{aligned} P_{Dt} &= \frac{1}{\varphi_D}\frac{1}{A_{Dt}}\frac{{W_t}^{1-\gamma}\left(\omega {P_{Dt}}^{1-\varepsilon}+(1-\omega){P_{Mt}}^{1-\varepsilon}\right)^{\frac{\gamma}{1-\varepsilon}}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}} \\ &= \frac{1}{\varphi_D}\frac{1}{A_{Dt}}\frac{{W_t}^{1-\gamma}P_{Dt}^{\gamma}\left(1+\frac{1-\omega}{\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{1-\varepsilon}\right)^{\frac{\gamma}{1-\varepsilon}}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}} \end{aligned}$$

Using the definition of H_t, we can write $1+\frac{1-\omega}{\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{1-\varepsilon}$ as

$$1+\frac{1-\omega}{\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{1-\varepsilon} = \frac{1-\gamma H_t}{1-H_t}$$

Thus, it can be re-written as:

$$P_{Dt} = \frac{1}{\varphi_D}\frac{1}{A_{Dt}}\frac{{W_t}^{1-\gamma}P_{Dt}^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\omega^{\frac{\gamma}{1-\varepsilon}}\left[\frac{1-\gamma H_t}{1-H_t}\right]^{\frac{\gamma}{1-\varepsilon}}$$

Given this expression for manufacturing prices, the non-linear goods and labor markets block is given by:

$$\begin{aligned} TB_t &= E_t P_{Xt}^{\$} X - \mu\gamma H_t P_{St} C_{St} \\ W_t L &= X_1\left(\chi_1 - \mu\gamma H_t\right) P_{St} C_{St} \\ P_{Dt} &= \frac{1}{\varphi_D}\frac{1}{A_{Dt}}\frac{{W_t}^{1-\gamma}P_{Dt}^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\omega^{\frac{\gamma}{1-\varepsilon}}\left[\frac{1-\gamma H_t}{1-H_t}\right]^{\frac{\gamma}{1-\varepsilon}} \\ \mathrm{P}_{St} &= \frac{1}{A_{St}}\frac{{W_t}^{1-\mu}{P_{Dt}}^{\mu}}{(1-\mu)^{1-\mu}\mu^{\mu}} \\ H_t &= \frac{1}{1+(1-\gamma)\frac{\omega}{1-\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{\varepsilon-1}} \end{aligned}$$

First-order linearization Linearizing the services price index, the labor market clearing condition and the trade balance condition is immediate. The linearized manufacturing price index is obtained by:

$$
\begin{aligned}
\ln\left(P_{Dt}\right) &= \ln\left(\frac{\omega^{\frac{\gamma}{1-\varepsilon}}}{\varphi_D}\frac{1}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right) - \ln(A_{Dt}) + (1-\gamma)\ln\left(W_t\right) + \gamma\ln\left(P_{Dt}\right) + \frac{\gamma}{1-\varepsilon}\ln\left(\frac{1-\gamma H_t}{1-H_t}\right) \\
p_{Dt} &= -a_{Dt} + (1-\gamma)w_t + \gamma p_{Dt} - \frac{\gamma}{\varepsilon-1}\left[-\frac{\gamma H}{1-\gamma H} + \frac{H}{1-H}\right]\eta_t \\
p_{Dt} &= -a_{Dt} + (1-\gamma)w_t + \gamma p_{Dt} - \frac{\gamma}{\varepsilon-1}\left[\frac{1-\gamma}{1-\gamma H}\frac{H}{1-H}\right]\eta_t
\end{aligned}
$$

where small letters indicate percentage deviations from the steady state: $\eta_t \equiv \frac{H_t - H}{H}$. The linearized definition of H_t is given by:

$$
\begin{aligned}
\ln\left(H_t\right) &= -\ln\left[1 + (1-\gamma)\frac{\omega}{1-\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{\varepsilon-1}\right] \\
\eta_t &= -\left(\varepsilon-1\right)\left[\frac{(1-\gamma)\frac{\omega}{1-\omega}\left(\frac{P_M}{P_D}\right)^{\varepsilon-1}}{1+(1-\gamma)\frac{\omega}{1-\omega}\left(\frac{P_M}{P_D}\right)^{\varepsilon-1}}\frac{1}{P_M}\left(P_{Mt}-P_M\right) - \frac{(1-\gamma)\frac{\omega}{1-\omega}\left(\frac{P_M}{P_D}\right)^{\varepsilon-1}}{1+(1-\gamma)\frac{\omega}{1-\omega}\left(\frac{P_M}{P_D}\right)^{\varepsilon-1}}\frac{1}{P_D}\left(P_{Dt}-P_D\right)\right] \\
\eta_t &= -\left(\varepsilon-1\right)\left(1-H\right)\left[p^{\$}_{Mt} + e_t - p_{Dt}\right]
\end{aligned}
$$

Therefore, the linearized system is given by:

$$
\begin{aligned}
tb_t &= e_t + p^{\$}_{Xt} - \eta_t + p_{St} + c_{St} \\
w_t &= -\frac{\mu\gamma H}{\chi_1 - \mu\gamma H}\eta_t + p_{St} + c_{St} \\
p_{Dt} &= -a_{Dt} + (1-\gamma)w_t + \gamma p_{Dt} - \frac{\gamma}{\varepsilon-1}\left[\frac{1-\gamma}{1-\gamma H}\frac{H}{1-H}\right]\eta_t \\
p_{St} &= -a_{St} + (1-\mu)w_t + \mu p_{Dt} \\
\eta_t &= -\left(\varepsilon-1\right)\left(1-H\right)\left[p^{\$}_{Mt} + e_t - p_{Dt}\right]
\end{aligned}
$$

General structure To obtain the general structure, we combine the equilibrium conditions in the following way. The price index for services yields an expression for real wages as a function of services productivity and the relative price of manufacturing goods:

$$
\begin{aligned}
p_{St} &= -a_{St} + (1-\mu)w_t + \mu p_{Dt} \\
0 &= -a_{St} + (1-\mu)\left(w_t - p_{St}\right) + \mu\left(p_{Dt} - p_{St}\right) \\
w_t - p_{St} &= \frac{1}{1-\mu}a_{St} - \frac{\mu}{1-\mu}\left(p_{Dt} - p_{St}\right)
\end{aligned}
$$

Given this expression for real wages, we can solve for manufacturing prices as a function of the shocks and η_t:

$$\begin{aligned}
(1-\gamma)p_{Dt} &= -a_{Dt} + (1-\gamma)w_t - \frac{\gamma}{\varepsilon-1}\left[\frac{1-\gamma}{1-\gamma H}\frac{H}{1-H}\right]\eta_t \\
(1-\gamma)\left(p_{Dt}-p_{St}\right) &= -a_{Dt} + (1-\gamma)\left(w_t - p_{St}\right) - \frac{\gamma}{\varepsilon-1}\left[\frac{1-\gamma}{1-\gamma H}\frac{H}{1-H}\right]\eta_t \\
&= -a_{Dt} + (1-\gamma)\left(\frac{1}{1-\mu}a_{St} - \frac{\mu}{1-\mu}\left(p_{Dt}-p_{St}\right)\right) - \frac{\gamma}{\varepsilon-1}\left[\frac{1-\gamma}{1-\gamma H}\frac{H}{1-H}\right]\eta_t \\
p_{Dt} &= a_{St} - \frac{1-\mu}{1-\gamma}a_{Dt} - \underbrace{\frac{1-\mu}{1-\gamma}\frac{\gamma}{(\varepsilon-1)(1-H)}\frac{(1-\gamma)H}{1-\gamma H}}_{\equiv \nu_{p\eta}}\eta_t
\end{aligned}$$

Now, use the labor market clearing condition to express final consumption

$$\begin{aligned}
c_{St} &= w_t - p_{St} + \underbrace{\frac{\mu\gamma H}{\chi_1 - \mu\gamma H}}_{\equiv \nu_{l\eta}}\eta_t \\
&= \frac{1}{1-\mu}a_{St} - \frac{\mu}{1-\mu}\left(p_{Dt}-p_{St}\right) + \nu_{l\eta}\eta_t \\
&= \frac{1}{1-\mu}a_{St} - \frac{\mu}{1-\mu}\left(a_{St} - \frac{1-\mu}{1-\gamma}a_{Dt} - \nu_{p\eta}\eta_t\right) + \nu_{l\eta}\eta_t \\
&= a_{St} + \frac{\mu}{1-\gamma}a_{Dt} + \underbrace{\left(\nu_{l\eta} + \frac{\mu}{1-\mu}\nu_{p\eta}\right)}_{\equiv \nu_{cH}}\eta_t
\end{aligned}$$

To obtain the expenditure switching expression, we combine the relative input equation with the expression for how manufacturing prices respond to changes in openness:

$$\begin{aligned}
\eta_t &= -(\varepsilon-1)(1-H)\left[p^{\$}_{Mt} + e_t - p_{Dt}\right] \\
&= -(\varepsilon-1)(1-H)\left[p^{\$}_{Mt} + e_t - p_{St} - \left(p_{Dt}-p_{St}\right)\right] \\
&= -(\varepsilon-1)(1-H)\left[p^{\$}_{Mt} + q_t - \left(a_{St} - \frac{1-\mu}{1-\gamma}a_{Dt} - \nu_{P\eta}\eta_t\right)\right] \\
&= \underbrace{-\frac{(\varepsilon-1)(1-H)}{1+(\varepsilon-1)(1-H)\nu_{p\eta}}}_{\equiv 1/\nu_{qH}}\left[p^{\$}_{Mt} + q_t - a_{St} + \frac{1-\mu}{1-\gamma}a_{Dt}\right]
\end{aligned}$$

D.2 Homogeneous Firms under Monopolistic Competition

In this section we derive the equilibrium system for the model with homogeneous producers that compete under monopolistic competition.

Rewriting in terms of H_t The non-linear equilibrium goods and labor markets block can be fully re-written in terms of H_t. In this case, only the manufacturing price index needs re-writing:

$$
\begin{aligned}
P_{Dt} &= \frac{\sigma}{\sigma-1}\frac{1}{\varphi_D}\frac{1}{A_{Dt}}\frac{{W_t}^{1-\gamma}\left(\omega {P_{Dt}}^{1-\varepsilon}+(1-\omega){P_{Mt}}^{1-\varepsilon}\right)^{\frac{\gamma}{1-\varepsilon}}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}} \\
&= \frac{\sigma}{\sigma-1}\frac{1}{\varphi_D}\frac{1}{A_{Dt}}\frac{{W_t}^{1-\gamma}P_{Dt}^{\gamma}\left(1+\frac{1-\omega}{\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{1-\varepsilon}\right)^{\frac{\gamma}{1-\varepsilon}}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}
\end{aligned}
$$

Using the definition of H_t, we can write $1+\frac{1-\omega}{\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{1-\varepsilon}$ as

$$
1+\frac{1-\omega}{\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{1-\varepsilon} = \frac{1-\gamma\frac{\sigma-1}{\sigma}H_t}{1-H_t}
$$

Thus, it can be re-written as:

$$
P_{Dt} = \frac{\sigma}{\sigma-1}\frac{1}{\varphi_D}\frac{1}{A_{Dt}}\frac{{W_t}^{1-\gamma}P_{Dt}^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\omega^{\frac{\gamma}{1-\varepsilon}}\left[\frac{1-\gamma\frac{\sigma-1}{\sigma}H_t}{1-H_t}\right]^{\frac{\gamma}{1-\varepsilon}}
$$

Given this expression for manufacturing prices, the non-linear goods and labor markets block is given by:

$$
\begin{aligned}
TB_t &= E_t P_{Xt}^{\$}X-\mu\gamma H_t P_{St}C_{St} \\
W_tL &= X_2\left(\chi_2-\mu\gamma H_t\right)P_{St}C_{St} \\
P_{Dt} &= \frac{\sigma}{\sigma-1}\frac{1}{\varphi_D}\frac{1}{A_{Dt}}\frac{{W_t}^{1-\gamma}P_{Dt}^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\omega^{\frac{\gamma}{1-\varepsilon}}\left[\frac{1-\gamma\frac{\sigma-1}{\sigma}H_t}{1-H_t}\right]^{\frac{\gamma}{1-\varepsilon}} \\
\mathrm{P}_{St} &= \frac{1}{A_{St}}\frac{{W_t}^{1-\mu}{P_{Dt}}^{\mu}}{(1-\mu)^{1-\mu}\mu^{\mu}} \\
H_t &= \frac{1}{1+(1-\gamma\frac{\sigma-1}{\sigma})\frac{\omega}{1-\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{\varepsilon-1}}
\end{aligned}
$$

First-order linearization Linearizing the services price index, the labor market clearing condition and the trade balance condition is immediate. The linearized manufacturing price index is obtained by:

$$
\begin{aligned}
\ln\left(P_{Dt}\right) &= \ln\left(\frac{\sigma}{\sigma-1}\frac{\omega^{\frac{\gamma}{1-\varepsilon}}}{\varphi_D}\frac{1}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right)-\ln(A_{Dt})+(1-\gamma)\ln\left(W_t\right)+\gamma\ln\left(P_{Dt}\right)-\frac{\gamma}{\varepsilon-1}\ln\left[\frac{1-\gamma\frac{\sigma-1}{\sigma}H_t}{1-H_t}\right] \\
p_{Dt} &= -a_{Dt}+(1-\gamma)w_t+\gamma p_{Dt}-\frac{\gamma}{\varepsilon-1}\left[-\frac{\gamma\frac{\sigma-1}{\sigma}H}{1-\gamma\frac{\sigma-1}{\sigma}H}+\frac{H}{1-H}\right]\eta_t \\
p_{Dt} &= -a_{Dt}+(1-\gamma)w_t+\gamma p_{Dt}-\frac{\gamma}{\varepsilon-1}\left[\frac{1-\gamma\frac{\sigma-1}{\sigma}}{1-\gamma\frac{\sigma-1}{\sigma}H}\frac{H}{1-H}\right]\eta_t
\end{aligned}
$$

where small letters indicate percentage deviations from the steady state: $\eta_t\equiv\frac{H_t-H}{H}$. The linearized definition of

H_t is given by:

$$
\begin{aligned}
\ln\left(H_t\right) &= -\ln\left[1+(1-\gamma\frac{\sigma-1}{\sigma})\frac{\omega}{1-\omega}\left(\frac{P_{Mt}}{P_{Dt}}\right)^{\varepsilon-1}\right] \\
\eta_t &= -\left(\varepsilon-1\right)\left[\frac{(1-\gamma\frac{\sigma-1}{\sigma})\frac{\omega}{1-\omega}\left(\frac{P_M}{P_D}\right)^{\varepsilon-1}}{1+(1-\gamma\frac{\sigma-1}{\sigma})\frac{\omega}{1-\omega}\left(\frac{P_M}{P_D}\right)^{\varepsilon-1}}\frac{1}{P_M}\left(P_{Mt}-P_M\right)-\frac{(1-\gamma\frac{\sigma-1}{\sigma})\frac{\omega}{1-\omega}\left(\frac{P_M}{P_D}\right)^{\varepsilon-1}}{1+(1-\gamma\frac{\sigma-1}{\sigma})\frac{\omega}{1-\omega}\left(\frac{P_M}{P_D}\right)^{\varepsilon-1}}\frac{1}{P_D}\left(P_{Dt}-P_D\right)\right] \\
\eta_t &= -\left(\varepsilon-1\right)\left(1-H\right)\left[p^{\$}_{Mt}+e_t-p_{Dt}\right]
\end{aligned}
$$

Therefore, the linearized system is given by:

$$
\begin{aligned}
tb_t &= e_t + p^{\$}_{Xt} - \eta_t + p_{St} + c_{St} \\
w_t &= -\frac{\mu\gamma H}{\chi_2-\mu\gamma H}\eta_t + p_{St} + c_{St} \\
p_{Dt} &= -a_{Dt} + (1-\gamma)w_t + \gamma p_{Dt} - \frac{\gamma}{\varepsilon-1}\left[\frac{1-\gamma\frac{\sigma-1}{\sigma}}{1-\gamma\frac{\sigma-1}{\sigma}H}\frac{H}{1-H}\right]\eta_t \\
p_{St} &= -a_{St} + (1-\mu)w_t + \mu p_{Dt} \\
\eta_t &= -\left(\varepsilon-1\right)\left(1-H\right)\left[p^{\$}_{Mt}+e_t-p_{Dt}\right]
\end{aligned}
$$

General structure To obtain the general structure, we combine the equilibrium conditions in the following way. The price index for services yields an expression for real wages as a function of services productivity and the relative price of manufacturing goods:

$$
\begin{aligned}
p_{St} &= -a_{St} + (1-\mu)w_t + \mu p_{Dt} \\
0 &= -a_{St} + (1-\mu)\left(w_t - p_{St}\right) + \mu\left(p_{Dt}-p_{St}\right) \\
w_t - p_{St} &= \frac{1}{1-\mu}a_{St} - \frac{\mu}{1-\mu}\left(p_{Dt}-p_{St}\right)
\end{aligned}
$$

Given this expression for real wages, we can solve for manufacturing prices as a function of the shocks and η_t:

$$
\begin{aligned}
(1-\gamma)p_{Dt} &= -a_{Dt} + (1-\gamma)w_t - \frac{\gamma}{\varepsilon-1}\left[\frac{1-\gamma\frac{\sigma-1}{\sigma}}{1-\gamma\frac{\sigma-1}{\sigma}H}\frac{H}{1-H}\right]\eta_t \\
(1-\gamma)\left(p_{Dt}-p_{St}\right) &= -a_{Dt} + (1-\gamma)\left(w_t-p_{St}\right) - \frac{\gamma}{\varepsilon-1}\left[\frac{1-\gamma\frac{\sigma-1}{\sigma}}{1-\gamma\frac{\sigma-1}{\sigma}H}\frac{H}{1-H}\right]\eta_t \\
&= -a_{Dt} + (1-\gamma)\left(\frac{1}{1-\mu}a_{St} - \frac{\mu}{1-\mu}\left(p_{Dt}-p_{St}\right)\right) - \frac{\gamma}{\varepsilon-1}\left[\frac{1-\gamma\frac{\sigma-1}{\sigma}}{1-\gamma\frac{\sigma-1}{\sigma}H}\frac{H}{1-H}\right]\eta_t \\
p_{Dt} &= a_{St} - \frac{1-\mu}{1-\gamma}a_{Dt} - \underbrace{\frac{1-\mu}{1-\gamma}\frac{\gamma}{\varepsilon-1}\frac{1}{1-H}\left[\frac{\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H}{1-\gamma\frac{\sigma-1}{\sigma}H}\right]}_{\equiv\nu_{p\eta}}\eta_t
\end{aligned}
$$

Now, use the labor market clearing condition to express final consumption

$$
\begin{aligned}
c_{St} &= w_t - p_{St} + \underbrace{\frac{\mu\gamma H}{\chi_2 - \mu\gamma H}}_{\equiv \nu_{l\eta}} \eta_t \\
&= \frac{1}{1-\mu} a_{St} - \frac{\mu}{1-\mu} \left(p_{Dt} - p_{St}\right) + \nu_{l\eta}\eta_t \\
&= \frac{1}{1-\mu} a_{St} - \frac{\mu}{1-\mu} \left(a_{St} - \frac{1-\mu}{1-\gamma} a_{Dt} - \nu_{p\eta}\eta_t\right) + \nu_{l\eta}\eta_t \\
&= a_{St} + \frac{\mu}{1-\gamma} a_{Dt} + \underbrace{\left(\nu_{l\eta} + \frac{\mu}{1-\mu}\nu_{p\eta}\right)}_{\equiv \nu_{cH}} \eta_t
\end{aligned}
$$

To obtain the expenditure switching expression, we combine the relative input equation with the expression for how manufacturing prices respond to changes in openness:

$$
\begin{aligned}
\eta_t &= -(\varepsilon-1)(1-H)\left[p^{\$}_{Mt} + e_t - p_{Dt}\right] \\
&= -(\varepsilon-1)(1-H)\left[p^{\$}_{Mt} + e_t - p_{St} - (p_{Dt} - p_{St})\right] \\
&= -(\varepsilon-1)(1-H)\left[p^{\$}_{Mt} + q_t - \left(a_{St} - \frac{1-\mu}{1-\gamma} a_{Dt} - \nu_{P\eta}\eta_t\right)\right] \\
&= \underbrace{-\frac{(\varepsilon-1)(1-H)}{1+(\varepsilon-1)(1-H)\nu_{p\eta}}}_{\equiv 1/\nu_{qH}} \left[p^{\$}_{Mt} + q_t - a_{St} + \frac{1-\mu}{1-\gamma} a_{Dt}\right]
\end{aligned}
$$

D.3 Homogeneous Firms under Monopolistic Competition and IRS Importing

In this section we derive the equilibrium system for the model with homogeneous producers that compete under monopolistic competition.

Rewriting in terms of H_t The non-linear equilibrium goods and labor markets block can be fully re-written in terms of H_t. Using the definition of H_t, we can write

$$
H_t = \frac{1 - \left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}}{1 - \frac{\sigma-1}{\sigma}\gamma\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}}
$$

$$
\left(1 - \frac{\sigma-1}{\sigma}\gamma H_t\right)\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}} = 1 - H_t
$$

$$
\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{\gamma(\sigma-1)}{\varepsilon-1-\gamma(\sigma-1)}} = \left(\frac{1 - \frac{\sigma-1}{\sigma}\gamma H_t}{1 - H_t}\right)^{\frac{\gamma}{1-\varepsilon}}
$$

Thus, aggregate manufacturing prices can be re-written as:

$$
\begin{aligned}
P_{Dt} &= \frac{\sigma}{\sigma-1}\frac{1}{\varphi_D}\frac{1}{A_{Dt}}\frac{{W_t}^{1-\gamma}\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{\gamma(\sigma-1)}{\varepsilon-1-\gamma(\sigma-1)}}\omega^{\frac{\gamma}{1-\varepsilon}}P_{Dt}^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}} \\
&= \frac{\sigma}{\sigma-1}\frac{1}{\varphi_D}\frac{1}{A_{Dt}}\frac{{W_t}^{1-\gamma}P_{Dt}^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\omega^{\frac{\gamma}{1-\varepsilon}}\left(\frac{1-\frac{\sigma-1}{\sigma}\gamma H_t}{1-H_t}\right)^{\frac{\gamma}{1-\varepsilon}}
\end{aligned}
$$

Next, we rewrite the productivity cut-off relation:

$$
\begin{aligned}
\varPhi_{Mt} &= \left(\frac{\sigma}{\sigma-1}\right)^{\frac{\sigma}{\sigma-1}}\left(\frac{\gamma(1-\omega)^{\frac{\gamma(\sigma-1)}{\varepsilon-1}}}{\varepsilon-1}\frac{{P_{Dt}}^{\sigma}(X_{St}+Q_{Dt})}{fW_t}\right)^{-\frac{1}{\sigma-1}}\frac{1}{A_{Dt}}\frac{{W_t}^{1-\gamma}\left(E_tP^{\$}_{Mt}\right)^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\left[\frac{\omega}{1-\omega}\left(\frac{P_{Dt}}{E_tP^{\$}_{Mt}}\right)^{1-\varepsilon}\right]^{\frac{\varepsilon-1-\gamma(\sigma-1)}{(\sigma-1)(\varepsilon-1)}} \\
&= \left(\frac{\sigma}{\sigma-1}\right)^{\frac{\sigma}{\sigma-1}}\left(\frac{\gamma(1-\omega)^{\frac{\gamma(\sigma-1)}{\varepsilon-1}}}{\varepsilon-1}\frac{P_{Dt}^{\sigma-1}}{fW_t}P_{Dt}(X_{St}+Q_{Dt})\right)^{-\frac{1}{\sigma-1}}\frac{1}{A_{Dt}}\frac{{W_t}^{1-\gamma}\left(E_tP^{\$}_{Mt}\right)^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\left[\frac{\omega}{1-\omega}\left(\frac{P_{Dt}}{E_tP^{\$}_{Mt}}\right)^{1-\varepsilon}\right]^{\frac{\varepsilon-1-\gamma(\sigma-1)}{(\sigma-1)(\varepsilon-1)}}
\end{aligned}
$$

$$
\varPhi_{Mt}^{\sigma-1} = \left(\frac{\sigma}{\sigma-1}\right)^{\sigma}\left(\frac{\gamma(1-\omega)^{\frac{\gamma(\sigma-1)}{\varepsilon-1}}}{\varepsilon-1}\frac{P_{Dt}^{\sigma-1}}{fW_t}\frac{1-\frac{\sigma-1}{\sigma}\gamma H_t}{1-\frac{\sigma-1}{\sigma}\gamma}\mu P_{St}C_{St}\right)^{-1}\left(\frac{1}{A_{Dt}}\frac{{W_t}^{1-\gamma}\left(E_tP^{\$}_{Mt}\right)^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right)^{\sigma-1}\left[\frac{\omega}{1-\omega}\left(\frac{P_{Dt}}{E_tP^{\$}_{Mt}}\right)^{1-\varepsilon}\right]^{\frac{\varepsilon-1-\gamma(\sigma-1)}{\varepsilon-1}}
$$

Given this expression for manufacturing prices and the productivity cut-off, the non-linear goods and labor markets block is given by:

$$
\begin{aligned}
TB_t &= E_tP^{\$}_{Xt}X - \mu\gamma H_tP_{St}C_{St} \\
W_tL &= X_3\left(\chi_3 - \mu\gamma H_t\right)P_{St}C_{St} \\
P_{Dt} &= \frac{\sigma}{\sigma-1}\frac{1}{\varphi_D}\frac{1}{A_{Dt}}\frac{{W_t}^{1-\gamma}P_{Dt}^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\omega^{\frac{\gamma}{1-\varepsilon}}\left(\frac{1-\frac{\sigma-1}{\sigma}\gamma H_t}{1-H_t}\right)^{\frac{\gamma}{1-\varepsilon}} \\
\mathrm{P}_{St} &= \frac{1}{A_{St}}\frac{{W_t}^{1-\mu}{P_{Dt}}^{\mu}}{(1-\mu)^{1-\mu}\mu^{\mu}} \\
\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{\gamma(\sigma-1)}{\varepsilon-1-\gamma(\sigma-1)}} &= \left(\frac{1-\frac{\sigma-1}{\sigma}\gamma H_t}{1-H_t}\right)^{\frac{\gamma}{1-\varepsilon}} \\
\varPhi_{Mt}^{\sigma-1} &= \left(\frac{\sigma}{\sigma-1}\right)^{\sigma}\left(\frac{\gamma(1-\omega)^{\frac{\gamma(\sigma-1)}{\varepsilon-1}}}{\varepsilon-1}\frac{P_{Dt}^{\sigma-1}}{fW_t}\frac{1-\frac{\sigma-1}{\sigma}\gamma H_t}{1-\frac{\sigma-1}{\sigma}\gamma}\mu P_{St}C_{St}\right)^{-1} \\
&\qquad \left(\frac{1}{A_{Dt}}\frac{{W_t}^{1-\gamma}\left(E_tP^{\$}_{Mt}\right)^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right)^{\sigma-1}\left[\frac{\omega}{1-\omega}\left(\frac{P_{Dt}}{E_tP^{\$}_{Mt}}\right)^{1-\varepsilon}\right]^{\frac{\varepsilon-1-\gamma(\sigma-1)}{\varepsilon-1}}
\end{aligned}
$$

First-order linearization Linearizing the services price index, the labor market clearing condition and the trade balance condition is immediate. The linearized manufacturing price index is obtained by:

$$\ln\left(P_{Dt}\right)=\ln\left(\frac{\sigma}{\sigma-1}\frac{\omega^{\frac{\gamma}{1-\varepsilon}}}{\varphi_D}\frac{1}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right)+\ln\left(\frac{W_t^{1-\gamma}P_{Dt}^{\gamma}}{A_{Dt}}\right)-\frac{\gamma}{\varepsilon-1}\ln\left[\frac{1-\gamma\frac{\sigma-1}{\sigma}H_t}{1-H_t}\right]$$

$$p_{Dt}=-a_{Dt}+(1-\gamma)w_t+\gamma p_{Dt}-\frac{\gamma}{\varepsilon-1}\left[-\frac{\gamma\frac{\sigma-1}{\sigma}H}{1-\gamma\frac{\sigma-1}{\sigma}H}+\frac{H}{1-H}\right]\eta_t$$

$$p_{Dt}=-a_{Dt}+(1-\gamma)w_t+\gamma p_{Dt}-\frac{\gamma}{\varepsilon-1}\left[\frac{1-\gamma\frac{\sigma-1}{\sigma}}{1-\gamma\frac{\sigma-1}{\sigma}H}\frac{H}{1-H}\right]\eta_t$$

where small letters indicate percentage deviations from the steady state: $\eta_t\equiv\frac{H_t-H}{H}$. Solving for φ_{Mt} as a function of η_t is executed using the definition of H_t:

$$\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}\ln\left(\frac{\Phi_{Mt}}{\Phi_D}\right)=-\ln\left[\left(\frac{1-\frac{\sigma-1}{\sigma}\gamma H_t}{1-H_t}\right)^{\frac{\gamma}{1-\varepsilon}}\right]$$

$$\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}\varphi_{Mt}=\left(-\frac{H}{1-H}+\frac{\gamma\frac{\sigma-1}{\sigma}H}{1-\gamma\frac{\sigma-1}{\sigma}H}\right)\eta_t$$

$$\varphi_{Mt}=-\frac{\varepsilon-1-\gamma(\sigma-1)}{(\sigma-1)(\varepsilon-1)}\frac{\left(1-\frac{\sigma-1}{\sigma}\gamma\right)H}{(1-H)\left(1-\frac{\sigma-1}{\sigma}\gamma H\right)}\eta_t$$

Next, the linearized cut-off equation is given by:

$$\begin{aligned}(\sigma-1)\ln\Phi_{Mt}=&\ln\left(\left(\frac{\sigma}{\sigma-1}\right)^{\sigma}\left(\frac{\gamma(1-\omega)^{\frac{\gamma(\sigma-1)}{\varepsilon-1}}}{\varepsilon-1}\frac{\mu}{f\left((1-\gamma)^{1-\gamma}\gamma^{\gamma}\right)^{1-\sigma}}\right)^{-1}\left(\frac{\omega}{1-\omega}\right)^{\frac{\varepsilon-1-\gamma(\sigma-1)}{\varepsilon-1}}\right)\\&-\ln\left(\frac{P_{Dt}^{\sigma-1}}{W_t}\frac{1-\frac{\sigma-1}{\sigma}\gamma H_t}{1-\frac{\sigma-1}{\sigma}\gamma}P_{St}C_{St}\right)+(\sigma-1)\ln\left(\frac{1}{A_{Dt}}W_t{}^{1-\gamma}\left(E_tP_{Mt}^{\$}\right)^{\gamma}\right)-\left(\varepsilon-1-\gamma(\sigma-1)\right)\ln\left(\frac{P_{Dt}}{E_tP_{Mt}^{\$}}\right)\end{aligned}$$

$$\begin{aligned}(\sigma-1)\varphi_{Mt}=&-(\sigma-1)p_{Dt}+w_t+\frac{\gamma\frac{\sigma-1}{\sigma}H}{1-\gamma\frac{\sigma-1}{\sigma}H}\eta_t-c_{St}-p_{St}+(\sigma-1)(1-\gamma)w_t+(\sigma-1)\gamma(p_{Mt}^{\$}+e_t)-(\sigma-1)a_{Dt}\\&+(\varepsilon-1-\gamma(\sigma-1)\left(p_{Mt}^{\$}+e_t-p_{Dt}\right)\end{aligned}$$

$$\begin{aligned}(\sigma-1)\varphi_{Mt}=&-(\sigma-1)\left(p_{Dt}-p_{St}-(1-\gamma)\left(w_t-p_{St}\right)-\gamma\left(p_{Mt}^{\$}+e_t-p_{St}\right)+a_{Dt}\right)\\&-\left(c_{St}-(w_t-p_{St})-\frac{\gamma\frac{\sigma-1}{\sigma}H}{1-\gamma\frac{\sigma-1}{\sigma}H}\eta_t\right)+(\varepsilon-1-\gamma(\sigma-1)\left(p_{Mt}^{\$}+e_t-p_{Dt}\right)\end{aligned}$$

Therefore, the linearized system is given by:

$$
\begin{aligned}
tb_t &= e_t + p^{\$}_{Xt} - \eta_t + p_{St} + c_{St} \\
w_t &= -\frac{\mu\gamma H}{\chi_3 - \mu\gamma H}\eta_t + p_{St} + c_{St} \\
p_{Dt} &= -a_{Dt} + (1-\gamma)w_t + \gamma p_{Dt} - \frac{\gamma}{\varepsilon-1}\left[\frac{1-\gamma\frac{\sigma-1}{\sigma}}{1-\gamma\frac{\sigma-1}{\sigma}H}\frac{H}{1-H}\right]\eta_t \\
p_{St} &= -a_{St} + (1-\mu)w_t + \mu p_{Dt} \\
\varphi_{Mt} &= -\frac{\varepsilon-1-\gamma(\sigma-1)}{(\sigma-1)(\varepsilon-1)}\frac{\left(1-\frac{\sigma-1}{\sigma}\gamma\right)H}{(1-H)\left(1-\frac{\sigma-1}{\sigma}\gamma H\right)} \\
(\sigma-1)\varphi_{Mt} &= -(\sigma-1)\left(p_{Dt} - p_{St} - (1-\gamma)\left(w_t - p_{St}\right) - \gamma\left(p^{\$}_{Mt} + e_t - p_{St}\right) + a_{Dt}\right) \\
&\quad - \left(c_{St} - (w_t - p_{St}) - \frac{\gamma\frac{\sigma-1}{\sigma}H}{1-\gamma\frac{\sigma-1}{\sigma}H}\eta_t\right) + (\varepsilon - 1 - \gamma(\sigma-1)\left(p^{\$}_{Mt} + e_t - p_{Dt}\right)
\end{aligned}
$$

General structure To obtain the general structure, we combine the equilibrium conditions in the following way. The price index for services yields an expression for real wages as a function of services productivity and the relative price of manufacturing goods:

$$
\begin{aligned}
p_{St} &= -a_{St} + (1-\mu)w_t + \mu p_{Dt} \\
0 &= -a_{St} + (1-\mu)\left(w_t - p_{St}\right) + \mu\left(p_{Dt} - p_{St}\right) \\
w_t - p_{St} &= \frac{1}{1-\mu}a_{St} - \frac{\mu}{1-\mu}\left(p_{Dt} - p_{St}\right)
\end{aligned}
$$

Given this expression for real wages, we can solve for manufacturing prices as a function of the shocks and η_t:

$$
\begin{aligned}
(1-\gamma)p_{Dt} &= -a_{Dt} + (1-\gamma)w_t - \frac{\gamma}{\varepsilon-1}\left[\frac{1-\gamma\frac{\sigma-1}{\sigma}}{1-\gamma\frac{\sigma-1}{\sigma}H}\frac{H}{1-H}\right]\eta_t \\
(1-\gamma)\left(p_{Dt} - p_{St}\right) &= -a_{Dt} + (1-\gamma)\left(w_t - p_{St}\right) - \frac{\gamma}{\varepsilon-1}\left[\frac{1-\gamma\frac{\sigma-1}{\sigma}}{1-\gamma\frac{\sigma-1}{\sigma}H}\frac{H}{1-H}\right]\eta_t \\
&= -a_{Dt} + (1-\gamma)\left(\frac{1}{1-\mu}a_{St} - \frac{\mu}{1-\mu}\left(p_{Dt} - p_{St}\right)\right) - \frac{\gamma}{\varepsilon-1}\left[\frac{1-\gamma\frac{\sigma-1}{\sigma}}{1-\gamma\frac{\sigma-1}{\sigma}H}\frac{H}{1-H}\right]\eta_t \\
p_{Dt} &= a_{St} - \frac{1-\mu}{1-\gamma}a_{Dt} - \underbrace{\frac{1-\mu}{1-\gamma}\frac{\gamma}{\varepsilon-1}\frac{1}{1-H}\left[\frac{\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H}{1-\gamma\frac{\sigma-1}{\sigma}H}\right]}_{\equiv\nu_{p\eta}}\eta_t
\end{aligned}
$$

Now, use the labor market clearing condition to express final consumption

$$\begin{aligned}
c_{St} &= w_t - p_{St} + \underbrace{\frac{\mu\gamma H}{\chi_3 - \mu\gamma H}}_{\equiv \nu_{l\eta}} \eta_t \\
&= \frac{1}{1-\mu} a_{St} - \frac{\mu}{1-\mu}\left(p_{Dt} - p_{St}\right) + \nu_{l\eta}\eta_t \\
&= \frac{1}{1-\mu} a_{St} - \frac{\mu}{1-\mu}\left(a_{St} - \frac{1-\mu}{1-\gamma} a_{Dt} - \nu_{p\eta}\eta_t\right) + \nu_{l\eta}\eta_t \\
&= a_{St} + \frac{\mu}{1-\gamma} a_{Dt} + \underbrace{\left(\nu_{l\eta} + \frac{\mu}{1-\mu}\nu_{p\eta}\right)}_{\equiv \nu_{cH}} \eta_t
\end{aligned}$$

To obtain the expenditure switching expression, combine the expression for how manufacturing prices respond to changes in openness and the labor market clearing condition to reduce the system:

$$\begin{aligned}
(\sigma-1)\varphi_{Mt} &= -(\sigma-1)\left(p_{Dt} - p_{St} - (1-\gamma)\left(w_t - p_{St}\right) - \gamma\left(p^{\$}_{Mt} + e_t - p_{St}\right) + a_{Dt}\right) \\
&\quad - \left(c_{St} - (w_t - p_{St}) - \frac{\gamma\frac{\sigma-1}{\sigma}H}{1-\gamma\frac{\sigma-1}{\sigma}H}\eta_t\right) + (\varepsilon - 1 - \gamma(\sigma-1)\left(p^{\$}_{Mt} + e_t - p_{Dt}\right) \\
&= -(\sigma-1)\Bigg(p_{Dt} - p_{St} - (1-\gamma)\left(p_{Dt} - p_{St}\right) - a_{Dt} - \frac{\gamma}{\varepsilon-1}\left[\frac{1-\gamma\frac{\sigma-1}{\sigma}}{1-\gamma\frac{\sigma-1}{\sigma}H}\frac{H}{1-H}\right]\eta_t \\
&\qquad - \gamma\left(p^{\$}_{Mt} + e_t - p_{St}\right) + a_{Dt}\Bigg) \\
&\quad - \left(c_{St} - (w_t - p_{St}) - \frac{\gamma\frac{\sigma-1}{\sigma}H}{1-\gamma\frac{\sigma-1}{\sigma}H}\eta_t\right) + (\varepsilon - 1 - \gamma(\sigma-1)\left(p^{\$}_{Mt} + e_t - p_{Dt}\right) \\
&= -(\sigma-1)\gamma\left(p_{Dt} - p_{St} - \left(p^{\$}_{Mt} + e_t - p_{St}\right)\right) - \frac{\gamma(\sigma-1)}{\varepsilon-1}\left[\frac{1-\gamma\frac{\sigma-1}{\sigma}}{1-\gamma\frac{\sigma-1}{\sigma}H}\frac{H}{1-H}\right]\eta_t \\
&\quad - \left(\nu_{l\eta} - \frac{\gamma\frac{\sigma-1}{\sigma}H}{1-\gamma\frac{\sigma-1}{\sigma}H}\right)\eta_t + (\varepsilon - 1 - \gamma(\sigma-1)\left(p^{\$}_{Mt} + e_t - p_{Dt}\right) \\
&= (\varepsilon-1)\left(p^{\$}_{Mt} + e_t - p_{St} - (p_{Dt} - p_{St})\right) - \frac{\gamma(\sigma-1)}{\varepsilon-1}\left[\frac{1-\gamma\frac{\sigma-1}{\sigma}}{1-\gamma\frac{\sigma-1}{\sigma}H}\frac{H}{1-H}\right]\eta_t - \left(\nu_{l\eta} - \frac{\gamma\frac{\sigma-1}{\sigma}H}{1-\gamma\frac{\sigma-1}{\sigma}H}\right)\eta_t
\end{aligned}$$

Now, note that:

$$\begin{aligned}
\frac{\gamma(\sigma-1)}{\varepsilon-1}\left[\frac{1-\gamma\frac{\sigma-1}{\sigma}}{1-\gamma\frac{\sigma-1}{\sigma}H}\frac{H}{1-H}\right]\eta_t &= \frac{(1-\gamma)\gamma(\sigma-1)}{\gamma(1-\mu)}\nu_{p\eta}\eta_t \\
-\frac{\varepsilon-1-\gamma(\sigma-1)}{(\sigma-1)(\varepsilon-1)}\frac{\left(1-\frac{\sigma-1}{\sigma}\gamma\right)H}{(1-H)\left(1-\frac{\sigma-1}{\sigma}\gamma H\right)}\eta_t &= -\frac{1-\gamma}{\gamma(1-\mu)}\frac{\varepsilon-1-\gamma(\sigma-1)}{(\sigma-1)}\nu_{p\eta}\eta_t \\
\frac{\gamma\frac{\sigma-1}{\sigma}H}{1-\gamma\frac{\sigma-1}{\sigma}H}\eta_t &= \frac{1-\gamma}{\gamma(1-\mu)}(\varepsilon-1)(1-\eta)\frac{\gamma\frac{\sigma-1}{\sigma}H}{1-\gamma\frac{\sigma-1}{\sigma}H}\nu_{p\eta}\eta_t
\end{aligned}$$

Then, we have that:

$$
\begin{aligned}
(\sigma-1)\frac{1-\gamma}{\gamma(1-\mu)}\frac{\varepsilon-1-\gamma(\sigma-1)}{(\sigma-1)}\nu_{p\eta}\eta_t &= -(\varepsilon-1)\left(p^{\$}_{Mt}+e_t-p_{St}-(p_{Dt}-p_{St})\right)+\frac{(1-\gamma)\gamma(\sigma-1)}{\gamma(1-\mu)}\nu_{p\eta}\eta_t \\
&\quad+\left(\nu_{l\eta}-\frac{1-\gamma}{\gamma(1-\mu)}(\varepsilon-1)(1-\eta)\frac{\gamma^{\frac{\sigma-1}{\sigma}}H}{1-\gamma^{\frac{\sigma-1}{\sigma}}H}\nu_{p\eta}\right)\eta_t \\
\frac{1-\gamma}{\gamma(1-\mu)}(\varepsilon-1)\nu_{p\eta}\eta_t &= -(\varepsilon-1)\left(p^{\$}_{Mt}+e_t-p_{St}-(p_{Dt}-p_{St})\right) \\
&\quad+\left(\nu_{l\eta}-\frac{1-\gamma}{\gamma(1-\mu)}(\varepsilon-1)(1-\eta)\frac{\gamma^{\frac{\sigma-1}{\sigma}}H}{1-\gamma^{\frac{\sigma-1}{\sigma}}H}\nu_{p\eta}\right)\eta_t \\
\left(\frac{1-\gamma}{\gamma(1-\mu)}(\varepsilon-1)\left(\frac{\left(1-\gamma^{\frac{\sigma-1}{\sigma}}\right)H}{1-\gamma^{\frac{\sigma-1}{\sigma}}H}\right)\nu_{p\eta}-\nu_{l\eta}\right)\eta_t &= -(\varepsilon-1)\left(p^{\$}_{Mt}+e_t-p_{St}-(p_{Dt}-p_{St})\right) \\
\left((\varepsilon-1)(1-H)\nu_{p\eta}+\frac{1-\gamma}{\gamma(1-\mu)}(\varepsilon-1)(1-H)\left(\frac{\left(1-\gamma^{\frac{\sigma-1}{\sigma}}\right)H}{1-\gamma^{\frac{\sigma-1}{\sigma}}H}\right)\nu_{p\eta}-(1-H)\nu_{l\eta}\right)\eta_t & \\
&= -(\varepsilon-1)(1-H)\left(p^{\$}_{Mt}+e_t-p_{St}-a_{St}+\frac{1-\mu}{1-\gamma}a_{Dt}\right) \\
\left((\varepsilon-1)(1-H)\nu_{p\eta}+H-(1-H)\nu_{l\eta}\right)\eta_t &= -(\varepsilon-1)(1-H)\left(p^{\$}_{Mt}+e_t-p_{St}-(p_{Dt}-p_{St})\right)
\end{aligned}
$$

where we used the expression for $\nu_{p\eta}$. Therefore, we have that the expenditure switching expression becomes.

$$
\eta_t=-\frac{(1-H)(\varepsilon-1)}{H-(1-H)\nu_{l\eta}+(1-H)(\varepsilon-1)\nu_{p\eta}}\left[p^{\$}_{Mt}+q_t-a_{St}+\frac{1-\mu}{1-\gamma}a_{Dt}\right]
$$

D.4 Heterogeneous Firms under Monopolistic Competition and IRS Importing

In this section we derive the equilibrium system for the model with heterogeneous producers that compete under monopolistic competition.

Rewriting in terms of H_t The non-linear equilibrium goods and labor markets block can be fully re-written in terms of H_t. Using the definition of H_t, we can write:

$$H_t = \left[\frac{1 - \frac{\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa}\left(\frac{1}{\kappa}-\frac{1}{\kappa-(\sigma-1)}\right)+\frac{1}{\sigma-1-\kappa}}{\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa}\left(\frac{1}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}-\frac{1}{\kappa-(\sigma-1)}\right)+\frac{1}{\kappa-(\sigma-1)}}}{1-\gamma\frac{\sigma-1}{\sigma}\frac{\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa}\left(\frac{1}{\kappa}-\frac{1}{\kappa-(\sigma-1)}\right)+\frac{1}{\sigma-1-\kappa}}{\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa}\left(\frac{1}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}-\frac{1}{\kappa-(\sigma-1)}\right)+\frac{1}{\kappa-(\sigma-1)}}} \right]$$

$$\begin{aligned}
\left(\frac{1-\gamma\frac{\sigma-1}{\sigma}H_t}{1-H_t}\right)\left(\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa}\left(\frac{1}{\kappa}-\frac{1}{\kappa-(\sigma-1)}\right)+\frac{1}{\sigma-1-\kappa}\right)&\\
=\left(\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa}\left(\frac{1}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}-\frac{1}{\kappa-(\sigma-1)}\right)+\frac{1}{\kappa-(\sigma-1)}\right)&\\
\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa} = -\frac{\frac{1}{\kappa-(\sigma-1)}\left(\frac{\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H_t}{1-H_t}\right)}{\frac{1-\gamma\frac{\sigma-1}{\sigma}H_t}{1-H_t}\left(\frac{1}{\kappa}-\frac{1}{\kappa-(\sigma-1)}\right)-\left(\frac{1}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}-\frac{1}{\kappa-(\sigma-1)}\right)}&\\
= -\frac{\frac{1}{\kappa-(\sigma-1)}\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H_t}{\left(\frac{1}{\kappa}-\frac{1}{\kappa-(\sigma-1)}\right)\left(1-\gamma\frac{\sigma-1}{\sigma}H_t\right)-\left(\frac{1}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}-\frac{1}{\kappa-(\sigma-1)}\right)(1-H_t)}&
\end{aligned}$$

Now define $\kappa_1 \equiv \frac{\frac{1}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}-\frac{1}{\kappa-(\sigma-1)}}{\frac{1}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}-\frac{1}{\kappa}}$ and $\kappa_2 \equiv \frac{\frac{1}{\kappa-(\sigma-1)}}{\frac{1}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}-\frac{1}{\kappa}}$, such that:

$$\begin{aligned}
\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa} &= \frac{\kappa_2\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H_t}{(1-\kappa_1)\left(1-\gamma\frac{\sigma-1}{\sigma}H_t\right)-\kappa_1(1-H_t)}\\
&= \frac{\kappa_2\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H_t}{1-H_t+(1-\kappa_1)\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H_t}
\end{aligned}$$

Aggregate manufacturing prices are given by

$$P_{Dt}^{\sigma-1} = \left(\frac{\sigma}{\sigma-1}\omega^{\frac{\gamma}{1-\varepsilon}}\frac{1}{A_{Dt}}\frac{{W_t}^{1-\gamma}{P_{Dt}}^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right)^{\sigma-1}\frac{\underline{\varphi}^{\sigma-1-\kappa}}{\kappa\underline{\varphi}^{\kappa}}\left[\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa}\left(\frac{1}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}-\frac{1}{\kappa-(\sigma-1)}\right)+\frac{1}{\kappa-(\sigma-1)}\right]^{-1}$$

Using the expresion for $\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)$, we obtain:

$$
\begin{aligned}
&\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa}\left(\frac{1}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}-\frac{1}{\kappa-(\sigma-1)}\right)+\frac{1}{\kappa-(\sigma-1)} \\
&=-\left(\frac{1}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}-\frac{1}{\kappa-(\sigma-1)}\right)\frac{\frac{1}{\kappa-(\sigma-1)}\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H_t}{\left(\frac{1}{\kappa}-\frac{1}{\kappa-(\sigma-1)}\right)\left(1-\gamma\frac{\sigma-1}{\sigma}H_t\right)-\left(\frac{1}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}-\frac{1}{\kappa-(\sigma-1)}\right)(1-H_t)}+\frac{1}{\kappa-(\sigma-1)} \\
&=-\frac{\frac{1}{\sigma-1-\kappa}\left(\frac{1}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}-\frac{1}{\kappa}\right)\left(1-\gamma\frac{\sigma-1}{\sigma}H_t\right)}{\left(\frac{1}{\kappa}-\frac{1}{\kappa-(\sigma-1)}\right)\left(1-\gamma\frac{\sigma-1}{\sigma}H_t\right)-\left(\frac{1}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}-\frac{1}{\kappa-(\sigma-1)}\right)(1-H_t)} \\
&=\frac{\frac{1}{\sigma-1-\kappa}\left(1-\gamma\frac{\sigma-1}{\sigma}H_t\right)}{\kappa_1(1-H_t)+(1-\kappa_1)\left(1-\gamma\frac{\sigma-1}{\sigma}H_t\right)} \\
&=\frac{\frac{1}{\sigma-1-\kappa}\left(1-\gamma\frac{\sigma-1}{\sigma}H_t\right)}{1-H_t+(1-\kappa_1)\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H_t}
\end{aligned}
$$

such that aggregate manufacturing prices can be written as:

$$
P_{Dt}^{\sigma-1}=\left(\frac{\sigma}{\sigma-1}\omega^{\frac{\gamma}{1-\varepsilon}}\frac{1}{A_{Dt}}\frac{{W_t}^{1-\gamma}{P_{Dt}}^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right)^{\sigma-1}\frac{\underline{\varphi}^{\sigma-1-\kappa}}{\frac{\kappa}{\kappa-(\sigma-1)}\underline{\varphi}^{\kappa}}\left(\frac{1-H_t+(1-\kappa_1)\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H_t}{1-\gamma\frac{\sigma-1}{\sigma}H_t}\right)
$$

Next, we rewrite the productivity cut-off relation:

$$
\begin{aligned}
\Phi_{Mt}&=\left(\frac{\sigma}{\sigma-1}\right)^{\frac{\sigma}{\sigma-1}}\left(\frac{\gamma(1-\omega)^{\frac{\gamma(\sigma-1)}{\varepsilon-1}}}{\varepsilon-1}\frac{{P_{Dt}}^{\sigma}(X_{St}+Q_{Dt})}{fW_t}\right)^{-\frac{1}{\sigma-1}}\frac{1}{A_{Dt}}\frac{{W_t}^{1-\gamma}\left(E_tP_{Mt}^{\$}\right)^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\left[\frac{\omega}{1-\omega}\left(\frac{P_{Dt}}{E_tP_{Mt}^{\$}}\right)^{1-\varepsilon}\right]^{\frac{\varepsilon-1-\gamma(\sigma-1)}{(\sigma-1)(\varepsilon-1)}} \\
&=\left(\frac{\sigma}{\sigma-1}\right)^{\frac{\sigma}{\sigma-1}}\left(\frac{\gamma(1-\omega)^{\frac{\gamma(\sigma-1)}{\varepsilon-1}}}{\varepsilon-1}\frac{P_{Dt}^{\sigma-1}}{fW_t}P_{Dt}(X_{St}+Q_{Dt})\right)^{-\frac{1}{\sigma-1}}\frac{1}{A_{Dt}}\frac{{W_t}^{1-\gamma}\left(E_tP_{Mt}^{\$}\right)^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\left[\frac{\omega}{1-\omega}\left(\frac{P_{Dt}}{E_tP_{Mt}^{\$}}\right)^{1-\varepsilon}\right]^{\frac{\varepsilon-1-\gamma(\sigma-1)}{(\sigma-1)(\varepsilon-1)}}
\end{aligned}
$$

$$
\Phi_{Mt}^{\sigma-1}=\left(\frac{\sigma}{\sigma-1}\right)^{\sigma}\left(\frac{\gamma(1-\omega)^{\frac{\gamma(\sigma-1)}{\varepsilon-1}}}{\varepsilon-1}\frac{P_{Dt}^{\sigma-1}}{fW_t}\frac{1-\frac{\sigma-1}{\sigma}\gamma H_t}{1-\frac{\sigma-1}{\sigma}\gamma}\mu P_{St}C_{St}\right)^{-1}\left(\frac{1}{A_{Dt}}\frac{{W_t}^{1-\gamma}\left(E_tP_{Mt}^{\$}\right)^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right)^{\sigma-1}\left[\frac{\omega}{1-\omega}\left(\frac{P_{Dt}}{E_tP_{Mt}^{\$}}\right)^{1-\varepsilon}\right]^{\frac{\varepsilon-1-\gamma(\sigma}{\varepsilon-1}}
$$

Given this expression for manufacturing prices and the productivity cut-off, the non-linear goods and labor markets block is given by:

$$
\begin{aligned}
TB_t &= E_t P^{\$}_{Xt} X - \mu\gamma H_t P_{St} C_{St} \\
W_t L &= X_4 \left(\chi_4 - \mu\gamma H_t\right) P_{St} C_{St} \\
P_{Dt}^{\sigma-1} &= \left(\frac{\sigma}{\sigma-1}\omega^{\frac{\gamma}{1-\varepsilon}}\frac{1}{A_{Dt}}\frac{{W_t}^{1-\gamma}{P_{Dt}}^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right)^{\sigma-1}\frac{\underline{\varphi}^{\sigma-1-\kappa}}{\frac{\kappa}{\kappa-(\sigma-1)}\underline{\varphi}^{\kappa}}\left(\frac{1-H_t+(1-\kappa_1)\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H_t}{1-\gamma\frac{\sigma-1}{\sigma}H_t}\right) \\
\mathrm{P}_{St} &= \frac{1}{A_{St}}\frac{{W_t}^{1-\mu}{P_{Dt}}^{\mu}}{(1-\mu)^{1-\mu}\mu^{\mu}} \\
\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa} &= \frac{\kappa_2\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H_t}{1-H_t+(1-\kappa_1)\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H_t} \\
\Phi_{Mt}^{\sigma-1} &= \left(\frac{\sigma}{\sigma-1}\right)^{\sigma}\left(\frac{\gamma(1-\omega)^{\frac{\gamma(\sigma-1)}{\varepsilon-1}}}{\varepsilon-1}\frac{P_{Dt}^{\sigma-1}}{fW_t}\frac{1-\frac{\sigma-1}{\sigma}\gamma H_t}{1-\frac{\sigma-1}{\sigma}\gamma}\mu P_{St}C_{St}\right)^{-1} \\
&\qquad \left(\frac{1}{A_{Dt}}\frac{{W_t}^{1-\gamma}\left(E_t P^{\$}_{Mt}\right)^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\right)^{\sigma-1}\left[\frac{\omega}{1-\omega}\left(\frac{P_{Dt}}{E_t P^{\$}_{Mt}}\right)^{1-\varepsilon}\right]^{\frac{\varepsilon-1-\gamma(\sigma-1)}{\varepsilon-1}}
\end{aligned}
$$

First-order linearization Linearizing the services price index, the labor market clearing condition and the trade balance condition is immediate. The linearized manufacturing price index is obtained by:

$$
\begin{aligned}
\ln\left(P_{Dt}\right) &= \ln\left(\frac{\sigma}{\sigma-1}\frac{\omega^{\frac{\gamma}{1-\varepsilon}}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\frac{\underline{\varphi}^{\sigma-1-\kappa}}{\frac{\kappa}{\kappa-(\sigma-1)}\underline{\varphi}^{\kappa}}\right)+\ln\left(\frac{W_t^{1-\gamma}P_{Dt}^{\gamma}}{A_{Dt}}\right)-\frac{1}{\sigma-1}\ln\left(\frac{1-H_t+(1-\kappa_1)\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H_t}{1-\gamma\frac{\sigma-1}{\sigma}H_t}\right) \\
p_{Dt} &= -a_{Dt}+(1-\gamma)w_t+\gamma p_{Dt}+\frac{1}{\sigma-1}\left(\frac{-H+(1-\kappa_1)\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H}{1-H+(1-\kappa_1)\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H}+\frac{\gamma\frac{\sigma-1}{\sigma}H}{1-\gamma\frac{\sigma-1}{\sigma}H}\right)\eta_t \\
&= -a_{Dt}+(1-\gamma)w_t+\gamma p_{Dt}+\frac{1}{\sigma-1}\left(\frac{(1-\kappa_1)\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H-\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H}{\left(1-H+(1-\kappa_1)\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H\right)\left(1-\gamma\frac{\sigma-1}{\sigma}H\right)}\right)\eta_t \\
&= -a_{Dt}+(1-\gamma)w_t+\gamma p_{Dt}-\frac{1}{\sigma-1}\left(\frac{\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H}{1-\gamma\frac{\sigma-1}{\sigma}H}\frac{\kappa_1}{1-H+(1-\kappa_1)\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H}\right)\eta_t
\end{aligned}
$$

Linearizing the relation between the productivity cut-off and H_t is given by:

$$
\begin{aligned}
-\left(\kappa-(\sigma-1)\right)\ln\left(\frac{\Phi_{Mt}}{\underline{\varphi}}\right) &= \ln\left(\frac{\kappa_2\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H_t}{1-H_t+(1-\kappa_1)\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H_t}\right) \\
-\left(\kappa-(\sigma-1)\right)\varphi_{Mt} &= \left(1-\frac{(1-\kappa_1)(1-\gamma\frac{\sigma-1}{\sigma}H-H)}{1-H+(1-\kappa_1)\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H}\right)\eta_t \\
\varphi_{Mt} &= -\frac{1}{\kappa-(\sigma-1)}\frac{1}{1-H+(1-\kappa_1)\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H}
\end{aligned}
$$

Next, the linearized cut-off equation is given by:

$$
\begin{aligned}
(\sigma-1)\ln\Phi_{Mt} &= \ln\left(\left(\frac{\sigma}{\sigma-1}\right)^{\sigma}\left(\frac{\gamma(1-\omega)^{\frac{\gamma(\sigma-1)}{\varepsilon-1}}}{\varepsilon-1}\frac{\mu}{f\left((1-\gamma)^{1-\gamma}\gamma^{\gamma}\right)^{1-\sigma}}\right)^{-1}\left(\frac{\omega}{1-\omega}\right)^{\frac{\varepsilon-1-\gamma(\sigma-1)}{\varepsilon-1}}\right) \\
&\quad -\ln\left(\frac{P_{Dt}^{\sigma-1}}{W_t}\frac{1-\frac{\sigma-1}{\sigma}\gamma H_t}{1-\frac{\sigma-1}{\sigma}\gamma}P_{St}C_{St}\right)+(\sigma-1)\ln\left(\frac{1}{A_{Dt}}W_t^{1-\gamma}\left(E_tP_{Mt}^{\$}\right)^{\gamma}\right)-(\varepsilon-1-\gamma(\sigma-1))\ln\left(\frac{P_{Dt}}{E_tP_{Mt}^{\$}}\right) \\
(\sigma-1)\varphi_{Mt} &= -(\sigma-1)p_{Dt}+w_t+\frac{\gamma\frac{\sigma-1}{\sigma}H}{1-\gamma\frac{\sigma-1}{\sigma}H}\eta_t-c_{St}-p_{St}+(\sigma-1)(1-\gamma)w_t+(\sigma-1)\gamma(p_{Mt}^{\$}+e_t)-(\sigma-1)a_{Dt} \\
&\quad +(\varepsilon-1-\gamma(\sigma-1)\left(p_{Mt}^{\$}+e_t-p_{Dt}\right) \\
(\sigma-1)\varphi_{Mt} &= -(\sigma-1)\left(p_{Dt}-p_{St}-(1-\gamma)\left(w_t-p_{St}\right)-\gamma\left(p_{Mt}^{\$}+e_t-p_{St}\right)+a_{Dt}\right) \\
&\quad -\left(c_{St}-(w_t-p_{St})-\frac{\gamma\frac{\sigma-1}{\sigma}H}{1-\gamma\frac{\sigma-1}{\sigma}H}\eta_t\right)+(\varepsilon-1-\gamma(\sigma-1)\left(p_{Mt}^{\$}+e_t-p_{Dt}\right)
\end{aligned}
$$

Therefore, the linearized system is given by:

$$
\begin{aligned}
tb_t &= e_t+p_{Xt}^{\$}-\eta_t+p_{St}+c_{St} \\
w_t &= -\frac{\mu\gamma H}{\chi_4-\mu\gamma H}\eta_t+p_{St}+c_{St} \\
p_{Dt} &= -a_{Dt}+(1-\gamma)w_t+\gamma p_{Dt}-\frac{1}{\sigma-1}\left(\frac{\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H}{1-\gamma\frac{\sigma-1}{\sigma}H}\frac{\kappa_1}{1-H+(1-\kappa_1)\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H}\right)\eta_t \\
p_{St} &= -a_{St}+(1-\mu)w_t+\mu p_{Dt} \\
\varphi_{Mt} &= -\frac{1}{\kappa-(\sigma-1)}\frac{1}{1-H+(1-\kappa_1)\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H}\eta_t \\
(\sigma-1)\varphi_{Mt} &= -(\sigma-1)\left(p_{Dt}-p_{St}-(1-\gamma)\left(w_t-p_{St}\right)-\gamma\left(p_{Mt}^{\$}+e_t-p_{St}\right)+a_{Dt}\right) \\
&\quad -\left(c_{St}-(w_t-p_{St})-\frac{\gamma\frac{\sigma-1}{\sigma}H}{1-\gamma\frac{\sigma-1}{\sigma}H}\eta_t\right)+(\varepsilon-1-\gamma(\sigma-1)\left(p_{Mt}^{\$}+e_t-p_{Dt}\right)
\end{aligned}
$$

General structure To obtain the general structure, we combine the equilibrium conditions in the following way. The price index for services yields an expression for real wages as a function of services productivity and the relative price of manufacturing goods:

$$
\begin{aligned}
p_{St} &= -a_{St}+(1-\mu)w_t+\mu p_{Dt} \\
0 &= -a_{St}+(1-\mu)\left(w_t-p_{St}\right)+\mu\left(p_{Dt}-p_{St}\right) \\
w_t-p_{St} &= \frac{1}{1-\mu}a_{St}-\frac{\mu}{1-\mu}\left(p_{Dt}-p_{St}\right)
\end{aligned}
$$

Given this expression for real wages, we can solve for manufacturing prices as a function of the shocks and η_t:

$$(1-\gamma)p_{Dt} = -a_{Dt} + (1-\gamma)w_t - \frac{1}{\sigma-1}\left(\frac{\left(1-\gamma^{\frac{\sigma-1}{\sigma}}\right)H}{1-\gamma^{\frac{\sigma-1}{\sigma}}H}\frac{\kappa_1}{1-H+(1-\kappa_1)\left(1-\gamma^{\frac{\sigma-1}{\sigma}}\right)H}\right)\eta_t$$

$$\begin{aligned}
(1-\gamma)\left(p_{Dt}-p_{St}\right) &= -a_{Dt} + (1-\gamma)\left(w_t - p_{St}\right) - \frac{\gamma}{\varepsilon-1}\frac{\varepsilon-1}{\gamma(\sigma-1)}\left(\frac{\left(1-\gamma^{\frac{\sigma-1}{\sigma}}\right)H}{1-\gamma^{\frac{\sigma-1}{\sigma}}H}\frac{\kappa_1}{1-H+(1-\kappa_1)\left(1-\gamma^{\frac{\sigma-1}{\sigma}}\right)H}\right)\eta_t \\
&= -a_{Dt} + (1-\gamma)\left(\frac{1}{1-\mu}a_{St} - \frac{\mu}{1-\mu}\left(p_{Dt}-p_{St}\right)\right) \\
&\quad - \frac{\gamma}{(\varepsilon-1)(1-H)}\frac{\varepsilon-1}{\gamma(\sigma-1)}\left(\frac{\left(1-\gamma^{\frac{\sigma-1}{\sigma}}\right)H}{1-\gamma^{\frac{\sigma-1}{\sigma}}H}\frac{\kappa_1}{1-H+(1-\kappa_1)\left(1-\gamma^{\frac{\sigma-1}{\sigma}}\right)H}\right)\eta_t \\
p_{Dt} &= a_{St} - \frac{1-\mu}{1-\gamma}a_{Dt} - \underbrace{\frac{1-\mu}{1-\gamma}\frac{\gamma}{(\varepsilon-1)(1-H)}\frac{\varepsilon-1}{\gamma(\sigma-1)}\left(\frac{\left(1-\gamma^{\frac{\sigma-1}{\sigma}}\right)H}{1-\gamma^{\frac{\sigma-1}{\sigma}}H}\frac{\kappa_1(1-H)}{1-H+(1-\kappa_1)\left(1-\gamma^{\frac{\sigma-1}{\sigma}}\right)H}\right)}_{\equiv\nu_{p\eta}}\eta_t
\end{aligned}$$

Now, use the labor market clearing condition to express final consumption

$$\begin{aligned}
c_{St} &= w_t - p_{St} + \underbrace{\frac{\mu\gamma H}{\chi_4-\mu\gamma H}}_{\equiv\nu_{l\eta}}\eta_t \\
&= \frac{1}{1-\mu}a_{St} - \frac{\mu}{1-\mu}\left(p_{Dt}-p_{St}\right) + \nu_{l\eta}\eta_t \\
&= \frac{1}{1-\mu}a_{St} - \frac{\mu}{1-\mu}\left(a_{St} - \frac{1-\mu}{1-\gamma}a_{Dt} - \nu_{p\eta}\eta_t\right) + \nu_{l\eta}\eta_t \\
&= a_{St} + \frac{\mu}{1-\gamma}a_{Dt} + \underbrace{\left(\nu_{l\eta} + \frac{\mu}{1-\mu}\nu_{p\eta}\right)}_{\equiv\nu_{cH}}\eta_t
\end{aligned}$$

To obtain the expenditure switching expression, combine the expression for how manufacturing prices respond to changes in openness and the labor market clearing condition to reduce the system:

$$
\begin{aligned}
(\sigma-1)\varphi_{Mt} &= -(\sigma-1)\left(p_{Dt}-p_{St}-(1-\gamma)\left(w_t-p_{St}\right)-\gamma\left(p^{\$}_{Mt}+e_t-p_{St}\right)+a_{Dt}\right) \\
&\quad -\left(c_{St}-(w_t-p_{St})-\frac{\gamma\frac{\sigma-1}{\sigma}H}{1-\gamma\frac{\sigma-1}{\sigma}H}\eta_t\right)+(\varepsilon-1-\gamma(\sigma-1)\left(p^{\$}_{Mt}+e_t-p_{Dt}\right) \\
&= -(\sigma-1)\Bigg(p_{Dt}-p_{St}-(1-\gamma)\left(p_{Dt}-p_{St}\right)-a_{Dt}-\frac{1}{\sigma-1}\left(\frac{\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H}{1-\gamma\frac{\sigma-1}{\sigma}H}\frac{\kappa_1}{1-H+(1-\kappa_1)\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H}\right)\eta_t \\
&\qquad\qquad -\gamma\left(p^{\$}_{Mt}+e_t-p_{St}\right)+a_{Dt}\Bigg) \\
&\quad -\left(c_{St}-(w_t-p_{St})-\frac{\gamma\frac{\sigma-1}{\sigma}H}{1-\gamma\frac{\sigma-1}{\sigma}H}\eta_t\right)+(\varepsilon-1-\gamma(\sigma-1)\left(p^{\$}_{Mt}+e_t-p_{Dt}\right) \\
&= -(\sigma-1)\gamma\left(p_{Dt}-p_{St}-\left(p^{\$}_{Mt}+e_t-p_{St}\right)\right)+\left(\frac{\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H}{1-\gamma\frac{\sigma-1}{\sigma}H}\frac{\kappa_1}{1-H+(1-\kappa_1)\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H}\right)\eta_t \\
&\quad -\left(\nu_{l\eta}-\frac{\gamma\frac{\sigma-1}{\sigma}H}{1-\gamma\frac{\sigma-1}{\sigma}H}\right)\eta_t+(\varepsilon-1-\gamma(\sigma-1)\left(p^{\$}_{Mt}+e_t-p_{Dt}\right) \\
&= (\varepsilon-1)\left(p^{\$}_{Mt}+e_t-p_{St}-(p_{Dt}-p_{St})\right)+\left(\frac{\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H}{1-\gamma\frac{\sigma-1}{\sigma}H}\frac{\kappa_1}{1-H+(1-\kappa_1)\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H}\right)\eta_t \\
&\quad -\left(\nu_{l\eta}-\frac{\gamma\frac{\sigma-1}{\sigma}H}{1-\gamma\frac{\sigma-1}{\sigma}H}\right)\eta_t
\end{aligned}
$$

Now, note that:

$$
\begin{aligned}
-\frac{\sigma-1}{\kappa-(\sigma-1)}\frac{1}{1-H+(1-\kappa_1)\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H}\eta_t &= -\frac{1-\gamma}{\gamma(1-\mu)}\frac{\gamma(\sigma-1)}{\frac{\kappa_1}{\sigma-1}\left(\kappa-(\sigma-1)\right)}\frac{1-\gamma\frac{\sigma-1}{\sigma}H}{\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H}\nu_{p\eta}\eta_t \\
\frac{\kappa_1}{1-H+(1-\kappa_1)\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H}\frac{\left(1-\frac{\sigma-1}{\sigma}\gamma\right)H}{\left(1-\frac{\sigma-1}{\sigma}\gamma H\right)}\eta_t &= \frac{1-\gamma}{\gamma(1-\mu)}\gamma(\sigma-1)\nu_{p\eta}\eta_t \\
\frac{\gamma\frac{\sigma-1}{\sigma}H}{1-\gamma\frac{\sigma-1}{\sigma}H}\eta_t &= \frac{1-\gamma}{\gamma(1-\mu)}\gamma(\sigma-1)\frac{\gamma\frac{\sigma-1}{\sigma}H}{1-\gamma\frac{\sigma-1}{\sigma}H}\frac{1-H+(1-\kappa_1)\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H}{\kappa_1}\nu_{p\eta}\eta_t
\end{aligned}
$$

Using these expressions, we get:

$$
\begin{aligned}
&-\frac{1-\gamma}{\gamma(1-\mu)}\frac{\gamma(\sigma-1)}{\frac{\kappa_1}{\sigma-1}\left(\kappa\left(\sigma-1\right)\right)}\frac{1-\gamma\frac{\sigma-1}{\sigma}H}{\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H}\nu_{p\eta}\eta_t \\
&\qquad = (\varepsilon-1)\left(p^{\$}_{Mt}+e_t-p_{St}-(p_{Dt}-p_{St})\right)+\frac{1-\gamma}{\gamma(1-\mu)}\gamma(\sigma-1)\nu_{p\eta}\eta_t \\
&\qquad\qquad -\left(\nu_{l\eta}-\frac{1-\gamma}{\gamma(1-\mu)}\gamma(\sigma-1)\frac{\gamma\frac{\sigma-1}{\sigma}H}{1-\gamma\frac{\sigma-1}{\sigma}H}\frac{1-H+(1-\kappa_1)\left(1-\gamma\frac{\sigma-1}{\sigma}\right)H}{\kappa_1}\nu_{p\eta}\right)\eta_t
\end{aligned}
$$

Now, we use a change of variables and define ξ as the difference between κ and its smallest possible value such that the moments of the firm-size distribution still exist. Therefore, we define $\kappa=\xi\frac{(\sigma-1)(\varepsilon-1)}{(\varepsilon-1)-\gamma(\sigma-1)}$. Given this definition, we can re-write κ_1 and $1-\kappa_1$ as

$$\kappa_1 = \frac{\xi\gamma(\sigma-1)}{(\xi-1)(\varepsilon-1)+\gamma(\sigma-1)}, \qquad 1-\kappa_1 = (\xi-1)\frac{(\varepsilon-1)-\gamma(\sigma-1)}{(\xi-1)(\varepsilon-1)+\gamma(\sigma-1)}$$

Using these substitutions, we get:

$$
\begin{aligned}
&-\frac{(1-\gamma)\gamma(\sigma-1)}{\gamma(1-\mu)}\frac{(\varepsilon-1)-\gamma(\sigma-1)}{\xi\gamma(\sigma-1)}\frac{1-\gamma^{\frac{\sigma-1}{\sigma}}H}{\left(1-\gamma^{\frac{\sigma-1}{\sigma}}\right)H}\nu_{p\eta}\eta_t \\
&\quad = (\varepsilon-1)\left(p^{\$}_{Mt}+e_t-p_{St}-(p_{Dt}-p_{St})\right)+\frac{(1-\gamma)\gamma(\sigma-1)}{\gamma(1-\mu)}\nu_{p\eta}\eta_t \\
&\quad -\Bigg(\nu_{l\eta}-\frac{(1-\gamma)\gamma(\sigma-1)}{\gamma(1-\mu)}\frac{\gamma^{\frac{\sigma-1}{\sigma}}H}{1-\gamma^{\frac{\sigma-1}{\sigma}}H} \\
&\qquad \frac{(1-H)\left((1-\xi)(\varepsilon-1)+\gamma(\sigma-1)\right)+(1-\xi)\left(\varepsilon-1-\gamma(\sigma-1)\right)\left(1-\gamma^{\frac{\sigma-1}{\sigma}}\right)H}{\xi\gamma(\sigma-1)}\nu_{p\eta}\Bigg)\eta_t
\end{aligned}
$$

$$
\begin{aligned}
&\Bigg[\Bigg\{\frac{(1-\gamma)\gamma(\sigma-1)}{(1-\mu)\gamma} \\
&\quad\Bigg(\frac{(1-\gamma^{\frac{\sigma-1}{\sigma}}H)(\varepsilon-1-\gamma(\sigma-1))+(1-H)\left((\xi-1)(\varepsilon-1)+\gamma(\sigma-1)\right)+(\xi-1)(\varepsilon-1-\gamma(\sigma-1))\left(1-\gamma^{\frac{\sigma-1}{\sigma}}\right)H}{(1-\gamma^{\frac{\sigma-1}{\sigma}})H\xi\gamma(\sigma-1)} \\
&\qquad +1\Bigg)\Bigg\}\nu_{p\eta}-\nu_{l\eta}\Bigg]\eta_t = -(\varepsilon-1)\left(p^{\$}_{Mt}+e_t-p_{St}-(p_{Dt}-p_{St})\right) \\
&\left(\frac{(\varepsilon-1-\gamma(\sigma-1))\left(1+(\xi-1)\gamma^{\frac{\sigma-1}{\sigma}}H\right)+\xi\gamma(\sigma-1)H}{(1-H)\left((1-H)(\varepsilon-1)+\gamma(\sigma-1)\right)+(\xi-1)(\varepsilon-1-\gamma(\sigma-1))\left(1-\gamma^{\frac{\sigma-1}{\sigma}}\right)H}-\nu_{l\eta}\right)\eta_t \\
&= -(\varepsilon-1)\left(p^{\$}_{Mt}+e_t-p_{St}-(p_{Dt}-p_{St})\right)
\end{aligned}
$$

where we used the expression for $\nu_{p\eta}$. Therefore, we have that the expenditure switching expression becomes:

$$\eta_t = -\frac{(1-H)(\varepsilon-1)}{\zeta(H)-(1-H)\nu_{l\eta}+(1-H)(\varepsilon-1)\nu_{p\eta}}\left[p^{\$}_{Mt}+q_t-a_{St}+\frac{1-\mu}{1-\gamma}a_{Dt}\right]$$

$$\text{where} \qquad \zeta(H) \equiv \frac{(\varepsilon-1-\gamma(\sigma-1))\left(1+(\xi-1)\gamma^{\frac{\sigma-1}{\sigma}}H\right)+\xi\gamma(\sigma-1)H}{(1-H)\left((1-H)(\varepsilon-1)+\gamma(\sigma-1)\right)+(\xi-1)(\varepsilon-1-\gamma(\sigma-1))\left(1-\gamma^{\frac{\sigma-1}{\sigma}}\right)H}$$

E General equilibrium elasticities

E.1 Financial Autarky

In financial autarky the trade balance conditions imposes the following equality

$$c_{St}+\eta_t = q_t + p_{Xt}$$

and when we substitute in the solutions for c_{St} and q_t we have

$$c_{St} = a_{St}+\frac{1}{1-\gamma}(\mu-\nu_c)+\nu_c(p_{Xt}-p_{Mt}) \quad \text{where} \quad \nu_c = \frac{\nu_{cH}}{1+\nu_{cH}-\nu_{e\eta}}$$

E.1.1 Perfect competition

$$L_S = \left(1 + (1-\gamma)\frac{\sigma-1}{\sigma}\frac{1-\gamma^{\frac{\sigma-1}{\sigma}}\eta}{1-\gamma^{\frac{\sigma-1}{\sigma}}}\frac{\mu}{1-\mu}\right)^{-1} = \left(1 + (1-\gamma\eta)\frac{\mu}{1-\mu}\right)^{-1} = \frac{1-\mu}{1-\mu\gamma\eta}$$

and labor allocated to manufacturing

$$L_D = 1 - L_S = \mu\frac{1-\gamma\eta}{1-\mu\gamma\eta}$$

We use these allocations to compute the partial elasticities that compose the main elasticities of the models, starting with $\nu_{w\eta}$

$$\nu_{w\eta} = \frac{\gamma^{\frac{\sigma-1}{\sigma}}\eta}{1-\gamma^{\frac{\sigma-1}{\sigma}}\eta}L_D = \frac{\gamma\eta}{1-\gamma\eta}\mu\frac{1-\gamma\eta}{1-\mu\gamma\eta} = \frac{\mu\gamma\eta}{1-\mu\gamma\eta}$$

and then onto $\nu_{p\eta}$

$$\nu_{p\eta} = \frac{1-\mu}{\mu}\frac{1}{(1-\eta)(\varepsilon-1)}\frac{\mu\gamma\eta}{1-\gamma\eta}\frac{1-\gamma^{\frac{\sigma-1}{\sigma}}}{1-\gamma}\frac{1-\gamma\eta}{1-\gamma^{\frac{\sigma-1}{\sigma}}\eta} = \frac{1-\mu}{\mu}\frac{1}{(1-\eta)(\varepsilon-1)}\frac{\mu\gamma\eta}{1-\gamma\eta}$$

These allow us to solve for the partial elasticity of consumption to imports

$$\nu_{cH} = \nu_{w\eta} + \frac{\mu}{1-\mu}\nu_{p\eta} = \frac{\mu\gamma\eta}{1-\mu\gamma\eta} + \frac{1}{(1-\eta)(\varepsilon-1)}\frac{\mu\gamma\eta}{1-\gamma\eta}$$

and the partial elasticity of the RER to imports

$$\nu_{qH} = -\frac{1}{(1-\eta)(\varepsilon-1)} - \nu_{p\eta} = \frac{1}{(1-\eta)(\varepsilon-1)}\left(1 + \frac{1-\mu}{\mu}\frac{\mu\gamma\eta}{1-\gamma\eta}\right) = -\frac{1}{(1-\eta)(\varepsilon-1)}\frac{1-\mu\gamma\eta}{1-\gamma\eta}$$

such that

$$\begin{aligned}
\nu_c = \frac{\nu_{cH}}{1+\nu_{cH}-\nu_{qH}} &= \frac{\frac{\mu\gamma\eta}{1-\gamma\eta}\left(\frac{1-\gamma\eta}{1-\mu\gamma\eta} + \frac{1}{(\varepsilon-1)(1-\eta)}\right)}{1 + \frac{\mu\gamma\eta}{1-\gamma\eta}\left(\frac{1-\gamma\eta}{1-\mu\gamma\eta} + \frac{1}{(\varepsilon-1)(1-\eta)}\right) + \frac{1}{(\varepsilon-1)(1-\eta)}\frac{1-\mu\gamma\eta}{1-\gamma\eta}} \\
&= \frac{\frac{\mu\gamma\eta}{1-\gamma\eta}\left(\frac{1-\gamma\eta}{1-\mu\gamma\eta} + \frac{1}{(\varepsilon-1)(1-\eta)}\right)}{1 + \frac{\mu\gamma\eta}{1-\mu\gamma\eta} + \frac{1}{(\varepsilon-1)(1-\eta)}\frac{1}{1-\gamma\eta}(\mu\gamma\eta + 1 - \mu\gamma\eta)} \\
&= \frac{\frac{\mu\gamma\eta}{1-\gamma\eta}\left(\frac{1-\gamma\eta}{1-\mu\gamma\eta} + \frac{1}{(\varepsilon-1)(1-\eta)}\right)}{\frac{1}{1-\mu\gamma\eta} + \frac{1}{(\varepsilon-1)(1-\eta)}\frac{1}{1-\gamma\eta}} = \mu\gamma\eta\frac{\frac{1}{1-\mu\gamma\eta} + \frac{1}{(\varepsilon-1)(1-\eta)}\frac{1}{1-\gamma\eta}}{\frac{1}{1-\mu\gamma\eta} + \frac{1}{(\varepsilon-1)(1-\eta)}\frac{1}{1-\gamma\eta}} = \mu\gamma\eta
\end{aligned}$$

leading finally to the equation for consumption

$$c_{St} = a_{St} + \frac{\mu(1-\gamma\eta)}{1-\gamma}a_{Dt} + \mu\gamma\eta(p_{Xt} - p_{Mt})$$

By the same token, we can solve for the dynamics of the nominal exchange rate as a function of exogenous variables only.

$$q_t = a_{St} - \frac{1}{1-\gamma}\big((1-\mu) - \nu_q\big)a_{Dt} - \nu_q p_{Xt} - (1-\nu_q)p_{Mt}$$

where

$$\nu_q = \frac{\nu_{qH}}{1+\nu_{cH}-\nu_{qH}} = \frac{\frac{1}{(\varepsilon-1)(1-\eta)}\frac{1-\mu\gamma\eta}{1-\gamma\eta}}{1+\frac{\mu\gamma\eta}{1-\gamma\eta}\left(\frac{1-\gamma\eta}{1-\mu\gamma\eta}+\frac{1}{(\varepsilon-1)(1-\eta)}\right)+\frac{1}{(\varepsilon-1)(1-\eta)}\frac{1-\mu\gamma\eta}{1-\gamma\eta}}$$
$$= \frac{\frac{1}{(\varepsilon-1)(1-\eta)}\frac{1-\mu\gamma\eta}{1-\gamma\eta}}{1+\frac{\mu\gamma\eta}{1-\mu\gamma\eta}+\frac{1}{(\varepsilon-1)(1-\eta)}\frac{1}{1-\gamma\eta}(\mu\gamma\eta+1-\mu\gamma\eta)}$$
$$= \frac{\frac{1}{(\varepsilon-1)(1-\eta)}\frac{1-\mu\gamma\eta}{1-\gamma\eta}}{\frac{1}{1-\mu\gamma\eta}+\frac{1}{(\varepsilon-1)(1-\eta)}\frac{1}{1-\gamma\eta}} = \frac{1-\mu\gamma\eta}{1+\frac{1-\gamma\eta}{1-\mu\gamma\eta}(1-\eta)(\varepsilon-1)}$$

E.1.2 Monopolistic competition, no IRS

Labor allocation in the services sector IRS

$$L_S = \left(1+(1-\gamma)\frac{\sigma-1}{\sigma}\frac{1-\gamma\frac{\sigma-1}{\sigma}\eta}{1-\gamma\frac{\sigma-1}{\sigma}}\frac{\mu}{1-\mu}\right)^{-1}$$

while in the manufacturing sector it is

$$L_D = \left(1+(1-\gamma)\frac{\sigma-1}{\sigma}\frac{1-\gamma\frac{\sigma-1}{\sigma}\eta}{1-\gamma\frac{\sigma-1}{\sigma}}\frac{\mu}{1-\mu}\right)^{-1}(1-\gamma)\frac{\sigma-1}{\sigma}\frac{1-\gamma\frac{\sigma-1}{\sigma}\eta}{1-\gamma\frac{\sigma-1}{\sigma}}\frac{\mu}{1-\mu}$$

We use these allocations to compute the partial elasticities that compose the main elasticities of the models, starting with $\nu_{w\eta}$

$$\nu_{w\eta} = \frac{\gamma\frac{\sigma-1}{\sigma}\eta}{1-\gamma\frac{\sigma-1}{\sigma}\eta}L_D = \left(1+(1-\gamma)\frac{\sigma-1}{\sigma}\frac{1-\gamma\frac{\sigma-1}{\sigma}\eta}{1-\gamma\frac{\sigma-1}{\sigma}}\frac{\mu}{1-\mu}\right)^{-1}(1-\gamma)\frac{\sigma-1}{\sigma}\frac{\gamma\frac{\sigma-1}{\sigma}\eta}{1-\gamma\frac{\sigma-1}{\sigma}}\frac{\mu}{1-\mu}$$
$$= \frac{\gamma\frac{\sigma-1}{\sigma}\eta}{1-\gamma\frac{\sigma-1}{\sigma}\eta}L_D = \left(1-\gamma\frac{\sigma-1}{\sigma}+(1-\gamma)\frac{\sigma-1}{\sigma}\left(1-\gamma\frac{\sigma-1}{\sigma}\eta\right)\frac{\mu}{1-\mu}\right)^{-1}(1-\gamma)\frac{\sigma-1}{\sigma}\gamma\frac{\sigma-1}{\sigma}\eta\frac{\mu}{1-\mu}$$
$$= \mu\gamma\eta\left[\frac{1}{1-\gamma}\left(\frac{\sigma}{\sigma-1}\right)^2\frac{1-\mu}{\mu}\left(1-\gamma\frac{\sigma-1}{\sigma}+(1-\gamma)\frac{\sigma-1}{\sigma}\left(1-\gamma\frac{\sigma-1}{\sigma}\eta\right)\frac{\mu}{1-\mu}\right)\right]^{-1}$$
$$= \mu\gamma\eta\left[\frac{1}{1-\gamma}\left(\frac{\sigma}{\sigma-1}\right)^2\frac{1-\mu}{\mu}\left(1-\gamma\frac{\sigma-1}{\sigma}+(1-\gamma)\frac{\sigma-1}{\sigma}\frac{\mu}{1-\mu}\right)-\mu\gamma\eta\right]^{-1}$$

such that finally

$$\nu_{w\eta} = \frac{\mu\gamma\eta}{\xi_1-\mu\gamma\eta} \quad \text{where} \quad \xi_1 = \frac{\sigma}{\sigma-1}\left(\frac{1-\gamma\frac{\sigma-1}{\sigma}}{1-\gamma}\frac{1-\mu}{\mu}+1\right) > 1$$

and now onto the elasticity of manufacturing prices to imports

$$\nu_{p\eta} = \frac{1-\mu}{\mu}\frac{1}{(1-\eta)(\varepsilon-1)}\frac{\mu\gamma\eta}{1-\gamma\eta}\xi_2 \quad \text{where} \quad \xi_2 = \frac{1-\gamma\frac{\sigma-1}{\sigma}}{1-\gamma}\frac{1-\gamma\eta}{1-\gamma\frac{\sigma-1}{\sigma}\eta} > 1$$

These allow us to solve for the partial elasticity of consumption to imports

$$\nu_{cH} = \nu_{w\eta} + \frac{\mu}{1-\mu}\nu_{p\eta} = \frac{\mu\gamma\eta}{\xi_1-\mu\gamma\eta} + \frac{1}{(1-\eta)(\varepsilon-1)}\xi_2$$

and the partial elasticity of the RER to imports

$$\nu_{qH} = -\frac{1}{(1-\eta)(\varepsilon-1)} - \nu_{p\eta} = \frac{1}{(1-\eta)(\varepsilon-1)}\left(1 + \frac{1-\mu}{\mu}\frac{\mu\gamma\eta}{1-\gamma\eta}\xi_2\right)$$
$$= -\frac{1}{(1-\eta)(\varepsilon-1)}\frac{1}{1-\gamma\eta}\left(1-\gamma\eta + (1-\mu)\gamma\eta\xi_2\right) = -\frac{1}{(1-\eta)(\varepsilon-1)}\frac{1-\gamma\eta(1-(1-\mu)}{1-\gamma\eta}$$

F Financial Markets

F.1 Financial autarky

In this appendix, we provide conditions under which the model is locally determinate. Given that the equilibrium conditions depend on the degree of financial market intergration, we discuss local determinacy seperately for the case of financial autarky and for the other cases. Under financial autarky, the equilibrium conditions are given by:

$$c_{St} = \frac{\mu}{1-\gamma}a_{Dt} + a_{St} + \nu_{cH}\eta_t$$
$$q_t = -\frac{1-\mu}{1-\gamma}a_{Dt} + a_{St} - p_{Mt} + \nu_{e\eta}\eta_t$$
$$c_{St} + H_t = q_t + p_{Xt}$$

where the first two conditions from market clearing in goods and labor markets. The third equation is the result of imposing trade balance period-by-period ($b_t = b_{t+1} = 0$) in financial autarky. Eliminating q_t and c_{St} from the system allows us to solve for η_t as a function of the structural shocks:

$$q_t - c_{St} = -\frac{1}{1-\gamma}a_{Dt} - p_{Mt} - (\nu_{cH} - \nu_{qH})\eta_t$$

Elminating c_{St} from the system leads to the following policy function for q_t:

$$q_t = \frac{\mu}{1-\gamma}\frac{1}{\nu_{cH}-\nu_{qH}+1}\left(\frac{\mu}{1-\gamma}a_{Dt} + a_{St}\right) + \frac{\nu_{qH}}{\nu_{cH}-\nu_{qH}+1}p_{Xt} + \frac{\mu\nu_{qH} + (1-\mu)\nu_{cH}}{(1-\gamma)\nu_{cH}+1}\frac{\nu_{qH}}{\nu_{cH}-\nu_{qH}+1}a_{Dt}$$
$$+ \frac{\nu_{cH}-\nu_{qH}}{\nu_{cH}-\nu_{qH}+1}\frac{\nu_{cH}+1}{\nu_{cH}}a_{St} - \frac{\nu_{cH}+1}{\nu_{cH}-\nu_{qH}+1}p_{Mt}$$

which can be plugged into the budget constraint

$$\eta_t = \frac{1}{\nu_{cH}-\nu_{qH}+1}\left(-\frac{1}{1-\gamma}a_{Dt} + p_{Mt} - p_{Xt}\right)$$

Because η_t is a linear function of the structural shocks, which are stationary by assumption, it is stationary as well. Furthermore, as all other endogenous variables can be written as a linear combination of η_t and the shocks, they are stationary as well.

F.2 Integrated and Segmented Financial Markets

To derive conditions under which the integrated and segmented financial markets versions of the model are stable, we take a two step approach. First, we show how each of the three versions admit the same state space form. Second, we derive the conditions under which this representative model is locally determinate.

F.2.1 Integrated financial markets with a ROW currency bond

Under integrated financial markets with a ROW currency bond, the equilibrium conditions are given by:

$$
\begin{aligned}
c_{St} &= \frac{\mu}{1-\gamma}a_{Dt} + a_{St} + \nu_{cH}H_t \\
q_t &= -\frac{1-\mu}{1-\gamma}a_{Dt} + a_{St} - p_{Mt} + \nu_{e\eta}H_t \\
\beta b_{t+1} - b_t &= \mu\gamma\eta(q_t - c_{St} + p_{Xt} - H_t) \\
\mathbb{E}_t\left[\Delta c_{St+1} - \Delta q_{t+1}\right] &= i_t^* - \psi b_{t+1} + \psi_t
\end{aligned}
$$

Eliminate q_t and c_{St} from the budget constraint and the Euler equation and use the AR(1)-structure of the shocks to obtain the following system in the (b_{t+1}, H_t) space:

$$
\begin{aligned}
\beta b_{t+1} - b_t &= \mu\gamma\eta\left(-\frac{1}{1-\gamma} + p_{Xt} - p_{Mt} - (\nu_{cH} + 1 - \nu_{qH})H_t\right) \\
(\nu_{cH} - \nu q\eta)\mathbb{E}_t\left[\Delta\eta_{t+1}\right] &= i_t^* - \psi b_{t+1} + \psi_t + \frac{1-\rho_D}{1-\gamma}a_{Dt} + (1-\rho_M)p_{Mt}
\end{aligned}
$$

Now define $\hat{b}_{t+1} \equiv \frac{b_{t+1}}{\mu\gamma\eta\frac{1}{\beta}(\nu_{cH}+1-\nu_{qH})}$ and re-write this system as:

$$
\begin{aligned}
\hat{b}_{t+1} &= \nu + \frac{1}{\beta}\hat{b}_t + \frac{1}{\nu_{cH}+1-\nu_{qH}}\left(-\frac{1}{1-\gamma} + p_{Xt} - p_{Mt}\right) \\
\mathbb{E}_t\left[\eta_{t+1}\right] + \hat{\chi}_{ROW}\hat{b}_{t+1} &= H_t + \frac{1}{\nu_{cH}-\nu_{qH}}\left(i_t^* + \psi_t + \frac{1-\rho_D}{1-\gamma}a_{Dt} + (1-\rho_M)p_{Mt}\right)
\end{aligned}
$$

where $\hat{\chi}_{ROW} \equiv \frac{\psi}{\nu_{cH}-\nu_{qH}}\mu\gamma\eta\frac{1}{\beta}(\nu_{cH}+1-\nu_{qH})$. Therefore, this system can be represented as follows:

$$
\begin{pmatrix} 1 & \hat{\chi}_{ROW} \\ 0 & 1 \end{pmatrix}\begin{pmatrix} \mathbb{E}_t\left[\eta_{t+1}\right] \\ \hat{b}_{t+1} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -1 & \frac{1}{\beta} \end{pmatrix}\begin{pmatrix} \eta_t \\ \hat{b}_t \end{pmatrix} + \boldsymbol{C}\boldsymbol{\Theta}_t
$$

where

$$
\begin{aligned}
\boldsymbol{C} &= \begin{pmatrix} \frac{1-\rho_D}{(1-\gamma)(\nu_{cH}-\nu_{qH})} & 0 & \frac{1-\rho_M}{\nu_{cH}-\nu_{qH}} & 0 & \frac{1}{\nu_{cH}-\nu_{qH}} & \frac{1}{\nu_{cH}-\nu_{qH}} \\ -\frac{1}{(1-\gamma)(\nu_{cH}+1-\nu_{qH})} & 0 & -\frac{1}{(\nu_{cH}+1-\nu_{qH})} & \frac{1}{(\nu_{cH}+1-\nu_{qH})} & 0 & 0 \end{pmatrix}, \\
\Theta_t &= \begin{pmatrix} a_{Dt} & a_{St} & p_{Mt} & p_{Xt} & i_t^* & \psi_t \end{pmatrix}'
\end{aligned}
$$

F.2.2 Integrated financial markets with a local currency bond

Under integrated financial markets with a local currency bond, the equilibrium conditions are given by (remember $P_{St} = 1$):

$$
\begin{aligned}
c_{St} &= \frac{\mu}{1-\gamma}a_{Dt} + a_{St} + \nu_{cH}H_t \\
q_t &= -\frac{1-\mu}{1-\gamma}a_{Dt} + a_{St} - p_{Mt} + \nu_{e\eta}H_t \\
\beta b_{t+1} - b_t &= \mu\gamma\eta(q_t - c_{St} + p_{Xt} - H_t) \\
\mathbb{E}_t\left[\Delta c_{St+1}\right] &= i_t^* - \psi b_{t+1} + \psi_t
\end{aligned}
$$

Eliminate q_t and c_{St} from the budget constraint and the Euler equation and use the AR(1)-structure of the shocks to obtain the following system in the (b_{t+1}, H_t) space:

$$\beta b_{t+1} - b_t = \mu\gamma\eta\left(-\frac{1}{1-\gamma} + p_{Xt} - p_{Mt} - (\nu_{cH} + 1 - \nu_{qH})H_t\right)$$
$$\nu_{cH}\mathbb{E}_t\left[\Delta\eta_{t+1}\right] = i_t^* - \psi b_{t+1} + \psi_t + \frac{(1-\rho_D)\mu}{1-\gamma}a_{Dt} + (1-\rho_S)a_{St}$$

Use the definition of $\hat{b}_{t+1}$ and define $\hat{\chi}_L \equiv \frac{\psi}{\nu_{cH}}\mu\gamma\eta\frac{1}{\beta}(\nu_{cH} + 1 - \nu_{qH})$, and write the system as follows:

$$\hat{b}_{t+1} = \nu + \frac{1}{\beta}\hat{b}_t + \frac{1}{\nu_{cH} + 1 - \nu_{qH}}\left(-\frac{1}{1-\gamma} + p_{Xt} - p_{Mt}\right)$$
$$\mathbb{E}_t\left[\eta_{t+1}\right] + \hat{\chi}_L\hat{b}_{t+1} = H_t + \frac{1}{\nu_{cH}}\left(i_t^* + \psi_t + \frac{(1-\rho_D)\mu}{1-\gamma}a_{Dt} + (1-\rho_S)a_{St}\right)$$

Therefore, this system can be represented as follows:

$$\begin{pmatrix} 1 & \hat{\chi}_L \\ 0 & 1 \end{pmatrix}\begin{pmatrix} \mathbb{E}_t\left[\eta_{t+1}\right] \\ \hat{b}_{t+1} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -1 & \frac{1}{\beta} \end{pmatrix}\begin{pmatrix} \eta_t \\ \hat{b}_t \end{pmatrix} + \boldsymbol{C}\boldsymbol{\Theta}_t$$

where

$$\boldsymbol{C} = \begin{pmatrix} \frac{(1-\rho_D)\mu}{(1-\gamma)\nu_{cH}} & \frac{1-\rho_S}{\nu_{cH}} & 0 & 0 & \frac{1}{\nu_{cH}} & \frac{1}{\nu_{cH}} \\ -\frac{1}{(1-\gamma)(\nu_{cH}+1-\nu_{qH})} & 0 & -\frac{1}{(\nu_{cH}+1-\nu_{qH})} & \frac{1}{(\nu_{cH}+1-\nu_{qH})} & 0 & 0 \end{pmatrix},$$
$$\Theta_t = \begin{pmatrix} a_{Dt} & a_{St} & p_{Mt} & p_{Xt} & i_t^* & \psi_t \end{pmatrix}'$$

F.2.3 Segmented financial markets

Under segmented financial markets, the equilibrium conditions are given by:

$$c_{St} = \frac{\mu}{1-\gamma}a_{Dt} + a_{St} + \nu_{cH}\eta_t$$
$$q_t = -\frac{1-\mu}{1-\gamma}a_{Dt} + a_{St} - p_{Mt} + \nu_{qH}\eta_t$$
$$\beta b_{t+1} - b_t = \mu\gamma\frac{\sigma-1}{\sigma}H(q_t - c_{St} + p_{Xt} - \eta_t)$$
$$\mathbb{E}_t\left[\Delta c_{St+1} - \Delta q_{t+1}\right] = \chi_1\delta_t + \delta_t^{\$} - \chi_2 b_{t+1}$$

where $\chi_1 = \omega\sigma_{\Delta e}^2$ and $\chi_2 = \beta P_S\bar{C}_S\sigma_{\Delta e}^2$. From the first two equations we have

$$c_{St} - q_t = \frac{1}{1-\gamma}a_{Dt} + p_{Mt} + (\nu_{cH} - \nu_{qH})\eta_t$$

and we plug it into the third equation

$$\beta b_{t+1} - b_t = \mu\gamma\frac{\sigma-1}{\sigma}H\left(-\frac{1}{1-\gamma}a_{Dt} + (p_{Xt} - p_{Mt}) - (1 + \nu_{cH} - \nu_{qH})\eta_t\right)$$

and into the fourth equation

$$\mathbb{E}_t\left[\frac{1}{1-\gamma}\Delta a_{Dt+1}+\Delta p_{Mt+1}+(\nu_{cH}-\nu_{qH})\Delta\eta_{t+1}\right]=\chi_1\delta_t+\delta_t^{\$}-\chi_2 b_{t+1}$$

which can then be written as

$$(\nu_{cH}-\nu_{qH})\mathbb{E}_t\Delta\eta_{t+1}=\frac{1-\rho_D}{1-\gamma}a_{Dt}+(1-\rho_M)p_{Mt}+\chi_1\delta_t+\delta_t^{\$}-\chi_2 b_{t+1}$$

Now we transform the system into hat notation

$$\hat{b}_{t+1}-\frac{1}{\beta}\hat{b}_t+\eta_t=\frac{1}{1+\nu_{cH}-\nu_{qH}}\left(-\frac{1}{1-\gamma}a_{Dt}+(p_{Xt}-p_{Mt})\right)$$

where

$$\beta\frac{1}{\mu\gamma\frac{\sigma-1}{\sigma}H}\frac{1}{1+\nu_{cH}-\nu_{qH}}b_t=\hat{b}_t$$

and the same with the fourth equation

$$\mathbb{E}_t\eta_{t+1}=\eta_t+\frac{1}{\nu_{cH}-\nu_{qH}}\left(\frac{1-\rho_D}{1-\gamma}a_{Dt}+(1-\rho_M)p_{Mt}+\chi_1\delta_t+\delta_t^{\$}-\chi_2 b_{t+1}\right)$$

which is transformed into

$$\mathbb{E}_t\eta_{t+1}=\eta_t+\frac{1}{\nu_{cH}-\nu_{qH}}\frac{1-\rho_D}{1-\gamma}a_{Dt}+\frac{1-\rho_M}{\nu_{cH}-\nu_{qH}}p_{Mt+1}+\frac{1}{\nu_{cH}-\nu_{qH}}\delta_t^{\$}-\hat{\chi}_2 b_{t+1}+\frac{\chi_1}{\nu_{cH}-\nu_{qH}}\delta_t$$

where

$$\hat{\chi}_2=\frac{1}{\beta}\mu\gamma\frac{\sigma-1}{\sigma}H(1+\nu_{cH}-\nu_{qH})\chi_2$$

We then set up the linear system

$$\begin{pmatrix}1 & \hat{\chi}_2\\ 0 & 1\end{pmatrix}\begin{pmatrix}\mathbb{E}_t\left[\eta_{t+1}\right]\\ \hat{b}_{t+1}\end{pmatrix}=\begin{pmatrix}1 & 0\\ -1 & \frac{1}{\beta}\end{pmatrix}\begin{pmatrix}\eta_t\\ \hat{b}_t\end{pmatrix}+\boldsymbol{C}\boldsymbol{\Theta}_t$$

where

$$\boldsymbol{C}=\begin{pmatrix}0 & \frac{1-\rho_D}{1-\gamma}\frac{1}{\nu_{cH}-\nu_{qH}} & 0 & \frac{1-\rho_M}{\nu_{cH}-\nu_{qH}} & \frac{1}{\nu_{cH}-\nu_{qH}} & \frac{\chi_1}{\nu_{cH}-\nu_{qH}}\\ 0 & -\frac{1}{1-\gamma}\frac{1}{1+\nu_{cH}-\nu_{qH}} & \frac{1}{1+\nu_{cH}-\nu_{qH}} & -\frac{1}{1+\nu_{cH}-\nu_{qH}} & 0 & 0\end{pmatrix},$$

$$\Theta_t=\begin{pmatrix}a_{St} & a_{Dt} & p_{Xt} & p_{Mt} & \delta_t\$ & \delta_t\end{pmatrix}'$$

Next, we show that as long as $\hat{\chi}_2 > 0$, the dynamic system is locally stable. To see this, diagionalize the system

$$\underbrace{\begin{pmatrix} 1 & \hat{\chi}_2 \\ 0 & 1 \end{pmatrix}}_{\boldsymbol{A}} \begin{pmatrix} \mathbb{E}_t\left[\eta_{t+1}\right] \\ \hat{b}_{t+1} \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 \\ -1 & \frac{1}{\beta} \end{pmatrix}}_{\boldsymbol{B}} \begin{pmatrix} \eta_t \\ \hat{b}_t \end{pmatrix} + \boldsymbol{C}\boldsymbol{\Theta}_t$$

$$\begin{pmatrix} \mathbb{E}_t\left[\eta_{t+1}\right] \\ \hat{b}_{t+1} \end{pmatrix} = \begin{pmatrix} 1 & -\hat{\chi}_2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1 & \frac{1}{\beta} \end{pmatrix} \begin{pmatrix} \eta_t \\ \hat{b}_t \end{pmatrix} + \tilde{\boldsymbol{C}}\boldsymbol{\Theta}_t$$

$$\begin{pmatrix} \mathbb{E}_t\left[\eta_{t+1}\right] \\ \hat{b}_{t+1} \end{pmatrix} = \underbrace{\begin{pmatrix} 1+\hat{\chi}_2 & -\frac{1}{\beta}\hat{\chi}_2 \\ -1 & \frac{1}{\beta} \end{pmatrix}}_{\tilde{\boldsymbol{B}}} \begin{pmatrix} \eta_t \\ \hat{b}_t \end{pmatrix} + \tilde{\boldsymbol{C}}\boldsymbol{\Theta}_t$$

where $\tilde{\boldsymbol{C}} \equiv \boldsymbol{A}^{-1}\boldsymbol{C}$. The eigenvalues of $\tilde{\boldsymbol{B}}$ are given by:

$$\lambda_1, \lambda_2 = \frac{1+\hat{\chi}_2+\frac{1}{\beta} \pm \sqrt{(1+\hat{\chi}_2+\frac{1}{\beta})^2 - \frac{4}{\beta}}}{2}$$

First, given the relevant range of the parameters, the eigenvalues are always real. To see this note that

$$(1+\hat{\chi}_2+\frac{1}{\beta})^2 - \frac{4}{\beta} = \beta^2\hat{\chi}_2^2 + (2\beta^2+2\beta)\hat{\chi}_2 + (\beta-1)^2$$

which describes a parabola that opens at the top in $\hat{\chi}_2$. The roots of this parabola are given by

$$\begin{aligned} \mu_1, \mu_2 &= \frac{-(\beta+1) \pm 2\sqrt{\beta}}{\beta} \\ &= -1 - \beta^{-1}\left(1 \pm \beta^{\frac{1}{2}}\right) \end{aligned}$$

Therefore, if $\beta = 1$, we have that $\mu_1, \mu_2 = (-4, 0)$. In addition,

$$\frac{\partial \mu_1}{\partial \beta}, \frac{\partial \mu_2}{\partial \beta} = \beta^{-2}\left(1 \pm \beta^{\frac{1}{2}}\right)$$

which shows that $\frac{\partial \mu_1}{\partial \beta} > 0, \frac{\partial \mu_2}{\partial \beta} > 0$ and thus that $\mu_1 < 0$ and $\mu_2 < 0$ when $\beta = [0, 1[$. This implies that $(1+\hat{\chi}_2+\frac{1}{\beta})^2 \geq \frac{4}{\beta}$ which ensures that the eigenvalues are real. Next, we show that $\lambda_1 < 1 < \lambda_2$ if $\hat{\chi}_2 > 0$. First, consider the largest eigenvalue

$$\begin{aligned} \lambda_2 &= \frac{1+\hat{\chi}+\frac{1}{\beta} + \sqrt{(1+\hat{\chi}+\frac{1}{\beta})^2 - \frac{4}{\beta}}}{2} \\ &\geq \frac{1+\frac{1}{\beta} + \sqrt{(1+\frac{1}{\beta})^2 - \frac{4}{\beta}}}{2} \\ &\geq \frac{1+\frac{1}{\beta}+1-\frac{1}{\beta}}{2} \\ &\geq 1 \end{aligned}$$

where we have used that $\hat{\chi}_2 \geq 0$. Therefore $\lambda_2 > 0$ if $\hat{\chi}_2 > 0$. Next, if $\hat{\chi}_2 > 0$

$$\begin{aligned}
4\hat{\chi}_2 &> 0 \\
2\hat{\chi}_2 - \frac{2}{\beta} + 1 + (\hat{\chi}_2 + \frac{1}{\beta})^2 &> -2\hat{\chi}_2 - \frac{2}{\beta} + 1 + (\hat{\chi}_2 + \frac{1}{\beta})^2 \\
(1 + \hat{\chi}_2 + \frac{1}{\beta})^2 - \frac{4}{\beta} &> (\hat{\chi}_2 + \frac{1}{\beta} - 1)^2 \\
\sqrt{(1 + \hat{\chi}_2 + \frac{1}{\beta})^2 - \frac{4}{\beta}} &> (\hat{\chi}_2 + \frac{1}{\beta} - 1) \\
\hat{\chi}_2 + \frac{1}{\beta} + 1 - \sqrt{(1 + \hat{\chi}_2 + \frac{1}{\beta})^2 - \frac{4}{\beta}} &< 2 \\
\frac{\hat{\chi}_2 + \frac{1}{\beta} + 1 - \sqrt{(1 + \hat{\chi}_2 + \frac{1}{\beta})^2 - \frac{4}{\beta}}}{2} &< 1 \\
\lambda_1 &< 1
\end{aligned}$$

We now head back to the main system

$$\begin{pmatrix} \mathbb{E}_t\left[\eta_{t+1}\right] \\ \hat{b}_{t+1} \end{pmatrix} = \begin{pmatrix} 1 + \hat{\chi}_2 & -\frac{1}{\beta}\hat{\chi}_2 \\ -1 & \frac{1}{\beta} \end{pmatrix} \begin{pmatrix} \eta_t \\ \hat{b}_t \end{pmatrix} + \tilde{\boldsymbol{C}}\boldsymbol{\Theta}_t$$

where

$$\tilde{\mathbf{C}} = \begin{pmatrix} 0 & \frac{1}{1-\gamma}\left[\frac{1-\rho_D}{\nu_{cH}-\nu_{qH}} + \frac{\hat{\chi}_2}{1+\nu_{cH}-\nu_{qH}}\right] & -\frac{\hat{\chi}_2}{1+\nu_{cH}-\nu_{qH}} & \frac{1-\rho_M}{\nu_{cH}-\nu_{qH}} + \frac{\hat{\chi}_2}{1+\nu_{cH}-\nu_{qH}} & \frac{1}{\nu_{cH}-\nu_{qH}} & \frac{\chi_1}{\nu_{cH}-\nu_{qH}} \\ 0 & -\frac{1}{1-\gamma}\frac{1}{1+\nu_{cH}-\nu_{qH}} & \frac{1}{1+\nu_{cH}-\nu_{qH}} & -\frac{1}{1+\nu_{cH}-\nu_{qH}} & 0 & 0 \end{pmatrix}$$

Now we notice that the left eigenvector of the main matrix associated with λ_2 is $v = (-1, (1/\beta) - \lambda_1)$ and so

$$\begin{pmatrix} -1 & (1/\beta) - \lambda_1 \end{pmatrix} \begin{pmatrix} \mathbb{E}_t\left[\eta_{t+1}\right] \\ \hat{b}_{t+1} \end{pmatrix} = \begin{pmatrix} -1 & (1/\beta) - \lambda_1 \end{pmatrix} \lambda_2 \begin{pmatrix} \eta_t \\ \hat{b}_t \end{pmatrix} + \begin{pmatrix} -1 & (1/\beta) - \lambda_1 \end{pmatrix} \tilde{\boldsymbol{C}}\boldsymbol{\Theta}_t$$

and we have $\begin{pmatrix} -1 & (1/\beta) - \lambda_1 \end{pmatrix}\tilde{\mathbf{C}} = \hat{\mathbf{C}}$ is the following

$$\hat{\mathbf{C}} = \begin{pmatrix} 0 & -\frac{1}{1-\gamma}\left[\frac{1-\rho_D}{\nu_{cH}-\nu_{qH}} + \frac{\lambda_2-1}{1+\nu_{cH}-\nu_{qH}}\right] & \frac{\lambda_2-1}{1+\nu_{cH}-\nu_{qH}} & -\left[\frac{1-\rho_M}{\nu_{cH}-\nu_{qH}} + \frac{\lambda_2-1}{1+\nu_{cH}-\nu_{qH}}\right] & -\frac{1}{\nu_{cH}-\nu_{qH}} & -\frac{\chi_1}{\nu_{cH}-\nu_{qH}} \end{pmatrix}$$

Iterating the system forward we have

$$-\eta_t + (1/\beta - \lambda_1)\hat{b}_t = -\sum_{j=0}^{\infty} \left(\frac{1}{\lambda_2}\right)^{j+1} \mathbb{E}_t \hat{\mathbf{C}}\boldsymbol{\Theta}_{t+j}$$

Now recall the third equation

$$\hat{b}_{t+1} - 1/\beta\hat{b}_t + \eta_t = \frac{1}{1 + \nu_{cH} - \nu_{qH}} \left(-\frac{1}{1-\gamma}a_{Dt} + (p_{Xt} - p_{Mt})\right)$$

such that

$$(1-\lambda_1 L)\hat{b}_{t+1} = -\eta_t + (1/\beta - \lambda_1)\hat{b}_t + \frac{1}{1+\nu_{cH}-\nu_{qH}}\left(-\frac{1}{1-\gamma}a_{Dt} + (p_{Xt}-p_{Mt})\right)$$

and

$$(1-\lambda_1 L)\hat{b}_{t+1} = \frac{1}{1+\nu_{cH}-\nu_{qH}}\left(-\frac{1}{1-\gamma}a_{Dt} + (p_{Xt}-p_{Mt})\right) - \sum_{j=0}^{\infty}\left(\frac{1}{\lambda_2}\right)^{j+1}\mathbb{E}_t\hat{\mathbf{C}}\mathbf{\Theta}_{t+j}$$

We then solve for the shock matrix

$$\begin{aligned}(1-\lambda_1 L)\hat{b}_{t+1} = {} & \frac{1}{1+\nu_{cH}-\nu_{qH}}\left(-\frac{1}{1-\gamma}a_{Dt} + (p_{Xt}-p_{Mt})\right) \\ & -\frac{1}{\lambda_2}\sum_{j=0}^{\infty}\left(\frac{1}{\lambda_2}\right)^{j}\mathbb{E}_t\Bigg[-\frac{1}{1-\gamma}\left(\frac{1-\rho_D}{\nu_{cH}-\nu_{qH}} + \frac{\lambda_2-1}{1+\nu_{cH}-\nu_{qH}}\right)a_{Dt+j} \\ & + \frac{\lambda_2-1}{1+\nu_{cH}-\nu_{qH}}p_{Xt+j} - \left(\frac{1-\rho_M}{\nu_{cH}-\nu_{qH}} + \frac{\lambda_2-1}{1+\nu_{cH}-\nu_{qH}}\right)p_{Mt+j} - \frac{1}{\nu_{cH}-\nu_{qH}}\left(\delta^{\$}_{t+j} + \chi_1\delta_{t+j}\right)\Bigg]\end{aligned}$$

and we use the shock structure to simplify as follows

$$\begin{aligned}(1-\lambda_1 L)\hat{b}_{t+1} = {} & \frac{1}{1+\nu_{cH}-\nu_{qH}}\left(-\frac{1}{1-\gamma}a_{Dt} + (p_{Xt}-p_{Mt})\right) \\ & -\frac{1}{\lambda_2}\sum_{j=0}^{\infty}\left(\frac{1}{\lambda_2}\right)^{j}\Bigg[-\frac{1}{1-\gamma}\left(\frac{1-\rho_D}{\nu_{cH}-\nu_{qH}} + \frac{\lambda_2-1}{1+\nu_{cH}-\nu_{qH}}\right)\rho_D^j a_{Dt} \\ & + \frac{\lambda_2-1}{1+\nu_{cH}-\nu_{qH}}\rho_X^j p_{Xt} - \left(\frac{1-\rho_M}{\nu_{cH}-\nu_{qH}} + \frac{\lambda_2-1}{1+\nu_{cH}-\nu_{qH}}\right)\rho_M^j p_{Mt} - \frac{1}{\nu_{cH}-\nu_{qH}}\rho_\delta^j\left(\delta^{\$}_{t} + \chi_1\delta_{t}\right)\Bigg]\end{aligned}$$

and the properties of infinite sums

$$\begin{aligned}(1-\lambda_1 L)\hat{b}_{t+1} = {} & \frac{1}{1+\nu_{cH}-\nu_{qH}}\left(-\frac{1}{1-\gamma}a_{Dt} + (p_{Xt}-p_{Mt})\right) \\ & + \frac{1}{\lambda_2}\frac{1}{1-\frac{\rho_D}{\lambda_2}}\frac{1}{1-\gamma}\left(\frac{1-\rho_D}{\nu_{cH}-\nu_{qH}} + \frac{\lambda_2-1}{1+\nu_{cH}-\nu_{qH}}\right)a_{Dt} \\ & - \frac{1}{\lambda_2}\frac{1}{1-\frac{\rho_X}{\lambda_2}}\frac{\lambda_2-1}{1+\nu_{cH}-\nu_{qH}}p_{Xt} + \frac{1}{\lambda_2}\frac{1}{1-\frac{\rho_M}{\lambda_2}}\left(\frac{1-\rho_M}{\nu_{cH}-\nu_{qH}} + \frac{\lambda_2-1}{1+\nu_{cH}-\nu_{qH}}\right)p_{Mt} \\ & + \frac{1}{\lambda_2}\frac{1}{1-\frac{\rho_\delta}{\lambda_2}}\frac{1}{\nu_{cH}-\nu_{qH}}\left(\delta^{\$}_{t} + \chi_1\delta_{t}\right)\end{aligned}$$

collecting terms we have

$$\begin{aligned}(1-\lambda_1 L)\hat{b}_{t+1} = {} & \frac{1-\rho_D}{\lambda_2-\rho_D}\frac{1}{1-\gamma}\left(\frac{1}{\nu_{cH}-\nu_{qH}} - \frac{1}{1+\nu_{cH}-\nu_{qH}}\right)a_{Dt} \\ & + \frac{1-\rho_X}{\lambda_2-\rho_X}\frac{1}{1+\nu_{cH}-\nu_{qH}}p_{Xt} + \frac{1-\rho_M}{\lambda_2-\rho_M}\left(\frac{1}{\nu_{cH}-\nu_{qH}} - \frac{1}{1+\nu_{cH}-\nu_{qH}}\right)p_{Mt} \\ & + \frac{1}{\lambda_2-\rho_\delta}\frac{1}{\nu_{cH}-\nu_{qH}}\left(\delta^{\$}_{t} + \chi_1\delta_{t}\right)\end{aligned}$$

Now we recall again that

$$(1-\lambda_1 L)\hat{b}_{t+1} = -\eta_t + (1/\beta - \lambda_1)\hat{b}_t + \frac{1}{1+\nu_{cH}-\nu_{qH}}\left(-\frac{1}{1-\gamma}a_{Dt} + (p_{Xt}-p_{Mt})\right)$$

which can now be rewritten as

$$\eta_t - (1/\beta - \lambda_1)\hat{b}_t = \frac{1}{1+\nu_{cH}-\nu_{qH}}\left(-\frac{1}{1-\gamma}a_{Dt} + (p_{Xt}-p_{Mt})\right) - (1-\lambda_1 L)\hat{b}_{t+1}$$

and we multiply both sides by $(1-\lambda_1 L)$ such that

$$\begin{aligned}(1-\lambda_1 L)\eta_t - (1/\beta - \lambda_1)L(1-\lambda_1 L)\hat{b}_{t+1} = {} & (1-\lambda_1 L)\frac{1}{1+\nu_{cH}-\nu_{qH}}\left(-\frac{1}{1-\gamma}a_{Dt} + (p_{Xt}-p_{Mt})\right) \\ & - (1-\lambda_1 L)(1-\lambda_1 L)\hat{b}_{t+1}\end{aligned}$$

leading to

$$\begin{aligned}(1-\lambda_1 L)\eta_t = {} & (1-\lambda_1 L)\frac{1}{1+\nu_{cH}-\nu_{qH}}\left(-\frac{1}{1-\gamma}a_{Dt} + (p_{Xt}-p_{Mt})\right) \\ & - \left(1-\lambda_1 L - (1/\beta - \lambda_1)L\right)(1-\lambda_1 L)\hat{b}_{t+1}\end{aligned}$$

and we substitute for bond holdings such that

$$\begin{aligned}(1-\lambda_1 L)\eta_t = {} & (1-\lambda_1 L)\frac{1}{1+\nu_{cH}-\nu_{qH}}\left(-\frac{1}{1-\gamma}a_{Dt} + (p_{Xt}-p_{Mt})\right) \\ & - (1-(1/\beta)L)\left[\frac{1-\rho_D}{\lambda_2-\rho_D}\frac{1}{1-\gamma}\left(\frac{1}{\nu_{cH}-\nu_{qH}} - \frac{1}{1+\nu_{cH}-\nu_{qH}}\right)a_{Dt}\right. \\ & + \frac{1-\rho_X}{\lambda_2-\rho_X}\frac{1}{1+\nu_{cH}-\nu_{qH}}p_{Xt} + \frac{1-\rho_M}{\lambda_2-\rho_M}\left(\frac{1}{\nu_{cH}-\nu_{qH}} - \frac{1}{1+\nu_{cH}-\nu_{qH}}\right)p_{Mt} \\ & \left. + \frac{1}{\lambda_2-\rho_\delta}\frac{1}{\nu_{cH}-\nu_{qH}}\left(\delta_t^{\$} + \chi_1\delta_t\right)\right]\end{aligned}$$

before we collect terms one last time

$$\begin{aligned}(1-\lambda_1 L)\eta_t = {} & -\frac{1}{1-\gamma}\frac{1}{1+\nu_{cH}-\nu_{qH}}\left((1-\lambda_1 L) + (1-(1/\beta)L)\,\frac{1-\rho_D}{\lambda_2-\rho_D}\frac{1}{\nu_{cH}-\nu_{qH}}\right)a_{Dt} \\ & + \frac{1}{1+\nu_{cH}-\nu_{qH}}\left((1-\lambda_1 L) - (1-(1/\beta)L)\,\frac{1-\rho_X}{\lambda_2-\rho_X}\right)p_{Xt} \\ & - \frac{1}{1+\nu_{cH}-\nu_{qH}}\left((1-\lambda_1 L) + (1-(1/\beta)L)\,\frac{1-\rho_M}{\lambda_2-\rho_M}\frac{1}{\nu_{cH}-\nu_{qH}}\right)p_{Mt} \\ & - (1-(1/\beta)L)\,\frac{1}{\lambda_2-\rho_\delta}\frac{1}{\nu_{cH}-\nu_{qH}}\left(\delta_t^{\$} + \chi_1\delta_t\right)\end{aligned}$$

Now using the fact that $\lambda_1 < 1$ we can use the inverse of the lag operator

$$\begin{aligned}\eta_t = & -\frac{1}{1-\gamma}\frac{1}{1+\nu_{cH}-\nu_{qH}}\left(1+\alpha_D\right)a_{Dt} - \alpha_\delta \frac{1}{1-\rho_\delta}\left(\delta_t^{\$} + \chi_1 \delta_t\right) \\ & + \frac{1}{1+\nu_{cH}-\nu_{qH}}\left(1-\alpha_X(\nu_{cH}-\nu_{qH})\right)p_{Xt} - \frac{1}{1+\nu_{cH}-\nu_{qH}}\left(1+\alpha_M\right)p_{Mt}\end{aligned}$$

where

$$\alpha_x = \left(1-\lambda_1 L\right)^{-1}\left(1-(1/\beta)L\right)\frac{1-\rho_x}{\lambda_2-\rho_x}\frac{1}{\nu_{cH}-\nu_{qH}}$$

such that finally

$$c_t = a_{St} + \frac{1}{1-\gamma}\left(\mu - \nu_c(1+\alpha_D)\right) + \nu_c\left(\left(1-\alpha_X(\nu_{cH}-\nu_{qH})\right)p_{Xt} - (1+\alpha_M)p_{Mt}\right) - \frac{\alpha_\delta \nu_{cH}}{1-\rho_\delta}\left(\delta_t^{\$} + \chi_1\delta_t\right)$$

G Calibration

G.1 Steady-state H

G.2 Input Shares

Table G.1: Total input purchases as a fraction of gross output per sector.

		2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018
Colombia	Manufacturing	0,64	0,65	0,64	0,66	0,65	0,66	0,68	0,67	0,66	-	-	-	-
	Services	0,39	0,39	0,39	0,40	0,39	0,39	0,40	0,40	0,39	-	-	-	-
Chile	Manufacturing	-	-	-	-	-	-	-	0,71	0,71	0,68	0,68	0,69	0,69
	Services	-	-	-	-	-	-	-	0,41	0,41	0,40	0,39	0,39	0,39

G.3 Shock processes

To calibrate the shock processes, we estimate AR(1)-processes for export and import prices series. To be consistent with different normalizations used in the model, we estimate the shock processes under different normalizations: (1) relative to USD (i.e. simply the price series), (2) relative to domestic CPI and (3) relative to the domestic price of services. To estimate the shock process for commodity prices, we rely on oil prices taken from the FRED St. Louis. Specifically, we use the monthly used price of one barrel WTI which we convert to quarterly data by averaging across months within the quarter. We take monthly data on the Colombian CPI and the price of Colombian services from the website of Colombia's DANE (Departamento Administrativo Nacional de Estadística) and average months within quarters to obtain quarterly data. To estimate the import price shock series, we rely on the monthly IPI from Colombia's DANE (Departamento Administrativo Nacional de Estadística) as well and also average months within quarters to obtain quarterly data.

H Aggregate Production Function

H.1 Defining aggregators

Before deriving the aggregate production function for manufacturing, we still need to define the aggregators for Y_{Dt} and X_{Dt}. We define Y_t as:

$$Y_{Dt} \equiv \left(\int_i Y_{it}^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}$$

which is consistent with market clearing on the variety level:

$$\begin{aligned}
Y_{Dt} &\equiv \left(\int_i Y_{it}^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}} \\
&= \left(\int_i \left(\left(\frac{P_{it}}{P_{D,t}} \right)^{-\sigma} (X_{St} + Q_{Dt}) \right)^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}} \\
&= (X_{St} + Q_{Dt}) \left(\int_i \left(\frac{P_{it}}{P_{D,t}} \right)^{1-\sigma} di \right)^{\frac{\sigma}{\sigma-1}} \\
&= (X_{St} + Q_{Dt})\, P_{D,t}^{\sigma} \left(\int_i (P_{it})^{1-\sigma} di \right)^{\frac{\sigma}{\sigma-1}} \\
&= (X_{St} + Q_{Dt})
\end{aligned}$$

where we have used the definition for the price index for manufacturing $P_{Dt} \equiv \left(\int_i P_{i,t}^{1-\sigma} di \right)^{\frac{1}{1-\sigma}}$.

We define X_{Dt} as

$$X_{Dt} \equiv \left(\int_i X_{D_i t}^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

In appendix I.2, we show that this choice for X_{Dt} makes sure that aggregate manufacturing productivity in the model without selection is equal to the level of degenerate productivity in the equivalent neoclassical model. Note that we did not need this definition to show the equivalence of the two models. Because we study a small open economy, there is no market clearing condition for foreign intermediate inputs and therefore we can freely choose the intermediate input aggregator that aggregates foreign intermediate use across domestic manufacturing firms.[43]. Define:

$$Q_{M,t} \equiv \left(\int_i Q_{M_i t}^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

H.2 Deriving the Aggregate Production Function

Manfacturing sector To derive the aggregate manufacturing production function, start from the individual production function for manufacturers: $Y_{it} = \varphi_i L_{D_i t}^{1-\gamma} X_{D_i t}^{\gamma}$ and insert into the aggregator:

$$\begin{aligned}
Y_{Dt} &\equiv \left(\int_i Y_{it}^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}} \\
&= \left(\int_i \left(A_{Dt} \varphi_i L_{D_i t}^{1-\gamma} X_{D_i t}^{\gamma} \right)^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}
\end{aligned}$$

[43] Note that this choice affects the productivity decomposition, but does affect the dynamics of the model

Consider the first order condition for L_{Dit}

$$\begin{aligned}
L_{Dit} &= (1-\gamma)\frac{\mathrm{MC}_{it}Y_{it}}{W_t} \\
&= (1-\gamma)\frac{\sigma-1}{\sigma}\frac{P_{it}}{W_t}\left(\frac{P_{it}}{P_{Dt}}\right)^{-\sigma}(X_{St}+Q_{Dt}) \\
&= (1-\gamma)\frac{\sigma-1}{\sigma}\frac{P_{Dt}}{W_t}(X_{St}+Q_{Dt})\left(\frac{P_{it}}{P_{Dt}}\right)^{1-\sigma} \\
&= L_{Dt}\left(\frac{P_{it}}{P_{Dt}}\right)^{1-\sigma}
\end{aligned}$$

where we have used the expression for aggregate labor demand from manufacturing for productive labor use. Insert and re-write:

$$\begin{aligned}
Y_{Dt} &= \left(\int_i\left(A_{Dt}\varphi_i\left(\frac{P_{it}}{P_{Dt}}\right)^{(1-\sigma)(1-\gamma)}L_{Dt}^{1-\gamma}X_{D_it}^{\gamma}\right)^{\frac{\sigma-1}{\sigma}}di\right)^{\frac{\sigma}{\sigma-1}} \\
&= A_{Dt}L_{Dt}^{1-\gamma}\left(\int_i\left(\varphi_i\left(\frac{P_{it}}{P_{Dt}}\right)^{(1-\sigma)(1-\gamma)}X_{D_it}^{\gamma}\right)^{\frac{\sigma-1}{\sigma}}di\right)^{\frac{\sigma}{\sigma-1}} \\
&= A_{Dt}L_{Dt}^{1-\gamma}X_{Dt}^{\gamma}\left(\int_i\left(\varphi_i\left(\frac{P_{it}}{P_{Dt}}\right)^{(1-\sigma)(1-\gamma)}\left(\frac{X_{D_it}}{X_{Dt}}\right)^{\gamma}\right)^{\frac{\sigma-1}{\sigma}}di\right)^{\frac{\sigma}{\sigma-1}}
\end{aligned}$$

Proving that this is the aggregate production function, boils down to showing that integral will only depend on parameters and the productivity cut-off for importing. First, rewrite $\frac{X_{D_it}}{X_{Dt}}$ as a function of productivity and the importing cut-off:

$$\begin{aligned}
\frac{X_{D_it}}{X_{Dt}} &= \frac{X_{D_it}}{\left(\int_i X_{D_it}^{\frac{\varepsilon-1}{\varepsilon}}di\right)^{\frac{\varepsilon}{\varepsilon-1}}} \\
&= \frac{\frac{\gamma\mathrm{MC}_{it}Y_{it}}{P_{X_it}}}{\left(\int_i\left(\frac{\gamma\mathrm{MC}_{it}Y_{it}}{P_{X_it}}\right)^{\frac{\varepsilon-1}{\varepsilon}}di\right)^{\frac{\varepsilon}{\varepsilon-1}}} \\
&= \frac{\frac{1}{\varphi_i}P_{X_it}^{\gamma-1}Y_{it}}{\left(\int_i\left(\frac{1}{\varphi_i}P_{X_it}^{\gamma-1}Y_{it}\right)^{\frac{\varepsilon-1}{\varepsilon}}di\right)^{\frac{\varepsilon}{\varepsilon-1}}} \\
&= \frac{\frac{1}{\varphi_i}P_{X_it}^{\gamma-1}P_{it}^{-\sigma}}{\left(\int_i\left(\frac{1}{\varphi_i}P_{X_it}^{\gamma-1}P_{it}^{-\sigma}\right)^{\frac{\varepsilon-1}{\varepsilon}}di\right)^{\frac{\varepsilon}{\varepsilon-1}}} \\
&= \frac{\varphi_i^{\sigma-1}P_{X_it}^{\gamma-1-\gamma\sigma}}{\left(\int_i\left(\varphi_i^{\sigma-1}P_{X_it}^{\gamma-1-\gamma\sigma}\right)^{\frac{\varepsilon-1}{\varepsilon}}di\right)^{\frac{\varepsilon}{\varepsilon-1}}}
\end{aligned}$$

Now re-write $\frac{P_{it}}{P_{Dt}}$ also as a function of productivity solely:

$$\begin{aligned}
\frac{P_{it}}{P_{Dt}} &= \frac{\frac{\sigma}{\sigma-1}\mathrm{MC}_{it}}{P_{Dt}} \\
&= \frac{\frac{\sigma}{\sigma-1}\frac{1}{\varphi_i A_{Dt}}\frac{W_t^{1-\gamma}P_{X_it}^{\gamma}}{1-\gamma^{1-\gamma}\gamma^{\gamma}}}{\frac{\sigma}{\sigma-1}\omega^{-\frac{\gamma}{\varepsilon-1}}\frac{1}{A_{Dt}}\frac{W_t{}^{1-\gamma}P_{Dt}{}^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\Gamma(\varphi_{Mt})} \\
&= \frac{1}{\varphi_i}\left(\frac{P_{X_it}}{P_{Dt}}\right)^{\gamma}\omega^{\frac{\gamma}{\varepsilon-1}}\kappa^{\sigma-1}\underline{\varphi}\Gamma(\varphi_{Mt})^{\frac{1}{\sigma-1}}
\end{aligned}$$

where $\Gamma(\varphi_{Mt}) \equiv \left[\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-\kappa}\left(\frac{1}{\sigma-1-\kappa}+\frac{1}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}\right)-\frac{1}{\sigma-1-\kappa}\right]$. Now plug these expressions back into the expression for Y_{Dt}:

$$\begin{aligned}
Y_{Dt} = A_{Dt}L_{Dt}^{1-\gamma}X_{Dt}^{\gamma}&\Bigg\{\int_i\Bigg\{\varphi\left[\frac{1}{\varphi_i}\left(\frac{P_{X_it}}{P_{Dt}}\right)^{\gamma}\omega^{\frac{\gamma}{\varepsilon-1}}\kappa^{\sigma-1}\underline{\varphi}\Gamma(\varphi_{Mt})^{\frac{1}{\sigma-1}}\right]^{(1-\gamma)(1-\sigma)} \\
&\left[\frac{\varphi_i^{\sigma-1}P_{X_it}^{\gamma-1-\gamma\sigma}}{\left(\int_i\left(\varphi_i^{\sigma-1}P_{X_it}^{\gamma-1-\gamma\sigma}\right)^{\frac{\varepsilon-1}{\varepsilon}}di\right)^{\frac{\varepsilon}{\varepsilon-1}}}\right]^{\gamma}\Bigg\}^{\frac{\sigma-1}{\sigma}}di\Bigg\}^{\frac{\sigma}{\sigma-1}} \\
= A_{Dt}L_{Dt}^{1-\gamma}X_{Dt}^{\gamma}&\left(\omega^{\frac{\gamma}{\varepsilon-1}}\kappa^{\sigma-1}\underline{\varphi}\Gamma(\varphi_{Mt})^{\frac{1}{\sigma-1}}\right)^{(1-\gamma)(1-\sigma)}\left[\omega^{\frac{1}{1-\varepsilon}}P_{D,t}\right]^{\gamma(1+\gamma(\sigma-1))}\left(k\underline{\varphi}^{\kappa}\right)^{\frac{-\varepsilon\gamma}{\varepsilon-1}}\underline{\varphi}^{-(\sigma-1)\gamma+\frac{\kappa\varepsilon\gamma}{\varepsilon-1}} \\
&\left\{\frac{1}{k-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon}}+\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon}-k}\left(\frac{1}{k-\frac{(\sigma-1)(\varepsilon-1)}{(\varepsilon-1)-\gamma(\sigma-1)}}+\frac{1}{k-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon}}\right)\right\}^{\frac{-\gamma\varepsilon}{\varepsilon-1}} \\
&\left\{\varphi\int_i\left\{\varphi\left[\frac{1}{\varphi_i}\left(\frac{P_{X_it}}{P_{Dt}}\right)^{\gamma}\omega^{\frac{\gamma}{\varepsilon-1}}\kappa^{\sigma-1}\underline{\varphi}\Gamma(\varphi_{Mt})^{\frac{1}{\sigma-1}}\right]^{(1-\gamma)(1-\sigma)}\left[\varphi_i^{\sigma-1}P_{X_it}^{\gamma-1-\gamma\sigma}\right]^{\gamma}\right\}^{\frac{\sigma-1}{\sigma}}di\right\}^{\frac{\sigma}{\sigma-1}} \\
= A_{Dt}L_{Dt}^{1-\gamma}X_{Dt}^{\gamma}&\left(\omega^{\frac{\gamma}{\varepsilon-1}}\kappa^{\sigma-1}\underline{\varphi}\Gamma(\varphi_{Mt})^{\frac{1}{\sigma-1}}\right)^{(1-\gamma)(1-\sigma)}\left[\omega^{\frac{1}{1-\varepsilon}}P_{D,t}\right]^{\gamma(1+\gamma(\sigma-1))}\left(k\underline{\varphi}^{\kappa}\right)^{\frac{-\varepsilon\gamma}{\varepsilon-1}}\underline{\varphi}^{-(\sigma-1)\gamma+\frac{\kappa\varepsilon\gamma}{\varepsilon-1}} \\
&\left\{\frac{1}{k-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon}}+\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon}-k}\left(\frac{1}{k-\frac{(\sigma-1)(\varepsilon-1)}{(\varepsilon-1)-\gamma(\sigma-1)}}+\frac{1}{k-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon}}\right)\right\}^{\frac{-\gamma\varepsilon}{\varepsilon-1}} \\
&P_{Dt}^{\gamma(\sigma-1)(1-\gamma)}\left(\omega^{\frac{1}{1-\varepsilon}}\right)^{-\gamma\sigma}\left(\kappa\underline{\varphi}^{\kappa}\right)^{\frac{\sigma}{\sigma-1}} \\
&\left\{\underline{\varphi}^{\sigma-1-\kappa}\left(\frac{1}{k-(\sigma-1)}+\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-k}\left(\frac{1}{k-\frac{(\sigma-1)(\varepsilon-1)}{(\varepsilon-1)-\gamma(\sigma-1)}}-\frac{1}{k-(\sigma-1)}\right)\right)\right\}^{\frac{\sigma}{\sigma-1}}
\end{aligned}$$

where we have used the Pareto distribution repeatedly and the fact that:

$$P_{Xit} = \begin{cases}\left(\frac{\varphi_{Mt}}{\varphi_i}\right)^{\frac{\sigma-1}{\varepsilon-1-\gamma(\sigma-1)}}\omega^{-\frac{1}{\varepsilon-1}}P_{Dt} & \text{if } \varphi_i \geq \varphi_{Mt} \\ \omega^{-\frac{1}{\varepsilon-1}}P_{Dt} & \text{if } \varphi_i < \varphi_{Mt}\end{cases}$$

which simplifies to

$$Y_{Dt} = KA_{Dt}L_{Dt}^{1-\gamma}X_{Dt}^{\gamma}\varphi(\varphi_{Mt})$$

where $K \equiv \underline{\varphi}^{\sigma-1+\frac{\kappa}{\sigma-1}}\kappa^{\frac{\sigma}{\sigma-1}-\frac{\varepsilon\gamma}{\varepsilon-1}-(1-\gamma)}$ is a constant and $\varphi(\varphi_{Mt})$ measures the extent to which aggregate productivity in the manufacturing sector is affected by reallocation across manufacturing firms.

$$\varphi(\varphi_{Mt}) \equiv \left\{ \frac{1}{k-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon}} + \left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon}-k} \left(\frac{1}{k-\frac{(\sigma-1)(\varepsilon-1)}{(\varepsilon-1)-\gamma(\sigma-1)}} + \frac{1}{k-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon}} \right) \right\}^{\frac{-\gamma\varepsilon}{\varepsilon-1}}$$
$$\left\{ \underline{\varphi}^{\sigma-1-\kappa} \left(\frac{1}{k-(\sigma-1)} + \left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\sigma-1-k} \left(\frac{1}{k-\frac{(\sigma-1)(\varepsilon-1)}{(\varepsilon-1)-\gamma(\sigma-1)}} - \frac{1}{k-(\sigma-1)} \right) \right) \right\}^{\frac{\sigma}{\sigma-1}-(1-\gamma)}$$

Services sector Next, we use the result from Appendix **??** and obtain an expression of the aggregate production function of our economy. First, we use the expression of intermediate input demand from the services sector for manufacturing goods: $X_{St} = \frac{1-\gamma\frac{\sigma-1}{\sigma}H_t}{1-\gamma\frac{\sigma-1}{\sigma}}Y_{Dt}$:

$$\begin{aligned}
Y_t &= A_{S,t}L_{S,t}^{1-\mu}X_{S,t}^{\mu} \\
&= A_{S,t}L_{S,t}^{1-\mu}\left(\frac{1-\gamma\frac{\sigma-1}{\sigma}H_t}{1-\gamma\frac{\sigma-1}{\sigma}}Y_{Dt}\right)^{\mu} \\
&= A_{S,t}L_{S,t}^{1-\mu}Y_{D,t}^{\mu}\left(\frac{1-\gamma\frac{\sigma-1}{\sigma}H_t}{1-\gamma\frac{\sigma-1}{\sigma}}\right)^{\mu} \\
&= A_{S,t}L_{S,t}^{1-\mu}\left(KA_{Dt}L_{Dt}^{1-\gamma}X_{Dt}^{\gamma}\varphi(\varphi_{Mt})\right)^{\mu}\left(\frac{1-\gamma\frac{\sigma-1}{\sigma}H_t}{1-\gamma\frac{\sigma-1}{\sigma}}\right)^{\mu} \\
&= K^{\mu}A_{S,t}A_{Dt}^{\mu}L_{S,t}^{1-\mu}L_{Dt}^{\mu(1-\gamma)}X_{Dt}^{\mu\gamma}\varphi(\varphi_{Mt})^{\mu}\left(\frac{1-\gamma\frac{\sigma-1}{\sigma}H_t}{1-\gamma\frac{\sigma-1}{\sigma}}\right)^{\mu}
\end{aligned}$$

Because X_{Dt} consists of both domestic and foreign intermediate input use, we now write X_{Dt} as a function of Q_{Mt} which solely measures foreign intermediate input use. Use the definition of Q_{Mt}:

$$Q_{M,t} \equiv \left(\int_i Q_{M_i t}^{\frac{\varepsilon-1}{\varepsilon}}di\right)^{\frac{\varepsilon}{\varepsilon-1}} = \left(\int_{\varphi_{Mt}}^{\infty} Q_{Mt}(\varphi)^{\frac{\varepsilon-1}{\varepsilon}}dG(\varphi)\right)^{\frac{\varepsilon}{\varepsilon-1}}$$

where we have changed the measure from i to φ and changed the boundaries to only include firms that import. To obtain the expression for $X_{D,t}$ as function of Q_{Mt}, plug in the first order condition for $Q_{Mt}(\varphi)$:

$$\begin{aligned}
Q_{M,t} &= \left(\int_{\varphi_{Mt}}^{\infty}\left((1-\omega)\left(\frac{P_{Xt}(\varphi)}{P_{Mt}(\varphi)}\right)^{\varepsilon}X_{Dt}(\varphi)\right)^{\frac{\varepsilon-1}{\varepsilon}}dG(\varphi)\right)^{\frac{\varepsilon}{\varepsilon-1}} \\
&= X_{Dt}\left(\int_{\varphi_{Mt}}^{\infty}(1-\omega)\left(\left(\frac{P_{Xt}(\varphi)}{P_{Mt}(\varphi)}\right)^{\varepsilon}\frac{X_{Dt}(\varphi)}{X_{Dt}}\right)^{\frac{\varepsilon-1}{\varepsilon}}dG(\varphi)\right)^{\frac{\varepsilon}{\varepsilon-1}} \\
&= X_{Dt}\frac{\left(\int_{\varphi_{Mt}}^{\infty}(1-\omega)\left(\left(\frac{P_{Xt}(\varphi)}{P_{Mt}(\varphi)}\right)^{\varepsilon}\varphi^{\sigma-1}P_{Xt}(\varphi)^{-[1+\gamma(\sigma-1)]}\right)^{\frac{\varepsilon-1}{\varepsilon}}dG(\varphi)\right)^{\frac{\varepsilon}{\varepsilon-1}}}{\left(\int_i\left(\varphi_i^{\sigma-1}P_{X_i t}^{-[1+\gamma(\sigma-1)]}\right)^{\frac{\varepsilon-1}{\varepsilon}}di\right)^{\frac{\varepsilon}{\varepsilon-1}}}
\end{aligned}$$

where we have plugged in the expression for $\frac{X_{Dt}(\varphi)}{X_{Dt}}$ from the previous section. Now, use the expression for $P_{Mt}(\varphi)$, given by:

$$P_{Mt}(\varphi) = E_t P^{\$}_{M,t} |\lambda_t(\varphi)|^{\frac{1}{\varepsilon-1}} = E_t P^{\$}_{M,t} \left[\frac{\omega}{1-\omega} \left(\frac{P_{E_t P^{\$}_{Mt}}}{P_{Dt}} \right)^{\varepsilon-1} \left(\left(\frac{\varphi}{\varphi_{Mt}} \right)^{\frac{(\sigma-1)(\varepsilon-1)}{(\varepsilon-1)-\gamma(\sigma-1)}} - 1 \right) \right]^{\frac{1}{1\varepsilon}}$$

and re-write using the expression for $P_{Xt}(\varphi)$:

$$\begin{aligned}
Q_{M,t} = {} & X_{Dt} \frac{1-\omega}{\left(\int_i \left(\varphi_i^{\sigma-1} P_{X_i t}^{-[1+\gamma(\sigma-1)]} \right)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}} \cdot \\
& \left(\int_{\varphi_{Mt}}^{\infty} \left(\varphi^{\sigma-1} P_{Xt}(\varphi)^{(\varepsilon-1)-\gamma(\sigma-1)} \left(\frac{\omega}{1-\omega} \right)^{-\frac{\varepsilon}{\varepsilon-1}} P_{Dt}^{-\varepsilon} \left[\left(\frac{\varphi}{\varphi_{Mt}} \right)^{\frac{(\sigma-1)(\varepsilon-1)}{(\varepsilon-1)-\gamma(\sigma-1)}} - 1 \right]^{\frac{\varepsilon}{\varepsilon-1}} \right)^{\frac{\varepsilon-1}{\varepsilon}} dG(\varphi) \right)^{\frac{\varepsilon}{\varepsilon-1}} \\
= {} & X_{Dt} \frac{(1-\omega)^{\frac{1}{1-\varepsilon}} \varphi_{Mt}^{\frac{(\sigma-1)[(\varepsilon-1)-\gamma(\sigma-1)]}{(\varepsilon-1)-\gamma(\sigma-1)}} \left(\int_{\varphi_{Mt}}^{\infty} \left[\left(\frac{\varphi}{\varphi_{Mt}} \right)^{\frac{(\sigma-1)(\varepsilon-1)}{(\varepsilon-1)-\gamma(\sigma-1)}} - 1 \right] dG(\varphi) \right)^{\frac{\varepsilon}{\varepsilon-1}}}{\left(\int_{\underline{\varphi}}^{\varphi_{Mt}} \varphi^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon}} dG(\varphi) + \int_{\varphi_{Mt}}^{\infty} \varphi^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon}} \left(\frac{\varphi_{Mt}}{\varphi}^{-\frac{(\sigma-1)[1+\gamma(\sigma-1)]}{\varepsilon-1-\gamma(\sigma-1)} \frac{\varepsilon-1}{\varepsilon}} \right) dG(\varphi) \right)^{\frac{\varepsilon}{\varepsilon-1}}}
\end{aligned}$$

Using the fact that $dG(\varphi) = \kappa \underline{\varphi}^{\kappa} \varphi^{-k-1}$, this expression becomes:

$$Q_{M,t} = X_{Dt} \frac{(1-\omega)^{\frac{1}{1-\varepsilon}} \varphi_{Mt}^{\sigma-1-\kappa} \underline{\varphi}^{\kappa} \left(\frac{\kappa}{\kappa - \frac{(\varepsilon-1)(\sigma-1)}{(\varepsilon-1)-\gamma(\sigma-1)}} - 1 \right)}{\left(\kappa \underline{\varphi}^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon}} \left(\left(\frac{\varphi_{Mt}}{\varphi} \right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon} - k} \left(\frac{\kappa}{\kappa - \frac{(\varepsilon-1)(\sigma-1)}{(\varepsilon-1)-\gamma(\sigma-1)}} - \frac{\kappa}{\kappa - \frac{(\varepsilon-1)(\sigma-1)}{\varepsilon}} \right) + \frac{\kappa}{\kappa - \frac{(\varepsilon-1)(\sigma-1)}{\varepsilon}} \right) \right)^{\frac{\varepsilon}{\varepsilon-1}}}$$

Therefore, we can write

$$X_{Dt} = Q_{M,t} \frac{\left(\kappa \underline{\varphi}^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon}} \left(\left(\frac{\varphi_{Mt}}{\varphi} \right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon} - k} \left(\frac{\kappa}{\kappa - \frac{(\varepsilon-1)(\sigma-1)}{(\varepsilon-1)-\gamma(\sigma-1)}} - \frac{\kappa}{\kappa - \frac{(\varepsilon-1)(\sigma-1)}{\varepsilon}} \right) + \frac{\kappa}{\kappa - \frac{(\varepsilon-1)(\sigma-1)}{\varepsilon}} \right) \right)^{\frac{\varepsilon}{\varepsilon-1}}}{(1-\omega)^{\frac{1}{1-\varepsilon}} \varphi_{Mt}^{\sigma-1-\kappa} \underline{\varphi}^{\kappa} \left(\frac{\kappa}{\kappa - \frac{(\varepsilon-1)(\sigma-1)}{(\varepsilon-1)-\gamma(\sigma-1)}} - 1 \right)}$$

Plugging this expression into the expression for Y_t, we obtain the expression in the text:

$$Y_t = K^{\mu} A_{S,t} A_{Dt}^{\mu} L_{S,t}^{1-\mu} L_{Dt}^{\mu(1-\gamma)} Q_{Mt}^{\mu\gamma} \varphi(\varphi_{Mt})^{\mu} \Gamma(\varphi_{Mt})^{\mu}$$

where

$$\Gamma(\varphi_{Mt}) \equiv \frac{1-\gamma\frac{\sigma-1}{\sigma} H_t}{1-\gamma\frac{\sigma-1}{\sigma}} \left(\frac{\left(\kappa \underline{\varphi}^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon}} \left(\left(\frac{\varphi_{Mt}}{\varphi} \right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon} - k} \left(\frac{\kappa}{\kappa - \frac{(\varepsilon-1)(\sigma-1)}{(\varepsilon-1)-\gamma(\sigma-1)}} - \frac{\kappa}{\kappa - \frac{(\varepsilon-1)(\sigma-1)}{\varepsilon}} \right) + \frac{\kappa}{\kappa - \frac{(\varepsilon-1)(\sigma-1)}{\varepsilon}} \right) \right)^{\frac{\varepsilon}{\varepsilon-1}}}{(1-\omega)^{\frac{1}{1-\varepsilon}} \varphi_{Mt}^{\sigma-1-\kappa} \underline{\varphi}^{\kappa} \left(\frac{\kappa}{\kappa - \frac{(\varepsilon-1)(\sigma-1)}{(\varepsilon-1)-\gamma(\sigma-1)}} - 1 \right)} \right)^{\gamma}$$

I The need for selection

In this section, we explain why heterogeneity in productive efficiency and fixed costs to import are only necessary and not sufficient ingredients to obtain dynamics that are distinct from a neoclassical setting. Instead, we show that selection is a sufficient ingredient and key for generating dynamics that are different for models with and without heterogeneity in productivity.

I.1 Model equivalence

To see this, we consider two nested specifications of the main model in which we do not allow for selection. This is implemented by assuming a minimum level of productivity that is above the importing cutoff, not only in steady state, but far enough from the cutoff that all firms in the economy are always importing ($\underline{\varphi} > \varphi_{Mt}$). To show how a model with heterogeneity and fixed costs, but without selection is dynamically equivalent to a neoclassical model, we specialize the heterogeneous firm model to a neoclassical model by letting $k \to \infty$ such that the productivity distribution becomes degenerate at some level φ_D. Next, we show that these two models are dynamically equivalent because they give rise to the same equilibrium conditions for the endogenous variables. Starting with the aggregate manufacturing price indices:

$$\text{Degenerate} \quad P_{Dt} = \frac{\sigma}{\sigma-1}\omega^{-\frac{\gamma}{\varepsilon-1}}\frac{1}{A_{Dt}}\frac{{W_t}^{1-\gamma}{P_{Dt}}^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\left({\varphi_D}^{\varepsilon-1}{\varphi_{Mt}}^{-\gamma(\sigma-1)}\right)^{-\frac{1}{\varepsilon-1-\gamma(\sigma-1)}}$$

$$\text{Pareto} \quad P_{Dt} = \frac{\sigma}{\sigma-1}\omega^{-\frac{\gamma}{\varepsilon-1}}\frac{1}{A_{Dt}}\frac{{W_t}^{1-\gamma}{P_{Dt}}^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\left[\frac{\kappa}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1))}}\right]^{-\frac{1}{\sigma-1}}\left(\underline{\varphi}^{\varepsilon-1}{\varphi_{Mt}}^{-\gamma(\sigma-1)}\right)^{-\frac{1}{\varepsilon-1-\gamma(\sigma-1)}}$$

The two latter expressions are equivalent whenever

$$\varphi_D = \underline{\varphi}\left[\frac{\kappa}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1))}}\right]^{\frac{\varepsilon-1-\gamma(\sigma-1))}{(\sigma-1)(\varepsilon-1)}} \tag{I.1.0.1}$$

and these equalities remain when we consider the other equations for these two different models. For example, in the model with degenerate heterogeneity we have

$$H_t = \frac{1-\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}}{1-\gamma\frac{\sigma-1}{\sigma}\left(\frac{\varphi_{Mt}}{\varphi_D}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}} = \frac{\kappa-\left(\kappa-\frac{(\sigma-1)(\epsilon-1)}{(\epsilon-1)-\gamma(\sigma-1)}\right)\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\frac{(\sigma-1)(\epsilon-1)}{(\epsilon-1)-\gamma(\sigma-1)}}}{\kappa-\gamma\frac{\sigma-1}{\sigma}\left(\kappa-\frac{(\sigma-1)(\epsilon-1)}{(\epsilon-1)-\gamma(\sigma-1)}\right)\left(\frac{\varphi_{Mt}}{\underline{\varphi}}\right)^{\frac{(\sigma-1)(\epsilon-1)}{(\epsilon-1)-\gamma(\sigma-1)}}}$$

which are the two placeholder variables that enter the trade balance equation

$$\varepsilon_t\frac{B^{\$}_{t+1}}{R_t} - \varepsilon_t B^{\$}_t = \varepsilon_t P^{\$}_{Xt}X - \mu\gamma\frac{\sigma-1}{\sigma}H_t P_{St}C_{St}$$

which is the same in both cases. The final check is to asses whether or not labor allocated to importing is expressed in the same equations in both cases. Under the Pareto distribution we have

$$
\begin{aligned}
L_{Mt} &= f\frac{\omega}{1-\omega}\left(\frac{P_{Dt}}{E_t P_{Mt}}\right)^{1-\varepsilon}\left[\left(\frac{\varphi}{\varphi_{Mt}}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}\frac{\kappa}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}-1\right] \\
&= \frac{\omega}{1-\omega}\left(\frac{P_{Dt}}{E_t P_{Mt}}\right)^{1-\varepsilon}\left[\left(\frac{\varphi_D}{\varphi_{Mt}}\right)^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}-1\right]
\end{aligned}
$$

which again means the frameworks are in concordance.

I.2 Aggregate production function

This section derives the aggregate production function in a model without selection. It also rationalizes the choice for $X_{D,t}$ as the one that makes aggregate productivity in the model without selection equal to the degenerate productivity level in a neoclassical model defined in equation I.1.0.1. To derive the aggregate production function use the definition of Y_t

$$
\begin{aligned}
Y_t &\equiv \left(\int_i Y_{it}^{\frac{\sigma-1}{\sigma}}\,di\right)^{\frac{\sigma}{\sigma-1}} \\
&= \left(\int_i \left(A_{Dt}\varphi_i L_{D_i t}^{1-\gamma} X_{D_i t}^{\gamma}\right)^{\frac{\sigma-1}{\sigma}}\,di\right)^{\frac{\sigma}{\sigma-1}}
\end{aligned}
$$

Consider the first order condition for L_{Dit}

$$
\begin{aligned}
L_{Dit} &= (1-\gamma)\frac{\mathrm{MC}_{it}Y_{it}}{W_t} \\
&= (1-\gamma)\frac{\sigma-1}{\sigma}\frac{P_{it}}{W_t}\left(\frac{P_{it}}{P_{Dt}}\right)^{-\sigma}(X_{St}+Q_{Dt}) \\
&= (1-\gamma)\frac{\sigma-1}{\sigma}\frac{P_{Dt}}{W_t}(X_{St}+Q_{Dt})\left(\frac{P_{it}}{P_{Dt}}\right)^{1-\sigma} \\
&= L_{Dt}\left(\frac{P_{it}}{P_{Dt}}\right)^{1-\sigma}
\end{aligned}
$$

where we have used the expression for aggregate labor demand from manufacturing for productive labor use. Insert and re-write:

$$
\begin{aligned}
Y_t &= \left(\int_i \left(A_{Dt}\varphi_i\left(\frac{P_{it}}{P_{Dt}}\right)^{(1-\sigma)(1-\gamma)} L_{Dt}^{1-\gamma}X_{D_i t}^{\gamma}\right)^{\frac{\sigma-1}{\sigma}}\,di\right)^{\frac{\sigma}{\sigma-1}} \\
&= A_{Dt}L_{Dt}^{1-\gamma}\left(\int_i \left(\varphi_i\left(\frac{P_{it}}{P_{Dt}}\right)^{(1-\sigma)(1-\gamma)} X_{D_i t}^{\gamma}\right)^{\frac{\sigma-1}{\sigma}}\,di\right)^{\frac{\sigma}{\sigma-1}} \\
&= A_{Dt}L_{Dt}^{1-\gamma}X_{Dt}^{\gamma}\left(\int_i \left(\varphi_i\left(\frac{P_{it}}{P_{Dt}}\right)^{(1-\sigma)(1-\gamma)} \left(\frac{X_{D_i t}}{X_{Dt}}\right)^{\gamma}\right)^{\frac{\sigma-1}{\sigma}}\,di\right)^{\frac{\sigma}{\sigma-1}}
\end{aligned}
$$

Now we obtain expression for $\frac{X_{D_i t}}{X_{Dt}}$ and $\frac{P_{it}}{P_{Dt}}$ as functions of φ only. Start by re-writing $\frac{X_{D_i t}}{X_{Dt}}$ as a function of productivity and P_{X_it}

$$\begin{aligned}
\frac{X_{D_it}}{X_{Dt}} &= \frac{X_{D_it}}{\left(\int_i X_{D_it}^{\frac{\varepsilon-1}{\varepsilon}}\, di\right)^{\frac{\varepsilon}{\varepsilon-1}}} \\
&= \frac{\frac{\gamma \mathrm{MC}_{it} Y_{it}}{P_{X_it}}}{\left(\int_i \left(\frac{\gamma \mathrm{MC}_{it} Y_{it}}{P_{X_it}}\right)^{\frac{\varepsilon-1}{\varepsilon}} di\right)^{\frac{\varepsilon}{\varepsilon-1}}} \\
&= \frac{\frac{1}{\varphi_i} P_{X_it}^{\gamma-1} Y_{it}}{\left(\int_i \left(\frac{1}{\varphi_i} P_{X_it}^{\gamma-1} Y_{it}\right)^{\frac{\varepsilon-1}{\varepsilon}} di\right)^{\frac{\varepsilon}{\varepsilon-1}}} \\
&= \frac{\frac{1}{\varphi_i} P_{X_it}^{\gamma-1} P_{it}^{-\sigma}}{\left(\int_i \left(\frac{1}{\varphi_i} P_{X_it}^{\gamma-1} P_{it}^{-\sigma}\right)^{\frac{\varepsilon-1}{\varepsilon}} di\right)^{\frac{\varepsilon}{\varepsilon-1}}} \\
&= \frac{\varphi_i^{\sigma-1} P_{X_it}^{\gamma-1-\gamma\sigma}}{\left(\int_i \left(\varphi_i^{\sigma-1} P_{X_it}^{\gamma-1-\gamma\sigma}\right)^{\frac{\varepsilon-1}{\varepsilon}} di\right)^{\frac{\varepsilon}{\varepsilon-1}}}
\end{aligned}$$

Use the definition of P_{X_it} to write the expression as a function of φ_{Mt}

$$P_{Xit} = \left(\frac{\varphi_{Mt}}{\varphi_i}\right)^{\frac{\sigma-1}{\varepsilon-1-\gamma(\sigma-1)}} \omega^{-\frac{1}{\varepsilon-1}} P_{Dt}$$

To obtain $\frac{X_{D_it}}{X_{Dt}}$ solely as a function of φ

$$\begin{aligned}
\frac{X_{D_it}}{X_{Dt}} &= \frac{\varphi_i^{\sigma-1}\left(\left(\frac{\varphi_{Mt}}{\varphi_i}\right)^{\frac{\sigma-1}{\varepsilon-1-\gamma(\sigma-1)}} \omega^{-\frac{1}{\varepsilon-1}} P_{Dt}^{\gamma-1-\gamma\sigma}\right)^{\gamma-1-\gamma\sigma}}{\left(\int_i \left(\varphi_i^{\sigma-1}\left(\left(\frac{\varphi_{Mt}}{\varphi_i}\right)^{\frac{\sigma-1}{\varepsilon-1-\gamma(\sigma-1)}} \omega^{-\frac{1}{\varepsilon-1}} P_{Dt}^{\gamma-1-\gamma\sigma}\right)^{\gamma-1-\gamma\sigma}\right)^{\frac{\varepsilon-1}{\varepsilon}} di\right)^{\frac{\varepsilon}{\varepsilon-1}}} \\
&= \frac{\varphi_i^{\sigma-1}\left(\left(\frac{\varphi_{Mt}}{\varphi_i}\right)^{\frac{\sigma-1}{\varepsilon-1-\gamma(\sigma-1)}}\right)^{\gamma-1-\gamma\sigma}}{\varphi_{Mt}^{\frac{(\sigma-1)(\gamma-1-\gamma\sigma)}{\varepsilon-1-\gamma(\sigma-1)}}\left(\int_i \varphi_i^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}} di\right)^{\frac{\varepsilon}{\varepsilon-1}}} \\
&= \frac{\varphi_i^{\frac{(\sigma-1)\varepsilon}{\varepsilon-1-\gamma(\sigma-1)}}}{\left(\int_i \varphi_i^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}} di\right)^{\frac{\varepsilon}{\varepsilon-1}}}
\end{aligned}$$

Now re-write $\frac{P_{it}}{P_{Dt}}$ also as a function of productivity solely:

$$
\begin{aligned}
\frac{P_{it}}{P_{Dt}} &= \frac{\frac{\sigma}{\sigma-1}\mathrm{MC}_{it}}{P_{Dt}} \\
&= \frac{\frac{\sigma}{\sigma-1}\frac{1}{\varphi_i A_{Dt}}\frac{W_t^{1-\gamma}P_{X_it}^{\gamma}}{1-\gamma^{1-\gamma}\gamma^{\gamma}}}{\frac{\sigma}{\sigma-1}\omega^{-\frac{\gamma}{\varepsilon-1}}\frac{1}{A_{Dt}}\frac{{W_t}^{1-\gamma}{P_{Dt}}^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\left[\frac{\kappa}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1))}}\right]^{-\frac{1}{\sigma-1}}\left(\underline{\varphi}^{\varepsilon-1}{\varphi_{Mt}}^{-\gamma(\sigma-1)}\right)^{-\frac{1}{\varepsilon-1-\gamma(\sigma-1)}}} \\
&= \frac{\frac{\sigma}{\sigma-1}\frac{1}{\varphi_i A_{Dt}}\frac{W_t^{1-\gamma}\left(\left(\frac{\varphi_{Mt}}{\varphi_i}\right)^{\frac{\sigma-1}{\varepsilon-1-\gamma(\sigma-1)}}\omega^{-\frac{1}{\varepsilon-1}}P_{Dt}\right)^{\gamma}}{1-\gamma^{1-\gamma}\gamma^{\gamma}}}{\frac{\sigma}{\sigma-1}\omega^{-\frac{\gamma}{\varepsilon-1}}\frac{1}{A_{Dt}}\frac{{W_t}^{1-\gamma}{P_{Dt}}^{\gamma}}{(1-\gamma)^{1-\gamma}\gamma^{\gamma}}\left[\frac{\kappa}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1))}}\right]^{-\frac{1}{\sigma-1}}\left(\underline{\varphi}^{\varepsilon-1}{\varphi_{Mt}}^{-\gamma(\sigma-1)}\right)^{-\frac{1}{\varepsilon-1-\gamma(\sigma-1)}}} \\
&= \frac{\frac{1}{\varphi_i}\left(\frac{\varphi_{Mt}}{\varphi_i}\right)^{\frac{\sigma-1}{\varepsilon-1-\gamma(\sigma-1)}\gamma}}{\left[\frac{\kappa}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1))}}\right]^{-\frac{1}{\sigma-1}}\left(\underline{\varphi}^{\varepsilon-1}{\varphi_{Mt}}^{-\gamma(\sigma-1)}\right)^{-\frac{1}{\varepsilon-1-\gamma(\sigma-1)}}} \\
&= \varphi_i^{-\frac{(\varepsilon-1)}{(\varepsilon-1)-\gamma(\sigma-1)}}\left[\frac{\kappa}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1))}}\right]^{\frac{1}{\sigma-1}}\underline{\varphi}^{-\frac{(\varepsilon-1)}{(\varepsilon-1)-\gamma(\sigma-1)}}
\end{aligned}
$$

We can put these pieces together as:

$$
\begin{aligned}
Y_t = A_{Dt}L_{Dt}^{1-\gamma}X_{Dt}^{\gamma}\Bigg(\int_i\Bigg\{\varphi_i\Bigg(&\varphi_i^{-\frac{(\varepsilon-1)}{(\varepsilon-1)-\gamma(\sigma-1)}}\left[\frac{\kappa}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1))}}\right]^{\frac{1}{\sigma-1}}\underline{\varphi}^{-\frac{(\varepsilon-1)}{(\varepsilon-1)-\gamma(\sigma-1)}}\Bigg)^{(1-\sigma)(1-\gamma)} \\
&\left(\frac{\varphi_i^{\frac{(\sigma-1)\varepsilon}{\varepsilon-1-\gamma(\sigma-1)}}}{\left(\int_i\varphi_i^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}di\right)^{\frac{\varepsilon}{\varepsilon-1}}}\right)^{\gamma}\Bigg\}^{\frac{\sigma-1}{\sigma}}di\Bigg)^{\frac{\sigma}{\sigma-1}} \\
= A_{Dt}L_{Dt}^{1-\gamma}X_{Dt}^{\gamma}&\left[\frac{\kappa}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1))}}\right]^{\frac{1}{\gamma-1}}\underline{\varphi}^{\frac{(\varepsilon-1)(1-\sigma)(1-\gamma)}{(\varepsilon-1)-\gamma(\sigma-1)}}\left(\int_i\varphi_i^{\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1)}}di\right)^{\frac{\sigma}{\sigma-1}-\gamma\frac{\varepsilon}{\varepsilon-1}} \\
= A_{Dt}L_{Dt}^{1-\gamma}X_{Dt}^{\gamma}&\left[\frac{\kappa}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1))}}\right]^{\frac{1}{\gamma-1}}\underline{\varphi}^{\frac{(\varepsilon-1)(1-\sigma)(1-\gamma)}{(\varepsilon-1)-\gamma(\sigma-1)}}\left(\frac{\kappa}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1))}}\underline{\varphi}^{\frac{(\varepsilon-1)(1-\sigma)}{(\varepsilon-1)-\gamma(\sigma-1)}}\right)^{\frac{\sigma}{\sigma-1}-\gamma\frac{\varepsilon}{\varepsilon-1}} \\
= A_{Dt}L_{Dt}^{1-\gamma}X_{Dt}^{\gamma}&\underline{\varphi}\left(\frac{\kappa}{\kappa-\frac{(\sigma-1)(\varepsilon-1)}{\varepsilon-1-\gamma(\sigma-1))}}\right)^{\frac{\varepsilon-1-\gamma(\sigma-1)}{(\sigma-1)(\varepsilon-1)}}
\end{aligned}
$$

Note that this expression yields two insights. First, the production function in a model with heterogeneous firms, fixed costs of importing and roundabout production, but without selection is equivalent to the production function obtained from neo-classical model with a degenerate productivity level given by equation I.1.0.1. Second, the combination of heterogeneity across firms, fixed costs of importing and roundabout production are not sufficient to generate changes in aggregate manufacturing productivity following aggregate shocks.[44] Instead, we show in Appendix **??** that selection into importing is a sufficient condition for aggregate productivity shocks.

[44] Here we refer to aggregate shocks that are not shocks to aggregate productivity in the manufacturing sector. These would trivially lead to changes in aggregate manufacturing productivity.