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Job Market Paper Comovement in Old and New Trade

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Abstract

International business-cycle models struggle to match aggregate output correlations between countries observed in the data. New Trade models have been proposed as a way of generating a positive association between correlation and bilateral trade intensity, but it remains an open question whether or not these models can match the levels of correlation. In this paper, I revisit this problem with a comparison between Ricardian and New Trade and their predictions for comovement. I provide common tractable expressions for the second moments of sectoral and aggregate output in the two most common frameworks of International Trade while allowing for an arbitrary number of countries, sectors and input-output linkages. In the symmetric case, I show that the correlation expressions are almost identical across frameworks, their differences converge sublinearly to zero in the number of countries, and that trade and input elasticities are the crucial parameters in matching the data. I calibrate the models using world input-output tables and I find that Ricardian Trade under-predicts the correlation of GDP by an order of magnitude, while New Trade predicts correlations between one-fifth and one-half of those observed in the data and helps solve the puzzles quantitatively in the one-sector case.

JEL Codes: F11, F12, F44

1 Introduction

It is a stylized fact of international macroeconomics that quarterly changes in gross domestic product are positively correlated across countries. Consequently, the correlation of output and value-added became standard moments researchers asked international business-cycle (IBC) models to match. What became clear early on in the literature is that standard models do not generate these stylized facts. The models' failure of generating high output correlations is part of what the literature has named the *quantity puzzle* (originally named the "quantity anomaly" in P. J. Kehoe et al. (1995)), while the models' failure to generate the positive slope between trade intensity and comovement is a phenomenon named the *trade comovement puzzle* (Kose & Yi (2006)).

In this paper, I revisit the quantity puzzle in light of recent work arguing for two different solutions to the comovement puzzle: i) low short-run trade elasticity (Boehm et al. (2020), Drozd et al. (2021)), and ii) the extensive margin of trade as a source of comovement (Liao & Santacreu (2015), de Soyres & Gaillard (2019)). I compare the predictions made about comovement in the two most common theories of International Trade, namely Ricardian Trade Theory and New Trade Theory. I develop a multi-country multi-sector model with input and final goods trade both in the Eaton & Kortum (2002) framework and the Melitz (2003) framework. I use the particular version of the Eaton & Kortum (2002) (henceforth EK) framework developed in Caliendo & Parro (2015) (henceforth CP) as representing Old Trade (Ricardo (1891)), and a version of Chaney (2008)'s Melitz framework extended to allow for multiple countries and sectors (henceforth Melitz-Pareto or MP), but also input-output linkages under restricted entry. The latter is the heterogeneous-firms extension of New Trade (Krugman (1979,1980)).

The first contribution of this paper is to provide (to a first-order approximation) an analytical representation of the *influence matrix*, which describes the response of the output of any country-sector pair in the world to a shock to any other country-sector pair, in both frameworks. I use this matrix to compute the covariance matrix of the world economy, and consequently the correlation of real output between any country pair. I show that the influence matrix has the same general form in both frameworks, and is itself composed of four other matrices: i) an augmented Leontief matrix that is informative about total spending exposure; ii) an expenditure switching matrix representing the extent to which input and final spending shares change following shocks; iii) a "trade-exposure" matrix which carries information on the extent to which bilateral trade shares change following shocks; iv) a Ghosh inverse matrix that carries information on supply-side exposure and input-output linkages.

Matrix (iv) represents the *supply*-side exposure, which stems from sectors using each others' inputs to produce. Following sectoral productivity shocks, country-sector pairs around the world are affected through their cost structure as relative wages and input prices change. Matrices (i)-(iii) represent the *demand*-side exposure, which stems from sectors responding to demand fluctuations for their outputs through market clearing. Following productivity shocks, country-sector pair outputs are affected by buyers substituting their purchases to other locations in the world, to other sectors, or outright reducing their demand.

I present equations representing the dynamics of trade shares and sector prices that clarify the different channels present in MP. In quarters when one (or many) foreign countries are more productive, there is downward pressure on the price of domestic consumption and/or inputs. To the extent that the domestic economy uses the cheaper foreign goods to produce more, this leads to positive comovement. In MP, a reduction in unit costs abroad is augmented because of increasing returns to scale in production. When prices are low, total spending at home increases, which incentivizes both foreign and domestic firms to enter the domestic market, exerting further downward pressure on domestic prices. However, this extra pressure dominates only to the extent that the fixed costs of entry abroad are not increasing as much.

The second contribution is to present a stylized (symmetric) version of both frameworks which allows for highly tractable expressions of the correlations, thus clarifying what it takes to get the level of comovement observed in the data. These correlations are functions of the number of countries, the number of sectors, the trade elasticity, the input elasticity and the trade intensity. The two forces that shape comovement, namely supply and demand-side exposures, are summarized into two eigenvalues of the influence matrix and are combined into expressions that are qualitatively almost identical between EK and MP. The only difference between frameworks is that the supply-side exposure in EK depends on the trade elasticity, while in MP it depends on the trade elasticity adjusted by one parameter alone, which can be interpreted as the steady-state share of entry-labor income of gross output.

In both frameworks, the level of comovement and its slope with trade intensity depends on the relative strength of the supply and demand-side forces. When the trade elasticity is low and/or the input elasticity is high, the supply-side force dominates and becomes an unambiguous source of positive comovement. High input elasticities imply high input shares in production, which creates a strong direct exposure between country-sector pairs, leading their outputs to rise and fall together. This is in line with the literature's prediction that input trade raises output correlations dating back to Ambler et al. (2002). When the trade elasticity is low the supply-side exposure is magnified because it is harder for any country-sector pair to substitute its demand for a specific input towards some other supplier located elsewhere in the world following a productivity shock. On the other hand, when the trade elasticity is high, the demand-side exposure is stronger which unambiguously lowers comovement. In this scenario, it is easier for buyers of a country-sector pair to substitute their purchases into or away from it, insulating their outputs from negative shocks while making it easier to take advantage of positive productivity shocks elsewhere.

A stylized fact in the data that perfectly illustrates the relative strengths of supply and demand-side forces, is the difference (or lack thereof) between the correlation of different sectors across countries and the correlation between similar sectors across countries. In particular, in the data, there are no pervasive sign differences between how the output of different sectors is correlated relative to the same sector across countries. For example, the correlation between the auto sector in the US and the auto sector in China is similar to the correlation between the auto sector in the US with the computer sector in China: both are positive. In theory, on the other hand, the relative strength of supply and demand forces leads to drastically different predictions for these moments. If both the intensity and elasticity of trade are high, then the output of the same sector is highly substitutable across countries. When a negative shock occurs, this leads consumers and producers to shift part of their demand to the same sector but in a different country, which increases its output and leads to negative comovement.

This suggests that low trade elasticities and high input elasticities are the common solutions to the puzzles in the literature. When these parameter restrictions are imposed, both frameworks generate higher aggregate output correlations, higher slopes of correlations to trade intensity, and similar sectoral output correlation between different sectors vis-a-vis the same sector across countries. Relative to EK, MP helps solve the puzzles quantitatively rather than qualitatively. For example, in both MP and EK there are producers' optimality conditions that equate total input spending in a sector to a fraction of gross output. In EK this fraction depends on the input elasticities of the production function, while in MP it depends also on the markups set by producers. To match the same moment in the data, i.e. total input spending over gross output, one calibrates larger input elasticities in MP.

The rest of the paper is structured as follows. In Section 2, I describe the assumptions that are made in the two frameworks and their equilibrium conditions. Both the demand system and the structure of variable trade costs are common, but they differ in the following dimensions: i) the distribution of productivity draws for individual producers; ii) market structure; iii) returns to scale in technology. I then structure the equilibrium conditions into a common set of equations that fully characterize both models, namely trade shares, sector prices and market clearing. These clarify how in MP there is an extra channel that depends on the ratio of total expenditure (market size) at the destination relative to entry costs at the source.

In Section 3, I rearrange the linearized equations into a linear system to prove the main theorem of the paper, which states that the vector of real output changes in any country-sector pair is solved as a function of exogenous shocks only, and that the matrix that maps shocks to real-output is a common combination of supply and demand-side exposure in both frameworks, but that the matrices in MP have to be adjusted to account for the extensive margin of trade. In Section 4, I use a symmetric version of the world economy developed in Section 3 to show that low trade elasticity and high input elasticity are the common solutions to the puzzles presented.

In Section 5, I calibrate the influence matrix according to the assumptions of each framework, while in Section 6 the models are taken to the data under the assumption that shocks are uncorrelated across countries and sectors. In the one sector case, MP predicts higher correlations than EK, with the former predicting correlations between one-fifth and one-half of those observed in G7 economies, while the latter misses the mark by an order of magnitude. These are in line with what is predicted in the stylized economy, and virtually all of the difference is due to the different calibrations for the input elasticity of technology that is calibrated such that producers' optimality conditions fit the data in both frameworks.

1.1 Literature Review

P. J. Kehoe et al. (1995) introduced the concept of the *quantity puzzle*, part of which is the inability of IBC models to generate cross-country correlation of output as high as in the data, but even since Backus et al. (1992) there has been a research agenda that tried to solve the puzzle[1] The literature also found that comovement is increasing in the intensity of trade. Frankel & Rose (1998) where the first to document the pattern, Clark & Van Wincoop (2001) verified that it holds across US regions, Baxter & Kouparitsas (2005) replicate the results for a larger dataset and Calderon et al. (2007) extends the results to developing countries. Kose & Yi (2001) is the first paper to propose that IBC models do not match the slope of trade intensity with output comovement, while Kose & Yi (2006) names this inability the *trade comovement puzzle*.

Since then, there have been many papers in the literature that studied business cycle transmission across countries, that report failing to achieve meaningful levels of correlations using models. For example, Cravino & Levchenko (2017) develop a model where multinationals have parent and affiliate firms that have correlated productivity inspired by Burstein et al. (2008), yet find that "(...) transmission of shocks across multinationals in and of itself cannot generate anything close to observed output correlations (pg. 954)". Johnson (2014) found that input trade might not even solve the comovement puzzle in the absence of correlated shocks.

There was progress in the comovement puzzle through the extensive margin of trade. Liao & Santacreu (2015) shows that the positive slope between trade intensity and comovement is dominated by the product extensive margin, while de Soyres & Gaillard (2019) shows that comovement is significantly associated with trade in inputs, but not in final goods. Both papers develop Melitz (2003) models with active extensive margins of trade and leverage the insights

[1] see, for example, Baxter and Crucini (1993, 1995), Heathcote & Perri (2002) and P. J. Kehoe & Perri (2002).

of A. C. Head (2002) and T. J. Kehoe & Ruhl (2008) to match the slope of comovement and trade intensity, but do not concern themselves with the magnitude of correlations. de Soyres & Gaillard (2019), in particular, reports a correlation of GDP close to zero (0.025 on average) when productivity shocks are uncorrelated. I extend their models to the multisector case, provide a tractable comparison with EK and explain the necessary modifications to get correlations closer to those observed in the data. I argue that, at least in the symmetric, the extensive margin strengthens the demand-side channel and lowers (not raises) comovement. In this scenario, Melitz (2003) helps solves the puzzle because it has more degrees of freedom than EK (more specifically, markups) and thus accommodates higher calibrated input elasticities.

Drozd et al. (2021) splits the puzzle into a demand-side and a supply-side effect (they name these the substitution effect and the income effect in the paper). It argues that following a positive productivity shock abroad, imported inputs become cheaper relative to domestic labor, increasing labor supply and investment and leading to positive comovement. However, the same shock increases income and asset payouts to the domestic economy, leading to negative comovement. A solution is a short-run complementarity in domestic and imported inputs, which forces the domestic economy to increase its output to make use of cheaper imports. In comparison to their paper, I build a model that allows for multiple countries, sectors and an extensive margin of trade. Their insights about the Armington elasticity being a crucial parameter in determining comovement still hold, but extend to the trade elasticity as well.

This paper is mostly related to Huo et al. (2019), which develops a multi-country multi-sector Armington model to study comovement. I extend their analysis, providing an expression for the influence matrix in EK and MP, while also providing a stylized version of these models that clarifies the conditions needed for higher comovement without imposing correlated shocks, namely high input elasticities and low trade elasticities. The similarities between EK and MP, especially in the symmetric setting, are reminiscent of Arkolakis et al. (2012) results that under the Pareto assumption the demand-side elasticity is the tail parameter of the productivity distribution.

This paper is also related to the literature that studies trade and the volatility of income. Caselli et al. (2020) develop a multi-sector, multi-country EK model and argue that the effect of trade on income volatility is ambiguous in theory[2]. I show that, in the multisector case, trade can increase volatility even in the absence of specialization. I also provide a theoretical counterpart to the predictions in Giovanni & Levchenko (2009) that sector-level volatility is increasing in the intensity of trade, while the correlation of each sector with the aggregate is not.

[2]The evidence for which dominates is mixed. Kose et al. (2006) and Giovanni & Levchenko (2009) provide evidence that trade increases the variability of income, while Buch et al. (2009), Burgess & Donaldson (2010) and Haddad et al. (2013) present evidence that trade decreases the variability of income.

This paper is tangentially related to the multisector real-business-cycles literature started by Long Jr & Plosser (1983). My framework allows for a multicountry version of Dupor (1999)'s exercise. The latter debated Horvath (1998 and 2000) on at what rate the volatility of aggregate value added declines as the level of disaggregation increases[3]. Acemoglu et al. (2012) proved that the variance of output declines with the Euclidean norm of the influence vector of the economy, as opposed to the square root of the number of sectors[4]. Atalay (2017) builds on the intuitions of Horvath (2000), but instead of the sparseness of the input-use matrix, it focuses on the short-run low elasticities of substitution between inputs. He argues that to achieve a given target level of aggregate variation, one either has to assume that elasticities of substitution are high and shocks are correlated, or that elasticities are low and the shocks are uncorrelated.

1.2 Facts about comovement

The stylized fact at the root of the comovement puzzle is that bilateral GDP correlations are positively associated with bilateral trade intensities, i.e. the more two countries trade with one another relative to their GDPs, the more their GDPs comove. The slope of comovement regressions is usually small, but it is statistically significant and replicated in a plethora of papers (Frankel & Rose (1998), Clark & Van Wincoop (2001), Calderon et al. (2007), Johnson (2014), Liao & Santacreu (2015), de Soyres & Gaillard (2019)).

	ρ_{ij}			
	coeff	se	t	p
$\log(\text{trade}_{ij})$	0.022	0.003	6.508	0.000

Table 1. Trade-intensity is defined as total trade (exports plus imports) over the sum of GDPs at the country-pair level. The correlations are the same as presented in Figure (1) and encompass $N = 930$ pairs in time.

[3]Dupor (1999) proved that under relative homogeneity of the input structure, the standard deviation of aggregate GDP generated by uncorrelated sectoral shocks declines with the square root of the number of sectors, while Horvath (2000) proved that the speed of decline is much slower depending on the sparsity of the intermediate input-use matrix.

[4]David Baqaee and Emmanuel Farhi (2019 and 2020) pushed the literature beyond the first-order Hulten economies to show how nonlinearities lead shocks to critical input sectors to have a disproportional effect in aggregate output and to asymmetries between positive and negative shocks. Up to Atalay (2017), the literature concerned itself more with amplification than comovement. All frameworks assumed perfectly competitive markets and CES preferences and technology. Long Jr & Plosser (1983), Dupor (1999) and Acemoglu et al. (2012) are even more restrictive by assuming the Cobb-Douglas case, which puts them in the realm of Hulten economies (Hulten (1978)) and in which the impact of a shock to one sector of the economy to aggregate output is summarized by its Domar weight (Domar (1961)), i.e. its sales share in GDP.

The stylized fact of the quantity puzzle is the level of correlations, rather than its relationship to trade intensity. Figure (1) illustrates the positive comovement of real value added across countries. The point estimates of the correlation between G7 economies are positive and lay between 0.20 for Canada-France and 0.50 for US-Canada, and are close to 0.40 between the three European G7 economies. Other estimates, such as those for Eastern European economies and developing economies in Latin America and Asia are less precise, but also indicate a general rule of positive comovement.

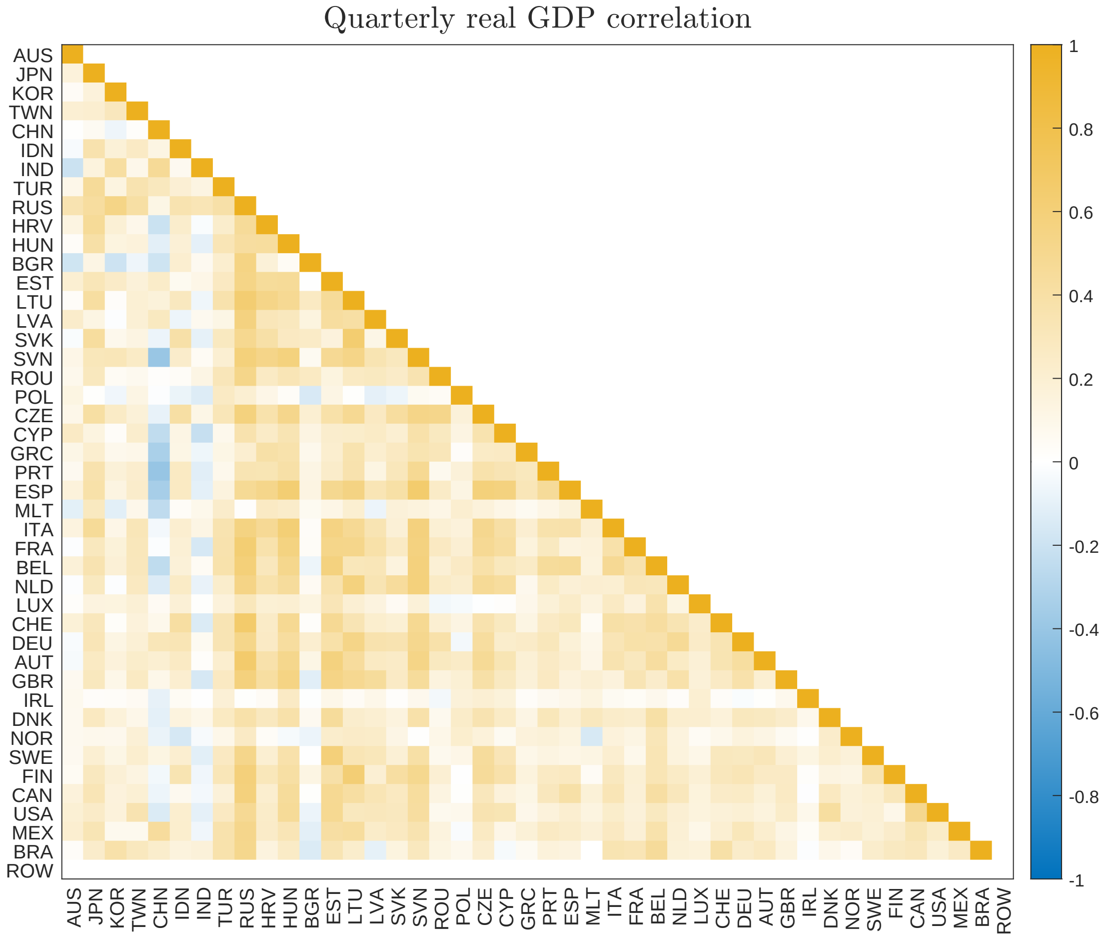

Figure 1. Real-GDP-growth correlation across countries listed in the WIOD data. Data for the US and the UK date back to 1947 and 1955, respectively. Data for Western Europe, Korea, Australia, Mexico and Japan starts in 1960, while for Eastern Europe, Brazil, India, Indonesia and Turkey ranges from 1990 to 2000. Russia and China's estimates are less precise as the data starts in 2008 and 2011, respectively. COVID quarters are omitted.

The third set of facts relates to the correlation between sectors within and across countries. Sector-level outputs are positively correlated within and across countries, but correlations are higher within countries, which is clear from a visual comparison between the charts in Figure (2). The fact that there is a positive correlation of output across sectors within a country is known at least since Hornstein & Praschnik (1997). The left-hand chart shows the correlation between sectors r and s in country i, while the right-hand chart shows the correlation between sectors r and s across countries i and j. On average, correlations between sectors within a country are two times higher than correlations between the same sectors across countries but could be as high as fifteen times higher.

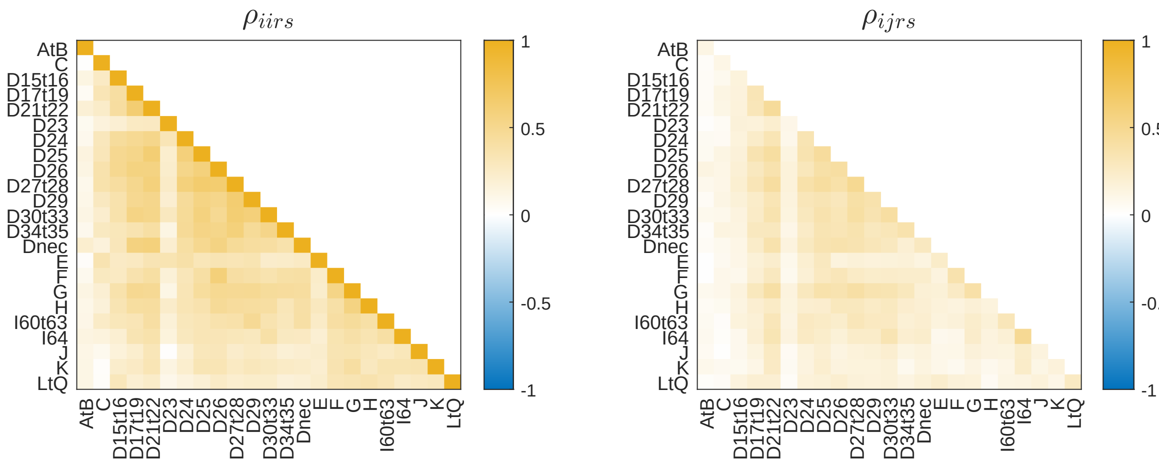

Figure 2. Sector-level real-output yearly growth correlations. Data comes from the Long-run WIOD and its social-economic accounts (Woltjer et al. (2021)). The figure presents the trade-intensity-weighted average of correlations across twenty-six countries and forty-five years.

Each data point in Figure (2) is generated as follows. I combine the long-run world input-output tables of Woltjer et al. (2021) with the world input-output tables in Timmer et al. (2015) into a dataset covering twenty-six countries over almost fifty years (1965-2014) that includes data on production and trade for twenty-three sectors. I compute the correlations of changes to sectoral output at 1995 prices[5] and weigh each observation using the trade intensity of each country-sector pair. Weights are normalized such that their sum is one, and guarantee that more importance is given to country-sector pairs that trade relatively more.

Sector-level output comovement between the same sector across countries and between different sectors across countries are not meaningfully different, which can be obersved through the differences between diagonal and non-diagonal elements of the right-hand chart of Figure (2). The data does not indicate that diagonal elements are on average smaller than their non-diagonal counterparts. A t-test indicates that, if anything, diagonal elements are slightly higher

[5]The authors use industry-level price data to compute real output. When the deflator of output is not available, the authors extrapolate the trend of the value-added deflator.

on average than off-diagonal elements. In this paper, I make the case that this stylized fact is extremely relevant as it can only be matched in theory when the short-run trade elasticity is very low, which in turn exacerbates the differences between frameworks with and without the extensive margin.

2 Theoretical framework

The world economy is comprised of J countries (indexed by i, j and k), each populated by a workforce of size L_j whose problem is aggregated into that of a representative consumer. In each country, there are S sectors (indexed by q, r and s) that use labor and intermediate inputs to produce, and of which outputs are used in the production of other sectors and/or are consumed by the representative consumer. In each sector s, varieties o_{js} can be produced, but are not necessarily produced. The set of varieties produced in j and available in country i is $O_{jis} \subseteq O_{is}$, where O_{is} is the set of varieties available in i from all countries including i itself. Time is discrete and denoted by t.

2.1 Model assumptions

Two sets of assumptions are common across frameworks: i) both the final and intermediate demand systems are CES; ii) transporting goods from one country to another is costly. In particular, there are *iceberg* costs which are paid in physical units of the transported good[6]. The cost associated with transporting sector-s goods from country i to j is τ_{ijs}.

Assumption 1 (Trade costs) *For a unit produced by sector s in country i to arrive in j, τ_{ijs} units are shipped, where $\tau_{ijs} \geq 1$ if $i \neq j$ and one otherwise. It is always less costly to send goods directly $\tau_{ijs} \leq \tau_{iks}\tau_{kjs}$ $\forall(i, j, k)$.*

The second set of assumptions imposes structure on preferences. Country j's representative consumer's problem is static and its aggregate consumption equals aggregate labor income plus any profits earned by producers, that is, $P_{jt}Q_{jt} = W_{jt}L_j + \bar{P}^j_{jt}$, where P^j_{jt} is the aggregate price index, W^j_{jt} are wages, $\bar{P}^j_{jt}$ are profits. Superscripts denote the currency of denomination.

Assumption 2 (Demand system) *The preferences of the representative consumer in any country j is represented by a CES utility function over varieties available in j and σ_s is the elasticity of*

[6]The assumption of iceberg costs are common in the trade literature, but also in international economics and finance in general. Samuelson (1954) uses them to study the transfer problem of Keynes-Ohlin and Obstfeld & Rogoff (2000) argue that these types of trade costs are a solution to common macroeconomic puzzles of the international real-business-cycles literature.

substitution between them. These varieties can be produced in any $i \in J$ countries in the world.

$$Q_{jst} = \left(\sum_i \int_{o_{is} \in O_{ijst}} Q_{ijst}(o_{is})^{\frac{\sigma_s - 1}{\sigma_s}} do_{is} \right)^{\frac{\sigma_s}{\sigma_s - 1}}$$

where $Q_{ijst}(o_{is})$ is the quantity of variety o_{is} from i and consumed in j. Each sector bundle is aggregated into aggregate consumption

$$Q_{jt} = \left(\sum_s \alpha_{js}{}^{\frac{1}{\chi}} Q_{jst}{}^{\frac{\chi - 1}{\chi}} \right)^{\frac{\chi}{\chi - 1}}$$

where each sector is valued according to a scalar α_{js} where $\sum_j \alpha_{js} = 1$, and where the elasticity of substitution between sectors is χ.

Following Assumption (2), expenditure-minimizing consumers purchase optimal quantities $Q_{ijst}(o_{is})$ of each variety and Q_{jst} of each sector-bundle according to the following demand functions

$$Q_{ijst}(o_{is}) = \left(\frac{P^j_{ijst}(o_{is})}{P^j_{jst}} \right)^{-\sigma_s} Q_{jst} \quad \text{and} \quad Q_{jst} = \alpha_{js} \left(\frac{P^j_{jst}}{P^j_{jt}} \right)^{-\chi} Q_{jt}$$

where $P^j_{ijst}(o_{is})$ is the price of variety o_{is} in j, P^j_{jst} sector-s's price index. Final spending in sector s as a share of total spending is defined as follows.

$$\Xi_{jst} \equiv \frac{P^j_{jst} Q_{jst}}{P^j_{jt} Q_{jt}} = \alpha_{js} \left(\frac{P^j_{jst}}{P^j_{jt}} \right)^{1-\chi}$$

The next set of assumptions imposes structure on technology. In particular, these are the production function and the assumption that intermediate inputs are aggregated in the same way as varieties are aggregated in the utility function.

Assumption 3 (Technology) *A potential producer of variety o_{js} has access to a technology that combines local labor $L_{Pjst}(o_{js})$ with inputs $M_{jrst}(o_{js})$ from all other sectors r, possibly sourced from all other countries i, and uses it to produce $Y_{jst}(o_{js})$ units of output*

$$Y_{jst}(o_{js}) = A_{jst} z_{js}(o_{js}) L_{Pjst}(o_{js})^{1-\gamma_{js}} M_{jst}(o_{js})^{\gamma_{js}}$$

where

$$M_{jst}(o_{js}) = \left(\sum_r \gamma_{jrs}{}^{\frac{1}{\chi}} M_{jrst}(o_{js})^{\frac{\chi - 1}{\chi}} \right)^{\frac{\chi}{\chi - 1}}$$

is the input bundle with γ_{jrs} determining the share of spending in each sector r. The input demand system has the same form as the consumer problem

$$M_{jrst}(o_{js}) = \left(\sum_i \int_{o_{ir} \in O_{ijrt}} M_{ijrst}(o_{ir})^{\frac{\sigma_r - 1}{\sigma_r}} do_{ir} \right)^{\frac{\sigma_r}{\sigma_r - 1}}$$

where $M_{ijrst}(o_{ir})$ is the quantity of a variety from sector r in country i used in sector s in country j. Every producer in sector s is subject to a Hicks-neutral-stochastic productivity shifter A_{jst}, which is the source of business cycles in the model. $z_{js}(o_{js})$ is the static level of productivity.

Following Assumption (3), cost-minimizing producers' optimal labor and input demand are described by the following equations

$$L_{Pjst}(o_{js}) = (1-\gamma_{js})\frac{MC^j_{jst}}{A_{jst}z_{js}(o_{js})}\frac{Y_{jst}(o_{js})}{W^j_{jt}} \quad \text{and} \quad M_{jrst}(o_{js}) = \gamma_{js}\Omega_{jrst}\frac{MC^j_{jst}}{A_{jst}z_{js}(o_{js})}\frac{Y_{jst}(o_{js})}{P^j_{jrt}}$$

where the input cost share of input r in sector s is defined as follows

$$\Omega_{jrst} \equiv \frac{P^j_{jrt}M_{jrst}}{P_{M}{}^j_{jst}M_{jst}} = \gamma_{jrs}\left(\frac{P^j_{jrt}}{P_{M}{}^j_{jst}}\right)^{1-\chi}$$

and where

$$MC^j_{jst} = \frac{W^j_{jt}{}^{1-\gamma_{js}}P_{M}{}^j_{jst}{}^{\gamma_{js}}}{(1-\gamma_{js})^{1-\gamma_{js}}\gamma_{js}{}^{\gamma_{js}}} \quad \text{and} \quad P_{M}{}^j_{jst} = \left(\sum_r \gamma_{jrs}P^j_{jrt}{}^{1-\chi}\right)^{\frac{1}{1-\chi}} \tag{2.1}$$

where MC^j_{jst} is the unit cost of producing in j[7]. Following these sets of common assumptions across the two frameworks, the first set of assumptions that differ depending on the framework is the distribution of productivity draws.

Assumption 4 (Productivity distribution) *In all countries j and sectors s, each variety o_{js} is produced with a specific efficiency $z_{js}(o_{js})$ drawn from a cumulative probability distribution $F_{js}(z)$.*
Assumption 4.1 *In the EK framework efficiencies are drawn from a Frechet distribution*

$$F_{js}(z) = e^{-T_{js}z^{-\theta_s}}$$

where $z > 0$ and $\theta_s > \sigma_s - 1$ and $T_{js} > 0$.
Assumption 4.2 *In the MP framework efficiencies are drawn from a Pareto distribution*

$$F_{js}(z) = 1 - \left(\frac{z}{\underline{z}_{js}}\right)^{-\kappa_s}$$

where $z > \underline{z}_{js} > 0$ and $\kappa_s > \sigma_s - 1$.

[7] Equations (2.1) are essential to finding analytical solutions in trade models that do not define an aggregate production function explicitly. The unit cost equations will replace the production function used in Huo et al. (2019) since they are equivalent by Shephard's Duality Theorem (Shephard (1953) and Uzawa (1964)). The Theorem does not necessarily hold without the assumption of perfect competition and with the assumption of increasing returns to scale, but in this particular case it remains true in MP.

As discussed extensively in the literature (see for example Costinot et al. (2012)), in the EK framework the scale parameters of the productivity distribution T_{js} are the source of absolute advantage in the model, while the shape parameters θ_s are the source of comparative advantage and higher values of θ_s increase the dispersion of productivity draws. In MP, $\underline{z}_{js}$ is the lowest efficiency value possible, and the shape parameter κ_s determines the dispersion of productivity draws across producers, with lower values of κ_s increasing the dispersion of productivity draws. The requirements $\kappa_s > \sigma_s - 1$ and $\theta_s > \sigma_s - 1$ guarantee the existence of an aggregate price index.

The final set of assumptions determines the market structure in the two different frameworks. In EK both consumers and producers search for the lowest-cost producer when sourcing final consumption and input, respectively. Perfect competition leads to marginal cost pricing, such that a producer will only sell to a market if it offers the lowest price among potential sources. In MP producers are monopolists and charge constant markups over marginal costs and pay fixed entry costs to operate in any given market.

Assumption 5 (Market Structure) *The market structure differs in the two frameworks.*
Assumption 5.1 *In EK buyers of variety o_s search the world for its cheapest source*

$$P^j_{jst}(o_s) = \min_i \left\{ P^j_{ijst}(o_s) \right\}$$

where

$$P^j_{ijst}(o_s) = \frac{1}{\varepsilon_{ijt}} \frac{MC^i_{ist}\tau_{ijs}}{A_{ist} z_{is}(o_s)}$$

is the price of sending sector-s goods from i to j and where ε_{ijt} is the nominal exchange rate that converts currency i into currency j. Perfect competition implies that these prices equal the unit costs adjusted for efficiency.
Assumption 5.2 *In MP there is a fixed measure of firms N_{is} that operates under restricted entry. A producer in i has to pay a sector-specific fixed cost f_{ijs} if it wants to sell in market j. These fixed costs are paid in labor at the source in every period*[8]*. Each producer charges a constant markup over efficiency-adjusted unit costs.*

$$P^j_{ijst}(o_s) = \frac{\sigma_s}{\sigma_s - 1} \frac{1}{\varepsilon_{ijt}} \frac{MC^i_{ist}\tau_{ijs}}{A_{ist} z_{is}(o_s)}$$

Assumption (5) part two implies that in MP only a fraction of N_{is} will be operating in any given country j. These are the producers that are above a cut-off productivity level $\underline{z}_{ijst}$ for which it is profitable to operate. Taken together with Assumption (4), it implies $N_{ijst} = N_{is}(\underline{z}_{ijst}/\underline{z}_{is})^{-\kappa_s}$ of firms from country i operating in country j.

[8]This assumption greatly simplifies the producer problem.

2.2 Nonlinear system and equilibrium conditions

A potential producer pays competitive prices P^j_{jrt} and W^j_{jt} to buy inputs from sector r and to rent labor in country j, respectively. The producers' first-order conditions pin down the demand for each input and factor at the variety level, and under CES aggregation these conditions determine factor and input demands at the sector level.

$$W_{jt}L_{Pjst} = {\mu_s}^{-1}(1-\gamma_{js})Y^j_{jst} \quad \text{and} \quad P^j_{jrt}M_{jrst} = {\mu_s}^{-1}\gamma_{js}\Omega_{jrst}Y^j_{jst}$$

Labor in MP is subscripted by P to denote that this is labor allocated to production, as opposed to labor allocated to pay the fixed costs of entering a market. μ_s are markups charged by producers, such that in MP $\mu_s = \sigma_s/(\sigma_s - 1)$ whereas in EK $\mu_s = 1$. Assumptions (**A**2)-(**A**5) lead to analytical expression both for sector-price indices and trade shares between countries. Sector s' price index in j depends on j's market access, which is an additive term of the product of productivity, unit costs and trade costs of all potential trading partners.

$$\text{Eaton-Kortum} \quad P^j_{jst} = \Upsilon_s \left[\sum_i T_{is}\left(\frac{MC^j_{ist}\tau_{ijs}}{A_{ist}}\right)^{-\theta_s}\right]^{-\frac{1}{\theta_s}} \tag{2.2a}$$

$$\text{Melitz-Pareto} \quad P^j_{jst} = \tilde{\Upsilon}_s \left[\sum_i \tilde{T}_{is}\left(\frac{MC^j_{ist}\tau_{ijs}}{A_{ist}}\right)^{-\kappa_s}\left(\frac{E^j_{jst}}{W^j_{it}f_{ijs}}\right)^{\frac{\kappa_s}{\sigma_s-1}-1}\right]^{-\frac{1}{\kappa_s}} \tag{2.2b}$$

where

$$\Upsilon_s = \Gamma\left(\frac{\theta_s-(\sigma_s-1)}{\theta_s}\right) \quad \text{and} \quad \tilde{\Upsilon}_s = (\sigma_s-1)^{\frac{1}{\sigma_s-1}-\frac{1}{\kappa_s}}\left(\frac{\sigma_s}{\sigma_s-1}\right)^{\frac{\sigma_s}{\sigma_s-1}-\frac{1}{\kappa_s}}\left(\frac{\kappa_s}{\kappa_s-(\sigma_s-1)}\right)^{-\frac{1}{\kappa_s}}$$

where $\tilde{T}_{is} = N_{is}{\underline{z}_{js}}^{\kappa_{js}}$. In EK the shape parameter of the Frechet distribution θ_s also determines the elasticity of prices to market access, while in MP that role is played by the shape parameter of the Pareto distribution κ_s. In MP, prices also depend on the ratio of j's total spending on good s to fixed costs paid on each potential source i to service j as well. Trade shares Π_{ijst} represent the fraction of j's total spending on sector s that is imported from country i.

$$\text{Eaton-Kortum} \quad \Pi_{ijst} = {\Upsilon_s}^{-\theta_s}T_{is}\left(\frac{MC^j_{ist}\tau_{ijs}}{A_{ist}P^j_{jst}}\right)^{-\theta_s} \tag{2.3a}$$

$$\text{Melitz-Pareto} \quad \Pi_{ijst} = \tilde{\Upsilon}_s^{-\kappa_s}\tilde{T}_{is}\left(\frac{MC^j_{ist}\tau_{ijs}}{A_{ist}P^j_{jst}}\right)^{-\kappa_s}\left(\frac{E^j_{jst}}{W^j_{it}f_{ijs}}\right)^{\frac{\kappa_s}{\sigma_s-1}-1} \tag{2.3b}$$

Trade shares are high when i) productivity in the source country is high; ii) the unit cost of production in the source country is low; iii) trade costs between countries are low; and/or iv) prices in the destination country are high. In addition, in MP trade shares also depend on total spending at the destination and fixed costs paid at the source. The latter follows from the cut-off productivity expression.

$$\underline{z}_{ijst} = \left[(\sigma_s - 1)\left(\frac{\sigma_s}{\sigma_s - 1}\right)^{\sigma_s} \frac{W^j_{it} f_{ijs}}{E^j_{jst}}\right]^{\frac{1}{\sigma_s - 1}} \frac{MC^j_{ist}\tau_{ijs}}{A_{ist}P^j_{jst}} \tag{2.4}$$

Aggregate price indices combine sector price indices according to the weight of a sector in preferences α_{js} and to the elasticity of substitution between sectors in final demand χ as follows.

$$P^j_{jt} = \left(\sum_s \alpha_{js} {P^j_{jst}}^{1-\chi}\right)^{\frac{1}{1-\chi}} \tag{2.5}$$

The final set of equations that describe the models impose that goods and labor markets are cleared in all countries. In the absence of any labor market frictions, all labor units must be employed at all points in time. This means that integrating over the labor units used in the production of all varieties o_{js} produced in j adds up to the labor units used by sector s and these in turn add up to the exogenous labor supply.

$$L_j = \sum_s L_{jst} = \sum_s \int_{o_{js} \in O_{js}} L_{jst}(o_{js}) do_{js}$$

The market for goods is assumed to clear so that the gross output of sector s in country j is used either as input and/or final consumption globally. The market clearing conditions form a system of $J \times S - 1$ independent equations which equate the j-currency amount produced to the sum of the j-currency amount consumed in the rest of the world weighted by their respective trade shares as they were defined in Equation (2.3).

$$Y^j_{jst} = \sum_i \Pi_{jist}\Big(P^j_{ist}Q_{ist} + \sum_r P^j_{ist}M_{isrt}\Big) \tag{2.6}$$

3 Equilibrium and linear solution

To find analytical expressions for the variance of output, I approximate to the first order the system of equations described in the previous section around a deterministic steady state. An equilibrium in these economies is a set of good prices and wages $\{P^j_{js}, W^j_j\}_{\forall j,s}$, labor allocations $\{L_{js}\}_{\forall j,s}$, nominal outputs $\{Y^j_{js}\}_{\forall j,s}$ and spending $\{Q^j_{js}, E^j_{jrs}\}_{\forall j,r,s}$ such that: i) consumers maximize utility, ii) firms maximize profits; and iii) goods and labor markets clear. Equations (2.2) - (2.4) and their linear approximations are described below.

3.1 Linear system

In what follows, lower-case letters represent percent changes to upper-case variables. The first set of equations equates percent changes in unit costs to changes in sectoral output weighted by sector labor shares, added to changes in input prices weighted by their respective input shares. The expression can be simplified to equate percent changes in unit costs to percent changes in output and sector productivity and prices only, by using the producers' first-order conditions.

$$mc^j_{jst} = (1-\gamma_{js}) \sum_r \Lambda_{jr} y^j_{jrt} + \gamma_{js} \sum_r \Omega_{jrs} p^j_{jrt} \tag{3.1}$$

where $\Lambda_{jr} = L_{jr}/L_j$ are employment shares. The next set of equations is the approximated version of Equations (2.2), and they equate percent changes in sector prices to changes in unit costs net of changes in productivity, weighted by trade shares. In this paper, I ignore changes in trade costs as a source of business-cycle fluctuations. In MP two additional forces are acting to change prices. The first is changes to the fixed costs that need to be paid to access foreign markets, which in this case amount to changes in wages in the source country. The second is changes in total spending in a sector in the destination country.

$$\text{Eaton-Kortum} \quad p^j_{jst} = \sum_i \Pi_{ijs} \left(mc^j_{ist} - a_{ist} \right) \tag{3.2a}$$

$$\text{Melitz-Pareto} \quad p^j_{jst} = \sum_i \Pi_{ijs} \left[mc^j_{ist} - a_{ist} - \phi_s \left(e^j_{jst} - w^j_{it} \right) \right] \tag{3.2b}$$

where $\phi_s = 1/(\sigma_s - 1) - 1/\kappa_s$. Equations (2.3) are approximated to equate percent changes in trade shares to changes in productivity net of changes in variable unit costs at the source country and changes in sectoral prices at the destination country. In MP, the assumption of increasing returns to scale implies additionally that trade shares move with changes in sector spending at the destination country net of percent changes in wages, which captures changes in the fixed costs of exporting (i.e. the extensive margin).

$$\text{Eaton-Kortum} \quad \pi_{ijst} = \theta_s \left(p^j_{jst} - mc^j_{ist} + a_{ist} \right) \tag{3.3a}$$

$$\text{Melitz-Pareto} \quad \pi_{ijst} = \kappa_s \left(p^j_{jst} - mc^j_{ist} + a_{ist} \right) + \kappa_s \phi_s \left(e^j_{jst} - w^j_{it} \right) \tag{3.3b}$$

The final set of equations combines the optimality conditions of both producers, consumers and the budget constraint to approximate the market clearing conditions as follows

$$\begin{aligned} y^j_{jst} = & \sum_i \Pi_{jis} \sum_r \left(E^j_{isr} / Y^j_{js} \right) y^j_{irt} + \sum_i \Pi_{jis} \left(E^j_{is} / Y^j_{js} \right) \pi_{jist} \\ & + (\chi - 1) \sum_i \Pi_{jis} \sum_r (1 - \gamma_{ir} \mu_{ir}{}^{-1}) \Xi_{is} \left(Y^j_{ir} / Y^j_{js} \right) \left(\sum_q \Xi_{ir} p^j_{iqt} - p^j_{ist} \right) \\ & + (\chi - 1) \sum_i \Pi_{jis} \sum_r \gamma_{ir} \mu_{ir}{}^{-1} \Omega_{isr} \left(Y^j_{ir} / Y^j_{js} \right) \left(\sum_q \Omega_{iqr} p^j_{iqt} - p^j_{ist} \right) \end{aligned} \tag{3.4}$$

where $E^j_{js} = \sum_r E^j_{jsr}$ and $E^j_{jsr} = \left((1 - \gamma_{jr} \mu_{jr}{}^{-1}) \Xi_{js} + \gamma_{jr} \mu_{jr}{}^{-1} \Omega_{jsr} \right) Y^j_{jr}$. Finally, equations (2.5) are linearized to equate changes in the price index to a sum of changes in sectoral prices weighted by expenditure shares.

$$p^j_{jt} = \sum_s \Xi_{js} p^j_{jst} \tag{3.5}$$

Equations (3.2) and (3.3) capture the main differences between the two frameworks. While in EK both trade shares and prices depend only on steady-state trade shares and the costs associated with producing abroad, in MP prices in country j are affected by j's own total spending in sector s. This happens because increasing returns to scale incentivize firms from other countries to enter j when it increases its spending, provided that wages in other countries have not risen enough to discourage entry. The importance of these different variables that play a dynamic role in MP depends on the value of ϕ_s. From its definition based on deep parameters, it is clear that $\phi_s \in (0,1)$ since $\kappa_s > \sigma_s - 1$. A more intuitive interpretation of ϕ_s follows from the next proposition.

Proposition 1 *Following assumptions (A2)-(A5) total spending on labor used for entry is a constant fraction of gross output at the sector level. In particular, it follows that in MP*

$$W^j_{jt} L_{Ejst} = {\mu_s}^{-1} \phi_s Y^j_{jst}$$

Proof *Appendix A.6.3* ■

Proposition (1) is true in MP with restricted entry and states that ϕ_s is the share of gross output that is paid to entry labor as labor income adjusted for markups. This means that the partial elasticity of trade shares to total spending at the destination and wages at the source depends on κ_s, which can be thought of as the equivalent of the trade elasticity θ_s in EK[9] but adjusted for the size of entry labor income in gross output.

3.2 Linear solution

Equations (3.1)-(3.4) form a set of $3(JS) - 1 + (JS)^2$ equations in $3(JS) + (JS)^2$ variables. Combined with a numeraire equation from (3.5), one can solve this system to express changes in real output quantities as a function of changes in sectoral TFP alone. Before proceeding with the steps used to solve for the influence matrix I define the main matrices that compose the system. Let $\mathbf{Y}$ be a $JS \times JS$ matrix with the vector of gross outputs Y^j_{js} in its diagonal and zero elsewhere. Let $\mathbf{\Pi}$ be a $JS \times JS$ matrix with domestic shares ($j = i$) in the main diagonal, bilateral trade shares in the main diagonals of the $J(J-1)$ non-diagonal blocks, and zeros

[9] θ determines how sensitive trade flows are to changes in trade costs, hence it is called the trade elasticity.

whenever $r \neq s$, such that its columns sum to one[10]. The matrix $\mathbf{\Omega}$ is a $JS \times JS$ block-diagonal matrix with its j-th diagonal block being a $S \times S$ matrix of input cost shares of country j and of which rows sum to one. With slight abuse of notation, let γ, κ, θ and ϕ be $JS \times JS$ diagonal matrices holding the input elasticities, trade elasticities and entry labor in their main diagonal, respectively.

There are two main steps leading to the influence matrix. The first step is to substitute the trade-share Equations (3.3) into the market-clearing Equations (3.4) and to sum across sources such that price Equations (3.2) solve for prices as a function of quantities alone[11]. These equations are then stacked in a linear system and combined with the stacked price equations to solve for prices as a function of quantities. In EK the price equation combined with market clearing leads to

$$p_t^j = -\big[\ \underbrace{\mathbf{\Pi}(\mathbf{I}-\mathbf{D})}_{\text{Demand exposure}} \quad + \quad \underbrace{(\mathbf{I}-\mathbf{\Pi}\mathbf{M})\,\theta}_{\text{Trade exposure}} \quad - \quad \underbrace{(\chi-1)\mathbf{\Pi}\mathbf{X}}_{\text{Expenditure switching}}\ \big]^{+}\mathbf{\Pi}(\mathbf{I}-\mathbf{D})y_t \tag{3.6}$$

which can be written simply as $p_t^j = -\mathbf{P}_{\text{EK}}y_t$, where $\mathbf{P}_{\text{EK}}$ collects the three components of the relationship between prices and quantities. These are: i) $\mathbf{I}_{JS} - \mathbf{D}$ is reminiscent of a Leontief inverse (Leontief (1936, 1986)), ii) $\mathbf{M}$ is a trade-exposure matrix, and iii) $\mathbf{X}$ is an expenditure switching matrix. The elements of matrix $\mathbf{D}$ represent the total spending exposure such that $(\mathbf{D})_{ijrs} = \Pi_{ijr}E_{jrs}^j/Y_{ir}^j$. It is reminiscent of a Leontief matrix since in the simple input-output model a similar clearing system solves for the matrix of input requirements that equates any given input vector to a final demand vector.

The elements of matrix $\mathbf{M}$ represent the trade exposure of sector s in j to the same sector s in some other country i. This part of $\mathbf{P}$ contains the effect on output that comes through changes in trade shares in response to shocks. Elements of total trade exposure are positive in the main diagonal, zero whenever $r \neq s$, and negative when $i \neq j$ and $r = s$. The elements are high in

[10]As an example, in a two-country two-sector world economy this matrix would look like the following

$$\mathbf{\Pi} = \begin{pmatrix} \Pi_{111} & 0 & \Pi_{211} & 0 \\ 0 & \Pi_{112} & 0 & \Pi_{212} \\ \Pi_{121} & 0 & \Pi_{221} & 0 \\ 0 & \Pi_{212} & 0 & \Pi_{222} \end{pmatrix}$$

[11]In the case of EK this is equivalent to

$$\sum_j \Pi_{jks}y_{jst}^j - \sum_{j,i}\Pi_{jks}\Pi_{jis}\frac{E_{is}^j}{Y_{js}^j}y_{ist}^j = -\theta_s \underbrace{p_{kst}^j}_{\sum_k \Pi_{jks}(mc_{jst}^j - a_{jst})} + \theta_s \sum_{j,i}\Pi_{jks}\Pi_{jis}\frac{E_{is}^j}{Y_{js}^j}p_{ist}^j$$

where it was imposed that $Y_{js}^j = \sum_i \Pi_{jis}\sum_r E_{isr}$ and where the percent change in gross output can be reexpressed as the sum of changes in prices and in quantities $p_{jst}^j + y_{jst}$.

absolute terms if both i and j are important destinations of good s, i.e Π_{kis} and Π_{kjs} are high for all k, and if j has high spending E^j_{js} relative to gross output Y^j_{ks} in all potential partners k.

$$(\mathbf{I}-\mathbf{\Pi M})_{ijrs} = \begin{cases} 0 & \text{if } r \neq s \\ 1 - \sum_k (\Pi_{kjs})^2 E^j_{js}/Y^j_{ks} & \text{if } i = j \\ -\sum_k \Pi_{kis}\Pi_{kjs} E^j_{js}/Y^j_{ks} & \text{otherwise} \end{cases}$$

Figure (3) illustrates that trade intensity rather than the absolute size of gross output dominates the elements of trade exposure: even though gross output in China has grown both in the manufacturing of motor vehicles and computers, only in the latter did trade exposure meaningfully change. Finally, matrix $\mathbf{X}$ is the expenditure switching matrix and its elements contain information on the extent to which input or final spending shares change following shocks.

$$(\mathbf{X})_{ijrs} = \begin{cases} \sum_k \Pi_{ikr}\Pi_{jks} \sum_q \left[\Omega_{krq}\Omega_{ksq}\gamma_{kq} + \Xi_{kr}\Xi_{ks}(1-\gamma_{kq})\right] \left(Y^j_{kq}/Y^j_{js}\right) & \text{if } r \neq s \\ -\sum_k \Pi_{ikr}\Pi_{jks} \sum_q \left[(1-\Omega_{krq})\,\Omega_{ksq}\gamma_{kq} + (1-\Xi_{kr})\,\Xi_{ks}(1-\gamma_{kq})\right] \left(Y^j_{kq}/Y^j_{js}\right) & \text{otherwise} \end{cases}$$

In MP the same substitution of the trade-share equations into market clearing leads to a different expression due to increasing returns to scale and the extensive margin. In this case, the price equations stacked in matrix form are the following.

$$p^j_t = -\phi e^j_t + \mathbf{\Pi}\left(mc^j_t - a_t + \phi w^j_t\right)$$

Expressing changes in total spending as a function of gross output and price changes allows one to rewrite the final system of equations in a similar form to that of EK, but matrices $\mathbf{D}$ and $\mathbf{X}$ are adjusted accordingly[12].

$$p^j_t = -\left[\mathbf{\Pi}\left(\mathbf{I}-\mathbf{\Psi}_D\mathbf{D}\right) + \left(\mathbf{I}-\mathbf{\Pi M}\right)\kappa - \mathbf{\Pi}\mathbf{\Psi}_D\mathbf{X}\right]^{+}\mathbf{\Pi}\left(\mathbf{I}-\mathbf{\Psi}_D\mathbf{D}\right)y_t \tag{3.7}$$

where the latter is written simply as $p^j_t = -\mathbf{P}_{\text{MP}}y_t$ and where $\mathbf{\Psi}_D = \mathbf{I} + \left(\mathbf{I} - (\mathbf{M\Pi})^{-1}\right)\kappa\phi$ reweights the elements of the demand-exposure and expenditure-switching matrices with the inverse of the trade-exposure matrix to account for the extensive margin effect acting through expenditures and wages.

Once prices are expressed as a function of quantities, the supply side of the model closes the system to express quantities as functions of exogenous shocks only. This is done by combining

[12]The substitution is as follows

$$\sum_k \Pi_{kjs} y^j_{jst} = \cdots - \kappa_s(\underbrace{p^j_{jst} + \phi_s e^j_{jst}}_{\sum_k \Pi_{kjs}(mc^j_{jst} - a_{jst} + \phi_s w^j_{it})}) + \sum_{k,i} \frac{\Pi_{kjs}\Pi_{jis}\kappa_s E^j_{is}}{Y^j_{js}}(p^j_{ist} + \phi_s e^j_{ist}) \cdots$$

where the difference between frameworks becomes clear as it introduces the extensive margin effect captured by the change in total expenditures and works through both a direct demand and an expenditure switching channel.

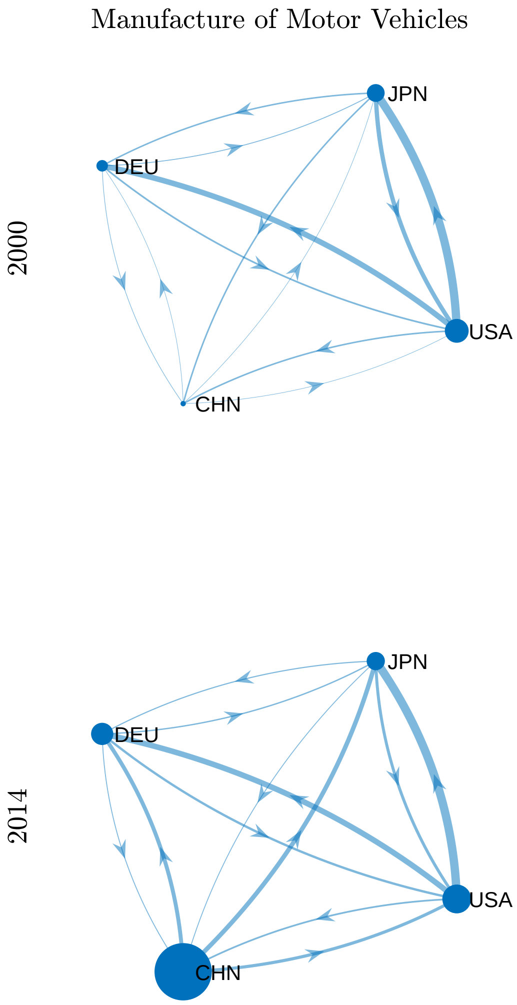

Figure 3. Elements of the trade-exposure matrix. The left panels display the elements of sector c29 “Manufacture of motor vehicles, trailers and semi-trailers", while the right panels display the elements of sector c26 “Manufacture of computer, electronic and optical products". The figure has select countries for the years 2000 and 2014 and node sizes are proportional to gross output.

the unit cost equations (3.1) and the price equations (3.2). The unit costs equations are the same across frameworks and are the following in matrix form

$$mc_t^j = (\mathbf{I} - \gamma)\mathbf{\Lambda}' y_t^j + \gamma \mathbf{\Omega}' p_t^j$$

where $\mathbf{\Lambda}$ is a block-diagonal matrix where each j block has column vectors of employment shares repeated S times. The solution of real output as a function of shocks alone involves two supply-side matrices. The matrix $\mathbf{G}$ is the Gosh inverse matrix (Gosh 1958) and describes the supply side exposure of country-sector pairs to one another by capturing all the paths of influence of any country-sector pair (i, r) on any other pair (j, s) through the international input-output network, and the form of this matrix is framework-specific. In the EK framework

$$\mathbf{G}_{\text{EK}} = \mathbf{\Pi} \left(\mathbf{I}_{JS} - \gamma \mathbf{\Omega}' \mathbf{\Pi}\right)^{-1}$$

while in the MP framework, the Ghosh inverse is augmented for expenditure switching, as changes in country-j's sector-s's price index depends on changes in j's own expenditure in sector s. In particular, an increase in expenditures decreases the price index as it leads to the entry of foreign firms into the domestic market, which is a market-size effect that is not present in EK. It follows that

$$\mathbf{G}_{\text{MP}} = \mathbf{\Psi}_G \mathbf{\Pi} \left(\mathbf{I}_{JS} - \gamma \mathbf{\Omega}' \mathbf{\Psi}_G \mathbf{\Pi}\right)^{-1}$$

where $\mathbf{\Psi}_G = \left(\mathbf{I}_{JS} + \phi \mathbf{\Pi} (\mathbf{M}\mathbf{\Pi})^{-1} \mathbf{X}\right)^{-1}$. This adjustment matrix again follows from prices moving with a country's own expenditure, but this time through expenditure switching. Finally, the matrix $\mathbf{H}$ is an employment share matrix. In EK $\mathbf{H}_{\text{EK}} = (\mathbf{I}_{JS} - \gamma)\mathbf{\Lambda}'$, while in MP $\mathbf{H}_{\text{MP}} = ((\mathbf{I}_{JS} - \gamma) + \phi)\, \mathbf{\Lambda}' - \phi (\mathbf{M}\mathbf{\Pi})^{-1} \mathbf{D}$. The first adjustment in MP is intuitive as ϕ can be thought of as the share of entry labor income in gross output, but the second adjustment is less intuitive as it involves demand-side matrices. The adjustment introduces a bilateral connection through the impact that entry labor introduces between source-destination pairs. From these definitions, a Theorem follows.

Theorem 1 (The influence matrix) *If assumptions **A2-A5** hold, then the elasticity of real output in any country-sector pair (j, s) to a shock in any other country-sector pair (i, r) is characterized by the following linear system.*

$$y_t = \left[\boldsymbol{I}_{JS} - (\boldsymbol{I}_{JS} - \mathbf{G}\ \mathbf{H})(\boldsymbol{I}_{JS} - \mathbf{P})\right]^{-1} \mathbf{G}\ a_t \tag{3.8}$$

Proof *Appendix B* ■

which can be written simply as $y_t = \mathbf{\Gamma} a_t$. The expression in Theorem 3.8 is true in both frameworks, albeit with matrices that differ in their components and its structure is common to what is found in the literature.

4 Stylized economy

To clarify the intuition of Theorem (1), in this section I develop a stylized version of the world economy inspired by the exercise in Dupor (1999). He developed a closed economy model where equal weights $1/S$ are given to each sector in preferences, where each sector produces with the same input elasticity γ and is subject to idiosyncratic productivity shocks. I assume all countries and sectors are symmetric such that each matrix composing the influence matrix is symmetric and has a maximum of four distinct elements, namely one that determines the exposure of a sector to itself, one which determines the exposure of a sector to other sectors in the same country, one that determines the exposure of a sector to the same sector in a different country, and finally one that determines the exposure of a sector to some other sector in some other country.

Assumption 6 (Stylized economy) *The stylized world economy is composed of J countries and S sectors. I assume that: i) trade costs are the same across all country-sector pairs $\tau_{ijs} = \tau \ \forall(i, j, s)$; ii) preferences are the same in all countries such that $\alpha_{js} = \alpha = 1/S \ \forall(j, s)$, $\sigma_s = \sigma \ \forall s$; iii) technology is the same is all country-sector pairs $\gamma_{js} = \gamma$, $\gamma_{jrs} = 1/S$; iv) producers draw their productivity parameters from the same distribution in all country sector pairs; v) there is no expenditure switching $\chi = 1$; and v) domestic entry cost to foreign entry costs is a constant $f_{iis} = f^{-1} f_{ijs}$, $f \geq 1$.*

The following lemma defines a proxy for trade intensity which is guaranteed to be between zero and one which aides in developing the intuition behind the influence matrix.

Lemma 1 (Trade intensity) *Define the trade intensity as $\eta = J\Pi$, that is, the number of countries times the trade share. If A6 then $0 < \eta < 1$.*
Proof: *It follows from the definitions that*

$$J\Pi = \frac{J\tau^{-\theta}}{1 + (J-1)\tau^{-\theta}} < 1 \Rightarrow J\tau^{-\theta} < 1 + (J-1)\tau^{-\theta} \Rightarrow 0 < 1 - \tau^{-\theta} \ \blacksquare$$

Following A6 all the matrices in the system leading to the influence matrix are symmetric. One can use these results to conjecture an analytical solution for all the elasticities in the influence matrix.

Conjecture 1 *Given Assumption (6) the elements of the influence matrix have the following structure. The impact of a sector on the same sector abroad ($r = s$) is*

$$(\mathbf{\Gamma})_{ijrr} = -\frac{1}{J}\frac{S-1}{S}\left(\lambda^D - 1\right) + \frac{1}{JS}\frac{1}{1-\gamma}\left(1 - \lambda^G\right)$$

while the impact of a sector on a different sector abroad ($r \neq s$) is

$$(\mathbf{\Gamma})_{ijrs} = \frac{1}{JS}\left(\lambda^D - 1\right) + \frac{1}{JS}\frac{1}{1-\gamma}\left(1 - \lambda^G\right)$$

The impact of a sector on itself is

$$(\mathbf{\Gamma})_{iirr} = 1 + \frac{1}{S}\frac{\gamma}{1-\gamma} + \frac{J-1}{J}\left(\frac{S-1}{S}\left(\lambda^D - 1\right) - \frac{1}{S}\frac{1}{1-\gamma}\left(1 - \lambda^G\right)\right)$$

while its impact on a different sector in the same country is

$$(\mathbf{\Gamma})_{iirs} = \frac{1}{S}\frac{\gamma}{1-\gamma} - \frac{J-1}{J}\left(\frac{1}{S}\left(\lambda^D - 1\right) + \frac{1}{S}\frac{1}{1-\gamma}\left(1 - \lambda^G\right)\right)$$

where in EK

$$\lambda^D_{EK} = 1 + \left[\theta(1-\eta) - (1-\theta)\right]\eta \quad \textit{and} \quad \lambda^G_{EK} = \frac{\theta + (1+\theta)(1-\eta)}{\theta + (1+\theta)(1-\eta) + \frac{\eta}{1-\gamma}}$$

and in MP

$$\lambda^D_{MP} = 1 + \left[\kappa(1-\eta) - (1-\kappa)\right]\eta \quad \textit{and} \quad \lambda^G_{MP} = \frac{\kappa(1+\phi) + (1+\kappa(1+\phi))(1-\eta)}{\kappa(1+\phi) + (1+\kappa(1+\phi))(1-\eta) + \frac{\eta}{1-\gamma}}$$

Intuition *The conjecture is based on induction, that is, solving the system for the first values of J and S and guessing the solution holds. The symmetry of $\mathbf{\Pi}$ and the fact that the sum of symmetric matrices is symmetric and that the inverse of a symmetric matrix is symmetric, both $\mathbf{P}$ and $\mathbf{\Gamma}$ are symmetric. These matrices are diagonalizable, that is, one can write $\mathbf{P} = \sum_n q^P_n(1/\lambda^P_n)(q^P_n)'$, where λ^P_n are the non-zero eigenvalues of $\mathbf{P}$ and q^P_n their associated orthonormal eigenvectors. The same is true for $\mathbf{\Gamma}$.*

Conjecture (1) describes how these elasticities vary across frameworks. The expressions of EK and MP are very similar, so much so that if $\phi = 0$ and if $\kappa = \theta$ both trade shares and elasticities in the influence matrix are the same. In the limit as $\tau^{-\theta}$ goes to zero (autarky) the two models are also equivalent, which follows directly from Lemma (1). These results are not surprising in light of Arkolakis et al. (2012). The conjecture also shows that in the stylized economy the influence matrix is simplified to four different elasticities, namely a sector's elasticity to its shocks, to shocks to different sectors in the same country, from the same sector abroad and different sectors abroad.

More importantly, supply and demand-side exposure in the stylized economy are each encoded into a function of the trade elasticity, the input elasticity and the trade intensity. λ^G aggregates all the information on supply-side exposure and it follows that

$$\lambda^G_{EK} \in \left[\frac{(1-\gamma)\theta}{1+(1-\gamma)\theta}, 1\right] \quad \lambda^G_{MP} \in \left[\frac{(1-\gamma)\kappa(1-\phi)}{1+(1-\gamma)\kappa(1-\phi)}, 1\right]$$

and it reaches its highest value of one in autarky, its lowest value of zero in free trade, and is always decreasing in the intensity of trade, i.e. $\partial\lambda^G/\partial\eta < 0$. Moreover, it is increasing in the trade elasticity $\partial\lambda^G/\partial\theta > 0$ and decreasing in the input elasticity $\partial\lambda^G/\partial\gamma < 0$. This force is always a source of positive comovement of sectors across countries and represents the direct impact of shocks through the cost structure. It is the same between different and similar sectors across countries due to sectors being undifferentiated in the production function. Whenever λ^G is low, the impact across countries is high, and the opposite is true when λ^G is low.

On the other hand, λ^D aggregates all information on demand-side exposure. It sits between one in autarky and θ (κ) in free trade. The demand-side exposure differentiates the response of different sectors versus the same sector across countries. It follows that $\partial\lambda^D/\partial\theta > 0$ such that if the trade elasticity is high, then a positive productivity shock to s in i should reduce the output of the same sector s abroad. In an open economy producers and consumers "insure" against negative shocks to specific sectors by buying from the same sector in other countries in the world. Trade intensity has an ambiguous effect on demand exposure, that is $\partial\lambda^D/\partial\eta \lesseqgtr 0$ and is more likely to be negative if the trade elasticity is low.

The elasticities to shocks to foreign countries are zero in autarky. $(\mathbf{\Gamma})_{ijrs}$ is positive, while $(\mathbf{\Gamma})_{ijrr}$ may be negative if trade intensity and elasticity are high. The elasticities of output to shocks within a country are also determined by λ^G and λ^D. Supply-side exposure of a sector to itself is the same as its exposure to other sectors of the same country since the assumption of symmetry imposes that a sector uses its own output as input with the same intensity as it uses other sectors'. When demand exposure λ^D is high, productivity shocks to a sector will lead to a higher response of this sector's output through a market stealing effect, that is, this sector will produce more at the expense of the same sector elsewhere.

4.1 Sector-level volatility and comovement

The forces that shape the elasticities that populate the influence matrix are the same forces that determine comovement both at the aggregate and at the sectoral level, that is, it also follows from the conjecture that the variance and covariance of any country-sector pair are also a function of the number of countries, the number of sectors and the eigenvalues of the influence matrix.

Corollary 1 (Sector variances and correlations) *The standard deviation of the output of a sec-*

tor s in country j is the following.

$$\frac{\sigma_{js}}{\sigma_a} = \left[\underbrace{\frac{S-1}{S} + \frac{1}{S}\left(\frac{1}{1-\gamma}\right)^2}_{\textit{Autarky}} + \frac{J-1}{J}\left(\frac{S-1}{S}\left(\left(\lambda^D\right)^2 - 1\right) - \frac{1}{S}\left(\frac{1}{1-\gamma}\right)^2\left(1 - \left(\lambda^G\right)^2\right)\right)\right]^{\frac{1}{2}}$$

The correlations of real output between any country pair and sectors r and s are

$$\rho_{iirs} = (\sigma_{is}\sigma_{ir})^{-1}\left[\frac{1}{S}\left(\frac{1}{1-\gamma}\right)^2 - \left(\left(\lambda^D\right)^2 - 1\right) + \frac{1}{S}\frac{J-1}{J}\left(\frac{1}{1-\gamma}\right)^2\left(1 - \left(\lambda^G\right)^2\right)\right]$$

$$\rho_{ijrr} = (\sigma_{ir}\sigma_{jr})^{-1}\left[-\frac{S-1}{S}\frac{1}{J}\left(\left(\lambda^D\right)^2 - 1\right) + \frac{1}{JS}\left(\frac{1}{1-\gamma}\right)^2\left(1 - \left(\lambda^G\right)^2\right)\right]$$

$$\rho_{ijrs} = (\sigma_{ir}\sigma_{js})^{-1}\left[\frac{1}{JS}\left(\left(\lambda^D\right)^2 - 1\right) + \frac{1}{JS}\left(\frac{1}{1-\gamma}\right)^2\left(1 - \left(\lambda^G\right)^2\right)\right]$$

Proof *Appendix C* ■.

Corollary (1) states that sectoral variance can be separated into an autarky component, which is common across frameworks, and a trade component which varies. An unambiguous prediction about sector-level volatility is that it increases with the trade elasticity, that is, $\partial\sigma^2_{js}/\partial\theta > 0$. When the trade elasticity is high it is easier to substitute to and away from any given country-sector pair, hence shocks lead to substantial reallocations across countries leading to higher volatility at the sector level. The higher the number of sectors the more like this is to be true. A second unambiguous prediction is that sector-level volatility is increasing in the input elasticity $\partial\sigma^2_{js}/\partial\gamma > 0$. The more sectors rely on inputs to produce, the more volatile each sector will be.

Corollary (1) also illustrates that in theory the relationship between trade intensity and sector-level volatility is ambiguous, that is $\partial\sigma^2_{js}/\partial\eta$ has an unclear sign[13]. Through the cost channel λ^G, trade lowers sector-level volatility by allowing sectors to have a more diversified set of suppliers. The flip side is that the demand for the output of a sector might become more volatile with trade through λ^D. The higher the trade elasticity the more likely the latter is to dominate and the more likely trade is to be a source of higher sector-level volatility[14].

[13]The slope has the following expression

$$2\frac{J-1}{JS}\left[(S-1)\lambda^D\underbrace{\frac{\partial\lambda^D}{\partial\eta}}_{\lessgtr 0} + \left(\frac{1}{1-\gamma}\right)^2\lambda^G\underbrace{\frac{\partial\lambda^G}{\partial\eta}}_{<0}\right]$$

[14]Giovanni & Levchenko (2009) estimate the impact of trade intensity on sector-level volatility as a linear relationship between sector-level variance and trade-to-GDP ratio and finds a slope ranging from 0.15 to 0.23. The estimates of Giovanni & Levchenko (2009) for a case of $S = 28$ sectors and $J = 61$ require a trade elasticity parameter of roughly 0.6, which is in line with the literature only as a short-run elasticity as in Boehm et al. (2020).

In the symmetric economy, there are three different sets of correlation parameters, namely the correlation between different sectors within a country, the correlation of the same sector across countries and the correlation of different sectors across countries, each of which is a combination of the same components λ^D and λ^G. Figure (4) plots the correlations of sector-level output across different countries in the stylized economy for both high and low levels of trade elasticity. Following Corollary (1), if the trade elasticity is high then the correlation of the same sector across countries should be negative even when the number of sectors is high. This follows from the market stealing effect present in trade models.

4.2 Aggregate volatility and comovement

Aggregate variances and correlations are combinations of sectoral variances and covariances. Consequently, they are also functions of λ^D and λ^G and thus are still combinations of the same two forces that determine variances and correlations at the sector level.

Corollary 2 (Aggregate variance and correlation) *The standard deviation of aggregate real output in country j is the following*

$$\frac{\sigma_j}{\sigma_a} = \frac{1}{\sqrt{S}} \left[\underbrace{\frac{S-1}{S} + \left(\frac{1}{1-\gamma}\right)^2}_{autarky} + \frac{S-1}{S}\frac{J-2}{J}\left((\lambda^D)^2 - 1\right) - \frac{J-1}{J}\left(\frac{1}{1-\gamma}\right)^2 \left(1 - (\lambda^G)^2\right) \right]^{\frac{1}{2}}$$

while aggregate comovement is the following

$$\rho_{ij} = \frac{1}{J} \frac{\left(\frac{1}{1-\gamma}\right)^2 \left(1 - (\lambda^G)^2\right)}{\frac{S-1}{S} + \left(\frac{1}{1-\gamma}\right)^2 + \frac{S-1}{S}\frac{J-2}{J}\left((\lambda^D)^2 - 1\right) - \frac{J-1}{J}\left(\frac{1}{1-\gamma}\right)^2 \left(1 - (\lambda^G)^2\right)}$$

Proof: *The proof uses $var(y_{jt}) = \left(\frac{1}{S}\right)^2 var\left(\sum_s y_{jst}\right)$[15] while covariances are calculated from $cov(y_{jt}, y_{it}) = \mathbb{E}(y_{jt}y_{it})$, see Appendix C.*

Just as in Dupor (1999), in the symmetric economy the baseline convergence rate of the standard deviation of aggregate is the square root of the number of sectors. This result is not changed in the open economy, which in itself is an interesting result, and the intuition for it is that countries in the open economy are repeated set of sectors of which impact to other countries decreases as the number of countries increases. A novel result in Corollary (2) is that the baseline rate of convergence of the correlation of aggregate output is the number of countries, which is intuitive: the more symmetric countries there are, the lower the impact of each country on any other and the lower their comovement. These results are made formal in what follows.

[15] A similar expression can be found in Koren & Tenreyro (2007).

correlations between sectors across countries, ρ_{ij}

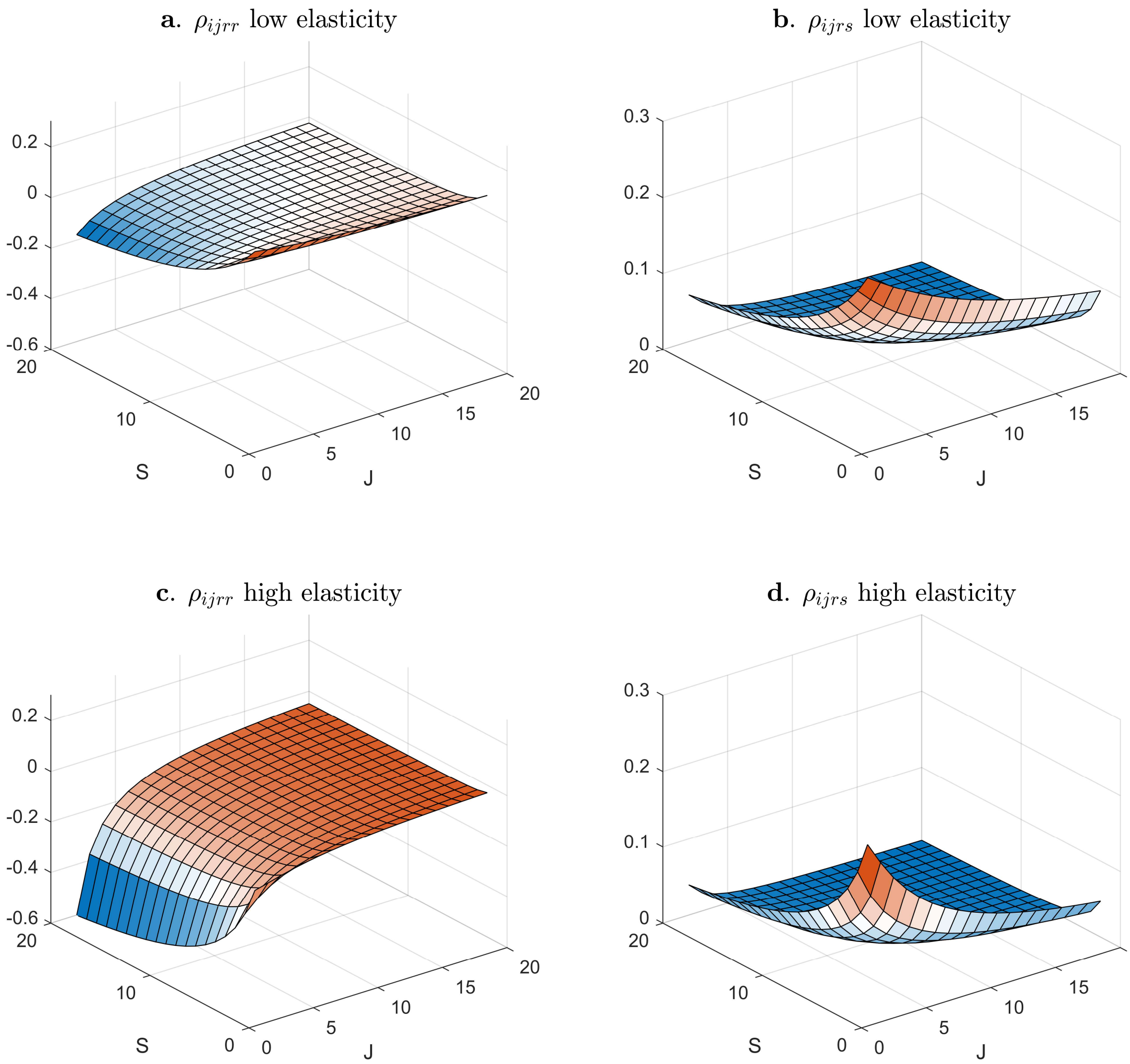

Figure 4. Panel (a) plots the correlation of the same sector in two different countries when $\theta = 1.5$. Panel (b) plots the correlation of a sector r and a sector s in two different countries when $\theta = 1.5$. Panel (c) plots the correlation of the same sector in two different countries when $\theta = 5$. Panel (d) plots the correlation of a sector r and a sector s in two different countries when $\theta = 5$. This particular exercise uses $\gamma = 0.7$, $\tau = 1.6$ when the trade elasticity is high and $\tau = 4$ when the trade elasticity is low to ensure comparable trade intensities.

Corollary 3 (Convergence of correlations) *The standard deviation of aggregate output and the correlation between two different countries $i \neq j$ converge Q-sublinearly*[16] *to zero in the number of sectors and countries, respectively, such that*

$$\lim_{S \to \infty} \sigma_j = 0 \quad \textit{and} \quad \lim_{J \to \infty} \rho_{ij} = 0$$

from which it follows that the difference between frameworks converges in the number of countries

$$\lim_{J \to \infty} (\rho_{MP} - \rho_{EK})_{ij} = 0$$

A second novel result that follows from Corollary (2), is that trade has an ambiguous effect on aggregate volatility and that the standard deviation of aggregate output might increase with trade even in the symmetric economy when there is no specialization. The intuition for this result is that even though sectors can diversify their input sourcing abroad, there is a direct substitution channel coming from the presence of foreign competitors. In other words, when countries produce in the same sector, shocks abroad will tend to move the domestic output more. This effect is highest when there is an intermediate number of sectors, trade intensity is high and trade elasticity is high.

This result contrasts Caselli et al. (2020), which argues in a one-sector case that trade decreases volatility through a diversification effect. This is also true in this paper's setup, however, it hinges on there being one sector only i.e. $\partial \sigma_j / \partial \eta$ conditional on $S = 1$. This diversification effect comes from countries using sectors abroad to hedge against negative shocks to domestic sectors. Caselli et al. (2020) then argues that specialization is the force in the multisector world that leads to increased volatility, as countries become more exposed to sector-specific shocks.

4.2.1 The comovement puzzle

The expression for ρ_{ij} in Corollary (2) contains key insights about the comovement puzzle. Following $\partial \lambda^G / \partial \eta < 0$, an increase in the intensity of trade always leads to an increase in ρ_{ij} (higher comovement) and leads to a positive slope between the two variables through the supply-side channel. The intuition for this result is that trade increases the direct exposure between countries and the higher the input elasticity and the lower the trade elasticity the stronger this channel will be as a source of higher comovement.

[16] The definition of convergence is standard. Let a sequence $\{a_n\}$ be such that $\lim_{n \to \infty} a_n = a$ then if

$$\lim_{n \to \infty} \frac{|a_{n+1} - a|}{|a_n - a|^{\alpha}} = b < \infty$$

then the sequence converges to zero. It converges linearly if $\alpha = 1$ and $b \in (0, 1)$, sublinearly if $\alpha = 1$ and $b = 1$, and superlinearly if $\alpha > 1$ and $b \in (0, 1)$.

However, it follows that $\partial\lambda^D/\partial\eta = 2(\theta-\eta)-1$ is ambiguous overall and it is positive when $\eta < 1/2\theta - 1$, which is more likely to hold when the trade elasticity is high and when the trade intensity is low. The intuition again is that through the demand side channel when the trade elasticity and intensity are high it is easy for the buyers of a sector to substitute in and away from it, which reduces the correlation of aggregate output. The only case where there is no ambiguity and the slope of trade and comovement is positive are the restrictive cases of $J = 2$ and/or $S = 1$, that is, when there is only one sector and hence section differentiation no longer exists, or in the two country case where for each sector at home there is only one like it abroad.

$$\frac{\partial\rho_{ij}}{\partial\eta} = \underbrace{-\left(\frac{1}{1-\gamma}\right)^2 2\lambda^G\frac{\partial\lambda^G}{\partial\eta}\left(1+\Psi^G\right)}_{>0 \text{ from } \frac{\partial\lambda^G}{\partial\eta}<0} \underbrace{-\frac{S-1}{S}(J-2)2\lambda^D\frac{\partial\lambda^D}{\partial\eta}\Psi^G}_{\lesseqgtr 0}$$

where

$$\Psi^G = \frac{(\rho_{ij})^2}{\left(\frac{1}{1-\gamma}\right)^2\left(1-(\lambda^G)^2\right)} > 0$$

4.2.2 The quantity puzzle

From Corollary (2) we know that the aggregate correlation is high when λ^G and λ^D are low. Following $\partial\lambda^G/\partial\theta > 0$ and $\partial\lambda^G/\partial\gamma < 0$, it is true in both EK and MP that supply-side exposure pushes correlation up when the trade elasticity is low and when the input elasticity is high. The lower the trade elasticity the more ossified the cost structure is and hence comovement is higher, but the more trade can be used as insurance leading to lower aggregate variance. The higher the input elasticity, the more countries depend on one another to produce relative to factors such as labor, and hence the more correlated they will be.

Figure (5) plots the predictions of the symmetric economy for the paths of the variance and correlation of aggregate output following the increase in the intensity of trade observed in the data since the 1960s. There are two scenarios plotted. In the first scenario (blue line), the trade elasticity is high and the input elasticity is one-half. In this scenario, trade flows are very sensitive to shocks which leads to the volatility of aggregate output to increase with the intensity of trade. At the same time, the fact that trade flows are sensitive to shocks implies that aggregate output is not correlated across countries. In the second scenario (orange line), the opposite is true. In it, trade flows are not sensitive to shocks and direct exposure is high due to a higher input elasticity. Trade in this scenario acts as an insurance mechanism and higher trade intensity lowers aggregate output variance, while at the same time increasing correlation across countries.

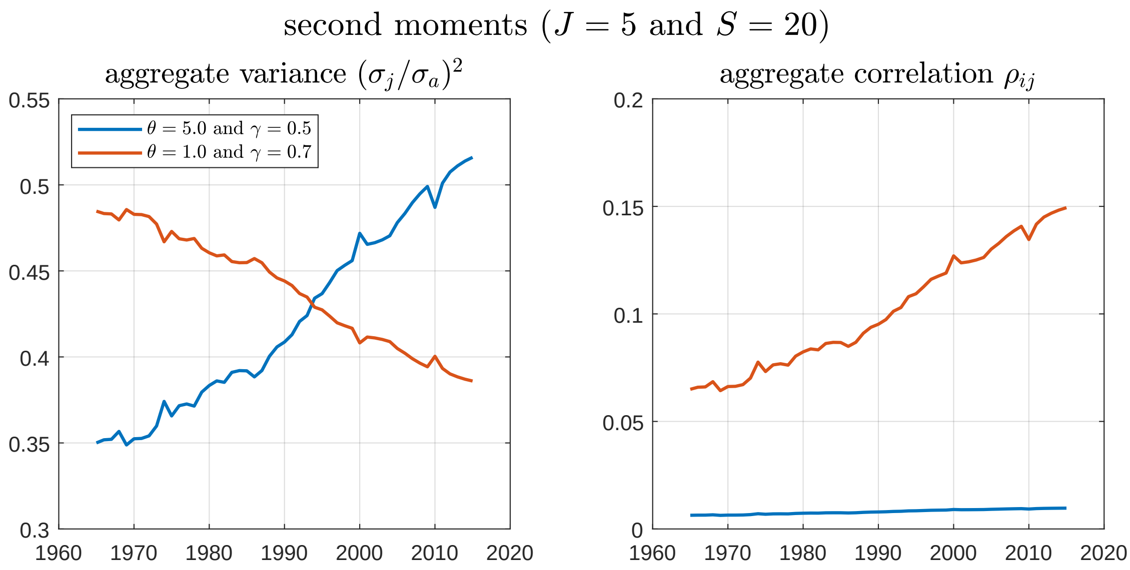

Figure 5. The trade and comovement puzzle visualized.

4.2.3 EK vs. MP

Following Corollary (2), aggregate comovement is higher in MP as long as $\lambda^G_{MP} < \lambda^G_{EK}$ and $\lambda^D_{MP} > \lambda^D_{EK}$. The former is true whenever $\kappa(1+\phi) < \theta$, but that means the latter is not. Conditional on trade shares, this holds when fixed entry costs are higher than relative variable trade costs. The following proposition helps to understand why that is.

Proposition 2 (Trade share equality) *Trade shares in each stylized economy are given by*

$$\Pi_{EK} = \frac{\tau^{-\theta}}{1+(J-1)\tau^{-\theta}} \quad \textit{and} \quad \Pi_{MP} = \frac{\left(\tau f^{\phi}\right)^{-\kappa}}{1+(J-1)\left(\tau f^{\phi}\right)^{-\kappa}}$$

and if $\tau_f = \ln f / \ln \tau$ then $\Pi_{EK} \leq \Pi_{MP}$ if and only if $\kappa \leq \theta(1+\phi\tau_f)^{-1}$.

Proof *From the expression of trade shares we have*

$$\Pi_{EK} \leq \Pi_{MP} \Rightarrow \tau^{-\theta} \leq \left(\tau f^{\phi}\right)^{-\kappa} \Rightarrow \tau^{\frac{\theta}{\kappa}-1} \leq f^{\phi} \Rightarrow \kappa\left(\phi\frac{\ln f}{\ln \tau}+1\right) \leq \theta \Rightarrow \kappa \leq \frac{1}{1+\phi\tau_f}\theta \;\blacksquare$$

This illustrates that a potential source of differences in comovement, in theory, are different equilibrium values for the trade shares. It is cheaper for producers in MP to enter the domestic market than any foreign market when compared to EK, where producers only have to pay variable trade costs. A necessary condition for $\Pi_{EK} \leq \Pi_{MP}$ is $\kappa < \theta$. Moreover, the higher fixed cost relative to variable trade cost ($\tau_f > 1$) the lower κ needs to be compared to θ such that trade shares are the same across frameworks.

If anything, the extensive margin channel is a force for lower comovement as it weakens the cost channel relative to the demand channel such that one shouldn't expect MP to generate higher aggregate comovement from the expressions alone. The second condition leading to higher correlations in MP is $\gamma_{MP} > \gamma_{EK}$. A higher input share increases real output correlation as it decreases the extent to which higher productivity abroad is transmitted directly through an income channel, which would be the case if goods were used only for final consumption.

4.3 Taking stock

One can predict from the stylized economy that i) any differences in the aggregate correlation between MP and EK are expected to appear in setups with a small number of countries; ii) differences are more likely to appear when either the trade elasticity in MP is lower than that in EK, or when the input elasticity in MP is higher than in EK; iii) aggregate correlation is higher in both frameworks when the number of countries is low; iv) aggregate correlation is higher when the trade elasticity is low and when the input elasticity is high.

5 Data and calibration strategy

In this section, I detail the calibration strategy used to populate the influence matrix in both frameworks. Apart from the different equations that separate both models, papers that start from different assumptions on market structure and returns to scale differ in their strategy for estimating the relevant parameters of each framework. These parameters include the trade elasticities $\{\theta_s\}_{\forall s}$ and $\{\kappa_s\}_{\forall s}$, the micro elasticities $\{\sigma_s\}_{\forall s}$, the Armington of substitution χ, as well as the output elasticities to inputs $\{\gamma_{js}\}_{\forall j,s}$ and $\{\gamma_{jrs}\}_{\forall j,r,s}$.

5.1 Data sources

The data sources used in the calibration are the World Input-Output Database (WIOD, Timmer et al. (2015)) and the Exporter Dynamics Database (EDD, Cebeci et al. (2012)).

5.1.1 World input-output data

Table (2) sketches the standard format of a world input-output table. Both rows and columns can be thought of as representing accounting identities which are best understood by subdividing the input-output matrix into smaller ones. The M block has information on input sales from each country-sector pair to another. I has information on the capital formation (investment flows) flowing between country pairs aggregated over sourcing sectors. Finally, F has information on final consumption flows. Each row of the input-output table documents all possible uses of a good produced by any sector s in any country j, so that summing across columns leads to a column vector of sector gross outputs. Subtracting from the latter the row sum of M equals a row vector of sector value added.

			Input Demand							Fixed Capital Formation			Final Demand			Gross Output
			Country 1			$\cdots$	Country J			Country 1	$\cdots$	Country J	Country 1	$\cdots$	Country J	
			Sector 1	$\cdots$	Sector S	$\cdots$	Sector 1	$\cdots$	Sector S							
		Sector 1	M_{11}^{11}	$\cdots$	M_{11}^{1S}	$\cdots$	M_{1J}^{11}	$\cdots$	M_{1J}^{1S}	I_{11}^{1}	$\cdots$	I_{1J}^{1}	F_{11}^{1}	$\cdots$	F_{1J}^{1}	Y_1^1
	Country 1	$\vdots$	$\vdots$	M_{ij}^{rs}	$\vdots$	M_{ij}^{rs}	$\vdots$	M_{ij}^{rs}	$\vdots$	$\vdots$	I_{ij}^{r}	$\vdots$	$\vdots$	F_{ij}^{r}	$\vdots$	$\vdots$
		Sector S	M_{11}^{S1}	$\cdots$	M_{11}^{SS}	$\cdots$	M_{11}^{S1}	$\cdots$	M_{1J}^{SS}	I_{11}^{S}	$\cdots$	I_{1J}^{S}	F_{11}^{S}	$\cdots$	F_{1J}^{S}	Y_1^S
Input Supply	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	M_{ij}^{rs}	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
		Sector 1	M_{J1}^{11}	$\cdots$	M_{J1}^{1S}	$\cdots$	M_{JJ}^{11}	$\cdots$	M_{JJ}^{1S}	I_{J1}^{1}	$\cdots$	I_{JJ}^{1}	F_{J1}^{1}	$\cdots$	F_{JJ}^{1}	Y_J^1
	Country J	$\vdots$	$\vdots$	M_{ij}^{rs}	$\vdots$	M_{ij}^{rs}	$\vdots$	M_{ij}^{rs}	$\vdots$	$\vdots$	I_{ij}^{r}	$\vdots$	$\vdots$	F_{ij}^{r}	$\vdots$	$\vdots$
		Sector S	M_{J1}^{S1}	$\cdots$	M_{J1}^{SS}	$\cdots$	M_{JJ}^{S1}	$\cdots$	M_{JJ}^{SS}	I_{J1}^{S}	$\cdots$	I_{JJ}^{S}	F_{J1}^{S}	$\cdots$	F_{JJ}^{S}	Y_J^S
Value Added			VA_1^1	$\cdots$	VA_1^S	$\cdots$	VA_J^1	$\cdots$	VA_J^S							
Gross Output			Y_1^1	$\cdots$	Y_1^S	$\cdots$	Y_J^1	$\cdots$	Y_J^S							

Table 2. World input-output table sketch

Some parameters can be calibrated regardless of the model assumptions. Each Ω_{jrs} equals a sector s' input purchases from r over its total input purchases[17] while spending shares are calibrated using ratios of sector consumption over total final consumption as follows.

$$\Omega_{jrs} = \frac{\sum_i M_{ijrs}^j}{\sum_i \sum_r M_{ijrs}^j} \quad \text{and} \quad \Xi_{js} = \frac{F_{js}^j}{\sum_s F_{js}^j}$$

Trade shares are calibrated to the ratio of input purchases and final consumption in j of sector s that originate in i over j's total input spending and final consumption of s.

$$\Pi_{ijs} = \frac{F_{ijs}^j + \sum_r M_{ijsr}^j}{\sum_i F_{ijs}^j + \sum_r M_{ijsr}^j}$$

5.1.2 Exporter Dynamics Database (EDD)

This dataset includes "(...) exporter characteristics and measures of exporter growth based on firm-level customs information from 38 developing and seven developed countries, primarily for the period between 2003 and 2008" Cebeci et al. (2012). The dataset has information on seven countries that are also present in the WIOD and this data is available aggregated at the third

[17]It is useful to transform the input-shares matrix into a block-diagonal matrix, with input-share matrices for each country j composing the diagonal blocks. To do so I start by adding across the rows of the input requirement matrix $A = M\hat{Y}$ to create a stacked matrix Ω_M using $[I_{1S} \cdots I_{JS}]$, a $S \times JS$ matrix composed of J identity matrices of dimension S side by side so that Ω_M is a $S \times JS$ matrix with J blocks of $S \times S$ matrices of input shares. I stack this matrix

$$\mathbf{\Omega} = \sum_j \tilde{D}_j' \Omega_M D_j$$

where $\tilde{D}_j$ is a $S \times JS$ matrix where all elements are zero except for an identity block for country j $\tilde{D}_j = [\mathbf{0}_S \cdots I_S^j \cdots \mathbf{0}_S]$. D_j is a $JS \times JS$ matrix of zeros with a $S \times S$ diagonal block for country j.

digit ISIC 3 codes. It contains information on export revenues divided into exporter size quartiles and on the share of total exports made up by the top one and top five percent of exporters, which can be used to calibrate κ_s and σ_s using the expression of the top share of the Pareto distribution of exports.

5.2 Elasticity parameters

The elasticity parameters necessary to calibrate the models are the trade elasticity θ_s in EK and its counterpart κ_s in MP, but also the micro elasticity of substitution across varieties σ_s. In addition, I separate these further into their long-run and short-run estimates. I use both long-run and short-run estimates in the literature to compare the predictions of the models about correlations.

5.2.1 Long-run elasticities

There are well-established methods developed to estimate the trade elasticity in EK, which usually involve using the gravity structure of EK models to transform either triad or tetrad trade flow ratios into log-linear equations that identify the elasticity, such as in Caliendo & Parro (2015)[18]. For example, Costinot et al. (2012) regress the adjusted product of bilateral trade flows on a proxy for observed productivity and trade costs, while Caliendo & Parro (2015) uses the tetrad method to regress the product of trilateral trade flows on trade cost only as a way to estimate θ_s. In the Eaton-Kortum tradition, the micro elasticity of substitution across varieties σ_s has no first-order relevance in the adjustment of variables to productivity shocks and is rarely estimated despite the condition $\theta_s > \sigma_s - 1$. In cases where the model is Armington such as in Huo et al. (2019), the authors will usually resort to a tetrad estimator as well. In this paper, I use the recent estimates of Fontagné et al. (2022) aggregated at the ISIC 2 level.

In MP the same procedure does not identify κ_s, and so papers resort to production function estimation methods (see, for example, Blaum et al. (2018)). Estimates of the micro elasticities of substitution σ_s are necessary as i) these parameters have first-order significance (e.g. they appear in ϕ_s); ii) these parameters determine sector-level markups. In this paper, I use the Blaum et al. (2018).

[18]In particular, θ_s is estimated from

$$\ln\left(\frac{\Pi_{ijs}}{\Pi_{kjs}}\frac{\Pi_{jks}}{\Pi_{jis}}\frac{\Pi_{kis}}{\Pi_{iks}}\right) = -\theta_s \ln\left(\frac{\tau_{ijs}}{\tau_{kjs}}\frac{\tau_{jks}}{\tau_{jis}}\frac{\tau_{kis}}{\tau_{iks}}\right)$$

where trade costs themselves are proxied with data on asymmetric trade barriers such as tariffs. The relationship holds as long as the unobservable part of trade costs is multiplicative and country-specific, not country-pair-specific. The inverse process can be used to back out trade costs using trade shares given knowledge of trade elasticities as in K. Head & Ries (2001)

As a robustness check, I leverage the stringent structure on the assumptions of fixed entry costs to calibrate κ_s and σ_s using the distribution of export revenues. As the producers' first-order conditions no longer uniquely identify production function parameters, one needs information on markups to calibrate the matrices used in Theorem 1. Price-cost margins are a way of estimating markups, which can be transformed into the ratio of gross output to variable costs. However, the labor income entry of Social Economic Accounts cannot be interpreted as variable labor costs in MP. In other words, total costs have to be used as social-economic accounts do not separate between fixed and variable labor costs, that is, it shows $W^j_{jt}(L_{Pjst} + L_{Ejst})$ instead of $W^j_{jt}L_{Pjst}$ alone. To do so, I use a corollary of the Proposition (1) which states that the ratio of gross output total costs can be expressed as a function of the markup μ_s and the income share of entry labor in gross output.

Proposition 3 *If total costs are defined as $TC^j_{jst} \equiv W^j_{jt}L_{jst} + \sum_r P^j_{jrt}M_{jrst}$ the ratio of gross output to total costs is a function of the micro elasticity σ_s and the tail parameter of the productivity distribution κ_s only.*

$$\frac{Y^j_{jst}}{TC^j_{jst}} = \frac{\sigma_s}{\sigma_s - \frac{\sigma_s - 1}{\kappa_s}}$$

Proof: *The proof follows directly from Proposition (1) and from the optimality conditions of producers.*

$$\frac{Y^j_{jst}}{W^j_{jt}L_{jst} + \sum_r P^j_{jrt}M_{jrst}} = \frac{Y^j_{jst}}{\left((1-\gamma_{js}) + \phi_s\right)\mu_s{}^{-1}Y^j_{jst} + \gamma_{js}\mu_s{}^{-1}Y^j_{jst}} = \frac{\mu_s}{1+\phi_s} = \frac{\frac{\sigma_s}{\sigma_s - 1}}{1 + \frac{1}{\sigma_s - 1} - \frac{1}{\kappa_s}} \quad \blacksquare$$

Figure (6) plots the left-hand side of the expression in Proposition (3) for a select sample of sectors. Note that capital compensation is not a measure of profits as it does not make any allowance for costs of depreciation. To recover σ_s and κ_s I calibrate the correction term in Proposition (3) using the following Proposition.

Proposition 4 *Following assumptions **A2-A5** the distribution of destination-specific exporter revenue $r_{ijst}(z)$ is characterized by the following probability density function.*

$$g(r) \propto \begin{cases} r^{-\frac{\kappa_s}{\sigma_s - 1} - 1} & \textit{if} \quad z > \underline{z}_{ijst} \\ 0 & \textit{otherwise} \end{cases}$$

Proof: *The proof uses the definition of firm revenues, optimal pricing and the Pareto distribution for productivity as follows*

$$\underbrace{P^j_{ijst}(z)Y_{ijst}(z)}_{\equiv r^j_{ijst}(z)} = \left(\frac{P^j_{ijst}(z)}{P^j_{jst}}\right)^{1-\sigma_s} E^j_{jst} = \left(\frac{\sigma_s}{\sigma_s - 1}\frac{1}{\varepsilon_{ijt}}\frac{c^i_{ist}\tau_{ijs}}{A_{ist}P^j_{jst}}\right)^{1-\sigma_s} E^j_{jst} z^{\sigma_s - 1} \equiv \Theta_{ijst} z^{\sigma_s - 1}$$

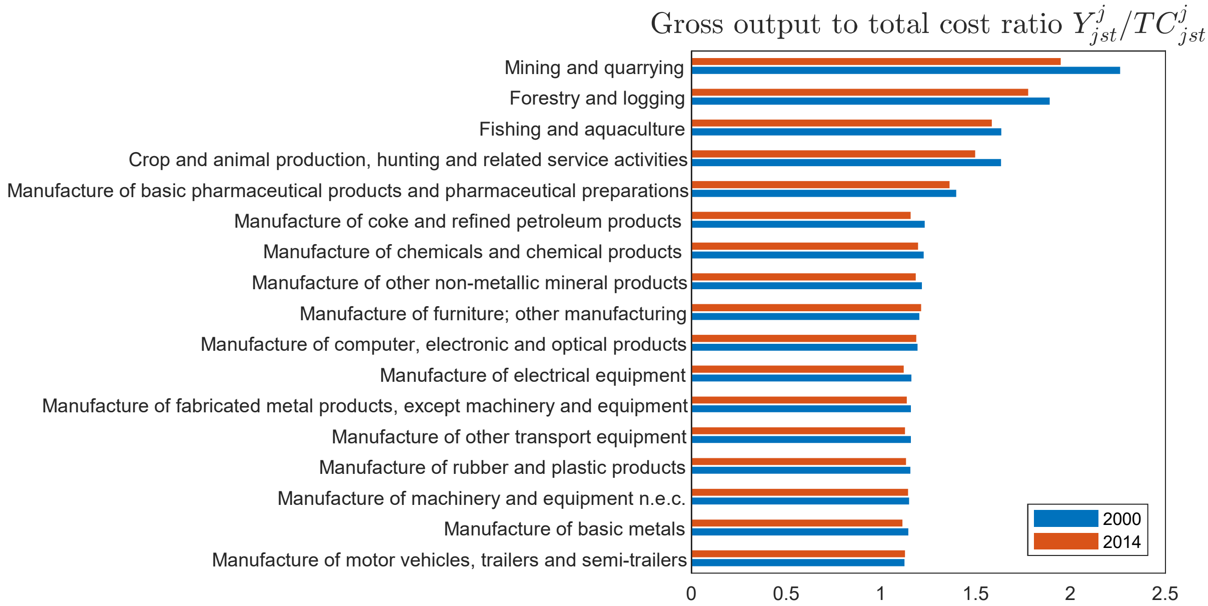

Figure 6. Total cost is the sum of labor compensation and total input purchases from the satellite accounts of the WIOD. Labor compensation is total compensation of employees (including subsidies) and the labor part of mixed-income, in line with Van Ark & Jäger (2017). The difference between gross output and total cost equals capital compensation, which is calculated as the sum of gross operating surplus and net taxes of production.

and the second part of the proof follows from the definition of a cumulative density function

$$G(r) = \Pr(R < r) = \Pr\left(\Theta_{ijst} z^{\sigma_s - 1} < r\right) = F\left(\Theta_{ijst}{}^{-\frac{1}{\sigma_s - 1}} r^{\frac{1}{\sigma_s - 1}}\right)$$

from which the proposition follows ■

Di Giovanni et al. (2011) describes the problems of using total exporting revenues per exporter to estimate the correction. Selection into importing and discontinuous market access lead to downward-biased estimates of the tail exponent of the productivity distribution because the sum of Pareto is not Pareto. Di Giovanni et al. (2011) advise either using the data on domestic sales only or using data on non-exporting firms only. Amand & Pelgrin (2016) point to "(...) just using the right tail (the highest order statistics) provided one has a sufficiently large sample" (pg. 15). I follow their advice and use the top one-percent share of the export distribution to back out the tail parameter. In the simple Pareto distribution, the top q-th percentile share x_q is used to back-out the tail parameter using the following expression.

$$\frac{\kappa_s}{\sigma_s - 1} = -\frac{\ln q}{\ln x_q - \ln q}$$

Table 3. Long-run elasticity estimates in the literature: Caliendo & Parro (2015), Fontagné et al. (2022), Blaum et al. (2018).

		EK		MP			
		CP	FGH	BLP		Pareto tails	
		θ_s		κ_s	ϕ_s	κ_s	ϕ_s
A01	Crop and animal		1.85				
A02	Forestry	8.11	0.52				
A03	Fishing		11.98				
B	Mining	15.72	13.97	2.60	0.35		
C10-C12	Food	2.55	4.16	3.86	0.14	4.68	0.06
C13-C15	Textiles	5.56	4.83	3.36	0.19	6.90	0.03
C16	Wood	10.83	5.33	4.65	0.09	6.24	0.05
C17	Paper	9.07	6.05	2.77	0.30	6.09	0.01
C18	Printing		3.65			5.13	0.03
C19	Petroleum	51.08	3.85				
C20	Chemicals	4.75	5.09	3.30	0.20	4.57	0.03
C21	Pharmaceuticals	8.98					
C22	Plastic	1.66	4.84	4.08	0.12	7.03	0.02
C23	Minerals	2.76	6.25	3.51	0.17	4.75	0.03
C24	Basic Metals	7.99	6.82	6.00	0.06	7.24	0.01
C25	Metal	4.30	7.23	3.27	0.20	6.73	0.03
C26	Electronics	12.79	4.55	7.47	0.04	5.11	0.03
C27	Electrical equipment	10.60	5.04	4.53	0.10	6.94	0.02
C28	Machinery n.e.c.	1.52	8.02	3.54	0.17	6.70	0.02
C29	Auto	1.01	9.59	7.06	0.04	8.07	0.02
C30	Other transport	0.37	3.27	1.90	0.85	6.29	0.04
C31_C32	Furniture	5.00	4.31	2.96	0.25	4.73	0.02
C33	Repair of machinery		4.98	3.95	0.13	6.16	0.03
mean		8.67	5.74	4.05	0.20	6.08	0.03
median		5.56	5.01	3.54	0.17	6.24	0.03

5.2.2 Short-run Elasticities

In the short run, trade may be inelastic. Initial estimates from Reinert & Roland-Holst (1992) challenge the standard Armington elasticity measures and estimate a much lower average short-run elasticity across 163 SIC sectors in the US. These estimates were in line with later work Blonigen & Wilson (1999), while recent work has focused on estimating trade elasticity over time. Yilmazkuday (2019) finds that the trade elasticity is one in the first year following a trade cost shock, while Boehm et al. (2020) finds similar estimates with a more robust identification strategy. In exercises where I use short-run trade elasticities, I calibrate θ_s and κ_s using the latter.

5.3 Production function parameters

Once σ_s and μ_s are calibrated, the calibration of production function parameters follows the same procedure in both frameworks. This procedure uses labor compensation and input spending to gross output ratio to back out γ's. Among the equilibrium conditions, there are relationships between input costs and total revenues. In particular, it follows that $P^j_{jrt}M_{jrst} = \gamma_{js}{\mu_s}^{-1}\Omega_{jrst}Y^j_{jst}$, such that sector value-added and markups recover input elasticities as follows

$$VA^j_{jst} = Y^j_{jst} - \sum_r P^j_{jrt}M_{jrst} = Y^j_{jst}(1 - \gamma_{js}\mu_s^{-1}\underbrace{\sum_r \Omega_{jrst}}_{=1})$$

such that can be calibrated using the moment $\gamma_{js} = \mu_s(1 - VA^j_{jst}/Y^j_{jst})$. The strategy for calibrating production function parameters in EK usually involve cost-revenue shares, a strategy that is present in Caliendo & Parro (2015), Caselli et al. (2020), Huo et al. (2019), Antras & Chor (2021) and others. In this case it follows that $\mu_s = 1\ \forall s$ and that $L_{Pjst} = L_{jst}\ \forall j, s$, and consequently $\gamma_{js} = VA^j_{jst}/Y^j_{jst}$ of which the right-hand-side is found in the input-output table.

5.4 Calibrated Matrices

Figure (7) illustrates the main empirical matrices that go into the influence matrix, the Ghosh inverse $\mathbf{G}$ and the demand-side inverse $\mathbf{P}$. The elements of the Ghosh-inverse are how much of a sector's value-added comes from other sectors' gross output such that all of its elements are positive. Figure (7) shows that the diagonal blocks have much higher numbers than bilateral blocks, illustrating that most demand exposure is domestic. The bottom panels of Figure (7) illustrate the differences between frameworks in matrices $\mathbf{G}$ and $\mathbf{P}$. As expected from the discussion in the previous section, there is very little difference between frameworks in $\mathbf{P}$. All the differences between the predictions in EK and MP hinge on $\mathbf{G}$, which differ less due to the presence of the extensive margin and increasing returns to scale, but due to input elasticities being calibrated to a higher value because of markups.

6 Output Correlation

The influence matrix computes percent changes to sectoral output in any pair (j, s) following a productivity shock to any other pair (i, r). In this section, I use this system to compute percent

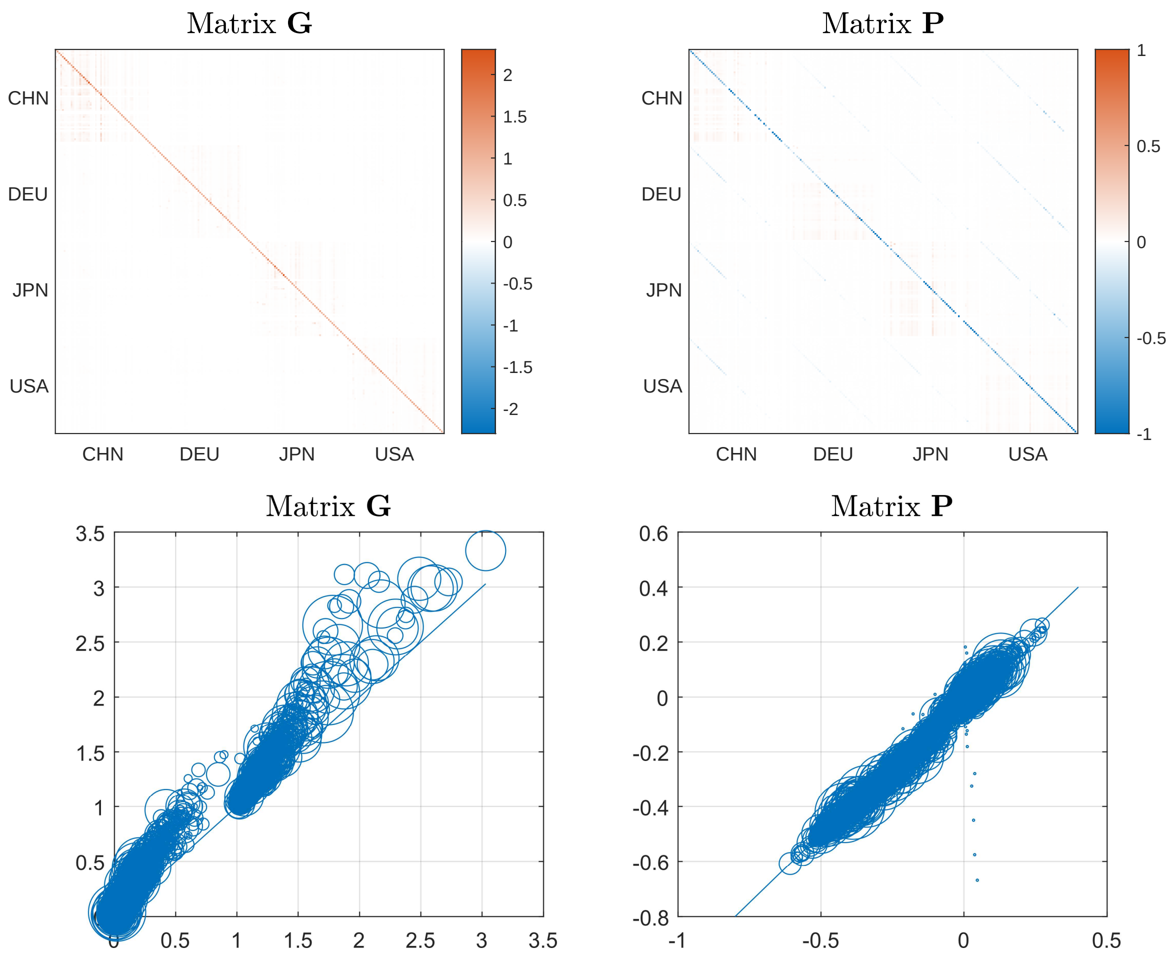

Figure 7. The top left panel illustrates the empirical Ghosh inverse $\mathbf{G}$ calibrated using the long-run elasticities, while the top-right panel illustrates the demand-side inverse $\mathbf{P}$, both in EK. The bottom panels illustrate the differences between entries of $\mathbf{G}$ (left panel) and $\mathbf{P}$ where EK entries are in the x-axis and MP entries in the y-axis.

changes in aggregate real value added and then its correlation across countries. Nominal value added equals the gross output of each sector net of input costs, i.e. $VA^j_{jt} = \sum_s (1 - \gamma_{js}\mu_s^{-1}) Y^j_{jst}$, which is linearized to $va^j_{jt} = \sum_s \nu_{js} y^j_{jst}$ where ν_{js} is the value-added share of sector s in the economy. The equations for each country-sector pair are stacked in a linear system $va^j_t = \nu'(\mathbf{I} + \mathbf{P})\mathbf{\Gamma} a_t$ where each specific shock component is assumed to be i.i.d and to follow an autoregressive process of order one with the following properties.

Assumption 7 (Exogenous stochastic productivity) *For all countries j and sectors s*

1. *The shock processes are AR(1)*

$$a_{jst+1} = (1 - \zeta_{js}) \sum_i \zeta_{ijs} a_{ist} + \epsilon_{jst+1}$$

where $\rho \in (0, 1)$ for all the shocks.

2. *$\mathbb{E}\epsilon_{js} = 0$ for all j and s.*

3. *Variances $\sigma^2_{\epsilon_{js}}$ are in the interval $(\underline{\sigma}^2, \bar{\sigma}^2)$ where $0 < \underline{\sigma} < \bar{\sigma} < \infty$ for all j and s.*

In matrix form, the shock process of Assumption (7) is written as $a_t = \tilde{\zeta} a_{t-1} + \epsilon_t$, where $\tilde{\zeta}$ is a $JS \times JS$ matrix that combines both the η_{js} persistence parameters and the ζ_{ijs} shock correlation parameter. The latter is assumed for completeness but in the empirical exercises, I assume that shocks are uncorrelated. Part three of Assumption (7) guarantees that the variance of idiosyncratic shocks is bounded above zero even in scenarios with a high number of sectors. Following this assumption, the variance-covariance matrix of the vector of aggregate value-added changes is the following.

$$\text{var}(va_t^j) = \big(\nu'(\mathbf{I} + \mathbf{P})\mathbf{\Gamma}\big) \left(\sum_i \tilde{\zeta}^i \mathbf{\Sigma}_\epsilon (\tilde{\zeta}^i)' \right) \big(\nu'(\mathbf{I} + \mathbf{P})\mathbf{\Gamma}\big)' \tag{6.1}$$

where ν is a $J \times (JS)$ matrix containing the vectors of sector value-added shares stacked to sum across sectors within each country. In the following subsections, I use Equation (6.1) to study the predictions of each framework in three scenarios: i) a one-sector $S = 1$ case where all trade flows are aggregated at the country level and serves as the baseline result of comparison; ii) a scenario where all data available is used, but where I compare multiple draws of random groupings of the $J = 44$ countries and $S = 56$ sectors.

6.1 One-sector baseline estimates

In this section, I aggregate trade flows from the sector to the country level and evaluate Theorem (1) and Equations (6.1) in EK and MP. In the one-sector case, there is a very simple expression for the output correlation in the stylized economy.

$$\rho_{ij} = \frac{1}{J} \frac{1 - \left(\lambda^G\right)^2}{1 - \frac{J-1}{J}\left(1 - (\lambda^G)^2\right)} \tag{6.2}$$

The lower the trade elasticity θ (or $\kappa(1 - \phi)$ in MP), the higher the input elasticity γ and/or the higher the trade intensity η, the higher the correlation. Equation (6.2) is a parsimonious way of knowing what to expect from each model. The average bilateral trade share times the number of countries ($J = 44$) in the year 2010 was 0.21, while the average input share across countries was 0.60. Conditional on these numbers, if the trade elasticity in EK is calibrated to its long-run average $\theta = 5$, one should expect a correlation of 0.0026.

Figure (8) illustrates that even when EK is calibrated to very low short-run trade elasticities values as in Boehm et al. (2020), the model underpredicts correlations in the data by over an

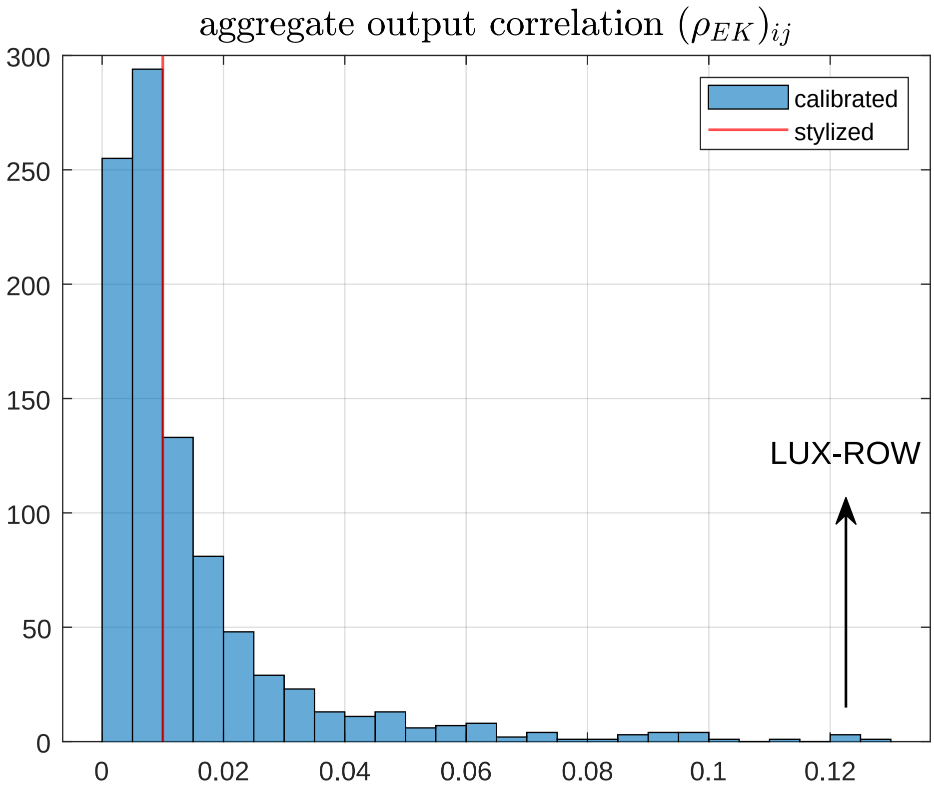

Figure 8. Histogram of correlations predicted in EK using Theorem (1) in the one sector $S = 1$ case. The trade elasticity calibrated is $\theta = 1$ and the data input-output table used is for the year 2010.

order of magnitude, that is, most of the statistically significant estimates in Figure (1) are over 0.2, while EK predicts that most correlations are under 0.02. Some correlations reach up to 0.13, but these are either between very small countries or between a small country and a big country, such as Luxembourg with the Rest of the World.

Following these results of EK calibrated to the data and the relative success of the stylized economy in predicting the median correlation, one can ask what would take to get past the 0.20 mark that is common between G7 economies in the data. If the trade elasticity is calibrated to its long-run value one can say that this mark is not possible to reach. If the trade elasticity is calibrated to its short-run value of one, then free trade is necessary to go over the 0.20 mark. To give the reader an idea, this would require a domestic absorption share of roughly two percent.

When the trade elasticity is high, MP is not guaranteed to do better. For example, de Soyres & Gaillard (2022) calibrate a model with $J = 14$ countries using $\kappa = 4.6$ and find an average correlation of 0.03 when shocks are assumed to be uncorrelated. Together with the assumption of $\sigma = 5$ and $\gamma = 0.80$[19], their calibration implies $\eta = 0.32$ and $\phi = 0.03$. When Equation (6.1)

[19]This is the upper bound of their calibration for when the capital elasticity in the production is added to the input

is fed these numbers it returns a predicted correlation of 0.028, which is extremely close to their prediction even in the absence of financial markets and capital.

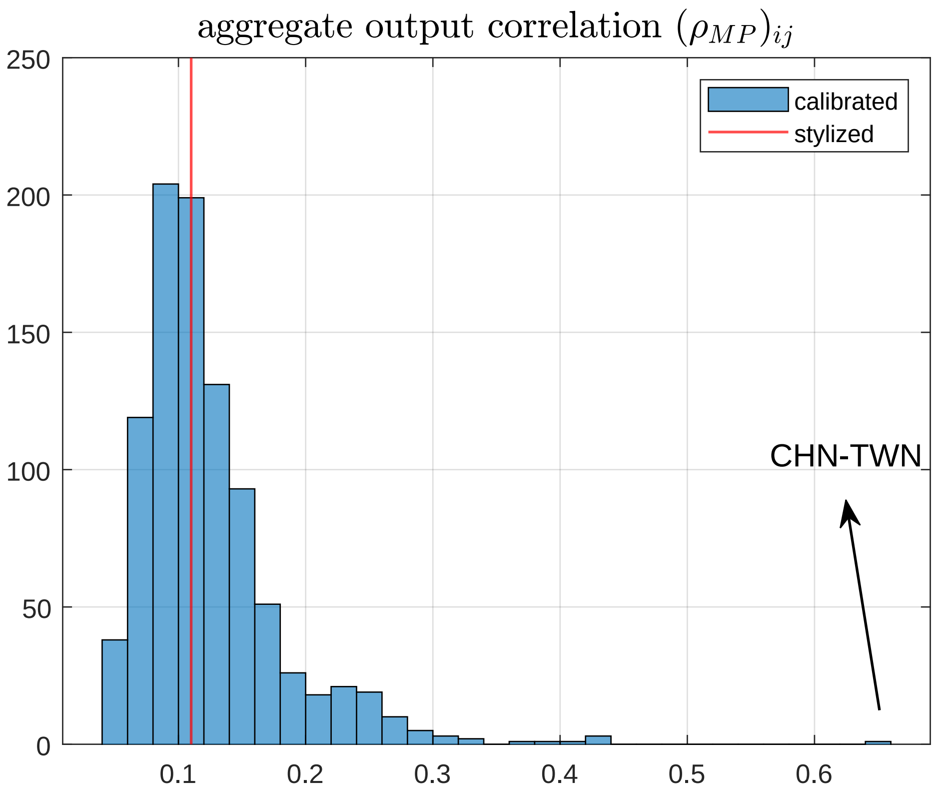

Figure 9. Histogram of correlations predicted in MP using Theorem (1) in the one sector $S = 1$ case. The trade elasticity calibrated is $\kappa = 1$ and the data input-output table used is for the year 2010.

In the case of short-run trade elasticities, MP will do much better than EK because of its higher implied input elasticities. When the long-run elasticity of substitution between varieties is $\sigma = 5$ and markups are 25%, input elasticities are on average 0.75 instead of the 0.60 in EK, and they may go as high as 0.86 for some sectors in the data. This leads to the biggest quantitative differences between EK and MP. From Equation (6.2) it follows that $\lim_{\gamma \to 1} \lambda^G = 0$ and hence $\lim_{\gamma \to 1} \rho_{ij} = 1$, such that higher input elasticities lead to correlations that are exponentially closer to one. For example, if $\kappa \leq 1$, that is, κ is calibrated to its short-run value following Boehm et al. (2020), then the stylized equation predicts a correlation of 0.11. This result is illustrated in Figure (9).

elasticity.

6.2 Multi sector estimates

I propose the following exercise to study comovement in the multisector case: starting from the data with $J = 44$ countries and $S = 56$ sectors, I randomly combine countries and sectors into each possible case from $J = 2$ and $S = 2$ up to $J = 20$ and $S = 20$ and compute the correlations between all country-sector pairs in all cases. For any J there are $44!/\left(J!(44-J)!\right)$ possible combinations of different countries, and for any S there are $56!/\left(S!(56-S)!\right)$ combinations. For example, in the two-by-two case, there are 946 possible combinations of countries and 1540 sector combinations. For all of these cases, I sample one thousand possibilities.

Figure (10) illustrates why the model cannot generate realistic correlations when the trade elasticity is high. Just as in the stylized economy, when the trade elasticity is high the demand channel is strong and the correlation between the same sector across countries is high and negative, eclipsing the supply-side force that pushes for positive comovement.

7 Conclusion

In this paper, I developed a multi-sector, multi-country version of both the Eaton-Kortum and Melitz-Pareto frameworks that allows for a complex input-output structure. In both cases, there is an analytical linear system that solves for changes in real output as a function of exogenous productivity shocks, and their structure is common. However, the matrices in the Melitz-Pareto model have to be adjusted for extensive margin effects. These matrices are the international Ghosh-inverse, the total expenditure-exposure matrix and a labor allocation matrix. In particular, the matrices are adjusted for the fact that both prices and trade shares move with a country's own total spending and for the additional role that wages play as the price of entry in source countries.

I use a stylized symmetric version of the models to build intuition for differences in the predictions in both frameworks. The first is that any differences should disappear as the number of sectors and countries increases, but this conversion can be much slower than what is implied by a simple law of large numbers. In the one-sector case, MP predicts higher correlations than EK as long as the trade elasticity adjusted for entry labor allocation is lower in MP than the trade elasticity in EK, and when input elasticities are higher in MP than in EK. The empirical correlations are in line with what is expected from the stylized economy in the one-sector case. In EK these estimates can be ten to one hundred times off, while in MP it can vary from one-fifth to one-half to the average correlation in the data.

There are plenty of reasons why these models might not be well suited for this task. First, there could be important non-linearities in the interaction between country-sector pairs that

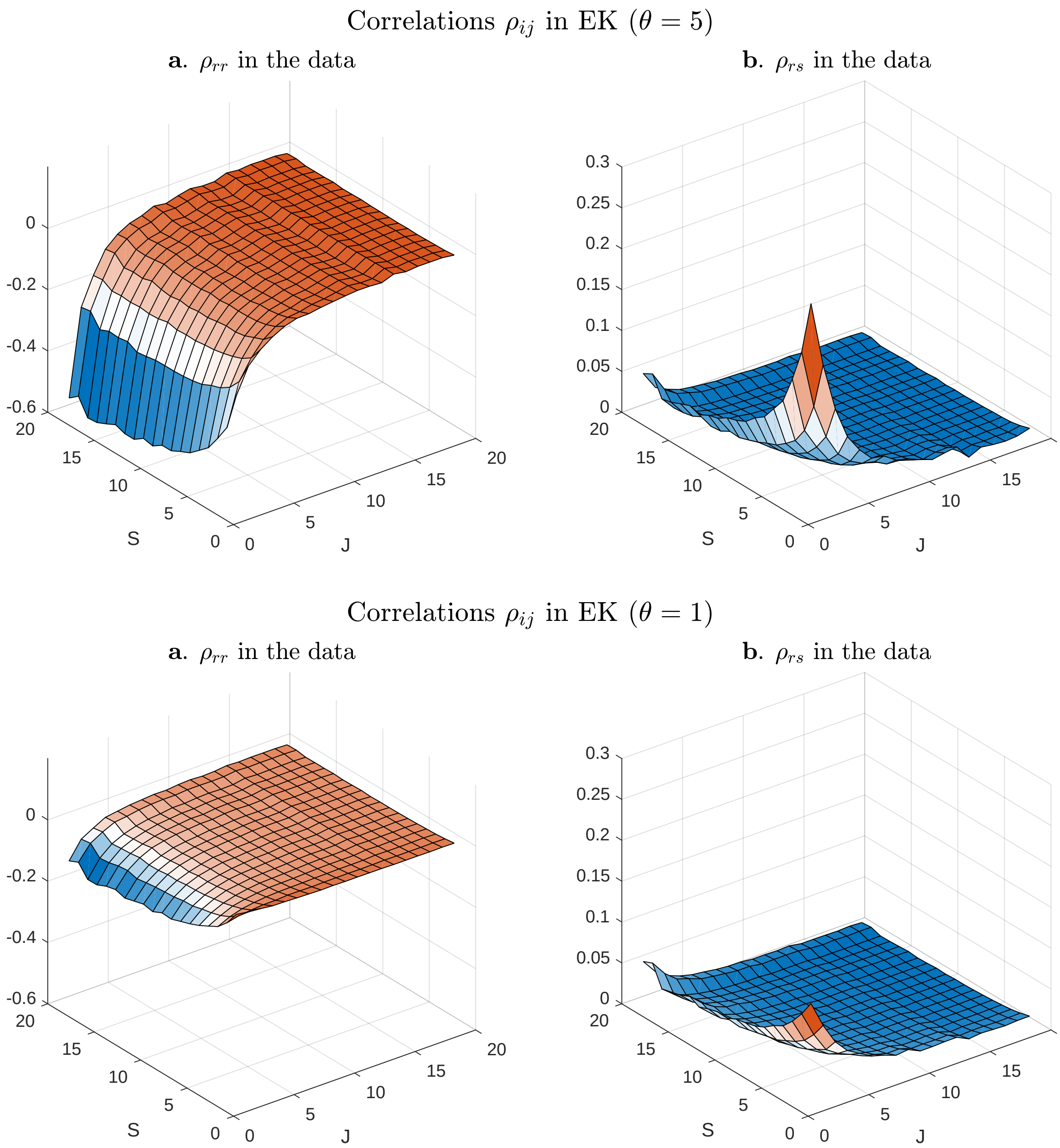

Figure 10. Trade-weighted average correlation between the same sector across countries (left panel) and between different sectors across countries (right panel).

are not playing a role in a first-order approximation. This could be particularly important if researchers have reasons to believe that non-linearities play a larger role in models with an active extensive margin of trade. Second, it could be that shocks are highly correlated across countries and sectors, such that correlations are high by construction. This is in line with previous results in the literature such as Koren & Tenreyro (2007) and Huo et al. (2019), but it leaves open the possibility that the frameworks are missing important endogenous mechanisms that lead to the correlations observed in the data. Third, and relatedly, it could be that international business cycles are driven by shocks in financial markets that are transmitted through net foreign asset positions which neither framework studied in this paper considers.

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Appendix

A Model solution

In this appendix, I present the problems and optimality conditions for agents in each model.

A.1 Consumers

The representative consumer's problem is the same in MP and EK and is subdivided into two parts. First, consumers in j minimize spending on each variety o_{is} in the set of varieties that are sourced from country i as follows.

$$\min_{Q_{ijst}(o_{is})} \int_{o_{is} \in O_{ijst}} P^j_{ijst}(o_{is}) Q_{ijst}(o_{is}) do_{is}$$

$$\text{such that } \left(\sum_i \int_{o_{is} \in O_{ijst}} Q_{ijst}(o_{is})^{\frac{\sigma_s - 1}{\sigma_s}} do_{is} \right)^{\frac{\sigma_s}{\sigma_s - 1}} \geq Q_{jst}$$

The solution to this problem is the following demand function

$$Q_{ijst}(o_{is}) = \left(\frac{P^j_{jst}}{P^j_{ijst}(o_{is})} \right)^{\sigma_s} Q_{jst}$$

Consumers also minimize spending across sectors as follows

$$\min_{Q_{jst}} \sum_s P^j_{jst} Q_{jst}$$

$$\text{such that } \left(\sum_s \alpha_{js}{}^{\frac{1}{\chi}} Q_{jst}{}^{\frac{\chi - 1}{\chi}} \right)^{\frac{\chi}{\chi - 1}} \geq Q_{jst}$$

which leads to a second demand function

$$P^j_{jst} Q_{jst} = \underbrace{\alpha_{js} \left(\frac{P^j_{jst}}{P^j_{jt}} \right)^{1-\chi}}_{\Xi_{jst}} P^j_{jt} Q_{jt}$$

which when plugged back into the consumption aggregator we can use the first-order condition to solve for the aggregate price index of country j, which is

$$P^j_{jt} = \left(\sum_s \alpha_{js} P^j_{jst}{}^{1-\chi} \right)^{\frac{1}{1-\chi}}$$

Finally, aggregate consumption equals labor income plus any profits rebated to the representative consumer.

$$P^j_{jt} Q_{jt} = W^j_{jt} L_j + \bar{P}^j_{jt}$$

A.2 Producers

I assume that the producer of variety o_{js} in country j and sector s is minimizing costs. The first problem consists of minimizing the total costs of hiring productive labor $L_{Pjst}(o_{js})$ and purchasing the input bundle $M_{jst}(o_{js})$ for a given level of production.

$$\min_{L_{Pjst}(o_{js}),M_{jst}(o_{is})} W_{jt}^{j} L_{Pjst}(o_{js}) + P_{M}{}_{jst}^{j} M_{jst}(o_{js})$$
$$\text{such that } A_{jst} z_{js}(o_{js}) L_{Pjst}(o_{js})^{1-\gamma_{js}} M_{jst}(o_{js})^{\gamma_{js}} \geq Y_{jst}(o_{js})$$

The first-order conditions of the problem are the following.

$$W_{jt}^{j} = (1-\gamma_{js})\lambda_{jst}^{j}(o_{js})\frac{Y_{jst}(o_{js})}{L_{Pjst}(o_{js})} \quad \text{and} \quad P_{M}{}_{jst}^{j} = \gamma_{js}\lambda_{jst}^{j}(o_{js})\frac{Y_{jst}(o_{js})}{M_{jst}(o_{js})}$$

Together with the production function, these can be used to solve for the shadow production cost.

$$\lambda_{jst}^{j}(o_{js}) = \underbrace{\frac{W_{jt}^{j\;1-\gamma_{js}} P_{M}{}_{jst}^{j\;\gamma_{js}}}{\gamma_{js}{}^{\gamma_{js}}(1-\gamma_{js})^{1-\gamma_{js}}}}_{MC_{jst}^{j}} \frac{1}{A_{jst} z_{js}(o_{js})}$$

The second problem of the producer of o_{js} is to minimize input spending across sectors

$$\min_{M_{jrst}(o_{js})} \sum_{r} P_{jrt}^{j} M_{jrst}(o_{js})$$
$$\text{such that } \left(\sum_{r} \gamma_{jrs}{}^{\frac{1}{\chi}} M_{jrst}(o_{js})^{\frac{\chi-1}{\chi}}\right)^{\frac{\chi}{\chi-1}} \geq M_{jst}(o_{js})$$

leading to the following input demand function

$$\begin{aligned} P_{jrt}^{j} M_{jrst}(o_{js}) &= \gamma_{jrs}\left(\frac{P_{jrt}^{j}}{P_{M}{}_{jst}^{j}}\right)^{1-\chi} P_{M}{}_{jst}^{j} M_{jst}(o_{js}) \\ &= \gamma_{js}\underbrace{\gamma_{jrs}\left(\frac{P_{jrt}^{j}}{P_{M}{}_{jst}^{j}}\right)^{1-\chi}}_{\Omega_{jrst}} \lambda_{jst}^{j}(o_{js}) Y_{jst}(o_{js}) \end{aligned}$$

where the second equality comes from the first-order condition of the first problem. When put back into the input aggregator the first-order condition it defines the input price index as follows.

$$P_{M}{}_{jst}^{j} = \left(\sum_{r} \gamma_{jrs} P_{jrt}^{j\;1-\chi}\right)^{\frac{1}{1-\chi}}$$

Finally, the producer of o_{js} in j has access to all varieties $o_{is} \in O_{ijst}$ coming from all other countries i. It then minimizes spending across all of these.

$$\min_{M_{ijrst}(o_{ir})} \int_{o_{ir}\in\Omega_{ijrt}} P^i_{irt}(o_{ir}) M_{ijrst}(o_{ir}) do_{ir}$$
$$\text{such that } \left(\sum_i \int_{o_{ir}\in\Omega_{ijrt}} M_{ijrst}(o_{ir})^{\frac{\sigma_r-1}{\sigma_r}} do_{ir}\right)^{\frac{\sigma_r}{\sigma_r-1}} \geq M_{jrst}(o_{js})$$

The solution to this problem is the following demand function.

$$M_{ijrst}(o_{ir}) = \left(\frac{P^i_{ijrt}(o_{ir})}{P^i_{jrt}}\right)^{-\sigma_r} M_{jrst}(o_{js})$$

A.3 Pricing

A.3.1 EK

Under perfect competition, producers are price takers and thus charge the shadow unit cost of a product, that is, the price of sending variety o_{is} from i to j is the following

$$P^j_{ijst}(o_{is}) = \frac{1}{\varepsilon_{ijt}} \frac{MC^i_{ist}\tau_{ijs}}{A_{ist} z_{is}(o_{is})}$$

which is just the unit costs multiplied by the iceberg cost of shipping from i to j.

A.3.2 MP

To determine prices in the case of MP, notice that if a variety o_{ir} is available in j then it is used by all producers in j as an input. I aggregate the demand function over destination varieties.

$$M_{ijrst}(o_{ir}) \int_{o_{js}\in\Omega_{jst}} do_{js} = \left(\frac{P^j_{ijrt}(o_{ir})}{P^j_{jrt}}\right)^{-\sigma_r} \int_{o_{js}\in\Omega_{jst}} M_{jrst}(o_{js}) do_{js}$$
$$M_{ijrst}(o_{ir}) = \left(\frac{P^j_{ijrt}(o_{ir})}{P^j_{jrt}}\right)^{-\sigma_r} \underbrace{\frac{1}{\int_{o_{js}\in\Omega_{jst}} do_{js}} \int_{o_{js}\in\Omega_{jst}} M_{jrst}(o_{js}) do_{js}}_{\equiv M_{jrst}}$$

and then impose market clearing for any given variety

$$Y_{ijst}(o_{is}) = Q_{ijst}(o_{is}) + \sum_r M_{ijsrt}(o_{is}) = \left(\frac{P^j_{ijst}(o_{is})}{P^j_{jst}}\right)^{-\sigma_s} Q_{jst} + \sum_r \left(\frac{P^j_{ijst}(o_{is})}{P^j_{jst}}\right)^{-\sigma_s} M_{jsrt}$$
$$= \left(\frac{P^j_{ijst}(o_{js})}{P^j_{jst}}\right)^{-\sigma_s} \underbrace{\left(Q_{jst} + \sum_r M_{jsrt}\right)}_{E_{jst}}$$

so that the profit maximization problem of a producer in i producing in sector s and selling to j is a price-setting problem where producers take into account the demand for their variety as follows.

$$\max_{P^j_{ijst}} \left(P^j_{ijst}(o_{is}) - \frac{1}{\varepsilon_{ijt}} \frac{MC^i_{ist}\tau_{ijs}}{A_{ist} z_{is}(o_{is})} \right) Y_{ijst}(o_{is}) - \frac{W^i_{it}}{\varepsilon_{ijt}} f_{ijs}$$

$$\text{such that} \quad Y_{ijst}(o_{is}) = \left(\frac{P^j_{ijst}(o_{is})}{P^j_{jst}} \right)^{-\sigma_s} E_{jst}$$

The first-order condition of the problem is a pricing strategy that places a constant markup over marginal costs

$$P^j_{ijst}(o_{is}) = \frac{\sigma_s}{\sigma_s - 1} \frac{1}{\varepsilon_{ijt}} \frac{MC^i_{ist}\tau_{ijs}}{A_{ist} z_{is}(o_{is})}$$

and a second result is a productivity threshold for producers in country j to operate in sector s in country i as an exporter

$$\underline{z}_{ijst} = \left[(\sigma_s - 1) \left(\frac{\sigma_s}{\sigma_s - 1} \right)^{\sigma_s} \frac{W^j_{it} f_{ijs}}{P^j_{jst}{}^{\sigma_s - 1} E^j_{jst}} \right]^{\frac{1}{\sigma_s - 1}} \frac{MC^j_{ist}\tau_{ijs}}{A_{ist}}$$

which comes from requiring profits net of fixed costs to be positive.

A.4 Trade shares and prices

A.4.1 EK

Following Assumption 4, the price paid by a buyer of variety o_s is the minimum found in the world $p^j_{jst}(o_s) = \min_i \{P^j_{ijst}(o_s)\}$. The probability of any given source country producing below a cutoff price is the following.

$$\Pr(P^j_{ijst} < P) = \Pr \left(\frac{1}{\varepsilon_{ijt}} \frac{MC^i_{ist}\tau_{ijs}}{A_{ist} z_{is}(o_{is})} < P \right) = 1 - \Pr \left(z_{is}(o_{is}) < \frac{1}{\varepsilon_{ijt}} \frac{MC^i_{ist}\tau_{ijs}}{A_{ist} P} \right)$$

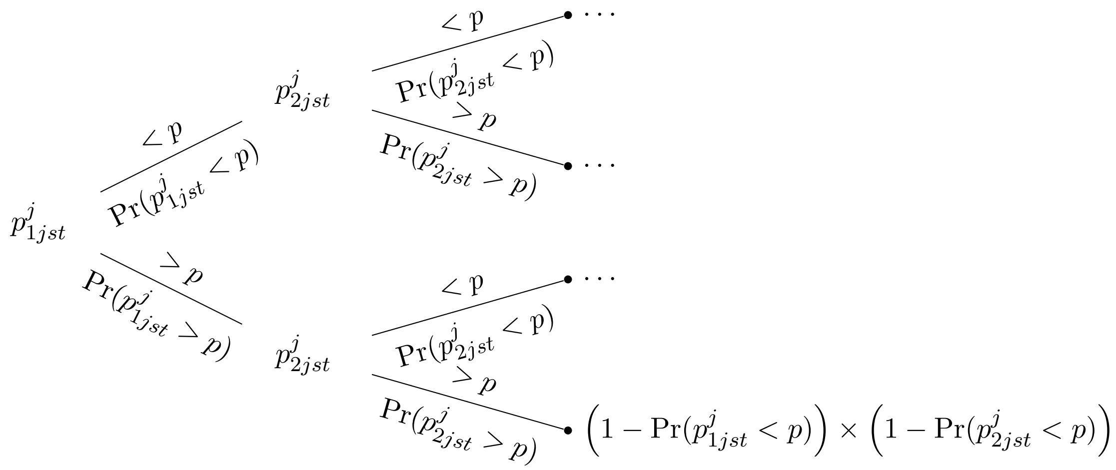
Given the assumption of Frechet distributed productivity draws $\Pr\left(z_{is}(o_{is}) < z\right) = e^{-T_{is} z^{-\theta_s}}$ the latter becomes

$$\Pr(P^j_{ijst} < P) = 1 - e^{-T_{is} \left(\frac{1}{\varepsilon_{ijt}} \frac{MC^i_{ist}\tau_{ijs}}{A_{ist} P} \right)^{-\theta_s}}$$

The distribution of prices faced by country j from all sources, that is the distribution of the minimum prices is then the following

$$\begin{aligned} \Pr(P_{jst} < P) &= \Pr(\min_i \{P_{ijst}(o_s) : i \in (1, ..., J)\} < P) \\ &= 1 - \prod_{i=1}^{J} (1 - \Pr(P^j_{ijst} < P)) = 1 - e^{\sum_i -T_{is} \left(\frac{1}{\varepsilon_{ijt}} \frac{MC^i_{ist}\tau_{ijs}}{A_{ist} P} \right)^{-\theta_s}} \end{aligned}$$

The intuition for these results can be seen in the following probability tree.



The probability that $P_{ijst}(o_s)$ is the smallest for variety o_s is the following

$$\Pi_{ijst} = \Pr\left(P_{ijst}(o_s) < \min\{P_{kjst}(o_s); k \neq i\}\right)$$

and can be computed by first representing the CDF of prices faced by country j as the product of country-specific CDFs

$$\begin{aligned}
G_{js}(P) &= 1 - \prod_{i=1}^{J}(1 - \Pr(P^j_{ijst} < P)) \\
&= 1 - (1 - \Pr(P^j_{ijst} < P)) \prod_{k \neq i}(1 - \Pr(P^j_{kjst} < P)) \\
&\Rightarrow \prod_{k \neq i}(1 - \Pr(P^j_{kjst} < P)) = \frac{1 - G_{js}(P)}{1 - \Pr(P^j_{ijst} < P)}
\end{aligned}$$

and then integrating it over all the possible price levels

$$\Pi_{ijst} = \int_0^\infty \frac{1 - G_{js}(P)}{1 - \Pr(P^j_{ijst} < P)} \underbrace{T_{is}\left(\frac{1}{\varepsilon_{ijt}} \frac{MC^i_{ist}\tau_{ijs}}{A_{ist}}\right)^{-\theta_s} \theta_s P^{\theta_s - 1} e^{-T_{is}\left(\frac{1}{\varepsilon_{ijt}} \frac{MC^i_{ist}\tau_{ijs}}{A_{ist}}\right)^{-\theta_s}}}_{\text{pdf}} dP$$

and hence

$$\Pi_{ijst} = \int_0^\infty \frac{e^{\sum_k -T_{ks}\left(\frac{1}{\varepsilon_{kjt}}\frac{MC_{kst}^k \tau_{kjs}}{A_{kst}P}\right)^{-\theta_s}}}{e^{-T_{is}\left(\frac{1}{\varepsilon_{ijt}}\frac{MC_{ist}^i \tau_{ijs}}{A_{ist}}\right)^{-\theta_s}}} T_{is}\left(\frac{1}{\varepsilon_{ijt}}\frac{MC_{ist}^i \tau_{ijs}}{A_{ist}P}\right)^{-\theta_s} \theta_s P^{\theta_s - 1} e^{-T_{is}\left(\frac{1}{\varepsilon_{ijt}}\frac{MC_{ist}^i \tau_{ijs}}{A_{ist}}\right)^{-\theta_s}} dP$$

$$= \int_0^\infty e^{\sum_k -T_{ks}\left(\frac{1}{\varepsilon_{kjt}}\frac{MC_{kst}^k \tau_{kjs}}{A_{kst}P}\right)^{-\theta_s}} T_{is}\left(\frac{1}{\varepsilon_{ijt}}\frac{MC_{ist}^i \tau_{ijs}}{A_{ist}}\right)^{-\theta_s} \theta_s P^{\theta_s - 1} dP$$

$$= T_{is}\left(\frac{1}{\varepsilon_{ijt}}\frac{MC_{ist}^i \tau_{ijs}}{A_{ist}}\right)^{-\theta_s} \int_0^\infty \theta_s P^{\theta_s - 1} e^{\sum_k -T_{ks}\left(\frac{1}{\varepsilon_{kjt}}\frac{MC_{kst}^k \tau_{ijs}}{A_{kst}P}\right)^{-\theta_s}} dP$$

$$= T_{is}\left(\frac{1}{\varepsilon_{ijt}}\frac{MC_{ist}^i \tau_{ijs}}{A_{ist}}\right)^{-\theta_s} \left.\frac{e^{\sum_k -T_{ks}\left(\frac{1}{\varepsilon_{kjt}}\frac{MC_{kst}^k \tau_{ijs}}{A_{kst}P}\right)^{-\theta_s}}}{\sum_k -T_{ks}\left(\frac{1}{\varepsilon_{kjt}}\frac{MC_{kst}^k \tau_{kjs}}{A_{kst}}\right)^{-\theta_s}}\right|_0^\infty$$

$$= T_{is}\left(\frac{1}{\varepsilon_{ijt}}\frac{MC_{ist}^i \tau_{ijs}}{A_{ist}}\right)^{-\theta_s} \frac{0-1}{\sum_k -T_{ks}\left(\frac{1}{\varepsilon_{kjt}}\frac{MC_{kst}^k \tau_{kjs}}{A_{kst}}\right)^{-\theta_s}}$$

$$= \frac{T_{is}\left(\frac{1}{\varepsilon_{ijt}}\frac{MC_{ist}^i \tau_{ijs}}{A_{ist}}\right)^{-\theta_s}}{\sum_k T_{ks}\left(\frac{1}{\varepsilon_{kjt}}\frac{MC_{kst}^k \tau_{kjs}}{A_{kst}}\right)^{-\theta_s}}$$

Following the original Eaton & Kortum (2002), one knows that a more productive source that produces at low costs or sends goods cheaply abroad will sell a wider range of goods. It does so exactly to the point where its price distribution in some sector s equals the price distribution of some destination j. Consequently, the solution to this problem also represents the trade shares between countries i and j in a sector s.

I solve for sectoral price indices, again using the assumption of Frechet distributed productivity draws.

$$\left(P_{jst}^j\right)^{1-\sigma} = \int_0^\infty P^{1-\sigma} G'_{js} dP$$

$$= \int_0^\infty \theta_s P^{\theta_s - \sigma_s} e^{\sum_k -T_{ks}\left(\frac{1}{\varepsilon_{kjt}}\frac{MC_{kst}^k \tau_{kjs}}{A_{kst}P}\right)^{-\theta_s}} \sum_k T_{ks}\left(\frac{1}{\varepsilon_{kjt}}\frac{MC_{kst}^k \tau_{kjs}}{A_{kst}}\right)^{-\theta_s} dP$$

To solve this integral one must use the following change of variable

$$m = P^{\theta_s} \sum_k T_{ks}\left(\frac{1}{\varepsilon_{kjt}}\frac{MC_{kst}^k \tau_{kjs}}{A_{kst}}\right)^{-\theta_s}$$

so that

$$P = \left(\frac{m}{\sum_k T_{ks}\left(\frac{1}{\varepsilon_{kjt}}\frac{MC_{kst}^k \tau_{kjs}}{A_{kst}}\right)^{-\theta_s}}\right)^{\frac{1}{\theta_s}} \Rightarrow \frac{dP}{dm} = \frac{1}{\theta_s}\left[\sum_k T_{ks}\left(\frac{1}{\varepsilon_{kjt}}\frac{MC_{kst}^k \tau_{kjs}}{A_{kst}}\right)^{-\theta_s}\right]^{-\frac{1}{\theta_s}} m^{\frac{1}{\theta_s}-1}$$

and then

$$
\begin{aligned}
\left(P^j_{jst}\right)^{1-\sigma} &= \int_0^\infty \theta_s P^{\theta_s-\sigma_s} e^{\sum_k -T_{ks}\left(\frac{1}{\varepsilon_{kjt}}\frac{MC^k_{kst}\tau_{kjs}}{A_{kst}P}\right)^{-\theta_s}} \sum_k T_{ks}\left(\frac{1}{\varepsilon_{kjt}}\frac{MC^k_{kst}\tau_{kjs}}{A_{kst}}\right)^{-\theta_s} dP \\
&= \int_0^\infty \theta_s P^{\theta_s-\sigma_s} e^{-m} \sum_k T_{ks}\left(\frac{1}{\varepsilon_{kjt}}\frac{MC^k_{kst}\tau_{kjs}}{A_{kst}}\right)^{-\theta_s} \frac{dP}{dm} dm \\
&= \int_0^\infty P^{-\sigma_s} m e^{-m} \theta_s \frac{dP}{dm} dm \\
&= \int_0^\infty P^{-\sigma_s} m e^{-m} \theta_s \frac{1}{\theta_s}\left[\sum_k T_{ks}\left(\frac{1}{\varepsilon_{kjt}}\frac{MC^k_{kst}\tau_{kjs}}{A_{kst}}\right)^{-\theta_s}\right]^{-\frac{1}{\theta_s}} m^{\frac{1}{\theta_s}-1} dm \\
&= \int_0^\infty m^{\frac{1-\sigma_s}{\theta_s}} e^{-m}\left[\sum_k T_{ks}\left(\frac{1}{\varepsilon_{kjt}}\frac{MC^k_{kst}\tau_{kjs}}{A_{kst}}\right)^{-\theta_s}\right]^{\frac{\sigma_s-1}{\theta_s}} dm \\
&= \underbrace{\int_0^\infty m^{\frac{1-\sigma_s}{\theta_s}} e^{-m} dm}_{\equiv\Gamma\left(\frac{1-\sigma_s}{\theta_s}+1\right)=\Gamma\left(\frac{\theta_s-(\sigma_s-1)}{\theta_s}\right)} \left[\sum_k T_{ks}\left(\frac{1}{\varepsilon_{kjt}}\frac{MC^k_{kst}\tau_{kjs}}{A_{kst}}\right)^{-\theta_s}\right]^{\frac{\sigma_s-1}{\theta_s}}
\end{aligned}
$$

such that finally

$$
P^j_{jst} = \underbrace{\left[\Gamma\left(\frac{\theta_s-(\sigma_s-1)}{\theta_s}\right)\right]^{\frac{1}{1-\sigma_s}}}_{\Upsilon_s} \left[\sum_k T_{ks}\left(\frac{1}{\varepsilon_{kjt}}\frac{MC^k_{kst}\tau_{kjs}}{A_{kst}}\right)^{-\theta_s}\right]^{-\frac{1}{\theta_s}}
$$

The exact equations in the paper are a combination of both prices and trade shares. For example, trade shares are written as

$$
\begin{aligned}
\Pi_{ijst} &= \frac{T_{is}\left(\frac{1}{\varepsilon_{ijt}}\frac{MC^i_{ist}\tau_{ijs}}{A_{ist}}\right)^{-\theta_s}}{\sum_k T_{ks}\left(\frac{1}{\varepsilon_{kjt}}\frac{MC^k_{kst}\tau_{kjs}}{A_{kst}}\right)^{-\theta_s}} = \frac{(\Upsilon_s)^{-\theta_s} T_{is}\left(\frac{1}{\varepsilon_{ijt}}\frac{MC^i_{ist}\tau_{ijs}}{A_{ist}}\right)^{-\theta_s}}{\underbrace{(\Upsilon_s)^{-\theta_s}\sum_k T_{ks}\left(\frac{1}{\varepsilon_{kjt}}\frac{MC^k_{kst}\tau_{kjs}}{A_{kst}}\right)^{-\theta_s}}_{(P^j_{jst})^{-\theta_s}}} \\
&= \Upsilon_s^{-\theta_s}\left(\frac{1}{\varepsilon_{ijt}}\frac{MC^i_{ist}\tau_{ijs}}{A_{ist}P^j_{jst}}\right)^{-\theta_s}
\end{aligned}
$$

A.5 MP

I use the equations that are part of the solution to the producer's problem in combination with the assumption of Pareto productivity draws $G_{is}(z) = 1 - (z/\underline{z}_{is})^{-\kappa_s}$ to find sectoral prices.

$$\begin{aligned}
\left(P^j_{jst}\right)^{1-\sigma_s} &= \sum_i \int_{o_{is}\in O_{ijst}} P^j_{ijst}(o_{is})^{1-\sigma_s} do_{is} \\
&= \sum_i \int_{o_{is}\in O_{ijst}} \left(\frac{\sigma_s}{\sigma_s-1}\frac{1}{\varepsilon_{ijt}}\frac{MC^i_{ist}\tau_{ijs}}{A_{ist}z_{is}(o_{is})}\right)^{1-\sigma_s} do_{is} \\
&= \left(\frac{\sigma_s}{\sigma_s-1}\right)^{1-\sigma_s} \sum_i \left(\frac{1}{\varepsilon_{ijt}}\frac{MC^i_{ist}\tau_{ijs}}{A_{ist}}\right)^{1-\sigma_s} \int_{o_{is}\in O_{ijst}} z_{ist}(o_{is})^{\sigma_s-1} do_{is} \\
&= \left(\frac{\sigma_s}{\sigma_s-1}\right)^{1-\sigma_s} \sum_i \left(\frac{1}{\varepsilon_{ijt}}\frac{MC^i_{ist}\tau_{ijs}}{A_{ist}}\right)^{1-\sigma_s} \int_{\underline{z}_{ijst}}^{\infty} z^{\sigma_s-1} N_{ijst} \underbrace{dG(z|\underline{z}_{ijst})}_{dG(z|\underline{z})=\frac{dG(z)}{1-G(\underline{z})}} \\
&= \left(\frac{\sigma_s}{\sigma_s-1}\right)^{1-\sigma_s} \sum_i \left(\frac{1}{\varepsilon_{ijt}}\frac{MC^i_{ist}\tau_{ijs}}{A_{ist}}\right)^{1-\sigma_s} \int_{\underline{z}_{ijst}}^{\infty} z^{\sigma_s-1} \underbrace{\frac{N_{ijst}}{1-G_{is}(\underline{z}_{ijst})}}_{N_{ijst}=N_{is}\left(\underline{z}_{ijst}/\underline{z}_{is}\right)^{-\kappa_s}} dG_{is}(z) \\
&= \left(\frac{\sigma_s}{\sigma_s-1}\right)^{1-\sigma_s} \sum_i \left(\frac{1}{\varepsilon_{ijt}}\frac{MC^i_{ist}\tau_{ijs}}{A_{ist}}\right)^{1-\sigma_s} \\
&\times \int_{\underline{z}_{ijst}}^{\infty} z^{\sigma_s-1} N_{is}\left(\frac{\underline{z}_{ijst}}{\underline{z}_{is}}\right)^{-\kappa_s}\left(1-1+\left(\frac{\underline{z}_{ijst}}{\underline{z}_{is}}\right)^{-\kappa_s}\right)^{-1} \kappa_s(\underline{z}_{is})^{\kappa_s} z^{-\kappa_s-1} dz \\
&= \left(\frac{\sigma_s}{\sigma_s-1}\right)^{1-\sigma_s} \sum_i \left(\frac{1}{\varepsilon_{ijt}}\frac{MC^i_{ist}\tau_{ijs}}{A_{ist}}\right)^{1-\sigma_s} \int_{\underline{z}_{ijst}}^{\infty} N_{is}\kappa_s \underline{z}_{is}^{\kappa_s} z^{\sigma_s-1-\kappa_s-1} dz \\
&= \left(\frac{\sigma_s}{\sigma_s-1}\right)^{1-\sigma_s} \sum_i \left(\frac{1}{\varepsilon_{ijt}}\frac{MC^i_{ist}\tau_{ijs}}{A_{ist}}\right)^{1-\sigma_s} \frac{N_{is}\kappa_s \underline{z}_{is}^{\kappa_s}}{\sigma_s-1-\kappa_s}\left(0-\underline{z}_{ijst}^{\sigma_s-1-\kappa_s}\right) \\
&= \left(\frac{\sigma_s}{\sigma_s-1}\right)^{1-\sigma_s} \frac{\kappa_s}{\kappa_s-(\sigma_s-1)} \sum_i N_{is}\underline{z}_{is}^{\kappa_s} \underline{z}_{ijst}{}^{\sigma_s-1-\kappa_s} \left(\frac{1}{\varepsilon_{ijt}}\frac{MC^i_{ist}\tau_{ijs}}{A_{ist}}\right)^{1-\sigma_s}
\end{aligned}$$

I then substitute for the exporting cutoff

$$\begin{aligned}\left(P^j_{jst}\right)^{1-\sigma_s} &= \left(\frac{\sigma_s}{\sigma_s-1}\right)^{1-\sigma_s}\frac{\kappa_s}{\kappa_s-(\sigma_s-1)}\\ &\times\sum_i N_{is}\underline{z}_{is}^{\kappa_s}\left[(\sigma_s-1)\left(\frac{\sigma_s}{\sigma_s-1}\right)^{\sigma_s}\frac{W^j_{it}f_{ijs}}{{P^j_{jst}}^{\sigma_s-1}E^j_{jst}}\right]^{\frac{\sigma_s-1-\kappa_s}{\sigma_s-1}}\\ &\times\left(\frac{MC^j_{ist}\tau_{ijs}}{A_{ist}}\right)^{\sigma_s-1-\kappa_s}\left(\frac{1}{\varepsilon_{ijt}}\frac{MC^i_{ist}\tau_{ijs}}{A_{ist}}\right)^{1-\sigma_s}\\ &= \left(\frac{\sigma_s}{\sigma_s-1}\right)^{1-\sigma_s}\frac{\kappa_s}{\kappa_s-(\sigma_s-1)}\left[(\sigma_s-1)\left(\frac{\sigma_s}{\sigma_s-1}\right)^{\sigma_s}\right]^{\frac{\sigma_s-1-\kappa_s}{\sigma_s-1}}\\ &\times\sum_i N_{is}\underline{z}_{is}^{\kappa_s}\left(\frac{W^j_{it}f_{ijs}}{{P^j_{jst}}^{\sigma_s-1}E^j_{jst}}\right)^{\frac{\sigma_s-1-\kappa_s}{\sigma_s-1}}\left(\frac{MC^j_{ist}\tau_{ijs}}{A_{ist}}\right)^{-\kappa_s}\end{aligned}$$

and after collecting terms on prices I find

$$\begin{aligned}\left(P^j_{jst}\right)^{-\kappa_s} &= \left(\frac{\sigma_s}{\sigma_s-1}\right)^{1-\sigma_s}\frac{\kappa_s}{\kappa_s-(\sigma_s-1)}\left[(\sigma_s-1)\left(\frac{\sigma_s}{\sigma_s-1}\right)^{\sigma_s}\right]^{\frac{\sigma_s-1-\kappa_s}{\sigma_s-1}}\\ &\times\sum_i N_{is}\underline{z}_{is}^{\kappa_s}\left(\frac{W^j_{it}f_{ijs}}{E^j_{jst}}\right)^{\frac{\sigma_s-1-\kappa_s}{\sigma_s-1}}\left(\frac{MC^j_{ist}\tau_{ijs}}{A_{ist}}\right)^{-\kappa_s}\end{aligned}$$

leading to the equation in the main body of the text

$$P^j_{jst} = \tilde{\Upsilon}_s\left[\sum_i \underbrace{N_{is}\underline{z}_{is}^{\kappa_s}}_{\tilde{T_{is}}}\left(\frac{W^j_{it}f_{ijs}}{E^j_{jst}}\right)^{\frac{\sigma_s-1-\kappa_s}{\sigma_s-1}}\left(\frac{MC^j_{ist}\tau_{ijs}}{A_{ist}}\right)^{-\kappa_s}\right]^{-\frac{1}{\kappa_s}}$$

where

$$\tilde{\Upsilon}_s = \left[\left(\frac{\sigma_s}{\sigma_s-1}\right)^{1-\sigma_s}\frac{\kappa_s}{\kappa_s-(\sigma_s-1)}\left[(\sigma_s-1)\left(\frac{\sigma_s}{\sigma_s-1}\right)^{\sigma_s}\right]^{\frac{\sigma_s-1-\kappa_s}{\sigma_s-1}}\right]^{-\frac{1}{\kappa_s}}$$

Once aggregate prices are found, the same equations can be used to compute aggregate trade shares per sector. Start with the volume of a particular variety o_{is} and impose market clearing in a given market j.

$$P^j_{ijst}(o_{is})Y_{ijst}(o_{is}) = \left(\frac{P^j_{ijst}(o_{is})}{P^j_{jst}}\right)^{1-\sigma_s}P^j_{jst}E_{jst}$$

Define trade shares as the sum across o_{is} over j's absorption

$$\Pi_{ijst} = \frac{\int_{o_{is}\in O_{ijst}} P^j_{ijst}(o_{is})Y_{ijst}(o_{is})do_{is}}{P^j_{jst}E_{jst}} = \int_{o_{is}\in O_{ijst}}\left(\frac{P^j_{ijst}(o_{is})}{P^j_{jst}}\right)^{1-\sigma_s} do_{is}$$

which from the definition of prices leads to

$$\Pi_{ijst} = \frac{N_{is}\underline{z}_{is}^{\kappa_s}\bar{z}_{ijst}^{\sigma_s-1-\kappa_s}\left(\frac{1}{\varepsilon_{ijt}}\frac{MC_{ist}^{i}\tau_{ijs}}{A_{ist}}\right)^{1-\sigma_s}}{\sum_k N_{ks}\underline{z}_{ks}^{\kappa_s}\bar{z}_{kjst}^{\sigma_s-1-\kappa_s}\left(\frac{1}{\varepsilon_{kjt}}\frac{MC_{kst}^{k}\tau_{kjs}}{A_{kst}}\right)^{1-\sigma_s}}$$

or without solving for prices is

$$\Pi_{ijst} = \frac{N_{is}\underline{z}_{is}^{\kappa_s}\bar{z}_{ijst}^{\sigma_s-1-\kappa_s}\left(\frac{1}{\varepsilon_{ijt}}\frac{MC_{ist}^{i}\tau_{ijs}}{A_{ist}}\right)^{1-\sigma_s}}{\left(\frac{\sigma_s-1}{\sigma_s}\right)^{1-\sigma_s}\frac{\kappa_s-(\sigma_s-1)}{\kappa_s}\left(P_{jst}^{j}\right)^{1-\sigma_s}}$$

Here I use the solution for the exporting cutoff

$$\Pi_{ijst} = \frac{N_{is}\underline{z}_{is}^{\kappa_s}\left[(\sigma_s-1)\left(\frac{\sigma_s}{\sigma_s-1}\right)^{\sigma_s}\frac{W_{it}^{j}f_{ijs}}{P_{jst}^{j\;\sigma_s-1}E_{jst}^{j}}\right]^{\frac{\sigma_s-1-\kappa_s}{\sigma_s-1}}\left(\frac{MC_{ist}^{j}\tau_{ijs}}{A_{ist}}\right)^{\sigma_s-1-\kappa_s}\left(P_{jst}^{j}\right)^{1-\sigma_s}}{\left(\frac{\sigma_s-1}{\sigma_s}\right)^{1-\sigma_s}\frac{\kappa_s-(\sigma_s-1)}{\kappa_s}\left(P_{jst}^{j}\right)^{1-\sigma_s}}$$

$$= N_{is}\underline{z}_{is}^{\kappa_s}\frac{\kappa_s}{\kappa_s-(\sigma_s-1)}\left[(\sigma_s-1)\left(\frac{\sigma_s}{\sigma_s-1}\right)^{\sigma_s}\frac{W_{it}^{j}f_{ijs}}{P_{jst}^{j\;\sigma_s-1}E_{jst}^{j}}\right]^{\frac{\sigma_s-1-\kappa_s}{\sigma_s-1}}$$

$$\times\left(\frac{\sigma_s}{\sigma_s-1}\right)^{1-\sigma_s}\left(P_{jst}^{j}\right)^{\sigma_s-1}\left(\frac{MC_{ist}^{j}\tau_{ijs}}{A_{ist}}\right)^{-\kappa_s}$$

$$= N_{is}\underline{z}_{is}^{\kappa_s}\frac{\kappa_s}{\kappa_s-(\sigma_s-1)}\left[(\sigma_s-1)\left(\frac{\sigma_s}{\sigma_s-1}\right)^{\sigma_s}\frac{W_{it}^{j}f_{ijs}}{E_{jst}^{j}}\right]^{\frac{\sigma_s-1-\kappa_s}{\sigma_s-1}}\left(P_{jst}^{j}\right)^{\kappa_s-(\sigma_s-1)}$$

$$\times\left(\frac{\sigma_s}{\sigma_s-1}\right)^{1-\sigma_s}\left(P_{jst}^{j}\right)^{\sigma_s-1}\left(\frac{MC_{ist}^{j}\tau_{ijs}}{A_{ist}}\right)^{-\kappa_s}$$

$$= \frac{N_{is}\underline{z}_{is}^{\kappa_s}\kappa_s}{\kappa_s-(\sigma_s-1)}\left[(\sigma_s-1)\left(\frac{\sigma_s}{\sigma_s-1}\right)^{\sigma_s}\right]^{\frac{\sigma_s-1-\kappa_s}{\sigma_s-1}}\left(\frac{\sigma_s}{\sigma_s-1}\right)^{1-\sigma_s}\left(\frac{W_{it}^{j}f_{ijs}}{E_{jst}^{j}}\right)^{1-\frac{\kappa_s}{\sigma_s-1}}\left(\frac{MC_{ist}^{j}\tau_{ijs}}{A_{ist}P_{jst}^{j}}\right)^{-\kappa_s}$$

now recall

$$\left(\tilde{\Upsilon}_s\right)^{-\kappa_s} = \left(\frac{\sigma_s}{\sigma_s-1}\right)^{1-\sigma_s}\frac{\kappa_s}{\kappa_s-(\sigma_s-1)}\left[(\sigma_s-1)\left(\frac{\sigma_s}{\sigma_s-1}\right)^{\sigma_s}\right]^{\frac{\sigma_s-1-\kappa_s}{\sigma_s-1}}$$

leading to

$$\Pi_{ijst} = \tilde{\Upsilon}_s^{-\kappa_s}\tilde{T}_{is}\left(\frac{W_{it}^{j}f_{ijs}}{E_{jst}^{j}}\right)^{1-\frac{\kappa_s}{\sigma_s-1}}\left(\frac{MC_{ist}^{j}\tau_{ijs}}{A_{ist}P_{jst}^{j}}\right)^{-\kappa_s}$$

which is the expression in the text.

A.6 Market Clearing

A.6.1 EK

Define aggregate nominal variables as $X^i_{ist} = \int_{o_{is}\in\Omega_{is}} X^i_{ist}(o_{is})do_{is}$ so that the first-order condition of the producer become the following

$$W^i_{it}\int_{o_{is}\in O_{ist}} L_{ist}(o_{is})do_{is} = (1-\gamma_{is})\int_{o_{is}\in O_{ist}} P^i_{ist}(o_{is})Y_{ist}(o_{is})do_{is}$$
$$W^i_{it}L_{Pist} = (1-\gamma_{is})Y^i_{ist}$$

and the first order condition for inputs is aggregated nominally as follows.

$$\int_{o_{is}\in O_{ist}} P^i_{irt}M_{irst}(o_{is})do_{is} = \gamma_{is}\Omega_{irst}\int_{o_{is}\in O_{ist}} P^i_{ist}(o_{is})Y_{ist}(o_{is})do_{is}$$
$$M^i_{irst} = \gamma_{is}\Omega_{irst}Y^i_{ist}$$

Market clearing for goods necessitates that gross output, the dollar amount that is produced, equals to total spending everywhere in the world.

$$Y^j_{jst} = \sum_i \Pi_{jist}\left(P^j_{ist}Q_{ist} + \sum_r M^j_{isrt}\right) = \sum_i \Pi_{jist}\sum_r \Big(\Xi_{ist}(1-\gamma_{ir}) + \Omega_{isrt}\gamma_{ir}\Big)Y^i_{irt}$$

where I've used the definition of trade shares as the percentage of total spending in some country i that orginates from j.

A.6.2 MP

Aggregate market clearing uses the definition of trade shares as follows

$$Y_{jist}(o_{js}) = Q_{jist}(o_{js}) + \sum_r M_{jisrt}(o_{js})$$
$$Y_{jist}(o_{js}) = \left(\frac{P^j_{jist}(o_{js})}{P^j_{ist}}\right)^{-\sigma^s} Q_{ist} + \sum_r \left(\frac{P^j_{jist}(o_{js})}{P^j_{ist}}\right)^{-\sigma^s} M_{isrt}$$
$$P^j_{jist}(o_{js})Y_{jist}(o_{js}) = \left(\frac{P^j_{jist}(o_{js})}{P^j_{ist}}\right)^{1-\sigma^s} P^j_{ist}Q_{ist} + \sum_r \left(\frac{P^j_{jist}(o_{js})}{P^j_{ist}}\right)^{1-\sigma^s} P^j_{ist}M_{isrt}$$
$$\underbrace{\int_{o_{js}\in O_{jist}} P^j_{jist}(o_{js})Y_{jist}(o_{js})do_{js}}_{Y^j_{jist}} = \underbrace{\left[\int_{o_{js}\in O_{jist}} \left(\frac{P^j_{jist}(o_{js})}{P^j_{ist}}\right)^{1-\sigma^s} do_{js}\right]}_{\Pi_{jist}} \left(P^j_{ist}Q_{ist} + \sum_r P^j_{ist}M_{isrt}\right)$$

which can be added across destination countries

$$Y^j_{jst} = \sum_i \Pi_{jist}\left[P^j_{ist}Q_{ist} + \sum_r P^j_{ist}M_{isrt}\right]$$

Using the fact that producers have a markup over marginal costs

$$\lambda^i_{ist}(o_{is}) = \frac{\sigma_s - 1}{\sigma_s} P^i_{ist}(o_{is})$$

I aggregate productive labor use

$$W^i_{it} \int_{o_{is}\in O_{ist}} L_{Pist}(o_{is}) do_{is} = (1-\gamma_{is})\frac{\sigma_s - 1}{\sigma_s}\int_{o_{is}\in O_{ist}} P^i_{ist}(o_{is}) Y_{ist}(o_{is}) do_{is}$$
$$W^i_{it} L_{Pist} = (1-\gamma_{is})\frac{\sigma_s - 1}{\sigma_s} Y^i_{ist}$$

while noticing that one still needs to calculate entry labor income, however this step is not necessary to express final spending as a function of gross output

$$\begin{aligned}
P^i_{ist}Q_{ist} &= \Xi_{ist}P^i_{it}Q_{it} = \Xi_{ist}(W^i_{it}L_{it} + \Pi^i_{it}) \\
&= \Xi_{ist}W^i_{it}(L_{Pit} + L_{Eit}) \\
&+ \Xi_{ist}\sum_r\sum_j \int_{o_{ir}\in O_{ijrt}} \left[\left(P^i_{ijrt}(o_{ir}) - \frac{c^i_{irt}\tau_{ijr}}{A_{irt}z_{ir}(o_{ir})}\right) Y_{ijrt}(o_{ir}) - W^i_{it} f_{ijr}\right] do_{ir} \\
&= \Xi_{ist}W^i_{it}(L_{Pit} + L_{Eit}) \\
&+ \Xi_{ist}\sum_r\sum_j \int_{o_{ir}\in O_{ijrt}} \left[\left(1 - \frac{\sigma_r - 1}{\sigma_r}\right) P^i_{ijrt}(o_{ir}) Y_{ijrt}(o_{ir}) - W^i_{it} f_{ijr}\right] do_{ir} \\
&= \Xi_{ist}W^i_{it}\sum_r L_{Pirt} + \Xi_{ist}\sum_r \frac{1}{\sigma_r}Y^i_{irt} = \Xi_{ist}\sum_r (1-\gamma_{ir})\frac{\sigma_r - 1}{\sigma_r}Y^i_{irt} + \Xi_{ist}\sum_r \frac{1}{\sigma_r}Y^i_{irt} \\
&= \sum_r \Xi_{ist}(1-{\mu_r}^{-1}\gamma_{ir})Y^i_{irt}
\end{aligned}$$

where $\mu_r = \sigma_r/(\sigma_r - 1)$ are input shares adjusted for inverse markups. These imply that market clearing can be written as

$$Y^j_{jts} = \sum_i \Pi_{jist} \sum_r \left(\Xi_{ist}(1 - {\mu_r}^{-1}\gamma_{ir}) + \Omega_{isrt}{\mu_r}^{-1}\gamma_{ir}\right) Y^i_{irt}$$

A.6.3 Entry labor income

The following is proof that the entry labor is a constant fraction of gross output.

$$\begin{aligned}
L_{Eist} &= \sum_j \int_{\underline{z}_{ijst}}^{\infty} f_{ijs}\kappa_s {\underline{z}_{is}}^{\kappa_s} z^{-\kappa_s - 1} dz = \sum_j f_{ijs}\kappa_s {\underline{z}_{is}}^{\kappa_s}\frac{1}{\kappa_s}{\underline{z}_{ijst}}^{-\kappa_s} \\
&= \sum_j f_{ijs}{\underline{z}_{is}}^{\kappa_s}\left[(\sigma_s - 1)\left(\frac{\sigma_s}{\sigma_s - 1}\right)^{\sigma_s}\frac{W^j_{it}f_{ijs}}{E^j_{jst}}\right]^{-\frac{\kappa_s}{\sigma_s - 1}}\left(\frac{MC^j_{ist}\tau_{ijs}}{A_{ist}P^j_{jst}}\right)^{-\kappa_s} \\
&= \sum_j \Pi_{ijst}{\Pi_{ijst}}^{-1} f_{ijs}{\underline{z}_{is}}^{\kappa_s}\left[(\sigma_s - 1)\left(\frac{\sigma_s}{\sigma_s - 1}\right)^{\sigma_s}\frac{W^j_{it}f_{ijs}}{E^j_{jst}}\right]^{-\frac{\kappa_s}{\sigma_s - 1}}\left(\frac{MC^j_{ist}\tau_{ijs}}{A_{ist}P^j_{jst}}\right)^{-\kappa_s}
\end{aligned}$$

Now recall

$$\Pi_{ijst} = \tilde{\Upsilon}_s^{-\kappa_s} \underline{z}_{js}{}^{\kappa_{js}} \left(\frac{1}{\varepsilon_{ijt}} \frac{MC_{ist}^i \tau_{ijs}}{A_{ist} P_{jst}^j} \right)^{-\kappa_s} \left(\frac{E_{jst}^j}{f_{ijs} W_{ist}^j} \right)^{\frac{\kappa_s}{\sigma_s - 1} - 1}$$

such that

$$\begin{aligned} L_{Eist} &= \sum_j \Pi_{ijst} \tilde{\Upsilon}_s^{\kappa_s} \underline{z}_{js}{}^{-\kappa_{js}} \left(\frac{MC_{ist}^j \tau_{ijs}}{A_{ist} P_{jst}^j} \right)^{\kappa_s} \left(\frac{E_{jst}^j}{f_{ijs} W_{ist}^j} \right)^{-\frac{\kappa_s}{\sigma_s - 1} + 1} \\ &\times f_{ijs} \underline{z}_{is}{}^{\kappa_s} \left[(\sigma_s - 1) \left(\frac{\sigma_s}{\sigma_s - 1} \right)^{\sigma_s} \frac{W_{it}^j f_{ijs}}{E_{jst}^j} \right]^{-\frac{\kappa_s}{\sigma_s - 1}} \left(\frac{MC_{ist}^j \tau_{ijs}}{A_{ist} P_{jst}^j} \right)^{-\kappa_s} \\ &= \sum_j \Pi_{ijst} \tilde{\Upsilon}_s^{\kappa_s} \left[(\sigma_s - 1) \left(\frac{\sigma_s}{\sigma_s - 1} \right)^{\sigma_s} \right]^{-\frac{\kappa_s}{\sigma_s - 1}} \frac{E_{jst}^j}{W_{it}^j} \end{aligned}$$

Now recall a second expression

$$\tilde{\Upsilon}_s = (\sigma_s - 1)^{\frac{1}{\sigma_s - 1} - \frac{1}{\kappa_s}} \left(\frac{\sigma_s}{\sigma_s - 1} \right)^{\frac{\sigma_s}{\sigma_s - 1} - \frac{1}{\kappa_s}} \left[\frac{\kappa_s}{\kappa_s - (\sigma_s - 1)} \right]^{-\frac{1}{\kappa_s}}$$

such that

$$\begin{aligned} L_{Eist} &= \sum_j \Pi_{ijst} (\sigma_s - 1)^{\frac{\kappa_s}{\sigma_s - 1} - 1} \left(\frac{\sigma_s}{\sigma_s - 1} \right)^{\kappa_s \frac{\sigma_s}{\sigma_s - 1} - 1} \left[\frac{\kappa_s}{\kappa_s - (\sigma_s - 1)} \right]^{-1} \left[(\sigma_s - 1) \left(\frac{\sigma_s}{\sigma_s - 1} \right)^{\sigma_s} \right]^{-\frac{\kappa_s}{\sigma_s - 1}} \frac{E_{jst}^j}{W_{it}^j} \\ &= \sum_j \Pi_{ijst} (\sigma_s - 1)^{-1} \left(\frac{\sigma_s}{\sigma_s - 1} \right)^{-1} \frac{\kappa_s - (\sigma_s - 1)}{\kappa_s} \frac{E_{jst}^j}{W_{it}^j} \\ &= \sum_j \Pi_{ijst} \phi_s \frac{\sigma_s - 1}{\sigma_s} \frac{E_{jst}^j}{W_{it}^j} \end{aligned}$$

finally leading to an expression of income paid to exporting labor

$$W_{it}^j L_{Eist} = \phi_s \frac{\sigma_s - 1}{\sigma_s} \sum_j \pi_{ijst} E_{jst}^j = \phi_s \mu_s{}^{-1} Y_{ist}^j$$

B Linear solution

B.1 Expenditure switching

In this appendix, I solve for the change in spending shares as changes in prices, which will then be a part of the expenditure switching matrix. Given the expression that defines input spending shares, it follows that

$$\Omega_{isrt} = \gamma_{isr} \left(\frac{P_{ist}^i}{P_{M}{}_{irt}^i} \right)^{1-\chi} \to \omega_{isrt} = (\chi - 1) \left(p_{M}{}_{irt}^i - p_{ist}^i \right)$$

and from the expression for the input price index, it follows that

$$P_{M}{}^{i}_{irt} = \left(\sum_q \gamma_{iqr} P_{iqt}{}^{1-\chi}\right)^{\frac{1}{1-\chi}} \rightarrow p_{M}{}^{i}_{irt} = \sum_q \underbrace{\gamma_{iqr} \left(\frac{P^i_{iq}}{P_{M}{}^{i}_{ir}}\right)^{1-\chi}}_{\Omega_{iqr}} p^i_{iqt}$$

such that finally there is an expression for the change in input spending shares as a function of prices

$$\omega_{isrt} = (\chi - 1)\left(\sum_q \Omega_{iqr} p^i_{iqt} - P^i_{ist}\right)$$

Expenditure switching in final consumption is also reexpressed as a function of prices first by linearizing final spending shares

$$\Xi_{jst} = \alpha_{js}\left(\frac{P^j_{jst}}{P^j_{jt}}\right)^{1-\chi} \rightarrow \xi_{jst} = (\chi - 1)\left(p^j_{jt} - p^j_{jst}\right)$$

and we have an equation for final prices

$$P^j_{jt} = \left(\sum_s \alpha_{js} P^j_{jst}{}^{1-\chi}\right)^{\frac{1}{1-\chi}} \rightarrow p^j_{jt} = \sum_s \underbrace{\alpha_{js}\left(\frac{P^j_{js}}{P^j_{j}}\right)^{1-\chi}}_{\Xi_{js}} p^j_{jst}$$

such that the change in final spending shares can be expressed as a function of the changes in sector prices only as follows.

$$\xi_{jst} = (\chi - 1)\left(\sum_r \Xi_{jr} p^j_{jrt} - p^j_{jst}\right)$$

B.2 Prices as functions of quantitities

The first step in recovering the influence matrix is to solve for prices as a function of quantities. This step requires using the market clearing condition, the trade shares equation and the price as a function of unit-cost equations, but these steps differ depending on the framework.

B.2.1 EK

I start with the first-order linearized market clearing equation where I use the expenditure switching equations.

$$
\begin{aligned}
y^j_{jst} &= (Y^j_{js})^{-1}\sum_i \Pi_{jis}E^j_{ir}\pi_{jist} + (Y^j_{js})^{-1}\sum_i \Pi_{jis}\sum_r \left((1-\gamma_{ir})\Xi_{is}+\gamma_{ir}\Omega_{isr}\right)Y^j_{ir}y^j_{irt} \\
&\quad + (Y^j_{js})^{-1}\sum_i \Pi_{jis}\Xi_{is}\Big(\sum_r (1-\gamma_{ir})Y^j_{ir}\Big)\xi_{ist} + \sum_i \Pi_{jis}\sum_r \gamma_{ir}\Omega_{isr}\frac{Y^j_{ir}}{Y^j_{js}}\omega_{isrt} \\
y^j_{jst} &= (Y^j_{js})^{-1}\sum_i \Pi_{jis}E^j_{ir}\theta_s\left(p^j_{ist} - mc^j_{jst} + a_{jst}\right) \\
&\quad + (Y^j_{js})^{-1}\sum_i \Pi_{jis}\sum_r \left((1-\gamma_{ir})\Xi_{is}+\gamma_{ir}\Omega_{isr}\right)Y^j_{ir}y^j_{irt} \\
&\quad + (Y^j_{js})^{-1}\sum_i \Pi_{jis}\Xi_{is}\Big(\sum_r (1-\gamma_{ir})Y^j_{ir}\Big)\xi_{ist} + (Y^j_{js})^{-1}\sum_i \Pi_{jis}\sum_r \gamma_{ir}\Omega_{isr}Y^j_{ir}\omega_{isrt} \\
y^j_{jst} &= -\theta_s(mc^j_{jst} + a_{jst}) + (Y^j_{js})^{-1}\sum_i \Pi_{jis}E^j_{ir}\theta_s p^j_{ist} \\
&\quad + (Y^j_{js})^{-1}\sum_i \Pi_{jis}\sum_r \left((1-\gamma_{ir})\Xi_{is}+\gamma_{ir}\Omega_{isr}\right)Y^j_{ir}y^j_{irt} \\
&\quad + (Y^j_{js})^{-1}\sum_i \Pi_{jis}\sum_r (1-\gamma_{ir})\Xi_{is}Y^j_{ir}(\chi-1)\left(\sum_q \Xi_{jq}p^j_{jrt} - p^j_{jst}\right) \\
&\quad + (Y^j_{js})^{-1}\sum_i \Pi_{jis}\sum_r \gamma_{ir}\Omega_{isr}Y^j_{ir}(\chi-1)\left(\sum_q \Omega_{iqr}p^i_{iqt} - p^i_{ist}\right)
\end{aligned}
$$

where I have also used the first-order linearized trade shares equation

$$
\pi_{ijst} = \theta_s\left(p_{jst} - mc_{ist} + a_{ist}\right)
$$

In matrix form, the market clearing equation can then be written as

$$
p^j_t + y_t = -\theta(mc^j_t - a_t) + \mathbf{Y}^{-1}\mathbf{\Pi}'\mathbf{E}\theta p^j_t + \mathbf{D}(p^j_t + y_t) + \mathbf{X}p^j_t
$$

which is then premultiplied by the trade shares matrix to use the following relationship between prices and units costs

$$
p^j_{jst} = \sum_i \Pi_{ijs}\left(mc^j_{ist} - a_{ist}\right) \Rightarrow p^j_t = \mathbf{\Pi}(mc_t - a_t)
$$

so that I can solve for prices as a function of quantities

$$
\mathbf{\Pi}(\mathbf{I}-\mathbf{D})y_t = -\mathbf{\Pi}(\mathbf{I}-\mathbf{D})p^j_t - \left(\mathbf{I} - \mathbf{\Pi}\underbrace{\mathbf{Y}^{-1}\mathbf{\Pi}'\mathbf{E}}_{\mathbf{M}}\right)\theta p^j_t + \mathbf{\Pi}\mathbf{X}p^j_t
$$

finally leading to the price-to-quantity equation in EK

$$
p^j_t = \underbrace{-\left[\mathbf{\Pi}(\mathbf{I}-\mathbf{D}) + (\mathbf{I}-\mathbf{\Pi}\mathbf{M})\,\theta - \mathbf{\Pi}\mathbf{X}\right]^{+}\mathbf{\Pi}(\mathbf{I}-\mathbf{D})}_{\mathbf{P}}\,y_t
$$

B.2.2 MP

I start with the first-order linearized market clearing equation where I use the expenditure switching equations.

$$
\begin{aligned}
y^j_{jst} &= (Y^j_{js})^{-1}\sum_i \Pi_{jis}E^j_{ir}\pi_{jist} + (Y^j_{js})^{-1}\sum_i \Pi_{jis}\sum_r\left((1-\gamma_{ir})\Xi_{is}+\gamma_{ir}\Omega_{isr}\right)Y^j_{ir}y^j_{irt} \\
&\quad + (Y^j_{js})^{-1}\sum_i \Pi_{jis}\Xi_{is}\Big(\sum_r(1-\gamma_{ir})Y^j_{ir}\Big)\xi_{ist} + (Y^j_{js})^{-1}\sum_i \Pi_{jis}\sum_r \gamma_{ir}\Omega_{isr}Y^j_{ir}\omega_{isrt} \\
y^j_{jst} &= (Y^j_{js})^{-1}\sum_i \Pi_{jis}E^j_{ir}\kappa_s\left[p^j_{ist} - mc^j_{jst} + a_{jst} + \phi_s\left(e^j_{ist} - w^j_{jst}\right)\right] \\
&\quad + (Y^j_{js})^{-1}\sum_i \Pi_{jis}\sum_r\left((1-\gamma_{ir})\Xi_{is}+\gamma_{ir}\Omega_{isr}\right)Y^j_{ir}y^j_{irt} \\
&\quad + (Y^j_{js})^{-1}\sum_i \Pi_{jis}\Xi_{is}\Big(\sum_r(1-\gamma_{ir})Y^j_{ir}\Big)\xi_{ist} + (Y^j_{js})^{-1}\sum_i \Pi_{jis}\sum_r \gamma_{ir}\Omega_{isr}Y^j_{ir}\omega_{isrt} \\
y^j_{jst} &= -\kappa_s(mc^j_{jst} + a_{jst} - \phi_s w^j_{jst}) + (Y^j_{js})^{-1}\sum_i \Pi_{jis}E^j_{ir}\kappa_s\left(p^j_{ist} + \phi_s e^j_{ist}\right) \\
&\quad + (Y^j_{js})^{-1}\sum_i \Pi_{jis}\sum_r\left((1-\gamma_{ir})\Xi_{is}+\gamma_{ir}\Omega_{isr}\right)Y^j_{ir}y^j_{irt} \\
&\quad + (Y^j_{js})^{-1}\sum_i \Pi_{jis}\sum_r(1-\gamma_{ir})\Xi_{is}Y^j_{ir}(\chi-1)\left(\sum_q \Xi_{jq}p^j_{irt} - p^j_{ist}\right) \\
&\quad + (Y^j_{js})^{-1}\sum_i \Pi_{jis}\sum_r \gamma_{ir}\Omega_{isr}Y^j_{ir}(\chi-1)\left(\sum_q \Omega_{iqr}p^j_{iqt} - p^j_{ist}\right)
\end{aligned}
$$

where I have also used the first-order linearized trade shares equation

$$
\pi_{ijst} = \kappa_s\left(p^j_{jst} - mc^j_{ist} + a_{ist}\right) + \kappa_s\phi_s\left(e^j_{jst} - w^j_{ist}\right)
$$

In matrix form, the market clearing equation can then be written as

$$
y^j_t = -\kappa(mc^j_t - a_t + \phi w^j_t) + \mathbf{Y}^{-1}\mathbf{\Pi}'\mathbf{E}\kappa(p^j_t + \phi e^j_t) + \mathbf{D}y^j_t + \mathbf{X}p^j_t
$$

which is then premultiplied by the trade shares matrix to use the following relationship between prices and units costs

$$
p^j_{jst} = -\phi_s e^j_{jst} + \sum_i \Pi_{ijs}\left(mc^j_{ist} - a_{ist} + \phi_s w^j_{ist}\right)
$$

so that only expenditures are left

$$
\mathbf{\Pi}y^j_t = \mathbf{\Pi}\mathbf{D}y^j_t - \left(\mathbf{I} - \mathbf{\Pi}\underbrace{\mathbf{Y}^{-1}\mathbf{\Pi}'\mathbf{E}}_{\mathbf{M}}\right)\kappa(p^j_t + \phi e^j_t) + \mathbf{\Pi}\mathbf{X}p^j_t
$$

Now I solve for expenditures as a function starting from $E^j_{jst} = \sum_r \left((1-{\mu_r}^{-1}\gamma_{jr})\Xi_{jst} + {\mu_r}^{-1}\gamma_{jr}\Omega_{jsrt}\right) Y^j_{jrt}$

$$\begin{aligned}
e^j_{jst} &= (E^j_{js})^{-1}\sum_r \left(\gamma_{Ljr}\Xi_{js} + \gamma_{Mjr}\Omega_{jsr}\right) Y^j_{jr} y^j_{jrt} \\
&\quad + (E^j_{js})^{-1}\sum_r \gamma_{Ljr}\Xi_{js}Y^j_{jr}\xi_{jst} + (E^j_{js})^{-1}\sum_r \gamma_{Mjr}\Omega_{jsr}Y^j_{jr}\omega_{jsrt} \\
e^j_{jst} &= (E^j_{js})^{-1}\sum_r \left((1-\gamma_{ir})\Xi_{is} + \gamma_{ir}\Omega_{isr}\right) Y^j_{ir} y^j_{irt} \\
&\quad + (E^j_{js})^{-1}\sum_r \gamma_{Ljr}\Xi_{ijs}Y^j_{jr}(\chi_F - 1)\left(\sum_q \Xi_{jq}p^j_{jrt} - p^j_{jst}\right) \\
&\quad + (E^j_{js})^{-1}\sum_r \gamma_{Mjr}\Omega_{jsr}Y^j_{jr}(\chi_M - 1)\left(\sum_q \Omega_{iqr}p^j_{jqt} - p^j_{jst}\right)
\end{aligned}$$

and which can be written in matrix form as follows

$$e^j_t = \mathbf{M}^{-1}\mathbf{D}y^j_t + \mathbf{M}^{-1}\mathbf{X}p^j_t$$

which is then put back into the market clearing system

$$\mathbf{\Pi}y^j_t = \mathbf{\Pi}\mathbf{D}y^j_t - \left(\mathbf{I} - \mathbf{\Pi}\mathbf{M}\right)\kappa p^j_t - \left(\mathbf{I} - \mathbf{\Pi}\mathbf{M}\right)\kappa\phi\left(\mathbf{M}^{-1}\mathbf{D}y^j_t + \mathbf{M}^{-1}\mathbf{X}p^j_t\right) + \mathbf{\Pi}\mathbf{X}p^j_t$$

where finally I use the interchangeability of $\kappa\phi\mathbf{M}^{-1} = \mathbf{M}^{-1}\kappa\phi$ to solve for prices as a function of quantities only as follows

$$p^j_t = \underbrace{-\left[\mathbf{\Pi}(\mathbf{I} - \mathbf{\Psi}_D\mathbf{D}) + \left(\mathbf{I} - \mathbf{\Pi}\mathbf{M}\right)\theta - \mathbf{\Pi}\mathbf{\Psi}_D\mathbf{X}\right]^+\mathbf{\Pi}(\mathbf{I} - \mathbf{\Psi}_D\mathbf{D})}_{\mathbf{P}_{\text{Melitz}}} y_t$$

where $\mathbf{\Psi}_D = \mathbf{I} + \left(\mathbf{I} - (\mathbf{M}\mathbf{\Pi})^{-1}\right)$.

B.3 Quantities as a function of shocks

B.3.1 EK

We start with the expression for changes in unit costs

$$mc_{jst} = (1-\gamma_{js})\left(y_{jst} - l_{jst}\right) + \gamma_{js}\sum_r \Omega_{jrs}p_{jrt}$$

which can be written as a function of gross output and prices only

$$mc_{jst} = (1-\gamma_{js})\sum_r \frac{L_{jr}}{L_j}y_{jrt} + \gamma_{js}\sum_r \Omega_{jrs}p_{jrt}$$

which can then be written in matrix form as follows

$$mc_t = (\mathbf{I} - \gamma)\mathbf{\Lambda}' y_t + \gamma\mathbf{\Omega}' p_t$$

Now recall that

$$p_{jst} = \sum_i \Pi_{ijs} \left(mc_{ist} - a_{ist} \right)$$

which can be written in matrix form as

$$p_t = \mathbf{\Pi}(mc_t - a_t)$$

so we can solve for changes in unit cost as a function of supply shocks

$$\begin{aligned} mc_t &= (\mathbf{I} - \gamma)\mathbf{\Lambda}' y_t + \gamma \mathbf{\Omega}' \mathbf{\Pi}(mc_t - a_t) \\ \left(\mathbf{I}_{JS} - \gamma \mathbf{\Omega}' \mathbf{\Pi}\right) mc_t &= (\mathbf{I} - \gamma)\mathbf{\Lambda}' y_t - \gamma \mathbf{\Omega}' \mathbf{\Pi} a_t \\ mc_t &= \left(\mathbf{I}_{JS} - \gamma \mathbf{\Omega}' \mathbf{\Pi}\right)^{-1} \left[(\mathbf{I} - \gamma)\mathbf{\Lambda}' y_t - \gamma \mathbf{\Omega}' \mathbf{\Pi} a_t\right] \end{aligned}$$

such that

$$\begin{aligned} p_t &= \mathbf{\Pi} \left(\mathbf{I}_{JS} - \gamma \mathbf{\Omega}' \mathbf{\Pi}\right)^{-1} \left[(\mathbf{I} - \gamma)\mathbf{\Lambda}' y_t - \gamma \mathbf{\Omega}' \mathbf{\Pi} a_t\right] - \mathbf{\Pi} a_t \\ p_t &= \mathbf{\Pi} \left(\mathbf{I}_{JS} - \gamma \mathbf{\Omega}' \mathbf{\Pi}\right)^{-1} (\mathbf{I} - \gamma)\mathbf{\Lambda}' y_t - \mathbf{\Pi} \big[\underbrace{\mathbf{I}_{JS} + \left(\mathbf{I}_{JS} - \gamma \mathbf{\Omega}' \mathbf{\Pi}\right)^{-1} \gamma \mathbf{\Omega}' \mathbf{\Pi}}_{(\mathbf{I}_{JS} - \gamma \mathbf{\Omega}' \mathbf{\Pi})^{-1}} \big] a_t \\ p_t &= \mathbf{\Pi} \left(\mathbf{I}_{JS} - \gamma \mathbf{\Omega}' \mathbf{\Pi}\right)^{-1} \left[(\mathbf{I} - \gamma)\mathbf{\Lambda}' y_t - a_t\right] \end{aligned}$$

Now I name $\mathbf{G} = \mathbf{\Pi} \left(\mathbf{I}_{JS} - \gamma \mathbf{\Omega}' \mathbf{\Pi}\right)^{-1}$ the Ghosh inverse and $\mathbf{H} = (\mathbf{I} - \gamma)\mathbf{\Lambda}'$ the labor allocation matrix such that I can expand the latter as follows

$$\begin{aligned} p_t &= \mathbf{G}\big[\mathbf{H}(y_t + p_t) - a_t\big] \\ (\mathbf{I} - \mathbf{GH}) p_t &= \mathbf{GH} y_t - \mathbf{G} a_t \\ y_t &= \big[\mathbf{GH} - (\mathbf{I} - \mathbf{GH})\mathbf{P}\big]^{-1} \mathbf{G} a_t \end{aligned}$$

or even

$$y_t = \big[\mathbf{I} - (\mathbf{I} - \mathbf{GH})(\mathbf{I} + \mathbf{P})\big]^{-1} \mathbf{G} a_t$$

B.3.2 MP

The process for finding the influence matrix in Melitz is the same. The variable unit cost equation is equivalent across frameworks.

$$mc_{jst} = (1 - \gamma_{js}) w^j_{jt} + \gamma_{js} \sum_r \Omega_{jrs} p_{jrt}$$

which is matrix form can be written as

$$mc^j_t = \gamma_L w^j_{jt} + \gamma \mathbf{\Omega}' p^j_t$$

The stacked equation for prices is again the following

$$p_t^j = -\phi e_t^j + \mathbf{\Pi}\left(mc_t^j - a_t + \phi w_t^j\right)$$

which is paired with the equation of expenditure as a function of gross output and prices

$$e_t^j = \mathbf{M}^{-1}\mathbf{D}y_t^j + \mathbf{M}^{-1}\mathbf{X}p_t^j$$

to form an updated set of equations

$$\begin{aligned}
p_t^j =& -\phi(\mathbf{M}^{-1}\mathbf{D}y_t^j + \mathbf{M}^{-1}\mathbf{X}p_t^j) + \mathbf{\Pi}\left(mc_t^j - a_t + \phi w_t^j\right)\\
(\underbrace{\mathbf{I} + \phi\mathbf{M}^{-1}\mathbf{X}}_{\mathbf{\Psi}_G{}^{-1}})p_t^j =& -\phi\mathbf{M}^{-1}\mathbf{D}y_t^j + \mathbf{\Pi}\left(mc_t^j - a_t + \phi w_t^j\right)\\
p_t^j =& -\mathbf{\Psi}_G\mathbf{\Pi}\phi(\mathbf{M\Pi})^{-1}\mathbf{D}y_t^j + \mathbf{\Psi}_G\mathbf{\Pi}\left(mc_t^j - a_t + \phi w_t^j\right)
\end{aligned}$$

such that the unit cost equation can be manipulated as follows

$$\begin{aligned}
mc_t^j =& \gamma_L w_{jt}^j + \gamma\mathbf{\Omega}'\big[-\mathbf{\Psi}_G\mathbf{\Pi}\phi(\mathbf{M\Pi})^{-1}\mathbf{D}y_t^j + \mathbf{\Psi}_G\mathbf{\Pi}\left(mc_t^j - a_t + \phi w_t^j\right)\big]\\
(\mathbf{I} - \gamma\mathbf{\Omega}'\mathbf{\Psi}_G\mathbf{\Pi})mc_t^j =& \gamma_L w_{jt}^j + \gamma\mathbf{\Omega}'\big[-\mathbf{\Psi}_G\mathbf{\Pi}\phi(\mathbf{M\Pi})^{-1}\mathbf{D}y_t^j + \mathbf{\Psi}_G\mathbf{\Pi}\left(-a_t + \phi w_t^j\right)\big]\\
=& \left(\gamma_L + \gamma\mathbf{\Omega}'\mathbf{\Psi}_G\mathbf{\Pi}\phi\right)w_t^j + \gamma\mathbf{\Omega}'\mathbf{\Psi}_G\mathbf{\Pi}\phi(\mathbf{M\Pi})^{-1}\mathbf{D}y_t^j - \gamma\mathbf{\Omega}'\mathbf{\Psi}_G\mathbf{\Pi}a_t
\end{aligned}$$

which can be put back into the equation for prices

$$\begin{aligned}
p_t^j =& -\mathbf{\Psi}_G\mathbf{\Pi}\phi(\mathbf{M\Pi})^{-1}\mathbf{D}y_t^j + \mathbf{\Psi}_G\mathbf{\Pi}\left(mc_t^j - a_t + \phi w_t^j\right)\\
p_t^j =& -\mathbf{\Psi}_G\mathbf{\Pi}\phi(\mathbf{M\Pi})^{-1}\mathbf{D}y_t^j + \mathbf{\Psi}_G\mathbf{\Pi}\left(-a_t + \phi w_t^j\right)\\
&+ \mathbf{\Psi}_G\mathbf{\Pi}\mathbf{G}\big[\left(\gamma_L + \gamma\mathbf{\Omega}'\mathbf{\Psi}_G\mathbf{\Pi}\phi\right)w_t^j - \gamma\mathbf{\Omega}'\mathbf{\Psi}_G\mathbf{\Pi}\phi(\mathbf{M\Pi})^{-1}\mathbf{D}y_t^j - \gamma\mathbf{\Omega}'\mathbf{\Psi}_G\mathbf{\Pi}a_t\big]\\
p_t^j =& -\mathbf{\Psi}_G\mathbf{\Pi}\big[\mathbf{I} + (\mathbf{I} - \gamma\mathbf{\Omega}'\mathbf{\Psi}_G\mathbf{\Pi})^{-1}\gamma\mathbf{\Omega}'\mathbf{\Psi}_G\mathbf{\Pi}\big]a_t\\
&- \mathbf{\Psi}_G\mathbf{\Pi}\big[\underbrace{\mathbf{I} + (\mathbf{I} - \gamma\mathbf{\Omega}'\mathbf{\Psi}_G\mathbf{\Pi})^{-1}\gamma\mathbf{\Omega}'\mathbf{\Psi}_G\mathbf{\Pi}}_{(\mathbf{I} - \gamma\mathbf{\Omega}'\mathbf{\Psi}_G\mathbf{\Pi})^{-1}}\big]\phi(\mathbf{M\Pi})^{-1}\mathbf{D}y_t^j\\
&+ \mathbf{\Psi}_G\mathbf{\Pi}(\mathbf{I} - \gamma\mathbf{\Omega}'\mathbf{\Psi}_G\mathbf{\Pi})^{-1}(\mathbf{I} - \gamma)w_t^j + \mathbf{\Psi}_G\mathbf{\Pi}\big[\mathbf{I} + (\mathbf{I} - \gamma\mathbf{\Omega}'\mathbf{\Psi}_G\mathbf{\Pi})^{-1}\gamma\mathbf{\Omega}'\mathbf{\Psi}_G\mathbf{\Pi}\big]\phi w_t^j\\
p_t^j =& -\mathbf{\Psi}_G\mathbf{\Pi}(\mathbf{I} - \gamma\mathbf{\Omega}'\mathbf{\Psi}_G\mathbf{\Pi})^{-1}a_t - \mathbf{\Psi}_G\mathbf{\Pi}(\mathbf{I} - \gamma\mathbf{\Omega}'\mathbf{\Psi}_G\mathbf{\Pi})^{-1}\phi(\mathbf{M\Pi})^{-1}\mathbf{D}y_t^j\\
&+ \mathbf{\Psi}_G\mathbf{\Pi}(\mathbf{I} - \gamma\mathbf{\Omega}'\mathbf{\Psi}_G\mathbf{\Pi})^{-1}((\mathbf{I} - \gamma) + \phi)w_t^j
\end{aligned}$$

Now I solve for w_t^j as functions of prices and quantities. From labor market clearing it follows that

$$L_{Pj}l_{Pjt} + L_{Ej}l_{Ejt} = 0$$

so that we have

$$w_{jt}^j = -l_{Pjt} + \sum_r \frac{L_{Pjr}}{L_{Pj}}y_{jrt} = \frac{L_{Ej}}{L_{Pj}}l_{Ejt} + \sum_r \frac{L_{Pjr}}{L_{Pj}}y_{jrt}$$

and we use the expression $W^j_{jt}L_{Xjt} = \sum_r \phi_r {\mu_r}^{-1} Y_{jrt}$ linearized to substitute in for l_{Ejt} as follows

$$\begin{aligned}
w^j_{it} &= \frac{L_{Ej}}{L_{Pj}}\left[-w^j_{it} + \sum_r \frac{L_{Xjr}}{L_{Ej}} y_{jrt}\right] + \sum_r \frac{L_{Pjr}}{L_{Pj}} y_{jrt} \\
\left(1+\frac{L_{Ej}}{L_{Pj}}\right) w^j_{it} &= \frac{L_{Ej}}{L_{Pj}} \sum_r \frac{L_{Xjr}}{L_{Ej}} y_{jrt} + \sum_r \frac{L_{Pjr}}{L_{Pj}} y_{jrt} \\
\frac{L_j}{L_{Pj}} w^j_{it} &= \sum_r \frac{L_{jr}}{L_{Pj}} y_{jrt}
\end{aligned}$$

such that finally

$$w^j_{jt} = \sum_r \frac{L_{jr}}{L_j} y_{jrt} \to w^j_t = \mathbf{\Lambda}' y^j_t$$

so the expression for prices becomes

$$\begin{aligned}
p^j_t =& \underbrace{\mathbf{\Psi}_G \mathbf{\Pi}(\mathbf{I} - \gamma \mathbf{\Omega}' \mathbf{\Psi}_G \mathbf{\Pi})^{-1}}_{\mathbf{G}} \big[- a_t - \phi(\mathbf{M\Pi})^{-1}\mathbf{D} y^j_t + ((\mathbf{I}-\gamma) + \phi)\mathbf{\Lambda}' y^j_t \big] \\
p^j_t =& - \mathbf{G} a_t + \mathbf{G}\big[\underbrace{((\mathbf{I}-\gamma)+\phi)\mathbf{\Lambda}' - \phi(\mathbf{M\Pi})^{-1}\mathbf{D}}_{\mathbf{H}}\big] y^j_t \\
p^j_t =& - \mathbf{G} a_t + \mathbf{GH}(y_t + p^j_t)
\end{aligned}$$

Now I solve for quantities as a function of shocks

$$\begin{aligned}
\mathbf{G} a_t =& \mathbf{GH} y_t + (\mathbf{GH} - \mathbf{I}) p^j_t \\
\mathbf{G} a_t =& \mathbf{GH} y_t + (\mathbf{GH} - \mathbf{I}) \mathbf{P}_{\text{Melitz}} y_t \\
\mathbf{G} a_t =& \big[\mathbf{I} - (\mathbf{I} - \mathbf{GH})(\mathbf{I} + \mathbf{P}_{\text{Melitz}})\big] y_t
\end{aligned}$$

such that finally

$$y_t = \big[\mathbf{I} - (\mathbf{I} - \mathbf{GH})(\mathbf{I} + \mathbf{P}_{\text{Melitz}})\big]^{-1} a_t$$

C Stylized Economy Appendix

C.1 Matrices in the stylized economy

The trade-exposure matrix has dimensions $JS \times JS$ and is composed of square blocks of size $S \times S$, with each of these blocks being a diagonal matrix itself. There are only three different entries. In the main diagonal we have the responses of a sector to a shock to its productivity, while in the main diagonal of bilateral blocks there are entries that describe a sector's response to the same sector elsewhere.

$$(\mathbf{I}-\mathbf{\Pi M})_{ijrs}=\begin{cases}(J-1)\delta_M & \text{if } i=j\ ,\ r=s\\ -\delta_M & \text{if } i\neq j\ ,\ r=s\\ 0 & \text{otherwise}\end{cases}\quad \text{where}\quad \delta_M=\pi(2-J\pi)$$

The analytical expressions of the components of the trade exposure matrix lead to two observations. First, if the matrices are calibrated to the same trade shares there will be no difference between EK and MP. Second, the main component δ_M is such that $\lim_{\tau\to 1^+}\delta_M=\theta$ and $\lim_{\tau\to\infty}\delta_M=0$, so that off-diagonal elements are highest when variable trade costs are low. In the limit where $J\to\infty$ the matrix converges to $\theta\mathbf{I}$. The trade exposure of a sector in country j to the same sector in some other country i is the average bilateral trade share plus a term that captures multilateral transmission times the trade elasticity.

The two frameworks differ in their demand exposure matrix as the elements in $(\mathbf{\Pi}(\mathbf{I}-\mathbf{D}))_{ijrs}$ in EK are augmented to $(\mathbf{\Pi}(\mathbf{I}-\mathbf{\Psi}_D\mathbf{D}))_{ijrs}$ to account for the extensive margin in MP. In the stylized economy, this is equivalent to adding the term $(J-1)\kappa\phi\delta_M$ to the $i=j\ ,\ r=s$ main diagonal elements, that is, augmenting the demand exposure of a sector to itself by ϕ times trade exposure times the number of remaining countries in the world. One also needs to subtract the same term from the $i=j\ ,\ r\neq s$ entries, while subtracting $S^{-1}\kappa\phi\delta_M$ from the $i\neq j$ entries. The elements of the international Ghosh-inverse are common across frameworks and three different entries present the value of any given pair (i,r) input per dollar of any (j,s) output.

$$(\mathbf{G})_{ijrs}=\begin{cases}1+\delta_G & \text{if } i=j\ ,\ r=s\\ \delta_G & \text{if } i=j\ ,\ r\neq s\\ \frac{1}{S}\frac{\gamma}{1-\gamma}\frac{\pi}{1-\gamma(1-J\pi)} & \text{otherwise}\end{cases}\quad \text{where}\quad \delta_G=\frac{1}{S}\frac{\gamma}{1-\gamma}\frac{1-(J-1)\pi-\gamma(1-J\pi)}{1-\gamma(1-J\pi)}$$

A second matrix that differs across frameworks is the labor allocation matrix $\mathbf{H}$ that mediates the supply-side response. The entries in this matrix reflect baseline productive labor allocation. In EK this is a block-diagonal matrix with elements $(\mathbf{H})_{ijrs}=S^{-1}(1-\gamma)$ when $i=j$ and zero otherwise. In MP the block-diagonal elements are net of non-productive (entry) labor, which is then distributed across all the other non-block-diagonal elements and reflect the extensive margin channel of supply-side exposure.

$$(\mathbf{H})_{ijrs}=\begin{cases}\frac{1}{S}\left((1-\gamma)-(J-1)\delta_H\right) & \text{if } i=j\\ \frac{1}{S}\delta_H & \text{if } i\neq j\end{cases}\quad \text{where}\quad \delta_H=\phi\frac{\pi}{1-J\pi}$$

Therefore, when it comes to labor allocation only the unilateral block is populated in EK, and the effects of domestic sectors on each other are proportional to the labor elasticity and inversely proportional to the number of sectors. In MP this effect is still present, but sectors abroad also impact domestic sectors through baseline entry labor allocation ϕ weighted by trade costs.

C.2 Comovement in the stylized economy

$$
\begin{aligned}
\rho_{ij} &= \frac{2\left(\frac{1}{J}\frac{1}{1-\gamma}\right)^2\left(1+(J-1)\lambda^G\right)(1-\lambda^G)+(J-2)\left(\frac{1}{J}\frac{1}{1-\gamma}\right)^2(1-\lambda^G)^2}{\left(\frac{1}{J}\frac{1}{1-\gamma}\right)^2\left(1+(J-1)\lambda^G\right)^2+(J-1)\left(\frac{1}{J}\frac{1}{1-\gamma}\right)^2(1-\lambda^G)^2} \\
&= \frac{2(1-\lambda^G+(J-1)\lambda^G-(J-1)(\lambda^G)^2)+(J-2)(1-2\lambda^G+(\lambda^G)^2)}{1+2(J-1)\lambda^G+(J-1)^2(\lambda^G)^2+(J-1)(1-2\lambda^G+(\lambda^G)^2)} \\
&= \frac{1-(\lambda^G)^2}{1+(J-1)(\lambda^G)^2} \blacksquare
\end{aligned}
$$