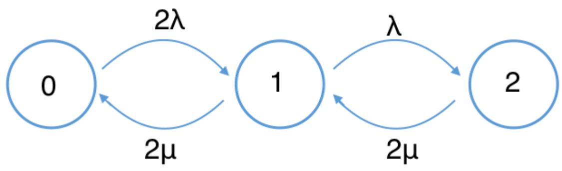
Q1.

$\lambda=1/10$ $\mu=1/8$

a. hire another repair man:



Stage0: $2\mu\ \pi 1 = 2\lambda\pi 0$

Stage1: $2\lambda\ \pi 0 + 2\mu\ \pi 2 = (2\mu + \lambda)\ \pi 1$

Stage2: $\lambda\ \pi 1 = 2\mu\ \pi 2$

Normalize: $\pi 0 + \pi 1 + \pi 2 = 1$

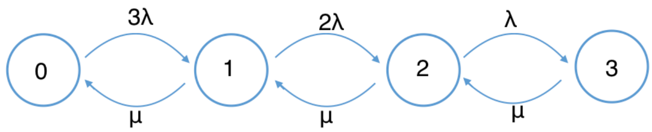
C0 = 1 $\qquad \pi 0 = \dfrac{1}{1 + 0.32 + 0.8} \approx 0.4717$

$C1 = \dfrac{\lambda 0}{\mu 1} = 0.8 \qquad \pi 1 = 0.8 * 0.4717 \approx 0.3774$

$C2 = \dfrac{\lambda 0 \lambda 1}{\mu 1 \mu 2} = 0.32 \qquad \pi 2 = 0.32 * 0.4717 \approx 0.1509$

Throughput.a = $2\pi 0 + 1\pi 1 = 2 * 0.4717 + 0.3774 = 1.3208$ (avg. # machines working)

b. buy another machine:



Stage0: $\mu\ \pi 1 = 3\lambda\pi 0$

Stage1: $3\lambda\ \pi 0 + \mu\ \pi 2 = (\mu + 2\lambda)\ \pi 1$

Stage2: $2\lambda\ \pi 1 + \mu\pi 3 = (\mu + \lambda)\ \pi 2$

Stage3: $\lambda\ \pi 2 = \mu\pi 3$

Normalize: $\pi 0 + \pi 1 + \pi 2 + \pi 3 = 1$

C0 = 1 $\qquad \pi 0 = \dfrac{1}{1 + 2.4 + 3.84 + 3.072} \approx 0.09697$

$C1 = \dfrac{\lambda 0}{\mu 1} = 2.4 \qquad \pi 1 = 0.9697 * 2.4 \approx 0.23273$

$C2 = \dfrac{\lambda 0 \lambda 1}{\mu 1 \mu 2} = 3.84 \qquad \pi 2 = 0.9697 * 3.84 \approx 0.3724$

$C3 = \dfrac{\lambda 0 \lambda 1 \lambda 2}{\mu 1 \mu 2 \mu 3} = 3.072 \qquad \pi 3 = 0.9697 * 3.072 \approx 0.2979$

Throughput.b = $3\pi 0 + 2\pi 1 + 1\pi 2 = 1.11288 < 1.3208$ (Throughput.a)

therefore hire another man would be more productive

Q2. λ=20arrivals/h μ=27arrivals/h

$$\omega = \frac{1}{\mu - \lambda} = \frac{1}{27-20} = 0.1429hour = 8.57min$$

$$L = \lambda\omega = 20 * \frac{1}{27-20} = 2.86$$

$$lq = \lambda\omega q = 20 * \frac{20}{27\,(27-20)} = 2.12$$

$$\omega q = \frac{\lambda}{\mu\,(\mu - \lambda)} = \frac{20}{27(27-20)} = 0.1058hour = 6.36min$$

Q3. λ'=25 arrivals/h μ'= 27arrivals/h

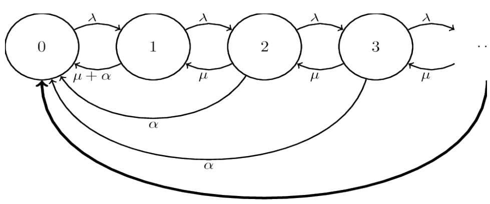
$$\omega' = \frac{1}{\mu' - \lambda'} = \frac{1}{27-20} = 0.5hour = 30min$$

$$L' = \lambda'\omega' = 25 * \frac{1}{27-25} = 12.5$$

$$\omega q' = \frac{\lambda'}{\mu'(\mu' - \lambda')} = \frac{25}{27(27-25)} = 0.463hour = 27.28min$$

$$Lq' = \lambda'\omega q' = 25 * \frac{25}{27(27-25)} = 11.57$$

Q4



$$\pi 0 = \frac{1-\rho}{1-\rho k+1} = \frac{1-\frac{\lambda}{\mu+\alpha}}{1-(\frac{\lambda}{\mu+\alpha})k}$$

$$\pi i = \pi 0 \rho k = \frac{1-\frac{\lambda}{\mu+\alpha}}{1-\left(\frac{\lambda}{\mu+\alpha}\right)k} * \left(\frac{\lambda}{\mu+\alpha}\right)k$$