

# PageRank

## **COMP3009J: Information Retrieval**

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# Introduction

- Using the models we have seen so far (e.g. Vector Space, BM25), each document is treated completely **separately** to other documents.
- The ranking score is calculated based entirely on the **content** of the document itself.
- This does not take into account everything a human judge would use in deciding whether or not a document is suitable for returning in response to a particular query.
- Some of these additional factors are very difficult for a computer algorithm to figure out, some are impossible.

# Introduction

- Some questions you might ask yourself are:
  - Though it contains the terms contained in the query, does the document actually satisfy the information need that is being expressed?
  - Is the document well-written and understandable?
  - How important and influential is this document?
- In most cases, these are not questions that a computer algorithm can answer easily, which is why we use human judges when evaluating IR systems.

# Document Importance

- There are, however, some areas where it may be possible to estimate how **important** a document is.
- For instance, in academic writing a paper will generally **cite** other works that influenced it.
- Some organisations use “citation counting” to measure the influence of a piece of work.
- The intuition is that the more times your paper is cited by others, the more influence it must have had in its field.
- Also, if a paper appears in an influential journal, it is likely to be important.

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# Citation Counts and the Web

- The World Wide Web contains the largest collection of documents in existence, and is a most suitable forum for IR.
- Unlike in traditional IR systems, the documents available on the web are **not standalone**.
- Because of the presence of **hypertext** (i.e. links) in documents on the web, there is an **interconnection** between these documents.
- A number of researchers have attempted to apply principles similar to citation counts to web pages.
- This operates on the assumption that a page that is linked to from many other pages is important and this should be reflected in search results.

# Citation Counts and the Web

- There are, however, a few notable differences between academic publishing and web publishing that must be taken into account:
- **Circular references** - in academic publishing, a paper can only cite a paper that has already been published. Later papers are not cited by earlier ones. On the web, two pages can link to one another.

# Circular References

## Republic of Ireland

From Wikipedia, the free encyclopedia

*This article is about the sovereign state. For the revolutionary republic of 1919–1922, see [Irish Republic](#). For other uses, see [Ireland \(disambiguation\)](#).*

**Ireland** (<sup>i</sup>/ˈaɪərlənd/; Irish: *Éire* [ˈeːɾʲə] (listen)), also known as the **Republic of Ireland** (*Poblacht na hÉireann*), is a [sovereign state](#) in north-western [Europe](#) occupying about five-sixths of the [island of Ireland](#). The capital and largest city is **Dublin**, which is located on the eastern part of the island, and whose metropolitan area is home to around a third of the country's 4.6 million inhabitants. The state shares its only land border with [Northern Ireland](#), a [part](#) of the [United Kingdom](#). It is otherwise surrounded by the Atlantic Ocean, with the [Celtic Sea](#) to the south, [Saint George's Channel](#) to the south-east and the [Irish Sea](#) to the east. It is a [unitary, parliamentary republic](#).<sup>[9]</sup> The legislature, the *Oireachtas*, consists of a [lower house](#), *Dáil Éireann*, an [upper house](#), *Seanad Éireann*, and an elected [President](#) (*Uachtarán*) who serves as the largely ceremonial [head of state](#), but with some important powers and duties. The [head of government](#) is the *Taoiseach* (Prime Minister, literally 'Chief', a title not used in English), who is elected by the *Dáil* and appointed by the President, and appoints other government ministers.

## Dublin

From Wikipedia, the free encyclopedia

*This article is about the capital of Ireland. For other uses, see [Dublin \(disambiguation\)](#).*

**Dublin** (<sup>i</sup>/dʌblɪn/, Irish: *Baile Átha Cliath* [ˈbˠalʲaːˈklʲiəh]) is the capital and largest city of Ireland.<sup>[9]</sup> Dublin is in the province of [Leinster](#) on Ireland's east coast, at the mouth of the [River Liffey](#). The city has an urban area population of 1,273,069.<sup>[10]</sup> The population of the [Greater Dublin Area](#), as of 2011, was 1,801,040 persons.

# Citation Counts and the Web

- **Quality control** - in academic publishing, a paper is peer-reviewed before it is approved for publication. Thus some quality is maintained in the papers that include citations. On the web, anybody can publish material and link to other documents. It would be a trivial task to write a program that would generate hundreds or thousands of pages containing links to somewhere else.



# PageRank

- In 1998, Sergey Brin and Larry Page published the PageRank algorithm, which to measure the importance of web pages\*.
- This later went on to be a core element in the success of the Google search engine.
- The algorithm itself has been modified since (secretly: Google rarely reveal anything about the search engine's inner workings anymore) to avoid situations where it was being exploited by malicious publishers.
- However, the core of how it functions has largely remained unchanged.

\* S. Brin and L. Page, "The anatomy of a large-scale hypertextual Web search engine", *Computer networks and ISDN systems*, 30(1-7), 107-117. 1998.

# PageRank

- Again, it is based on the premise that documents that are linked to by many other documents are important, and should receive a boost in search engine rankings as a result.
- A document will tend to have a high PageRank score if:
  - It is linked to by many documents and/or
  - It is linked to by documents that themselves have a high PageRank.

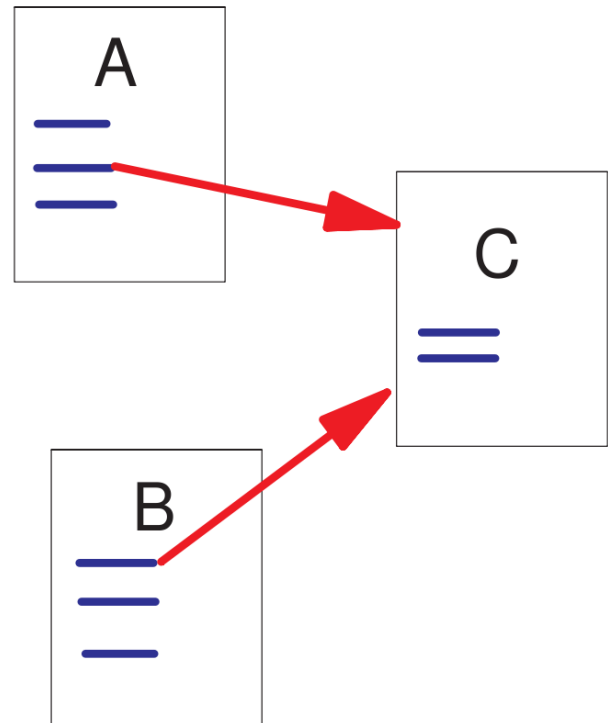
# PageRank - Link Structure

The web is made up of HTML pages that are connected using hyperlinks.

We can think of this as a **directed graph**.

Pages A, B, C are vertices in the graph.

A and B have links (edges) to Page C.



# PageRank - Link Structure

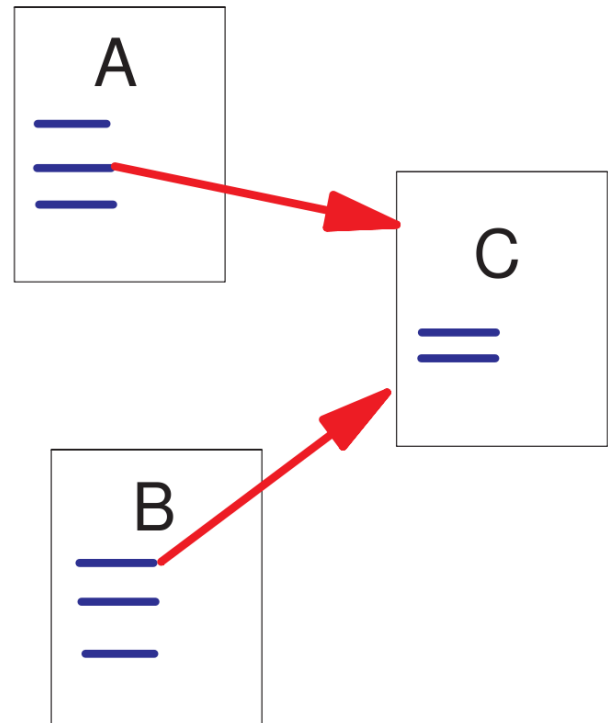
In their 1998 crawl, Brin and Page had 150 million vertices (pages) and 1.7 billion edges (links).

We say that A and B are **backlinks** of C.

We say that A and B have **outlinks** to C.

A document will have high PageRank if it has:

- Many backlinks
- Backlinks with high PageRank



# PageRank: Simplified Version

- At a basic level, PageRank works using a formula similar to the following...
- $$R(u) = \sum_{v \in B_u} \frac{R(v)}{N_v}$$
  - $R(u)$  is the PageRank score for document  $u$ .
  - $B_u$  is the set of all backlinks of document  $u$ .
  - $R(v)$  is the PageRank score for document  $v$ .
  - $N_v$  is the number of outlinks in document  $v$ .

# PageRank: Simplified Version

What does this mean?

A document contributes  $\frac{R(v)}{N_v}$  to the PageRank of each document it links to.

That is, if a document links to 4 pages, its contribution to each of those pages is  $\frac{1}{4}$  of its own PageRank.

- $R(u) = \sum_{v \in B_u} \frac{R(v)}{N_v}$
- $R(u)$  is the PageRank score for document  $u$ .
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- $R(v)$  is the PageRank score for document  $v$ .
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# PageRank: Simplified Version

So if a backlink has high PageRank (and few outlinks), this will have a beneficial effect.

A document's final PageRank score is the sum of each of these contributions from backlinks.

The more backlinks a document has, the higher its PageRank will be.

- $R(u) = \sum_{v \in B_u} \frac{R(v)}{N_v}$
- $R(u)$  is the PageRank score for document  $u$ .
- $B_u$  is the set of all backlinks of document  $u$ .
- $R(v)$  is the PageRank score for document  $v$ .
- $N_v$  is the number of outlinks in document  $v$ .

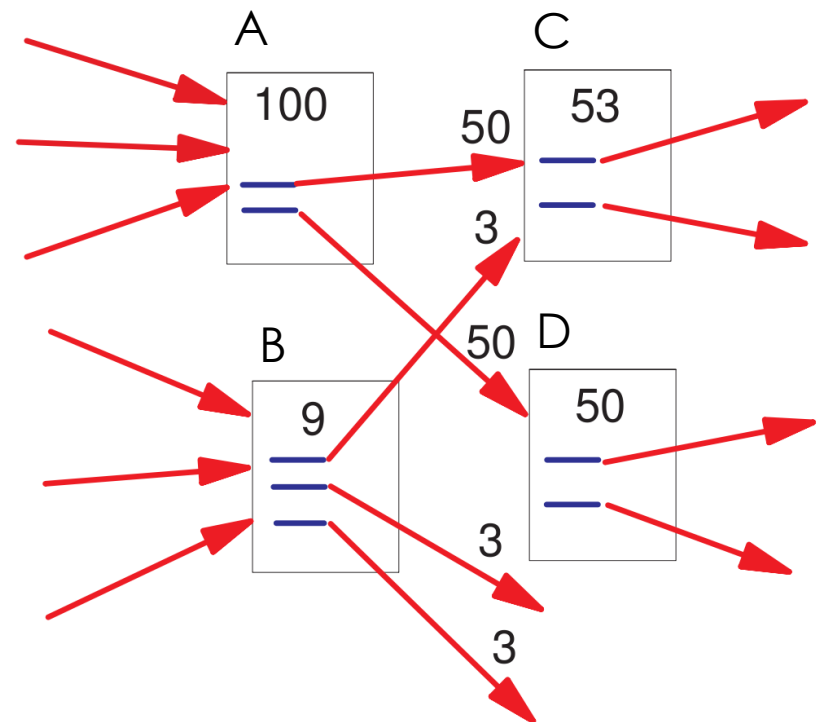
# PageRank - Simplified Version

Document A has PageRank of 100 and 2 outlinks:

- It sends 50 to C and 50 to D

Document B has PageRank of 9 and 3 outlinks:

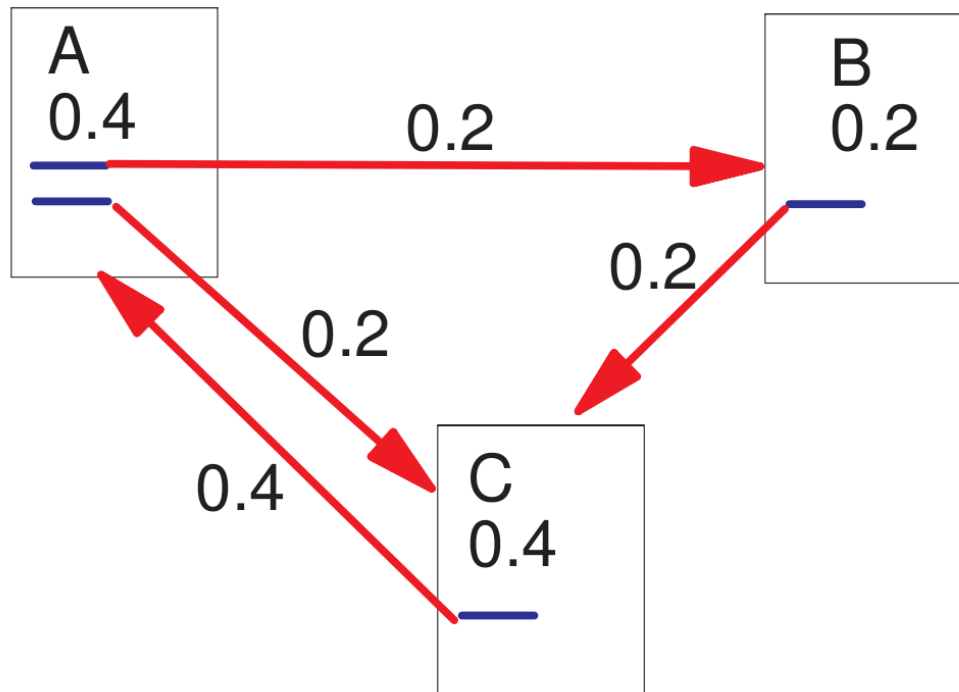
- It contributes 3 to the PageRank of each of its outlinks (including C)





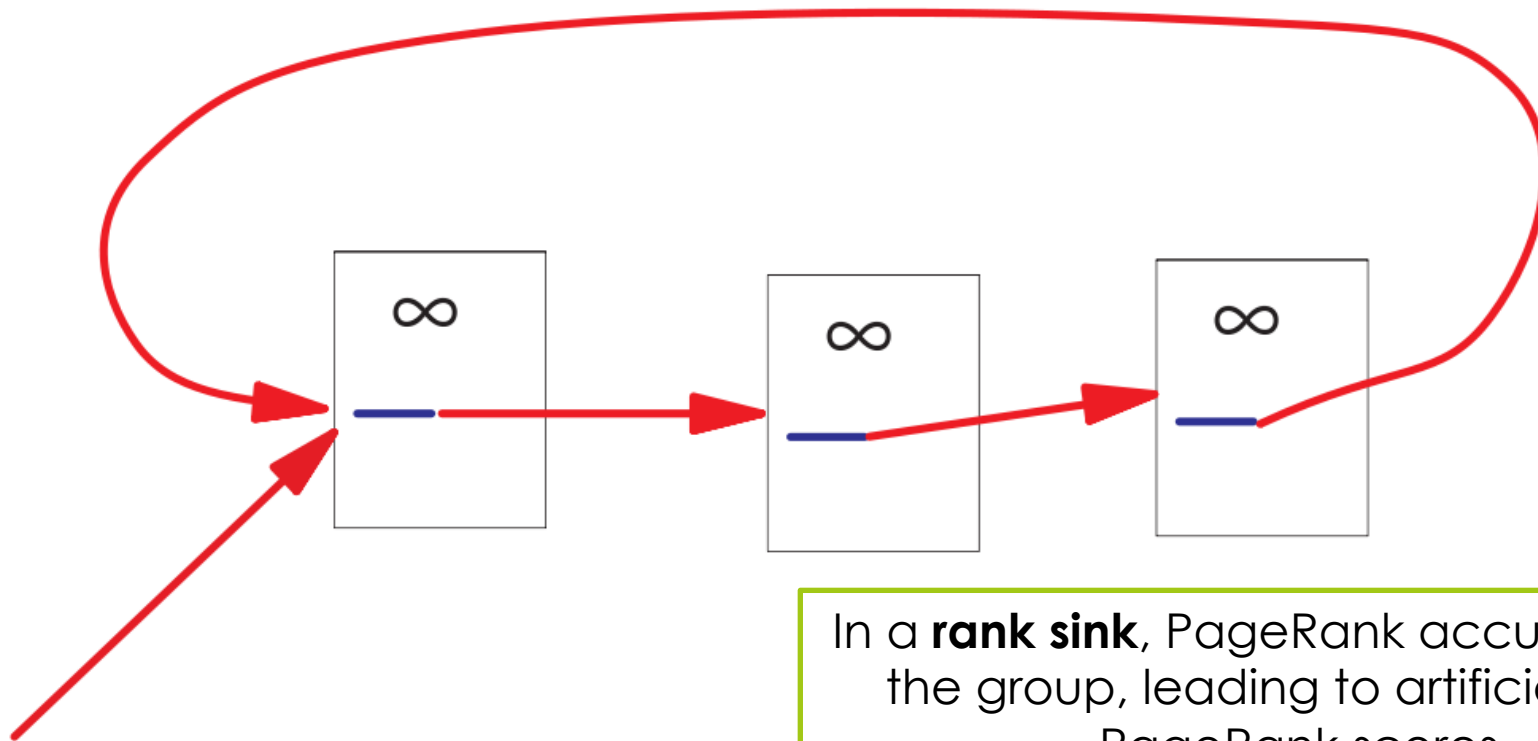
# PageRank - Simplified Version

- Question: If we need PageRank to calculate PageRank, where does the initial PageRank come from?
  - At the beginning, we can give an **arbitrary score** to every document.
  - The formula we have seen can then be used to calculate new scores.
  - These **continue to be recalculated** until the scores **converge** (i.e. calculating again does not change the scores, or changes them very little).
    - The scores use as the inputs for each iteration of the algorithm are the scores from the previous iteration.



## PageRank - Simplified Version

This image shows a stable state: no matter how many times PageRank is recalculated, the scores for A, B and C will always be the same.



In a **rank sink**, PageRank accumulates in the group, leading to artificially high PageRank scores.

## PageRank - Problems

Although this simple example illustrates how PageRank works, it does not deal with certain situations very well.

One such situation is a **rank sink** which refers to a group of pages that have at least one backlink and link to one another, but do not link to anywhere else outside the group.

# Combating Rank Sinks

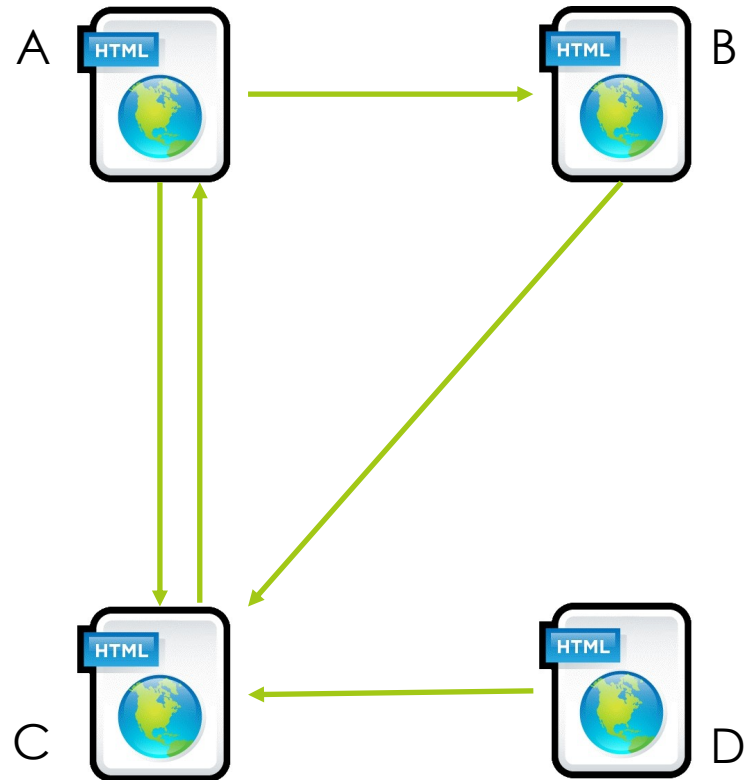
- To combat this type of situation, a new equation is used:
- $$R(u) = (1 - d) + d \times \sum_{v \in B_u} \frac{R(v)}{N_v}$$
- The formula is the very similar to the one we have seen before.
- The difference is the addition of a **damping factor** ( $d$ ), which ensures that not all of a document's PageRank is passed on via its outlinks.

# PageRank - No Backlinks

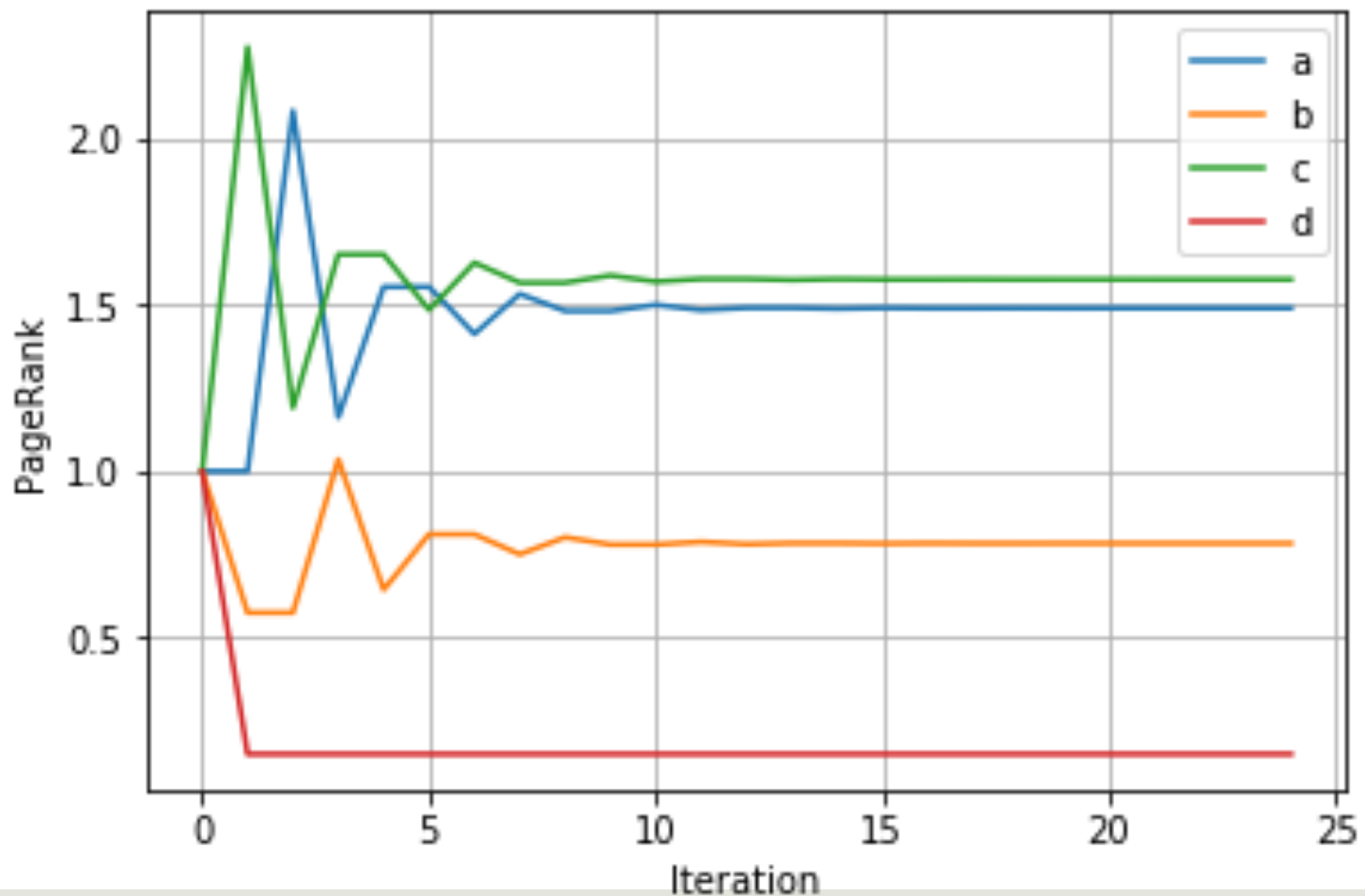
- In the original formula, the only source of PageRank for a document is from its backlinks.
- This meant that a document with no backlinks would have a PageRank of zero.
- This is perfectly acceptable when considering the importance of that document itself.
- However, this also means that it would not contribute anything to the PageRank of documents it links to.
- With the modified formula, a document with no backlinks has a PageRank of  $(1-d)$  to contribute to the documents it links to.

# PageRank - Example: 4 pages

- Consider the following simple page structure:
  - Page A: links to B and C
  - Page B: links to C
  - Page C: links to A
  - Page D: links to C
- Starting with an initial PageRank of 1 and using a damping factor of 0.85 (which Google appears to use), calculate the PageRank of each document.



# PageRank - Example: 4 Pages

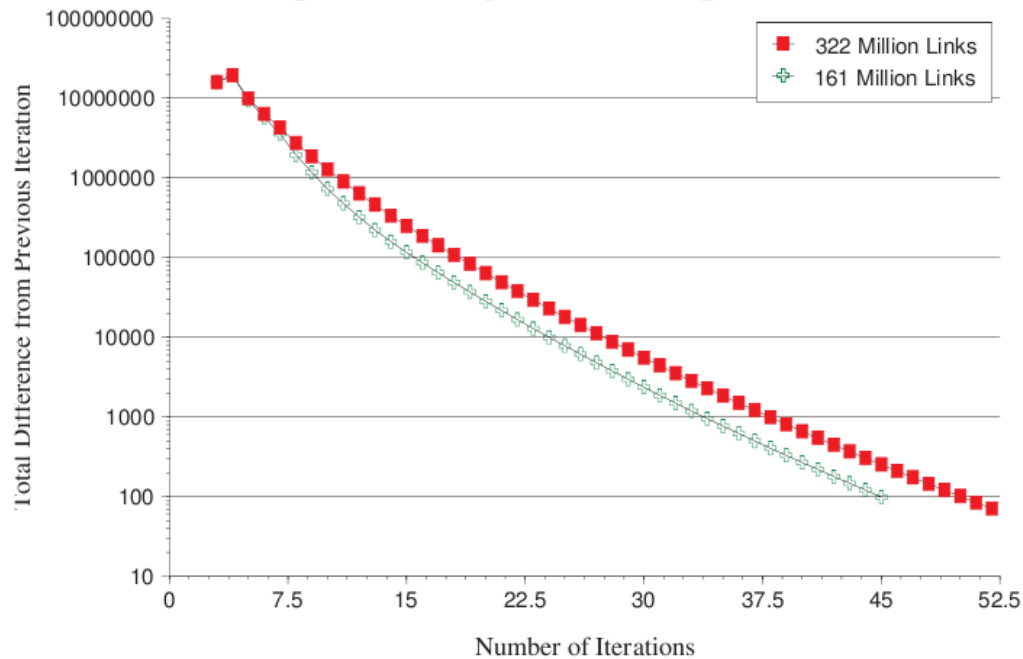


# PageRank - Example: 4 Pages

- With this simple system, the PageRank scores have **converged** perfectly after 20 iterations.
- Even after 8 or 9 iterations, the values are very similar to their ultimate values.
- Brin and Page were using a database of 322 million links and believed that the convergence had reached a reasonable level after 52 iterations.
- Calculations on half the data took 45 iterations to get to the same stage.
- This suggests that PageRank scales very well to very large-scale data collections.



## Convergence of PageRank Computation



## PageRank - Convergence

Doubling the size of the document collection does not double the time taken to converge.

In fact, the increase in the number of iterations required is very small (52 vs. 45): very efficient for larger collections.

Search

Showing Results From 0 to 10

74.79% <http://www.stanford.edu/>  
# - 2501003 - 11/01/307

65.78% <http://www.stanford.edu/home/administration/portfolio.html>  
36 - 2521993 - 0103927

73.26% 136 - 139/195 - 116/139/27

68.38% <http://www.indiana.edu/>  
1k - 04228425 - 01035427

68.07% <http://www.uci.edu/>  
3K - 13333336 - 0123327

62.05% <http://www.umn.edu/>  
106 - 13316326 - 01692323

66.66% <http://www.iastate.edu>  
 36 - 13/18/96 - 01/03/97

66.35%  $fk = -3591993 = -11913927$

66.35% <http://www.msstate.edu/>  
36 - 3591993 - 01623927

66.15% <http://www.nwu.edu/>  
36 - 13/14/95 - 01/05/97

<http://optich.mcgill.ca/> - size 1K - 16 Dec 96

<http://www.net.cmu.edu/> - size 4K - 19 Apr 95

<http://www.cw.westview.edu/> - size 3K - 15 Apr 96

<http://www.sfc.bea.gov/> - since 3K - 5 Feb 97

<http://www.chem.su.se/~suu/> - size 4K - 25 Feb 97

<http://www.mankato.mn.us.edu/> - size 3K - 27 Nov 96

<http://www.sau.edu/> - size 2K - 4 Feb 97

University of Washington ECSEL Projects

# PageRank - Consequences

- Google's use of PageRank to help rank documents led to them dominating web search in the English-speaking world, which they continue to do today.
- Other IR techniques are also used (full-text search, title search, proximity search etc.) and a fusion process is used to merge the results of these different kinds of search.
- Specific details about how exactly Google does its searching are not generally available anymore, such is the competitive nature of the online search business.
  - Other search companies have their own version of PageRank.