Chapter 27a : Different ways to parameterise lead to different results.

We are given f[0..M), g[0..N) of int. We are told that f is ascending and g is descending. We are asked to construct a program to compute the number of pairs f.i and g.j whose sum exceeds 37.

$$r = \langle + \, i,j : 0 \le i < M \wedge 0 \le j < N : h.(f.i).(g.j) \rangle$$

where

* (0) h.x.y	=	1	⇐	x + y > 37
* (1) h.x.y	=	0	⇐	x + y ≤ 37

We begin by modelling our domain.

* (2) C.m.n	=	$\langle + \, i,j : m \le i < M \wedge n \le j < N : h.(f.i).(g.j) \rangle$	
- (3) C.M.n	=	0	, 0 ≤ n ≤ N
- (4) C.m.N	=	0	, 0 ≤ m ≤ M
- (5) C.m.n	=	C.(m+1).n + D.n	, 0 ≤ m < M
- (6) C.m.n	=	C.m.(n+1) + E.m	, 0 < n ≤ N
* (7) D.n	=	$\langle + \, j : n \le j < N : h.(f.m).(g.j) \rangle$	
-*(8) E.m	=	$\langle + \, i : m \le i < M : h.(f.i).(g.n) \rangle$	
- (9) D.n	=	? ⇐ f.m + g.n > 37	
- (10) D.n	=	0 ⇐ f.m + g.n ≤ 37	
- (11) E.m	=	M-m ⇐ f.m + g.n > 37	
- (12) E.m	=	? ⇐ f.m + g.n ≤ 37	

We note that (9), (10), (11) and (12) do allow us to cover all cases so the model is adequate for constructing a program. This is of course what was done in Chapter 27.

Now I want to look at the other possibilities. I am only going to record the theorems and not their proofs.

* (2) C.m.n	=	$\langle + i,j : m \leq i < M \wedge 0 \leq j < n : h.(f.i).(g.j) \rangle$	
- (3) C.M.n	=	0	, $0 \leq n \leq N$
- (4) C.m.0	=	0	, $0 \leq m \leq M$
- (5) C.m.n	=	C.(m+1).n + D.n	, $0 \leq m < M$
- (6) C.m.n	=	C.m.(n-1) + E.m	, $0 < n \leq N$
* (7) D.n	=	$\langle + j : 0 \leq j < n : h.(f.m).(g.j) \rangle$	
-*(8) E.m	=	$\langle + i : m \leq i < M : h.(f.i).(g.(n-1)) \rangle$	
- (9) D.n	=	n $\quad \Leftarrow f.m + g.(n-1) > 37$	
- (10) D.n	=	? $\quad \Leftarrow f.m + g.(n-1) \leq 37$	
- (11) E.m	=	M-m $\quad \Leftarrow f.m + g.(n-1) > 37$	
- (12) E.m	=	? $\quad \Leftarrow f.m + g.(n-1) \leq 37$	

(9), (10), (11) and (12) do not cover all cases so the model is not adequate.

* (2) C.m.n	=	$\langle + i,j : 0 \le i < m \wedge 0 \le j < n : h.(f.i).(g.j) \rangle$	
- (3) C.0.n	=	0	, $0 \le n \le N$
- (4) C.m.0	=	0	, $0 \le m \le M$
- (5) C.m.n	=	C.(m-1).n + D.n	, $0 \le m < M$
- (6) C.m.n	=	C.m.(n-1) + E.m	, $0 < n \le N$
* (7) D.n	=	$\langle + j : 0 \le j < n : h.(f.(m-1)).(g.j) \rangle$	
-*(8) E.m	=	$\langle + i : 0 \le i < m : h.(f.i).(g.(n-1)) \rangle$	
- (9) D.n	=	n $\Leftarrow f.(m-1) + g.(n-1) > 37$	
- (10) D.n	=	? $\Leftarrow f.(m-1) + g.(n-1) \le 37$	
- (11) E.m	=	? $\Leftarrow f.(m-1) + g.(n-1) > 37$	
- (12) E.m	=	0 $\Leftarrow f.(m-1) + g.(n-1) \le 37$	

(9), (10), (11) and (12) do cover all cases so the model is adequate.

* (2) C.m.n	=	$\langle + i,j : 0 \leq i < m \wedge n \leq j < N : h.(f.i).(g.j) \rangle$		
- (3) C.0.n	=	0		, $0 \leq n \leq N$
- (4) C.m.N	=	0		, $0 \leq m \leq M$
- (5) C.m.n	=	C.(m-1).n + D.n		, $0 \leq m < M$
- (6) C.m.n	=	C.m.(n+1) + E.m		, $0 < n \leq N$
* (7) D.n	=	$\langle + j : n \leq j < N : h.(f.(m-1)).(g.j) \rangle$		
-*(8) E.m	=	$\langle + i : 0 \leq i < m : h.(f.i).(g.(n)) \rangle$		
- (9) D.n	=	?	$\Leftarrow f.(m-1) + g.n > 37$	
- (10) D.n	=	0	$\Leftarrow f.(m-1) + g.n \leq 37$	
- (11) E.m	=	?	$\Leftarrow f.(m-1) + g.n > 37$	
- (12) E.m	=	0	$\Leftarrow f.(m-1) + g.n \leq 37$	

(9), (10), (11) and (12) do not cover all cases so the model is not adequate.