## Chapter 35: The Celebrity problem.

*In which we continue to apply Searching by Elimination.* 

Among N people, a celebrity is someone who is known to everyone but who doesn't know anyone. We are given a group of N people, which contains a celebrity, and asked to identify them. We will use the symmetric linear search algorithm.

We represent the problem domain using a 2 dimensional Boolean matrix

Where

$$R.i.j = "person i knows person j"$$

For a celebrity x we define

\* (0) 
$$F.x = \langle \forall i : i \neq x : R.i.x \land \neg R.x.i \rangle$$

We can now specify our problem as follows

Pre: 
$$\langle \exists \ k : 1 \le k \le N : F.k \rangle$$

Post: F.x

We now calculate the guards

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F.a \Rightarrow F.b

\Leftrightarrow \qquad \{\text{Predicate calculus, } \neg P \Rightarrow (P \Rightarrow Q)\} \\
\neg F.a \\
= \qquad \{\text{definition of } F\} \\
\neg \langle \forall i : i \neq a : R.i.a \land \neg R.a.i \rangle \\
= \qquad \{\text{deMorgan}\} \\
\langle \exists i : i \neq a : \neg R.i.a \lor R.a.i \rangle \\
\Leftrightarrow \qquad \{a \neq b, \text{predicate calculus}\} \\
\neg R.b.a \lor R.a.b
```

Symmetrically,  $(F.b \Rightarrow F.a) \Leftarrow \neg R.a.b \lor R.b.a$ 

## This yields our finished program

$$a, b := 1, N$$
  
 $;do a \neq b \Rightarrow \{M \leq a < b \leq N\}$   
 $if \neg R.b.a \lor R.a.b \Rightarrow a := a + 1$   
 $[] \neg R.a.b \lor R.b.a \Rightarrow b := b - 1$   
 $fi$   
od  
 $; x := a$