# Chapter 8
# The Special Relativaty

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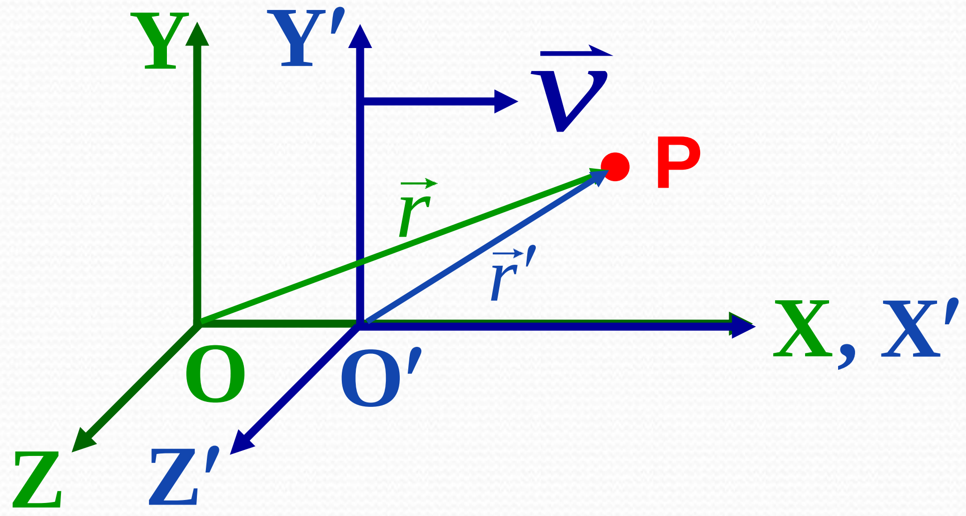
# § 8.1 Galilean Relativity

## 1. Galilean transformation




**Consider two coordinate systems K and K′ , where K′ moves along the $x$ axis with a constant velocity $\vec{v}$ relative to K**

**Galilean transformation of coordinates**

$$x' = x - vt$$

$$y' = y$$

$$z' = z$$

$$t' = t$$

**Taking the derivative with respect to time gives**

$$v'_x = v_x - v$$

$$v'_y = v_y$$

$$v'_z = v_z$$

**This transformation indicates that time is independent on the space, and the measurement of time interval and length has nothing to do with coordinate system.**

Taking the derivative with respect to time again gives

$$\vec{a}\,' = \vec{a}$$

## 2. Galilean principle of relativity

Mass is independent of the movement

$$m' = m$$

Force has nothing to do with coordinate system.

$$\vec{F}\,' = \vec{F}$$

**So** **All the laws of mechanics are the same in all inertial frames.**

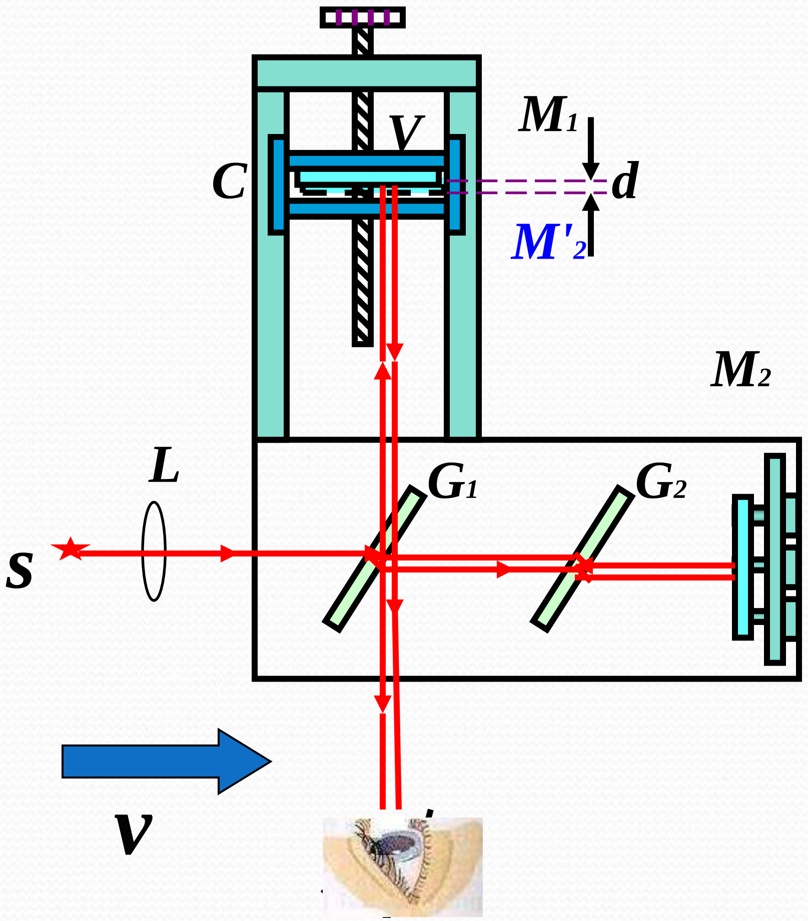
# § 8.2 The Speed of light

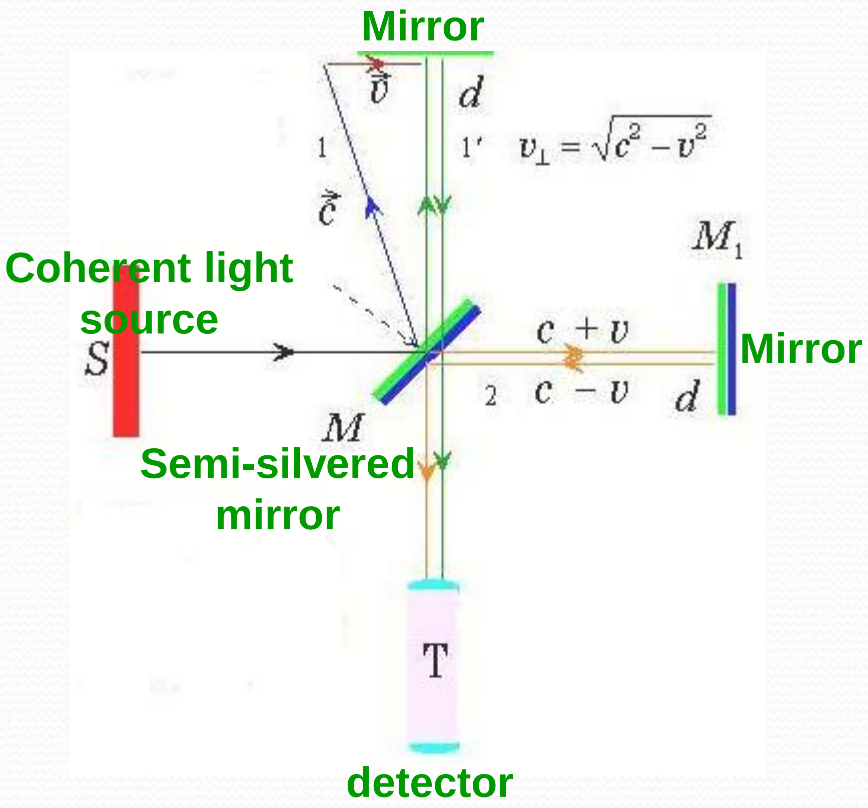
- In 1873, Maxwell first understood that light was an electromagnetic wave.
- According to maxwell's equations, a pulse of light emitted from a source at rest would spread out at velocity c in all directions.
- But what would happen if the pulse was emitted from a source that was moving?
- This possibility confused physicists until 1905.

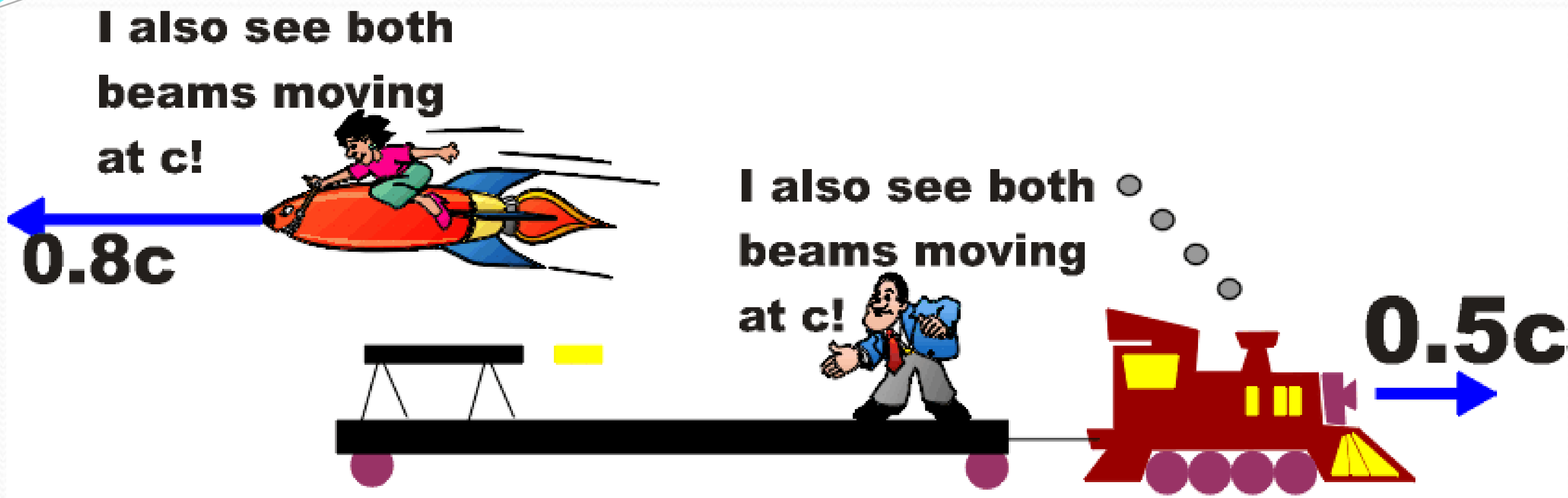
## Michelson-Morley Experiment

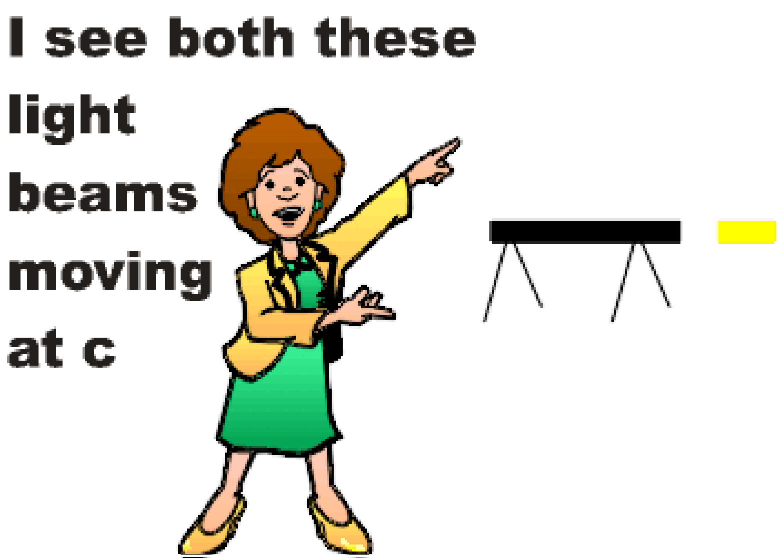
- **Albert Michelson(迈克尔逊) and Edward Morley(莫雷) were two American physicists working at Case Western Reserve University(凯斯西保留地大学) in Cleveland(克利夫兰市(美国俄亥俄州东北部城市) )**
- **They constructed a device which compared the velocity of light traveling in different directions (1887).**
- **If the aether (以太) theory were correct, light would thus move more slowly against the aether wind and more quickly downwind. The Michelson-Morley apparatus should easily be able to detect this difference.**

# 1887-1907: Find the velocity of the Aether






**The result is: light always moves at the same speed regardless of the velocity of the source or the observer or the direction that the light is moving!**

- In 18 years after the Michelson-Morley experiment, the smartest people in the world attempted to explain it away
- In particular C.F. FitzGerald(菲茨杰拉德) and H.A. Lorentz(洛仑兹) constructed a mathematical formulation (called the Lorentz transformation) which seemed to explain things but no one could figure out which it all meant.
- In 1905, Albert Einstein proposed the theory of Special Relativity which showed that the only way to explain the experimental result is to suppose that space and time as seen by one observer are distorted when observed by another observer (in such a way as to keep c invariant)

# § 8.3 The Principles of the Special Relativity

- **The laws of physics are identical in all inertial frames of reference.**
- **The speed of light in vacuum has the same value, C, in all inertial frames regardless of the source of the light and the direction it moves.**

**Nothing can move faster than c, the speed of light in vacuum.**

**Example 8.1** A pilot on a space ship moving at 0.86C away from the Earth sends a laser beam signal to the Earth.

(a) The pilot measures the speed $v$ of the laser beam signal to be

(A) $v<c$ (B) $v=c$ (C) $v>c$

(b)The people on the Earth measure the speed of the laser beam signal to be

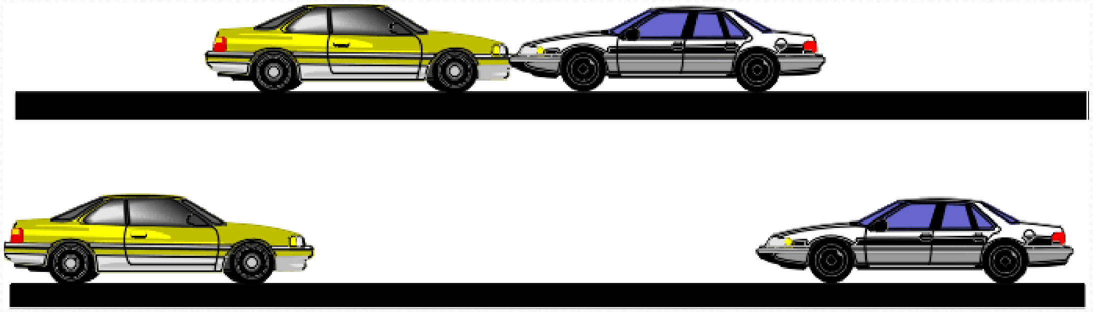
(A) $v<c$ (B) $v=c$ (C) $v>c$

**Solution**

(a) (B) (b) (B)

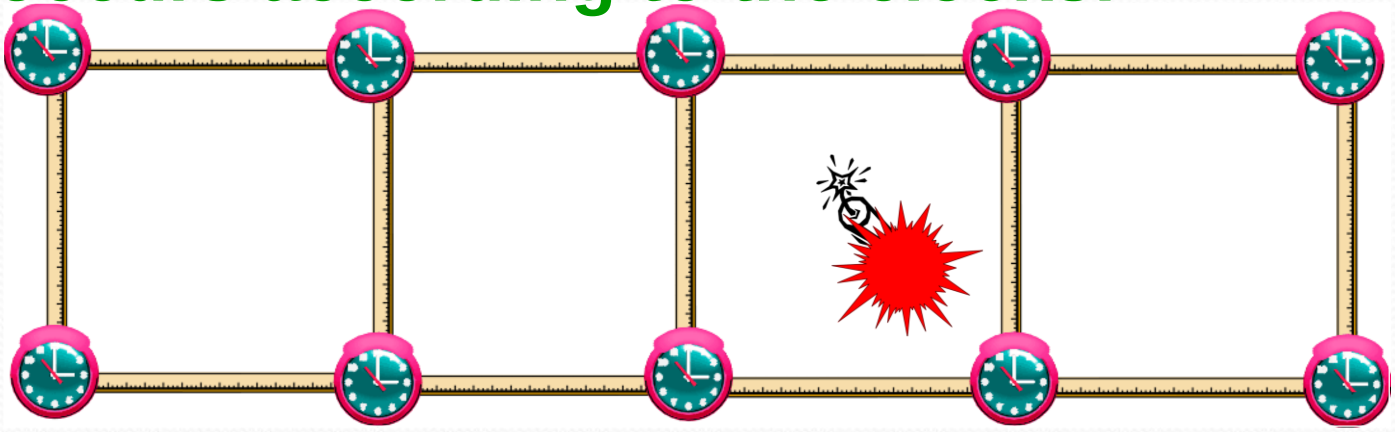
## Event(事件)

- In physics jargon(术语), the word event has about the same meaning as it’s everyday usage.
- An event occurs at a specific location in space at a specific moment in time.



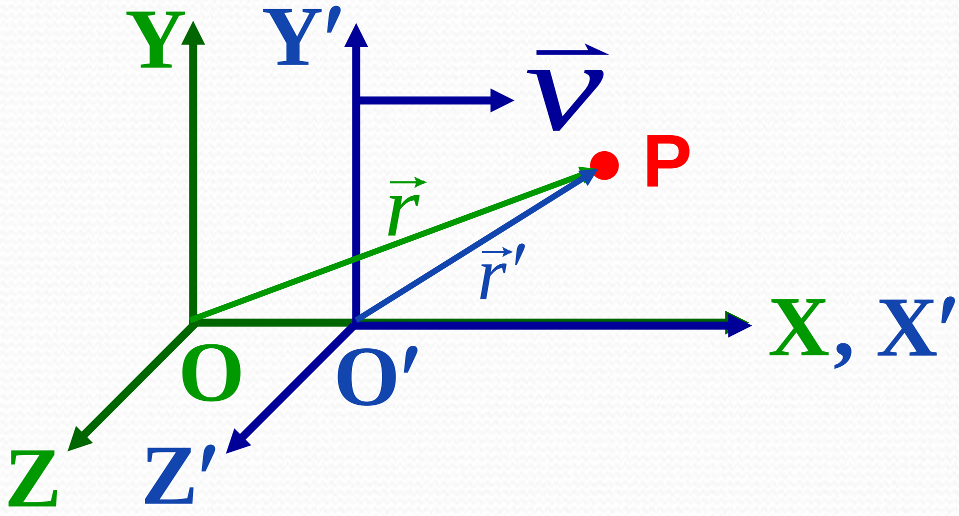
## Reference Frame

- A reference frame is a means of describing the location of an event in space and time.
- To construct a reference frame, lay out a bunch of rulers and synchronized(同步的) clocks.
- You can then describe an event by where it occurs according to the rulers and when it occurs according to the clocks.



## ■ Lorentz CoordinateTransformation

Two coordinate systems K and K′ , where K′ moves along the *x* axis with a constant velocity $\vec{v}$ relative to K.




Space and time are not absolute as in Newtonian physics and everyday experience. The mathematical relation between the description of two different observers is called the Lorentz transformation.

**Lorentz transformation equations**

$$\begin{cases} x' = \gamma(x - vt) \\ y' = y \\ z' = z \\ t' = \gamma(t - \frac{v}{c^2}x) \end{cases}$$

**Inverse Lorentz transformation equations**

$$\begin{cases} x = \gamma(x' + Vt') \\ y = y' \\ z = z' \\ t = \gamma(t' + \frac{V}{c^2}x') \end{cases}$$

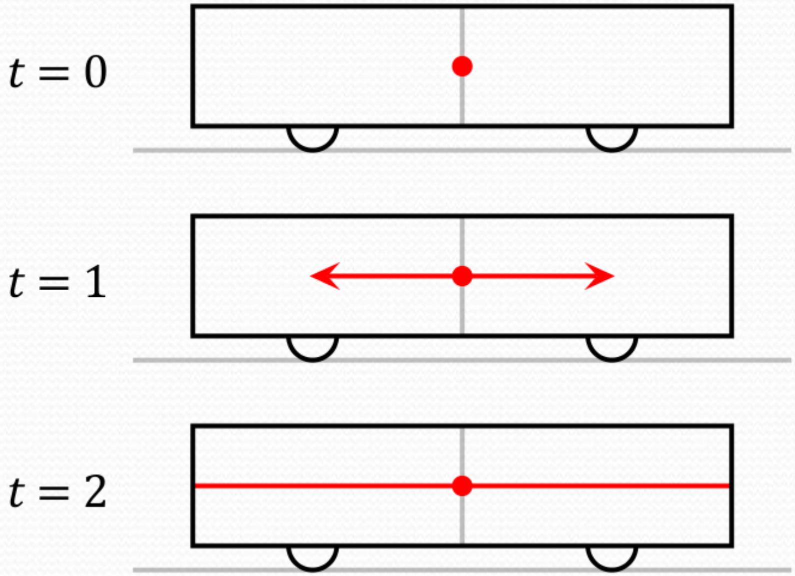
**Lorentz factor** $\gamma = \dfrac{1}{\sqrt{1 - \dfrac{v^2}{c^2}}}$

# § 8.4 Consequences of the Special Relativity

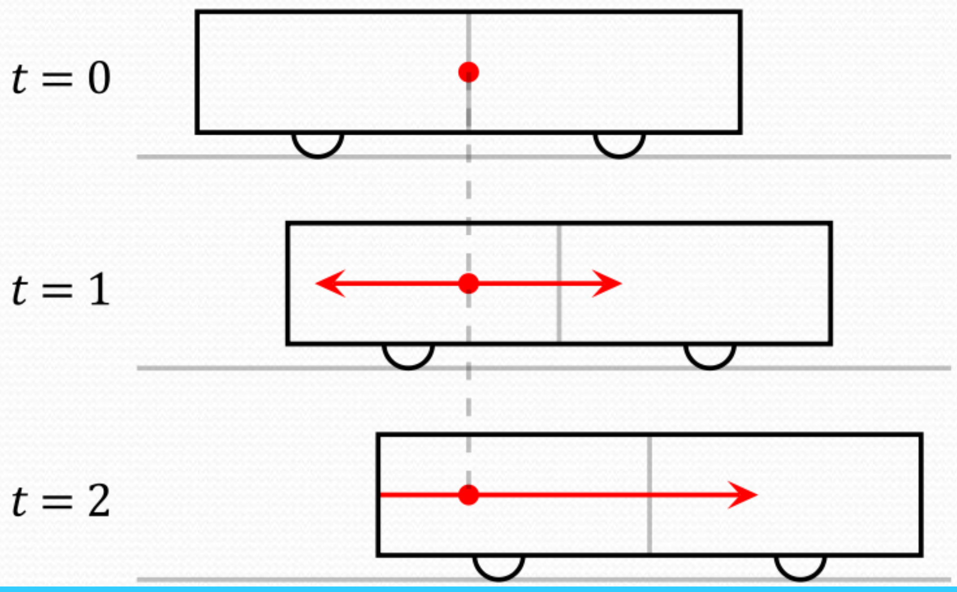
**(狭义相对论时空观)**

- **Events which are simultaneous to a stationary observer are not simultaneous to a moving observer.**
- **A stationary observer will see a moving clock running slow.**
- **A moving object will be contracted along its direction of motion.**
- **Mass can be shown to be a frozen form of energy according to the relation $E=mc^2$.**

## ■ Relativity of Simultaneity（同时的相对性）




**The train and platform experiment from the reference frame of an observer onboard the train.**




**The train and platform experiment from the reference frame of an observer on the platform.**

- **Thus two events which are simultaneous to the observer on the train are not simultaneous to an observer on the ground.**

**We can get this result by Lorentz transformation**

**In K' ( onboard the train) :**

**Event 1** $(x_1', t_1')$

**Event 2** $(x_2', t_2')$

**The observer on the train sees**

$$\Delta t' = t_2' - t_1' = 0$$

**In K ( on the platform) :**

**Event 1** $(x_1, t_1)$

**Event 2** $(x_2, t_2)$

**What does the observer on the platform see?**

$$\Delta t = t_2 - t_1 = ?$$

**According to Lorentz transformation**

$$t_1 = \gamma(t_1' + \frac{v}{c^2}x_1'); \qquad t_2 = \gamma(t_2' + \frac{v}{c^2}x_2')$$

$$\because \Delta t' = 0; \qquad \Delta x' = x_2' - x_1' \neq 0$$

$$\therefore \Delta t = \gamma(\Delta t' + \frac{v}{c^2}\Delta x') \neq 0$$

**Note** **This occurs only when the two events occur at different locations. If the two events occur at the same location and are simultaneous according to one reference frame, they are simultaneous to another reference frame as well.**

## ■ Time Dilation(时间膨胀)

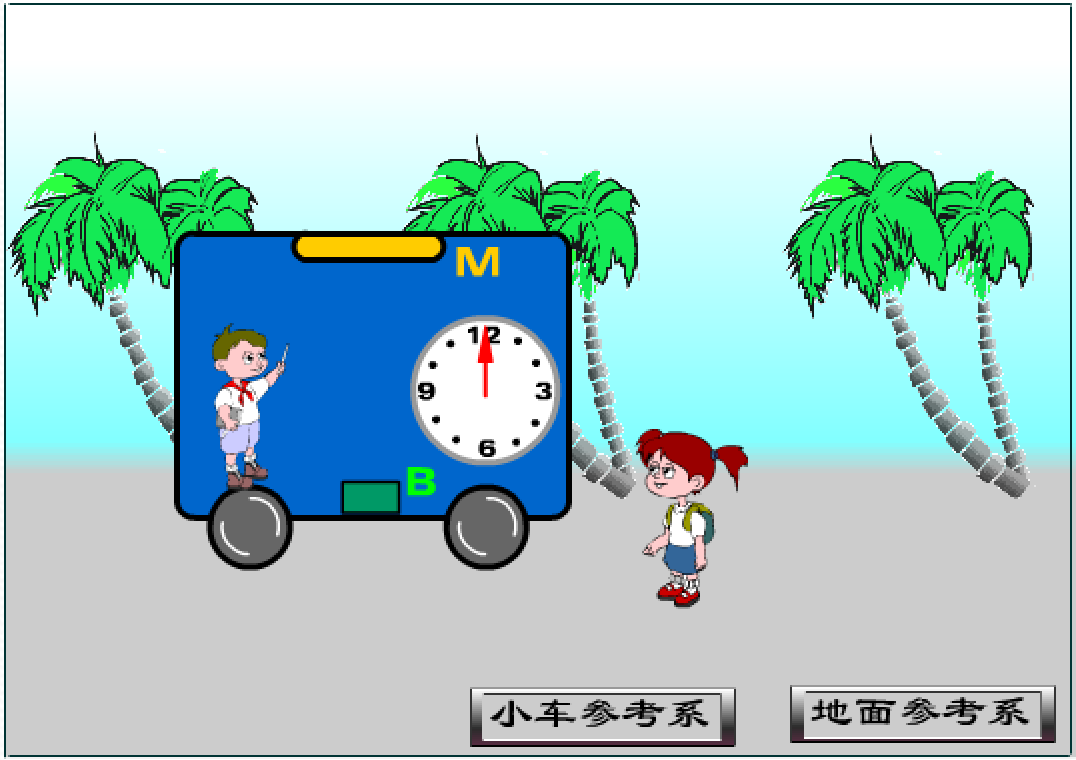
**Let us now consider the relation between time interval as measured by moving and stationary observers .**

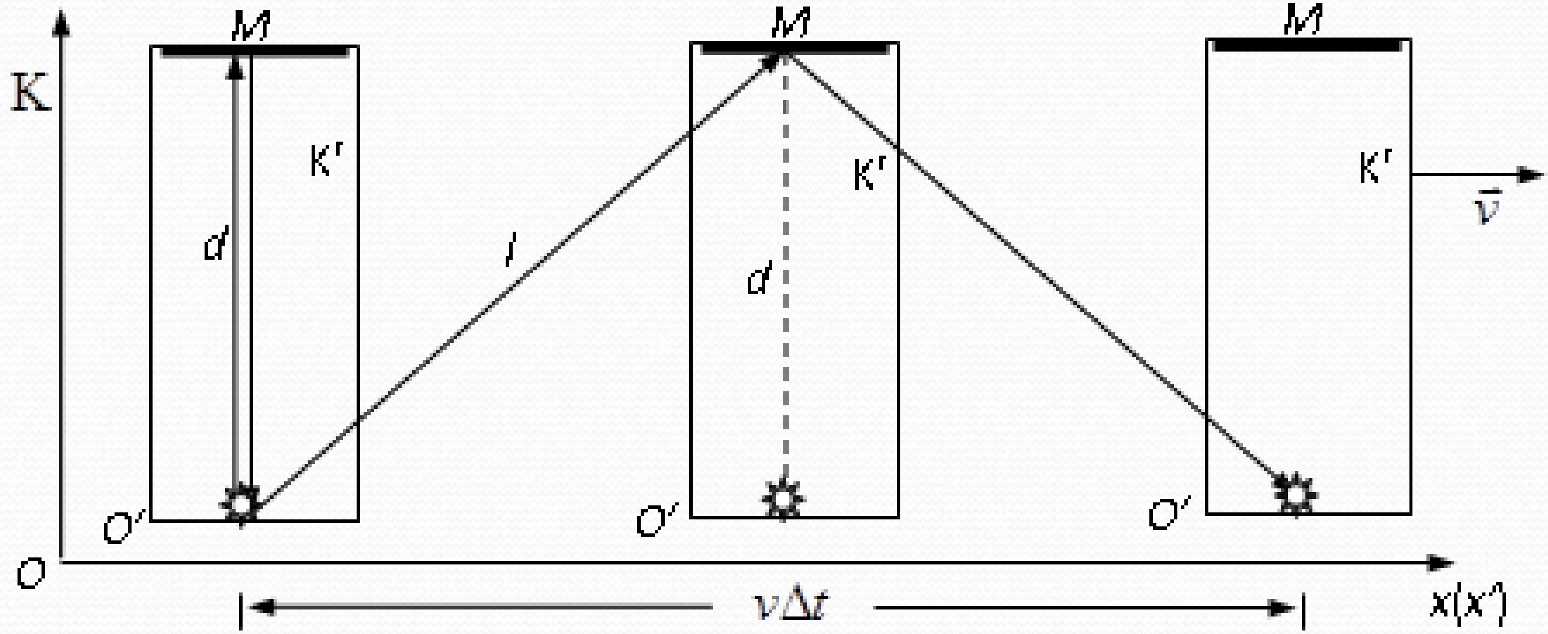
### Event 1

**The light source emits a flash.**

### Event 2

**The flash having been reflected from a mirror returns to the light source.**






**What is the relationship of Δt and Δt′?**

**In K':** $\Delta x' = 0 \quad \Delta t' = \dfrac{2d}{c}$

**In K:** $\Delta x \neq 0 \quad \Delta t = \dfrac{2l}{c}$

**Where** $$l=\sqrt{d^2+\left(\frac{v\Delta t}{2}\right)^2}$$

**Then we get** $$\Delta t=\frac{\Delta t'}{\sqrt{1-\frac{v^2}{c^2}}}$$

**We can get this result by Lorentz transformation**

| **In K'** ( on the train) **:** | **In K** ( on the ground) **:** |
|---|---|
| **Event 1** $(x_1', t_1')$ | $(x_1, t_1)$ |
| **Event 2** $(x_2', t_2')$ | $(x_2, t_2)$ |

$$\Delta t = \frac{\Delta t' + \frac{v}{c^2}\Delta x'}{\sqrt{1-\frac{v^2}{c^2}}}$$

**Where $\Delta x' = 0$ , so we get the same result.**

$$\boxed{\Delta t = \frac{\Delta t'}{\sqrt{1-\frac{v^2}{c^2}}}}$$

proper time

**Proper time（原时/固有时） is defined to be the time interval between two events those occur at the same point.**

**1971 U. S. Naval Observatory（海军天文台） flew jets around the world**

**Some East, with Earth's rotation**

**Some West, against Earth's rotation**

| | **East** | **West** |
|---|---|---|
| **Predictions** | $40 \pm 23ns$ | $275 \pm 21ns$ |
| **Result from experiment** | $59 \pm 10ns$ | $273 \pm 7ns$ |

**After each flight, they compared the time on the clocks in the aircraft to the time on the clock at the Observatory. Their experimental data agreed within error to the predicted effects of time dilation. Of course, the effects were quite small since the planes were flying nowhere near the speed of light.**

**Example 8.2** The average lifetime of a pi($\pi$) meson($\pi$介子) in its own frame of reference is $2.5\times10^{-8}$s, and then decays into a muon meson($\mu$介子) and a neutrino(中微子). There is a beam of pi mesons and the speed of them is $u$=0.99c. The distance the pi mesons traveled before decay is about 52m measured by an observer on the Earth. Is this result be consistent with the special relativity?

**Solution**

$$l = 2.5\times10^{-8}\times0.99c = 7.4\text{m}$$

This result is far away from the experimental result 52m.

**The average lifetime of a pi meson measured by an observer on the Earth is**

$$\Delta t = \frac{\tau}{\sqrt{1-u^2/c^2}} = \frac{2.5\times10^{-8}}{\sqrt{1-(0.99)^2}} = 1.8\times10^{-7}(s)$$

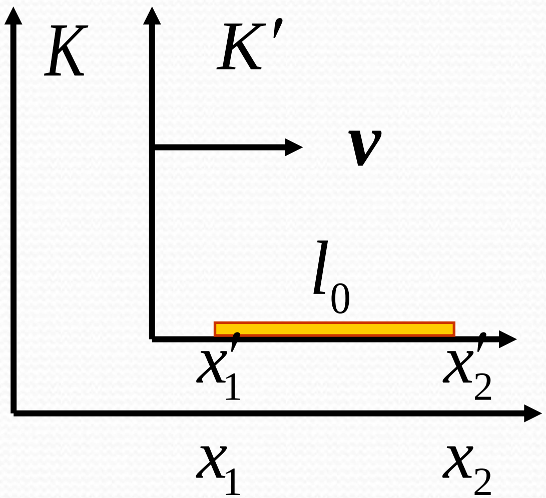
**The average distance measured on the earth is**

$$l = 1.8\times10^{-7}\times0.99c = 53.4\mathrm{m}$$

**This result is consistent with the experimental result.**

## ■ Length contraction（尺度收缩）

- **Just as relativity tells us that different observers will experience time differently, the same is also true of length.**




**Consider a rod at rest in K′. An observer at rest with the rod measures the length of the rod is**

$$\Delta x' = x_2' - x_1' = l_0$$

**proper length**
（原长/固有长度）

**How about the length measured in K system?**

$$\Delta x = x_2 - x_1 = ?$$

## According to Lorentz transformation

$$\Delta x = x_2 - x_1 = \frac{\Delta x' + v\Delta t'}{\sqrt{1-\frac{v^2}{c^2}}}$$

?
An arbitrary value

To measure the length of the rod in the frame of K, we must make a simultaneous determination of the coordinates $x_1$ and $x_2$ of the ends of the rod. That is to say, when $\Delta t = t_2 - t_1 = 0$ , gives the length of the rod.

**So**

$$\Delta x' = \frac{\Delta x - v\Delta t}{\sqrt{1-\frac{v^2}{c^2}}}$$

**Where** $\Delta x' = x_2' - x_1' = l_0$ **, and** $\Delta t = 0$

$$\boxed{l = l_0\sqrt{1-\frac{v^2}{c^2}}}$$

- **In fact, a stationary observer will observe a moving object shortened by a factor of which is the same as the time dilation factor.**

**Example 8.4** Two airships fly past each other with speed 0.98C. The observer in airship I measured the length of airship II is 2/5 of the length of airship I. What is the ratio of proper length of airship II and airship I?

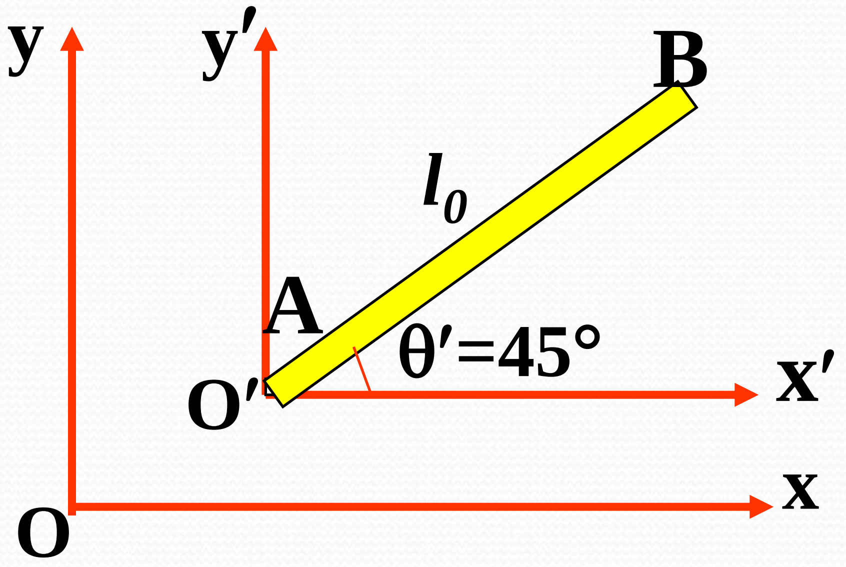
**Solution** Suppose airship I in K and airship II in K′, And $l_{10}$ and $l_{20}$ are the proper length of airship I and II, so

$$\frac{l_2}{l_{10}} = \frac{l_{20}\sqrt{1-\frac{v^2}{c^2}}}{l_{10}} = \frac{2}{5} \quad \Rightarrow \frac{l_{20}}{l_{10}} = \frac{2}{1}$$

**Example 8.5** **Inertial frame K′ moves along the common x-x′ axis with constant speed 0.8Crelative to inertial frame K. A 1m rigid rod is placed in K′ and the angle between the rod and the O′x′ is 45°. What is the length of the rod in K and what is the angle between the rod and the axis Ox?**

**Solution**

$$l = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$$




**where**

$$(y_B - y_A)^2 = l_0^{\ 2} \sin^2 \theta'$$

$$(x_B - x_A)^2 = (x'_B - x'_A)^2 (1 - \frac{v^2}{c^2})$$

$$= l_0{}^2 \cos^2\theta' (1 - \frac{v^2}{c^2})$$

**so** $$l = l_0 \sqrt{1 - \frac{v^2}{c^2} \cos^2\theta'} = 0.825 l_0 = 0.825m$$

**and** $$\theta = arctg(\frac{y_B - y_A}{x_B - x_A})$$

$$= arctg(\frac{l_0 \sin\theta'}{l_0 \cos\theta' \sqrt{1 - v^2 / c^2}}) \approx 59^\circ$$

**Example 8.6** **A spaceship is flying in a straight line with a constant speed *v*. An astronaut sends a light signal from the head of the spaceship to a rear receiver, and the signal is received after $\Delta t$. What is the proper length of the spaceship?**

$(A) \quad c\Delta t$

$(B) \quad v\Delta t$

$(C) \quad c\Delta t\sqrt{1-(v/c)^2}$

$(D) \quad c\Delta t/\sqrt{1-(v/c)^2}$

**Solution** **(A)**

## ■ The Lorentz Velocity Transformation

——The relation between two inertial reference frame

**The speed v in O′:**

$$u'_x = \frac{dx'}{dt'} = \frac{d(x - vt)}{d(t - \frac{v}{c^2}x)} = \frac{dx - v\,dt}{dt - \frac{v}{c^2}dx}$$

$$= \frac{\frac{dx}{dt} - v}{1 - \frac{v}{c^2}\frac{dx}{dt}} = \frac{u_x - v}{1 - \frac{v}{c^2}u_x}$$

# The Lorentz Velocity Transformation

$$u'_x = \frac{u_x - v}{1 - \frac{v}{c^2} u_x}$$

$$u_x = \frac{u'_x + v}{1 + \frac{v}{c^2} u'_x}$$

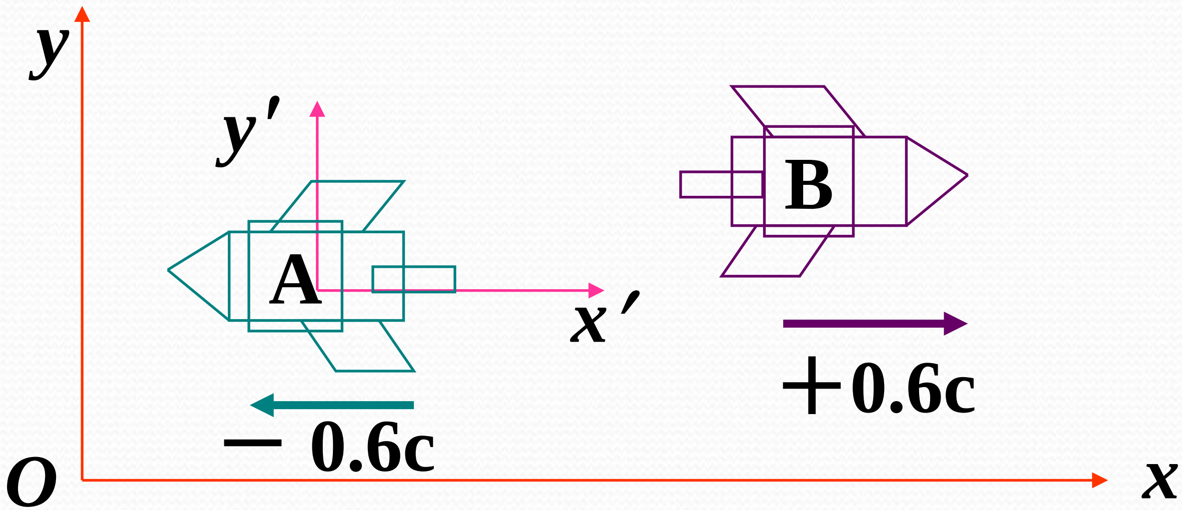
$$u'_y = \frac{u_y \sqrt{1 - \frac{v^2}{c^2}}}{1 - \frac{v}{c^2} u_x}$$

$$u_y = \frac{u'_y}{1 + \frac{v}{c^2} u'_x} \sqrt{1 - \frac{v^2}{c^2}}$$

$$u'_z = \frac{u_z \sqrt{1 - \frac{v^2}{c^2}}}{1 - \frac{v}{c^2} u_x}$$

$$u_z = \frac{u'_z}{1 + \frac{v}{c^2} u'_x} \sqrt{1 - \frac{v^2}{c^2}}$$

**Example 8.7** Two spaceships A and B are flying in opposite direction, as shown in figure. What is the speed of spaceship B relative to A?




## Solution

The ground is reference O and spaceship A is reference O′.

**the speed of spaceship B relative to A**

$$u_x' = \frac{u_x - v}{1 - \frac{v}{c^2} u_x} = \frac{0.6c - (-0.6c)}{1 - \frac{(-0.6c)}{c^2} 0.6c} = 0.882c$$

**According to the Galilean transformation we get**

$$u_x' = u_x - v = 0.6c - (0.6c) = 1.2c > c$$

**Of course, this result is wrong.**

**Example 8.8** Reference O′ is moving with 0.5C along -x direction relative to O frame. A light pulse is emitted from point O′ to +x direction. What is the speed of this light pulse measured in O frame?

**Solution** According to The Lorentz Velocity Transformation, we have

$$u_x = \frac{u'_x + v}{1 + \frac{v}{c^2} u'_x} = \frac{c + (-0.5c)}{1 + \frac{(-0.5c)}{c^2} c}$$

$$= c$$

# § 8.5 Relativistic Dynamics

- **The speed of light is the ultimate speed limit**
- **If you accelerate a particle continuously towards *c*, it's velocity gets closer to *c* but never reaches it;**
- **The amount of work required to do this is thus greater than $\frac{1}{2}mv^2$ （Kinetic energy theorem）**
- **The amount of impulse will be greater than mc（Theorem of momentum）**

## Relativistic Mass and Relativistic Momentum

- **The relationship between the relativistic mass and speed:**

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

Relativistic mass

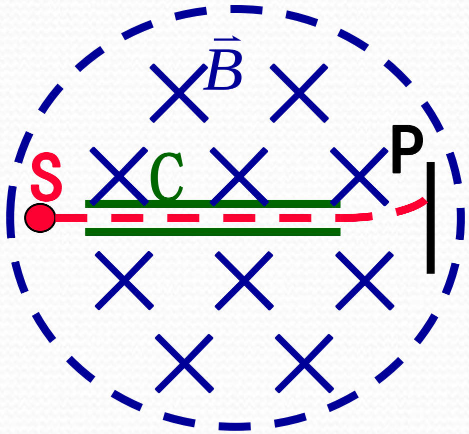
Rest mass

Speed of the particle

**Note**

- **When $v$ is much smaller than $c$, $m \approx m_0$**
- **When $v = c \Rightarrow m_0 = 0$, the rest mass of photon(光子) is 0.**

**Equipment:**




**S**——radioactive source

**C**——Capacitor

**P**——photographic plate

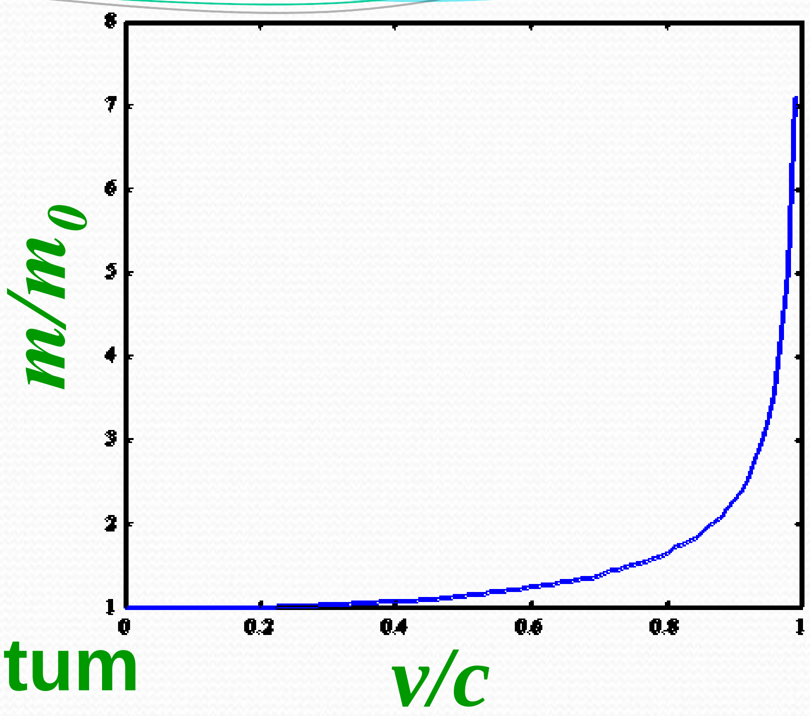
$\vec{B}$——uniform magnetic field

**Principle:**

① $qvB=qE \rightarrow v$

② $R=mv/qB \rightarrow m$

**W.Kaufmann**(考夫曼,Germany) **verified this relationship by experiments in 1901.**




- **The relativistic momentum**

$$p = mv = \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}}$$

**Momentum reduces to the classical expression when *v* is much smaller than *c*.**

**Example 8.7** **The linear density of a rod (length *L* and mass *m*) is $\rho$. (a) Suppose the rod moves along the length direction with a speed of *v*, what is the linear density of the rod？(b) If the rod moves perpendicular to the length direction with a speed of *v*, what is the linear density of the rod？**

**Solution** **(a)When the rod moves along the length direction with speed *v*, we have**

$$L' = L\sqrt{1-(v/c)^2} \qquad m' = \frac{m}{\sqrt{1-(v/c)^2}}$$

**So the linear density of the rod is**

$$\rho' = \frac{m'}{L'} = \frac{m}{L(1-\frac{v^2}{c^2})} = \frac{\rho}{(1-\frac{v^2}{c^2})}$$

**(b) If the rod moves perpendicular to the length direction with speed *v*, we have**

$$L'' = L \qquad m'' = \frac{m}{\sqrt{1-(v/c)^2}}$$

**So the linear density of the rod is**

$$\rho'' = \frac{m''}{L''} = \frac{m}{L\sqrt{(1-\frac{v^2}{c^2})}} = \frac{\rho}{\sqrt{(1-\frac{v^2}{c^2})}}$$

## Relativistic Kinetic Energy and the Equivalence of Mass and Energy

**The kinetic energy theorem of a mass point can be expressed as**

$$E_K = \int_0^r \vec{F} \cdot d\vec{r} = \int_0^r \frac{d(m\vec{v})}{dt} \cdot d\vec{r} = \int_0^{mv} \vec{v} \cdot d(m\vec{v})$$

**where**

$$\vec{v} \cdot d(m\vec{v}) = m\vec{v} \cdot d\vec{v} + \vec{v} \cdot \vec{v} dm = mvdv + v^2 dm$$

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow m^2\left(1 - \frac{v^2}{c^2}\right) = m_0^2$$

$$\boxed{m^2c^2 - m^2v^2 = m_0^2c^2}$$

**Take the derivative of the both sides**

$$2mc^2dm - 2mv^2dm - 2m^2vdv = 0$$

$$\Downarrow$$

$$c^2dm = v^2dm + mvdv$$

$$\Downarrow$$

$$E_K = \int_{m_0}^{m} c^2dm = mc^2 - m_0c^2$$

Rest energy

Total energy

——**Relativistic Kinetic Energy**(相对论动能)

**Note**

- $E_0 = m_0 c^2$ **is independent of the speed of the object. It can be regarded as the internal energy of a particle or a system of particles at rest.**

- $E = mc^2$ **depends on the object's speed and is the sum of the kinetic energy and rest energy.**

$$E = mc^2 = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}} = E_K + E_0$$

**——Mass-Energy equivalence equation(质能方程)**

- **When v<<c, $E_k$ reduces to the classical expression.**

$$\frac{1}{\sqrt{1+x}}=1-\frac{1}{2}x+\frac{1\cdot 3}{2\cdot 4}x^2-\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}x^3+\cdots$$

$$\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}=1+\frac{1}{2}\frac{v^2}{c^2}+\cdots\cdots\approx 1+\frac{1}{2}\frac{v^2}{c^2}$$

$$E_K=mc^2-m_0c^2=m_0\left(1+\frac{v^2}{2c^2}\right)c^2-m_0c^2$$

$$=m_0c^2+\frac{m_0}{2}v^2-m_0c^2$$

$$=\frac{1}{2}m_0v^2$$

- It for the first time showed that energy is related to the mass of an object at rest. If energy is stored in the object, its rest mass increased. It also implies that mass can be destroyed to release energy.

**Example8.8** A kind of nuclear reaction

$$ {}_{1}^{2}\mathrm{H} + {}_{1}^{3}\mathrm{H} \rightarrow {}_{2}^{4}\mathrm{He} + {}_{0}^{1}\mathrm{n} $$

Deuteron(氘核)： $m_{10}=3.3437\times10^{-27}\mathrm{kg}$

Triton(氚核)： $m_{20}=5.0049\times10^{-27}\mathrm{kg}$

Helion(氦核)： $m_{30}=606425\times10^{-27}\mathrm{kg}$

Neutron(中子)： $m_{40}=1.6750\times10^{-27}\mathrm{kg}$

Find the energy released in the nuclear reaction.

**Solution** **According to the mass-energy equation, we have**

$$\Delta E = \Delta m_0 c^2$$

**The mass loss（质量亏损） is**

$$\Delta m_0 = (m_{10} + m_{20}) - (m_{30} + m_{40})$$

$$= \left[(3.3437 + 5.0049) - (6.6425 + 1.6750)\right] \times 10^{-27}$$

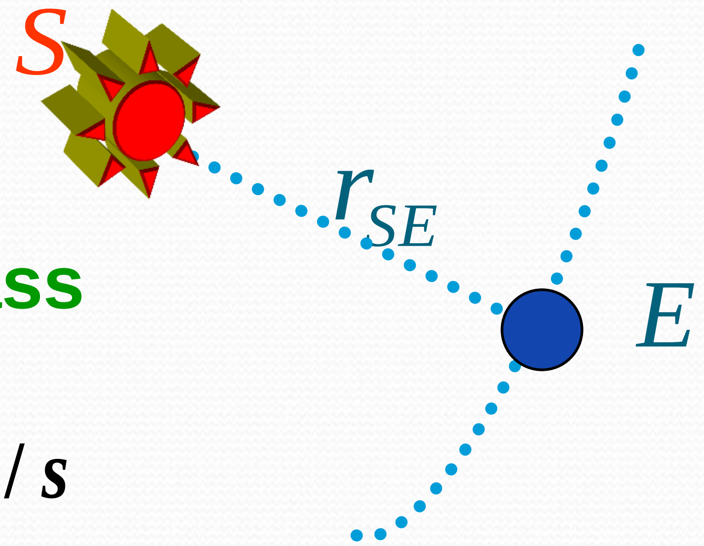
$$= 0.0311 \times 10^{-27} \mathrm{kg}$$

**So the energy released in the nuclear reaction is**

$$\Delta E = \Delta m_0 c^2 = 0.0311 \times 10^{-27} \times 9 \times 10^{16} = 2.799 \times 10^{-12} \mathrm{J}$$

The Sun is a nuclear reactor. The energy released per second is

$$\frac{\Delta E}{\Delta t} \approx 4.92 \times 10^{26} J/s$$




This is equivalent to loss mass

$$\frac{|\Delta m|}{\Delta t} = \frac{\Delta E}{c^2 \Delta t} = 5.4 \times 10^9 kg/s$$

This mass loss account for very very small proportion of the total mass of the sun.

$$\boxed{\frac{\Delta m}{m} = 8.5 \times 10^{-14}}$$

**Example8.9** **What is the relativistic mass of a particle if (a) its kinetic energy is $4m_0c^2$ and (b) its total energy is $4m_0c^2$. $m_0$ is the rest mass of the particle.**

**(A) $5m_0$ (B) $6m_0$ (C) $4m_0$ (D) $8m_0$**

**Solution (a)**

$$\because \ E_K = mc^2 - m_0c^2 = 4m_0c^2$$

$$\therefore \ mc^2 = 5m_0c^2$$

**That is** $m = 5m_0$

**(b)** $\because \ mc^2 = 4m_0c^2 \qquad \therefore \ m = 4m_0$

**Example8.10** **Two particles A and B move along the same straight line but opposite directions. The speed and the rest mass of the two particles are v and $m_0$ respectively. They become one large particle after collision. The rest mass of this large particle is**

（A） $2m_0$　　（B） $2m_0\sqrt{1-(\upsilon/c)^2}$

（C） $\dfrac{m_0}{2}\sqrt{1-(\upsilon/c)^2}$　　（D） $\dfrac{2m_0}{\sqrt{1-(\upsilon/c)^2}}$

**Solution (D)**

See you next time!