

# Data Structures and Algorithms

## The List Abstract Data Type - Doubly-Linked List

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# Learning outcomes

After this lecture and the related practical students should...

- be able to implement an doubly linked list
- understand the use of polymorphism to implement a data structure that can store any type of data

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- 1 The List Abstract Data Type
- 2 Doubly-Linked List Implementation
  - Algorithmic Complexity
  - Comparing Complexity

# The List Abstract Data Type

## Concept

- The List ADT models a sequence of **positions**
- Each position store a piece of data
- There is a before/after relation between positions
- This allows for efficient insertion into the middle of a list

# The Position Abstract Data Type

## Concept

Before we can fully understand the list abstract data type we need to look at the position abstract data type

- The Position abstract data type models the idea of a place within a data structure where a single piece of data is stored
- Positions provide a simple view of different ways of storing data
  - ▶ An element in an array
  - ▶ A Node in a linked list

# The Position Abstract Data Type

## Specification

- Operation:
  - ▶ `element()`: This returns the piece of data that is stored in this position

# The Position Abstract Data Type

## Interface

```
1 public interface Position{  
2     public Object element();  
3 }
```

# The List Abstract Data Type

## Specification

### Operations:

- **first()**: returns the first position in the list
- **last()**: return the last position in the list
- **before(p)**: returns the position in the list before p
- **after(p)**: returns the position in the list after p
- **insertBefore(p, d)**: inserts the value d into the position in the list before p
- **insertAfter(p, d)**: inserts the value d into the position in the list after p
- **insertFirst(d)**: inserts the value d into the first position in the list
- **insertLast(d)**: inserts the value d into the last position in the list
- **remove(p)**: removes the position p from the list
- **size()**: returns the number of elements stored in the list
- **isEmpty()**: is the list empty?



# The List Abstract Data Type

## Interface

```
1 public interface List {  
2     public Position first();  
3     public Position last();  
4     public Position before(Position p);  
5     public Position after(Position p);  
6     public Position insertBefore(Position p,  
7         Object d);  
8     public Position insertAfter(Position p,  
9         Object d);  
10    public Position insertFirst(Object d);  
11    public Position insertLast(Object d);  
12    public Object remove(Position p);  
13    public int size();  
14    public boolean isEmpty();  
15 }
```

# The List Abstract Data Type

## Implementation Strategies

- Array based implementation
  - ▶ An array of `Position` objects
  - ▶ Search through the array to find the correct position
  - ▶ Not very efficient
- Link based implementations
- There are two versions
  - ▶ Singly-Linked List
    - ★ Each `Position` object keeps a reference to the next `Position` in the sequence
  - ▶ Doubly-Linked List
    - ★ Each `Position` object keeps a reference to the next and previous `Positions` in the sequence

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# Position Abstract Data Type

## Doubly-Linked Implementation

- We create a `Node` class that implements the `Position` interface
- We add functionality to the class to store the next `Node` in the sequence
- We also add functionality to the class to store the previous `Node` in the sequence

# Position Abstract Data Type

## Doubly-Linked Implementation

```
1 public class Node implements Position {  
2     private Object element;  
3     Node next;  
4     Node previous;  
5  
6     public Node(int e) {  
7         this.element = e;  
8     }  
9  
10    public Object element() {  
11        return element;  
12    }  
13 }
```

# Doubly-Linked List Implementation

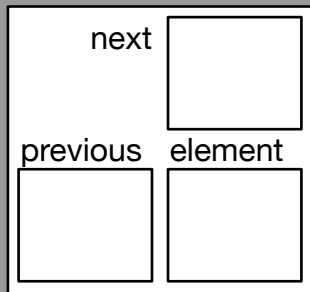
- We keep a reference to the first position in the list
- We keep a reference to the last position in the list
- We update the references when necessary
- We keep count of the number of positions in the list

## Variables:

- A reference to the first position in the list  
`private Node first;`
- A reference to the last position in the list  
`private Node last;`
- A number to keep track of the size  
`private int size;`

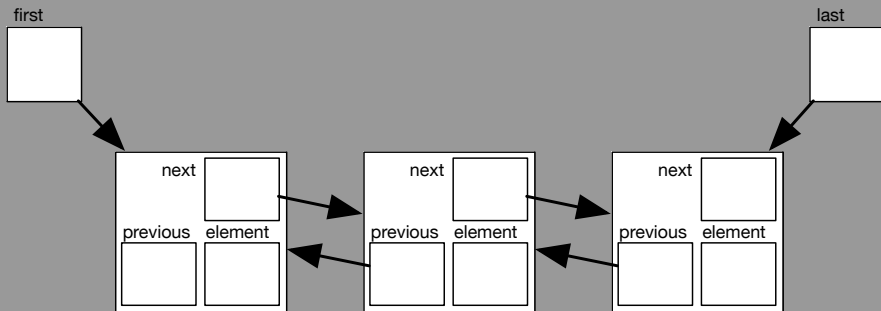
# Representation of a Node Object

## Doubly-Linked List Implementation



# Representation of a List

## Doubly-Linked List Implementation





# Doubly-Linked List Operations

- `first()`
  - ▶ Return the reference that is stored in the variable `first`
- `last()`
  - ▶ Return the reference that is stored in the variable `last`
- `size()`
  - ▶ Return the value of the `size` variable
- `isEmpty()`
  - ▶ Return the result of the expression `size == 0`
- `after(p)`
  - ▶ Convert `p` to a node
  - ▶ Return the next reference of `p`
- `before(p)`
  - ▶ Convert `p` to a node
  - ▶ Return the previous reference of `p`

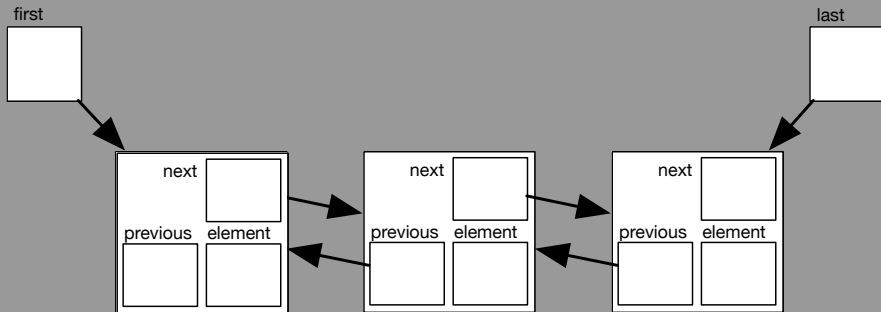
# Doubly-Linked List Operations

## insertFirst(d)

- Insert the value into the first position
  - ▶ Construct a new Node object, called `n`, containing the value
  - ▶ Change the `next` reference in `n` so that it points to `first`
  - ▶ Change the `previous` reference in `first` so that it points to `n`
  - ▶ Change the `first` reference so that it points to `n`
  - ▶ Increment the size
- What happens if the list is empty?

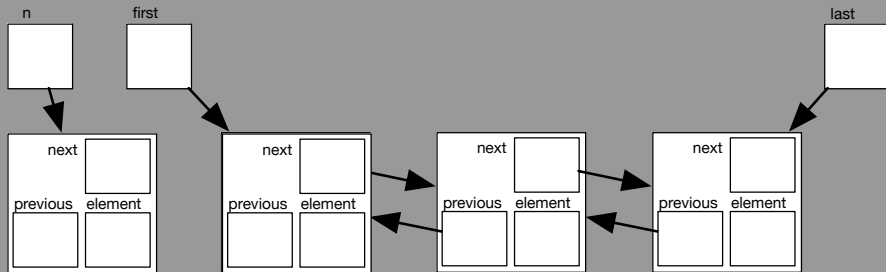
# Doubly-Linked List Operations

insertFirst(d)



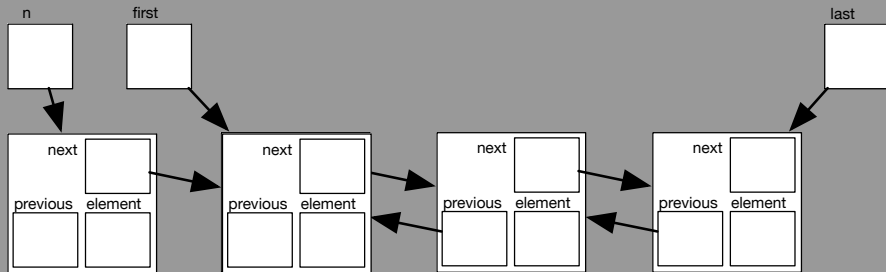
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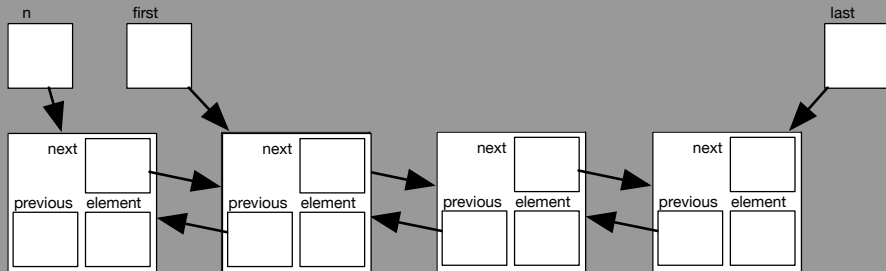
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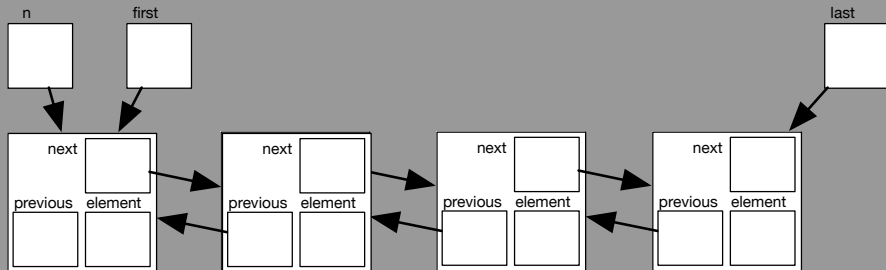
# Doubly-Linked List Operations

insertFirst(d)



# Doubly-Linked List Operations

insertFirst(d)



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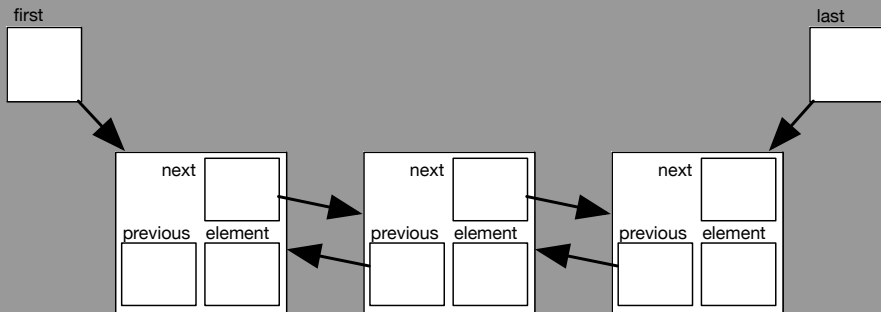
insertLast(d)

- Insert the value into the last position
  - ▶ Construct a new Node object, called `n`, containing the value
  - ▶ Change the `previous` reference of `n` so it points to `last`
  - ▶ Change the `next` reference of `last` so that it points to `n`
  - ▶ Change the `last` reference so that it points to `n`
  - ▶ Increment the size
- What happens if the list is empty?



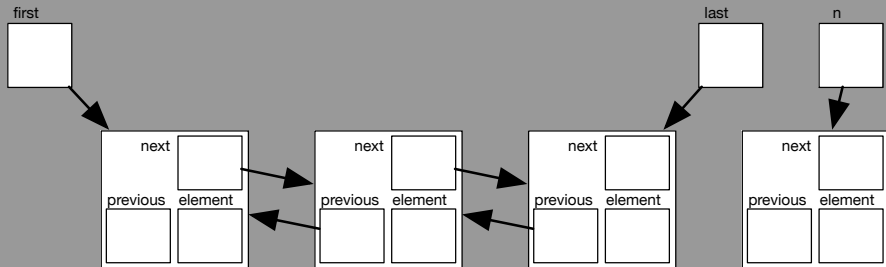
# Doubly-Linked List Operations

insertLast(d)



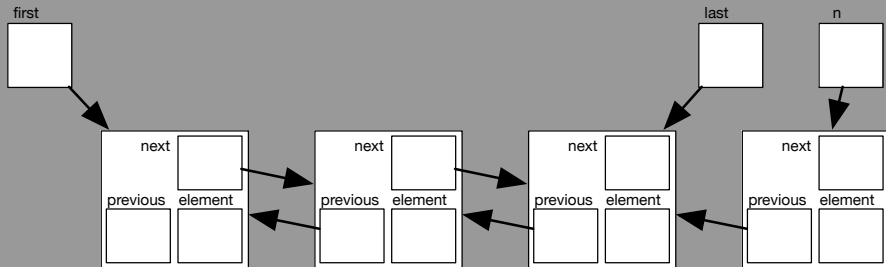
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insertLast(d)



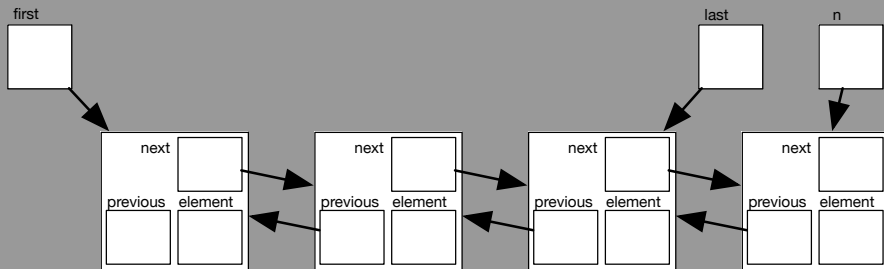
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insertLast(d)



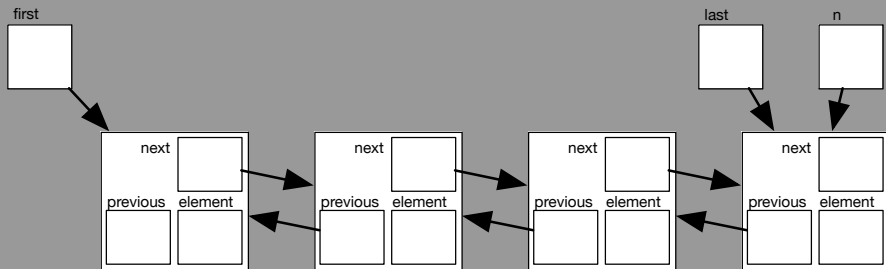
# Doubly-Linked List Operations

insertLast(d)



# Doubly-Linked List Operations

insertLast(d)



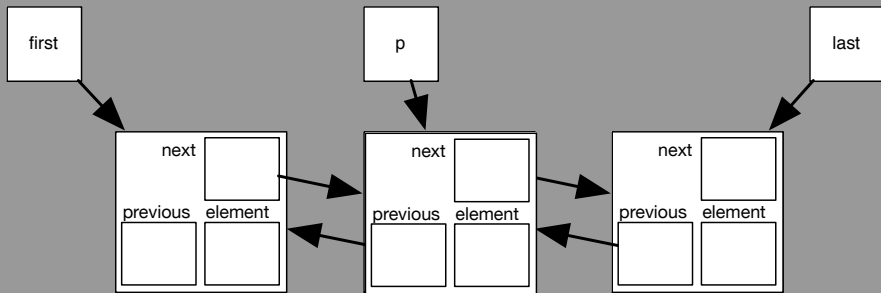
# Doubly-Linked List Operations

insertAfter(p, d)

- Insert the value into the position after p
  - ▶ Construct a new Node object, called n, containing the value
  - ▶ Convert p to a Node
  - ▶ Change next reference in n so it points to the next of p
  - ▶ Change previous reference in n so it points to p
  - ▶ Change previous reference in the next of p so it points to n
  - ▶ Change the next of p so it points to n
  - ▶ Increment the size
- What happens if p is in the last position?

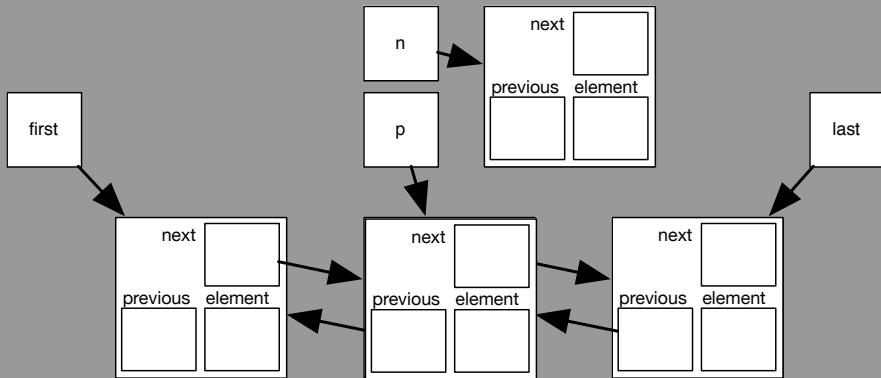
# Doubly-Linked List Operations

`insertAfter(p, d)`



# Doubly-Linked List Operations

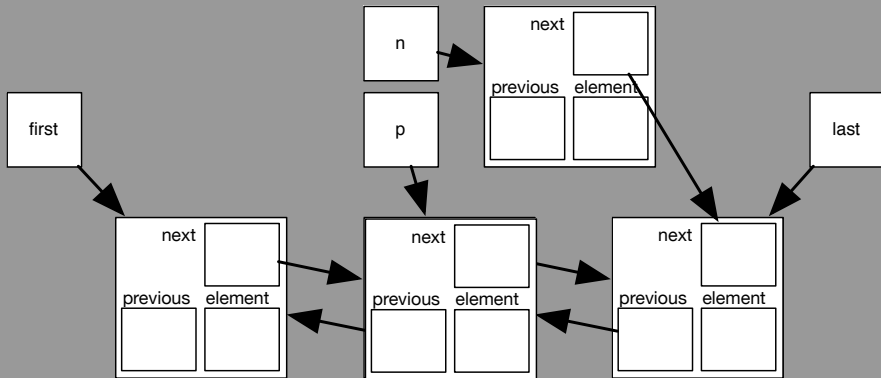
insertAfter(p, d)





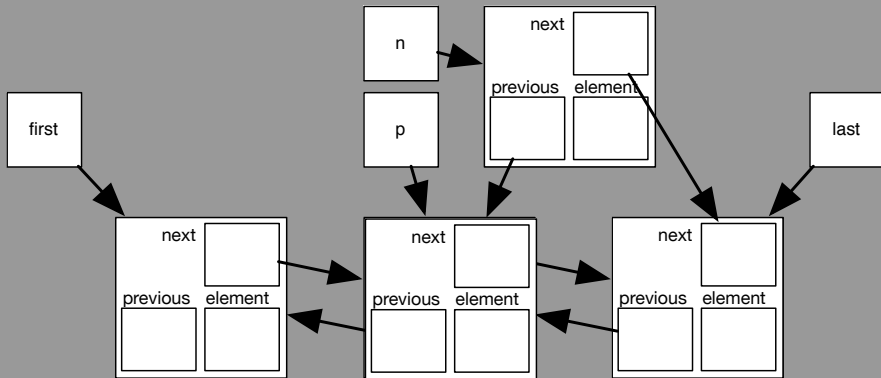
# Doubly-Linked List Operations

insertAfter(p, d)



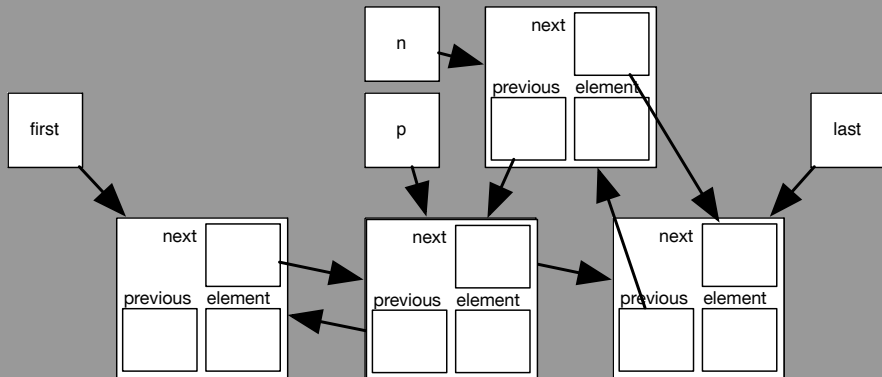
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insertAfter(p, d)



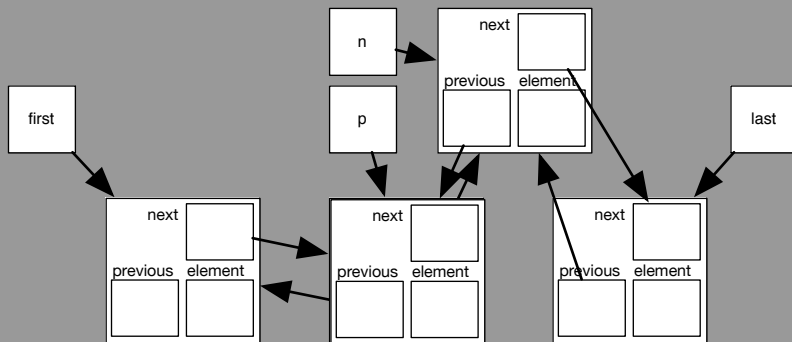
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insertAfter(p, d)



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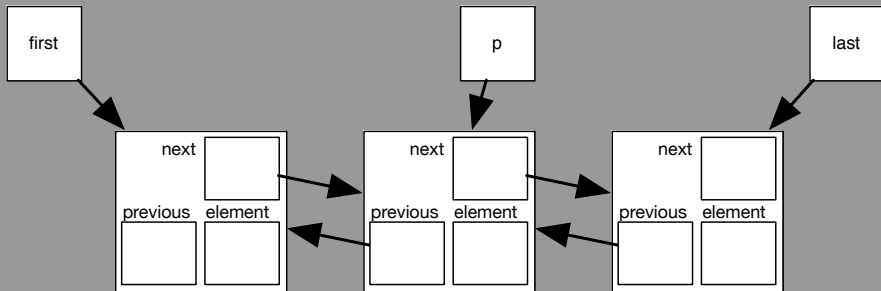
# Doubly-Linked List Operations

`insertBefore(p, d)`

- Insert the value into the position before p
  - ▶ Construct a new Node object, called n, containing the value
  - ▶ Convert p to a Node
  - ▶ Change previous reference in n so it points to the previous of p
  - ▶ Change next reference in n so it points to p
  - ▶ Change next reference in the previous of p so it points to n
  - ▶ Change the previous of p so it points to n
  - ▶ Increment the size
- What happens if p is the first position?

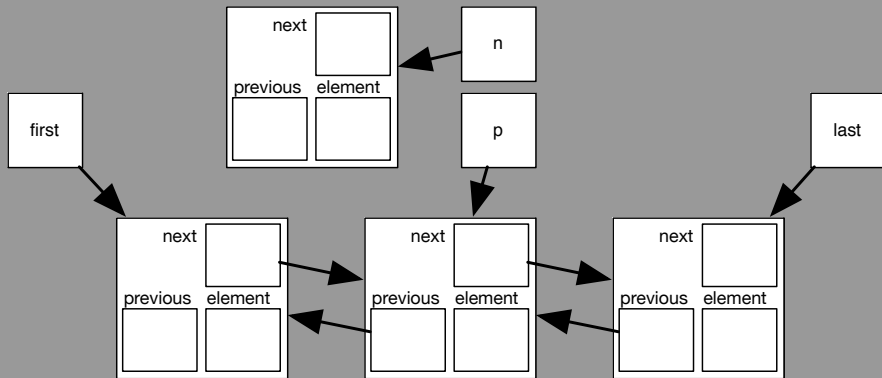
# Doubly-Linked List Operations

insertBefore(p, d)



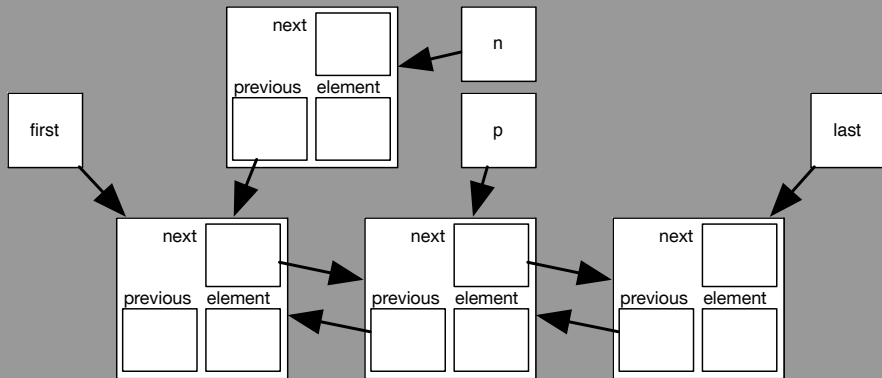
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insertBefore(p, d)



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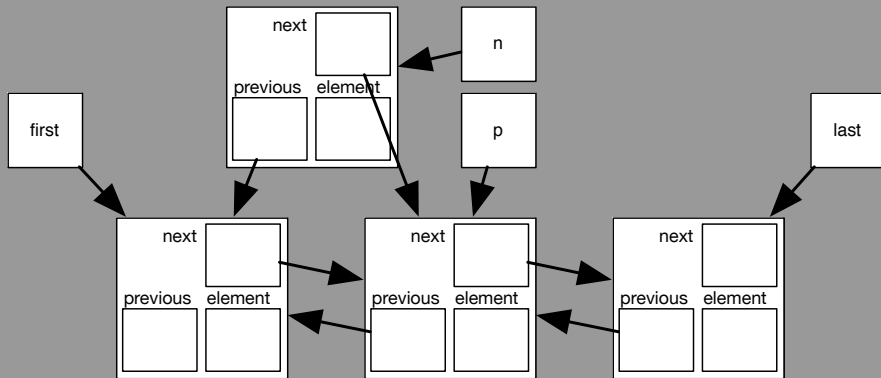
insertBefore(p, d)





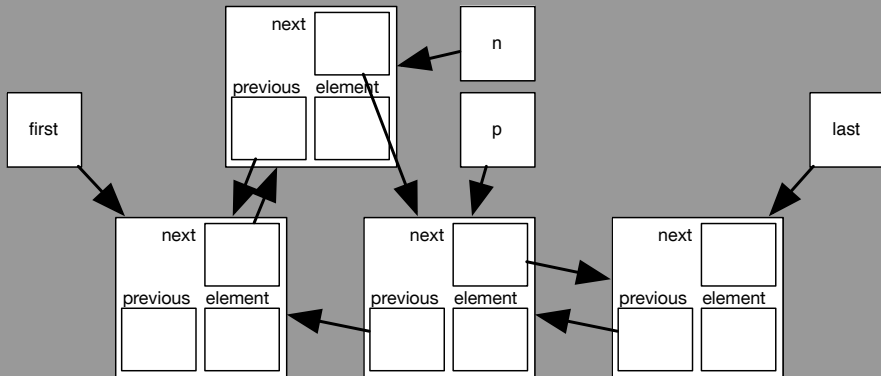
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insertBefore(p, d)



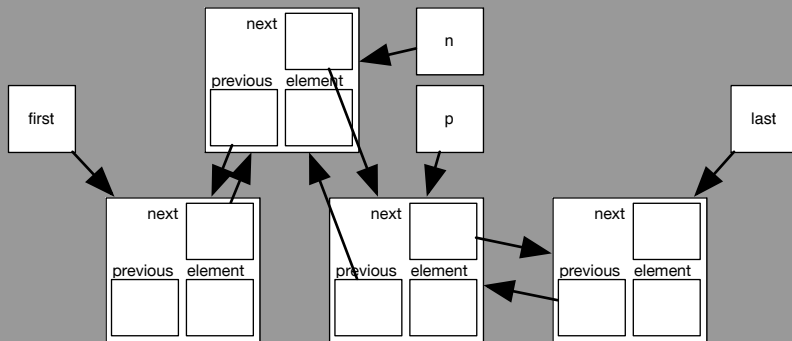
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insertBefore(p, d)



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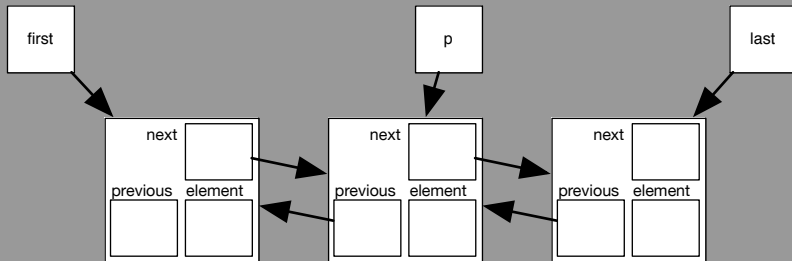
# Doubly-Linked List Operations

`remove(p)`

- Remove the object `p` from the list
  - ▶ Copy the data from inside the position `p` to variable `d`
  - ▶ Convert `p` to a Node
  - ▶ Change `next` of `previous` of `p` so that it points to the `next` of `p`
  - ▶ Change `previous` of `next` of `p` so that it points to the `previous` of `p`
  - ▶ Decrement the size
  - ▶ return `d`
- What happens if `p` is the first position?
- What happens if `p` is the last position?
- What happens if `p` is the only element?

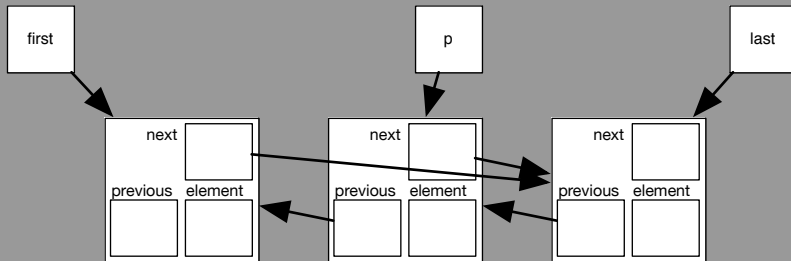
# Doubly-Linked List Operations

remove(p)



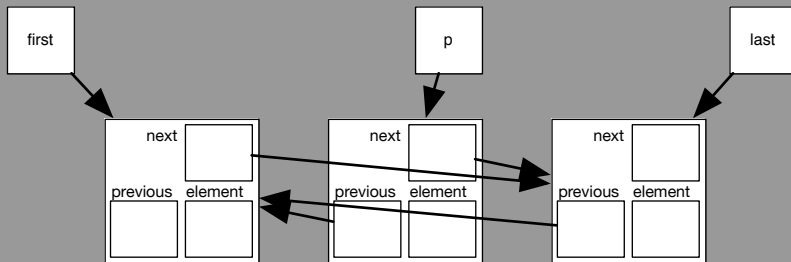
# Doubly-Linked List Operations

remove(p)



# Doubly-Linked List Operations

remove(p)



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# Algorithmic Complexity

- `first()`
  - ▶  $O(1)$
- `size()`
  - ▶  $O(1)$
- `isEmpty()`
  - ▶  $O(1)$
- `after(p)`
  - ▶  $O(1)$
- `last()`
  - ▶  $O(1)$

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# Comparing Complexity of Different Implementations

Operation	Array-Based	Singly-Linked	Doubly-Linked
first()	$O(1)$	$O(1)$	$O(1)$
last()	$O(1)$	$O(n)$	$O(1)$
size()	$O(1)$	$O(1)$	$O(1)$
isEmpty()	$O(1)$	$O(1)$	$O(1)$
after()	$O(1)$	$O(1)$	$O(1)$
before()	$O(1)$	$O(n)$	$O(1)$
insertFirst()	$O(n)$	$O(1)$	$O(1)$
insertLast()	$O(1)$	$O(n)$	$O(1)$
insertBefore()	$O(n)$	$O(n)$	$O(1)$
insertAfter()	$O(n)$	$O(1)$	$O(1)$
remove()	$O(n)$	$O(n)$	$O(1)$