IN THE CHAPTERS ON SEARCHING BY ELIMINATION I APPEALED TO SOME THEOREMS. SOME OF YOU HAVE ASKED TO SEE PROOFS.

HERE GOES

- $[\neg P \Rightarrow (P \Rightarrow Q)]$

PROOF.

$$\neg P \Rightarrow (P \Rightarrow Q)$$
$$= \quad \{\text{defn } \Rightarrow \; [P \Rightarrow Q \equiv \neg P \vee Q]\}$$
$$\neg P \Rightarrow (\neg P \vee Q)$$
$$= \quad \{\text{defn } \Rightarrow \text{ AGAIN}\}$$
$$\neg\neg P \vee (\neg P \vee Q)$$
$$= \quad \{\neg\neg\}$$
$$P \vee (\neg P \vee Q)$$
$$= \quad \{\vee \text{ ASSOCIATIVE}\}$$
$$(P \vee \neg P) \vee Q$$
$$= \quad \{\text{EXCLUDED MIDDLE } [P \vee \neg P \equiv \text{true}]\}$$
$$\text{true} \vee Q$$
$$= \quad \{\text{IDENTITY of } \vee\}$$
$$\text{true}$$

- $[P \vee Q \Rightarrow Q \;\equiv\; P \Rightarrow Q]$

Proof

$P \vee Q \Rightarrow Q$

$=$ $\{$defn $\Rightarrow\}$

$\neg(P \vee Q) \vee Q$

$=$ $\{$de MORGAN $\neg(P \vee Q) \equiv \neg P \wedge \neg Q\}$

$(\neg P \wedge \neg Q) \vee Q$

$=$ $\{\vee / \wedge\}$

$(\neg P \vee Q) \wedge (\neg Q \vee Q)$

$=$ $\{$EXCLUDED MIDDLE$\}$

$(\neg P \vee Q) \wedge$ TRUE

$=$ $\{$IDENTITY $\wedge\}$

$\neg P \vee Q$

$=$ $\{$defn $\Rightarrow\}$

$P \Rightarrow Q$

- $[(P \Rightarrow Q) \Rightarrow (P \Rightarrow (Q \vee R))]$

Proof.

$P \Rightarrow (Q \vee R)$

$=$ $\{$defn $\Rightarrow\}$

$\neg P \vee (Q \vee R)$

$=$ $\{\vee$ ASSOCIATIVE$\}$

$(\neg P \vee Q) \vee R$

$\Leftarrow$ $\{[X \Rightarrow X \vee Y]\}$

$\neg P \vee Q$

$=$ {defn $\Rightarrow$}

$P \Rightarrow Q$

If you have any questions please ask.