Numeral Systems and Base Conversion

Background

What is a numeral system?

A numeral system is any of **various sets of symbols and the rules** for using them to represent numbers, which are used to express how many objects are in a given set.

What is a number base?

When it became necessary to count frequently to numbers larger than 10 or so, the numeration had to be systematized and simplified; this was commonly done through use of a group unit or **base**, just as might be done today counting 43 eggs as three dozen and seven.

Example numeral bases

Base 10: The special position occupied by base 10, from the number of human fingers, of course, and it is still evident in modern usage not only in the logical structure of the decimal number system but in the English names for the numbers. Thus, eleven comes from Old English endleofan, literally meaning “[ten and] one left [over],” and twelve from twelf, meaning “two left”; the endings -teen and -ty both refer to ten, and hundred comes originally from a pre-Greek term meaning “ten times [ten].”

Base 2: Binary is a number system that builds numbers from elements called bits. Each bit can be represented by any two mutually exclusive states. Generally, when we write it down or code bits, we represent them with 1 and 0. We also talk about them being true and false, and the computer internally represents bits with high and low voltages.

Base 8: The octal numeral system, or oct for short, is the base-8 number system, and uses the digits 0 to 7. Octal is used as a shorthand for representing file permissions on UNIX systems. For example, file mode `rwxr-xr-x` would be 0755 base 8.

Base 16: **Hexadecimal** (also base 16, or hex) is a positional numeral system with a base of 16. It uses sixteen distinct symbols, most often the symbols 0-9 to represent values zero to nine, and A, B, C, D, E, F to represent values ten to fifteen. It is nowadays used for representing IPv6 addresses, or a networking hardware MAC address.
To avoid confusion while using different numeral systems, the base of each individual number may be specified by writing it as a subscript of the number. For example, the binary number 10011100 may be specified as "base two" by writing it as 10011100_2. The decimal number 197 may be written as 197_{10} and read as "one hundred ninety- seven, base ten".

Since the binary system is the internal language of electronic computers, serious computer programmers should understand how to convert from decimal to binary, octal, and hexadecimal.

Decimal to Binary Conversion

We will describe one method for converting from binary to decimal. The method is known as the division by two with remainder. A step-by-step description of the method is provided below, for the conversion of 197_{10} to base 2:

1. Write the decimal number (e.g. “197”) as the dividend and to its right draw a “L” with its horizontal base longer, so it can accommodate a few digits, like in the example below.

2. Inside the “L” write the base of the destination system (in our case, “2” for binary) as the divisor.

3. Take the first digit on the left-hand-side of the dividend, which is “1” and divide it by the divisor.

4. If the integer answer (quotient) of the division is “0” we consider also the next digit of the dividend, which is “9”, thus obtaining “19”.

5. The quotient of “19” divided by “2” is “9”, so we write the “9” under the L’s base.

6. Multiply “2” (our desired base) with the obtained “9” and we get “18”. Write the “18” directly under the “19” of the dividend.

7. Pad “18” to its right with as many zeros (“0”s) needed to make it a number of the length of the dividend. The dividend “197” has 3 digits, so we need to pad “18” with a single “0” to make it a 3 digit number.

8. Subtract the obtained “180” from “197” and you get “17”.

9. Use “17” in the same way we used “197”. That is, take its first digit, “1”, and divide by “2”. If the quotient is “0” take also the next digit, thus obtaining “17”.

10. Divide “17” to divisor “2”, and you get “8” as a quotient.

11. Write the quotient “8” under the L’s base to the right of the already existing “9”.

12. Multiply “2” (our desired base) with “8” and we get “16”. Write the

“16” directly under the “17”.

13. Subtract “16” from “17” and we get a “1”.

14. If the result of the subtraction gets smaller than “2” (our desired base), than that is the remainder of the division, and we make sure to highlight it.

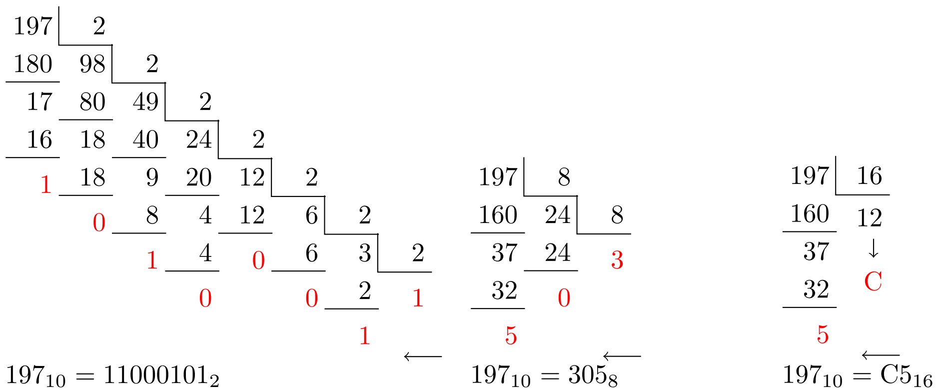
15. We take “98” as the new dividend.

16. Repeat the division by “2” in the same manner as presented above.

17. Stop dividing when the new dividend becomes smaller than “2” (our desired base).

18. Starting with the last new dividend smaller than “2”, “1” in our case, and read the sequence of remainders from right to left. That is 11000101, which is 197 base 10 converted to base 2.

Examples



Note: This method can be modified to convert from decimal to any base. The divisor is 2 because the desired destination is base 2. If the desired destination is a different base, replace the 2 in the method with the desired base. For example, if the desired destination is base 8, replace the 2 with 8. The final result will then be in the desired base. Below are two examples of how to convert 197 base 10, to base 8 and to base 16.

Binary to Decimal Conversion

We will describe one method for converting from binary to decimal, called the Positional Notation method. We convert this time the number 11000101 base 2, to base 10:

1. Write the number to be converted (11000101) leaving some space between the digits.

2. List below each digit its position in the number, starting from right to left. Start the numbering from "0".

3. Write below each digit's position, the base "2" to the power of the digit's position multiplied by the digit's value.

4. Add the numbers from step 3 and it will evaluate as the number value in base 10.

Example

Original number:	11000101							
Original base:	2							
Digits of the number to be converted:	1	1	0	0	0	1	0	1
Digit's Position:	7	6	5	4	3	2	1	0
(Original Base)$^{\text{(Digit's Position)}}$ * Digit:	2^7*1	2^6*1	2^5*0	2^4*0	2^3*0	2^2*1	2^1*0	2^0*1
The number converted to base 10 is:	$128 + 64 + 0 + 0 + 0 + 4 + 0 + 1 = 197_{10}$							

Note: This method can be used to convert a number in any base to base 10, by using that number's actual digits and replacing "2" with the base in which the number is given. Below are two examples showing how to convert 305_8 and $C5_{16}$ to base 10.

Original number:	305		
Original base:	8		
Digits of the number to be converted:	3	0	5
Digit's Position:	2	1	0
(Original Base)$^{\text{(Digit's Position)}}$ * Digit:	8^2*3	8^1*0	8^0*5
The number converted to base 10 is:	$192 + 0 + 5 = 197_{10}$		

Conversion from binary to octal or hexadecimal, and reverse

We saw in the previous examples that $197_{10} = 11000101_2 = 305_8 = C5_{16}$.

The conversion form **binary to octal** is done by:

1. Grouping digits into groups of three, starting from right to left, and we will obtain something like this: 11 000 101.

2. Each group is converted to octal, like this: 112=310=38 and 0002=010=08 and 1012=510=58.

3. The obtained values are concatenated, and we obtain 305 base 8. The conversion from binary to hexadecimal is done by:

Grouping the digits into groups of **four** from right to left, and we will obtain something like this: 1100 0101.
Each group is converted to hexadecimal, like this: $1100_2=12_{10}=C_{16}$ and $0101_2=5_{10}=5_{16}$.
We concatenate these, and obtain C5 base 16.

The reverse:

Octal to binary: take each digit of the octal number, convert it into base 10, and then convert that number in base 2, making sure to pad each group to the left with zeroes (“0”s) up to three digits. Concatenate the obtained binary numbers and you will obtain the initial number converted to binary. For example: 3058 38=310=0112 and 08=010=0002 and 58=510=1012 Put these together and you’ll obtain: 3058 = 0110001012. Notice the first 0 form the left can be ignored as it does not change the value of the number, thus 3058 = 0110001012 = 110001012.

Hexadecimal to binary: take each digit of the hexadecimal number, convert it into base 10, and then convert that number in base 2, making sure to pad to the left with zeroes (“0”s) up to four digits. Concatenate the obtained binary numbers and you will obtain the initial number converted to binary. For example: C516 C16=1210=11002 and 516=510=01012 Put these together and you’ll obtain: C516 = 110001012.