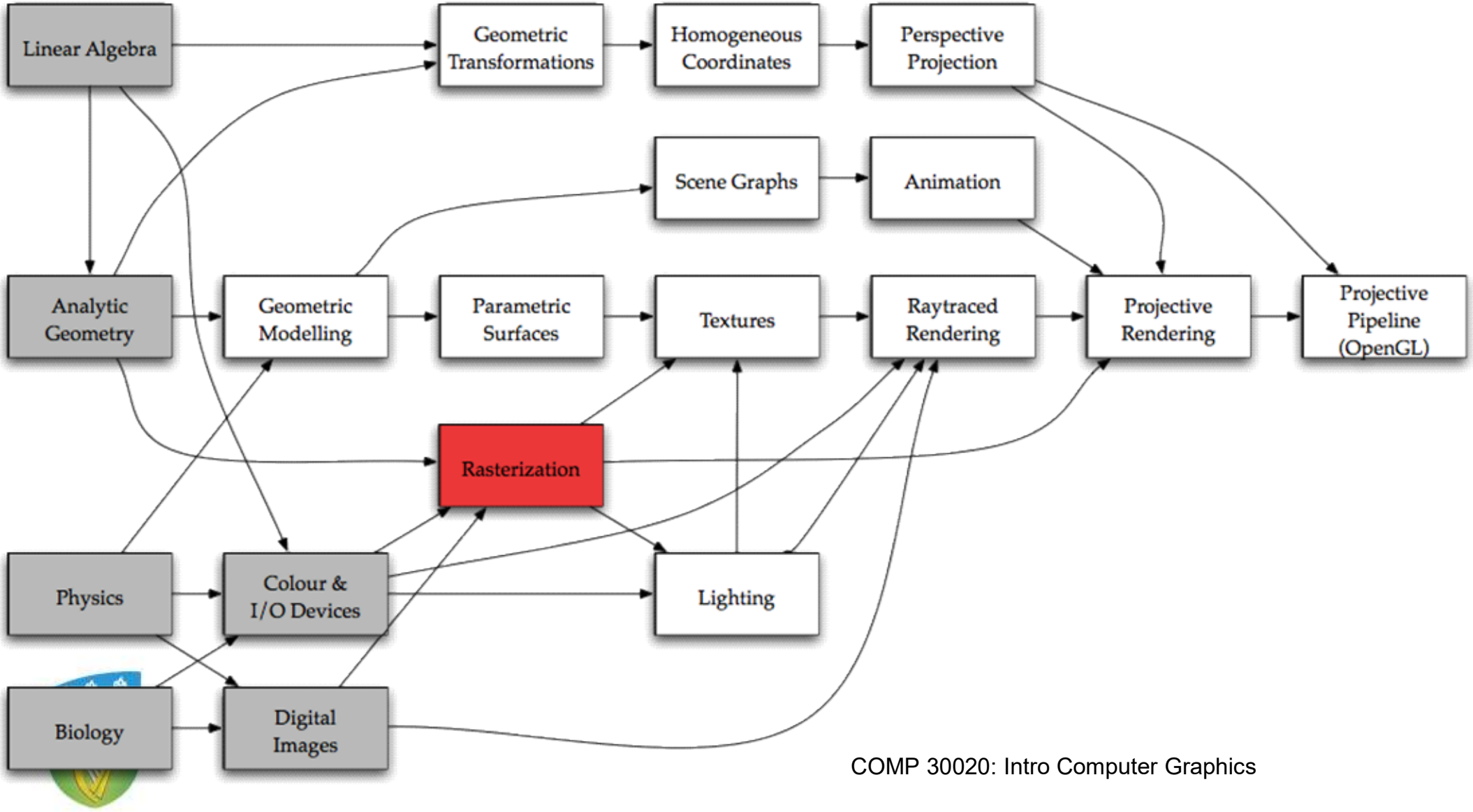
# Rasterizing Lines & Triangles

# Where We Are


Linear Algebra
Geometric Transformations
Homogeneous Coordinates
Perspective Projection
Scene Graphs
Animation
Analytic Geometry
Geometric Modelling
Parametric Surfaces
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Projective Rendering
Projective Pipeline (OpenGL)
Rasterization
Physics
Colour & I/O Devices
Lighting
Biology
Digital Images

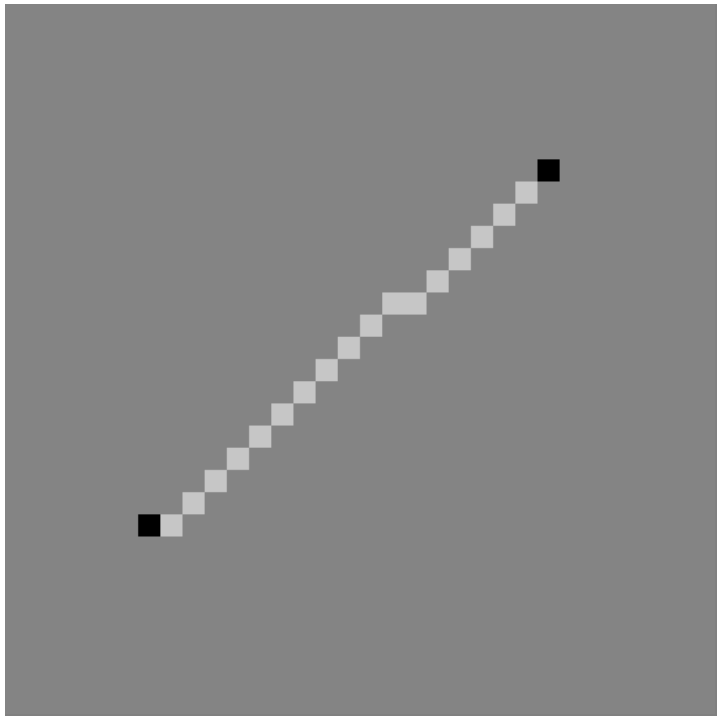
# Drawing A Line

- We want to draw a line (segment)
  - from $(x_0, y_0)$ to $(x_1, y_1)$
  - algorithms exist for:
    - explicit form
    - implicit / normal form
    - parametric form

# Explicit Form

- Use equation of line to connect points:

```
for (x = x0; x < x1; x++)
        {
        y = mx + c;
        setPixel(x, y);
        }
```



# Problems

- Doesn't work for slope > 1
  - Solution: use x = ny+d instead
- Doesn't work when $x_1 < x_0$ or $y_1 < y_0$
  - Solution: use descending loop
- These add complexity to the algorithm

# Bresenham's Algorithm

```
rise = y1 - y0; run = x1 - x0;
if (run > 0)
    if (rise > 0)
        if (run > rise)
            for (x = x0; x < x1; x++)   {
                y = (rise/run) x + c;
                setPixel(x, y);         }
        else
            for (y = y0; y < y1; y++)   {
                x = (run/rise) y + b;
                setPixel(x, y);         }
    else
        if (run > -rise)
            for (x = x0; x < x1; x++)   {
                y = (rise/run) x + c;
                setPixel(x, y);         }
        else
            for (y = y0; y < y1; y++)   {
                x = (run/rise) y + b;
                setPixel(x, y);         }
&c., &c., &c.
```

# Implicit Form

- We want to draw a line 1 pixel wide
  - all pixels within 0.5 pixels of line
  - we know how to measure distance
- But this draws a line, not a segment

```
for (x = xMin; x < xMax; x++)
    for (y = yMin; y < yMax; y++)
        if (abs(distance((x, y), (x0, y0), (x1, y1))) < 0.5)
            setPixel(x, y);
```

# Parametric Form

- This is the easiest one yet!
- Walk along the line one step at a time:

```
for (t = 0.0; t <= 1.0; t += 0.001)
    {
    point_r = point_p + (point_q - point_p) * t;
    setPixel(round(r_x), round(r_y));
    }
```

# Comparison

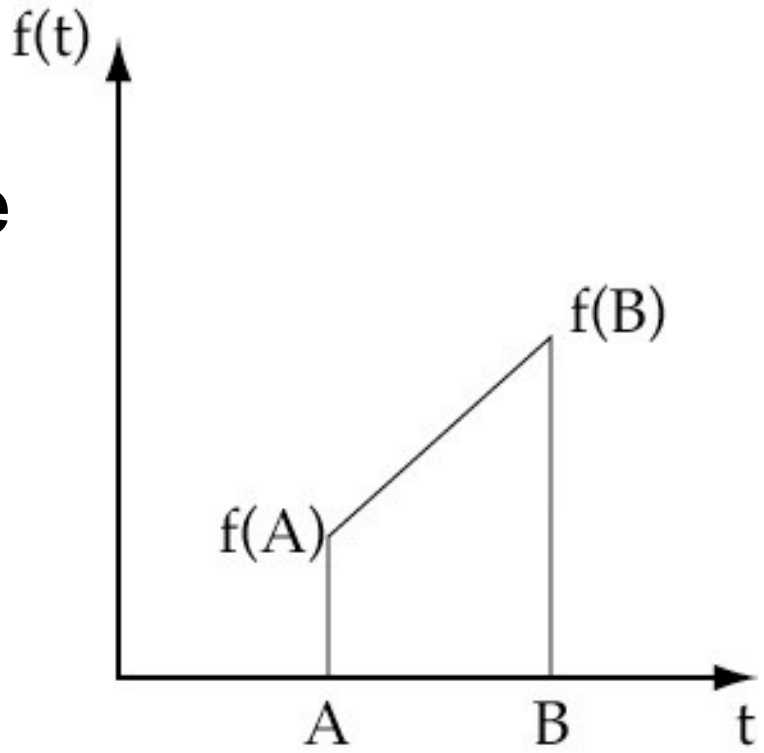
- Explicit: conceptually easy, but messy
- Implicit: easy, but lines not segments
- Parametric: easy to code
- Which is most efficient?

# Colour Interpolation

- What if we want a coloured gradient?
  - At p, the line is 100% red, 0% blue
  - At q, the line is 0% red, 100% blue
  - In between, it varies smoothly
- This process is called interpolation

# Linear Interpolation

- Assume parametric line
- Let f(t) be the colour
  - we use a straight line
  - f changes linearly:




$$f(t) = f(A) + \left(\frac{t - A}{B - A}\right)\left(f(B) - f(A)\right)$$

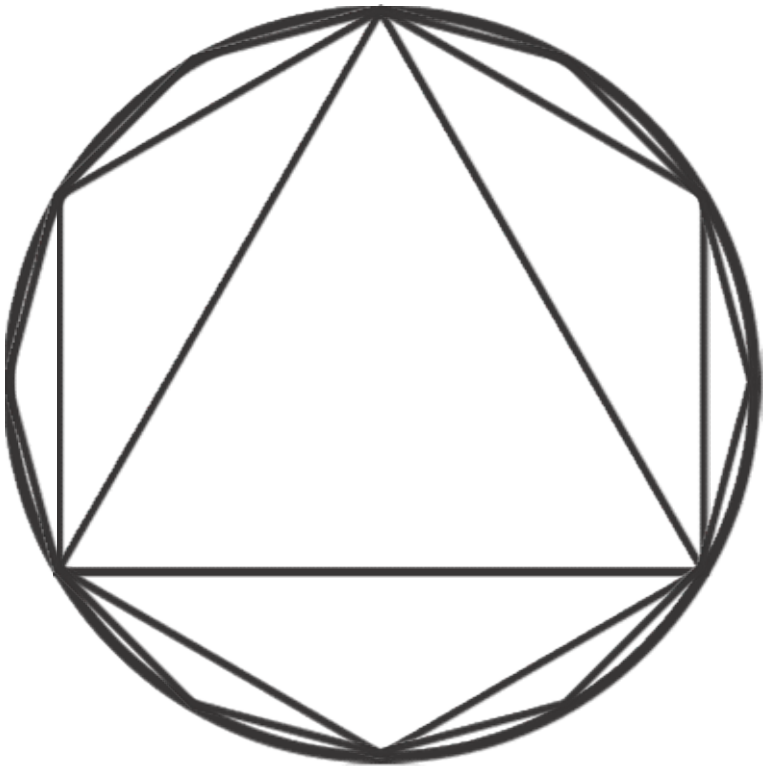
# Interpolating Colour

- Easiest in parametric form:

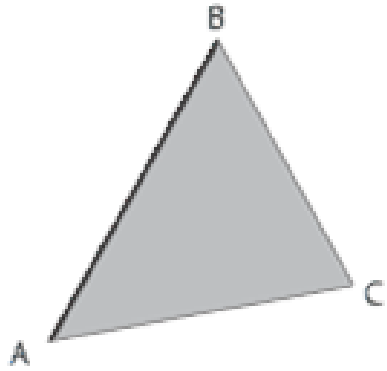
```
for (t = 0.0; t <= 1.0; t += 0.001)
    {
    point_r = point_p + (point_q - point_p) * t;
    colour = colour_p + (colour_q - colour_p) * t;
    setColour(colour);
    setPixel(round(r_x), round(r_y));
    }
```

# Lines & Curves

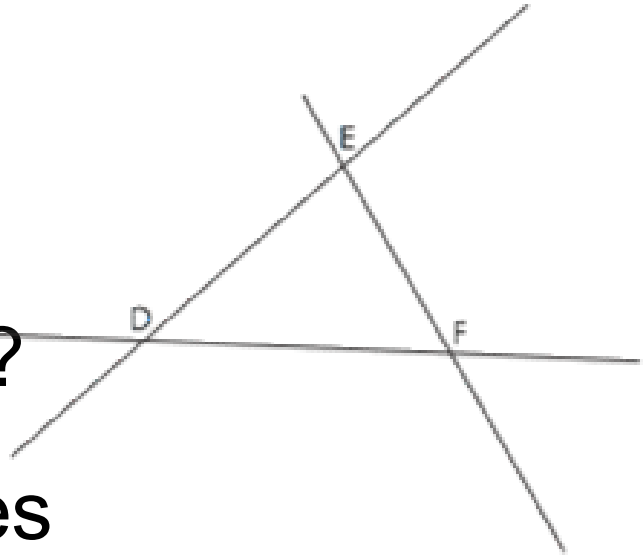
- We approximate curves
  - with many short lines
- Not always the best way!
  - we'll come back to this



# Triangles




- Defined by 3 points:
  - Or by 3 lines
- Drawing three lines is easy
- But what about filled triangles?
- Start with equations of triangles



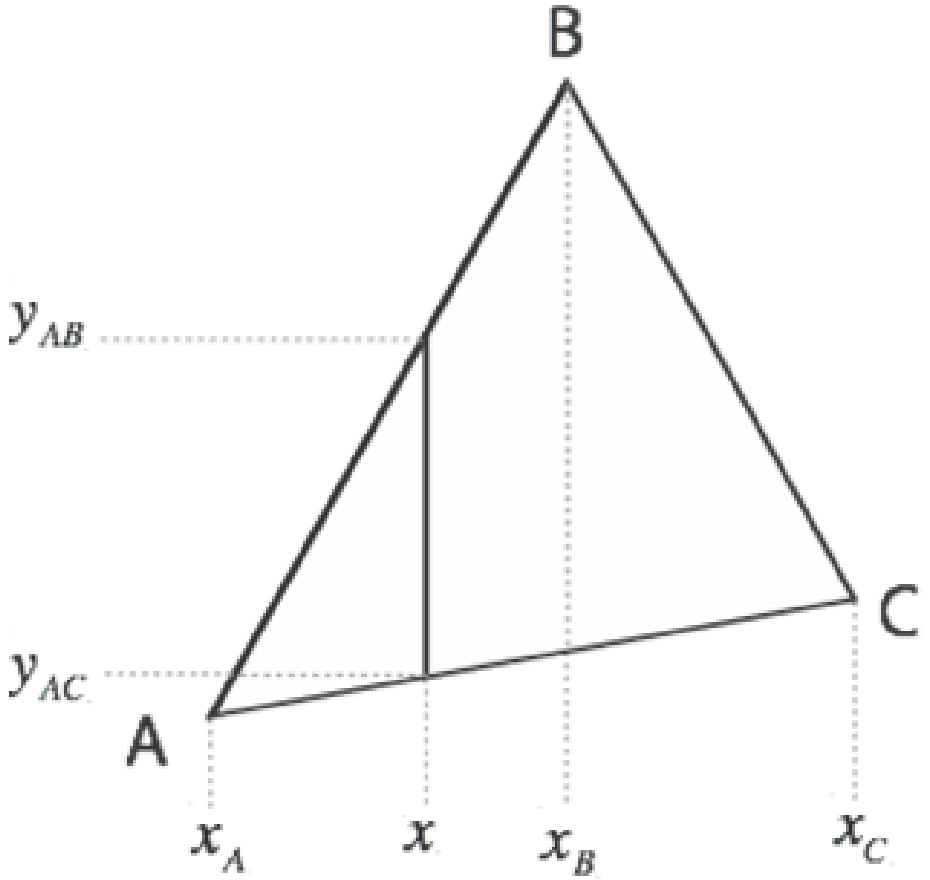
# Explicit Form

- For any x, specify valid y

$$y_{AC} \le y \le y_{AB} \quad if \quad x_A \le x \le x_B$$

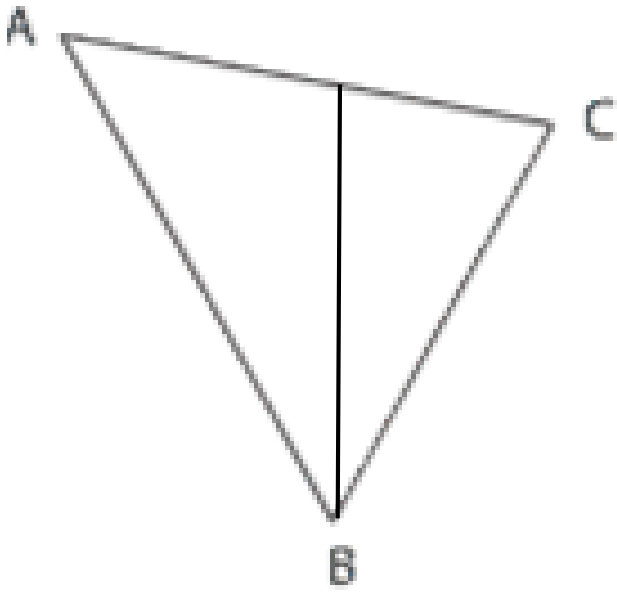
$$y_{AC} \le y \le y_{BC} \quad if \quad x_B \le x \le x_C$$

- Assumes B is above AC



# Explicit Form, II

- If B is below AC:

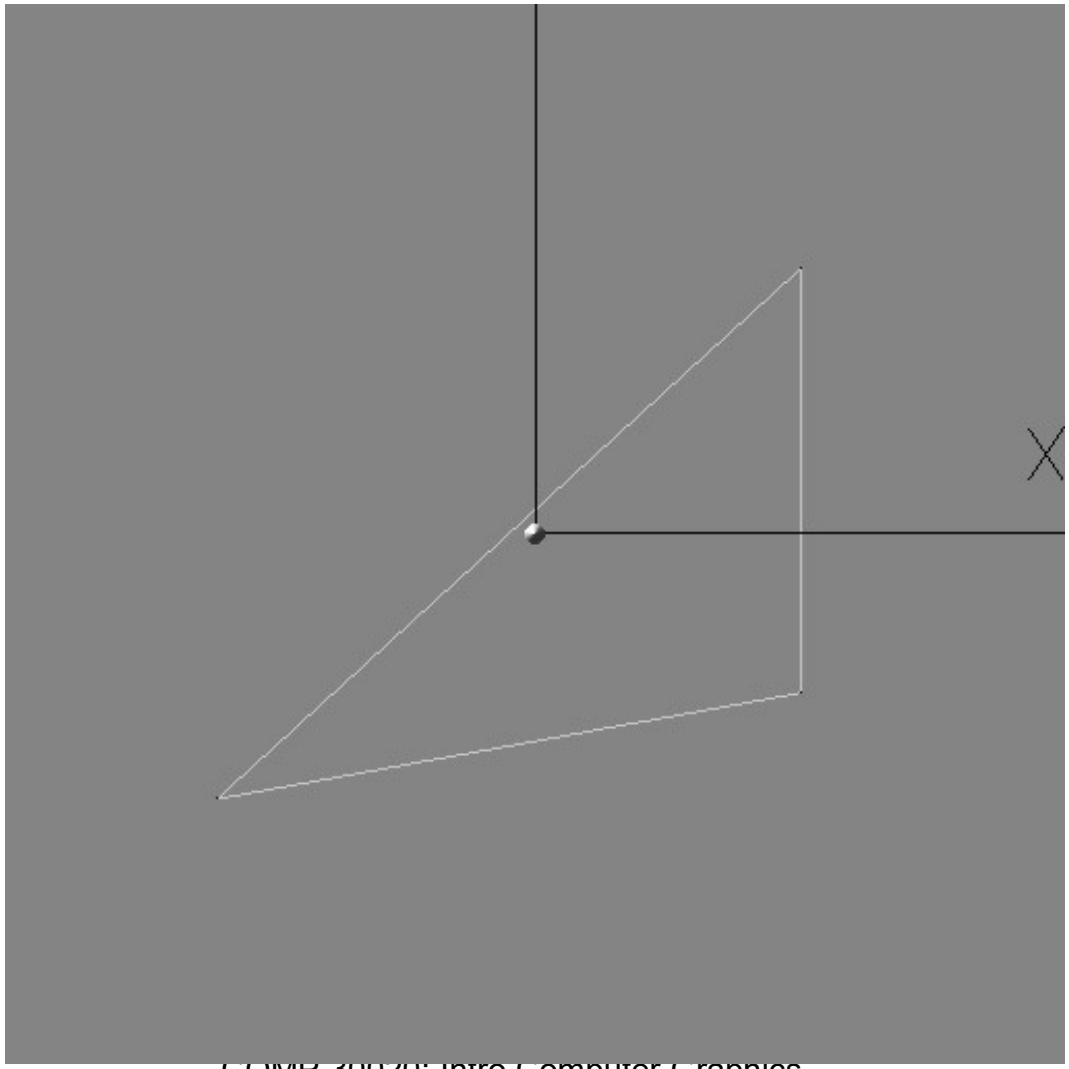



$$y_{AB} \le y \le y_{AC} \quad if \; x_A \le x \le x_B$$

$$y_{BC} \le y \le y_{AC} \quad if \; x_B \le x \le x_C$$

# Raster Scan Algorithm

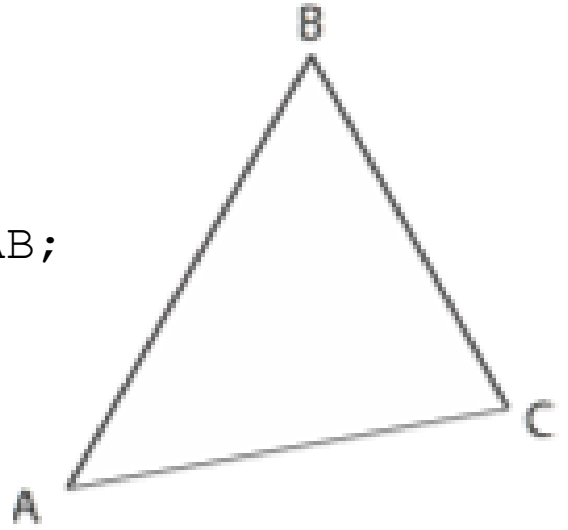
- Algorithm scans one line at a time
  - raster scan (raster means a rake)
  - scan conversion of triangles to pixels



# Explicit Algorithm

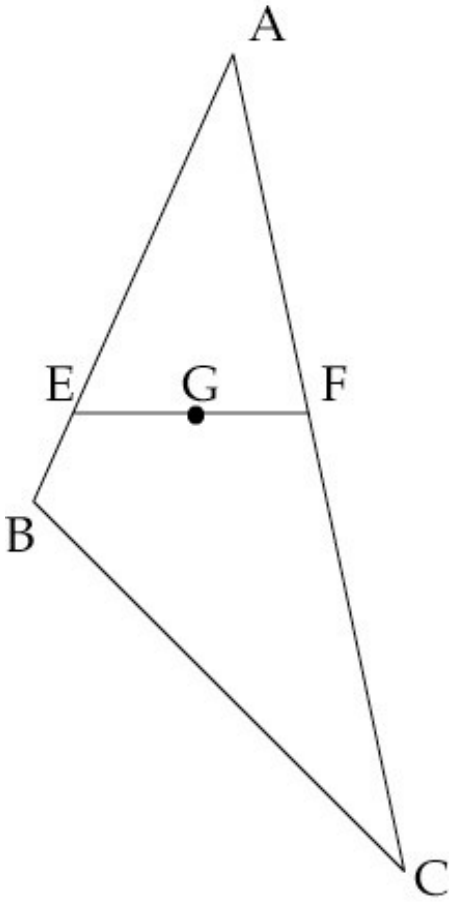
- Also called linewise scan
- Usually loops horizontally, not vertically

```
Sort A, B, C so Ax < Bx < Cx
Find slopes mAB, mAC, mBC,
Find y-intercepts cAB, cAC, cBC
for (x = Ax; x <= Bx; x++)
  { // for each column
  yMin = mAC * x + cAC; yMax = mAB * x + cAB;
  if (yMin < yMax)
    swap(yMin, yMax);
  for (y = yMin; y <= yMax; y++)
      setPixel(x,y);
} // for each column
```



# Linewise Interpolation

- To compute f(G):
  - Interpolate f(E) from f(A), f(B)
  - Interpolate f(F) from f(A), f(C)
  - Interpolate f(G) from f(E), f(F)
- Perform for each of R,G,B



# Implicit / Normal Form

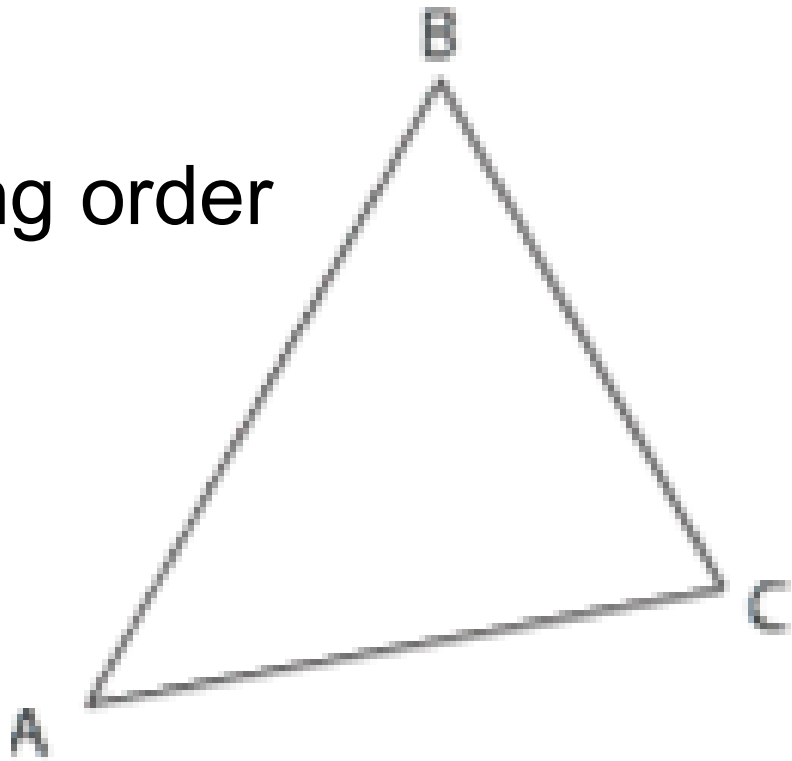
- Based on normal form of lines:

$$\vec{n} \cdot p - c = \begin{cases} - & \text{to } \textit{left} \text{ of line} \\ 0 & \textit{on} \text{ line} \\ + & \text{to } \textit{right} \text{ of line} \end{cases}$$

- Also known as the half-plane test

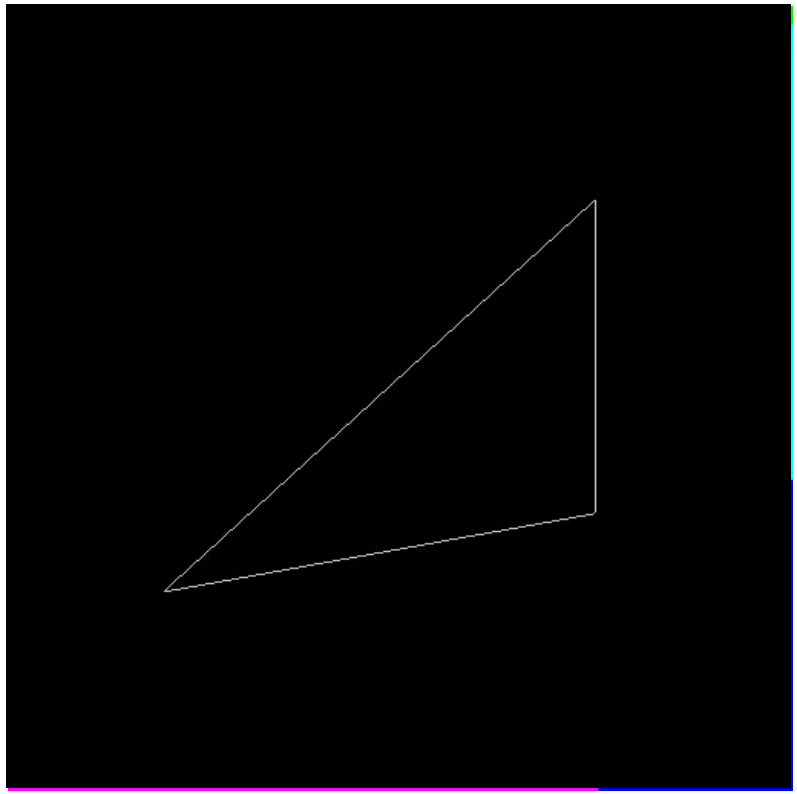
# Winding Order

- Inside depends on the winding order
  - which direction we wind
  - ABC is clockwise (CW)
    - inside on right
  - ACB is counterclockwise (CCW)
    - inside on left



# Half-Plane Test

- Each test divides plane in half:
  - Red vs. Not-Red
  - Green vs. Not-Green
  - Blue vs. Not-Blue
- Triangle is inside each



# Implicit Algorithm

- Assume CCW winding order (left is inside)

```
for (x = xMin; x < xMax; x++)
    for (y = yMin; y < yMax; y++)
        if (  (x,y) leftOf (A,B) &&
              (x,y) leftOf (B,C) &&
              (x,y) leftOf (C,A))
            setPixel(x,y);
```

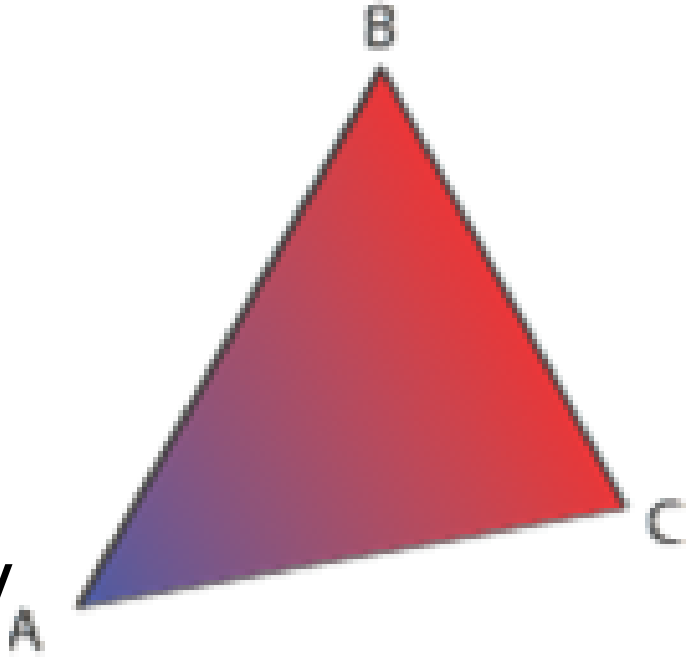
- But what about colour interpolation?
  - As with lines, we need parametric form

# Parametric Form

- For a line pq, t = 0.0 at p, t = 1.0 at q
- How can we parameterize a triangle?
- We need at least two parameters
  - Start with one parameter
  - Use it to interpolate colour as well

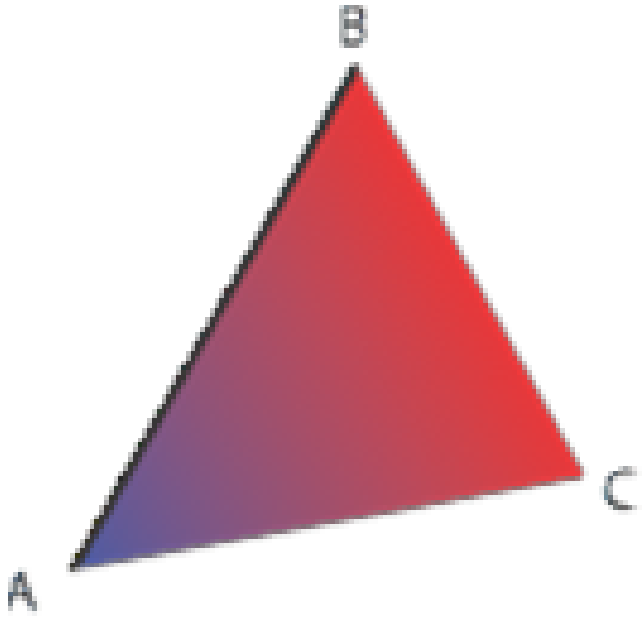
# Triangle Interpolation

- Pick a vertex A
  - Set 100% blue at A
  - Set 0% blue at CB
- In between, varies linearly
  - perpendicular to CB



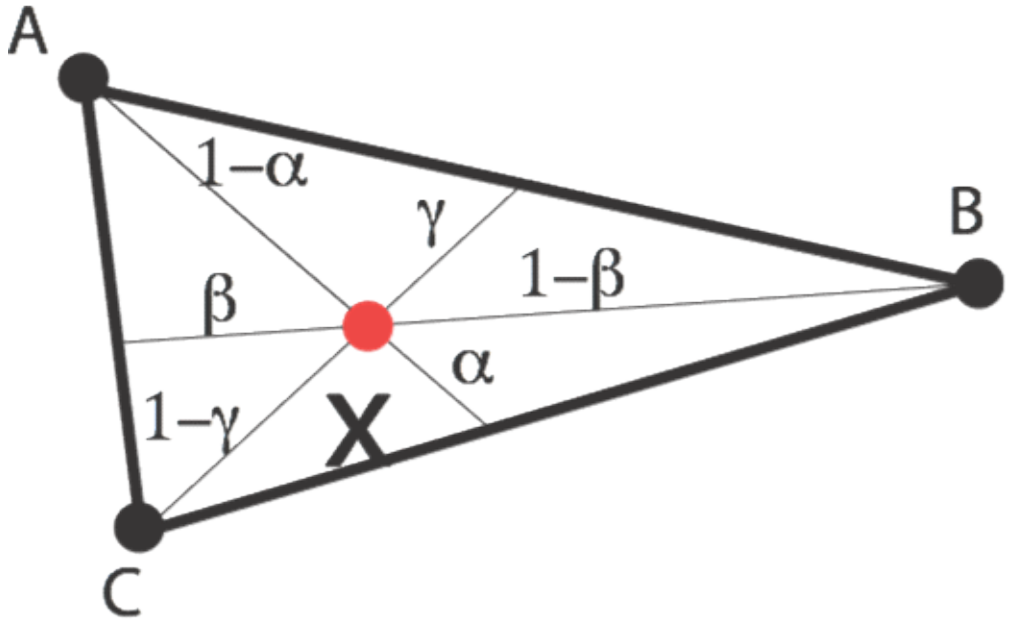
# The Parameter α

- Colour depends on distance from CB
- Call this distance α
  - Parametrize so that:
    - α = 1.0 at A
    - α = 0.0 at BC




$$\alpha = \frac{dist(P,CB)}{dist(A,CB)}$$

# Do it Three Times




$$f(x) = \alpha f(A) + \beta f(B) + \gamma f(C)$$

$$\alpha + \beta + \gamma = 1$$

$$\alpha = \frac{dist(P, CB)}{dist(A, CB)}$$

$$\beta = \frac{dist(P, AC)}{dist(B, AC)}$$

$$\gamma = \frac{dist(P, BA)}{dist(C, BA)}$$

# Barycentric Coordinates

- α,β,γ  are called barycentric coordinates
- Conveniently, $\alpha + \beta + \gamma = 1.0$
- So we really only have two parameters
- But we have three weights
  - This lets us interpolate from three vertices
    - to get colour, normals, textures, &c.

# Parametric Algorithm

```
for (x = xMin; x < xMax; x++)
    for (y = yMin; y < yMax; y++)
        alpha = distance((x,y), BC) / distance(A, BC);
       gamma = distance((x,y), AB) / distance(C, AB);
        beta = 1 - alpha - gamma;
if ((alpha < 0.0) || (beta < 0.0) || (gamma < 0.0))
            continue;
        colour = alpha * colour(A) + beta * colour(B)
                  + gamma * colour(C);
        setColour(colour);
        setPixel(x,y);
```

# Comparison

- Which is best?
- Explicit form is easiest to understand
- Half-plane (implicit) is easiest to code
- Barycentric (parametric) also computes weights for colour interpolation