Chapter 28 : The Co-incidence count.

Given f[0..M), g[0..N) of int, where both f and g are increasing, we are asked to establish the following postcondition

Post : $r = \langle + i,j : 0 \le i < M \wedge 0 \le j < N : h.(f.i).(g.j)\rangle$

Where $h.x.y \quad = \quad 1 \quad \Leftarrow \; x = y$

$h.x.y \quad = \quad 0 \quad \Leftarrow \; x \ne y$

*(0) $h.x.y \quad = \quad 1 \quad \Leftarrow \; x = y$

*(1) $h.x.y \quad = \quad 0 \quad \Leftarrow \; x \ne y$

*(2) $C.m.n \quad = \quad \langle + i,j : m \le i < M \wedge n \le j < N : h.(f.i).(g.j)\rangle$

-(3) $C.M.n \quad = \quad 0$

-(4) $C.m.N \quad = \quad 0$

We observe,

$C.m.n$

= {(2)}

$\langle + i,j : m \le i < M \wedge n \le j < N : h.(f.i).(g.j)\rangle$

= { split off i = m term }

$\langle + i,j : m+1 \le i < M \wedge n \le j < N : h.(f.i).(g.j)\rangle + \langle + j : n \le j < N : h.(f.m).(g.j)\rangle$

= { (2) (7) }

$C.(m+1).n + D.n$

-(5) $C.m.n \quad = \quad C.(m+1).n + D.n$

We observe,

$$C.m.n$$

$=$ $\{(2)\}$

$$\langle + i,j : m \le i < M \wedge n \le j < N : h.(f.i).(g.j) \rangle$$

$=$ $\{$ split off $j = n$ term $\}$

$$\langle + i,j : m \le i < M \wedge n{+}1 \le j < N : h.(f.i).(g.j) \rangle + \langle + i : m \le i < M : h.(f.i).(g.n) \rangle$$

$=$ $\{ (2)\ (8) \}$

$$C.m.(n{+}1) + G.m$$

-(6) $C.m.n \quad = \quad C.m.(n{+}1) + G.m.n$

*(7) $D.n \quad = \quad \langle + j : n \le j < N : h.(f.m).(g.j) \rangle$

*(8) $G.m \quad = \quad \langle + i : m \le i < M : h.(f.i).(g.n) \rangle$

Now we use the increasing property to attempt to determine the values of D.m.n and G.m.n

-(9) $D.n \quad = \quad 0 \quad \Leftarrow \quad f.m < g.n$

-(10) $D.n \quad = \quad 1 \quad \Leftarrow \quad f.m = g.n$

-(11) $D.n \quad = \quad ? \quad \Leftarrow \quad f.m > g.n$

-(12) $G.m \quad = \quad ? \quad \Leftarrow \quad f.m < g.n$

-(13) $G.m \quad = \quad 1 \quad \Leftarrow \quad f.m = g.n$

-(14) $G.m \quad = \quad 0 \quad \Leftarrow \quad f.m > g.n$

This completes our model. Now we move on to constructing the program.

Post $r = C.0.0$

Invariants

P0: $r + C.m.n = C.0.0$

P1: $0 \le m \le M \wedge 0 \le n \le N$

Establish invariants

$r,m.n := 0.0.0$

Termination.

We note that at the end

$P0 \wedge P1 \wedge (m = M \vee n = N) \Rightarrow Post$

Guard

$m \ne M \wedge n \ne N$

Loop body

We observe

P0

= {definition P0}

$r + C.m.n = C.0.0$

= {(5)}

$r + C.(m+1).n + D.n = C.0.0$

= {case analysis, $f.m < g.n$ (9) }

$r + C.(m+1).n + 0 = C.0.0$

= {WP}

$(m := m+1).P0$

So if $f.m < g.n$ -> $m := m + 1$

We observe

P0

= {definition P0}

r + C.m.n = C.0.0

= {(5}

r + C.(m+1).n + D.n = C.0.0

= {case analysis, f.m = g.n (10) }

r + C.(m+1).n + 1= C.0.0

= {WP}

(r,m := r+1, m+1).P0

So if f.m = g.n -> r,m := r+1, m+1

We observe

P0

= {definition P0}

r + C.m.n = C.0.0

= {(6)}

r + C.m.(n+1) + G.m = C.0.0

= {case analysis, f.m = g.n (13) }

r + C.m.(n+1) + 1 = C.0.0

= {WP}

(r, n := r+1, n+1).P0

So if f.m = g.n -> r,n := r+1, n+1

We observe

$$P0$$

$=$ {definition P0}

$$r + C.m.n = C.0.0$$

$=$ {(6)}

$$r + C.m.(n+1) + G.m = C.0.0$$

$=$ {case analysis, $f.m > g.n$ (14) }

$$r + C.m.(n+1) + 0 = C.0.0$$

$=$ {WP}

$$(n := n+1).P0$$

So if $f.m > g.n \rightarrow n := n+1$

Finished Algorithm.

```
 r,m.n := 0.0.0
; do m ≠ M ∧ n ≠ N ->
      if f.m < g.n -> m := m+1
      [] f.m = g.n -> r,m := r+1,m+1
      [] f.m = g.n -> r,n := r+1, n+1
      [] f.m > g.n -> n := n+1
      fi
 od
{r = C.0.0}
```