

Chapter 37 : The starting pit problem. (Problem statement only)

In which we tackle a difficult problem.

There are N pits located along a circular race track. They are numbered $1..N$. At pit i there are $p.i$ litres of fuel available. To race from pit i to its clockwise neighbour we require $q.i$ litres of fuel. We are asked to find a pit from which it is possible to race a complete lap starting with an empty fuel tank.

To guarantee the existence of such a pit we are given

$$* (0) \langle + i : 1 \leq i \leq N : p.i \rangle = \langle + i : 1 \leq i \leq N : q.i \rangle$$

We introduce some notation.

$$* (1) D.i.j = \langle + k : i \leq k < j : p.k - q.k \rangle$$

This is the difference between the number of litres available and the number of litres required when racing from pit i to pit j .¹

Here are a few properties of D

$$- (2) D.i.k = D.i.j + D.j.k \quad , i, j, k \in \{1..N\}$$

$$- (3) D.i.i = 0$$

$$- (4) D.i.j + D.j.i = 0$$

NOW, CAN YOU TRY TO FINISH THIS?????????

¹ As the race track is circular we can have $D.2.1$ which is of course $D.2.N + D.N.1$. We will not complicate our notation by introducing modular arithmetic.