Chapter 2

Newton's Laws

Instructor：Liu Fengyan

Office：数理楼3303

Tel：67392201-202

Email: Liufengyan@bjut.edu.cn

Isaac Newton (1643-1727), an English physicist, mathematician, astronomer and natural philosopher. His "Mathematical Principles of Natural Philosophy", published

in 1687, is one of the most important scientific books ever written. In this work, Newton described universal gravitation and the three laws of motion, which lays the groundwork for most of classical mechanics.

§ 2.1 Newton′s First Law

"Every body continues in its state of rest, or in uniform motion in a straight line unless it is compelled to change that state by forces impressed upon it."

The mathematical form of Newton′s first law is

$$\sum_i \vec{F}_i = 0 \quad \rightarrow \quad \vec{v} = \text{constant vector}$$

A net force of zero could mean no forces at all, or, in most cases, that the vector sum of forces is zero.

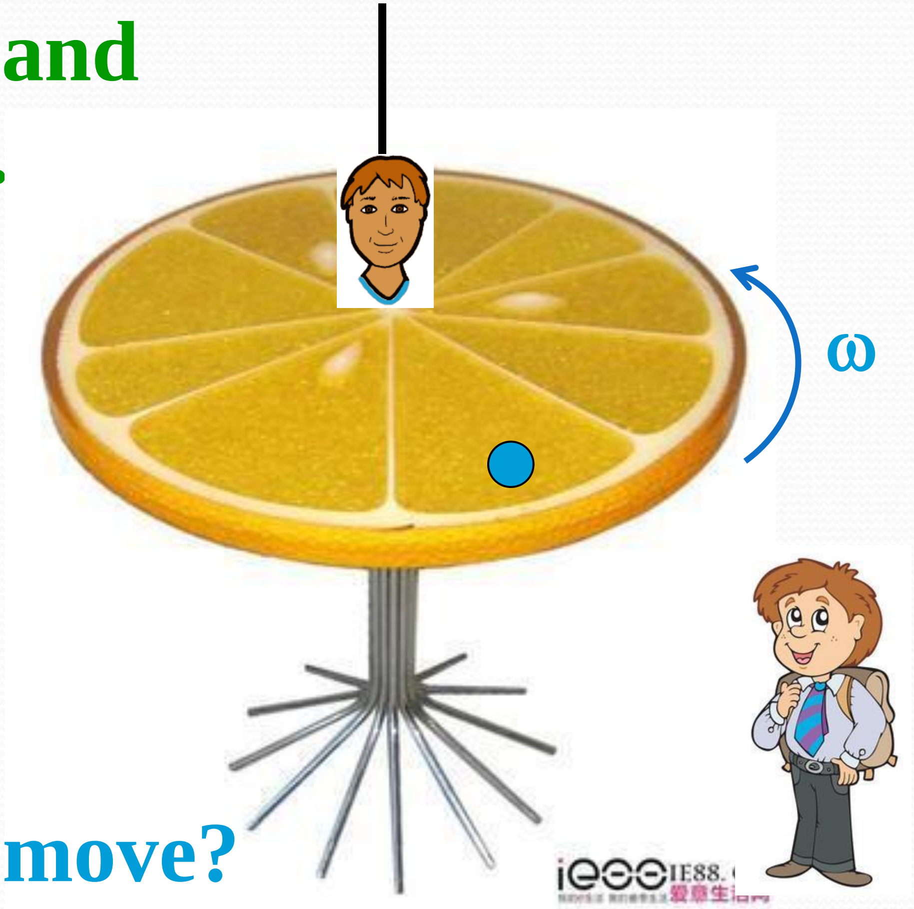
Note

◆ **Newton's first law is a definition about inertial frame of reference(IRF). A reference frame in which the first law is valid is called inertial system.**

◆ **An IRF is a reference frame that is not accelerating (or rotating) with respect to the "fixed stars".**

◆ **If one IRF exists, infinitely many exist since they are related by any arbitrary constant velocity vector!**

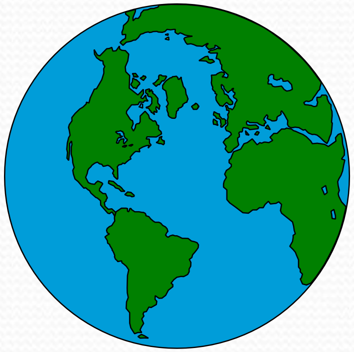
The table top is smooth and the friction is negligible.



What is the net force on the little ball?

How does the little ball move?

Is Beijing a good IRF?



- Is Beijing accelerating?
- YES!

  Beijing is on the Earth. The Earth is rotating.
- What is the centripetal acceleration of Beijing?

$$a_{\text{Beijing}} = \frac{v^2}{R} = \omega^2 R = \left(\frac{2\pi}{T}\right)^2 R$$

$$a_{\text{Beijing}} = \frac{v^2}{R} = \omega^2 R = \left(\frac{2\pi}{T}\right)^2 R$$

T = 1 day = 8.64 x 104 sec

R ~ R_E = 6.4 × 10^6 meters

Plug these in:

$$a_{\text{Beijing}} = 0.034\text{m/s}^2 \approx \frac{g}{300}$$

Close enough to *0* that we will ignore it.

Beijing is a pretty good IRF.

§ 2.2 Newton′s Second Law

The acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass.

$$\vec{a} = \frac{\vec{F}}{m}$$

vector sum of all forces acting on the object

inertial mass

Mass is herein defined as the measure of a body’s inertia. More correctly this is the definition for inertial mass. Or its resistance to a change in its motion.

More precisely, the Newton′s second Law can be stated as

"The change of motion is proportional to the motive force impressed, and is made in the direction of the right line in which that force is impressed."

$$\vec{F} = \frac{\mathrm{d}\vec{p}}{\mathrm{d}t} = \frac{\mathrm{d}(m\vec{v})}{\mathrm{d}t}$$

This expression is valid if the "mass" is also changing

Note

- **The 2nd Law provides a precise definition of the force exerted.**
- **Choosing a fundamental unit of mass now gives us a unit for force**

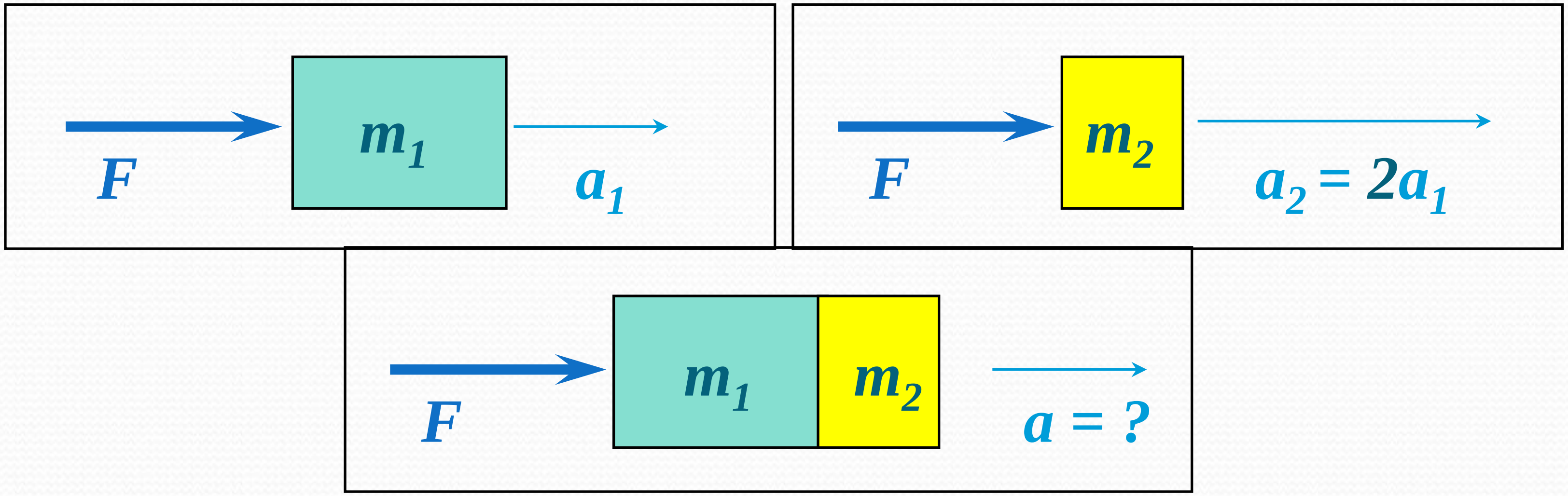
$$1\ \mathrm{N} = 1\ \mathrm{kg} \cdot \mathrm{m/s}^2$$

Dimension of force [ML / T²]

- **It tells us that some quantity varying like**

$$m_1 a_1 = m_2 a_2 = \cdots = m_n a_n$$

Example2.1 **A force F acting on a mass m_1 results in an a_1. The same force acting on a mass m_2 results in an $a_2 = 2a_1$. If m_1 and m_2 are glued together and the same force F acts on this combination, what is the resulting acceleration?**



(a) $2/3\ a_1$ **(b)** $3/2\ a_1$ **(c)** $3/4\ a_1$

Solution

$$a = \frac{F}{m_1 + m_2}$$

Since $a_2 = 2a_1$ for the same applied force,

$$m_2 = \frac{1}{2} m_1 \quad \text{and} \quad m_1 + m_2 = \frac{3}{2} m_1$$

So

$$a = \frac{F}{m_1 + m_2} = \frac{2}{3}\frac{F}{m_1} = \frac{2}{3} a_1$$

Example2.2 **A mass point m is moving along the $+x$ axis. The relationship between position x and velocity v is $v=kx^2$(k is a constant). Find the function of F and position x.**

Solution

$$F = m\frac{dv}{dt} = m\frac{dv}{dx}\cdot\frac{dx}{dt}$$

$$= m\cdot 2kx\cdot kx^2$$

$$= 2mk^2x^3$$

Example2.3 **Assume the radius of the Earth decreases by 1%， and the mass is invariant，the gravity acceleration on the surface of the Earth will increase by**

(A)1%　　(B)2%　　(C)3%　　(D)4%

Solution

$$g = \frac{GM}{R^2} \qquad \rightarrow dg = -\frac{2GM}{R^3} dR$$

$$\rightarrow \frac{dg}{g} = -2\frac{dR}{R} = -2 \cdot (-1\%) = 2\%$$

Example2.4 **A 0.5kg mass point is acted on by a force $\vec{F} = 3t\vec{i}\,\mathrm{N}$, At beginning, the initial position vector is $\vec{r}_0 = 0$, and the initial velocity is $\vec{v}_0 = 2\vec{j}\,\mathrm{m/s}$. What is the position vector $\vec{r}(t)$?**

Solution

Newton′s second law gives $$\frac{\mathrm{d}\vec{v}}{\mathrm{d}t} = \frac{\vec{F}}{m} = 6t\vec{i}$$

It can be written as $$\mathrm{d}\vec{v} = 6t\mathrm{d}t\vec{i}$$

Integrating two sides $$\int_{\vec{v}_0}^{\vec{v}} \mathrm{d}\vec{v} = \vec{i}\int_0^t 6t\mathrm{d}t$$

So $$\vec{v}=\frac{\mathrm{d}\vec{r}}{\mathrm{d}t}=3t^2\vec{i}+2\vec{j}$$

It can be written as $$\mathrm{d}\vec{r}=(3t^2\vec{i}+2\vec{j})\mathrm{d}t$$

Then， integrating two sides again

$$\int_{\vec{r}_0}^{\vec{r}}\mathrm{d}\vec{r}=\vec{i}\int_0^t 3t^2\mathrm{d}t+\vec{j}\int_0^t 2\mathrm{d}t$$

We obtain $$\vec{r}=(t^3\vec{i}+2t\vec{j})\mathrm{m}$$

§ 2.2.1 Some forces often seen

◆ **Contact force**

Friction:

Static friction acts between a surface and a stationary object on that surface to prevent motion. The magnitude of static friction is

$$f_s \le \mu_s N$$

Coefficient of static friction

It has a maximum value —if this is overcome, the object starts to move.

Kinetic (or sliding)friction acts between a surface and a moving object opposing the motion of the object.

$$f_k = \mu_k N$$

Coefficient of kinetic friction

Its roughly constant value is smaller than the maximum static friction

Elastic Force: $\boldsymbol{F= -kx}$

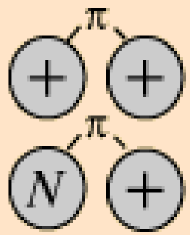
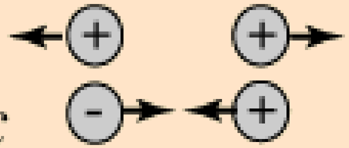
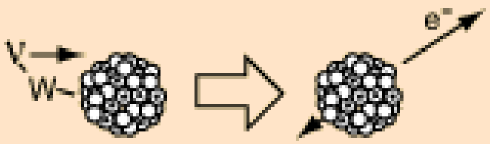
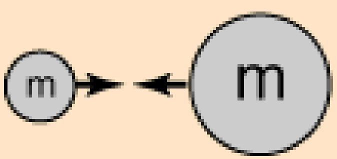
- **Action at a distance**

Universal gravitational force $F = G\dfrac{m_1 m_2}{r^2}$

Magnetic force

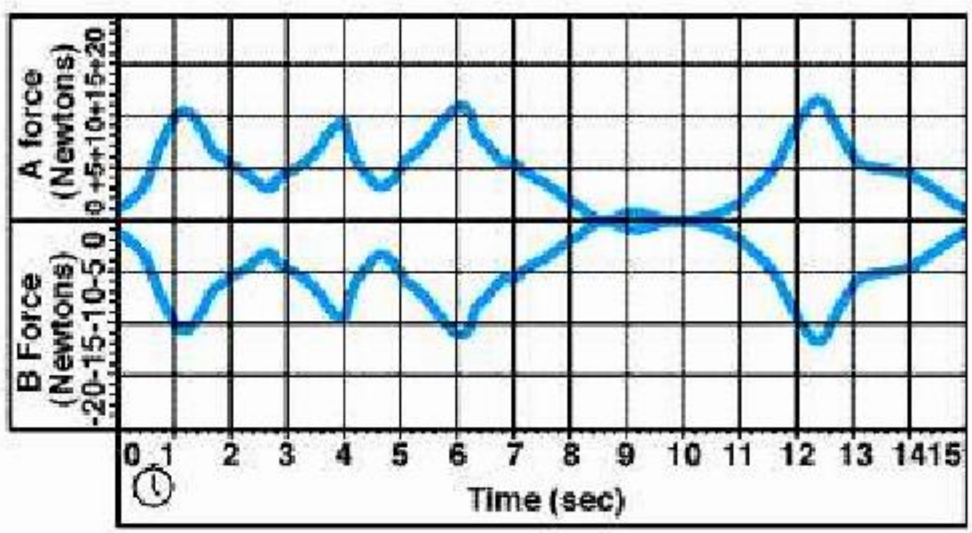
Electric force

§ 2.2.2 Fundamental Interaction

Interaction		Strength	Range (m)	Particle
Strong	 Force which holds nucleus togeher	1	10^{-15} (diameter of a medium sized nucleus)	gluons, π(nucleons)
Electro-magnetic		$\frac{1}{137}$	Infinite	photon mass = 0 spin = 1
Weak	 neutrino interaction induces beta decay	10^{-6}	10^{-18} (0.1% of the diameter of a proton)	Intermediate vector bosons W^+, W^-, Z_0, mass > 80 GeV spin =1
Gravity		6×10^{-39}	Infinite	graviton ? mass = 0 spin = 2

§ 2.3 Newton′s Third Law

The active force and the reactive force between two objects act upon each other, and the force are equal in magnitude, opposite in direction, and along the same straight line.



Example2.5 **Since $\boldsymbol{F}_{m,b} = -\boldsymbol{F}_{b,m}$, why isn't $\boldsymbol{F}_{net} = 0$ and $\boldsymbol{a} = 0$?**

Solution **Consider *only the box* as the system!**

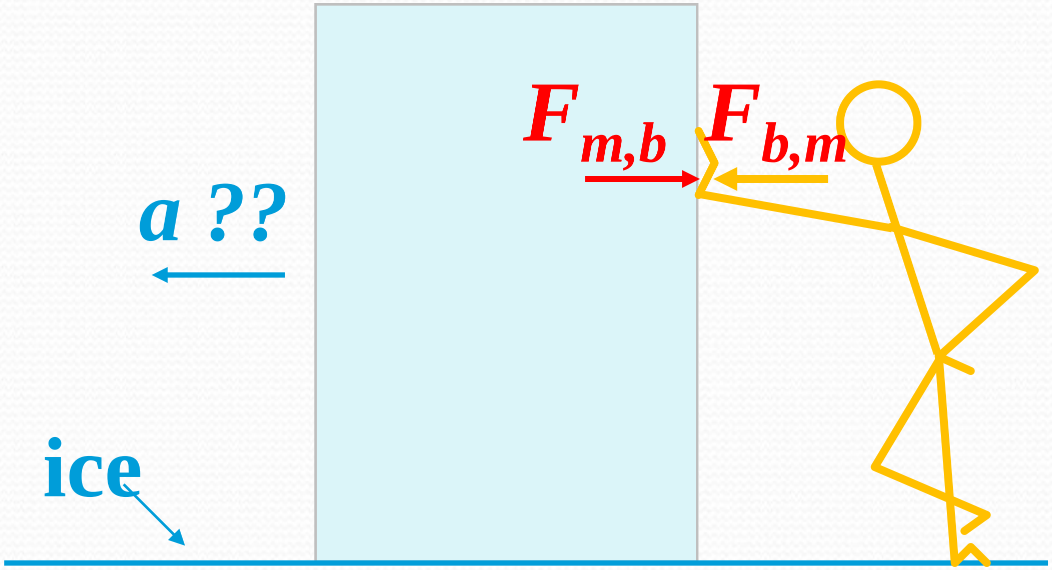
$$\boldsymbol{F}_{\text{on box}} = m\boldsymbol{a}_{box} = \boldsymbol{F}_{b,m}$$

So

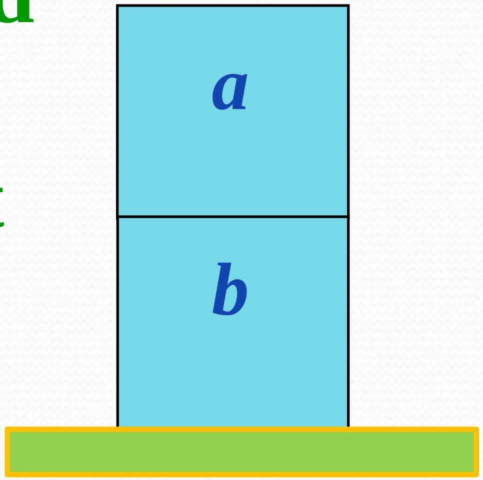
$$\boldsymbol{F}_{net} \neq 0$$

And

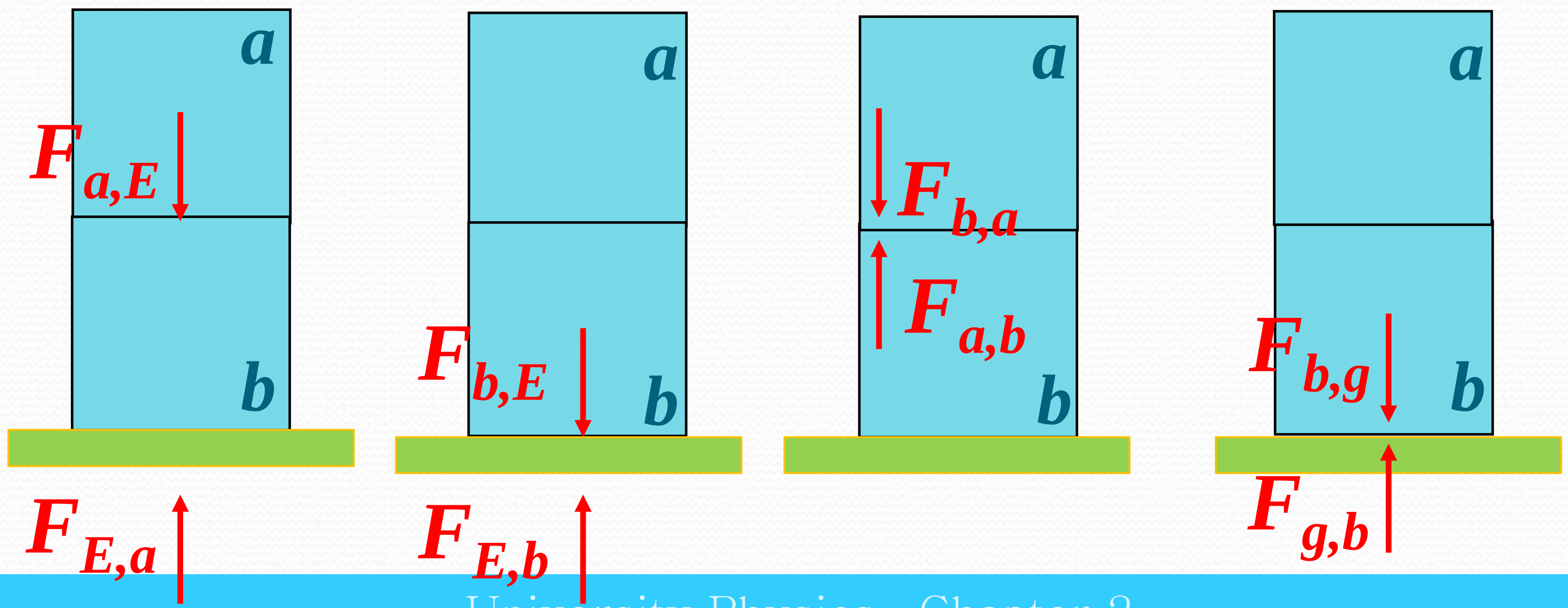
$$\boldsymbol{a} \neq 0$$



Example2.6 **Two blocks are stacked on the ground. How many action-reaction pairs of forces are present in this system?**



(a) 2 **(b) 3** **(c) 4**



§ 2.4 Non-inertial frames and inertia forces

Accelerating

A 

B 

B sees no motion of the stop sign. No force acting on it.

But A will see stop sign accelerating.

Interpretation 1:

I am accelerating. The stop sign only seems to be accelerating. There is actually no net force on it.

Interpretation 2:

I am at rest. There is some unusual force acting on the stop sign causing the stop sign to accelerate.

The latter view involves a inertia force.

$$F = -ma$$

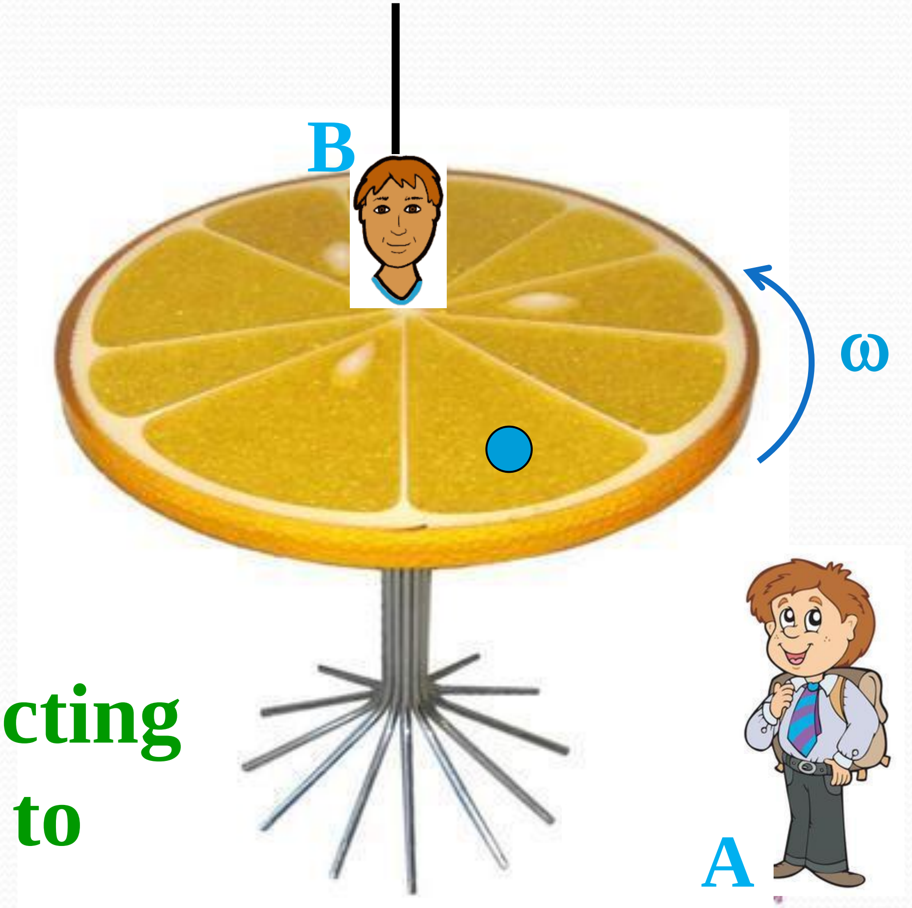
a **is the acceleration of the non-inertial frame.**

Interpretation A:

The ball is at rest. There is actually no net force on it.

Interpretation B:

I am at rest. There is some unusual force acting on the ball causing it to accelerate.



The inertia force is $$\vec{F}' = -m\omega^2\vec{r}$$

◆ **It is sometimes difficult to distinguish a real force from a inertia force for some distant "source".**

◆ **Equivalently, it is difficult to tell whether the frame of the distant "source" is inertial or not.**

	Source	Dependence on motion of the observer
Fictitious	×	√
real	√	×

See you next time!