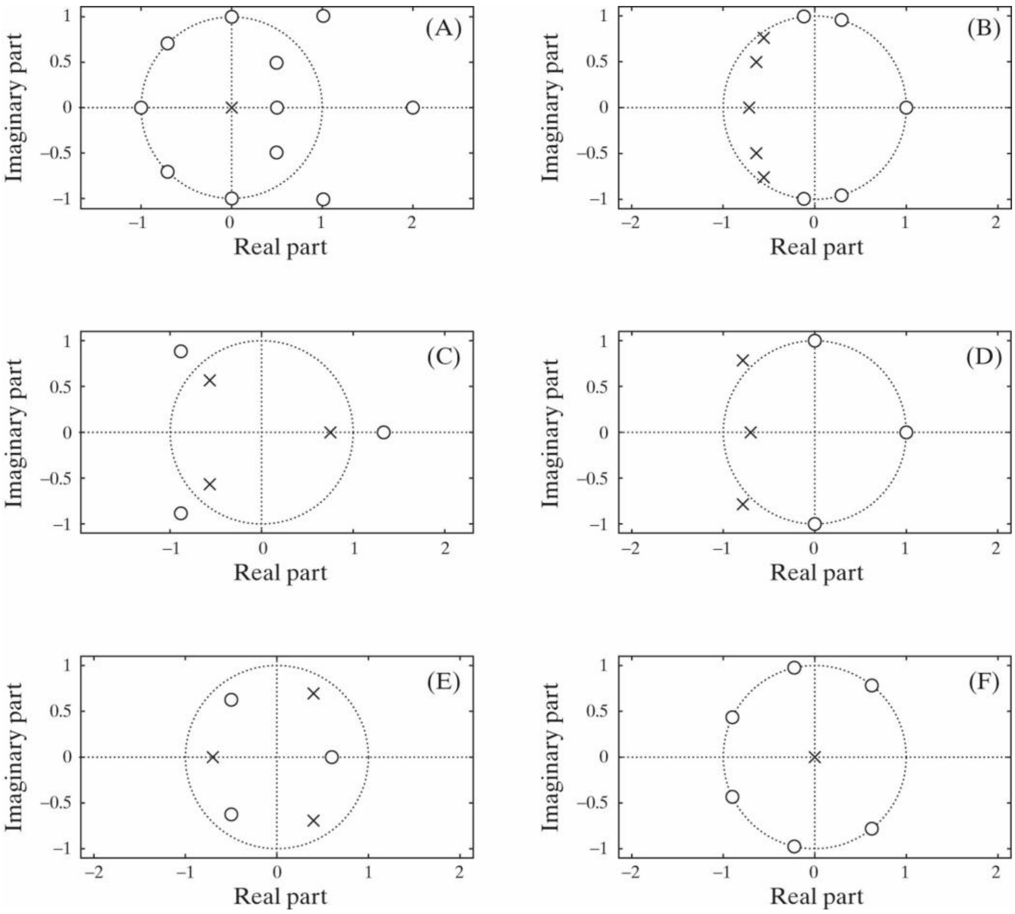
Digital Filters & Spectral Analysis
Lecture 11

Design of Filters using Pole Zero Placement

Problem sheet

Solutions

1. The pole zero plots in the figure below describe six different causal digital filters (A to F). Answer the following questions about these filters:



<table>
<tr><td>(a)</td><td>Which filters are Infinite Impulse Response (IIR) Filters? Explain why.</td></tr>
<tr><td colspan="2">Filters B,C,D, E are IIR because they have poles at places other than the origin.</td></tr>
<tr><td>(b)</td><td>Which filters are Finite Impulse Response (FIR) filters? Explain why.</td></tr>
<tr><td colspan="2">Filters A and F are FIR because they have poles only at the origin.</td></tr>
<tr><td>(c)</td><td>Which filters are stable? Explain why.</td></tr>
<tr><td colspan="2">All filters are stable except D which has poles outside the unit circle.</td></tr>
<tr><td>(d)</td><td>Which filter has the shortest impulse response? Explain why.</td></tr>
<tr><td colspan="2">Filter F has the shortest impulse response with seven non-zero samples. Filter A has 12 non-zero samples and the remaining filters are IIR</td></tr>
<tr><td>(e)</td><td>Which two filters are clearly NOT low pass filters and why?</td></tr>
<tr><td colspan="2">Filters B and D are clearly not low pass filters as they have zeros at ω=0.</td></tr>
</table>

2. Design a suitable IIR digital filter for removing narrow-band interference at 50Hz from a signal sampled at 500Hz. For your design:
   a) Give the transfer function.
   b) Sketch the pole zero diagram.
   c) Sketch the magnitude of the frequency response in the range $0 \le \Omega < 2\pi$.

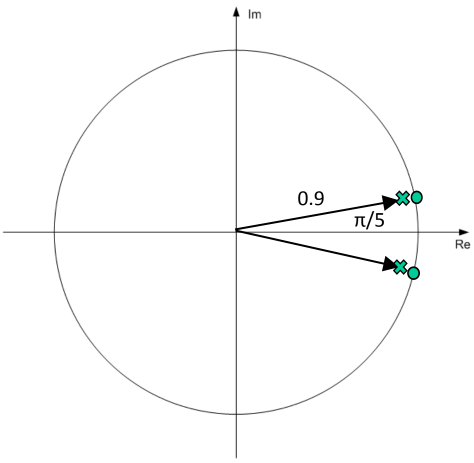
a) Notch filter with notch at :

$$\Omega_c = 2\pi \frac{f_c}{f_s} = 2\pi \frac{50}{500} = \frac{\pi}{5}$$

Need 2 zeros at $e^{\pm j\Omega_c}$ and 2 poles at $re^{\pm j\Omega_c}$ where *r* is the pole radius and is just less than 1 (say 0.9).

$$H(z) = \frac{(1-e^{j\Omega_c}z^{-1})(1-e^{-e^{j\Omega_c}}z^{-1})}{(1-re^{j\Omega_c}z^{-1})(1-re^{-e^{j\Omega_c}}z^{-1})} = \frac{1-2\cos(\Omega_c)z^{-1}+z^{-2}}{1-2r\cos(\Omega_c)z^{-1}+r^2z^{-2}} = \frac{1-1.61z^{-1}+z^{-2}}{1-1.46z^{-1}+0.81z^{-2}}$$

b)



c)