Digital Filters & Spectral Analysis

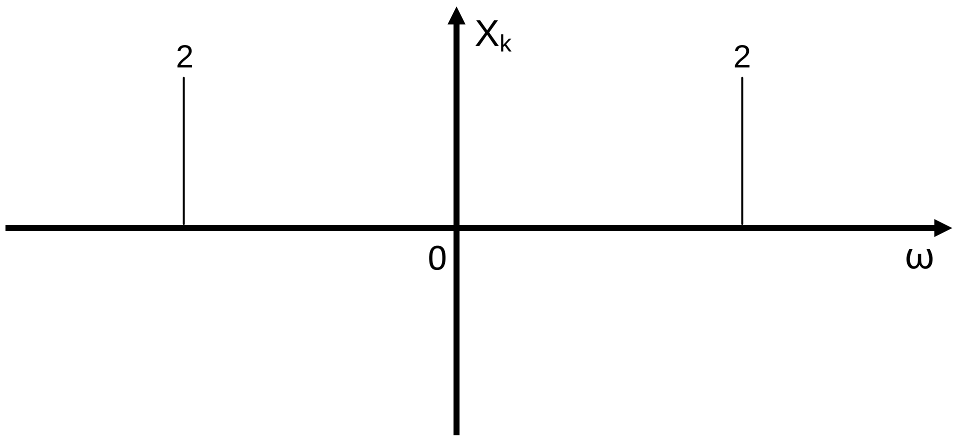
Lecture 2

The Fourier Series

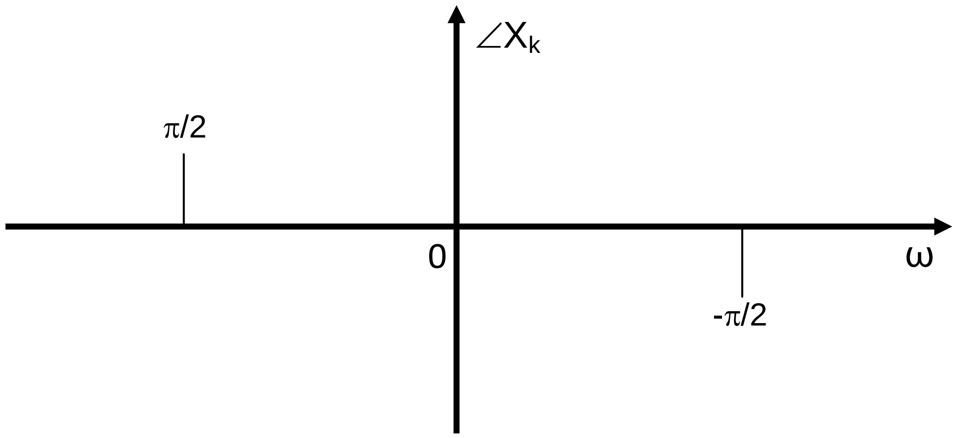
Problem sheet

Solutions

1. Plot the spectrum (amplitude and phase) of the following signal: *x(t) = 4sin(7t)* . Use Euler's identity to find the spectrum. Explain the values on your phase vs. frequency plot.



Magnitude vs. Frequency



Phase vs. Frequency

---

1. $x(t) = 4\sin(7t) = 4\left(\dfrac{e^{j7t} - e^{-j7t}}{2j}\right) = \dfrac{4}{2}\dfrac{1}{j}e^{j7t} - \dfrac{4}{2}\dfrac{1}{j}e^{-j7t} = 2\dfrac{1}{j}\dfrac{j}{j}e^{j7t} - 2\dfrac{1}{j}\dfrac{j}{j}e^{-j7t} = 2(-j)e^{j7t} + 2je^{-j7t}$

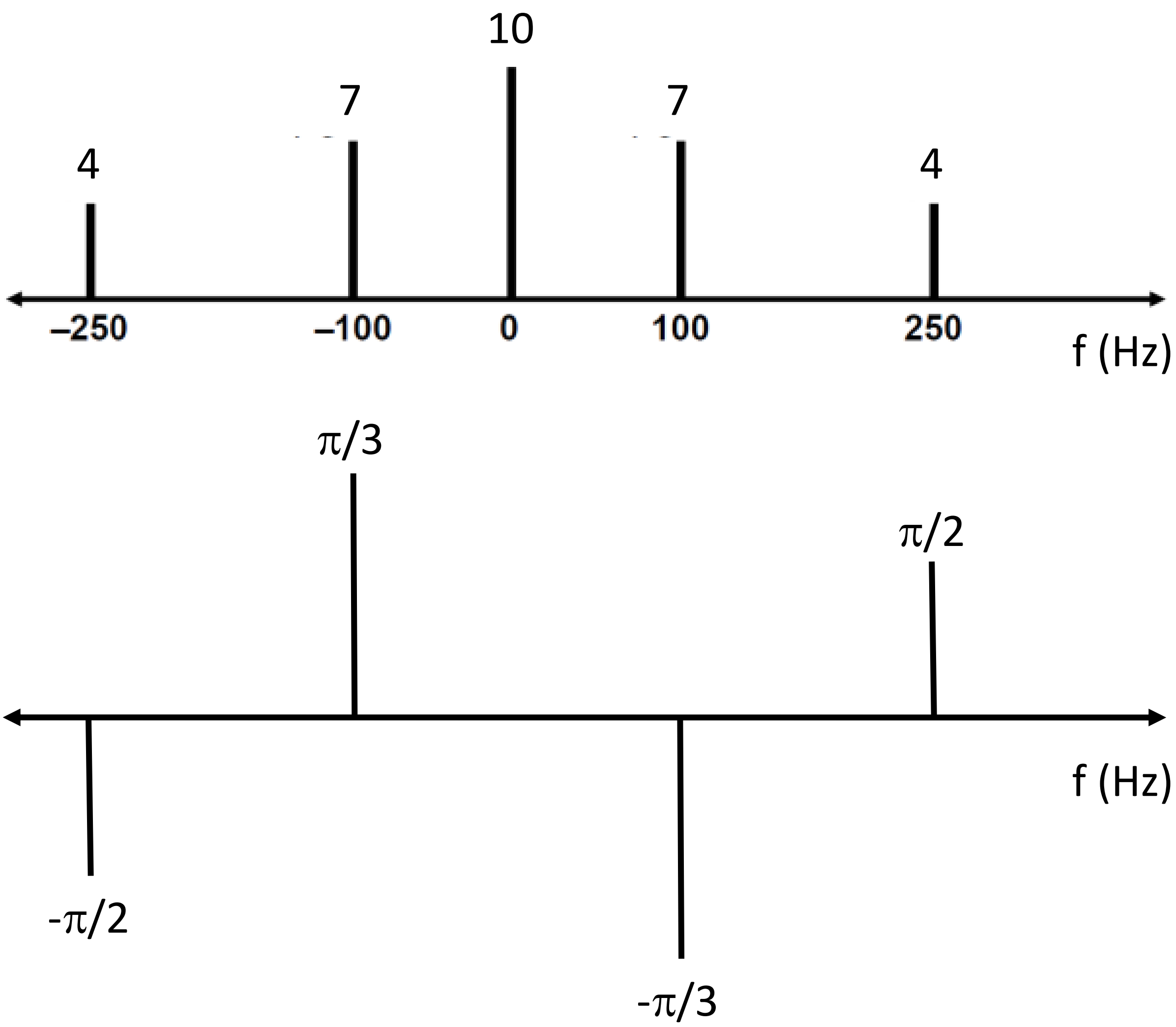
2. $j = e^{j\frac{\pi}{2}}, \; -j = e^{j(-\frac{\pi}{2})}$

From 1 & 2

3. $x(t) = 4\sin(7t) = 2e^{j(-\frac{\pi}{2})}e^{j7t} + 2e^{j\frac{\pi}{2}}e^{-j7t}$

Positive frequencies have a negative phase
Negative frequencies have a positive phase

2. Find the formula of the real signal *x(t)* whose spectrum is shown below. What is the fundamental frequency f_0 of the signal?



1. $$\begin{aligned} x(t) &= 10 + 7e^{-j\pi/3}e^{j2\pi(100)t} + 7e^{j\pi/3}e^{-j2\pi 100t} + 4e^{j\pi/2}e^{j2\pi(250)t} + 4e^{-j\pi/2}e^{-j2\pi 250t} \\ &= 10 + 7(e^{j(2\pi 100t-\pi/3)} + e^{-j(2\pi 100t-\pi/3)}) + 4(e^{j(2\pi 250t+\pi/2)} + e^{-j(2\pi 250t+\pi/2)}) \end{aligned}$$

2. $(e^{j(\omega t+\theta)} + e^{-j(\omega t+\theta)}) = 2\cos(\omega t+\theta)$

   From 1 & 2

3. $x(t) = 10 + 14\cos(2\pi 100t - \pi/3) + 8\cos(2\pi 250t + \pi/2)$

The fundamental frequency f_0 is 50Hz (suppressed)