Digital Filters & Spectral Analysis
Lecture 8

Spectral Smearing

Problem sheet

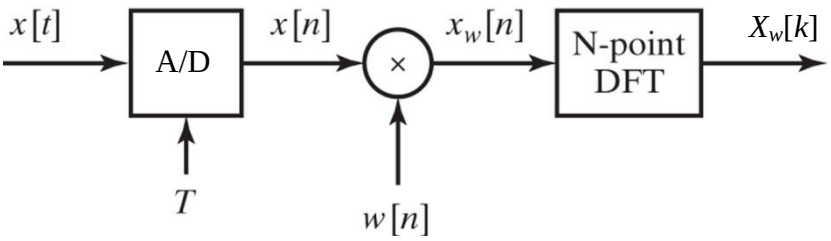
1. An audio signal sampled at 44.1KHz is to be analysed with the DFT using a rectangular window.
    a. Estimate the minimum length of DFT which could be used in order to be able to distinguish between different frequency components separated by at least 10 Hz.
    b. How would your answer change if a smoother window function (e.g. Hamming) was used?

2. A signal $x[n] = a_1 \cos(n\Omega_1) + a_2 \cos(n\Omega_2)$ is analysed using an N point DFT. Under what circumstances will the two cosine functions be clearly distinguishable in the frequency domain?

3. Consider the system shown in the figure below. The input signal $x(t) = e^{-j\left(\frac{3\pi}{8}\right)10^4 t}$, is sampled with a period $T = 10^{-4}$ and windowed with

   $w[n] = \begin{cases} 1, 0 \leq nN - 1 \\ 0, \ otherwise \end{cases}$

   What is the smallest nonzero value of N such that $X_w[k]$ is non-zero at exactly one value of k?



4. We want to estimate the spectrum of a discrete-time signal $x[n]$ using the DFT with a Hamming window applied to $x[n]$. We wish to be able to resolve sinusoidal signals that are separated by as little as $\pi/100$ in frequency. The window length L is constrained to be a power of 2. What is the minimum length $L=2^v$ that will meet our resolution requirement?