

Think about the following:

Interpolation

Would you ever get overlapping spectral copies due to interpolation? With interpolation we scale the normalised frequency axis with $1/L$ (L being the number of times we increase the sampling rate or equally decrease the sampling period). As a result the spectrum shrinks and that includes all the copies that we have due to the discrete nature of the signal (due to sampling). Overlap of the copies is not the problem – it won't happen.

The problem with interpolation is that the periodic copies of the spectrum that were once centred at $k2\pi$ and were outside the fundamental frequency range ($-\pi$ to π) will now be centred at $k2\pi/L$ thus entering the fundamental frequency range and introducing high frequency noise to our signal (frequencies outside $-\pi$ to π are ignored as they are indistinguishable from those inside the fundamental range). If L is 2 then what was at 2π will now be centred at π , if L is 3 it will be centred at $2\pi/3$ if L is 4 at $\pi/2$ etc. But as L increases these copies shrink as does the original and they never overlap.

With interpolation we need a good low pass filter that will keep the original copy and will attenuate the copies that have entered the fundamental range. In practice the frequency response of the filter will contain some side lobes outside the pass-band hence the filter will not fully attenuate the extra frequencies that have crept in the fundamental range. Moreover the filters don't have a perfect response at the pass-band and hence alter slightly the original spectrum (e.g. some of the original high frequencies will be attenuated).

As a result your interpolated image will be less sharp (will look smoother-more blurry) and can suffer from some high frequency noise. The content of the original signal can make the life of the filter easier or more difficult. If the original spectrum isn't close to π (i.e. there aren't many high frequencies included – or the original sampling rate was a lot greater than the Nyquist rate) then its spacing to the next copy when that copy enters the fundamental range will be relatively large thus relaxing the requirements of the filter (the spacing of the copies will remain the same albeit scaled).

Downsampling

With downsampling, the spectrum stretches, as we scale the normalised frequency axis with M (M being the number of times we decrease the sampling rate). So if the spectrum was originally extending up to $\pi/2$ then halving the sampling rate (downsampling with $M=2$) will stretch the spectrum to π . Downsampling with $M=3$ would extend the spectrum to $3\pi/2$ which means that we will get aliasing (the frequencies around π – the high frequencies in our signal- will be distorted because of overlap of the spectral copies).

As long as Ω_N is less than π/M we will not suffer from aliasing when we downsample with M . So for example a very smooth image is less likely to suffer from aliasing when downsampled compared to an image containing many edges. Filtering prior to downsampling with an antialiasing filter (a digital low pass filter) with cut-off frequency of π/M means that downsampling M times will not stretch the spectrum past the fundamental range ($-\pi$ to π) i.e. we will not get aliasing. However filtering will alter the original signal and mainly the high frequency content of it. Hence the signal after downsampling will longer be an exact representation of the original.