Digital Filters & Spectral Analysis
Lecture 4

Sampling

Problem sheet

1. Consider the signal : $x(t) = \cos(\omega_1 t) - \cos(\omega_2 t)$ where $\omega_2 > \omega_1$

   a. Sketch the spectrum $X(\omega)$ of the signal

   b. The signal is sampled with a sampling period T_s resulting in the discrete time signal: $x[n] = x(nT_s)$. Sketch the spectrum for the sampled signal, assuming that the sampling frequency $\omega_s = 2\pi / T >> \omega_2$.

   c. What is the minimum sampling frequency $\omega_s = 2\pi / T$ required to avoid aliasing?

   d. Sketch the spectrum obtained when the sampling frequency is just below this value.

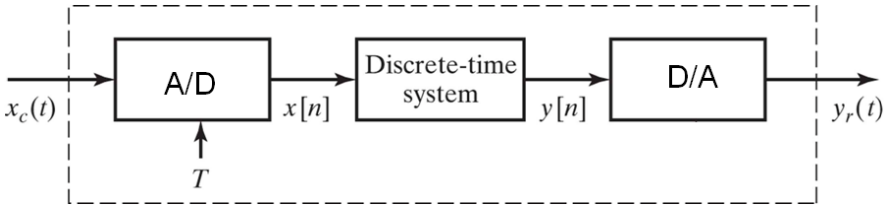
   e. Show that at a certain sampling frequency the sampled signal $x[n]$ will be 0.

2. The signal $x_c(t) = \sin(2\pi 100t)$ was sampled with sampling period *T = 1/400* second to obtain a discrete-time signal x[n]. What is the resulting sequence *x[n]*?

3. The sequence $x[n] = \cos(\frac{\pi}{4}n)$ was obtained by sampling the continuous-time signal $x_c(t) = \cos(\omega_0 t)$ at a sampling rate of 1000 samples/sec. What are two possible positive values of ω_0 that could have resulted in the sequence *x[n]*

4. The continuous-time signal $x_c(t) = \cos(4000\pi t)$ is sampled with a sampling period T to obtain the discrete-time signal $x[n] = \cos(\frac{\pi n}{3})$
   a. Determine a choice of T consistent with this information
   b. Is your choice for T in part (a) unique? If so explain why. If not specify another choice of T consistent with the information given.

5. Consider the system shown below with the discrete time system being an ideal low pass filter with cut-off frequency $\pi/8$ rads/sec



   a. If $x_c(t)$ is band-limited to 5kHz what is the maximum value of *T* that will avoid aliasing in the A/D converter?

   b. If 1/T = 10kHz what will the cut-off frequency of the effective continuous-time filter be?

   c. Repeat part b for 1/T = 20kHz

6. A continuous time signal $x_a(t)$ is composed of a linear combination of sinusoidal signals of frequencies 250 Hz, 450 Hz, 1.0 k Hz, 2.75 k Hz and 4.05 kHz. The signal $x_a(t)$ is sampled at a 1.5 kHz rate and the sampled sequence is passed through an ideal low pass filter with a cut-off frequency of 750Hz, generating a continuous time signal $y_a(t)$. What are the frequency components present in $y_a(t)$?