Digital Filters & Spectral Analysis

Lecture 3

The Fourier Transform

Problem sheet

Solutions

1. Use the frequency shift property to determine the Fourier transform of $f(t)sin(\omega_0 t)$

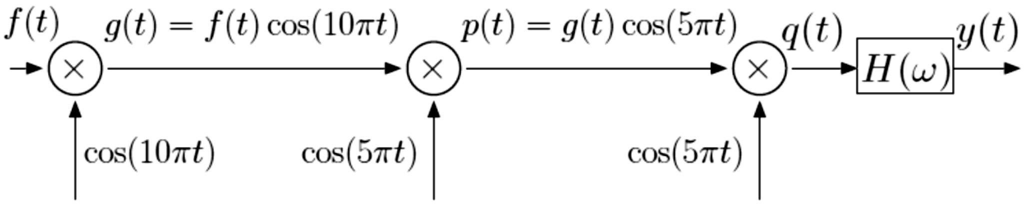
Using Euler's identity, we have

$$f(t)\sin(\omega_o t) = f(t)\left(\frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{j2}\right) = \frac{j}{2}f(t)e^{-j\omega_0 t} - \frac{j}{2}f(t)e^{j\omega_0 t}.$$

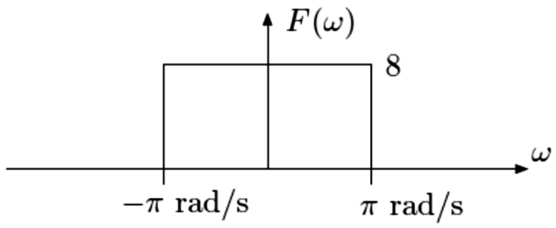
Therefore, using the given Fourier transform pair, we obtain

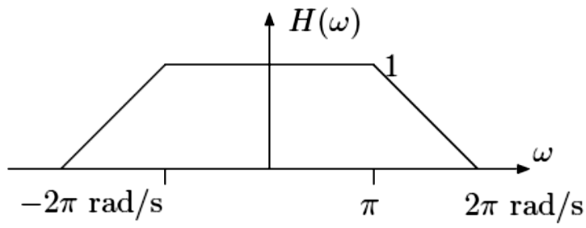
$$f(t)\sin(\omega_o t) \leftrightarrow \frac{j}{2}F(\omega+\omega_0) + \frac{j}{2}F(\omega-\omega_0)$$

2. Consider the following system



where $F(\omega)$ and $H(\omega)$ are as shown below:





a) Express $q(t)$ in terms of $p(t)$.

b) Sketch the Fourier transforms $G(\omega)$, $P(\omega)$, $Q(\omega)$, and $Y(\omega)$.

c) Express $y(t)$ in terms of $f(t)$.

a) Clearly, after the third mixer, we have

$$q(t) = p(t)\cos(5\pi t).$$

b) Since, $g(t) = f(t)\cos(10\pi t)$, then

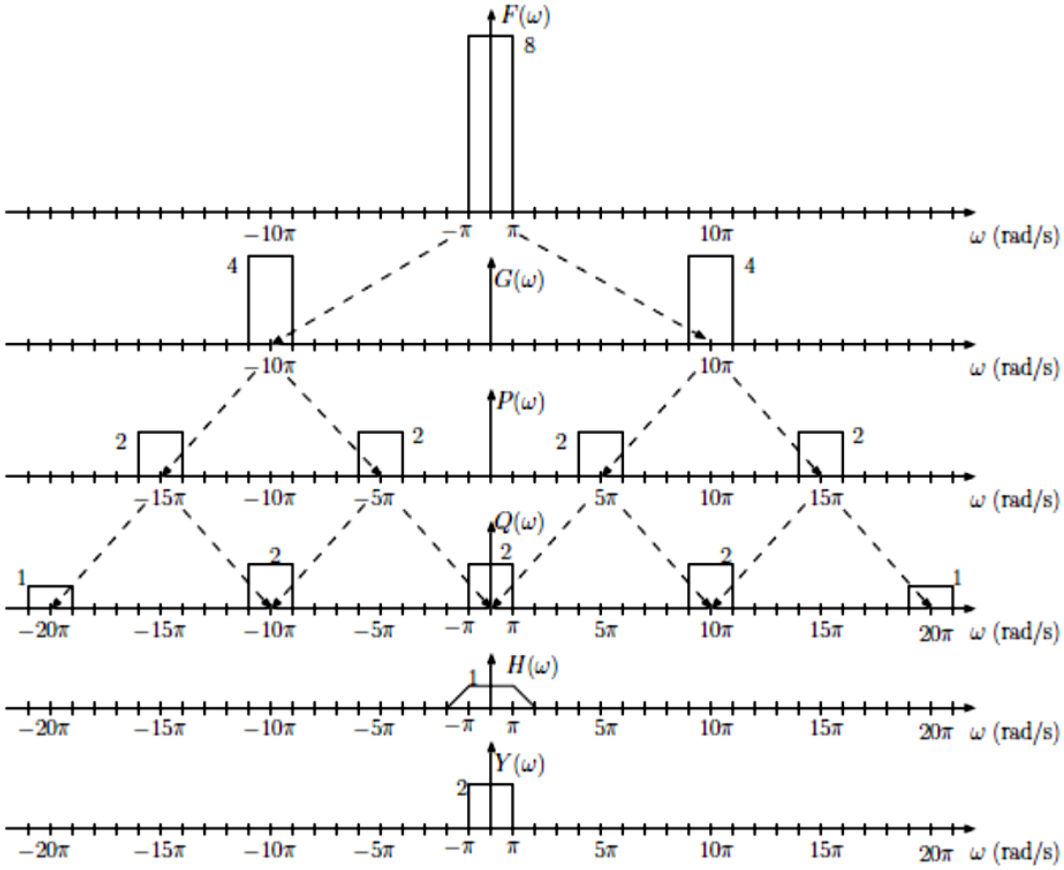
$$G(\omega) = \frac{1}{2}F(\omega - 10\pi) + \frac{1}{2}F(\omega + 10\pi).$$

Similarly,

$$P(\omega) = \frac{1}{2}G(\omega - 5\pi) + \frac{1}{2}G(\omega + 5\pi)$$

and

$$Q(\omega) = \frac{1}{2}P(\omega - 5\pi) + \frac{1}{2}P(\omega + 5\pi).$$



c) From the figure we notice that

$$Y(\omega) = \frac{1}{4}F(\omega).$$

Hence,

$$y(t) = \frac{1}{4}f(t).$$