Digital Filters & Spectral Analysis
Lecture 5

DTFT

Problem sheet

1. The analogue signal $x(t) = \cos(2\pi f_0 t) - j\sin(2\pi f_0 t)$ comprising a cosine with frequency f_0 = 200*Hz* is sampled at a frequency $f_s = 300Hz$ to give a discrete time signal $x[n] = x(n/f_s)$. Sketch, over the range $0 \le \Omega < 2\pi$ the magnitude $|X(\Omega)|$ of the DTFT of the discrete time signal.

2. Find the DTFT for the following signal and sketch its magnitude and phase response:

$$x[n] = a^{|n|} \quad 0 < a < 1$$

3. Find an expression for the DTFT of a discrete time signal:

$$x[n]=\begin{cases}1 & 0\le n<N_1\\ -1 & N_1\le n<2N_1\\ 0 & elsewhere\end{cases}$$

You may use the knowledge that:

$$x[n]=\begin{cases}1 & 0\le n<N\\ 0 & elsewhere\end{cases} \quad\Rightarrow\quad X(\Omega)=e^{-j\Omega(N-1)/2}\frac{\sin(\Omega N/2)}{\sin(\Omega/2)}$$