Discrete Time Sampling

Problem sheet

Solutions

1 Figure 1.1 shows a general system for changing the sampling rate of a discrete time signal $x[n]$ by a non-integer factor.

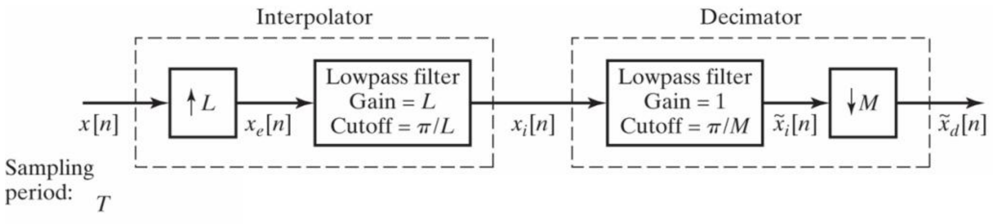


Figure 1.1

(a) What is the sampling period of signal $x_i[n]$ and of the final signal $\tilde{x}_d[n]$ assuming that the original sampling period of signal $x[n]$ is T.

Sampling period $x_i[n]$: T/L

Sampling period $\tilde{x}_d[n]$: $(T \times M)/L$

(b) The system can be simplified by combining the decimation and interpolation filters into one lowpass filter. What would the cut-off frequency of this combined filter be?

Combined filter : cut-off = min $(\pi/L, \pi/M)$.

(c) Consider the discrete-time system shown in Figure 1.2

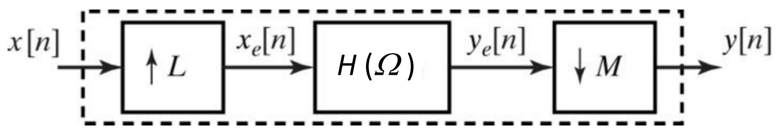


Figure 1.2

where

- L and M are positive integers
- $x_e[n] = \begin{cases} x[n/L] & n = kL,\ k\ integer \\ 0 & otherwise \end{cases}$
- $y[n] = y_e[nM]$
- $H(\Omega) = \begin{cases} M & |\Omega| \leq \frac{\pi}{4} \\ 0 & \frac{\pi}{4} < |\Omega| \leq \pi \end{cases}$

Assume that L = 2 and M = 4 and that $X(\Omega)$, the DTFT of $x[n]$, is real and is as shown in Figure 1.3.

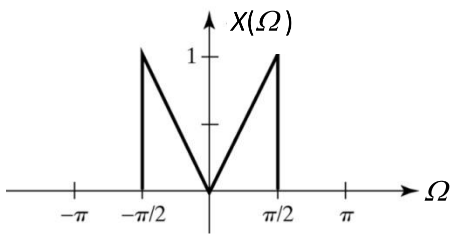
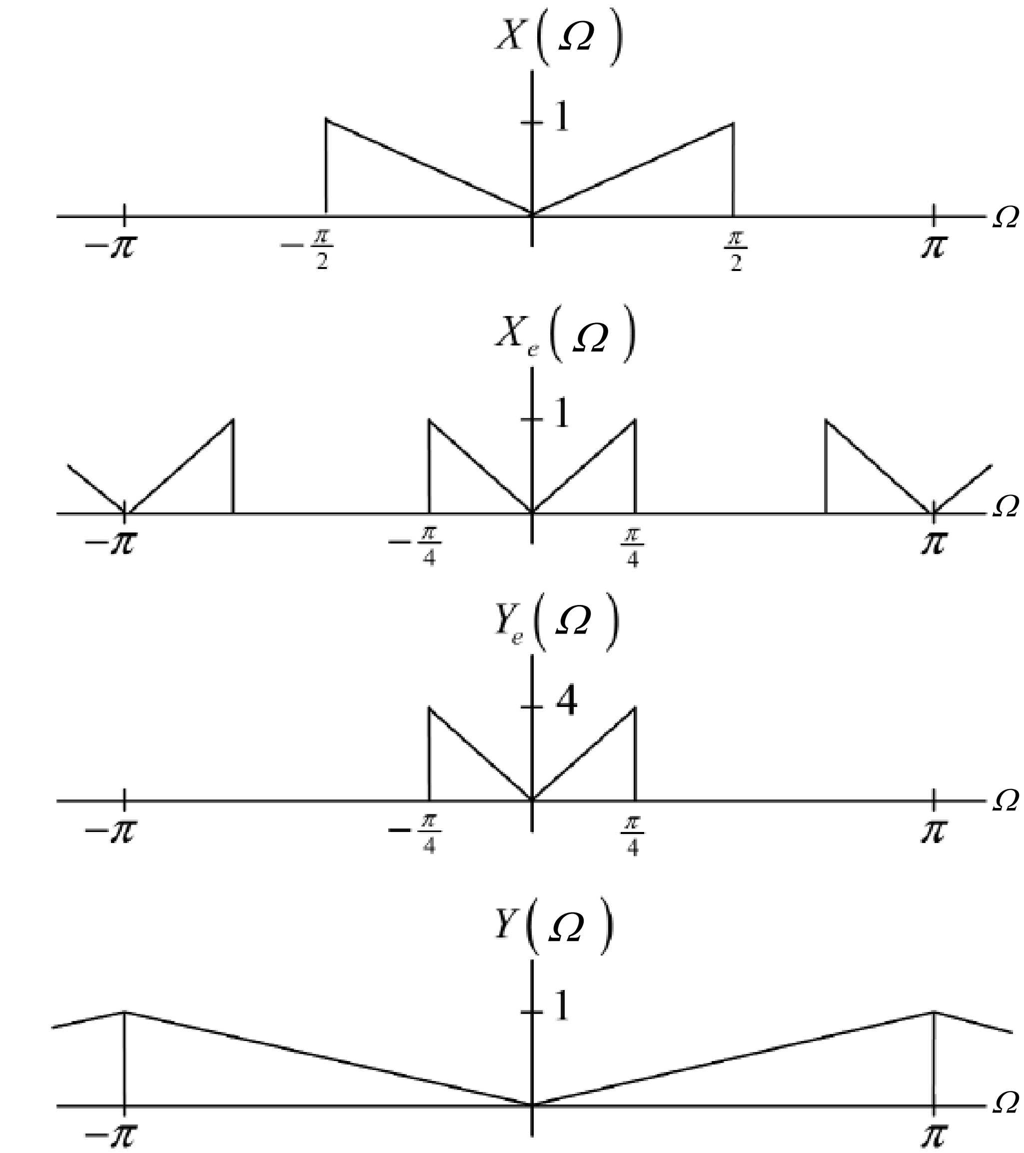
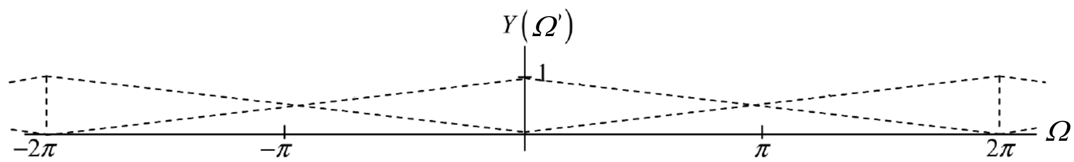


Figure 1.3

Sketch the following DTFTs on the special sheet provided:
$X_e(\Omega)$, the DTFT of $x_e[n]$,
$Y_e(\Omega)$, the DTFT of $y_e[n]$,
$Y(\Omega)$, the DTFT of $y[n]$,
Make sure you clearly label amplitudes and frequencies on your sketches.



(d) Now assume that L=2 and M=8. Sketch $Y(\Omega)$, the DTFT of $y[n]$. What is $y[n]$ equal to in this case?



$Y(\Omega)$ is equal to 1 for all Ω. Which means that $y[n] = \delta[n]$.