Lecture 12

Linear Phase Response

Designing FIR Filters with Linear Phase Response

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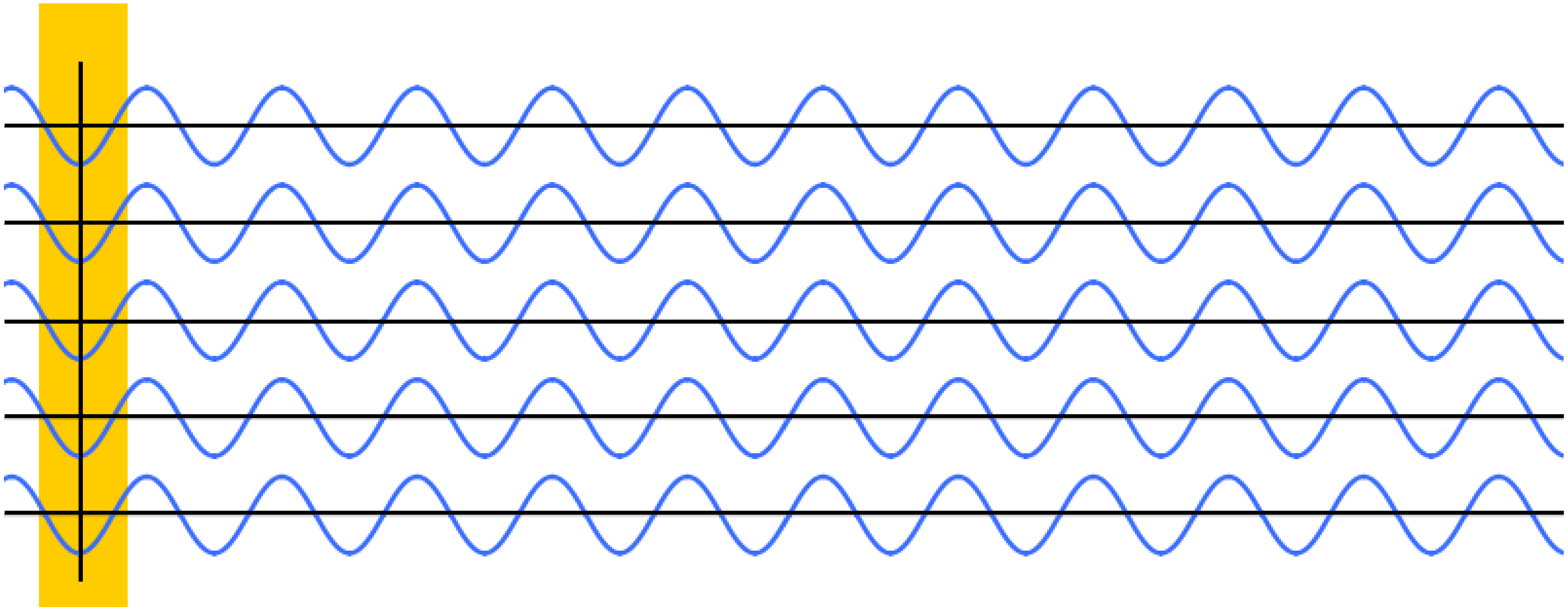
Linear Phase Response

Phase of Sinusoidal Signals

The fraction of the period that has elapsed relative to the origin

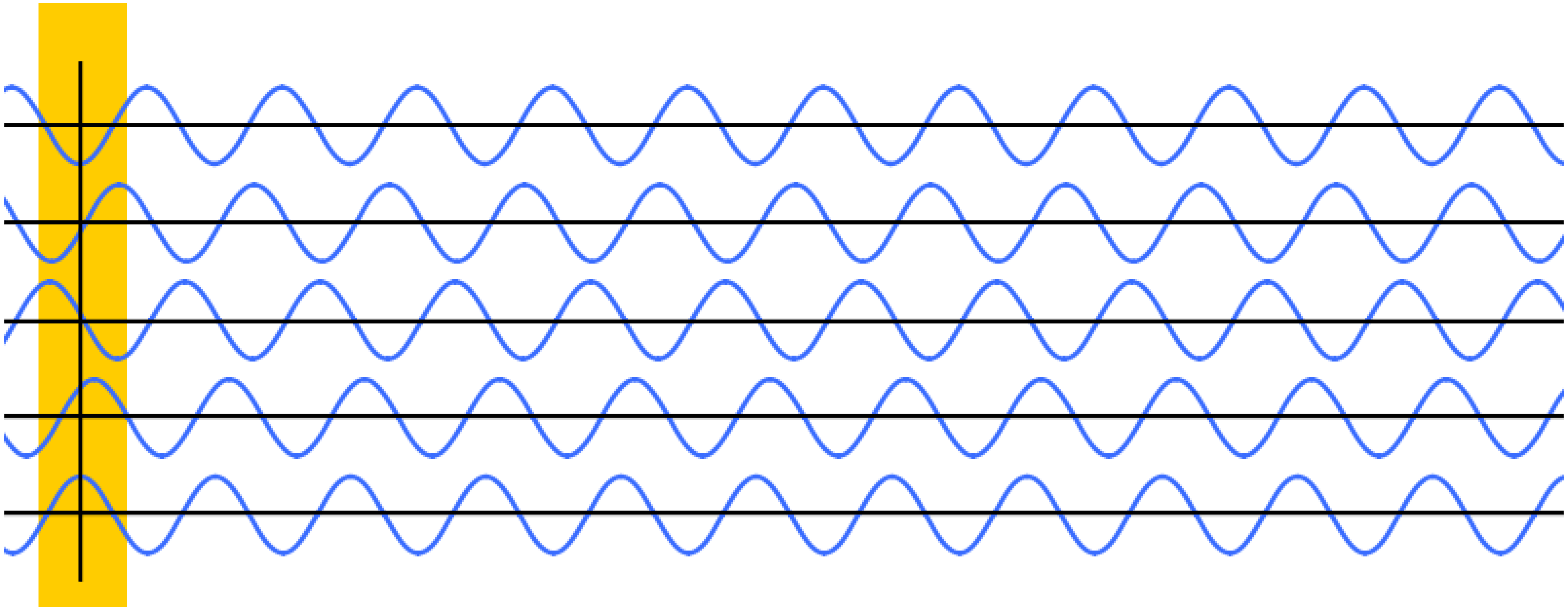


Sinusoids of the same amplitude, frequency and phase

Linear Phase Response

Phase of Sinusoidal Signals

The fraction of the period that has elapsed relative to the origin

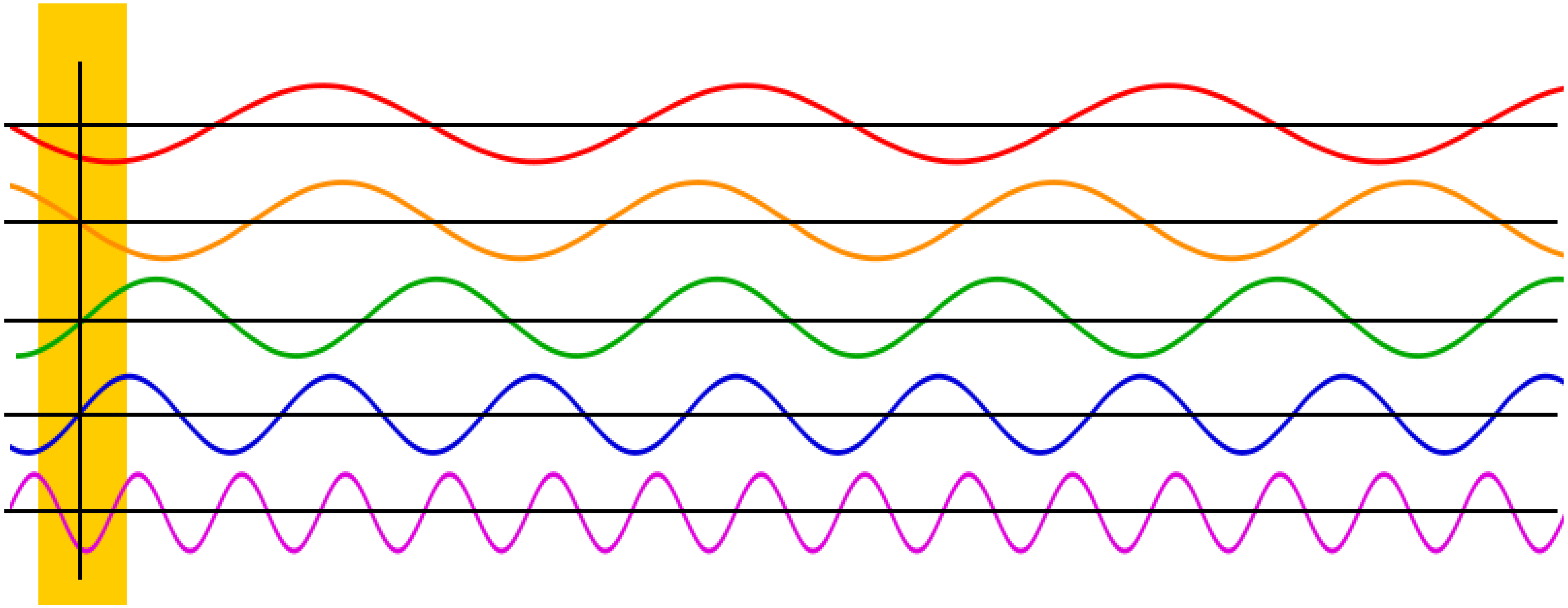


Sinusoids of the same amplitude and frequency but of different phase

Linear Phase Response

Phase of Sinusoidal Signals

The fraction of the period that has elapsed relative to the origin

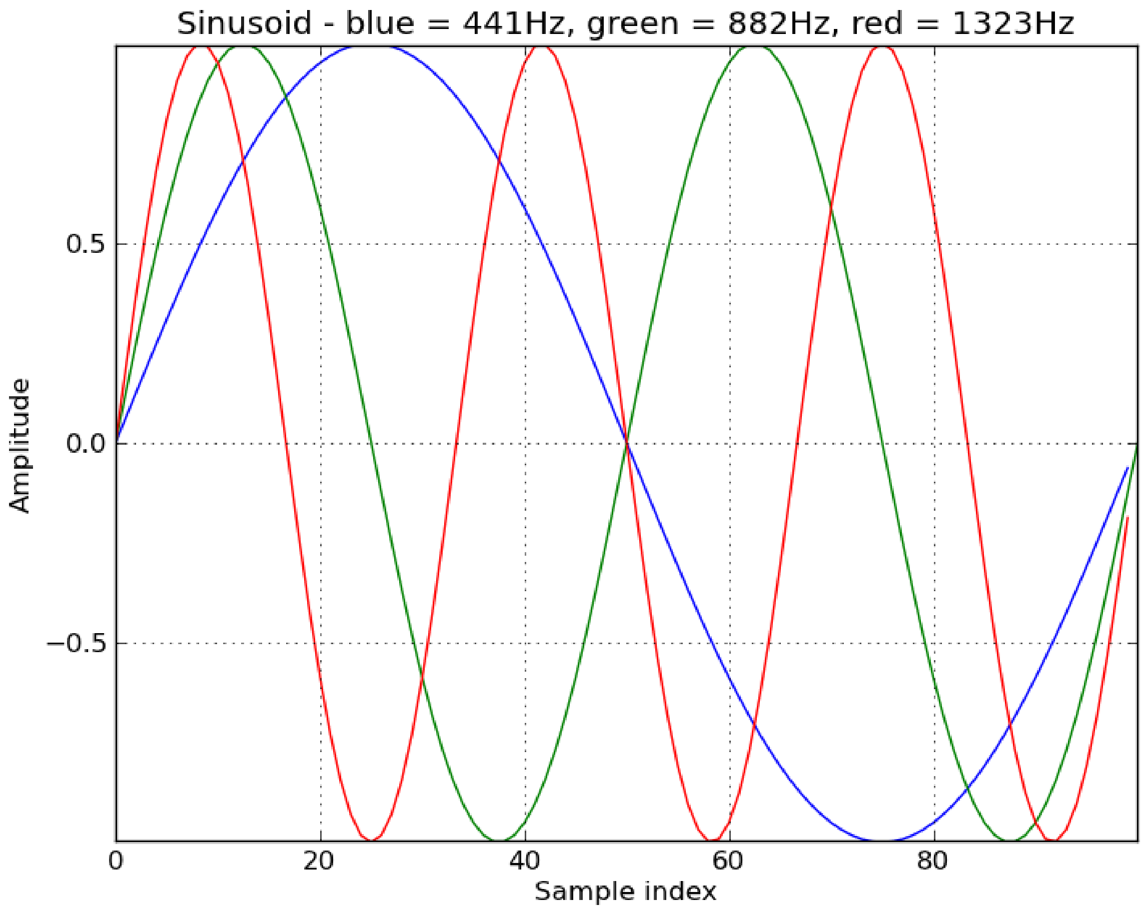


Sinusoids of the same amplitude but of different frequency and phase

Linear Phase Response

Fourier Transform

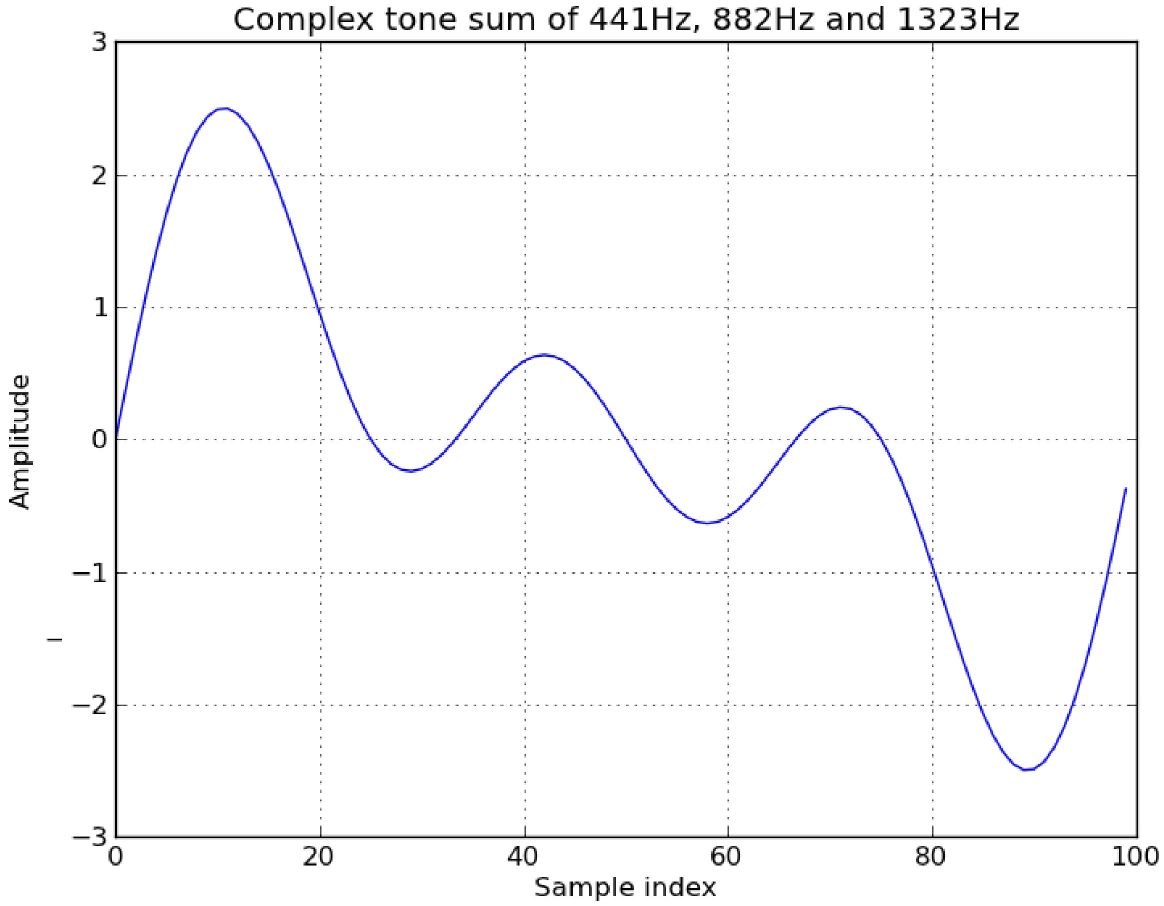
Synthesising/decomposing signals with/into sinusoids of different frequencies



Linear Phase Response

Fourier Transform

Synthesising/decomposing signals with/into sinusoids of different frequencies



Linear Phase Response

Fourier Transform

Synthesising/decomposing signals with/into sinusoids of different frequencies

DT Signal $x[n]$

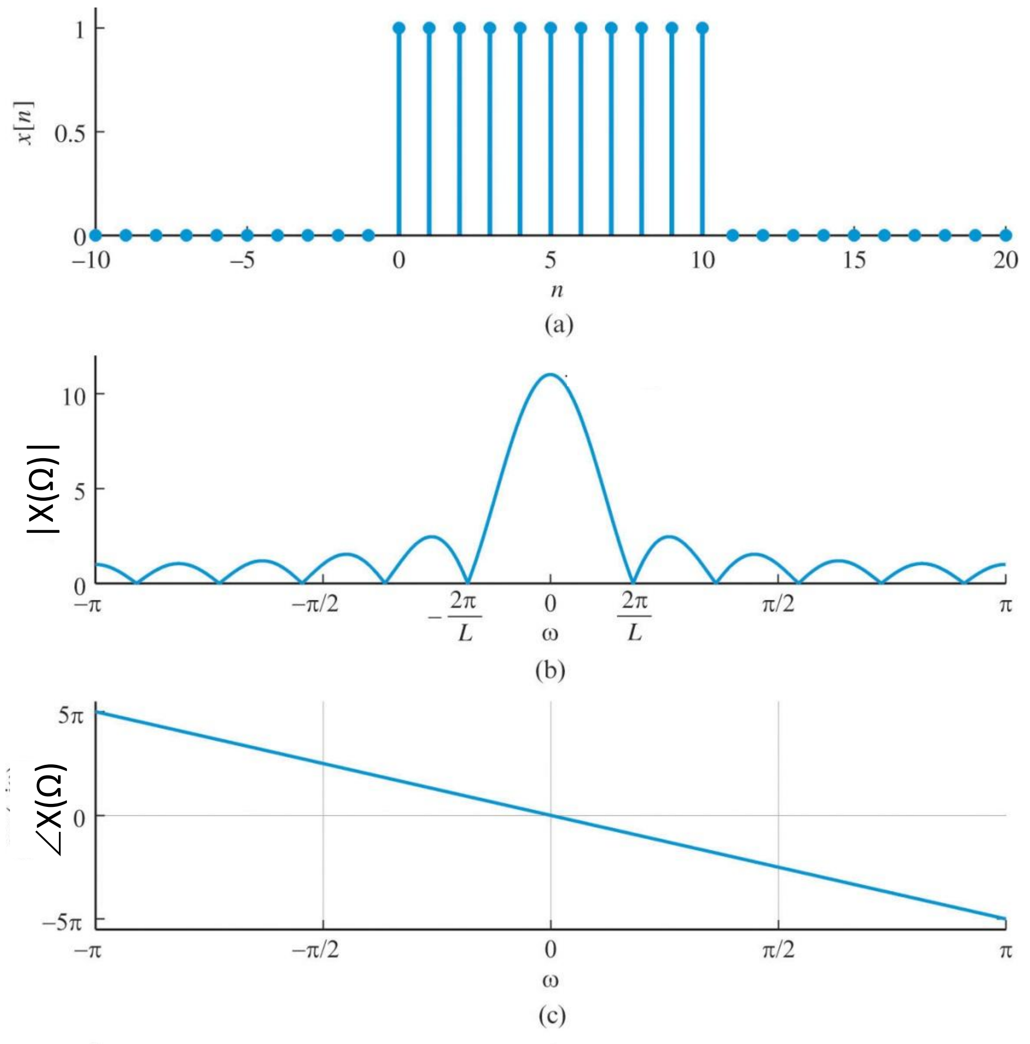
DTFT: $$X(\Omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\Omega n}$$

Magnitude Spectrum $|X(\Omega)|$

What proportion of a complex exponential (sinusoid) of certain frequency should be used to synthesise the signal $x[n]$

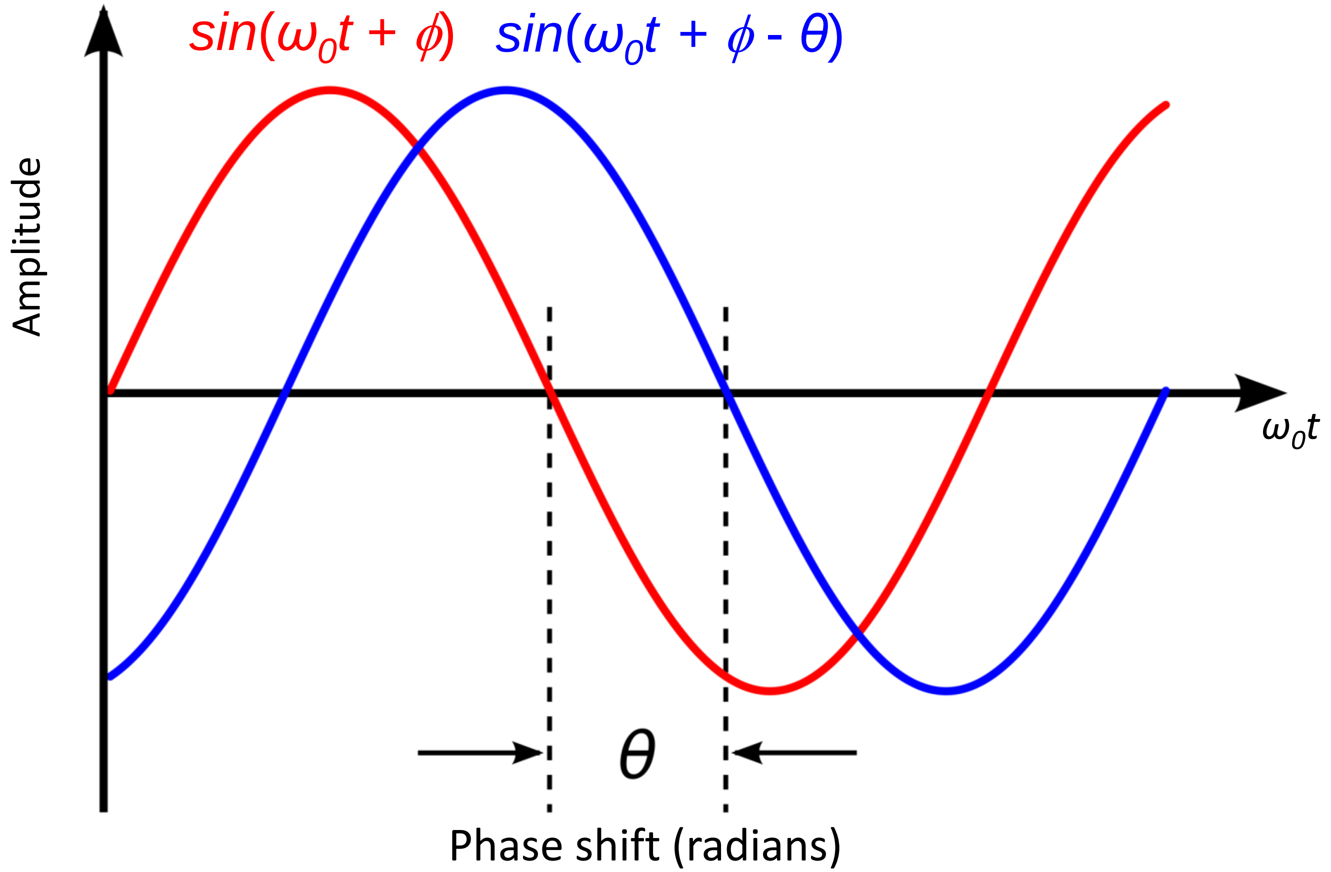
Phase Spectrum $\angle X(\Omega)$

What should the initial angle of each complex exponential (starting point of each sinusoid) be to synthesise the signal $x[n]$



Phase Shift

Change in the initial angle of a sinusoid

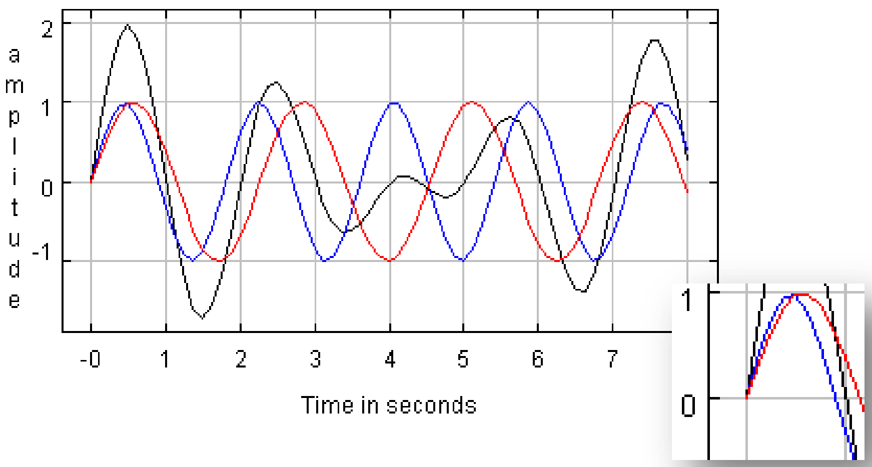


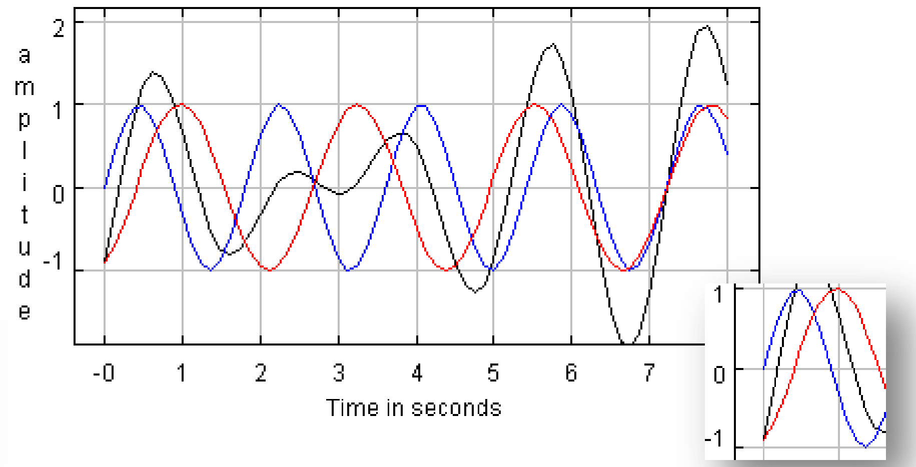
What is the effect of relative phase shifts on the shape of the signal in the time domain?

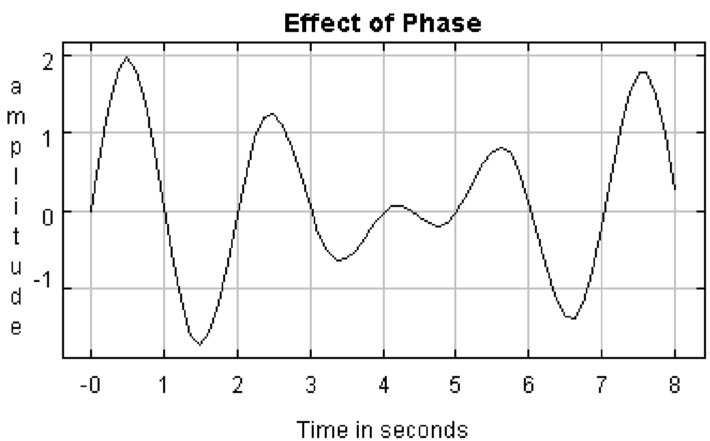
Linear Phase Response

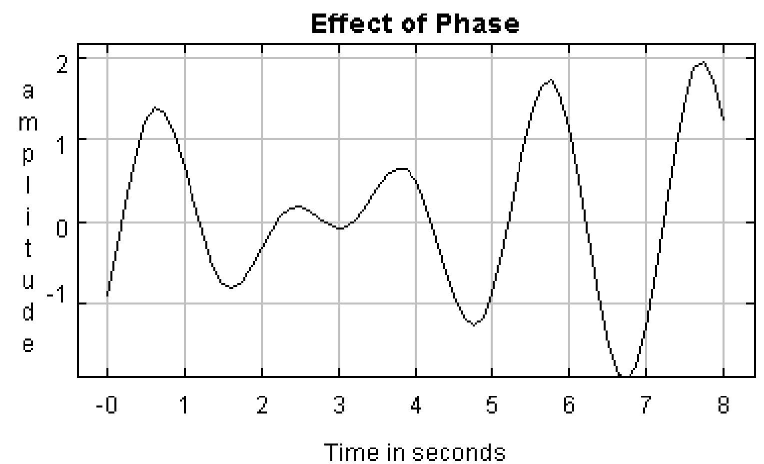
Effect of Phase

How the phase of constituent sinusoids affects the shape of the signal



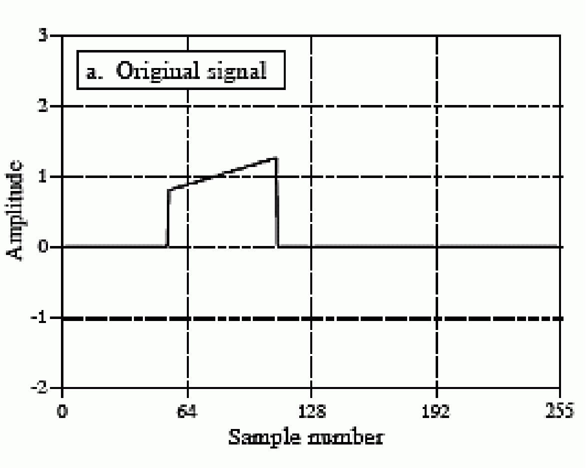






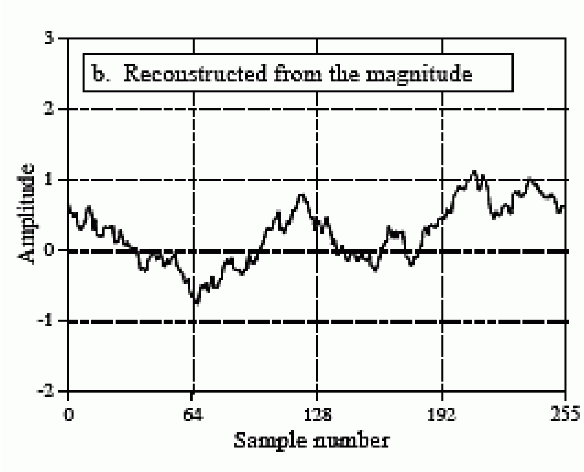
Effect of Phase

Information contained in the phase spectrum of a signal



1. DFT of signal
2. **Replace phase with random numbers**
3. IDFT

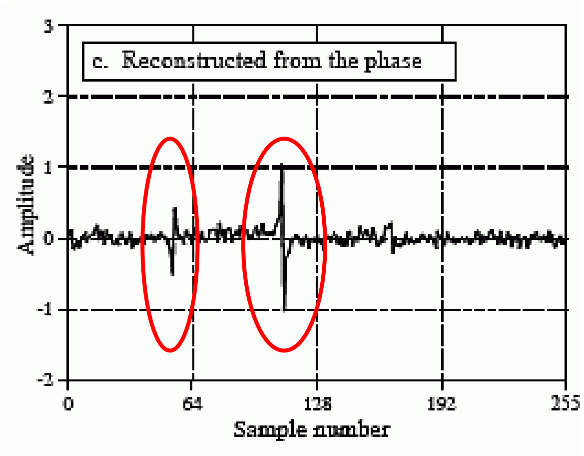
reconstructed signal based only on info contained in the magnitude



Information about the shape of the time domain waveform is contained in the phase rather than the magnitude

1. DFT of signal
2. **Replace magnitude with random numbers**
3. IDFT

reconstructed signal based only on information contained in the phase



Linear Phase Response

Effect of Phase

Information contained in the phase spectrum of a signal

Image 1

1. DFT of signal

   $|X_1(\Omega)|$, $\angle X_1(\Omega)$

2. **Keep the magnitude spectrum**
   Replace phase spectrum with the phase spectrum of Image 2

   $|X_1(\Omega)|$, $\angle X_2(\Omega)$

3. IDFT

Magnitude Image 1 , Phase Image 2

Image 2

1. DFT of signal

   $|X_2(\Omega)|$, $\angle X_2(\Omega)$

2. **Keep the magnitude spectrum**
   Replace phase spectrum with the phase spectrum of Image 1

   $|X_2(\Omega)|$, $\angle X_1(\Omega)$

3. IDFT

Magnitude Image 2 , Phase Image 1

Linear Phase Response

Effect of Phase

Information contained in the phase spectrum of a signal

Image 1

1. DFT of signal

   $\left|X_1(\Omega)\right|,\ \angle X_1(\Omega)$

2. **Magnitude Spectrum set to 1**

   (reconstructed signal based only on information contained in phase)

   $\left|X_1(\Omega)\right| = 1,\ \angle X_1(\Omega)$

3. IDFT

Phase of Image 1 $\left|X_1(\Omega)\right| = 1$

Image 2

1. DFT of signal

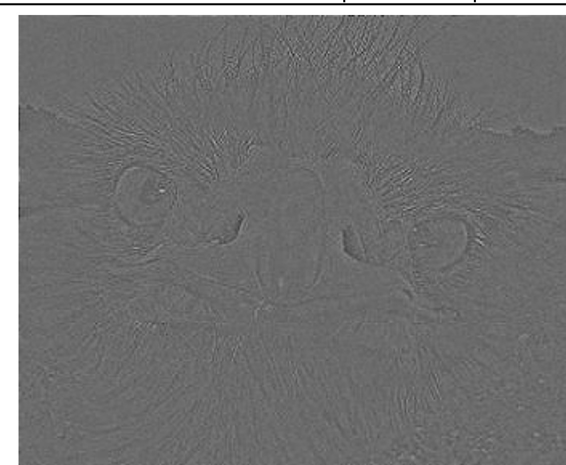
   $\left|X_2(\Omega)\right|,\ \angle X_2(\Omega)$

2. **Magnitude Spectrum set to 1**

   (reconstructed signal based only on information contained in phase)

   $\left|X_2(\Omega)\right| = 1,\ \angle X_2(\Omega)$

3. IDFT



Phase of Image 2 $\left|X_2(\Omega)\right| = 1$

Linear Phase Response

Frequency Response of LTI Systems

Response of LTI systems to sinusoidal input

- LTI system input : complex exponential $x[n] = e^{j\Omega n}$, $-\infty < n < \infty$

- LTI system output given by convolution

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = \sum_{k=-\infty}^{\infty} h[k]x[n-k] = \sum_{k=-\infty}^{\infty} h[k]e^{j\Omega(n-k)} = \underbrace{\boxed{\sum_{k=-\infty}^{\infty} h[k]e^{-j\Omega k}}}_{H(\Omega)} e^{j\Omega n}$$

$$y[n] = \underbrace{\boxed{H(\Omega)}}_{\text{Eigenvalue}} \; \underbrace{\boxed{e^{j\Omega n}}}_{\text{Eigenfunction}}$$

Output to complex exponential is complex exponential of same frequency with altered magnitude and phase

- LTI system input : linear combination of complex exponentials $x[n] = \sum_k a_k e^{j\Omega_k n}$

- LTI system output due to linearity principle : $y[n] = \sum_k a_k \, H(\Omega_k) \, e^{j\Omega_k n}$

Frequency Response of LTI Systems

Magnitude and Phase of the Frequency Response

$$H(z)=\frac{b}{1-az^{-1}} \overset{z=e^{j\Omega}}{\Rightarrow} H(\Omega)=\frac{b}{1-ae^{-j\Omega}}$$

$$\boxed{\left|H(\Omega)\right|}=\frac{|b|}{\left|1-ae^{-j\Omega}\right|}\overset{(1)}{=}\frac{|b|}{\sqrt{1+a^2-2a\cos\Omega}}$$

$$\boxed{\angle H(\Omega)}=\angle b-\angle(1-ae^{-j\Omega})\overset{(2)}{=}-\tan^{-1}\left[\frac{a\sin\Omega}{1-a\cos\Omega}\right]$$

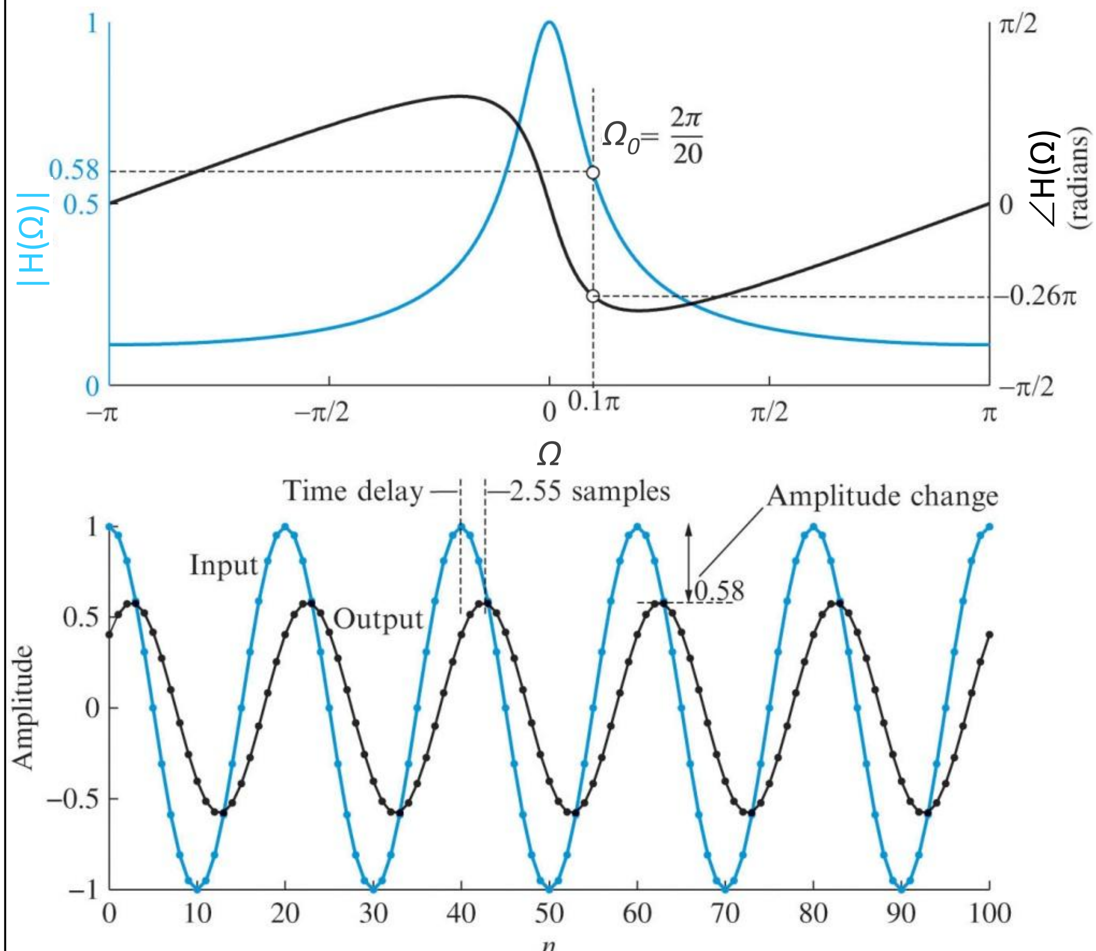
(1) $$\left|1-ae^{-j\Omega}\right|=\left|1-a(\cos\Omega-j\sin\Omega)\right|=$$

$$\left|(1-a\cos\Omega)-j(a\sin\Omega)\right|$$

$$\sqrt{(1-a\cos\Omega)^2+(a\sin\Omega)^2}=\sqrt{1+a^2-2a\cos\Omega}$$

(2) $$\angle b-\angle\left(1-ae^{-j\Omega}\right)\overset{(1)}{=}0-\tan^{-1}\left[\frac{a\sin\Omega}{1-a\cos\Omega}\right]$$

$a=0.8, b=0.2$



Linear Phase Response

Frequency Response of LTI Systems

Magnitude and Phase of the Frequency Response

$$Y(z) = H(z)\,X(z) \overset{z=e^{j\Omega}}{\Rightarrow} Y(e^{j\Omega}) = H(e^{j\Omega})\,X(e^{j\Omega})$$

Express the frequency response of the filter in Polar Form

$$H(e^{j\Omega}) = H_{re}(e^{j\Omega}) + j\,H_{im}(e^{j\Omega}) \;=\; \left|H(\Omega)\right| e^{j\theta(\Omega)} \;, \theta(\Omega) = \angle H(e^{j\Omega})$$

Magnitude of Frequency Response

$$\left|H(e^{j\Omega})\right| = \sqrt{\left(H_{re}(e^{j\Omega})\right)^2 + \left(H_{im}(e^{j\Omega})\right)^2} \quad , \quad \left|Y(e^{j\Omega})\right| = \left|H(e^{j\Omega})\right|\,\left|X(e^{j\Omega})\right|$$

Phase of Frequency Response

$$\angle H(e^{j\Omega}) = \tan^{-1}\left[\frac{H_{im}(e^{j\Omega})}{H_{re}(e^{j\Omega})}\right] \quad , \quad \angle Y(e^{j\Omega}) = \angle H(e^{j\Omega}) + \angle X(e^{j\Omega})$$

Frequency Response of LTI Systems

Distortion of signals passing through LTI systems

Distortionless response: Input and output signal have the same shape

$$y[n] = G\,x[n-n_d],\; G > 0$$

In the frequency domain this becomes (take the Fourier Transform of both sides):

$$Y(\Omega) = G\,e^{-j\Omega n_d}\,X(\Omega)$$

For a distortionless response the frequency response of the filter should be:

$$H(\Omega) = \frac{Y(\Omega)}{X(\Omega)} = G\,e^{-j\Omega n_d}$$

Which means that

$$|H(\Omega)| = G$$

The magnitude of the frequency response must be constant (flat)

$$\angle H(\Omega) = -\Omega n_d$$

The phase of frequency response must be linear function of frequency

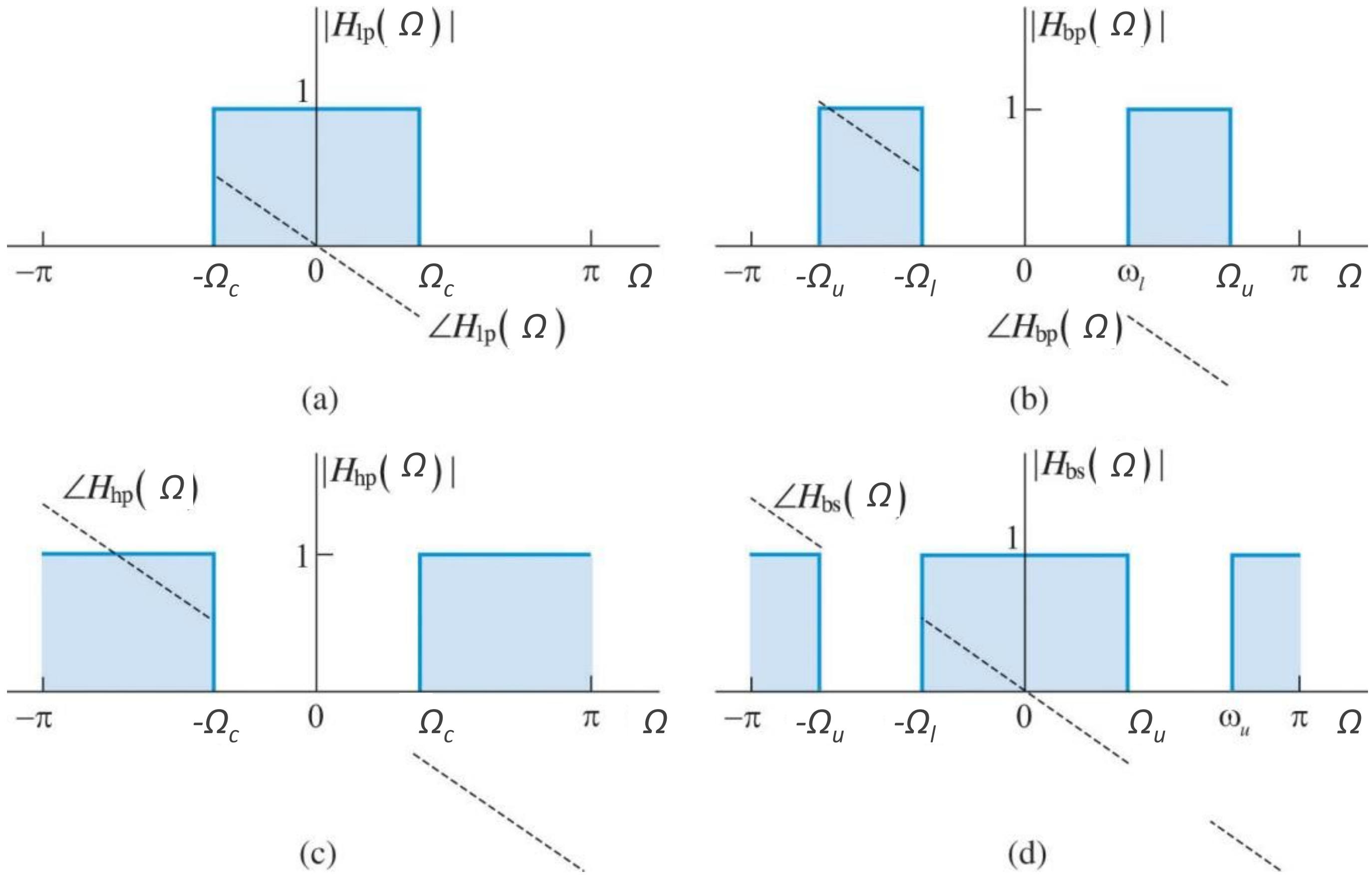
Linear Phase Response

Frequency Response of Digital Filters

Distortion of signals passing through frequency selective filters

Distortionless response: No magnitude or phase distortion in the passband

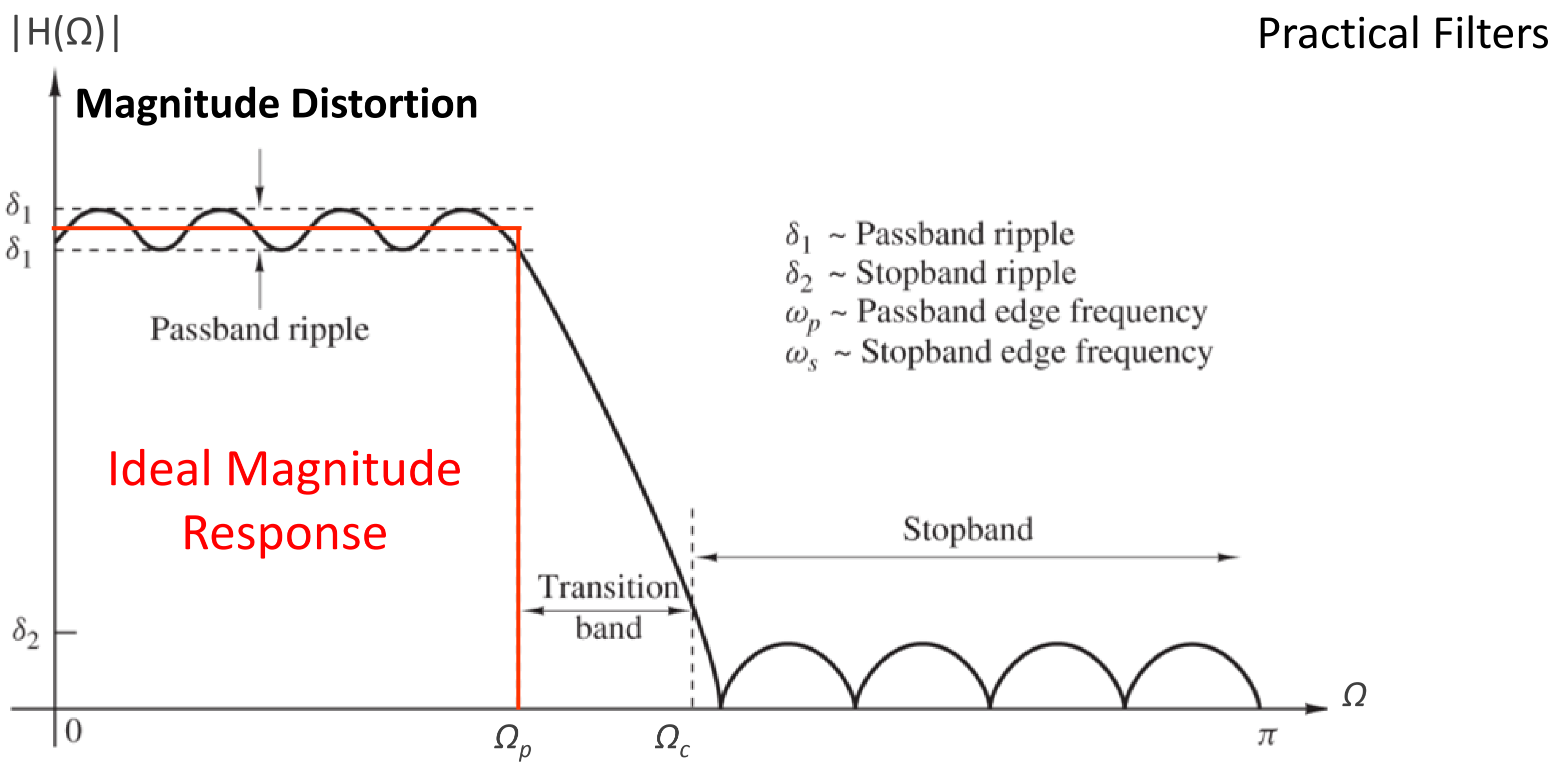
Ideal Filters



Frequency Response of Digital Filters

Magnitude distortion (frequency domain)

$$|Y(\Omega)| = |H(\Omega)|\,|X(\Omega)|$$



Magnitude Distortion: distortion of the magnitude of the signal's spectrum in the passband due to a filter's variable magnitude response (ripples) in the passband

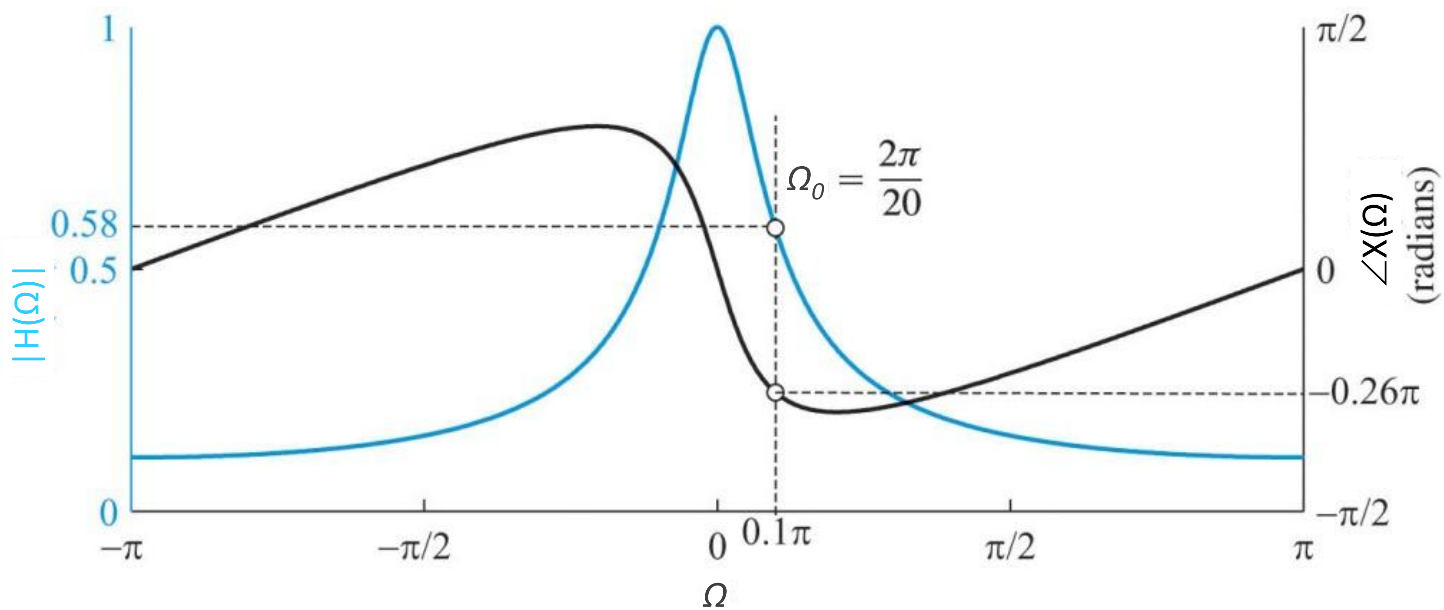
Linear Phase Response

Frequency Response of Digital Filters

Phase distortion (frequency domain)

$$\angle Y(\Omega) = \angle H(\Omega) + \angle X(\Omega)$$

Practical Filters



Phase Distortion: distortion of the phase of the signal's spectrum in the passband due to a filter's non-linear phase response in the passband

Linear Phase Response

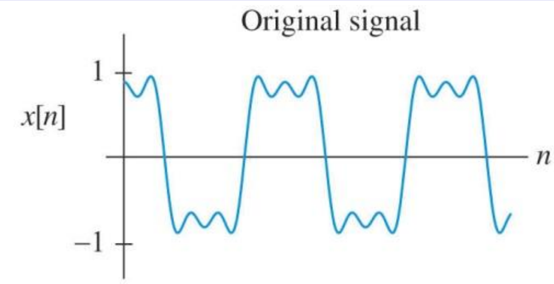
Frequency Response of Digital Filters

Effect of filter's magnitude and phase response on the shape of the signal

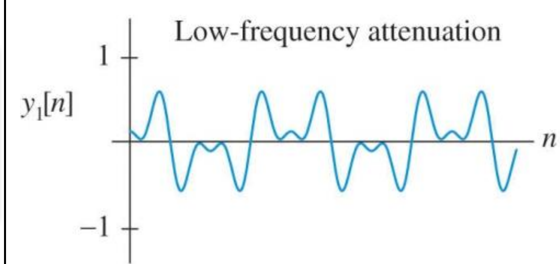
$$x[n] = \cos(\Omega_0 n) - \tfrac{1}{3}\cos(3\Omega_0 n) + \tfrac{1}{5}\cos(5\Omega_0 n)$$

$$y[n] = c_1 \cos(\Omega_0 n + \varphi_1) + c_2 \cos(3\Omega_0 n + \varphi_2) + c_3 \cos(5\Omega_0 n + \varphi_3)$$

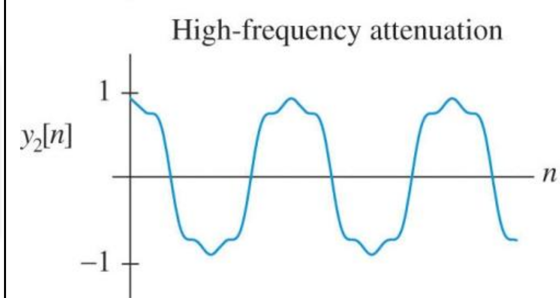
$|H(\Omega)| \neq G$ Magnitude Response Phase Response



Signal	c_1	c_2	c_3	ϕ_1	ϕ_2	ϕ_3	Amplitude
$y_1[n]$	1/4	-1/3	1/5	0	0	0	Highpass



Signal	C_1	c_2	c_3	ϕ_1	ϕ_2	ϕ_3	Amplitude
$y_2[n]$	1	-1/6	1/10	0	0	0	Lowpass



$\Omega_0 = 0.004\pi$

Linear Phase Response

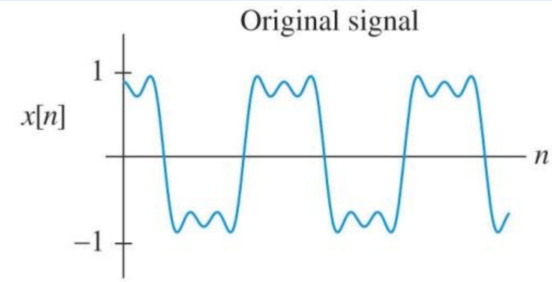
Frequency Response of Digital Filters

Effect of filter's magnitude and phase response on the shape of the signal

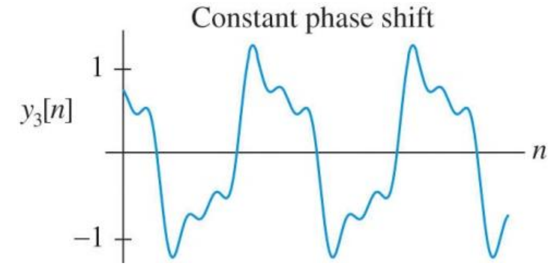
$$x[n] = \cos(\Omega_0 n) - \tfrac{1}{3}\cos(3\Omega_0 n) + \tfrac{1}{5}\cos(5\Omega_0 n)$$

$$y[n] = c_1 \cos(\Omega_0 n + \boxed{\varphi_1}) + c_2 \cos(3\Omega_0 n + \boxed{\varphi_2}) + c_3 \cos(5\Omega_0 n + \boxed{\varphi_3})$$

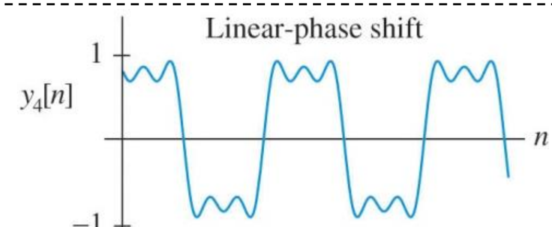
$$\angle H(\Omega)$$



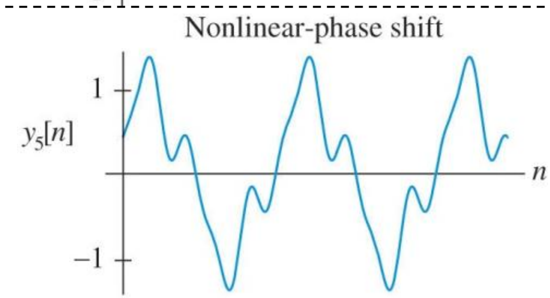
Signal	C_1	C_2	c_3	ϕ_1	ϕ_2	ϕ_3	Phase Shift
$y_3[n]$	1	-1/3	1/5	π/6	π/6	π/6	constant



Signal	C_1	c_2	c_3	ϕ_1	ϕ_2	ϕ_3	Phase Shift
$y_4[n]$	1	-1/3	1/5	-π/4	-3π/4	-5π/4	linear



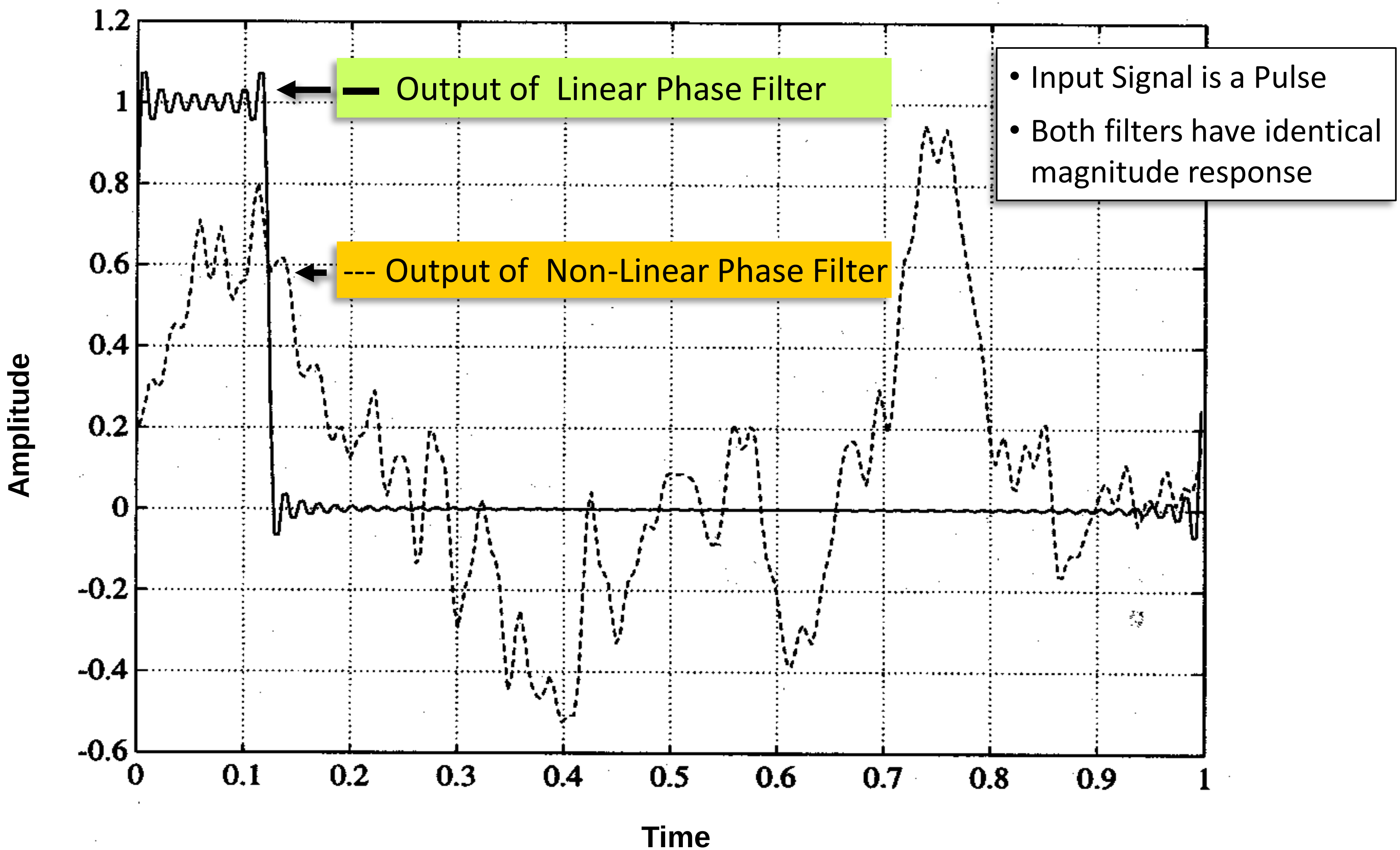
Signal	C_1	c_2	c_3	ϕ_1	ϕ_2	ϕ_3	Phase Shift
$y_5[n]$	1	-1/3	1/5	-π/3	π/4	π/7	nonlinear



$\Omega_0 = 0.004\pi$

Frequency Response of Digital Filters

Phase distortion due to non-linear phase response



Linear Phase Response

Phase Response and Distortion

Phase Shift, Phase Delay, Group Delay, Linear Phase Response

Phase Shift: The phase response gives the phase shift (in radians) experienced by each sinusoidal component of the input signal

$$y[n] = A_x \left|H(e^{j\Omega})\right| \cos\left(\Omega n + \varphi_x + \angle H(e^{j\Omega})\right)$$

Phase Delay: The phase response divided by the frequency gives the time shift (in number of samples) experienced by each sinusoidal component of the input signal

$$y[n] = A_x \left|H(e^{j\Omega})\right| \cos\left\{\Omega\left(n + \frac{\varphi_x}{\Omega} + \frac{\angle H(e^{j\Omega})}{\Omega}\right)\right\} \quad , \quad \tau_{pd} = -\frac{\angle H(e^{j\Omega})}{\Omega}$$

Linear Phase Response : $\angle H(\Omega) = -\Omega n_d \Rightarrow \tau_{pd} = -\angle H(e^{j\Omega}) / \Omega = \Omega n_d / \Omega \Rightarrow \tau_{pd} = n_d$

No phase distortion as all sinusoidal components of input signal experience the same delay (n_d samples)

Group Delay: The negative of the slope of the continuous (unwrapped) phase response $\Psi(\Omega)$

$$\tau_{gd} = -\frac{d\,\Psi(\Omega)}{d\Omega}$$

Constant group delay more relaxed condition than constant phase delay

Generalised linear phase response: $\angle H(\Omega) = \theta - \Omega n_d \Rightarrow \tau_{gd} = n_d$

Linear Phase Response

Phase Response

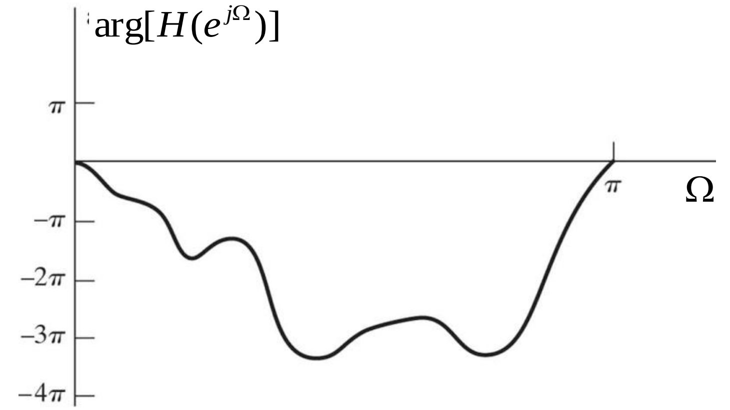
Unwrapped & wrapped phase (continuous & principal phase)

Unwrapped Phase (continuous phase)

Continuous phase response evaluated on the unit circle

$$\Psi(\Omega) = \arg[H(e^{j\Omega})] = ARG[H(e^{j\Omega})] + 2\pi r(\Omega)$$

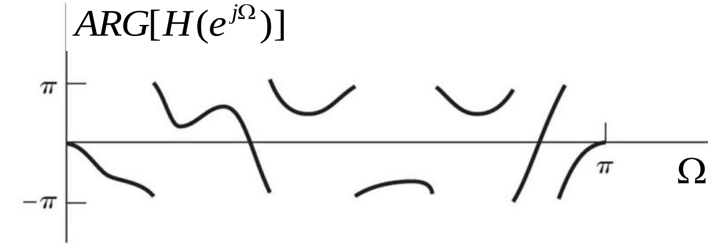
$r(\Omega)$ integer



Wrapped Phase (principal phase)

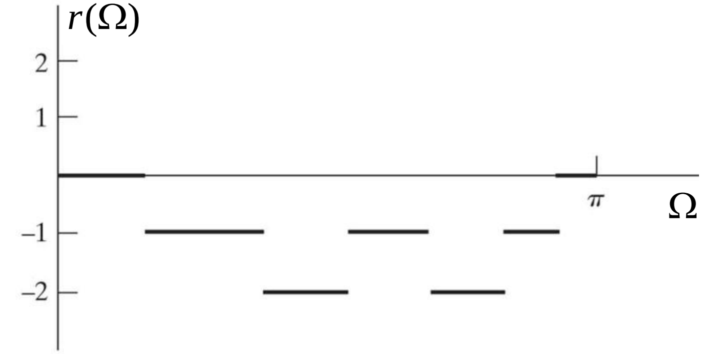
Principal value of the phase

$$\angle H(e^{j\Omega}) = ARG[H(e^{j\Omega})], \ -\pi < ARG[H(e^{j\Omega})] \leq \pi$$



From principal to continuous phase

Integer values of 2π to be added to the above wrapped phase in order to obtain the continuous phase



Linear Phase Response

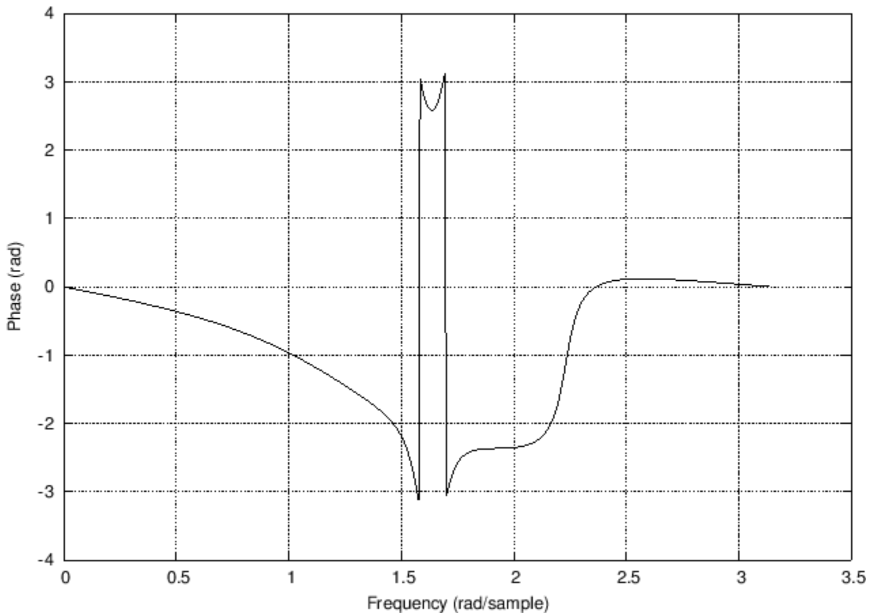
Phase Response

Unwrapped & wrapped phase (continuous & principal phase)

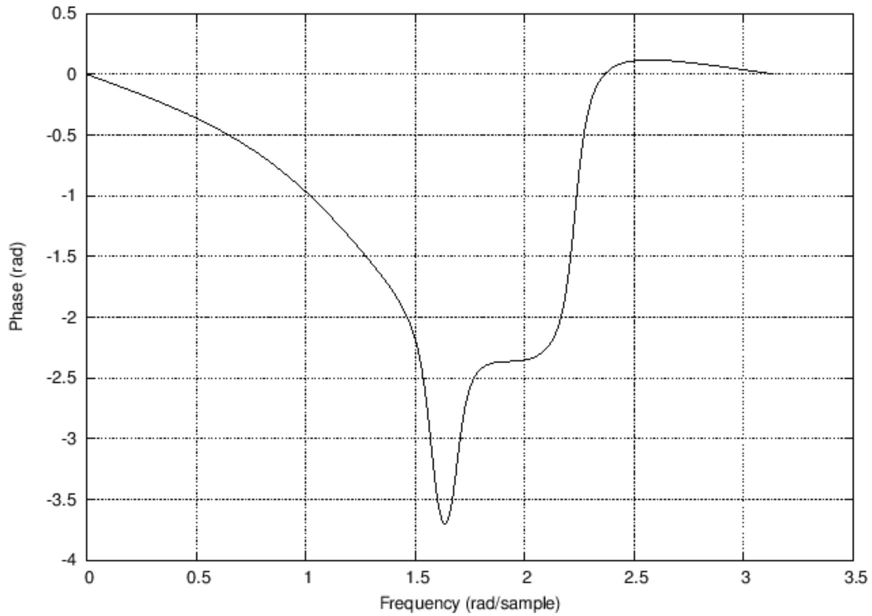
Matlab Example

```
[B,A] = ellip(4,1,20,0.5);          % design lowpass filter
B = B .* (0.95).^[1:length(B)];     % contract zeros by 0.95
[H,w] = freqz(B,A);                 % frequency response
theta = angle(H);                   % phase response
thetauw = unwrap(theta);            % unwrapped phase response
```

Wrapped Phase Response



Unwrapped Phase Response



Linear Phase Response

Zero Phase Response

No group delay

Magnitude Response: $\left|Y(e^{j\Omega})\right| = \left|H(e^{j\Omega})\right|\left|X(e^{j\Omega})\right|$

Phase Response: $\angle Y(e^{j\Omega}) = \angle H(e^{j\Omega}) + \angle X(e^{j\Omega})$

Generalised linear phase response: $\angle H(\Omega) = \theta - \Omega n_d \Rightarrow \tau_{gd} = -\dfrac{d\,\Psi(\Omega)}{d\Omega} = \boxed{n_d}$ Constant group delay

No Distortion

Zero Phase Response - No Distortion, No Delay: $\angle H(e^{j\Omega}) = 0 \Rightarrow \begin{cases} \angle Y(e^{j\Omega}) = \angle X(e^{j\Omega}) \\ H(e^{j\Omega}) = H_{re}(e^{j\Omega}) \end{cases}$

$H(e^{j\Omega}) = H_{re}(e^{j\Omega})$ Zero phase response implies a purely real non-negative frequency response

Linear Phase Response

FIR Filters with Zero Phase (No Delay) Response

Real Non-negative Frequency Response

- One way to avoid any phase distortion and any delay to the output is to make sure the frequency response of the filter does not delay any of the spectral components
- **Zero Phase frequency response** (constant group delay of zero) is needed for the above.
- A zero phase frequency response has no phase component, that is, **the spectrum is purely real (no imaginary component) and non-negative**
- However, it is NOT possible to design a causal digital filter with a zero phase. (Why?)

  Hint: What do we require from the impulse response to ensure that the frequency response is real and non-negative?

DTFT Property : Symmetric Signals

$$\boxed{x[n] = x^*[-n]} \overset{DTFT}{\longleftrightarrow} \boxed{X(\Omega) = X^*(\Omega)}$$

Symmetry about zero — Real Frequency Response

- The impulse response of the filter has to be symmetric about zero for the frequency response to be real i.e. for the phase response (and hence group delay) to be zero
- Symmetry about zero means the filter is non causal

Linear Phase Response

Causal Linear Phase FIR Filters

FIR Filters with constant group delay

Linear phase response can be achieved by filters with conjugate symmetry or anti-symmetry

Symmetric filter coefficients (real) : Even Symmetry

Phase response

even magnitude response

$$h[n] = \begin{cases} h[N-n], & 0 \le n \le N \\ 0, & \text{otherwise} \end{cases} \Rightarrow H(e^{j\Omega}) = \boxed{G_e}(e^{j\Omega})e^{-j\Omega N/2}$$

$$\Psi(\Omega) = -\frac{\Omega N}{2}$$

Anti-symmetric filter coefficients (real) : Odd Symmetry

Phase response

odd magnitude response

$$h[n] = \begin{cases} -h[N-n], & 0 \le n \le N \\ 0, & \text{otherwise} \end{cases} \Rightarrow H(e^{j\Omega}) = \boxed{G_o}(e^{j\Omega})e^{-j\Omega N/2 + j\pi/2}$$

$$\Psi(\Omega) = \frac{\pi}{2} - \frac{\Omega N}{2}$$

Group delay in both cases $\tau_{gd} = -\frac{d\Psi(\Omega)}{d\Omega} = N/2$, N=filter order and N+1 = filter length

Filter coefficients (complex) with complex conjugate symmetry $h[k] = h^*[N-k]$

Filter coefficients (complex) with complex conjugate anti-symmetry $h[k] = -h^*[M-k]$

Linear Phase Response

Causal Linear Phase FIR Filters

Frequency response

$$H(z=e^{j\Omega})=\sum_{k=0}^{N} b_k e^{-jk\Omega}$$, where b_k are the filter coefficients

Even Symmetry (symmetric) $b_k = b_{N-k}$	Odd Symmetry (anti-symmetric) $b_k = -b_{N-k}$
$H(e^{j\Omega})=\sum_{k=0}^{N} b_k e^{-jk\Omega}$	$H(e^{j\Omega})=\sum_{k=0}^{N} b_k e^{-jk\Omega}$
$=\frac{1}{2}\sum_{k=0}^{N} b_k e^{-jk\Omega}+\frac{1}{2}\sum_{k=0}^{N} b_{N-k} e^{-j(N-k)\Omega}$	$=\frac{1}{2}\sum_{k=0}^{N} b_k e^{-jk\Omega}+\frac{1}{2}\sum_{k=0}^{N} b_{N-k} e^{-j(N-k)\Omega}$
$=\frac{1}{2}\sum_{k=0}^{N} b_k e^{-jk\Omega}+b_k e^{-j(N-k)\Omega}$	$=\frac{1}{2}\sum_{k=0}^{N} b_k e^{-jk\Omega}-b_k e^{-j(N-k)\Omega}$
$=\frac{1}{2}e^{\frac{-jN\Omega}{2}}\sum_{k=0}^{N} b_k\left(e^{j\left(\frac{N}{2}-k\right)\Omega}+e^{-j\left(\frac{N}{2}-k\right)\Omega}\right)$	$=\frac{1}{2}e^{\frac{-jN\Omega}{2}}\sum_{k=0}^{N} b_k\left(e^{j\left(\frac{N}{2}-k\right)\Omega}-e^{-j\left(\frac{N}{2}-k\right)\Omega}\right)$
$=e^{\frac{-jN\Omega}{2}}\sum_{k=0}^{N} b_k\cos\left(\left(\frac{N}{2}-k\right)\Omega\right)$	$=je^{\frac{-jN\Omega}{2}}\sum_{k=0}^{N} b_k\sin\left(\left(\frac{N}{2}-k\right)\Omega\right)$
$=e^{\frac{-jN\Omega}{2}}\sum_{k=0}^{N} b_k\cos\left((N-2k)\Omega/2\right)$	$=je^{\frac{-jN\Omega}{2}}\sum_{k=0}^{N} b_k\sin\left((N-2k)\Omega/2\right)$

Linear Phase Response

Causal Linear Phase FIR Filters

Restrictions on the location of zeros for symmetry & anti-symmetry

	Odd Length (N+1)	Even Length (N+1)
	Even Order (N)	Odd Order (N)
Symmetric (Even Symmetry)		Zeros at $z = -1$
Anti-symmetric (Odd Symmetry)	Zeros at $z = \pm 1$	Zeros at $z = 1$

Zero at z =1 (Ω=0) :Anti-symmetric filters cannot be low pass

Zero at z=-1 (Ω=π) :Even length symmetric and odd length anti-symmetric filters can't be high pass

To see the relation between the location of zeros and the type of symmetry and length check what happens to the frequency response at Ω=0 and Ω=π for N odd and N even bearing in mind that N-2k is odd if N is odd and even if N is even

Frequency Response

Symmetric Filter (even symmetry)

$$H(\Omega) = e^{\frac{-jN\Omega}{2}} \sum_{k=0}^{N} b_k \cos\left(\left(N-2k\right)\Omega/2\right)$$

Anti-symmetric filter (odd symmetry)

$$H(\Omega) = je^{\frac{-jN\Omega}{2}} \sum_{k=0}^{N} b_k \sin\left(\left(N-2k\right)\Omega/2\right)$$

Linear Phase Response

Causal Linear Phase FIR Filters

Types of linear phase FIR filters based on type of symmetry and length

	Odd Length (N+1)	Even Length (N+1)
	Even Order (N)	Odd Order (N)
Symmetric (Even Symmetry)		Zeros at $z = -1$
Anti-symmetric (Odd Symmetry)	Zeros at $z = \pm 1$	Zeros at $z = 1$

Zero at z =1 (Ω=0) :Anti-symmetric filters cannot be low pass

Zero at z=-1 (Ω=π) :Even length symmetric and odd length anti-symmetric filters can't be high pass

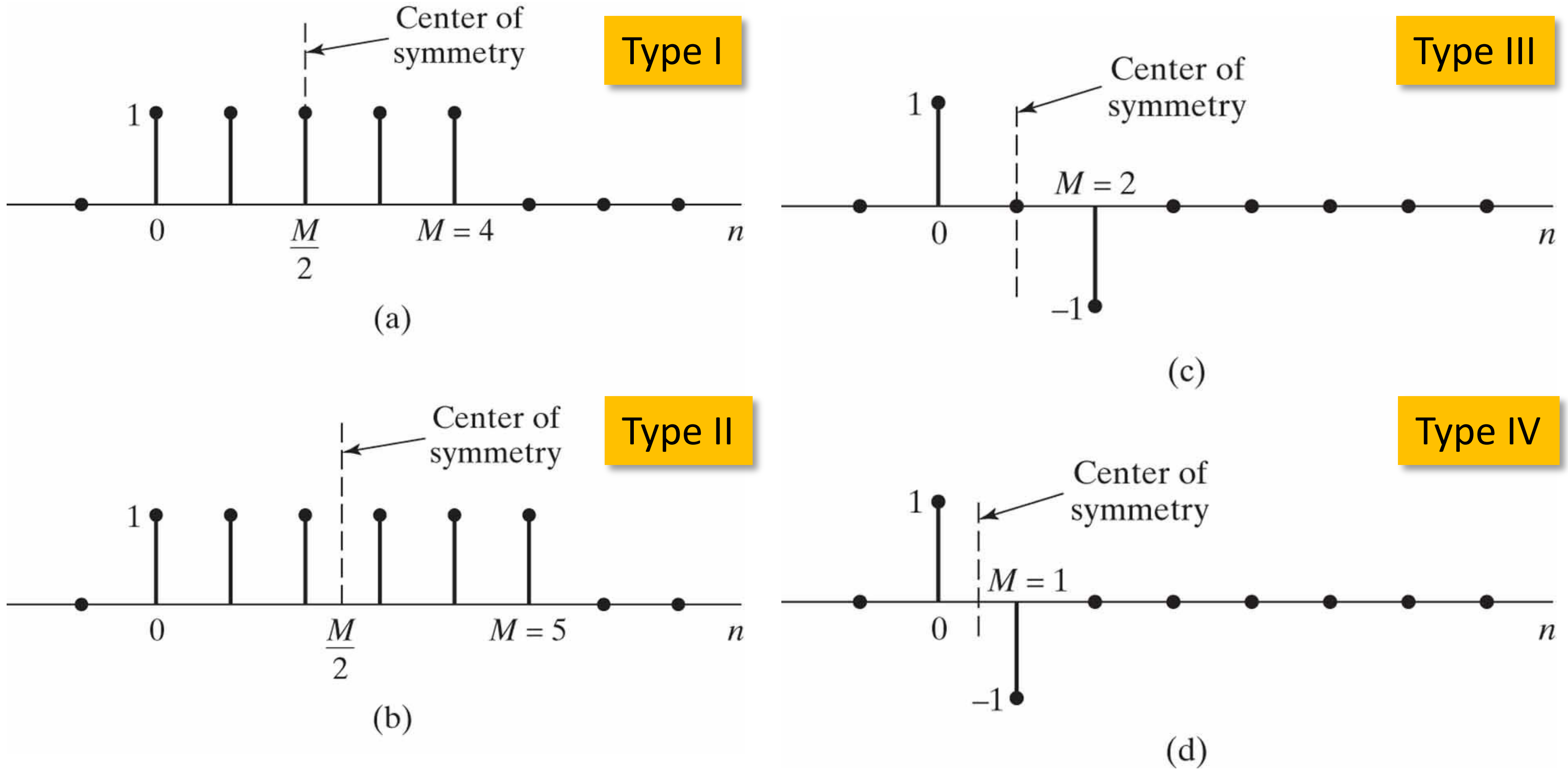
Type I FIR Filters most flexible

	Odd Length (N+1)	Even Length (N+1)
	Even Order (N)	Odd Order (N)
Symmetric (Even Symmetry)	Type I	Type II
Anti-symmetric (Odd Symmetry)	Type III	Type IV

Linear Phase Response

Causal Linear Phase FIR Filters

Types of Linear Phase FIR filters



(a) Type I, M even, $h[n] = h[M - n]$.

(b) Type II, M odd, $h[n] = h[M - n]$.

(c) Type III, M even, $h[n] = -h[M - n]$.

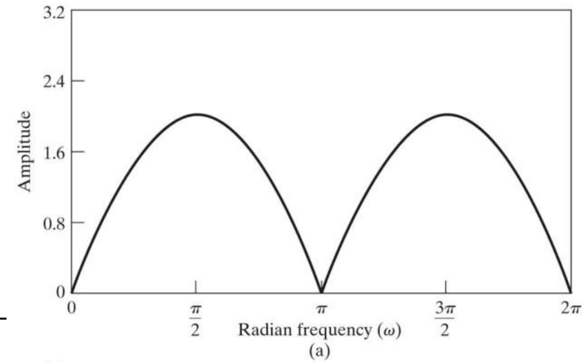
(d) Type IV, M odd, $h[n] = -h[M - n]$.

Causal Linear Phase FIR Filters

Example : Type III FIR Filter (odd length, anti-symmetric)

Amplitude Response

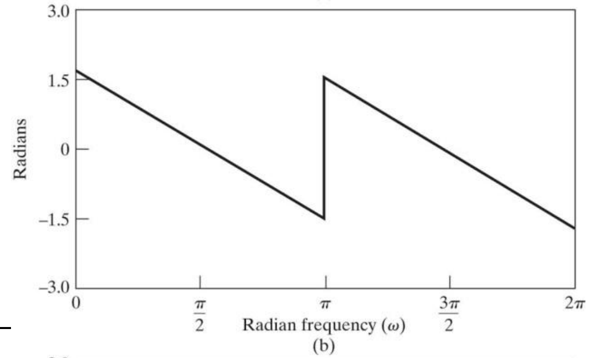
$$h[n] = \delta[n] - \delta[n-2], \qquad \left|H(e^{j\Omega})\right| = \left|1 - e^{-j2\Omega}\right|$$



Phase Response (radians)

$$\Psi(\Omega) = \frac{\pi}{2} - \frac{\Omega M}{2}$$

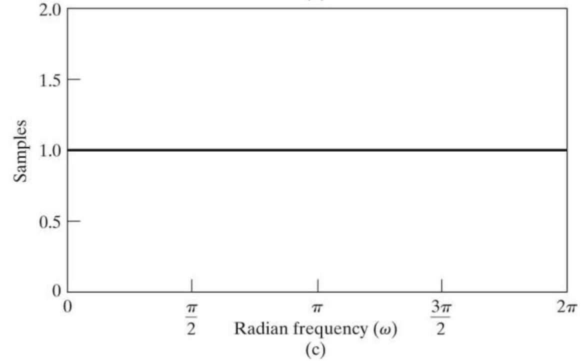
$M = 2$ (filter order)



Group Delay (samples)

$$\tau_{gd}(\Omega) = -\frac{d\Psi(\Omega)}{d\Omega} = M/2$$

$\tau_{gd}(\Omega) = 1$ (constant group delay as a result of linear phase)



Linear Phase Response

Causal Linear Phase FIR Filters

Example : Type IV FIR Filter (even length, anti-symmetric)

Amplitude Response

$$h[n] = \delta[n] - \delta[n-1], \quad \left|H(e^{j\Omega})\right| = \left|1 - e^{-j\Omega}\right|$$

Phase Response (radians)

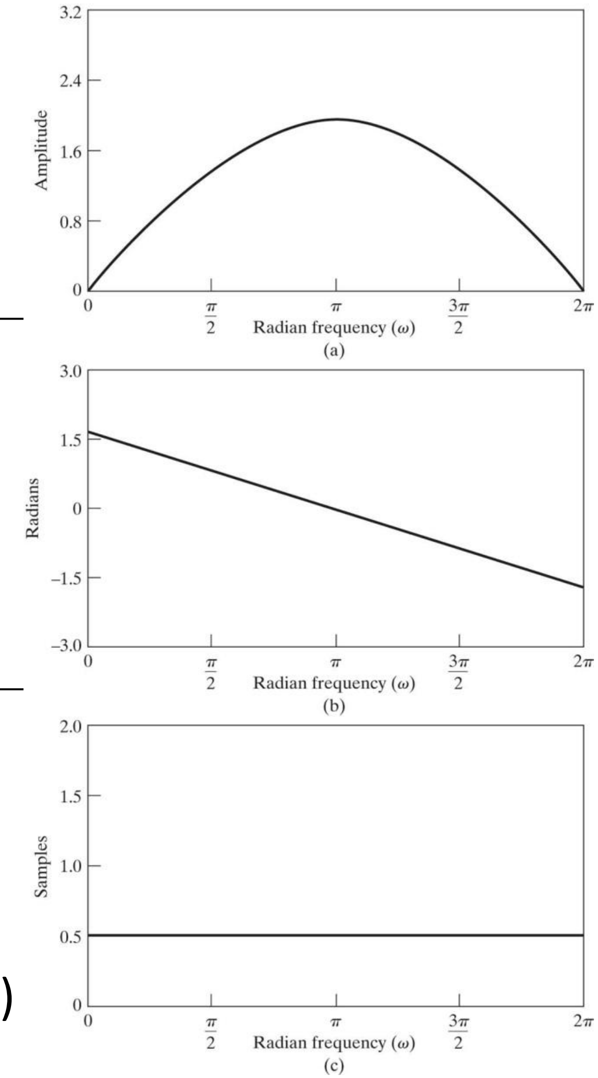
$$\Psi(\Omega) = \frac{\pi}{2} - \frac{\Omega M}{2}$$

$M = 1$ (filter order)

Group Delay (samples)

$$\tau_{gd}(\Omega) = -\frac{d\Psi(\Omega)}{d\Omega} = M/2$$

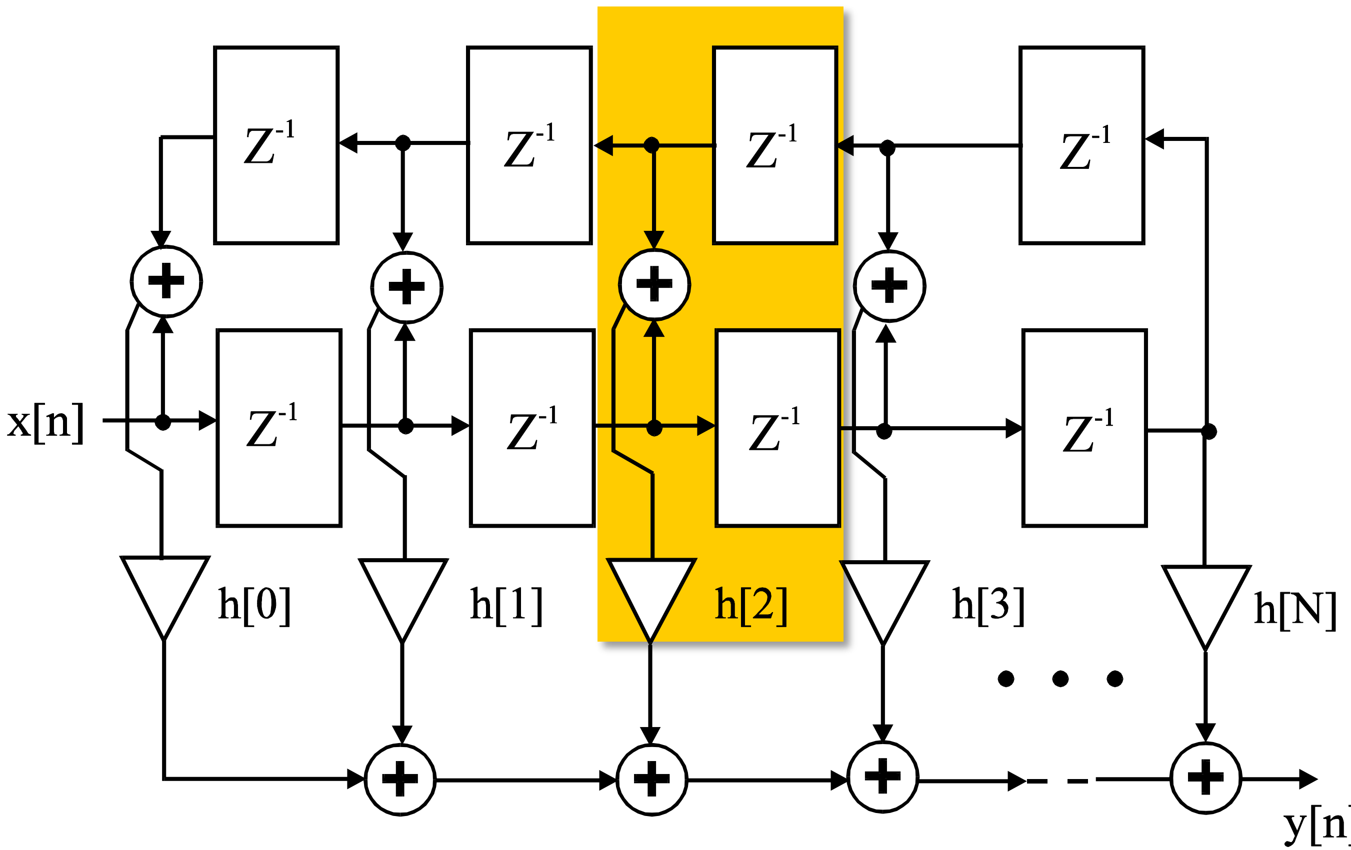
$\tau_{gd}(\Omega) = 0.5$ (constant group delay as a result of linear phase)



Linear Phase Response

Causal Linear Phase FIR Filters

Implementation savings due to symmetry



One multiplication for two filter taps due to symmetry

Linear Phase Response

Zero Phase IIR Filtering

Non-Causal IIR filtering with zero phase response

A zero-phase filtering scheme can be obtained by the following procedure:

1. Process the input data (finite length) with a causal real-coefficient filter H(z).
2. Time reverse the output of this filter (flip)…
3. .. and process again by the same filter.
4. Time reverse once again the output of the second filter

x[n] → H(z) → v[n] → Flip → v[-n]=u[n] → H(z) → w[n] → Flip → w[-n]=y[n]

$$\left.\begin{aligned} V(\Omega) &= H(\Omega)\,X(\Omega) \\ u[n] &= v[-n] \end{aligned}\right\} \; U(\Omega) = V^*(\Omega)$$

(1) (2)

$$\left.\begin{aligned} W(\Omega) &= H(\Omega)\,U(\Omega) \\ y[n] &= w[-n] \end{aligned}\right\} \; Y(\Omega) = W^*(\Omega) = H^*(\Omega)U^*(\Omega)$$

(3)

$$Y(\Omega) \overset{(2,3)}{=} H^*(\Omega)V(\Omega) \overset{(1)}{=} H^*(\Omega)H(\Omega)X(\Omega) = \boxed{|H(\Omega)|^2} X(\Omega)$$

Real Frequency Response => Zero Phase

Offline IIR filtering with no phase distortion (Matlab : *filtfilt*() function – see Matlab help)

Linear Phase Response

Phase Distortion and Audio Signals

Can you hear any difference?