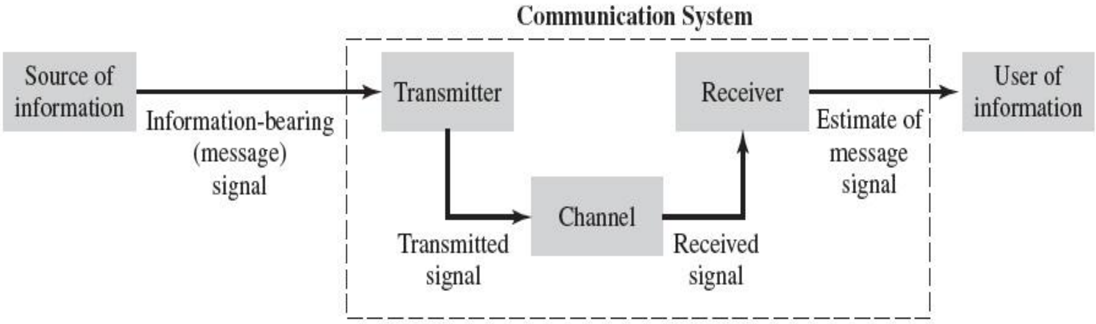
Digital Communications System



- Information
- Transmitter
- Channel & Noise
- Receiver
- Use of Information

Digital communication is reliable transmission of bits over an unreliable physical medium. The block in the above diagram shows a high level representation of a typical communication system.

There are two types of communications:

-*Broadcasting*, which involves the use of a single powerful transmitter and numerous receivers that are relatively inexpensive to build. In that case information bearing signals flow only in one direction from the transmitter to each of the receivers out there in the field.

-*Point-to-point communications*, in which the communication process takes place over a link between a single transmitter and a single receiver. In this class of communications systems there is usually bidirectional flow of information- bearing signals which in effect requires a transmitter and a receiver at each end of the link (transceiver)

BASEBAND DATA COMMUNICATIONS

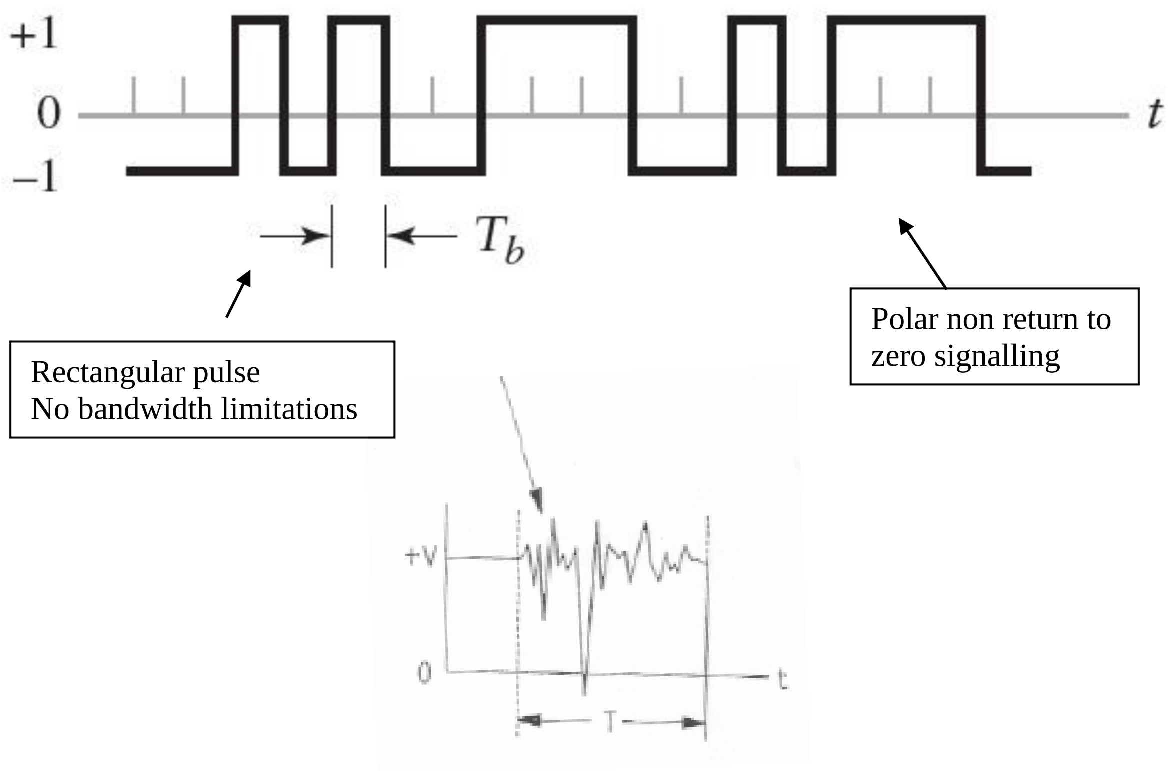
Data transmission:

Data Set	Representation	
1	+V (+1) held for T seconds	$S_1(t)$
0	-V (-1) held for T seconds	$S_2(t)$

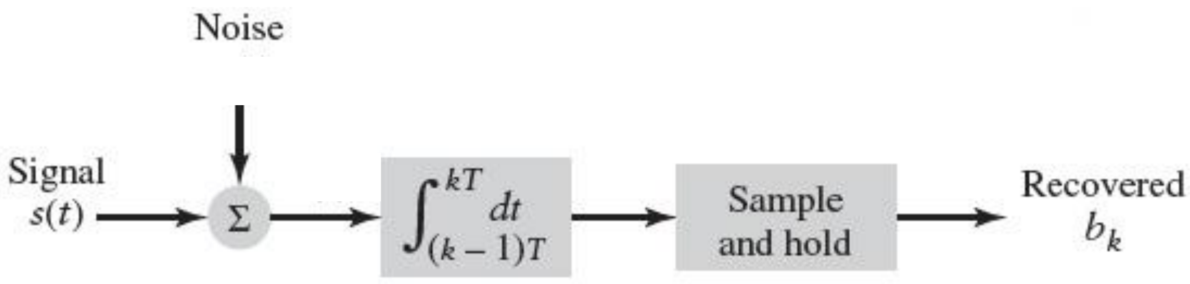
Binary logic levels

- Usually this signal is used to modulate the carrier (eg BPSK).
- Upon reception, the signal is corrupted by noise. Noise is modelled as AWGN

$\therefore$ Finite probability that an error will be made in determining which data value was transmitted



INTEGRATE & DUMP FILTER



Passing the signal through an integrator improves SNR

ANALYSIS OF INTEGRATE & DUMP FILTER

Let τ = RC (integration constant), then

$$v_o(T) = \frac{1}{\tau}\int_0^T [s(t) + n(t)]dt$$

$$= \frac{1}{\tau}\int_0^T s(t)dt + \frac{1}{\tau}\int_0^T n(t)dt$$

Output signal voltage is a RAMP

$$s_o(T) = \frac{1}{\tau}\int_0^T V(t) = VT/\tau$$

Output noise voltage is a Gaussian random variable, the variance of which is equal to the average Noise Power:

$$\sigma_o^2 = \frac{N_o T}{2\tau^2}$$

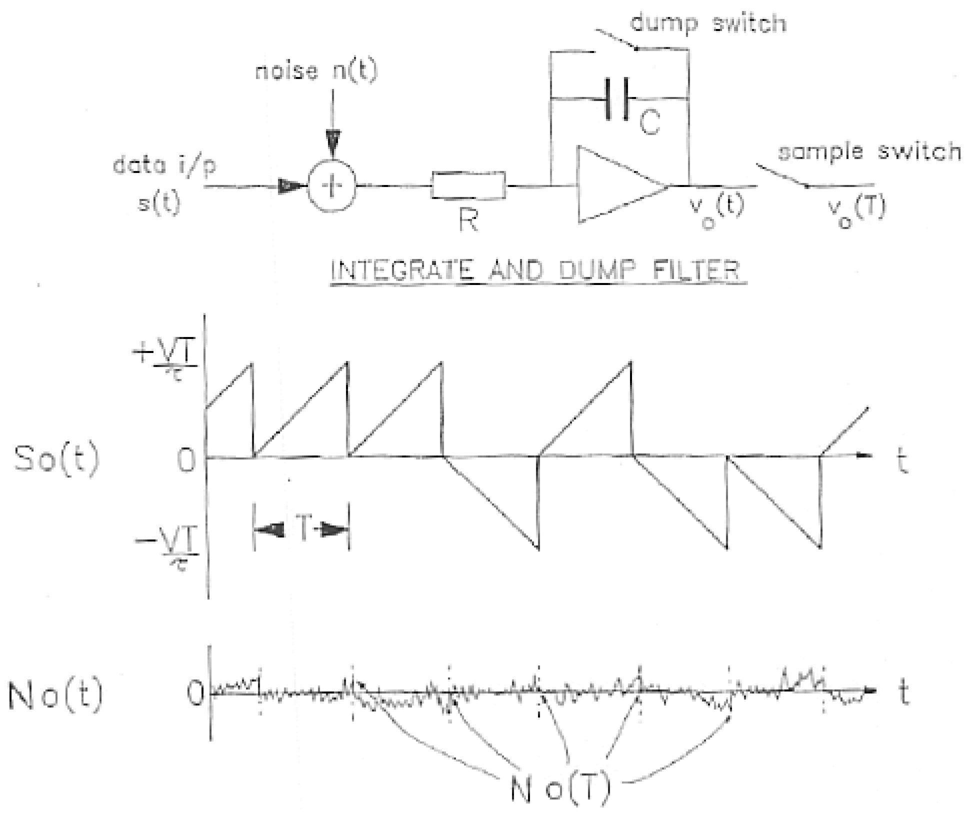
Remember that $N_o/2$ is the two-sided power spectral density of gaussian white noise.

<u>Figure of Merit</u> – Signal to Noise Ratio

$$\mathrm{SNR}_o = \frac{[s_o(T)]^2}{[n_o(T)]^2} = \frac{V^2T^2/\tau^2}{N_oT/2\tau^2}$$

$$= \frac{2}{N_o}V^2T = \frac{2Es}{N_o}$$

We reset the integrator at the beginning of each new symbol



- $\mathrm{SNR_o}$ increases with increasing symbol duration (T) and increasing level (V) . It is proportional to the symbol energy $\mathrm{V^2T}$.

- Sampled signal voltage s_o(T) increases linearly with T.

- Sampled Noise voltage increases as $\sqrt{T}$

- Signal processing advantage of filter is $\sqrt{\mathrm{T}}$

DECISION ERROR PROBABILITY
(for an integrate & dump filter)

- The probability density of the noise sample $n_0(T)$ is gaussian, with a density:

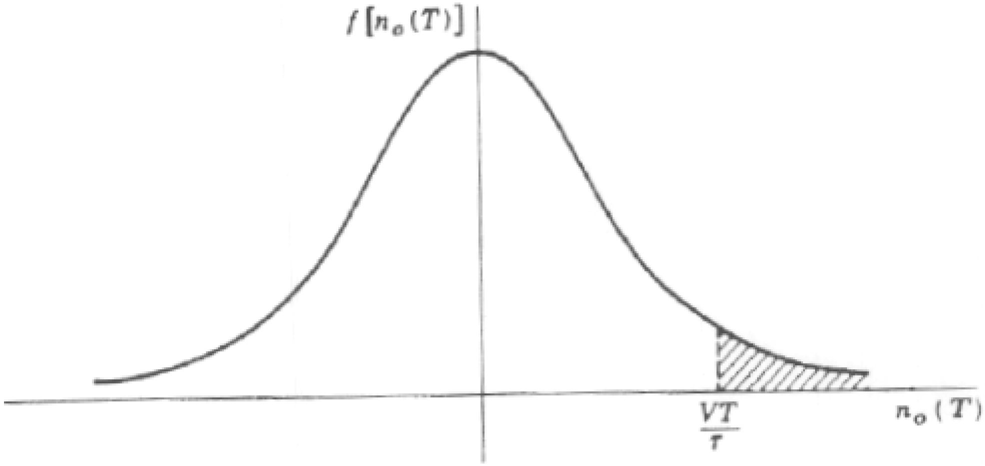
$$f[n_o(T)] = \frac{e^{-n^2_o(T)/2\sigma^2_o}}{\sqrt{2\pi\sigma_o^{\,2}}}$$

where $\sigma_o^2 = \text{variance} = \overline{n_o^2(T)}$ is the variance after the integrate and dump filter

- A received negative symbol (-V) will be detected in error, if the noise sample $n_0(T)$ is positive and larger in magnitude than the sampled symbol $s_o(T)$, (VT/τ). Giving the error probability as:

$$P_e = \int_{VT/\tau}^{\infty} f[n_o(T)]dn_o(T)$$

$$= \int_{VT/\tau}^{\infty} \frac{e^{-n_o^2(T)/2\sigma_o{}^2}}{\sqrt{2\pi\sigma_o^{\,2}}} dn_o(T)$$



To solve this integral, make subs: $x = n_o(T)/\sqrt{2}\,\sigma_o$

$$P_e = \frac{1}{\sqrt{\pi}} \int_{x=V\sqrt{T/N_o}}^{\infty} e^{-x^2}\,dx \qquad \text{as} \qquad \sigma_o^2 = \frac{N_o T}{2\tau^2}$$

THE ERROR FUNCTION ERF (X)

Def ^{n}

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_{o}^{x} e^{-t^{2}}\, dt$$

Also, the complementary error function erfc (x)

Def ^{n}

$$\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_{x}^{\infty} e^{-t^{2}}\, dt = 1-\operatorname{erf}(x)$$

DECISION ERROR PROBABILITY cont

So

$$P_e = \frac{1}{2}\operatorname{erfc}\left(V\sqrt{\frac{T}{N_o}}\right) = \frac{1}{2}\operatorname{erfc}\left(\frac{V^2 T}{N_o}\right)^{1/2}$$

$$= \frac{1}{2}\operatorname{erfc}\left(\frac{E_s}{N_o}\right)^{1/2}$$

- If +V is transmitted, by symmetry the P_e is identical to that given above for –V.

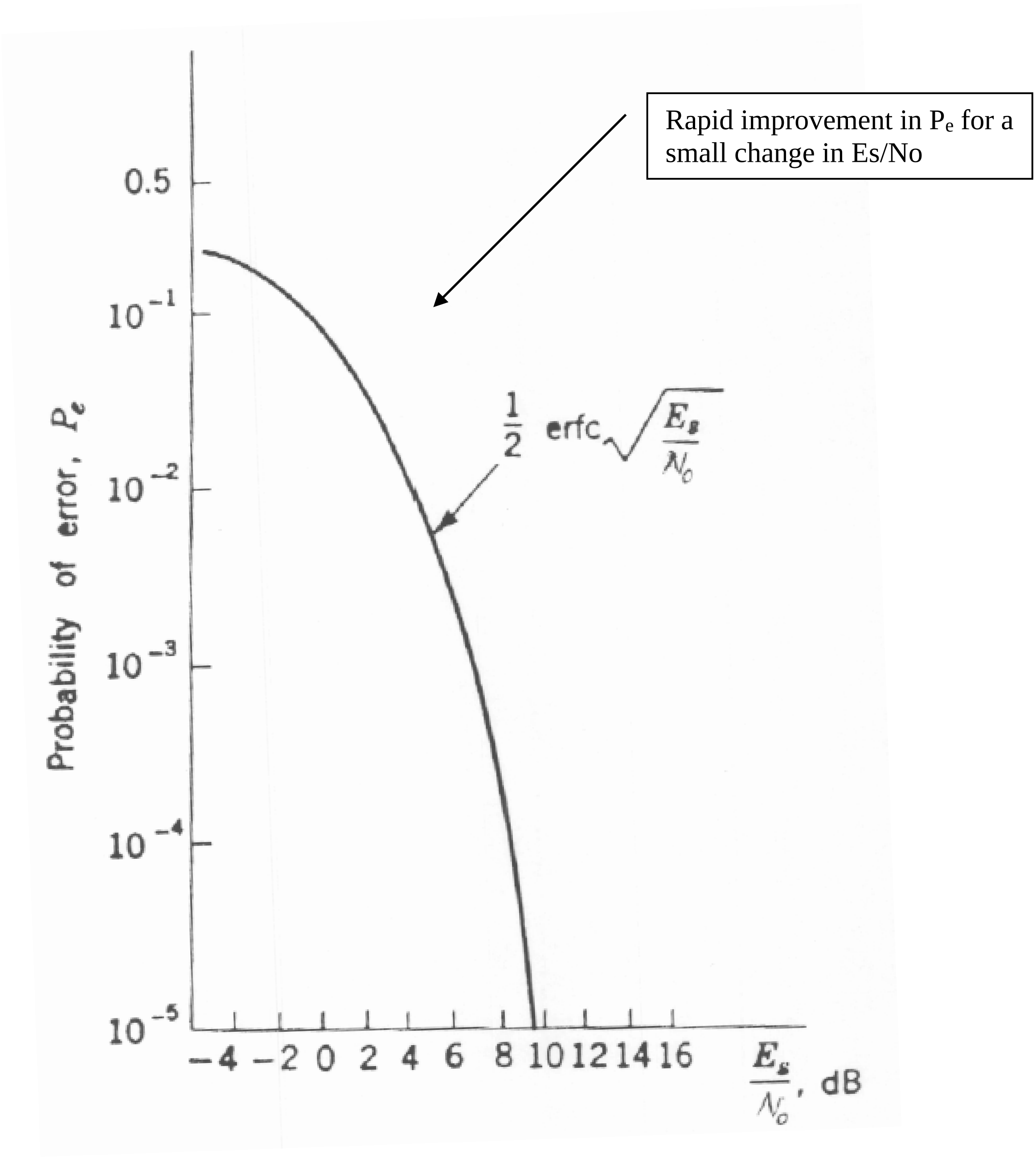
Refer to the following P_e paragraph

- P_e rapidly decreases as E_s/N_o increases

- Maximum value of P_e = 0.5. (Thus, even if the signal is entirely lost in the noise, so that any determination by the receiver is a sheer guess, the receiver cannot be wrong more than half the time on average)

- The average probability of symbol depends solely on E_s/N_o

VARIATION OF Pe VERSUS Es/No



Optimum & Matched Filter Detection

- Optimum is the filter that maximises SNR. Hence the ideal filter that minimises the effect of noise.

- Is the Integrate & Dump Filter the Optimum Filter for the detection of a data signal in Noise?

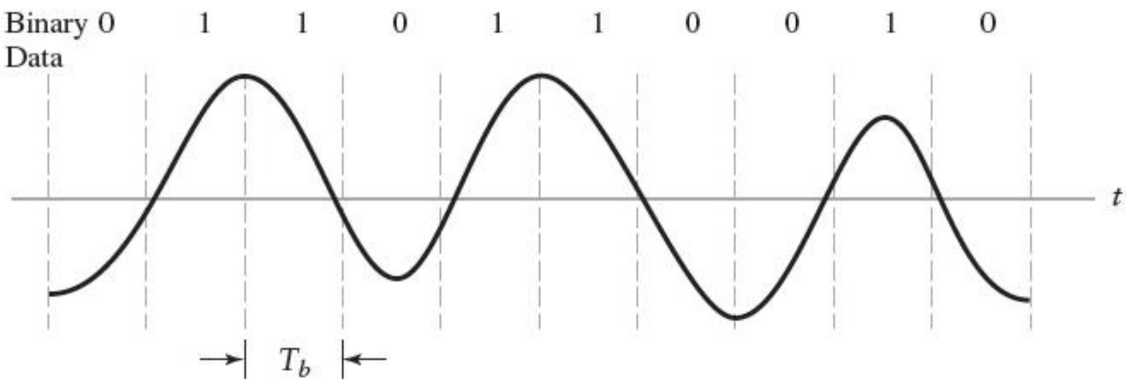
- For unfiltered rectangular data pulses in white noise the integrate & dump filter is the optimum or matched detector

- In the case of filtered data transmission, the detection filter must be matched to the pulse shaping employed. If s(t) is the signal then:

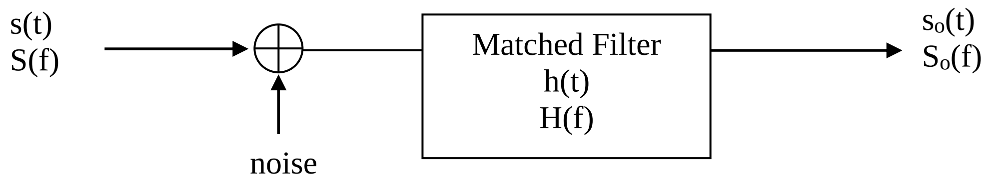
$$h_{opt} = s(T - t)$$

- In that case again SNR = $2 E_s / N_o$ and the BER is as before.

- The term correlation receiver is often used in the case of coherent reception



Matched Filter Detection



Impulse response of a matched filter

$$h_{opt}(t) = s(T - t)$$

and hence in the frequency domain:

$$H_{opt}(f) = S^{*}(f)e^{-j2\pi fT}$$

The signal after the filter is given by:

$$S_o(f) = S(f)H_{opt}(f) = S(f)S^{*}(f)e^{-j2\pi fT} = |S(f)|^2 e^{-j2\pi fT}$$

and in the time domain

$$s_o(t) = \int_{-\infty}^{\infty} S_o(f)e^{j2\pi fT}df = \int_{-\infty}^{\infty} |S(f)|^2 e^{-j2\pi fT}e^{j2\pi fT}df$$

$$= E_s$$

Power = $E_s{}^2$

$$E\left(n^2(t)\right) = \frac{N_o}{2}\int_{-\infty}^{\infty} |H_{opt}(f)|^2 df = \frac{N_o}{2}\int_{-\infty}^{\infty} |S(f)|^2 df = \frac{N_o}{2}E_s$$

$$SNR = \frac{2E_s^2}{N_oE_s} = \frac{2E_s}{N_o}$$

- Independent of the waveform
- Same result as the integrate and dump filter for square pulses

COHERENT RECEPTION: CORRELATION

For the correlator receiver:

$$s_o(t) = \int_0^T r(t)s(t)dt$$

where r(t) is the received signal s(t)+noise

As discussed the matched filter:

$$s_o = r(t) * h_{opt} = \int_0^T r(\lambda)h_{opt}(T-\lambda)d\lambda$$

$$= \int_0^T r(\lambda)s(T-(T-\lambda))d\lambda = \int_0^T r(\lambda)s(\lambda)d\lambda$$

Therefore, the correlator is another technique for realising an optimum filter for detecting s(t). (The output is sampled at t=T)

$$\int_0^T s(t)s(t)dt + \int_0^T n(t)s(t)dt$$

Power E_s^2 | Power $\frac{N_o}{2}E_s$

$$SNR = \frac{2E_s^2}{N_oE_s} = \frac{2E_s}{N_o}$$

<u>PHASE SHIFT KEYING</u>

- Passband Data Transmission (Data stream modulated onto a carrier)

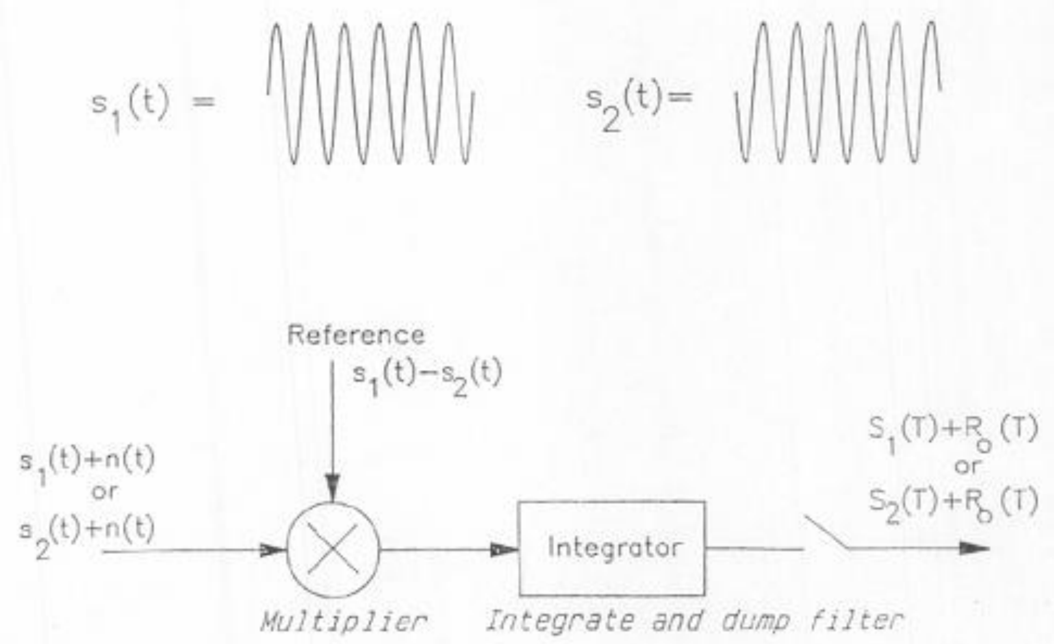
- Application of coherent reception techniques (Antipodal states)

  $s_1(t) = A\cos\omega_o t$ logic 1

  $s_2(t) = -A\cos\omega_o t$ logic 0

- Coherent reference employed in receiver

$$s_1(t) - s_2(t) = 2A\cos\omega_o t$$



CORRELATOR

Pe OF PHASE SHIFT KEYING

- Now since in PSK $S_1(t) = -S_2(t)$, then the error probability can be obtained from the previous case studies (matched filter/ correlator)

$$P_e = \frac{1}{2}\operatorname{erfc}\sqrt{E_s / N_o}$$

- Assumes matched data system

- If bit duration is T, and this encompasses a whole number of carrier cycles, then

  $E_s = A^2T/2$ Carrier Power

  Giving

$$P_e = \frac{1}{2}\operatorname{erfc}\sqrt{A^2T / 2N_o}$$

PRATICAL PSK OPERATION

(a) Imperfect Carrier Recovery

- Correlator reference has a phase error ϕ due to noise etc, therefore output of correlator is reduced by $\cos\phi$.

  Note: only symbol magnitude affected

- Sampled energy reduced by $\cos^2\varphi$

- Resultant error probability

$$P_e(\phi) = \frac{1}{2}\operatorname{erfc}\left(\frac{E_s\cos^2\phi}{N_o}\right)^{1/2}$$

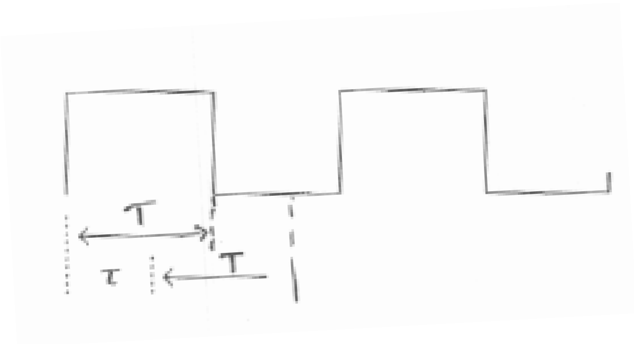
- Typical P_e from 10^{-4} to 10^{-7}

- A phase error of 25^o, will increase symbol error probability by a factor of 10.

PRATICAL PSK OPERATION cont

(b)Imperfect Sample Timing

- Bit synchronisation
- Integration period extends from τ to T + τ, then a portion of the adjacent symbol will be included in the output sample.



- If adjacent symbol is of opposite sign to current symbol, then magnitude of output is reduced:

$$s(T+\tau) = \int_{\tau}^{T} A\cos\omega_o t[2A\cos\omega_o t]dt - \int_{T}^{\tau+T} A\cos\omega_o t[2A\cos\omega_o t]dt$$

$$= A^2T\left[1-\frac{2\tau}{T}\right]$$

PRATICAL PSK OPERATION cont

- Symbol energy is reduced by $\left[1 - \frac{2|\tau|}{T}\right]^2$

- Giving symbol error probability as

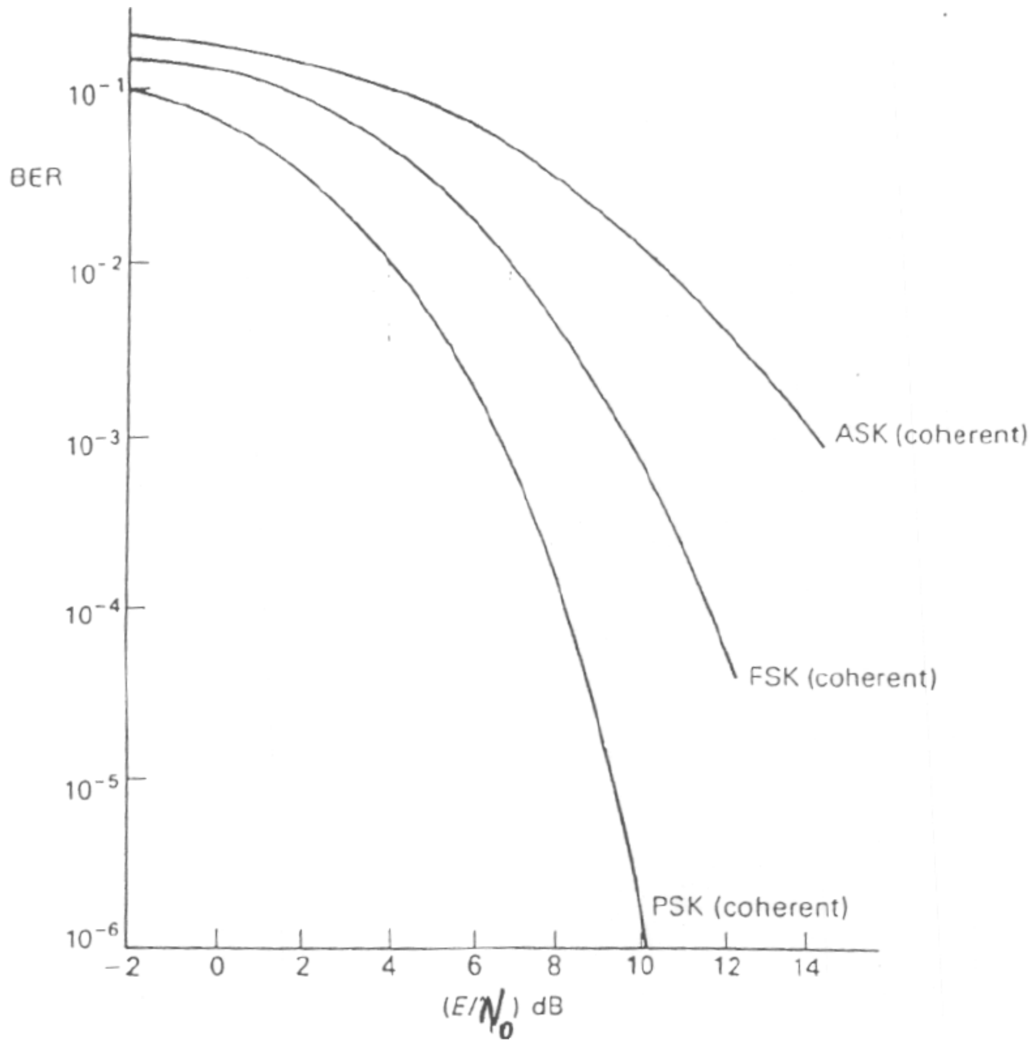
$$P_e(\tau) = \frac{1}{2}\mathrm{erfc}\left[\left(\frac{E_s}{N_o}\right)\left(1 - \frac{2|\tau|}{T}\right)^2\right]^{1/2}$$

- Combining both carrier phase and timing errors gives

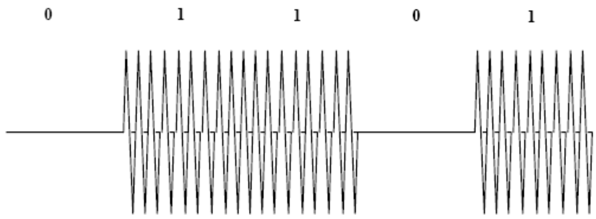
$$P_e(\phi, \tau) = \frac{1}{2}\mathrm{erfc}\left[\left(\frac{E_s}{N_o}\right)\cos^2\phi\left(1 - \frac{2|\tau|}{T}\right)^2\right]^{1/2}$$

- Example, if τ = 0.05, the $P_e\uparrow$ X10

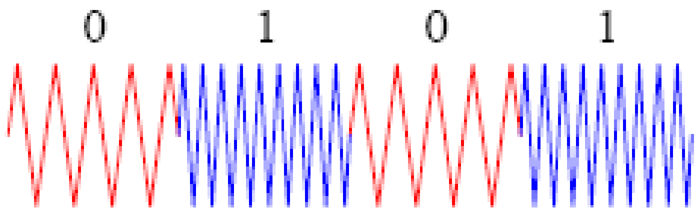
SUMMARY OF BER VERSUS Eb/ No FOR COHERENT MODULATION



System	P_e
ASK Coherent	$0.5\mathrm{erfc}\left(E_b/4N_o\right)^{1/2}$
FSK Coherent	$\sim 0.5\mathrm{erfc}\left(E_b/2N_o\right)^{1/2}$
PSK Coherent	$\sim 0.5\mathrm{erfc}\left(E_b/N_o\right)^{1/2}$

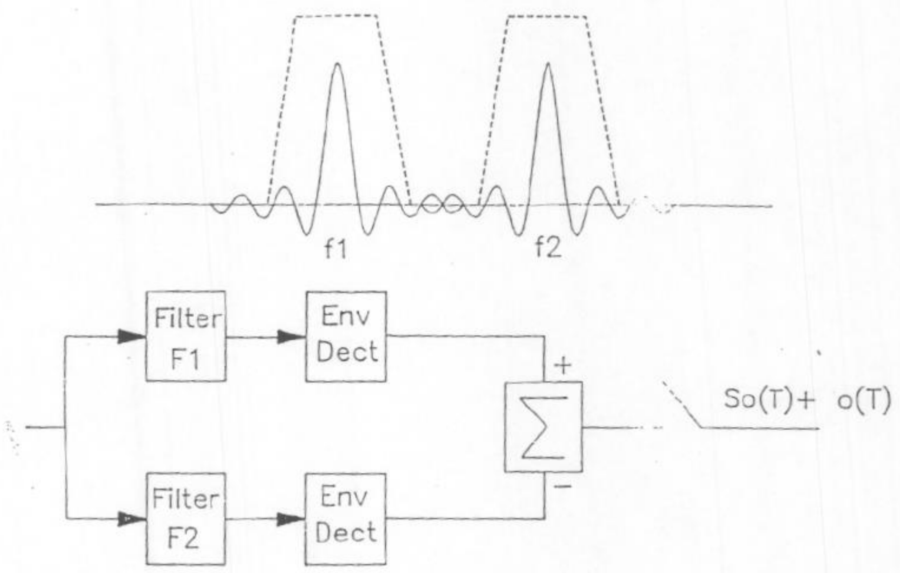


Binary ASK or OOK



Binary FSK

NON – COHERENT FSK DETECTION

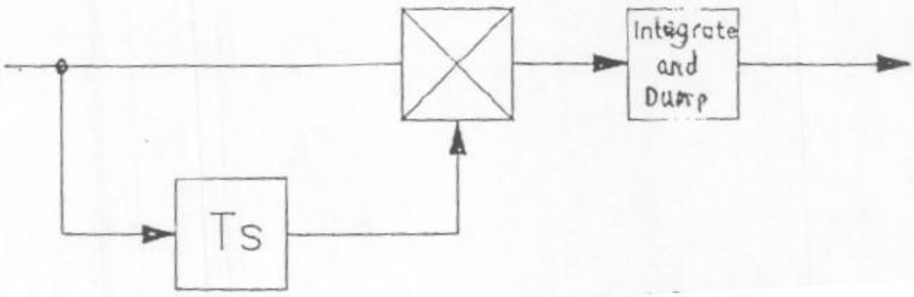


Detects amplitude only, hence inferior to coherent FSK

$$Pe = 0.5*\exp(-Es/2No)$$

DIFFERENTIAL PSK (DPSK)

- Simple circuit, no need to generate local carrier.
- Reference is obtained directly from input waveform.



At the transmitter

Input data stream	1	1	1	0	0	1	0	1	0	1
Delayed Data stream	1	0	1	0	0	0	1	1	0	0
Exclusive Or	0	1	0	0	0	1	1	0	0	1
At the receiver										
Input to receiver	0	1	0	0	0	1	1	0	0	1
Delayed Data	1	0	1	0	0	0	1	1	0	0
Output	1	1	1	0	0	1	0	1	0	1

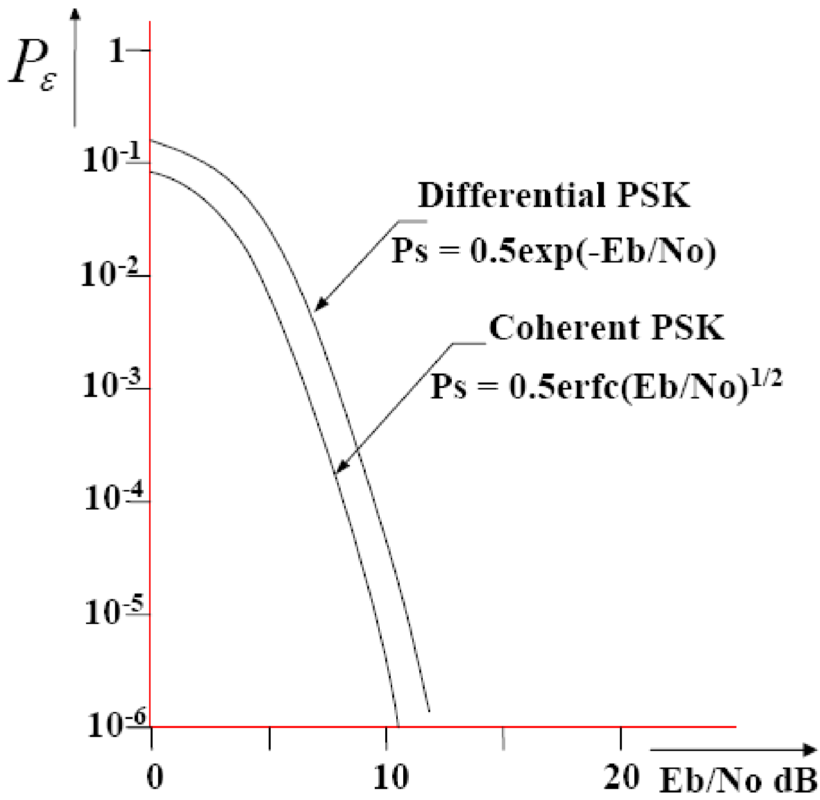
Arbitary starting bit

Output from transmitter

- DPSK is sub-optimum compared with PSK

$$P_e = \frac{1}{2} e^{-Es/No}$$

<u>DIFFERENTIAL PSK (DPSK) PERFORMANCE</u>



M-ARY DATA SYTEMS

- Why
  - Enhanced transmission rate
  - Conserve both power & bandwidth
  - Maintain low P_e

- Many messages in the interval O → T

- Techniques
  - Multi-amplitude eg. 16QAM
  - Multi-phase eg. QPSK
  - Multi-frequency eg. M-ary FSK

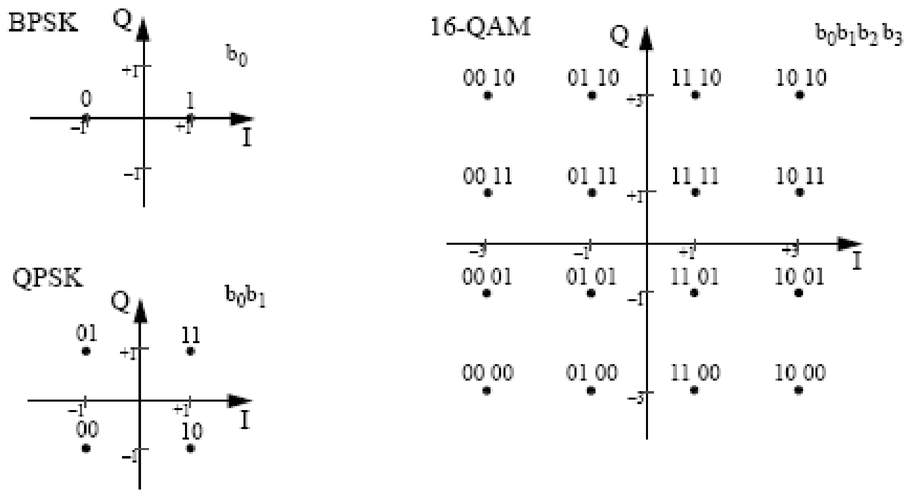
- M-ary systems allow us to conserve power or bandwidth

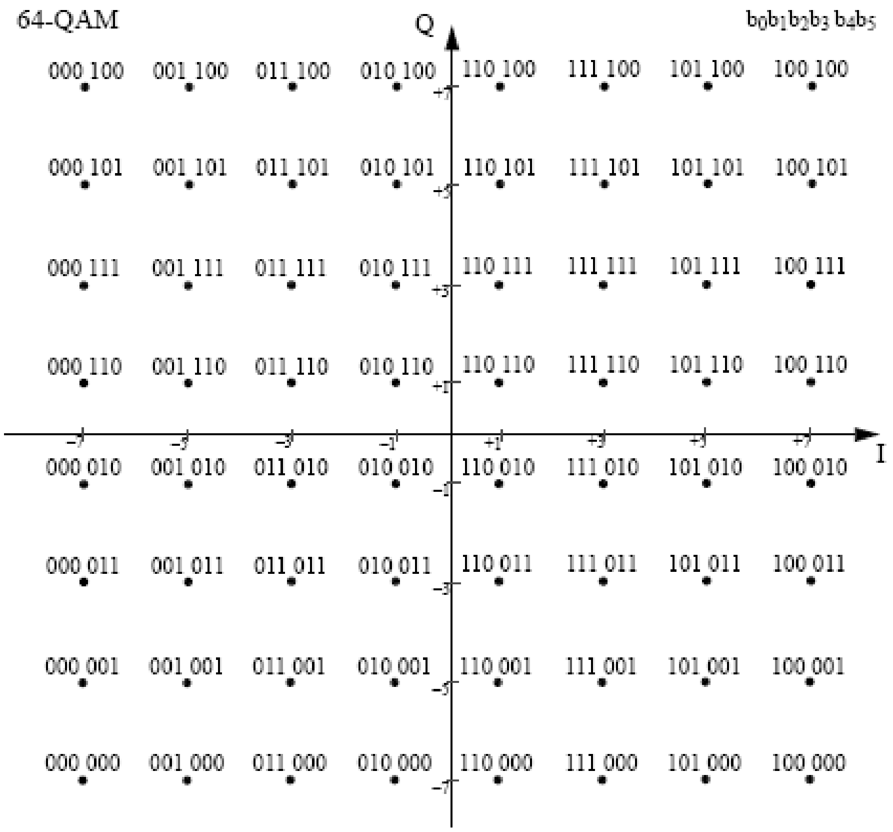
  Eg. QPSK only requires ½ bandwidth of PSK for the same transmission rate and error performance.

- However as the number of symbol states is increased the tolerance to noise is reduced. QPSK is an exception.

- M-ary QAM is used in many of the current wireless communications systems (example WIFI, 3G LTE and WiMax)

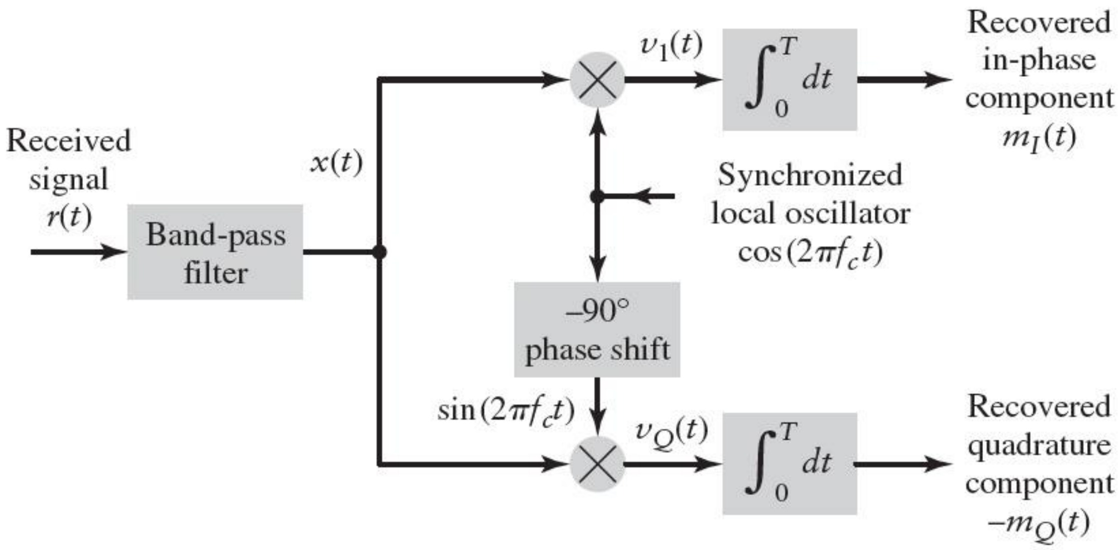
BPSK, QPSK, 16QAM AND 64QAM CONSTELLATION DIAGRAMS





QPSK

It is possible to transmit orthogonal signals at the same transmission link without one set of signals affecting the detection of the other. Cos and Sin waves are orthogonal.



QPSK improves spectral efficiency of BPSK (double)

<u>SYMBOL & BIT ERROR PROBABILTY FOR QPSK</u>

- Two orthogonal BPSK receivers

- Due to orthogonality, bit error probability is

$$P_b = \frac{1}{2}\operatorname{erfc}\sqrt{E_b/N_o} = P_b(\text{BPSK})$$

- For correct symbol detection both correlator outputs must be correct

$$P_c = (1 - P_b)(1 - P_b) = 1 - 2P_b + P_b^2$$

- Now assume that $P_b << 1$, then probability of a symbol error is

$$P_s = 1 - P_c \approx 2P_b =$$
$$\operatorname{erfc}\left(E_b/N_o\right)^{1/2} = \operatorname{erfc}\left(E_S/2N_o\right)^{1/2}$$

- P_s (QPSK) = 2 P_s (BPSK)
- P_b (QPSK) = P_b (BPSK)

- W (QPSK) = W (BPSK)
- R (QPSK) = 2R (BPSK)

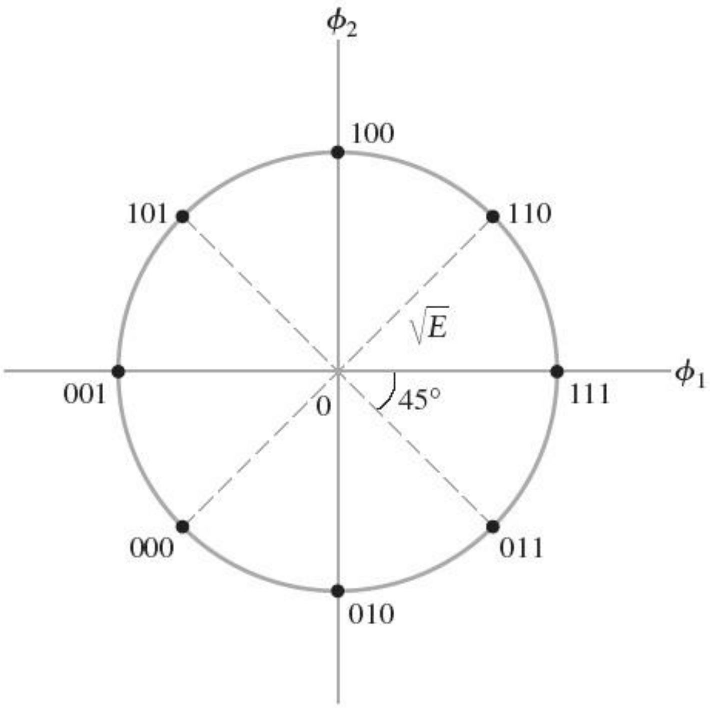
M-ARY PSK – HIGHER ORDER SCHEMES

- For example, 8-PSK and 16-PSK

  - Separation between signal states is reduced when compared with QPSK.

  - Noise immunity worse than QPSK, and hence P_e

  - Can be shown that SNR increases in an exponential manner with N for constant P_e

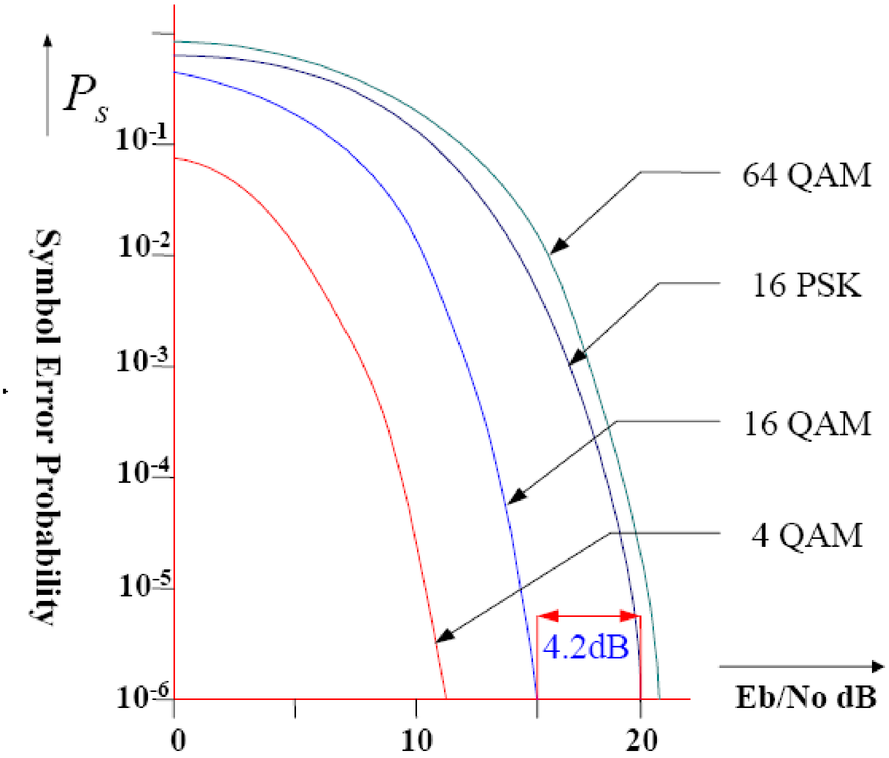


- 16 QAM

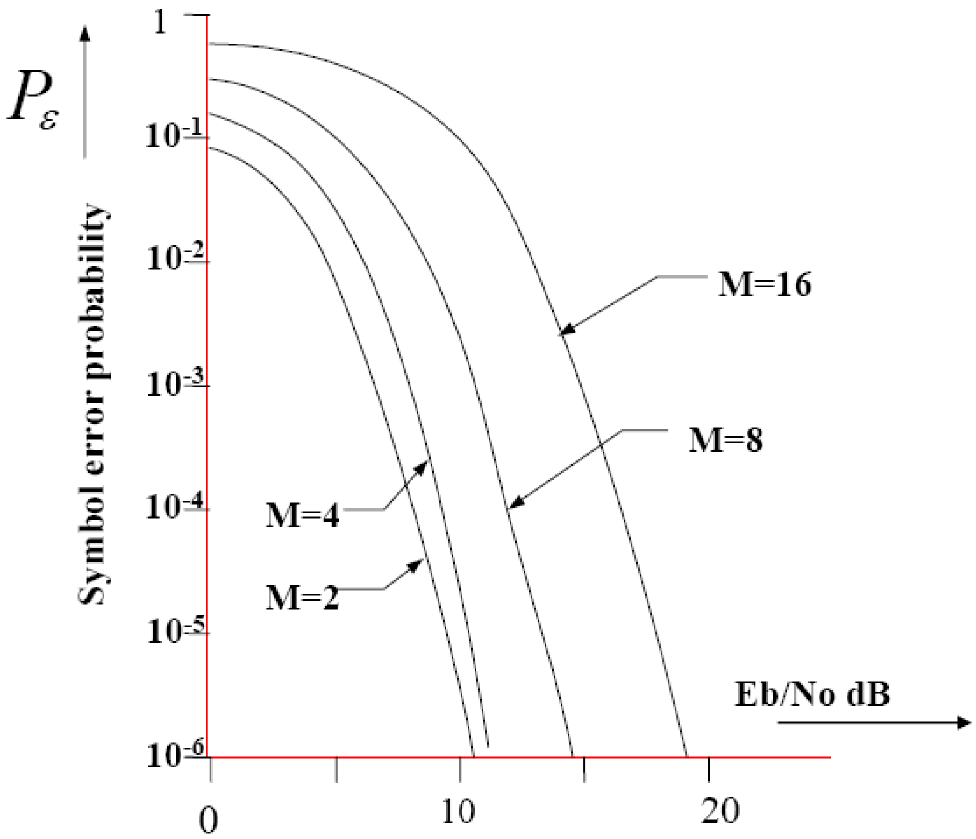
  - Can be shown that noise immunity of 16QAM is better than 16PSK for the same signal powers.

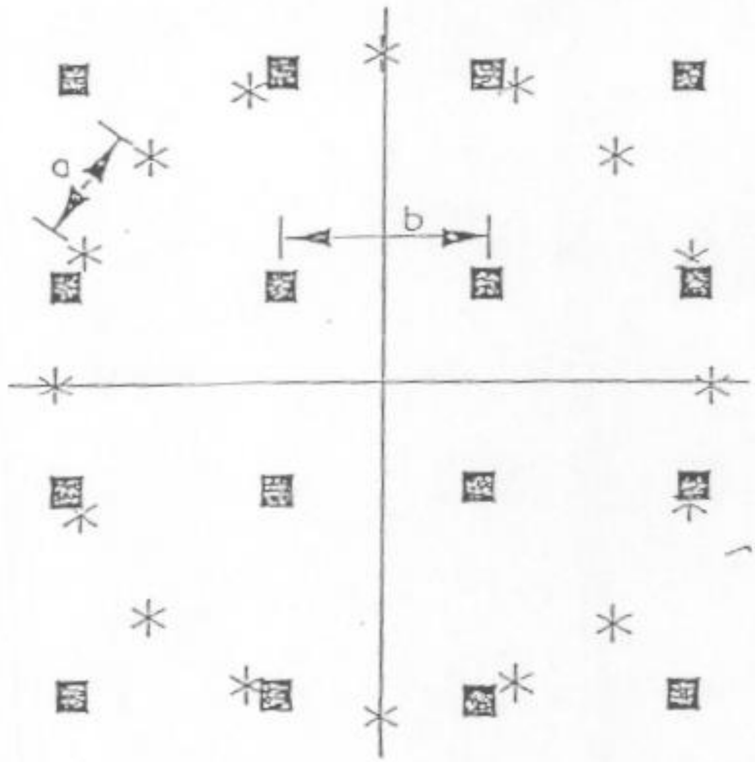
  - Hence P_e (16-QAM) < P_e (16-PSK)

HIGHER ORDER SCHEMES



M-ARY PSK





- Constellation diagram for 16PSK and 16QAM
- Down to scale for equal average symbol power

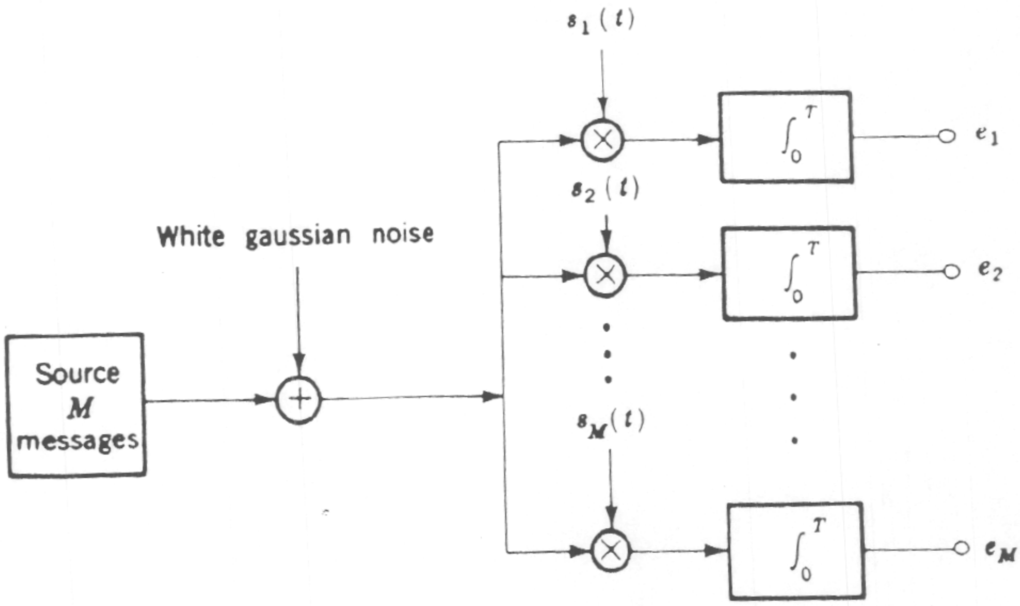
- Spacing between symbol states for QAM is greater than that for PSK
- PSK is restricted to having symbol states of equal amplitude (on a circle equidistant from the origin)

ORTHOGONAL SIGNALLING

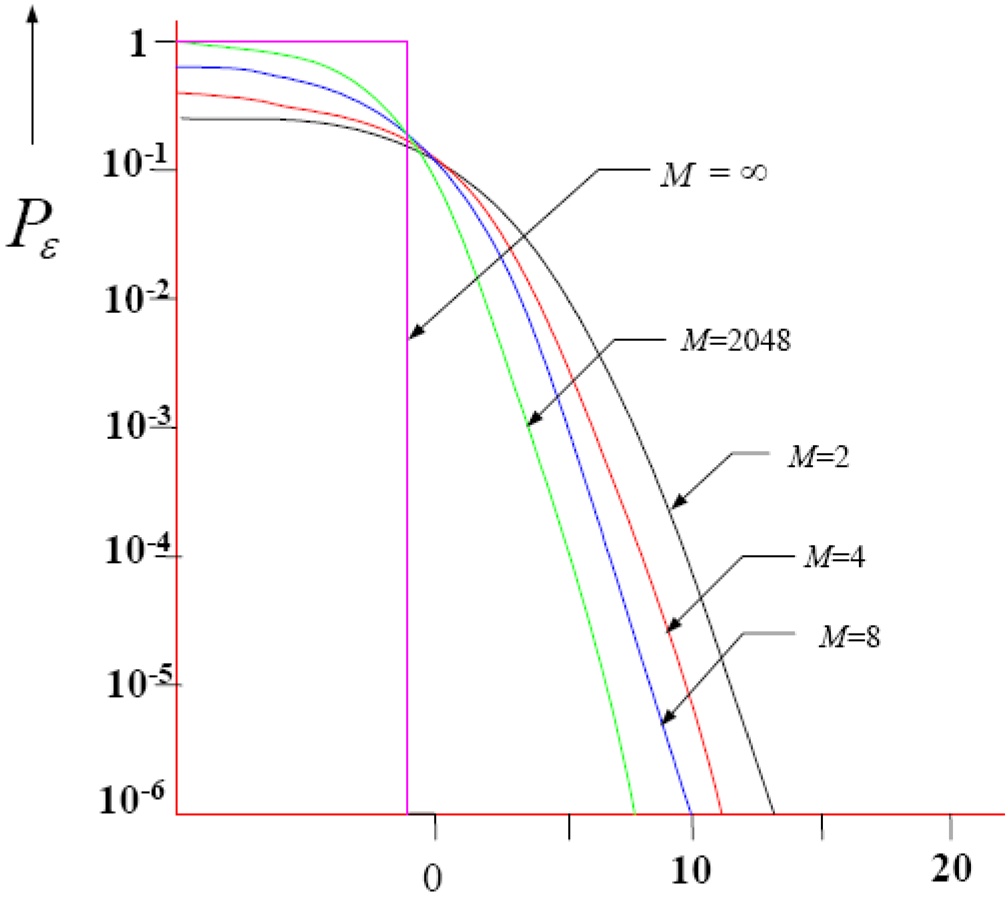
- A signal (symbol) set is defined to be orthogonal over the interval 0 to T if it has the following property:

$$\int_0^T s_i(t)s_j(t)dt = 0 \quad \text{for} \quad i \neq j$$

- A common set of orthogonal signals is that of M-ary FSK, where the frequencies used are multiplies of mωt (m= 1, 2 ,3…….)



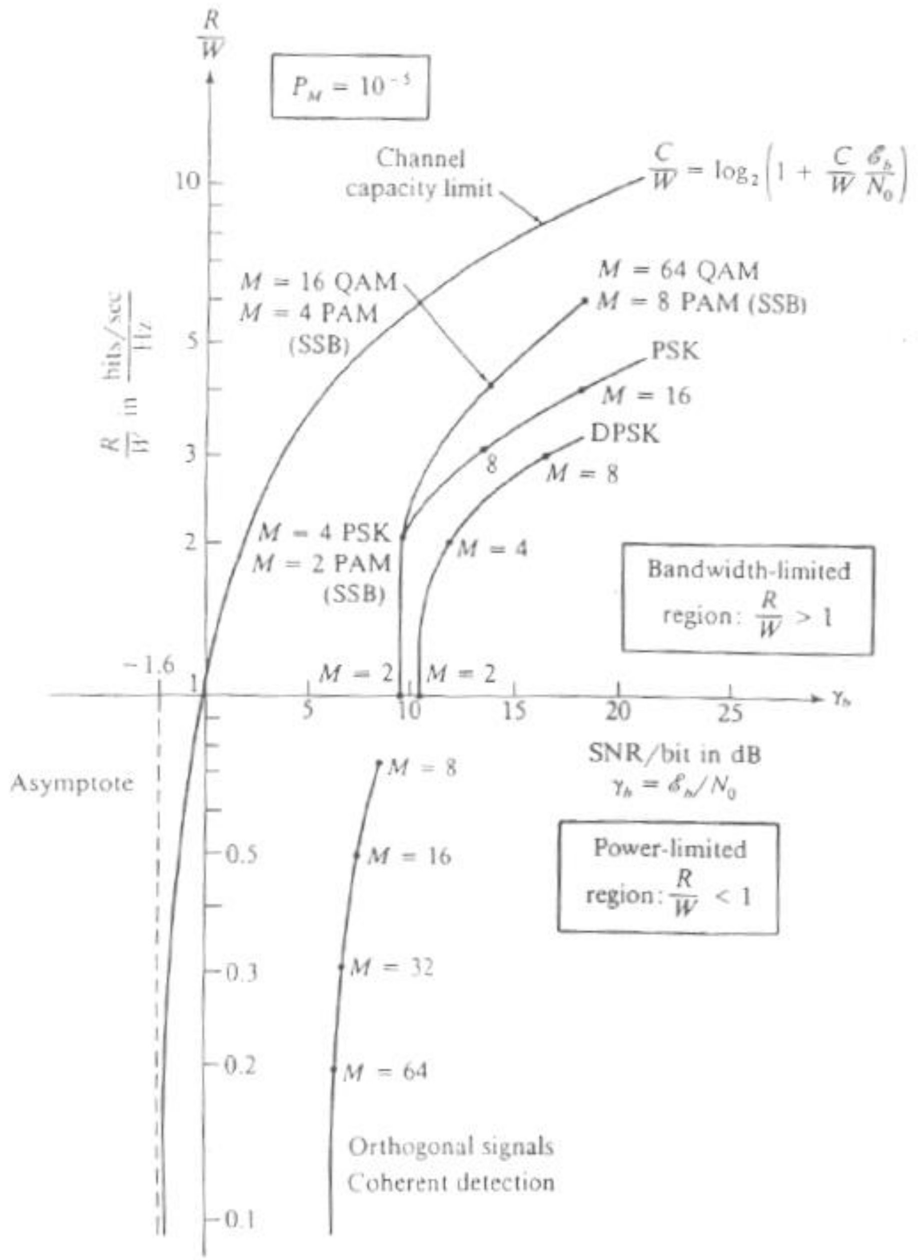
ORTHOGONAL FSK



- As M is increased, the error probability decreases, at the expense of bandwidth.
- Spectral efficiency decreases with M
- In the limit, $M \rightarrow \infty$, the error probability tends to zero for an E_b / N_o of -1.6dB. This is the Shannon Limit. Note at this point, the bandwidth is infinite.

COMPARISON OF MODULATION TECHNIQUES

- Comparison based on SNR are meaning-less, unless results are normalised to a fixed data rate or bandwidth.



ERROR CONTROL CODING