Angle Modulation & Frequency Modulation

Angle Modulation

$\rightarrow$ phase $\rightarrow$ Frequency Modulation

- A modulated signal can be described by the equation:

$$x(t) = a(t).\cos(\omega_c.t + \theta(t))$$

- If $\theta(t)$ is constant and $a(t)$ is variable we have amplitude modulation which we have discussed previously.
- If $a(t)$ is constant and $\theta(t)$ is variable we have *angle modulation*.

University of BRISTOL

continuous

Angle Modulation

- $\theta(t)$ can be made proportional to the message signal $m(t)$.
- This proportionality can take 2 forms.
  - The phase is proportional to the message signal, in which case we have phase modulation.

$$\boxed{\theta(t) = K_p . m(t)}$$

  - The rate of change of phase is proportional to the message signal in which case we have $\boxed{\text{frequency modulation.}}$ FM

Frequency Modulation

Const.

$$\boxed{\frac{d\theta(t)}{dt} = K_f . m(t) \qquad \theta(t) = K_f \int m(t) . dt}$$

Frequency Modulation

- Consider that the message signal is sinusoidal in nature. The message signal can then be written as:

$$\boxed{m(t) = a.\sin(\omega_m.t)}$$

- Which means that the angle modulated signal can be written as:

Fixed Carrier

$$x(t) = A.\cos(\omega_c.t + K_p.\boxed{a.\sin(\omega_m.t)})$$

$$= A.\cos(\omega_c.t + \beta.\sin(\omega_m.t))$$

- β is called the modulation index

Frequency Modulation

- The instantaneous frequency of *x(t)* is given by the derivative (with respect to time) of the argument of *x(t)*.

  That is:

$$\boxed{\frac{d(\omega_c.t+\beta.\sin(\omega_m.t))}{dt}}$$

$$=\omega_c+\beta.\omega_m.\cos(\omega_m.t)$$

This can be written:

$$\boxed{f_c+\beta.f_m.\cos(\omega_m.t)}$$

Frequency Modulation

- The maximum frequency deviation (of the modulated signal from the carrier frequency) is given by:

$$f_{dev} = \beta . f_m$$

$\beta = 5$ for FM broadcast Radio.

- Thus the modulated signal can be expressed as:

Constant Carrier

$$x(t) = A.\cos\left(\omega_c .t + \frac{f_{dev}}{f_m} \sin(\omega_m .t) \right)$$

FM Spectrum

Spectrum of frequency modulated signal

- As with all modulation schemes, it is important to be able to predict the spectrum of a frequency modulated signal.
- Given the basic FM equation.

$$\boxed{x(t) = A.\cos(\omega_c.t + \beta.\sin(\omega_m.t))}$$

This can be expanded to:

$$x(t) = A.\cos(\omega_c.t).\cos(\beta.\sin(\omega_m.t)) - A.\sin(\omega_c.t).\sin(\beta.\sin(\omega_m.t))$$

Spectrum of frequency modulated signal

- As we are interested in determining the spectral components of the modulated signal we can use the Fourier series to each of the components in the previous expression.
- The Fourier series expansion of $\cos(\beta \sin(\omega_m.t))$ is given by: Bessel Function

$$X(t) = J_0(\beta) + 2J_2(\beta).\cos(2.\omega_m.t) + 2J_4(\beta).\cos(4.\omega_m.t) + \ldots + 2J_{2n}(\beta).\cos(2.n.\omega_m.t)$$

DC + even harmonics → ∞ !!

Spectrum of frequency modulated signal

- The Fourier series expansion of $\boxed{\sin(\beta \sin(\omega_m.t))}$ is given by:

$$X(t) = 2J_1(\beta).\sin(\omega_m.t) + 2J_3(\beta).\sin(3.\omega_m.t) + \ldots + 2J_{2n-1}(\beta).\sin((2.n-1).\omega_m.t)$$

Odd harmonic of ω_m $\longrightarrow \infty$

- $J_n(\beta)$ is known as a Bessel Function of the first kind order n.
- Tables of Bessel functions are available in most communications text books

Spectrum of frequency modulated signal

- A table of Bessel functions of the first kind is given below:

Modulation index

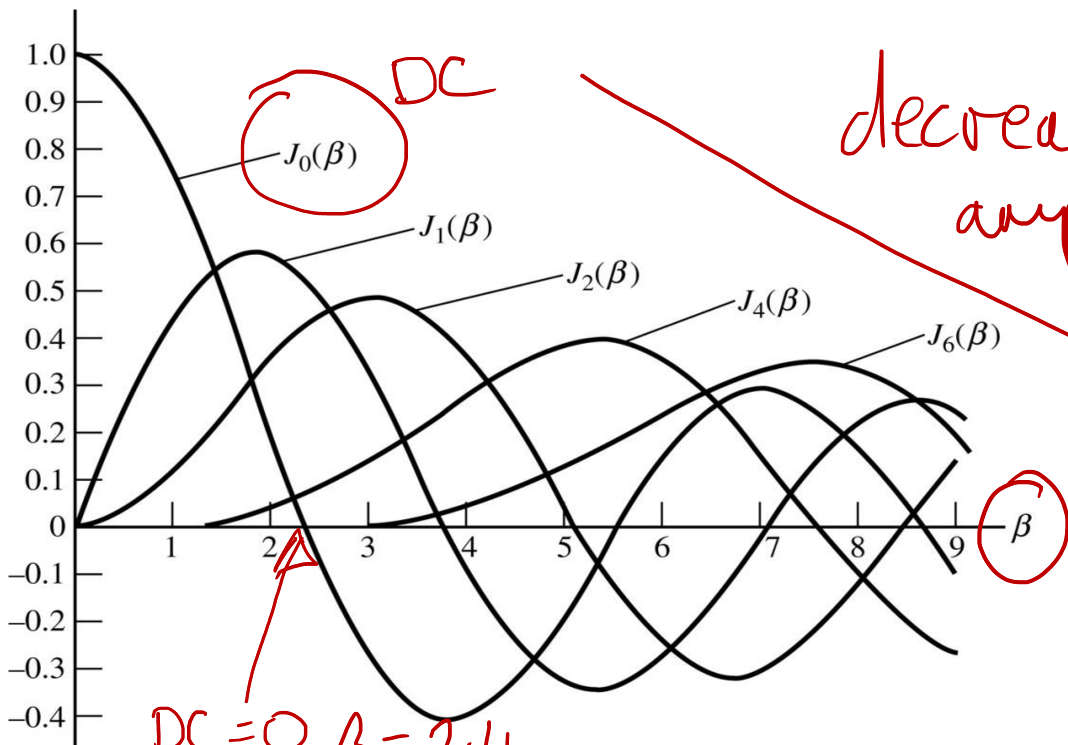
β	J0	J1	J2	J3	J4	J5	J6	J7	J8	J9	J10	J11	J12	J13	J14
0.00	1.00														
0.25	0.98	0.12													
0.50	0.94	0.24	0.03												
1.00	0.77	0.44	0.11	0.02											
1.50	0.51	0.56	0.23	0.06	0.01										
2.00	0.22	0.58	0.35	0.13	0.03										
2.40	0.00	0.52	0.43	0.2	0.06	0.02									
2.50	-0.05	0.50	0.45	0.22	0.07	0.02	0.01								
3.00	-0.26	0.34	0.49	0.31	0.13	0.04	0.01								
4.00	-0.40	-0.07	0.36	0.43	0.28	0.13	0.05	0.02							
5.00	-0.18	-0.33	0.05	0.36	0.39	0.26	0.13	0.05	0.02						
6.00	0.15	-0.28	-0.24	0.11	0.36	0.36	0.25	0.13	0.06	0.02					
7.00	0.30	0.00	0.3	-0.17	0.16	0.35	0.34	0.23	0.13	0.06	0.02				
8.00	0.17	0.23	0.11	-0.29	-0.1	0.19	0.34	0.32	0.22	0.13	0.06	0.03			
9.00	-0.09	0.25	0.14	-0.18	-0.27	-0.06	0.20	0.33	0.31	0.21	0.12	0.06	0.03	0.01	
10.00	-0.25	0.05	0.25	0.06	-0.22	-0.23	-0.01	0.22	0.32	0.29	0.21	0.12	0.06	0.03	0.01

high β — Significant number of harmonic terms

Spectrum of frequency modulated signal

- Plots of Bessel functions

Amplitude



decreasing amplitude of harmonics

Amplitude can be -ve

DC=0 $\beta = 2.4$

Spectrum of frequency modulated signal

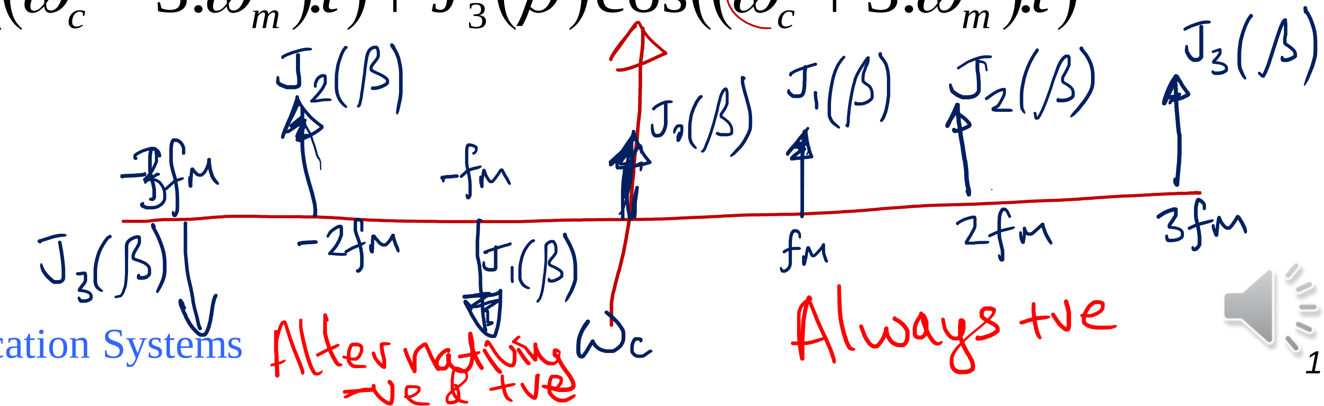
- Combining the 2 equations gives:

$$X(t) =$$
$$A.\big(J_0(\beta)\cos(\omega_c.t)$$
$$-J_1(\beta)\cos((\omega_c-\omega_m).t)+J_1(\beta)\cos((\omega_c+\omega_m).t)$$
$$+J_2(\beta)\cos((\omega_c-2.\omega_m).t)+J_2(\beta)\cos((\omega_c+2.\omega_m).t)$$
$$-J_3(\beta)\cos((\omega_c-3.\omega_m).t)+J_3(\beta)\cos((\omega_c+3.\omega_m).t)$$
$$+\ldots..\big)$$

1st harmonic

2nd harmonic

3rd harmonic



Spectrum of frequency modulated signal

- Note the consistent + sign on the upper and lower sidebands of the even harmonics, and the alternating + and - on the upper and lower sidebands of the odd harmonics.
- This results from the even harmonics being an expansion of cos(A).cos(B), and the odd harmonics being an expansion of sin(A).sin(B).

Spectrum of frequency modulated signal

- When $\beta = 0$ i.e. no modulation, $J_0(\beta) = 1$ $\Rightarrow$ ωc only present.
  $J_1(\beta)$,etc, =0
- When $\beta << 1$ and $J_0(\beta)$ and $J_1(\beta)$ are significant, but $J_2(\beta)$, etc, are very small.
- This is called "Narrow Band FM" $\rightarrow$ No Capture effect
- For various values of modulation index (β) $J_0(\beta)$ goes to zero, e.g. for β=2.4, $J_0(\beta) = 0$, and there is no carrier component.

Power in FM Spectrum

Power of an FM signal

- As would be expected, the power of an FM signal is independent of modulation index. This is because the modulation process does not affect the signal amplitude, and therefore it can have no effect on the power.
- FM Power = $\dfrac{A^2}{2} = A^2\left[\dfrac{J_0{}^2(\beta)}{2} + 2\sum_{n=1}^{n=\infty}\dfrac{J_n{}^2(\beta)}{2}\right]$

Spectrum of frequency modulated signal

Message

- *Example:* A 10kHz signal frequency modulates a 1MHz carrier that has an amplitude of 20 volts (peak). This modulating signal causes a maximum frequency deviation of 50kHz. Plot the spectrum of this modulation.

β is determined as follows.

$$\boxed{\beta = \frac{f_{dev}}{f_m} = \frac{50}{10} = 5}$$

University of BRISTOL

Spectrum of frequency modulated signal

- Extracting the Bessel functions for n = 0 to 14 gives:

β	J0	J1	J2	J3	J4	J5	J6	J7	J8	J9	J10	J11	J12	J13	J14
5.00	-0.18	-0.33	0.05	0.36	0.39	0.26	0.13	0.05	0.02						

No Significant energy

- The magnitude of the carrier component is given by
- The minus sign indicates a phase reversal

$$A \times J_0(5) = 20 \times (-)0.18 = -3.6 volts$$

Spectrum of frequency modulated signal

- At 10 kHz away from the carrier the magnitude is given by: $A\times J_1(5) = 20\times(-)0.33 = -6.6volts$
- The upper sideband (ie the signal at a frequency of 1.01MHz) a negative sign associated with it because $J_1(5)$ is negative.
- The lower sideband will have a plus sign associated with it.

odd

$\beta_1(5) = -0.33$

- A plus sign indicates zero phase shift and a negative sign indicates 180^0 phase shift.

Complex valued Spectrum

Spectrum of frequency modulated signal

Carrier

Bandwidth = 160 KHz

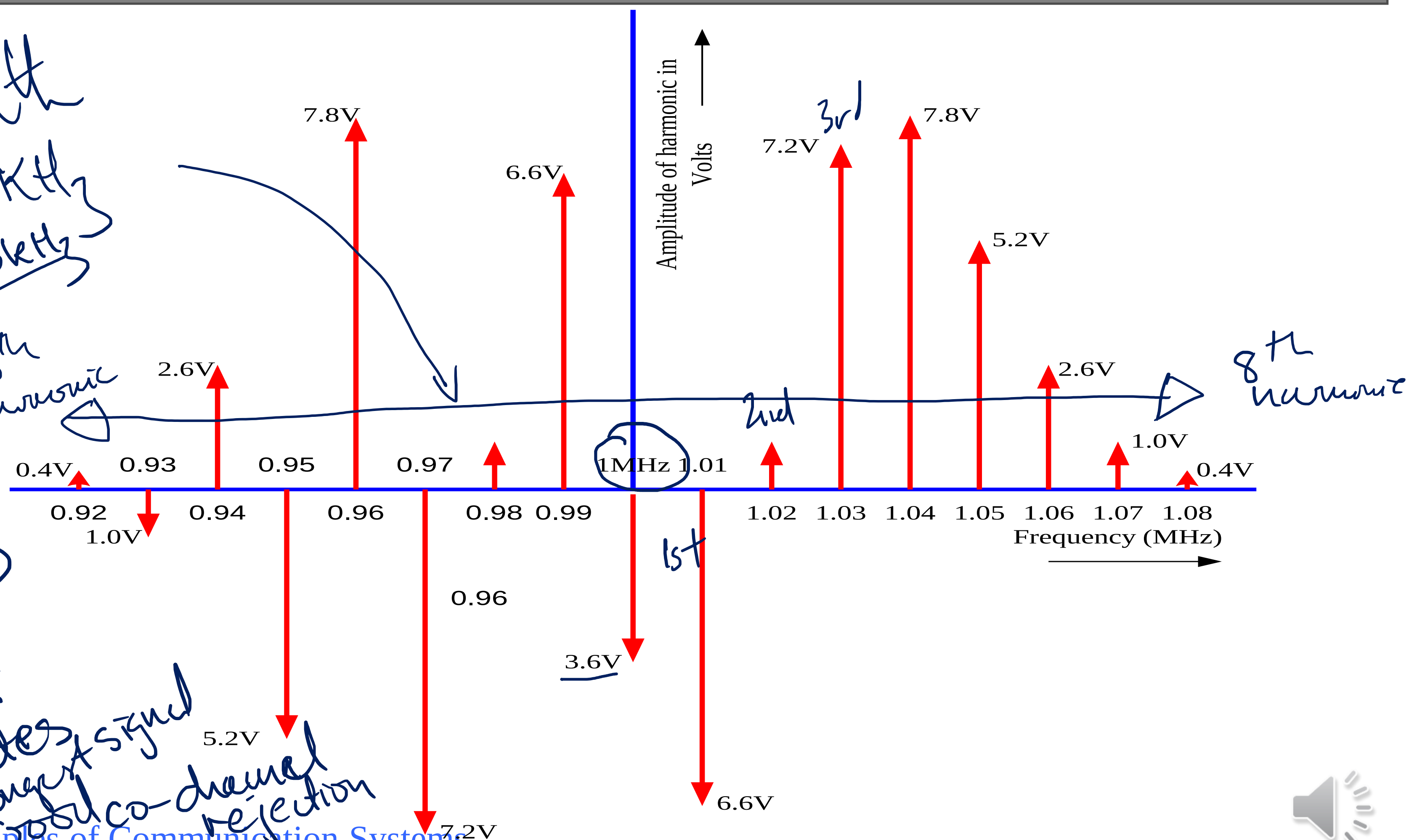
Message was 10 KHz

Better fidelity than AM

Capture effect

Demodulates stronger signal

Good co-channel rejection



Narrow band FM

- Returning to our expression for an FM signal:

$$x(t) = A.\cos(\omega_c.t).\cos(\beta.\sin(\omega_m.t)) - A.\sin(\omega_c.t).\sin(\beta.\sin(\omega_m.t))$$

- For $\beta \langle\langle 1$

$$\cos(\beta.\sin(\omega_m.t)) \to 1$$

$$\sin(\beta.\sin(\omega_m.t)) \to \beta.\sin(\omega_m.t)$$

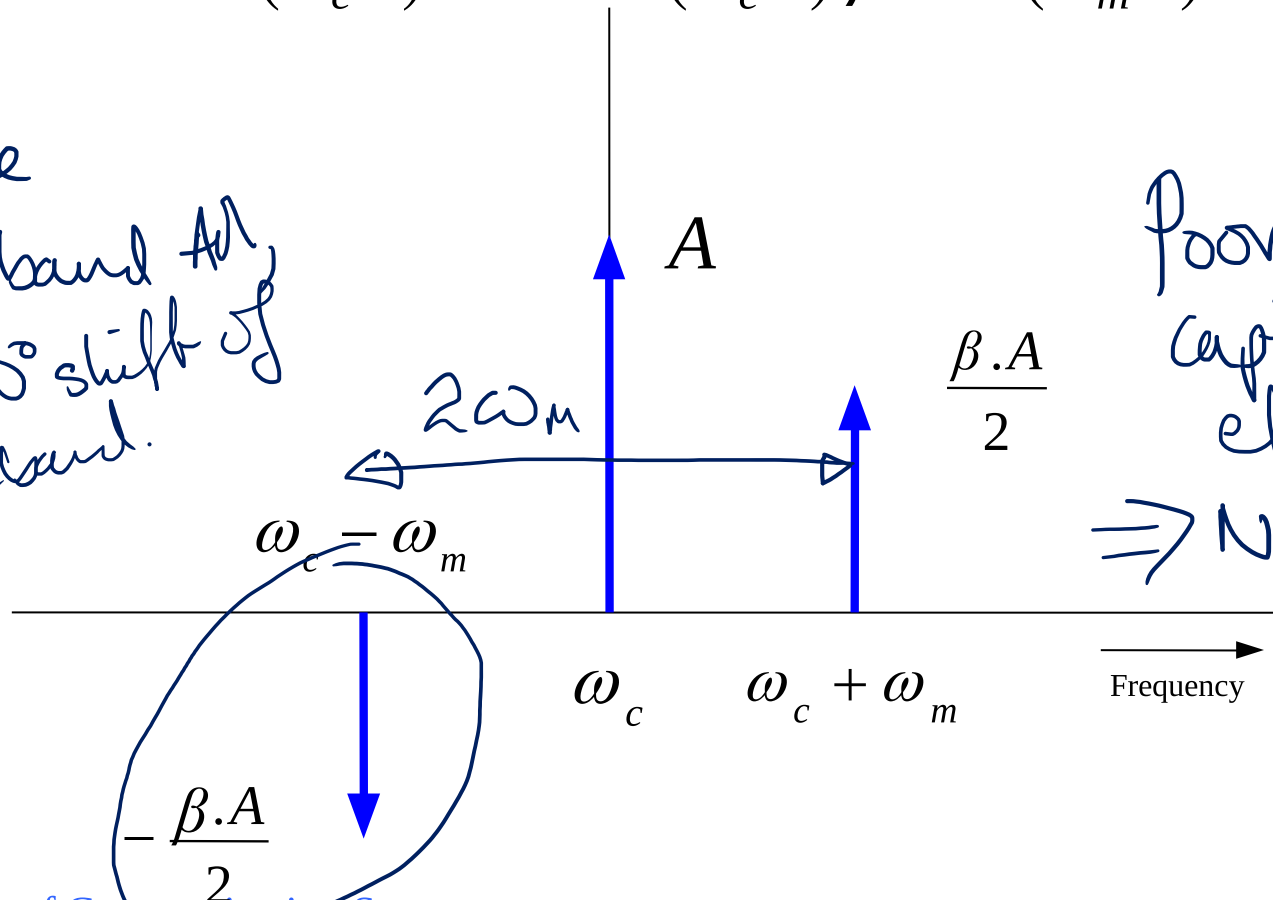
- Therefore: For $\beta \ll 1$

$$x(t) = A.\cos(\omega_c.t) - A.\sin(\omega_c.t).\beta.\sin(\omega_m.t)$$

Narrow band FM

$$x(t) = A.\cos(\omega_c.t) - A.\sin(\omega_c.t).\beta.\sin(\omega_m.t)$$

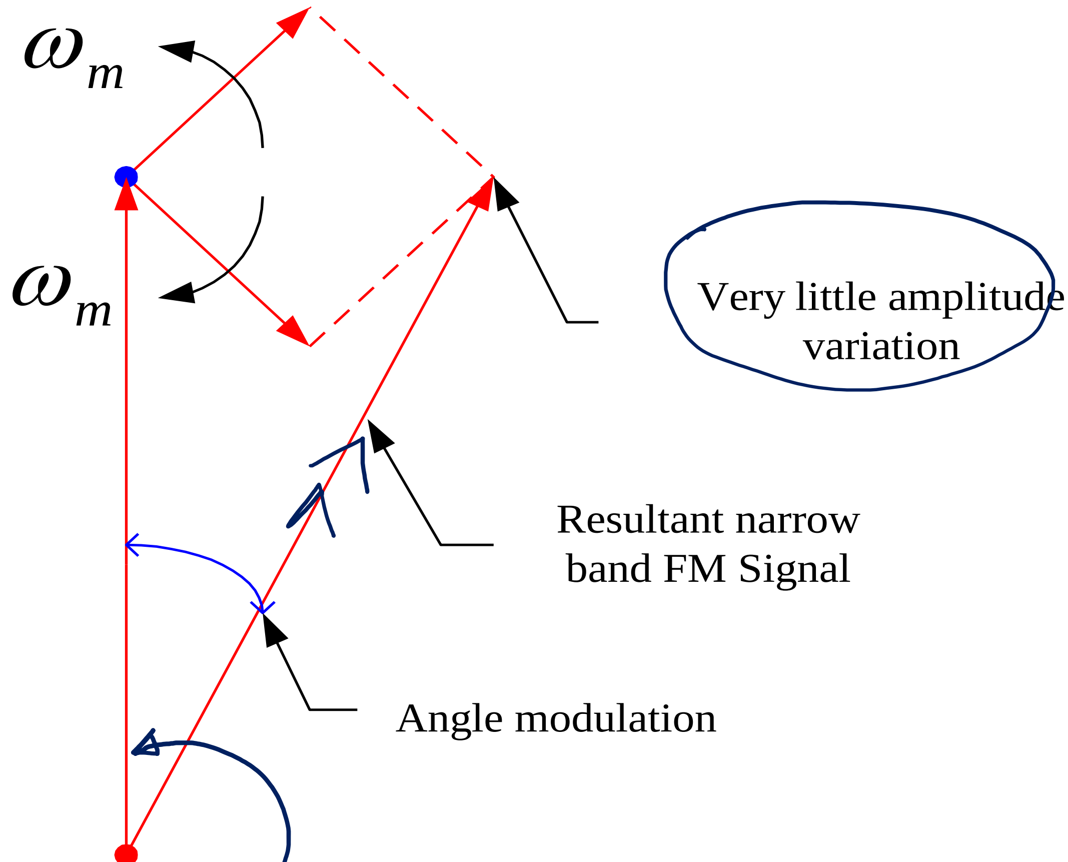
Looks like Double side band AM, except 180° shift of lower sideband.



Poor capture effect

$\Rightarrow$ No benefits from FM

Narrow band FM Phasor diagram



University of BRISTOL

Analogue Modulation

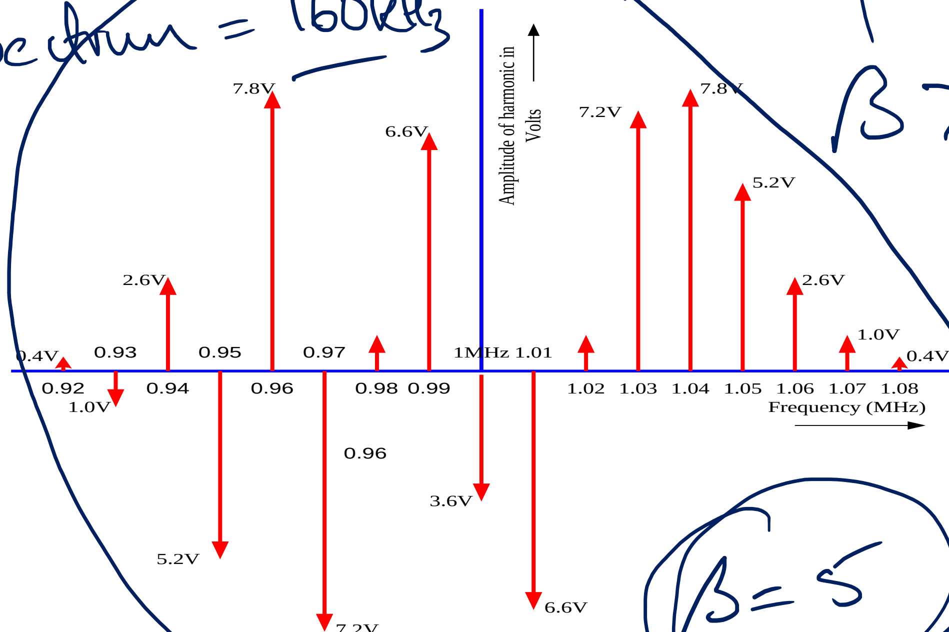
AM & FM

Message BW = 10kHz

Spectrum = 160kHz

FM Spectrum

$\beta > 1$



$\beta = 5$

Narrowband FM