Homework 8

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A propositional 2-CNF expression is a conjunction of clauses, each containing exactly 2 literals, e.g.,

$$(\mathbf{A} \vee \mathbf{B}) \wedge (\neg \mathbf{A} \vee \mathbf{C}) \wedge (\neg \mathbf{B} \vee \mathbf{D}) \wedge (\neg \mathbf{C} \vee \mathbf{G}) \wedge (\neg \mathbf{D} \vee \mathbf{G}).$$

a. Prove using resolution that the above sentence entails G.

Proof We prove $(A \vee B) \wedge (\neg A \vee C) \wedge (\neg B \vee D) \wedge (\neg C \vee G) \wedge (\neg D \vee G)$ entails G by proving that $(A \vee B) \wedge (\neg A \vee C) \wedge (\neg B \vee D) \wedge (\neg C \vee G) \wedge (\neg D \vee G) \wedge \neg G$ is unsatisfiable.

According to the soundness of resolution, we only need to prove that the resolution closure of the sentence above contains $\emptyset$.

Process of resolution is as followings:

$$\frac{A \vee B, \neg A \vee C}{B \vee C}, \frac{B \vee C, \neg B \vee D}{C \vee D}, \frac{C \vee D, \neg C \vee G}{D \vee G}, \frac{D \vee G, \neg D \vee G}{G}, \frac{G, \neg G}{\emptyset}$$

Thus we have $(A \vee B) \wedge (\neg A \vee C) \wedge (\neg B \vee D) \wedge (\neg C \vee G) \wedge (\neg D \vee G) \vdash G$, then we have $(A \vee B) \wedge (\neg A \vee C) \wedge (\neg B \vee D) \wedge (\neg C \vee G) \wedge (\neg D \vee G) \wedge \neg G$ is unsatisfiable, finally we prove that $(A \vee B) \wedge (\neg A \vee C) \wedge (\neg B \vee D) \wedge (\neg C \vee G) \wedge (\neg D \vee G) \models G$.

b. Two clauses are semantically distinct if they are not logically equivalent. How many semantically distinct 2-CNF clauses can be constructed from n proposition symbols?

Solution n proposition symbols contains 2n different literals which could construct $\begin{pmatrix} 2n \\ 2 \end{pmatrix} = 2n^2 - n$ different 2-CNF clauses. However, they contain n valid clauses like $A \vee \neg A$ which should be eliminated, so $2n^2 - n - n = 2n^2 - 2n$ semantically distinct 2-CNF clauses can be constructed from n proposition symbols.

c. Using your answer to (b), prove that propositional resolution always terminates in time polynomial in n given a 2-CNF sentence containing no more than n distinct symbols.

Proof 2-CNF clauses are **closed** under resolution, trivially. Given a 2-CNF sentence containing no more than n distinct symbols, according to **(b)**, the clauses that can be resolved by this sentence are no more than $2n^2 - 2n$. That is to say, resolution will terminates in $2n^2 - 2n - 1(O(n))$ steps in the worst case.

Thus, propositional resolution always terminates in time polynomial in n given a 2-CNF sentence containing no more than n distinct symbols.

d. Explain why your argument in (c) does not apply to 3-CNF.

Proof Each 3-CNF clause has an additional literal which makes its resolution termination time in the worst different with 2-CNF clause's termination time. What's more, 3-CNF clauses are **closed** under resolution, $\frac{A \vee B \vee \neg C, C \vee D \vee E}{A \vee B \vee D \vee E}$, for example.

Therefore, resolution in 3-CNF is quite different with 2-CNF's. Argument in **(c)** cannot be applied to 3-CNF.