Homework 1

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Theorem 1 定理*2.6.4*

(i) $A \to B, A \vdash B$

(ii) $A \to B, B \to C \vdash A \to C$

(iii) $A \to (B \to C), A \to B \vdash A \to C$

Proof 1 证明如下：

(i)

$(1) A \to B, A \vdash A \to B$	$(\in)$
$(2) A \to B, A \vdash B$	$(\in)$
$(3) A \to B, A \vdash B$	$((1)(2)(\to -))$ $\square$

(ii)

$(1) A, B \vdash A$	$(\in)$
$(2) A \vdash B \to A$	$((1)(\to +))$ $\square$

(iii)

$(1) A \to (B \to C), A \to B, A \vdash A$	$(\in)$
$(2) A \to (B \to C), A \to B, A \vdash A \to B$	$(\in)$
$(3) A \to (B \to C), A \to B, A \vdash B$	$((1)(2)(\to -))$
$(4) A \to (B \to C), A \to B, A \vdash A \to (B \to C)$	$(\in)$
$(5) A \to (B \to C), A \to B, A \vdash B \to C$	$((1)(4)(\to -))$
$(6) A \to (B \to C), A \to B, A \vdash C$	$((3)(5)(\to -))$
$(7) A \to (B \to C), A \to B \vdash A \to C$	$((6)(\to +))$ $\square$

Theorem 2 定理*2.6.9*

(i) $A \vdash B, AB.$

(ii) $A \vee B \vdash\dashv B \vee A.(\vee\Theta)$

(iii) $(A \vee B) \vee C \vdash\dashv A \vee (B \vee C).(\vee()$

(iv) $A \vee B \vdash\dashv \neg A \to B$

(v) $A \to B \vdash\dashv \neg A \vee B$

(vi) $\neg(A \vee B) \vdash\dashv \neg A \wedge \neg B.(DeMorgen)$

(vii) $\neg(A \wedge B) \vdash\dashv \neg A \vee \neg B.(DeMorgen)$

(viii) $\emptyset \vdash A \vee \neg A.()$

Proof 2 证明如下：

(i)

(1)$A \vdash A$ $(\in)$

(2)$A \vdash A \vee B, \quad A \vdash B \vee A$ $((1)(\vee+))$

(3)$A \vdash A \vee B, B \vee A$ (2) $\square$

(ii)

先证$A \vee B \vdash B \vee A$

(1)$A \vdash B \vee A, \quad B \vdash B \vee A$ *(*由定理2.6.9(i)*)*

(2)$A \vee B, A \vdash B \vee A, \quad A \vee B, B \vdash B \vee A$ $((1)(\vee+))$

(3)$A \vee B, A \vee B \vdash B \vee A$，即$A \vee B \vdash B \vee A$ $((2)(\vee-))$

再证$B \vee A \vdash A \vee B$

(1)$B \vdash A \vee B, \quad A \vdash A \vee B$ *(*由定理2.6.9(i)*)*

(2)$B \vee A, B \vdash A \vee B, \quad B \vee A, A \vdash A \vee B$ $((1)(\vee+))$

(3)$B \vee A, B \vee A \vdash A \vee B$，即$B \vee A \vdash A \vee B$ $((2)(\vee-))$

综上

$$A \vee B \vdash\dashv B \vee A \qquad \square$$

(iii)

先证$(A \vee B) \vee C \vdash A \vee (B \vee C)$

(1)$A \vdash A \vee (B \vee C)$ *(*由定理2.6.9(i)*)*

(2)$B \vdash B \vee C$ *(*由定理2.6.9(i)*)*

(3)$B \vdash A \vee (B \vee C)$ $((2)(\vee+))$

(4)$A \vee B \vdash A \vee (B \vee C)$ $((1)(3)(\vee-))$

(5)$C \vdash B \vee C$ *(*由定理2.6.9(i)*)*

(6)$C \vdash A \vee (B \vee C)$ $((5)(\vee+))$

(7)$(A \vee B) \vee C \vdash A \vee (B \vee C)$ $((4)(6)(\vee-))$

再证$A \vee (B \vee C) \vdash (A \vee B) \vee C$

(1)$C \vdash (A \vee B) \vee C$ *(*由定理2.6.9(i)*)*

(2)$B \vdash A \vee B$ *(*由定理2.6.9(i)*)*

(3)$B \vdash (A \vee B) \vee C$ $((2)(\vee+))$

(4)$B \vee C \vdash (A \vee B) \vee C$ $((1)(3)(\vee-))$

(5)$A \vdash A \vee B$ *(*由定理2.6.9(i)*)*

(6)$A \vdash (A \vee B) \vee C$ $((5)(\vee+))$

(7)$A \vee (B \vee C) \vdash (A \vee B) \vee C$ $((4)(6)(\vee-))$

综上

$$(A \vee B) \vee C \vdash\dashv A \vee (B \vee C) \qquad \square$$

(iv)

先证$A \vee B \vdash \neg A \to B$

(1)$A \vdash \neg A \to B$ *(*由定理2.6.5(v)*)*

(2)$B \vdash \neg A \to B$ *(*由定理2.6.4(ii)*)*

(3)$A \vee B \vdash \neg A \to B$ $((1)(2)(\vee-))$

再证$\neg A \to B \vdash A \vee B$

(1)$A \vdash A \vee B$	(由本定理(i))
(2)$\neg(A \vee B) \vdash \neg A$	(由定理2.6.6(v), (1))
(3)$\neg A \to B, \neg(A \vee B) \vdash \neg A$	((2)(+))
(4)$\neg A \to B, \neg(A \vee B) \vdash \neg A \to B$	($\in$)
(5)$\neg A \to B, \neg(A \vee B) \vdash B$	((4), (3), ($\to -$))
(6)$\neg A \to B, \neg(A \vee B) \vdash A \vee B$	((5), ($\vee +$))
(7)$\neg A \to B, \neg(A \vee B) \vdash \neg(A \vee B)$	($\in$)
(8)$\neg A \to B \vdash A \vee B$	((6), (7), ($\neg -$)) □

(v)

先证$A \to B \vdash \neg A \vee B$

(1)$\neg A \vdash \neg A \vee B$	(由本定理(i))
(2)$\neg(\neg A \vee B) \vdash A$	(由定理2.6.6(v), (1))
(3)$A \to B, \neg(\neg A \vee B) \vdash A$	((2)(+))
(4)$A \to B, \neg(\neg A \vee B) \vdash A \to B$	($\in$)
(5)$A \to B, \neg(\neg A \vee B) \vdash B$	((4), (3), ($\to -$))
(6)$A \to B, \neg(\neg A \vee B) \vdash \neg A \vee B$	((5), ($\vee +$))
(7)$A \to B, \neg(\neg A \vee B) \vdash \neg(\neg A \vee B)$	($\in$)
(8)$A \to B \vdash \neg A \vee B$	((6), (7), ($\neg -$))

再证$\neg A \vee B \vdash A \to B$

(1)$\neg A \vdash A \to B$	(由定理2.6.5(v))
(2)$B \vdash A \to B$	(由定理2.6.4(ii))
(3)$\neg A \vee B \vdash A \to B$	((1)(2)($\vee -$)) □

(vi)

先证$\neg(A \vee B) \vdash \neg A \wedge \neg B$

(1)$\neg A \to B \vdash A \vee B$	(由本定理(iv))
(2)$\neg(A \vee B) \vdash \neg(\neg A \to B)$	(由定理2.6.6(v))
(3)$\neg(A \vee B), A \vdash \neg(\neg A \to B)$	(($\in$))
(4)$A \vdash \neg A \to B$	(由定理2.6.5(v))
(5)$\neg(A \vee B), A \vdash \neg A \to B$	(($\in$))
(6)$\neg(A \vee B) \vdash \neg A$	((3), (5), ($\neg -$))
(7)$\neg B \to A \vdash B \vee A$	(由定理2.6.9(iv))
(8)$\neg(A \vee B) \vdash \neg(\neg B \to A)$	(由定理2.6.6(v))
(9)$\neg(A \vee B), B \vdash \neg(\neg B \to A)$	(($\in$))
(10)$B \vdash \neg B \to A$	(由定理2.6.5(v))
(11)$\neg(A \vee B), B \vdash \neg B \to A$	(($\in$))
(12)$\neg(A \vee B), B \vdash \neg B$	((9), (11), ($\neg -$))
(13)$\neg(A \vee B), B \vdash \neg A \wedge \neg B$	((6), (12), ($\wedge +$))

再证$\neg(A \vee B) \dashv \neg A \wedge \neg B$

(1)$\neg A \to B \vdash \neg(\neg A \wedge \neg B)$	(由定理2.6.8(v))
(2)$\neg A \wedge \neg B \vdash \neg(\neg A \to B)$	(由定理2.6.6(ii))
(3)$\neg A \wedge \neg B, A \vee B \vdash \neg(\neg A \to B)$	(($\in$))
(4)$A \vdash \neg A \to B$	(由定理2.6.5(v))
(5)$B \vdash \neg A \to B$	(由定理2.6.4(ii))
(6)$A \vee B \vdash \neg A \to B$	((4), (5), ($\vee -$))
(7)$\neg A \wedge \neg B, A \vee B \vdash \neg A \to B$	(($\in$))
(8)$\neg A \wedge \neg B \vdash \neg(A \vee B)$	((3), (7), ($\neg -$)) □

(vii)

先证$\neg(A \wedge B) \vdash \neg A \vee \neg B$

$(1) A \to \neg B \vdash \neg A \vee \neg B$ (由本定理(iv))
$(2) \neg(\neg A \vee \neg B) \vdash \neg(A \to \neg B)$ (由定理2.6.6(v))
$(3) \neg(\neg A \vee \neg B), \neg A \vdash \neg(A \to \neg B)$ $((\in))$
$(4) \neg A \vdash A \to \neg B$ (由定理2.6.5(v))
$(5) \neg(\neg A \vee \neg B), \neg A \vdash A \to \neg B$ $((\in))$
$(6) \neg(\neg A \vee \neg B) \vdash A$ $((3), (5), (\neg -))$
$(7) B \to \neg A \vdash \neg B \vee \neg A$ (由定理2.6.9(iv))
$(8) \neg(\neg A \vee \neg B) \vdash \neg(B \to \neg A)$ (由定理2.6.6(v))
$(9) \neg(\neg A \vee \neg B), \neg B \vdash \neg(B \to \neg A)$ $((\in))$
$(10) \neg B \vdash B \to \neg A$ (由定理2.6.5(v))
$(11) \neg(\neg A \vee \neg B), \neg B \vdash B \to A$ $((\in))$
$(12) \neg(\neg A \vee \neg B) \vdash B$ $((9), (11), (\neg -))$
$(13) \neg(\neg A \vee \neg B) \vdash A \wedge B$ $((6), (12), (\wedge +))$
$(14) \neg(A \wedge B) \vdash \neg A \vee \neg B$ (由定理2.6.6$(vii), (13)$)

再证$\neg(A \wedge B) \dashv \neg A \vee \neg B$

$(1) A \to \neg B \vdash \neg(A \wedge B)$ (由定理2.6.8(v))
$(2) A \wedge B \vdash \neg(A \to \neg B)$ (由定理2.6.6(ii))
$(3) A \wedge B, \neg A \vee \neg B \vdash \neg(A \to \neg B)$ $((\in))$
$(4) \neg A \vdash A \to \neg B$ (由定理2.6.5(v))
$(5) \neg B \vdash A \to \neg B$ (由定理2.6.4(ii))
$(6) \neg A \vee \neg B \vdash A \to \neg B$ $((4), (5), (\vee -))$
$(7) A \wedge B, \neg A \vee \neg B \vdash A \to \neg B$ $((\in))$
$(8) A \wedge B \vdash \neg(\neg A \vee \neg B)$ $((3), (7), (\neg -))$
$(9) \neg A \vee \neg B \vdash \neg(A \wedge B)$ (由定理2.6.6$(vi), (8)$) □

(viii)

$(1) \emptyset \vdash \neg(A \wedge \neg A)$ (由定理2.6.8(vii))
$(2) \neg(\neg A \vee A) \vdash A \wedge \neg A$ (由定理2.6.9(vi))
$(3) \emptyset, \neg(\neg A \vee A) \vdash A \wedge \neg A$ $(\in)$
$(4) \emptyset \vdash \neg A \vee A$ $((1)(3)(\neg -))$
$(5) \emptyset \vdash A \vee \neg A$ ($\vee$ 交换律) □