Unilateral Control for Social Welfare of Iterated Game in Mobile Crowdsensing

APPENDIX

Proof of Theorem 1:

Proof. In the discrete case, the actions of platform and ISP are finite. If there is no existence of pure actions, mixed actions must exist because every finite strategic-form game has a mixed-strategy equilibrium [1]. For the continuous model; we fit the continuous payoff functions $F_{\mathrm{g}}(x_{\mathrm{e}}, x_{\mathrm{r}})$ and $F_{\mathrm{t}}(x_{\mathrm{e}}, x_{\mathrm{r}})$ for the platform and ISP under real trace, then the equilibrium exists in the convex space [2]. □

Proof of Theorem 2:

Proof. For the continuous case, inspired by the work [2], we first construct a function $\sigma(x_{\mathrm{e}}, x_{\mathrm{r}}, \vec{s}) = s_1 \times F_{\mathrm{g}}(x_{\mathrm{e}}, x_{\mathrm{r}}) + s_2 \times F_{\mathrm{t}}(x_{\mathrm{e}}, x_{\mathrm{r}}), s_i \geq 0$, where $\boldsymbol{s}$ is an nonnegative vector. We proof $\sigma(x_{\mathrm{e}}, x_{\mathrm{r}}, \vec{s})$ is diagonally strictly concave [2] if the symmetric matrix $(G(x_{\mathrm{e}}, x_{\mathrm{r}}, \vec{s}) + G^{'}(x_{\mathrm{e}}, x_{\mathrm{r}}, \vec{s}))$ be negative definite for $x_{\mathrm{e}}, x_{\mathrm{r}} \in \mathcal{R}$, where $\mathcal{R}$ is the domain of the actions vector, $G(x_{\mathrm{e}}, x_{\mathrm{r}}, \vec{s})$ is the Jacobian with respect to x_{e} and x_{r}. Then we obtain that the equilibrium point is unique. □

Proof of Theorem 3:

Proof. We first divide the continuous action space into η parts. Then the action of the platform and ISP satisfy $x_e \in \{l^{\mathrm{e}}, l^{\mathrm{e}} + \delta, l^{\mathrm{e}} + 2\delta, \cdots, l^{\mathrm{e}} + \eta\delta\}$, and $x_{\mathrm{r}} \in \{l^{\mathrm{r}}, l^{\mathrm{r}} + \delta, l^{\mathrm{r}} + 2\delta, \cdots, l^{\mathrm{r}} + \eta\delta\}$, respectively, where δ is small enough and η is sufficiently large, satisfying $l^{\mathrm{e}} + \eta\delta = h^{\mathrm{e}}$ and $l^{\mathrm{r}} + \eta\delta = h^{\mathrm{r}}$. When $\delta \to 0$, the action space is approximately continuous. Thus, the payoffs of the platform is $\boldsymbol{M}^{\mathrm{e}} = (F_{\mathrm{g}}(l^{\mathrm{e}}, l^{\mathrm{r}}), \cdots, F_{\mathrm{g}}(l^{\mathrm{e}}, l^{\mathrm{r}} + \eta\delta), \cdots, F_{\mathrm{g}}(l^{\mathrm{e}} + \eta\delta, l^{\mathrm{r}}), \cdots, F_{\mathrm{g}}(l^{\mathrm{e}} + \eta\delta, l^{\mathrm{r}} + \eta\delta)) = (F_{\mathrm{g}00}, \cdots, F_{\mathrm{g}0\eta} \cdots, F_{\mathrm{g}\eta 0} \cdots, F_{\mathrm{g}\eta\eta})$; the payoffs of the ISP is $\boldsymbol{M}^{\mathrm{r}} = (F_{\mathrm{t}}(l^{\mathrm{e}}, l^{\mathrm{r}}), \cdots, F_{\mathrm{t}}(l^{\mathrm{e}}, l^{\mathrm{r}} + \eta\delta), \cdots, F_{\mathrm{t}}(l^{\mathrm{e}} + \eta\delta, l^{\mathrm{r}}), \cdots, F_{\mathrm{t}}(l^{\mathrm{e}} + \eta\delta, l^{\mathrm{r}} + \eta\delta)) = (F_{\mathrm{t}00}, \cdots, F_{\mathrm{t}0\eta} \cdots, F_{\mathrm{t}\eta 0} \cdots, F_{\mathrm{t}\eta\eta})$. The platform's mixed strategy at current round is $p_{ij-k}, \forall i, j, k \in \{0, 1, \cdots, \eta\}$, which indicates the probability of the platform chooses $x_e = l^{\mathrm{e}} + k\delta$ in the current round when the previous actions are $x'_{\mathrm{e}} = l^{\mathrm{e}} + i\delta$ and $x'_{\mathrm{r}} = l^{\mathrm{r}} + j\delta$. Similarly, ISP's mixed strategy at current round is q_{ij-k}.

According to the above partition on the action space and utility space, we get the Markov state transition matrix as:

$$\boldsymbol{H}_{\mathrm{d}} = (\boldsymbol{H}_{00}, \cdots, \boldsymbol{H}_{0\eta}, \boldsymbol{H}_{10}, \cdots, \boldsymbol{H}_{1\eta}, \boldsymbol{H}_{\eta 0}, \cdots, \boldsymbol{H}_{\eta\eta}),$$

where each element $\boldsymbol{H}_{ij}, \forall i, j \in \{0, 1, \cdots, \eta\}$ is a vector, which contains the transition probability from all the possible combinations of the previous state $x'_{\mathrm{e}}x'_{\mathrm{r}}$ to the current state $x_{\mathrm{e}} = l^{\mathrm{e}} + i\delta$ and $x_{\mathrm{r}} = l^{\mathrm{r}} + j\delta$. Each element is written as:

$$\begin{aligned}\boldsymbol{H}_{ij} =&(p_{00-i}q_{00-j}, \cdots, p_{0\eta-i}q_{0\eta-j}, p_{10-i}q_{10-j}, \cdots,\\ &p_{1\eta-i}q_{1\eta-j}, \cdots, p_{\eta 0-i}q_{\eta 0-j}, \cdots, p_{\eta\eta-i}q_{\eta\eta-j})^{\mathrm{T}}.\end{aligned}$$

We assume the stable vector of $\boldsymbol{H}_{\mathrm{d}}$ is $\boldsymbol{v}_{\mathrm{d}}$, we have $\boldsymbol{v}_{\mathrm{d}}^{\mathrm{T}}\boldsymbol{H}_{\mathrm{d}} = \boldsymbol{v}_{\mathrm{d}}^{\mathrm{T}}$, and the expected utilities of the platform and ISP are $U^{\mathrm{e}} = \boldsymbol{v}_{\mathrm{d}}^{\mathrm{T}}\boldsymbol{M}^{\mathrm{e}}$ and $U^{\mathrm{r}}\boldsymbol{v}_{\mathrm{d}}^{\mathrm{T}}\boldsymbol{M}^{\mathrm{r}}$, respectively.

We suppose $\boldsymbol{H}'_{\mathrm{d}} = \boldsymbol{H}_{\mathrm{d}} - \boldsymbol{I}$, and have $\boldsymbol{v}_{\mathrm{d}}\boldsymbol{H}'_{\mathrm{d}} = 0$. With the similar calculation as the one under the discrete model, $\boldsymbol{v}_{\mathrm{d}}^{\mathrm{T}}$ is proportional to each row of $Adj(\boldsymbol{H}'_{\mathrm{d}})$. Thus, for any vector $f = (f_{00}, f_{01}, \cdots, f_{\eta\eta})$, with the known condition $\sum_{k=0}^{\eta} q_{ij-k} = 1$, we can calculate its dot product with $\boldsymbol{v}_{\mathrm{d}}$ as follows:

$$\boldsymbol{v}_{\mathrm{d}}^{\mathrm{T}} \cdot \boldsymbol{f} = D(\boldsymbol{p}, \boldsymbol{q}, \boldsymbol{f})$$
$$= det \begin{pmatrix} p_{00-0}q_{00-0} & \cdots & p_{00-\eta} & f_{00} \\ \vdots & \vdots & \vdots & \vdots \\ p_{(\eta-1)\eta-0}q_{(\eta-1)\eta-0} & \cdots & p_{(\eta-1)\eta-\eta} & f_{(\eta-1)\eta} \\ p_{\eta 0-0}q_{\eta 0-0} & \cdots & p_{\eta 0-\eta}-1 & f_{\eta 0} \\ \vdots & \vdots & \vdots & \vdots \\ p_{\eta\eta-0}q_{\eta\eta-0} & \cdots & p_{\eta\eta-\eta}-1 & f_{\eta\eta} \end{pmatrix}. \quad (1)$$

It is obvious that the penultimate column of equation (1) is decided only by the platform, denoted as $\tilde{\boldsymbol{p}}$. When $f = (\alpha \boldsymbol{M}^{\mathrm{e}} + \beta \boldsymbol{M}^{\mathrm{r}} + \gamma \mathbf{1})$, we obtain $\boldsymbol{v}_{\mathrm{d}}^{\mathrm{T}} \cdot \boldsymbol{f} = (\alpha \boldsymbol{M}^{\mathrm{e}} + \beta \boldsymbol{M}^{\mathrm{r}} + \gamma \mathbf{1}) = \alpha U^{\mathrm{e}} + \beta U^{\mathrm{r}} + \gamma$. Therefore, if $\tilde{\boldsymbol{p}} = \phi(\alpha \boldsymbol{M}^{\mathrm{e}} + \beta \boldsymbol{M}^{\mathrm{r}} + \gamma \mathbf{1})$, we have $\alpha U^{\mathrm{e}} + \beta U^{\mathrm{r}} + \gamma = 0$. When the small number $\delta \to 0$, the theorem is proven. □

References

[1] Nash J F et al. Equilibrium points in n-person games. *Proceedings of the national academy of sciences*, 1950.

[2] Rosen J B. Existence and uniqueness of equilibrium points for concave n-person games. *Econometrica: Journal of the Econometric Society*, 1965.