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RESEARCH ARTICLE

Congestion Game in Spectrum and Power Allocation in Cellular Networks

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ABSTRACT

In this chapter, we will see the application of congestion game in solving the channel allocation problem in the context of TV white space. The channel allocation problem we will address is a general problem, as the transmission power is not identical for every transmitter and on each channel, actually, the transmission power could be unique for each transmitter-channel combination. With the suitable utility function designed for transmitters, the behaviours of the transmitters can be described by a congestion game. The algorithm of channel allocation is derived from the dynamics of the transmitter in the game, which reaches Nash equilibrium quickly.
Furthermore, we provide a complete solution to fully exploit TV white space complying with IEEE 802.22 standard. We propose a centralized methods to regulate the upper bound of transmission power, so that to strictly protect the primary users. The the distributed channel allocation and power control are conducted sequentially.

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1. INTRODUCTION

Secondary users working with TV white space is promising to cope with the scarcity of spectrum resources [1]. Firstly, more unused TV white frequencies become vacant than ever with the ongoing transition from analog to digital broadcasts. Secondly, the frequencies of TV bands enable broadband access over larger geographic ranges compared to higher frequency bands. Nevertheless, services on TV receivers need to be protected with so called interference margin [*] [2] which should not be exceeded by the accumulated interference caused by all secondary users working on the the channel.

FCC and ECC have announced rules on the transmission power of secondary users working in TV white space in US and Europe respectively [1, 3]. FCC requires a minimum distance between secondary user and TV service area, besides, the transmission power for fixed secondary users is set as 4 W, which is a conservative setting. FCC believes with these prudent measures, the interference margin can not be exceeded by interference from secondary users. But it may not be the case when there are multiple secondary equipments transmitting at the same time, which is pointed in [4]. ECC requires the secondary users to adapt their maximum transmission power according to the distance away from the TV receivers.

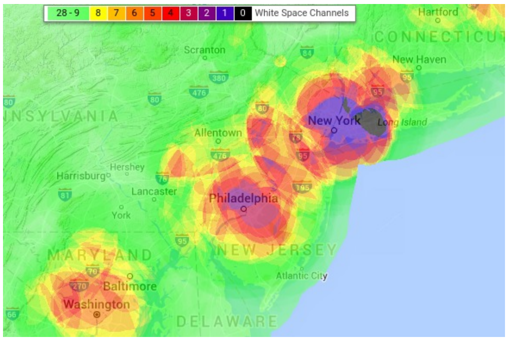

Figure 1. Variability of available channels in a densely populated area. This figure is obtained from [?]

FCC issued a memorandum [1, 5] in 2010, which removes the mandatory rigid sensing requirements, and prompts the usage of geolocations[†]. FCC regulates a

[*]interference margin is the maximal interference caused by secondary users, which doesn't violate TV service.

[†]Geolocation means both geographic location and terrain.

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centralized database, which registers all the secondary users within one certain area, and decides on the available channels for them to use. The secondary users should access the database to obtain the list of available channels for the to use. The authors of [6] validate this regulation and demonstrate the feasibility of only using geolocations and propagation model. They adopt a central database which contains the geolocations of all TV stations. Then with sophisticated propagation model (Longley-Rice), the central database calculates the received signal strength index (RSSI) levels of TV UHF signals in a vast area. If RSSI on a channel is below a certain threshold on a location, TV service is regarded to be idle on that channel there and the secondary users there are allowed to use. The calculated results on channel availability is very close to the measurement results, which gives big impetus to the application of database mode in the exploration of TV white space. The FCC memorandum [1,5] and the work [6] initialize a new and easier way to utilize the TV white space, and the work [6] illustrates it is feasible to decide the RSSI level only with appropriate propagation model and geolocation.

In this paper, we investigate the efficient way to exploit the TV spectrum in a wireless regional area network which complies with IEEE 802.22 network. The secondary users are assumed to be cellular base stations and associated terminals, all of which work on TV white spectrum. The base station is referred as WBS. Some cellular networks, i.e., GSM or LTE network, work on licensed spectrum and emphasis on providing satisfactory services to their end terminals by choosing proper transmission channel and power. As to cellular network working on TV white spectrum, they have to keep one eye on the primary users to make sure that TV service is not violated, which makes the problem of channel and power selection difficult. With the existence of central database, it is natural to utilize it as a central controller to assign channel and power usage for secondary users, but the secondary users may belong to different commercial groups and they may not contend with the assigned resource. Hence, the spectrum sharing of the secondary users in IEEE 802.22 network should be decided in distributed manner and each secondary user takes care of its own interest, i.e., to maximize its preferred utility.

Given all the other WBSs' selection on channel and transmission power, a WBS is interested in choosing the channel which brings it the best performance, i.e., the data rate of its end users. A WBS prefers to choose the channel which experiences the minimum interference, and the transmission power allowed on that channel is higher, so as to obtain better SINR on its terminals and meanwhile maximize their coverage [7, 8]. Nevertheless, high transmission power causes significant co-channel interference to other secondary users operating on the same channel. Hence, a secondary cell has to balance its transmission power and the caused interference on other cells, meanwhile to choose working channel to decrease the experienced interference on its terminals. The goal of this chapter is to protect the primary users from harmful interference, meanwhile to find a strategy for WBSs to choose channel and power level in order to acquire good SINR on end terminals.

The rest of the paper is organized as follows. We elucidate the system model in Section II, afterwards related work and problem formulation is presented in Section III. In Section IV, we discuss how to utilize the white space sufficiently by setting the transmission powers based on a convex problem formulation. We analyze the spectrum allocation problem under game theoretical framework and propose an algorithm in Section V, thereafter performance evaluation is presented in Section VI. Finally, we conclude our work and point out directions of future research in Section VII.

2. SYSTEM MODEL AND PROBLEM STATEMENT

According to the IEEE 802.22 standard, the primary systems considered in this chapter are digital TV (DTV) stations which use the TV spectrum legally. TV stations provide service to passive TV receivers. The secondary users are IEEE 802.22 Wireless Regional Area Network base stations utilizing the TV spectrum with senseless mode [6]. DTV's service should not be interfered by secondary systems. WBSs locate in one area which is surrounded by areas where TV service is delivered. WBSs serve a set of end users/terminals. These secondary systems are distributed over a certain area A and is surrounded by multiple DTV service areas, as Fig. 2 shows. The set of DTV stations is denoted as $\mathcal{K}$ and the collection of WBSs is denoted as $\mathcal{N}$ where $|\mathcal{N}| = N$. The set of TV white spectrum contains multiple channels which are denoted as $\mathbb{C}$, they are assumed to be identical in terms of attenuation and shadowing on the same path. Let $c(i)$ denote the channel used by a WBS $i \in \mathcal{N}$.

When two WBSs working on the same channel, co-channel interference is caused on each, while, neighbouring channel interference is not considered in our model. To simplify the analysis, we assume that each DTV station as well as each WBS utilizes exactly one channel.[‡] We represent the usage of channel for WBS i with a binary vector $X_i^{|\mathbb{C}|\times 1} = \{\cdots, x_{ik}, \cdots\} \in \{0,1\}^{|\mathbb{C}|}$, where $k \in \mathbb{C}$ and binary variable x_{ik} denotes whether channel k is used by user i. As each node can only uses one channel, for X_i, there is $\sum_{k=1}^{|\mathbb{C}|} x_{ik} = 1$. The transmission power of WBS i on channel c is P_i^c.

In the rest of the chapter, we use WBS and secondary base station interchangeably. There are interference

[‡]The assumption that one WBS only utilizes one channel is for convenience of analysis. In reality multiple channel usage (channel bonding) is requisite as one single TV channel's bandwidth is 6 MHz which is not adequate for a WBS to fulfil system requirement.

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measurement equipments deployed on the contours of TV service areas (as bold rectangles in Fig. 2), which represent the worst located TV receivers in the TV service areas. For these interference measurement devices, an interference threshold should not be violated by the noise generated by the secondary users. The deployment of the interference measurement devices is decided by the TV operators, which are usually along the contour of the area where TV receivers reside. Thus, the locations of interference measurement devices vary according to the concrete location, geographic terrain and possible deployment of secondary networks. WBSs are deemed to be static. We assume the secondary base stations are not under the same operators, thus there is no scheduling mechanism available among WBSs.

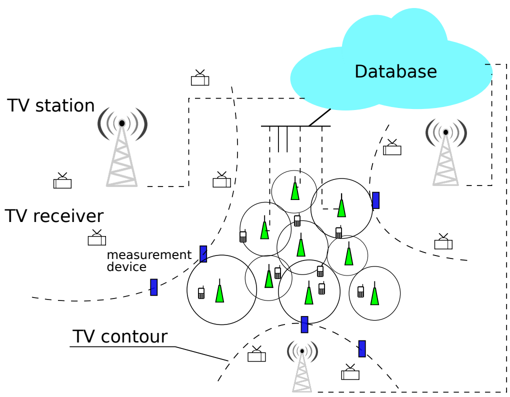

Figure 2. System model: WBS cells and DTV systems

WBSs are interested in payload data communication with their associated terminals. As to performance metric for the QoS provisioning, we choose the signal to noise and interference ratio (SINR) on the terminals. SINR is the ratio between the received power of signal of interest and the summed interference experienced by the terminal. As to a terminal m associated to WBS i, the attenuation between its serving WBS i and itself is denoted as h_{im}, and the attenuation between the interfering WBS j and m is denoted as h_{jm}. The path loss is dependent on the distance between the corresponding equipments, e.g. $h_{im} = K \cdot d_{im}^{-\alpha}$, where α is the path loss exponent, d_{im} is the distance between i and m, K is a constant which models the reference loss over a single unit of distance. N_0 denotes the thermal noise power. Shadowing without fading is considered in our model. z_{im} models the zero-mean log-normally distributed shadow fading between i and m, and the standard deviation is σ_{SH}.

The sum of all disturbing radio frequency effects (including interference) on terminal m (we assume the working channel is c) is as following,

$$f_m^c = \sum_{\bar{i}} (P_j^c \cdot h_{jm} \cdot z_{jm}) + N_0, \qquad j \in \mathcal{N} \setminus i, c(j) = c \tag{1}$$

where P_j^c denotes the transmission power of interfering WBS j. The SINR on end terminal m is,

$$\gamma_m = \frac{P_i^c \cdot h_{im} \cdot z_{im}}{f_m^c} \tag{2}$$

2.1. Problem Statement

Our goal is to design distributed solution for WBSs to choose channel and transmission power, so as to improve the SINR of their associated end terminals. Each WBS's utility is a function of the SINR on all its end terminals, i.e., the utility can be the average SINR at all its terminals. When adopting a function of SINR on all terminals as utility, as the terminals are mobile and they are influenced by many factors, i.e., the type of service provided to the terminals, the utility may diverge from the real performance of the terminals. On the other hand, it is not appropriate to choose one [9] or more fixed terminals, and use their SINRs to represent the SINR for all the other terminals in that cell, because their location could diverge greatly with the locations of the other terminals. Thus, we propose a metric *QuasiSINR* to represent WBS's performance on providing services to its end terminals, which is independent on the actual locations of end terminals.

QuasiSINR of WBS

Instead of improving the SINR on each end users of WBSs, we propose a metric QuasiSINR to represent the services provided by WBSs to its end users. Then the utility of WBS becomes a function of the and try to improve this metric. QuasiSINR of a WBS is the ratio between the weakest signal of interest on a reference point and the summation of the strongest interference caused on the relevant reference point.

We need an auxiliary circle to construct the reference point for each WBS, which is shown in Figure 3. As to WBS i, the auxiliary circle is the dashed circle centred at WBS i, whose radius is δ. Assume WBS i and all the other WBSs work on the same channel c, then co-channel interference are caused on its end users by all the other WBSs. The intersection of the auxiliary circle and the connecting line between WBS i and one interfering WBS j, which is shown as red dot, is a reference point which corresponds to the interfering WBS j. There are multiple reference points on the auxiliary circle, which corresponds to the co-channel interfering WBSs respectively. The power of signal from i on the auxiliary circle, the green dot, is the reference point for the power of signal of interest. We can see both of the reference point of interference and the reference point for the power of signal of interest are largely decided by the radius of the auxiliary circle δ.

The co-channel interference on the reference point of WBS i from WBS j is,

$$f_{ji}^c = P_j^c \cdot h_{ji} \cdot z_{ji} = P_j^c \cdot (d_{ji} - \delta)^{-\alpha} \cdot z_{ji} \tag{3}$$

where d_{ji} is the distance between WBS i and j, while, h_{ji} and z_{ji} are the attenuation and shadowing from WBS

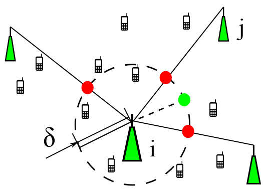

Figure 3. QuasiSINR is the ratio between the power of signal of interest on the green point with the sum of co-channel interference on the red points

j to the relevant interference reference point. The sum of interference on WBS i' interference reference points is denoted as f_i^c.

$$f_i^c = \sum_{j \in \mathcal{N}, c(j)=c} f_{ji}^c \tag{4}$$

The power of the signal of interest on auxiliary circle is expressed as,

$$\tilde{P_i}^c = P_i^c \cdot h_i \cdot z_i = P_i^c \cdot \delta^{-\alpha} \cdot z_i \tag{5}$$

where h_i and z_i are the attenuation and shadowing from i to any point on the auxiliary circle.

Then the quasiSINR of WBS i is denoted as γ_i,

$$\begin{aligned} \gamma_i &= \frac{\tilde{P_i}^c}{f_i^c + N_0} \\ &= \frac{P_i^c \cdot h_i \cdot z_i}{\sum_{\substack{j \neq i, j \in \mathcal{N} \\ c(j)=c(i)}} (P_j^c \cdot h_{ji} \cdot z_{ji}) + N_0} \\ &= \frac{P_i^c \cdot \delta^{-\alpha} \cdot z}{\sum_{\substack{j \neq i, j \in \mathcal{N} \\ c(j)=c(i)}} (P_j^c \cdot (d_{ji} - \delta)^{-\alpha} \cdot z_{ji}) + N_0} \end{aligned} \tag{6}$$

The radius of the auxiliary circle δ can be adjusted to let WBS foster better service to the terminals in certain area. For instance, to take care of the SINR on the border area of the cell, the radius δ can be set as the distance between WBS i and the furthest associated terminal. When the terminals concentrate towards to the WBS, δ can be set smaller to better fit to the terminals' distribution.

Because of the auxiliary circle, the interaction between co-channel WBSs are independent on the location of individual end terminals, and WBSs only take care the co-channel WBSs. As a result, the concrete terminals are excluded from the channel and power allocation problem, which simplifies the problem to be discussed. In the following part of this chapter, the notations are exclusively about the WBSs. We clarify the meaning of some notations here.

Based on Formula 4, as to WBS i, when the information is given, which include the locations of other co-channel WBSs, the radius of auxiliary circle and the standard deviation of shadow fading, then the co-channel inference caused on its interference reference points by other co-channel WBSs can be obtained, besides, WBS i is also aware of the interference it causes on the interference reference points associated with other co-channel WBSs.

According to our system model, WBSs are able to access the central database which stores all WBSs' geolocations i.e., working channel, transmission power, the characteristics of radio frequency environment such as parameters of attenuation and shadowing. To obtain quasiSINR, one WBS doesn't need to measure the signal strength on the reference points which is needed in Formula 6, instead, it can can make use of the propagation model [4] along with the operating parameters of all the other WBSs, which are stored in the central database to calculate the quasiSINR of any specific WBS.

Table I. Notations

Symbol	Description
γ_i	QuasiSINR of WBS i
f_{ji}^c	The interference caused by WBS j on the interference reference point of WBS i, and both of them work on channel c
f_i^c	The sum of interference caused on the interference reference points of WBS i
$\tilde{P_i}^c$	The power of the signal of interest on auxiliary circle of WBS i which works on channel c
p_i^k	The transmission power of WBS i
P_i^k	The maximal transmission power of WBS i in scheme I
P_μ, P_{op}	The minimum and maximal possible transmission power of WBS i in scheme II
h_{ij}	The attenuation from the co-channel interfering WBS j to the corresponding interference reference point of WBS i.
h_i	The attenuation from WBS i to its auxiliary circle.
z_{ij}	The shadowing from the co-channel interfering WBS j to the corresponding interference reference point of WBS i.
z	The shadowing from WBS i to auxiliary circle.
$\alpha_{ij}^k, \beta_{ij}^k$ y_i, z_i	4 binary auxiliary variables in the optimization in Section 5.

Problem Formulation

Our goal can be illustrated in the form of a constrained optimization problem. To ensure fairness among WBSs, instead of maximizing the sum of quasiSINR of all WBSs, we minimize the sum of inverted quasiSINR.

$$\begin{aligned} &\text{Minimize} && \sum_{i \in \mathcal{N}} \frac{1}{\gamma_i} \\ &\text{subject to} && \sum_{k=1}^{|\mathbb{C}|} x_{ik} = 1 \\ &&& P_{i,min}^c \leq P_i^c \leq P_{i,max}^c, c \in \mathbb{C}, i \in \mathcal{N} \end{aligned} \tag{7}$$

where $P_{i,min}^c$ and $P_{i,max}^c$ are the minimal and maximal transmission power of the transmitter of WBS i, where are

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restricted by the hardware configuration or capabilities. We assume $P^c_{i,min}$ and $P^c_{i,max}$ are identical for all WBSs and over all channels.

When a WBS works on different channels, the co-channel interference received by its end users from other WBSs is different. In order to provide better service to its end users, WBS is motivated to choose the channel which either permits higher transmission power or experiences less interference, or the channel compromising the two factors according to Formula 2. Achieving optimal white spectrum allocation in a distributed style is the goal of this work, furthermore, this distributed solution should converge fast and lead to an efficient and stable solution.

3. PROBLEM DECOMPOSITION AND RELATED WORKS

In related works, the protection on primary users is taken care in the same time when channel and power selection are conducted. But according to the current regulations and standards, there exist no communication means between the secondary users and the primary users. Besides, when assuming such communication media is available and preventing primary users from being interfered during secondary users' power and channel allocation, the communication overhead between primary users and second users is considerable.

Utilization of TV White Space

Here we introduce the solutions proposed on the utilization of TV white space, which includes regulations, proposed standards and recent research advances. In accordance with the regulations of FCC, there are some prototype applications proposed in both cellular network [10, 11] and WiFi-like network [12]. The secondary users access a centralized data base to know the allowed channels and transmission power. Standardization bodies are also working on TVWS utilization, including IEEE 802.22 [13] for Wireless Regional Area Networks (WRAN), IEEE 802.11af [14] for WLAN, IEEE 802.15.4m [15] for 802.15.4 wireless networks in TVWS and 802.19.1 [16] for coexistence methods among local and Metropolitan Area Networks (MAN).

Scientific research on utilization of TVWS goes on in parallel with the regulatory agencies. Feng et al. [17] investigate the business model of TV spectrum utilization in database involved network structure, emphasis on the price policy of the channels approved by FCC. Spectrum sharing in TVWS is formulated as a series of optimization problems. The guarantee that TV receivers should not be affected by the aggregate interferences form TVBDs is one constraint. The objective can be maximizing TVBD's downlink transmission power [2], uplink transmission power [18], or best geographic distribution of TVBDs [19]. A series of works [20–24] emphasise on interference mitigation among TVBDs via spectrum allocation. Vehicular networks operating with TVWS assisted by TV database and cooperative sensing is discussed in [25]. Work [26] steps further from the database paradigm and makes efforts to utilize the *grey space*, where TVDB is allowed to operate even within the TV service area.

Related Works on Maximal Transmission Power Planning

To protect the TV receivers from harmful interference, the aggregate interference caused by WBSs at the contours of TV receivers should not exceed the interference margin. Work [27] proposes detailed calculations which a geolocation database performs in order to derive location-specific maximum permitted EIRP levels for white-space devices (WSDs) which operate in digital terrestrial TV bands. [2] considers the maximum permitted transmission power for the network which complies with IEEE 802.22 standard. The standard requires a centralized database to store the available channels for each secondary base station, thus centralized scheme can be conducted there after trivial modification. The sufficient condition for the TV receivers not be interfered in the context of TV white space is formulated into a centralized linear programming program (LP) in [2]. The objective function is to maximize the summation of all secondary base stations' transmission power, and the constraints are formed to satisfy the sufficient condition for every interference measuring device for the TV receivers. However, this approach doesn't take the channel assignment problem into account.

Related Works on Channel Allocation with Fixed Transmission Power Level

In our proposed solution, after obtaining the maximum transmission power on each channel, WBSs need to decide one channel to use, and the transmission power is the maximum transmission power, so as to mitigate interference among WBSs and provide the best SINR for their associated end users. Note that in our problem, the transmission power is different for two interfering users when they work on the same channel.

Channel allocation problem dealing with mitigating co-channel interference via channel allocation, which has been attracting plenty of research efforts in the past decade, from multiple channel mesh network [28], Ad hoc network [29] up to cognitive radio network [21, 30].

Channel assignment problem tries to mitigate co-channel interference among users, which can be converted into colouring problem thus is NP hard [28]. Authors of [29] propose heuristic algorithms utilizing best response to improve its welfare, but the transmission power is assumed identical and path loss is deemed as symmetric, which renders this method problematic for our problem where transmission is non-identical and the path loss is asymmetric. [31] formulates channel assignment problem in ad-hoc cognitive radio network into potential game which leads to pure NE, a learning scheme achieving slightly better performance is provided for comparison, but they assume the transmission power is identical and there is no noise in the secondary network, and the proposed random access mechanism demands a huge amount of information to be exchanged, which is a burden for network in ad-hoc structure. [32, 33] investigate the channel allocation problem under game framework in same collision domain, the authors propose algorithms to converge to pure Nash equilibrium (NE) and strongly dominate strategy equilibrium respectively.

Authors of [29] propose heuristic algorithms utilizing best response based on the welfare on itself to assign channels among users. Simulated annealing is applied to mitigate co-channel interferences in [21]. For the same purpose, no-regret learning [30, 34] is exploit to optimize the choice on channel.

All the available channel allocation schemes are designed under the same assumption, that the transmission power levels are identical, and the attenuation between any pair is reciprocal. As to our knowledge, there is no work dealing with channel allocation problem where transmission power is different.

In this paper we propose two schemes to exploit the TV white space.

4. CHANNEL AND POWER ALLOCATION SCHEME COMPLYING WITH ECC RULES

In this section we will discuss the channel and power allocation problem complies with the rules regulated by ECC. According to the ECC regulations, the transmission power of the WBSs are strictly regulated, so that the TV receivers are affected even when all the WBSs operate on the same channel.

4.1. The Maximum Permitted Transmission Power

The WBSs work in underlay manner and coexist with primary TV stations and receivers, the aggregate generated interference from WBSs on each channel should not exceed the threshold of the TV receivers. We adopt the interference model and the optimization methodology from the work of [2] to plan the maximum transmission power on each channel for WBSs. Having a global view of the propagation parameters, geolocations of WBSs and interference threshold at interference measuring devices which locate on the contour of TV service area, linear programming is implied in the database to calculate the maximum permitted power over each channel.

For WBS $i \in \mathcal{N}$, the maximum transmission power allowed on channel $c \in \mathbb{C}$ is denoted as P_i^c. As to each channel c, the generated interference on each interference measuring device should be within a predefined interference margin I_{pt}^c. The interference margin in a slow fading environment is decided according to [35].

Then the maximum permitted transmission power on channel c for each WBS can be obtained by solving the following optimization problem,

$$\begin{aligned} \text{Maximize} \quad & \sum_{i \in \mathcal{N}} P_i^k \\ \text{subject to} \quad & \sum_{i \in \mathcal{N}} (P_i^k \cdot h_{i,pt} \cdot z) < I_{pt}^k, \\ & P_{min}^k \leq P_i^k \leq P_{max}^k \end{aligned} \tag{8}$$

P_{min}^c is the prudent transmission power. P_{max}^c is the maximum transmission power which is restricted by the hardware. z is shadow fading as introduced in 2. Here we only consider the interference caused by WBSs, and omit the interferences from end terminals. Since WBSs' transmission power is higher and their altitude is higher [2], the downlink transmission contributes the major part of interference [36]. The first constraint indicates that the interference margin will not be exceeded even when all the WBSs work on the same channel.

Formula 8 will be solved for each channel $c \in \mathbb{C}$. After solving the $|\mathbb{C}|$ problems, the maximum permitted transmission power vector $\boldsymbol{P^c} = \{P_1, \cdots, P_{|\mathcal{N}|}\}, \forall c \in \mathbb{C}$ is obtained.

When working with the same transmission power, the WBSs locating closer to the TV interference measuring devices contribute more to the aggregate interference comparing with the WBSs which locate far from the TV interference measuring devices. Thus when implying linear programming to decide the maximal transmission power, the transmission power used by WBSs which are closer to the TV interference measuring devices is much higher than other WBSs. As a result the maximum permitted transmission power on each channel obtained with LP is seriously unbalanced.

To address this fairness issue, we maximize the sum of the logarithmic value of every WBS's transmission power, and formulate the problem into a convex optimization problem.

$$\begin{aligned} \text{Maximize} \quad & \sum_{i \in \mathcal{N}} \log p_i^k \\ \text{subject to} \quad & \sum_{i \in \mathcal{N}} (p_i^k \cdot h_{i,pt} \cdot z) < I_{pt}^k, \end{aligned} \tag{9}$$

This optimization will be solved for each channel $c \in \mathbb{C}$.

Figure 4 depicts the distribution of maximum permitted transmission power levels obtained in 100 simulations. In each simulation the locations of TV interference measuring devices are randomly decided around the WBSs. In Figure 4, It shows that when applying optimization 8, WBSs' transmission power levels are either the minimum transmission power or the maximum power allowed by the equipment hardware. When applying convex programming, the planed maximum permitted

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transmission power levels are distributed evenly in between the minimum and maximum power. The gain of SINR on end terminals by applying convex optimization to decide the maximal transmission power is illustrated in the simulation section.

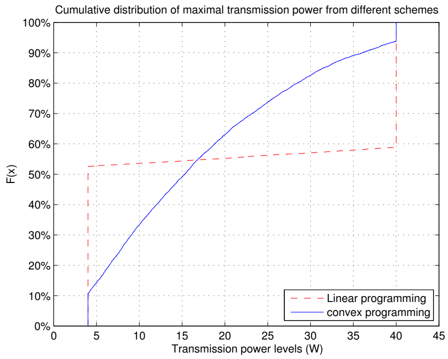

Figure 4. Distribution of maximum permitted transmission power levels obtained from convex and linear programming formulations

Optimization problem 9 provides the maximum permitted transmission power for every WBS over each channel. When all the WBSs working on the same channel, the generated interference doesn't exceed the threshold on the interference measurement devices at the contour of TV service area. If there are multiple channels available and WBSs are free to choose their preferred channels, the aggregate interference on one channel will be smaller than that when all WBSs work on that channel. Thus, there is exists a interference margin created by using multiple channels, which provides a room for network dynamics such as new WBS starting to work or increased interference on TV contour due to the variance of broadcast path condition.

In the following subsections, we firstly present the centralized solution to obtain the global optimum, then introduce the decentralized scheme under the game theoretic framework.

4.2. Centralized Optimization for Channel Allocation

We formulate the channel allocation problem into a binary quadratic programming problem which can be solved in a centralized way. Let $X_i = \{x_i^1, \cdots, x_i^k, \cdots, x_i^{|\mathbb{C}|}\}$ denote the vector of channel usage, there is $|X_i| = |\mathbb{C}|$ and binary element x_i^k represent whether WBS i occupies channel k. Given two WBSs i and j, there is,

$$X_i^T X_j = \sum_{k=1}^{|\mathbb{C}|} x_i^k \cdot x_j^k = \begin{cases} 1 & \text{if } c(i) = c(j) \\ 0 & \text{if } c(i) \neq c(j) \end{cases} \tag{10}$$

$c(i)$ means the working channel which is chosen by WBS i. The transmission power levels on all channels for WBS i are denoted by a constant vector $\mathbf{P}_i = \{P_i^1, \cdots, x_i^k, \cdots, x_i^{|\mathbb{C}|}\}$. The transmission power adopted by WBS i is $\mathbf{P}_i^T X_i = \sum_{k=1}^{|\mathbb{C}|} P_i^k \cdot x_i^k$.

Problem 7 can be modeled via general purpose nonlinear optimization:

$$\begin{aligned} \text{minimize} \quad & \sum_{i=1}^{n} \frac{\sum_{j \in \mathcal{N}, j \neq i} \mathbf{P}_j^T X_j (X_j^T X_i) h_{ji} z_{ji} + N_0}{\mathbf{P}^T X_i h_i z_i} \\ \text{subject to} \quad & \sum_{k=1}^{|\mathbb{C}|} x_i^k = 1, x_{ik} \in X_i \in \{0,1\}^{|\mathbb{C}|} \end{aligned} \tag{11}$$

Problem 20 is a non-linear problem with binary variables, but it can be reformulated into a quadratic programming problem as,

$$\begin{aligned} \text{minimize} & \sum_{i=1}^{n} \Big(\sum_{j \in \mathcal{N}, j \neq i} \sum_{k \in \mathbb{C}} \frac{P_j^k \cdot h_{ji} \cdot z_{ji}}{P_i^k \cdot h_i \cdot z_i} \cdot x_j^k \cdot x_i^k \\ & + \sum_{k \in \mathbb{C}} \frac{N_0}{P_i^k \cdot h_i \cdot z_i} \cdot x_i^k \Big) \\ \text{subject to} & \sum_{k=1}^{|\mathbb{C}|} x_i^k = 1, x_i^k \in X_i \in \{0,1\}^{|\mathbb{C}|} \end{aligned} \tag{12}$$

The reformulation is available in Appendix **??**. We use LINDO [37] which is a state of art non-linear problem solver to solve the problem, which employs Branch-And-Reduce method to get the global optimum for the problem. The result of this centralized channel assignment will be evaluated in the simulation section with other schemes.

4.3. Distributed White Space Channel Allocation (WhiteCat)

In this section a distributed scheme for WBSs to allocate channels is proposed, which is named as white space channel allocation technology (WhiteCat).

4.3.1. Algorithm and Protocol

WhiteCat adopts the best response process, where each WBS (referred as i) chooses the channel which brings the bigger utility u_i as the response of other WBSs' choices on channels. WhiteCat is depicted by algorithm 1.

$$u_i = \frac{\sum_{\substack{j \in \mathcal{N}, j \neq i, \\ c(\sigma_j) = c(\sigma_i)}} f_{ji}}{2 \cdot \tilde{P}_i} + \frac{1}{2} \sum_{\substack{j \in \mathcal{N}, j \neq i, \\ c(\sigma_j) = c(\sigma_i)}} \frac{f_{ij}}{\tilde{P}_j} + \sum_{\substack{\mathcal{S}: i, j \in \mathcal{S}, \\ c(\sigma_j) = c(\sigma_i)}} \frac{N_0}{C \cdot \tilde{P}_i} \tag{13}$$

where $f_{ij} = P_i \cdot h_{ij} \cdot z$ and $f_{ji} = P_j \cdot h_{ij} \cdot z$. Note that f_{ij} is the sum of interference on WBS i's interference reference points. Overlooking the constant coefficient 2, the first item of u_i is a part of the inverted QuasiSINR of station i. To minimize the first item, WBS i needs to choose a channel either permits higher transmission power or experiences less interference, whereas the higher power increases the second item which is a part of inverted QuasiSINR of other co-channel WBSs. Hence, the cost function presents a reasonable comprise between the welfare of one WBS and others.

When WBS only emphasizes on its own utility (e.g. the first part of Formula 13), the best response process doesn't converge. We have following theorem:

Theorem 4.1. With non-identical transmission power, if every WBS updates its channel based on Algorithm 1 with utility based on its own interests, the process doesn't always converge.

The proof is in Appendix **??**.

Algorithm 1: Spectrum selection by WBS i

Input: the distance, path lose and shadowing parameter between WBS i to WBS $j \in \mathcal{N} \setminus i$; radius of auxiliary circle, noise N_0, total number of WBSs N;
for $j \in \mathcal{N} \setminus i$, the maximal transmission power $P_j^c, c \in \mathbb{C}$ and the working channel $c(j)$.

```
for c ∈ ℂ \ c(i) do
    calculate u_i(c) based on Formula 13 if u_i(c) < u_i(c(i)) then
        c(i) ← c
    else
        keep c(i) unchanged
    end
end
Notify database of its channel usage, which further notifies the other WBSs
```

Some parameters needed to calculate the utility are identical for all WBSs, such as quasi distance e, the total number of WBSs N, number of channels C, attenuation factor α, standard deviation σ_{WBS} in flat shadowing and noise N_0, albeit the following information is further needed to calculate u_i:

- $\sum_{\substack{j \in \mathcal{N}, j \neq i, \\ c(\sigma_j)=c(\sigma_i)}} f_{ji}^c, c \in \mathbb{C}$: the received interference on i' virtual measurement point from other WBSs j working on the same channel for $\forall c \in \mathbb{C}$.
- f_{ij}^c: the interference caused by i on j's virtual measurement point when i works on channel $\forall c \in \mathbb{C}$.
- P_j^c: transmission power of j for using $\forall c \in \mathbb{C}$.

Unfortunately, it is difficult to measure the interference of interested, i.e., for WBS i, it is not efficient to scan all channels and obtain the interference f_{ji} on virtual measurement point for each channel, furthermore, it is impossible to split the interference f_{ij} from the total interference received on WBS j' virtual measurement point.

We refer [31] to decide WBSs' sequence to update their channels. [31] proposes a method which is akin to the random access mechanism of CSMA/DA, where the access for broadcast medium is changed to get access to the centralized center to retrieve the current channel usage and update its new channel. All WBSs are able to access the database in one round (with random or predetermined sequence). As WBSs are connected with database, the control messages needed to decide the sequence will not become a burden. Update of channels can happen in the boot phase, or when the quality of services (the SINR on its end users) of WBSs falls below a threshold, or a fixed time duration comes to end, or a new WBS joins in the network.

Similar with [6], we let every WBS store the location information and maximal power map of all other WBSs, i.e., $P_i^c, i \in \mathcal{N}, c \in \mathbb{C}$, and each WBS retrieves information about channel usage of other WBSs from centralized base station. After executing Algorithm 1, it reports to centralized database of its channel if it updates the working channel. As the location of WBSs and TV stations and the transmission channel and power of TV stations are usually static (entries of TV station change averagely once in 2 days [6]), except for the channel usage in the network, the change of the other data stored in WBS is infrequent.

4.3.2. Analysis in Game Theoretical Framework

In this section, We give the proof on whiteCat's convergence in the framework of congestion game theory. Formulating a spectrum sharing problem into a congestion game and the concept of *virtual resources* are firstly proposed in [38]. This work reversely engineers the distributed channel allocation schemes proposed in [29, 39], i.e., unifies the algorithms with congestion game. But the problem analysed in [38] assume the transmission power is identical, which is a major difference from the channel allocation problem discussed here.

We have introduced congestion game in Chapter **??**, thus we only recap the essence of congestion game here. In congestion game, each player acts selfishly and aims at choosing strategy $\sigma_i \in \Sigma_i$ to minimize their individual cost. The gain (loss) caused by any player's unilateral move is exactly the same as the gain (loss) in the potential, which may be viewed as a global objective function. For problems where the potential of the problem is the same with the summation of the cost of all users, the cost function can be used as a utility function directly. This equivalence doesn't exist in our problem, but by carefully choosing the cost function for players, we can make sure that the change of individuals' cost is in the same direction with that of the global utility.

4.3.3. From WhiteCat to Congestion Game

We utilize the conception of virtual resource which is firstly introduced in [38]. Virtual resource is a triplet $\{i, j, c\}$, where i, j are two WBSs and $c \in \mathbb{C}$ is one channel. This piece of resource is regarded used by i when

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both i and j use channel c, otherwise, $\{i,j,c\}$ is not used by any WBS.

In the following, we list the element of the congestion game which emulates Algorithm 1. In this section, player and base station are used interchangeably.

- Player i' strategy space is $\Sigma_i = \{(i,j,c), j \in \mathcal{N}, j \neq i, c(\sigma_j) = c, c = 1, 2, \cdots, N\}$, and i has C admissible strategies, one strategy related with channel $c \in \mathbb{C}$ is described by the set of virtual resources it uses: $\sigma_i = \{(i,j,c), j \in \mathcal{N}, j \neq i, c(\sigma_j) = c\}$, note that virtual resource $(i,j,c) \neq (j,i,c)$.
- Under the strategy profile $\sigma = (\sigma_1, \sigma_2, \cdots \sigma_N)$, player i obtains a total cost of

$$g^i(\sigma) = \sum_{\substack{j \in \mathcal{N}, j \neq i,\\ c = c(\sigma_i) = c(\sigma_j)}} (g_{(i,j,c)}(n_{(i,j,c)}(\sigma)) + g_{(j,i,c)}(n_{(j,i,c)}(\sigma))) \tag{14}$$

The transmission power over all channels of player i is $\{p_i^1, p_i^2, \cdots, p_i^{|\mathbb{C}|}\}$. We define the cost function for virtual recourses (i,j,c) as follows,

$$g_{(i,j,c)}(k) = \begin{cases} \dfrac{f_{ji}}{2\tilde{P}_i} + \dfrac{f_{ij}}{2\tilde{P}_j} + \dfrac{C \cdot N_0}{N \cdot \tilde{P}_i} & \text{if } k = 2 \\ 0 & \text{otherwise} \end{cases} \tag{15}$$

As resource (i,j,c) only lies in the strategy space of player i and j, thus can only be accessed by this two players. More specifically, according to Formula 15, the cost of resource (i,j,c) is only decided by the number of players using it, which is either 0 or 2. At the first glance, this is a player specific congestion game, as $g_{(i,j,c)}$ is decided by the relevant players' transmission power and inference. But actually the resource (i,j,c) excludes the players except for i and j from using it, thus the cost happened on this resource is only dependant on how many of players from the set $\{i,j\}$ to use it. Hence, the cost is a function of the number of players using the resource, and this is a canonical congestion game.

Now we substitute Formula 15 to Formula 14, the total cost for user i under strategy profile σ .

$$\begin{aligned} g^i(\sigma) &= \sum_{\substack{j \in \mathcal{N} \setminus i,\\ c = c(\sigma_j) = c(\sigma_i)}} (g_{(i,j,c)}(2) + g_{(j,i,c)}(2)) \\ &= \sum_{\substack{j \in \mathcal{N} \setminus i,\\ c(\sigma_j) = c(\sigma_i)}} \left(\frac{f_{ji}}{\tilde{P}_i} + \frac{f_{ij}}{\tilde{P}_j} + \frac{C \cdot N_0}{N}\left(\frac{1}{\tilde{P}_i} + \frac{1}{\tilde{P}_j}\right)\right) \\ &= \frac{\sum_{\substack{j \in \mathcal{N} \setminus i,\\ c(\sigma_j) = c(\sigma_i)}} f_{ji}}{\tilde{P}_i} + \sum_{\substack{j \in \mathcal{N} \setminus i,\\ c(\sigma_j) = c(\sigma_i)}} \frac{f_{ij}}{\tilde{P}_j} + \frac{CN_0}{N} \sum_{\substack{j \in \mathcal{N} \setminus i,\\ c(\sigma_j) = c(\sigma_i)}} \left(\frac{1}{\tilde{P}_i} + \frac{1}{\tilde{P}_j}\right) \\ &= \frac{\sum_{\substack{j \in \mathcal{N} \setminus i,\\ c(\sigma_j) = c(\sigma_i)}} f_{ji}}{\tilde{P}_i} + \sum_{\substack{j \in \mathcal{N} \setminus i,\\ c(\sigma_j) = c(\sigma_i)}} \frac{f_{ij}}{\tilde{P}_j} + \frac{2CN_0}{N} \sum_{\substack{i \in \mathcal{S} \subset \mathcal{N},\\ \mathcal{S}: \forall i \in \mathcal{S}\\ c(\sigma_i) = c}} \frac{1}{\tilde{P}_i} \end{aligned} \tag{16}$$

where $\mathcal{S}$ denotes the set of WBSs whose working channel is the same with WBS i.

Now we are going to have a look at the *potential* of the network. According to the expression of Rosenthal's potential in Formula 19, the potential is accumulated by adding the players' cost sequentially, in particular, the value which is added is the cost that player experiences when it starts to use the relevant resource, and the value is not changed when other players come to use that resource. Back to our problem, for two WBSs $i, j \in \mathcal{S}$, we assume WBS i's index is smaller than j's index, then the potential increased by i using the resource $\{i,j,c\}$ is 0 according to Formula 14, and the increase brought in by j using the resource $\{i,j,c\}$ is $g_{(i,j,c)}(2) + g_{(j,i,c)}(2)$. In other words, for each interfering pair of WBSs, only the WBS with bigger index contributes to the potential. Then the total potential is,

$$\begin{aligned} G(\sigma) &= \sum_{r \in \mathcal{R}} \sum_{i=1}^{n_r(\sigma)} g_r(i) = \sum_{i \in \mathcal{N}} \sum_{r \in \sigma_i} g_r(n_r^i(\sigma)) \\ &= \sum_{i \in \mathcal{N}} \frac{\sum_{\substack{j \in \mathcal{N}, j \neq i,\\ c(\sigma_j) = c(\sigma_i)}} f_{ji}}{\tilde{P}_i} + \frac{CN_0}{N} \sum_{\substack{\mathcal{S} \subset \mathcal{N},\\ \forall i \in \mathcal{S}, c(\sigma_i) = c}} |\,\mathcal{S}\,| \sum_{i \in \mathcal{S}} \frac{1}{\tilde{P}_i} \end{aligned} \tag{17}$$

note that the summation of one WBS's congestion is related to its index.

When players minimize their utilities (cost or potential) illustrated by Formula 16, the total congestion in the secondary network given by Formula 17 decreases monotonically before reaching one Nash equilibrium. Players' greedy update in the game to minimize its cost Function16, which ceases finally in pure Nash Equilibrium. The strategy and cost function of players in the game is transplanted as Algorithm 1 and utility Function 13 respectively.

4.3.4. Comparison between the Potential in the Congestion Game and the Objective of WhiteCat

It is natural to raise the question, is the sum of the final utilities of all WBSs exactly the same with the value of potential when the game converges to a Nash equilibrium, which is represented by 17? The answer is, they are identical when N_0 is zero, and there will be a little difference when N_0 is not zero. Recall the target objective we want to minimize in Problem 7 is,

$$\sum_{i\in\mathcal{N}}\frac{f_i}{\tilde{P}_i}=\sum_{i\in\mathcal{N}}\frac{\sum_{\substack{j\in\mathcal{N},j\neq i,\\ c(\sigma_j)=c(\sigma_i)}} f_{ji}+N_0}{\tilde{P}_i}$$
$$=\sum_{i\in\mathcal{N}}\frac{\sum_{\substack{j\in\mathcal{N},j\neq i,\\ c(\sigma_j)=c(\sigma_i)}} f_{ji}}{\tilde{P}_i}+\sum_{i\in\mathcal{N}}(\frac{N_0}{\tilde{P}_i}) \tag{18}$$

We notice that only the last items of the objective 18 and the potential of the congestion game 17 are different. When $N_0 = 0$, the potential is exactly the same with the object we want to minimize. When $N_0 \neq 0$, if channels are evenly distributed and there is $C/N* \mid \mathcal{S} \mid= 1$, then Formula 18 and 17 are also the same. In both cases, the sum of utilities 18 decreases monotonically with every update of WBSs before the system reaches Nash Equilibrium. When $N_0 \neq 0$ and Formula 18 and 17 are thus different, the monotonicity on the decrease of sum of utilities 18 is not perceived, whereas the system will still cease to NE.

Based on above analysis, we can see the assumption that each WBS only occupies one channel can be easily removed. If we regard one WBS as multiple ones which locate at the same place, and each WBS works on one distinct channel, then the proof on convergence of whiteCat can be applied directly to this case.

Note that the convergence of the game is independent on the the concrete form of the cost function. We adopt the function 16 to let the potential of the game be the same with the total utility of all WBSs, so that by executing Algorithm 1, the system objective experiences a monotonic decreasing process before the system reaching NE. The algorithm has potential to solve many other problems, where one user's decision affects others. In this case, the utility of one user can be formulated to incorporate the information of its own utility and others', then the congestion game theory can be used to analogize.

4.3.5. Communication Overhead of WhiteCat

The problem of channel allocation with different and fixed transmission power is NP hard. WhiteCat is a distributed scheme but certain information of the other WBSs is needed. The centralized base station is piggybacked to provided the needed information. As to one WBS, the number of such inquiries is the number of steps before convergence.

In our formulated congestion game, a player i is allowed to access up to $(N-1)$ resources in the same time, i.e., $\{i, j_1, c(i)\}, \{i, j_2, c(i)\} \cdots \{i, j_{N-1}, c(i)\}$, thus the upper bound of converge steps can not be obtained from the conclusion 19 for singleton congestion game. But our problem is special because for each resource, the possible number of players allowed to use each resource is either 2 or 0. Thus we can refer the method used in Section **??** to analyse the update times for our problem. Firstly, we sort the cost values in increasing order. Although a WBS

$$\begin{aligned}\phi(\sigma) &= \sum_{r\in\mathcal{R}}\sum_{i=1}^{n_r(\sigma)} g_r(i)\\ &\leq \sum_{r\in\mathcal{R}}\sum_{i=1}^{n_r(\sigma)} n\\ &\leq n^2 m\end{aligned} \tag{19}$$

The upper bound of total update steps is $2n^2$, thus averagely, the upper bound of update steps for each WBS is $2n$.

5. CHANNEL AND POWER ALLOCATION SCHEME COMPLYING WITH FCC RULES

The second scheme is in line with the FCC proposals in terms of transmission power, i.e., WBSs operate with the same transmission power P_{op}. When the number of transmitters increases, the accumulated interference may cause interference to TV receivers. In this scenario, the number of WBSs operating need to be restricted. This being said, when the number of WBSs is large, some of them need to stop transmitting.

5.1. Distributed Scheme

To protect the TV receivers from harmful interference, some WBSs are not allowed to operate, and the operating WBSs choose channels in distributed manner. We name this scheme as white space Secondary User Selection and Spectrum Allocation (WhiteSussa).

5.2. Centralized Optimization for Channel Allocation

Due to the existence of certain WBSs which don't transmit, we can not adopt the problem formulation 7 directly because some denominators in the objective function, which are related with the idle WBSs are zero. In order to solve this problem, we allow the idle WBSs to work with a very low transmission power. That being said, the power used by user i can be presented as $\mathbf{P}_i^T X_i = \sum_{k=1}^{|\mathbb{C}|} p_i^k \cdot x_{ik}$. There is $p_i^k \in \{P_\mu, P_{\text{op}}\}$, where P_μ represents the very weak transmission power. The transmission power levels are the same as for all the WBSs, so we omit the superscript when we refer the transmission power.

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As all the WBS transmit with the same transmission power, the Problem 7 is modeled as following:

$$
\begin{aligned}
\text{Minimize} \quad & \sum_{i=1}^{n} \frac{\sum_{j\in\mathcal{N},j\neq i} \mathbf{P}_j^T X_j (X_j^T X_i) h_{ji} z_{ji} + N_0}{\mathbf{P}_i^T X_i h_i z_i} \\
\text{subject to} \quad & \sum_{k=1}^{|\mathbb{C}|} x_i^k = 1, x_i^k \in X_i \in \{0,1\}^{|\mathbb{C}|} \\
& p_i^k \in \{P_\mu, P_{\text{op}}\}, \texttt{for} \forall i \in \mathcal{N} \\
& \sum_{i\in\mathcal{N}} (p_i^k \cdot h_{i,pt} \cdot z) < I_{pt}^c, \texttt{for} \forall k \in \mathbb{C}
\end{aligned} \tag{20}
$$

Problem 20 is a non-linear problem with binary variables, with the technique in Section 4.2, it can be written as,

$$
\begin{aligned}
\text{minimize} \quad & \sum_{i=1}^{n} \Big(\sum_{j\in\mathcal{N},j\neq i} \sum_{k=1}^{|\mathbb{C}|} \frac{p_j \cdot h_{ji} \cdot z_{ji}}{p_i \cdot h_{ii} \cdot z_{ii}} \cdot x_j^k \cdot x_i^k \\
& + \sum_{k=1}^{|\mathbb{C}|} \frac{N_0}{p_i \cdot h_{ii} \cdot z_{ii}} \cdot x_i^k \Big) \\
\text{subject to} \quad & \sum_{k=1}^{|\mathbb{C}|} x_i^k = 1, x_i^k \in X_i \in \{0,1\}^{|\mathbb{C}|} \\
& p_i \in \{P_\mu, P_{\text{op}}\} \texttt{for} \forall i \in \mathcal{N} \\
& \sum_{i=1}^{n} x_i^k \cdot p_i \cdot h_{i,pt} \cdot z \leq I_{pt}^k, \texttt{for} \forall k \in \mathbb{C}
\end{aligned} \tag{21}
$$

Formulation 21 is nonlinear but we can transform it into a quadratic optimization with auxiliary variables. We introduce binary number α_{ij}^k, real number β_{ij}^k and q_i, where

$$x_j^k \cdot x_i^k = \alpha_{ij}^k \tag{22}$$

$$\beta_{ij}^k = p_j \cdot \alpha_{ij}^k \tag{23}$$

$$\frac{1}{p_i} = q_i \tag{24}$$

Now the objective function is is quadratic as shown in Formula 25,

$$\sum_{i=1}^{n} (h_{ji} z_{ji} \sum_{j\in\mathcal{N},j\neq i} \sum_{k=1}^{|\mathbb{C}|} \beta_{ij}^k q_i + N_0 \sum_{k=1}^{|\mathbb{C}|} x_i^k q_i) \tag{25}$$

Constraint 23 can be transformed as a set of linear constraints:

$$\beta_{ij}^k - p_{\text{op}} \cdot \alpha_{ij}^k \leq 0 \tag{26}$$

$$-\beta_{ij}^k + p_\mu \cdot \alpha_{ij}^k \leq 0 \tag{27}$$

$$\beta_{ij}^k - p_j - p_\mu \cdot \alpha_{ij}^k \leq -p_\mu \tag{28}$$

$$-\beta_{ij}^k + p_j + p_{\text{op}} \cdot \alpha_{ij}^k \leq p_{\text{op}} \tag{29}$$

The constraint 24 is a quadratic equality, which makes the problem to be non-convex. Thus we need to relax it into a linear inequalities. As p_i is either P_{op} or P_μ, we can replace p_i and q_i with the help of two binary variables y_i and z_i respectively:

$$p_i = y_i P_\mu + (1 - y_i) P_{\text{op}} \tag{30}$$

$$q_i = z_i \frac{1}{P_{\text{op}}} + (1 - z_i) \frac{1}{P_\mu} \tag{31}$$

Here we put a restriction on P_μ and P_{op} with $P_\mu P_{\text{op}} = 1$, Function 31 becomes

$$q_i = P_\mu z_i + (1 - z_i) P_{\text{op}} \tag{32}$$

Now we conduct convex relaxation by applying McCormick envelope [40]. As we want to get the lower bound of the objective function, we get the over-estimators so as to expand the search space as follows,

$$1 = p_i \cdot q_i \leq q_i P_{\text{op}} + p_i P_\mu - P_{\text{op}} P_\mu \tag{33}$$

$$1 = p_i \cdot q_i \leq p_i P_{\text{op}} + q_i P_\mu - P_{\text{op}} P_\mu \tag{34}$$

$$1 = p_i \cdot q_i \geq q_i P_\mu + p_i P_\mu - P_\mu^2 \tag{35}$$

$$1 = p_i \cdot q_i \geq q_i P_{\text{op}} + p_i P_{\text{op}} - P_{\text{op}}^2 \tag{36}$$

By bringing Formulas 30, 32 into the Formulas 33 to 36, the binary linear constrains with respect to the transmission powers are obtained, which are inequalities 47 to 50 in the transformed optimization problem 21. Meanwhile, inequalities 41 to 42 are relaxation of Formula 22. Inequalities 43 to 46 are relaxation of Formula 23 when we bring Formula 30 into Formula 28 and 29.

$$\text{minimize} \tag{37}$$

$$\sum_{i=1}^{n} \sum_{j\in\mathcal{N}, j\neq i} \sum_{k=1}^{|\mathbb{C}|} ((P_\mu - P_{\text{op}}) h_{ji} z_{ji} \beta_{ij}^k z_i \tag{38}$$

$$+ (P_\mu - P_{\text{op}}) N_0 x_i^k z_i + P_{\text{op}} h_{ji} z_{ji} \beta_{ij}^k + P_{\text{op}} N_0 x_i^k) \tag{39}$$

$$\text{subject to:} \tag{40}$$

$$x_i^k + x_j^k - \alpha_{ij}^k \leq 1 \tag{41}$$

$$- x_i^k - x_j^k + 2\alpha_{ij}^k \leq 0 \tag{42}$$

$$\beta_{ij}^k - P_{\text{op}} \alpha_{ij}^k \leq 0 \tag{43}$$

$$- \beta_{ij}^k + P_\mu \alpha_{ij}^k \leq 0 \tag{44}$$

$$\beta_{ij}^k + (P_{\text{op}} - P_\mu) y_j - P_\mu \alpha_{ij}^k \leq P_{\text{op}} - P_\mu \tag{45}$$

$$- \beta_{ij}^k + (P_\mu - P_{\text{op}}) y_j + P_{\text{op}} \alpha_{ij}^k \leq 0 \tag{46}$$

$$P_\mu y_i + P_{\text{op}} z_i \leq \frac{P_{\text{op}}^2 - 1}{P_{\text{op}} - P_\mu} \tag{47}$$

$$P_{\text{op}} y_i + P_\mu z_i \leq \frac{P_{\text{op}}^2 - 1}{P_{\text{op}} - P_\mu} \tag{48}$$

$$- y_i - z_i \leq \frac{1 + P_\mu^2 - 2P_{\text{op}} P_\mu}{P_\mu (P_{\text{op}} - P_\mu)} \tag{49}$$

$$- y_i - z_i \leq \frac{1 - P_{\text{op}}^2}{P_{\text{op}} (P_{\text{op}} - P_\mu)} \tag{50}$$

$$\sum_{k=1}^{|\mathbb{C}|} x_i^k = 1, x_i^k \in X_i \in \{0,1\}^{|\mathbb{C}|} \tag{51}$$

$$\sum_{i=1}^{n} (((P_\mu - P_{\text{op}}) x_i^k y_i + P_{\text{op}} x_i^k) h_{i,pt} z_{i,pt}) \leq I_{pt}^k \tag{52}$$

$$x_i^k, \alpha_{ij}^k, \beta_{ij}^k, y_i, z_i \in \{0,1\} \tag{53}$$

$$\forall i, j \in \mathcal{N}, i \neq j, k \in \mathbb{C} \tag{54}$$

Inequalities 41 and 42 are equivalent to Formula 22. Inequalities 43 - 46 are equivalent to Formula 23. Inequalities 47 - 50 are relaxation of the equality quadratic 24. The objective function is quadratic and there is only linear and quadratic inequality constraints, so we use GUROBI Mixed-Integer Programming solver to solve it get the lower bound of the objective function.

5.3. Distributed White Space Channel Allocation (WhiteSussa)

Being aware of the accumulated interference on the interference measurement devices for the primary users, each WBS can decide whether to switch from idle state to operation state and work on a channel which brings it with the highest utility, or to change its operating channel to get higher utility value. The operation of the WBSs should not result in harmful interference onto the primary system.

The pseudo code is as follows,

When a WBS switches its state from idle to operation, new interference is generated on the other WBSs which work with the same channel. Due to this disturbance , the affected WBSs may change their channels to achieve better γ.

Algorithm 2: Spectrum selection by WBS i

Input: For each WBS, the complete graph of the network, which includes: (1) the distance and signal shadowing between it and any other WBS and TV interference measurement device; (2)Transmission power and channel of the WBS $i \in \mathcal{N}$.

while *Operation profile is different from previous round* **do**
  for $i \in \mathcal{N}$ **do**
    if *i is idle* **then**
      if $\exists c$, ***where*** $\sum f_t^c \leq Thold_t^c, \forall t \in \mathcal{T}$, $\gamma_i^c > \gamma_i^{c'}$, $c' \in \mathbb{C} \setminus c$ **then**
        $c(i) = c$
      end
    end
    if *i is operating* **then**
      if $\exists c', f_i^{c'} < f_i^{c(i)}$ ***and*** $\sum f_t^{c'} \leq Thold_t^{c'}, \forall t \in \mathcal{T}$ **then**
        $c(i) = c'$
      end
    end
  end
end

This process can be formulated as a congestion game which eventually converges, furthermore, the dynamics initiated by the WBS starting operation eventually ceases. Here we have Lemma 5.1.

Lemma 5.1. The dynamics caused by a WBS changing its state from idle to operation ceases after finite steps.

Proof
After a WBS switches on, all the WBSs will choose the channel which brings the highest γ, i.e., , the lowest co-channel interference considering the transmission power is identical to each WBS. This process is the same as the problem of channel assignment with identical transmission power, which is discussed in [38]. In [38], WBSs change operating channels in order to minimize the received co-channel interference is formulated into a congestion game, then the convergence of the channel assignment is proved. □

Note that according to Algorithm 2, after a WBS starts to operate, it will never change back to idle state. Now we have the following theorem.

Theorem 5.1. *The whole process ends after finite updates.*

Proof
Whenever there is a WBS changes its state from idle

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to operation, a new congestion game among the WBSs begins. As the convergence is guaranteed as shown in lemma 5.1 and there is only a limited number of WBSs which change their state from idle to operation, the whole process, which consists of a series of concatenated congestion games, will eventually end. □

6. PERFORMANCE EVALUATION

Performance evaluation consists two parts, in the first part whiteCat are compared with other distributed schemes proposed for the problem of channel allocation with different fixed transmission power, where the transmission power levels associated with WBSs and channels are different. In the second part, we compare the distributed joint channel and power allocation solution with other solutions for this problem. To illustrate the structure, we list the contents in Figure 5.

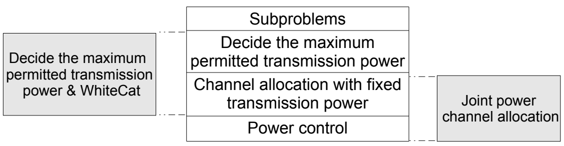


Figure 5. The evaluation contents in this section, the left part is discussed in Section 6.1 and the right part is in Section 6.2.

The evaluation setting is as follows. A square area which is 60km x 60km is divided evenly into 16 square blocks. There is one WBS locating in the middle of each block. Same amount of end terminals are distributed in each block, where the end terminals don't necessarily choose the WBS which is nearest to them to obtain service, in stead they choose the WBS which provides them with the strongest RSSI. There is a 20km wide rim area around the square area, where the interference measurement devices for the TV receivers are randomly located. As to each channel there is one interference measurement device. The locations of WBSs and TV contours are illustrated in Fig. 6. WBSs' locations are fixed, but the locations of interference measurement devices for the TV receivers, the end terminals, and the sequence for WBS to update are randomly decided in each run. Simulations are conducted for 50 times.

The other parameters are listed in Table 14.

6.1. Maximal Permitted Power Decision and the Distributed Channel Allocation Schemes

In this section, we will firstly evaluate the convex optimization and linear optimization proposed in Section 4.1 to see which is the better choice to decide the maximum permitted transmission power, the adopted metrics are

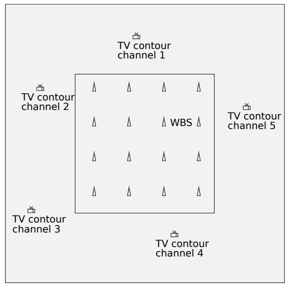


Figure 6. Layout of WBSs and TV contours

Number of channels	5
Number of WBSs	16
Noise	10^{-12}W
Length of side the square to locate WBSs	60km
Distance between quasai terminal and WBS	7km
Interference threshold on TV contour	10^{-7}W
Path loss factor	2
Standard deviation in flat shadowing	8
Minimal WBS transmission power [§]	4W
Maximal WBS transmission power	40W
Number of end terminals in network	800

Table II. Simulation parameters

average power consumption, and SINR on end users where channel allocation is executed. Then with the decided better method, we compare given the power map, how do the channel allocation schemes perform. We compare our proposed channel allocation scheme *whiteCat* with three other distributed schemes, the random allocation scheme, *whiteCase* and No-regret learning, besides, centralized optimization is used to obtain global optima.

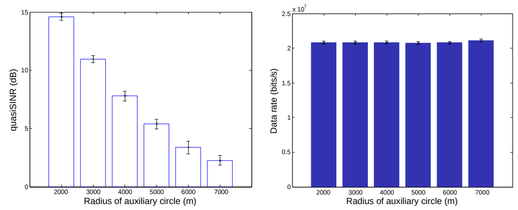
- *WhiteCase*: Whitespace channel allocation selfish, where each WBS selfishly updates its channel to achieve the best (as to the considered problem, smallest) possible utility based on Formula **??**.
- *Noregret learning*: Each WBS maps the probability of choosing each strategy to a certain proportion of the regret which the WBS may have if it doesn't choose that strategy, and the WBS choose the strategy with the biggest probability. WBSs update such mapping dynamically and this approach converges to correlated equilibrium. Please refer the original paper [34] for details.
- *Quadratic optimization*: centralized quadratic optimization introduced in Section 4.2.

The Choice of Radius of Auxiliary Circles, quasiSINR of WBS, and SINR on End Users

The usage of quasiSINR exempts WBSs from taking care the SINR on the end terminals. A WBS's quasiSINR is related with WBS's location and the radius of auxiliary circle. Figure 7 illustrates the effect of using different radii

[§]minimal and maximal power here denote the power level restricted by the specification of hardware.

of the auxiliary circle on the data rate can be achieved by end terminals.



(a) quasiSINR of WBSs (b) Data rate of end terminals when applying whiteCat

Figure 7. The effects of different radii of auxiliary circle on end terminals' data rate. Maximum permitted power is obtained by solving convex optimization. WhiteCat is used to assign the channels.

Subfigure 7(a) shows WBSs' quasiSINR decreases when the radii of auxiliary circles increase. Subfigure 7(b) illustrates the choice on radius of auxiliary circle don't influence the performance of whiteCat. In the following simulation, we fixed the radius at 6000 m.

Performance of Channel Allocation Schemes

In Section 4.1, two different optimization formulations are introduced to obtain the maximum permitted transmission power for WBSs, i.e., convex optimization and linear optimization respectively. In Figure 4, we have seen that the convex optimization generates power levels which distribute evenly between the minimum and maximum transmission power levels configured by the hardware, while, the majority of the power levels generated by linear optimization are either the minimum or maximum transmission power. In this section we run the channel allocation schemes with the maximum permitted power levels obtained from convex and linear optimization respectively. The simulation in this subsection carries twofold meanings. The first is to see which maximum permitted power decision method outperforms the other, the second is to evaluate the performance of the channel allocation schemes. The adopted metrics are the SINR on end terminals and transmission power consumption.

Comparison of the Methods for Maximum Permitted Transmission Power

Figure 8 depicts the power consumption of the channel allocation schemes which work with the two groups of maximum permitted transmission power decided by linear and convex problems respectively. When given maximum permitted transmission power, whiteCat and the centralized optimization scheme consume the least energy. The schemes utilize less transmission power with the maximum permitted transmission power decided by convex optimization. Figure 9 shows the quasiSINR of WBSs. The centralized optimization scheme achieves the highest quasiSINR, because the optimization formulation 20 obtains the global optima.

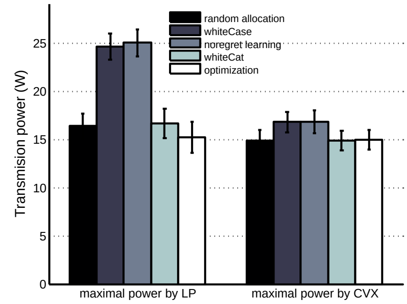


Figure 8. Power consumed of WBSs by different distributed spectrum allocation schemes under different ways deciding the maximal transmission power map

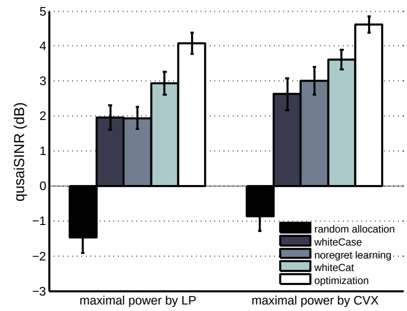


Figure 9. QuasiSINR of WBSs achieved by different distributed spectrum allocation schemes under different ways deciding the maximal transmission power map

The average SINR on the end terminals is depicted in Figure 10. When the given maximum permitted transmission power, whiteCat and the centralized optimization achieve similar and the best performance among the schemes. It is also noticed that, the maximum permitted transmission power decided by linear optimization helps the channel allocation schemes achieve better SINR.

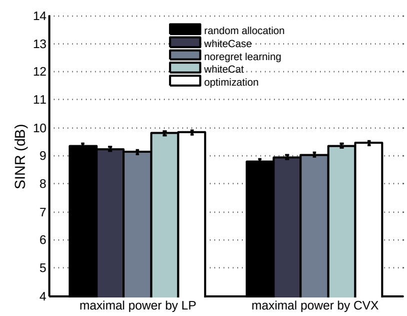


Figure 10. SINR on end terminals achieved by different distributed spectrum allocation schemes under different ways deciding the maximal transmission power map

The empirical cumulative distribution function curve of SINR on end terminals is drawn in Figure 11. The SINR

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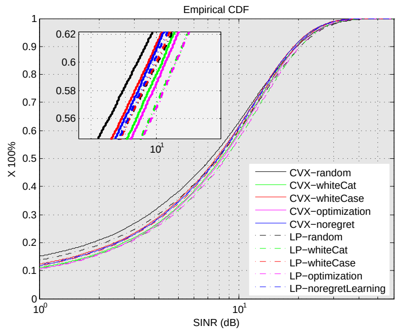

Figure 11. CDF of SINR on end users obtained by different CA schemes under different methods to decide the maximal transmission power map

achieved by WhiteCat and the centralized optimization is stably higher than that obtained from other schemes. For example, the 20% and 80% percentile of the SINR achieved by WhiteCat and the centralized optimization are 0.5 to 1 dB higher than the other channel allocation schemes.

Convergence Speed

In the congestion game where scheme whiteCat is derived, each player (WBS) has at most $(n-1) * |\mathbb{C}|$ resources available for usage, thus there is no polynomial steps converging to NE, while, simulation shows the algorithm can quickly converge to NE when the number of WBS is up to 100. Table III shows the average number of steps needed before convergence in 100 runs of simulations. As to whiteCat, we account each WBS accessing the base station (refer to 4.3) as *one step*. We compare the convergence speed of WhteCat with no-regret learning, the scheme derived from potential game [41] and whiteCase. Note that the potential game scheme is to solve joint power and channel allocation problem, as it is developed with game theory, it is reasonable to see its convergence speed. As there is no guarantee for WhiteCase to converge, we stop the channel allocation process after 16000 steps (1000 rounds).

Table III tells that whiteCat is two times faster than the scheme derived from potential game, and 20 times faster than no-regret learning scheme. The relatively smaller confidence interval shows that whiteCat's convergence is not affected by different network configurations. Fast converge is attributed to the working style of WBSs whcih access the database to get the information of other WBSs, thus the distributable decision involves a part of the global information of the network. Thus, we can see that the speed up of convergence is due to the overhead caused by accessing the database.

Figure 12 depicts one instance of the convergence processes of three schemes. The Y axis is the summed utility of all WBSs. We can see whiteCat decreases the summed utility constantly, and the channel allocation process ceases after 38 times of updates. Whereas, noregret learning scheme takes 120 steps before convergence, and whiteCase fails to converge.

Scheme	Average steps	95% CI	Average time (s)
whiteCat	58	5.6	2
noregret	1916	1541	144
PotentialGame [41]	120	10	4
optimization-LINDO	-	-	40
whiteCase	4587	2742	50

Table III. Convergence speed of the distributed channel allocation schemes. As to the distributed scheme, the time involved to communicate with database is note considered and included.

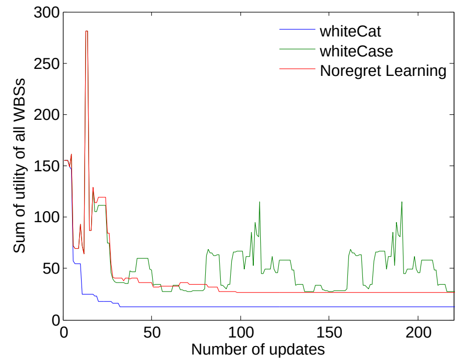

Figure 12. Convergence process of three different schemes in one simulation.

Stability of SINR in Convergence Process

WBS provides service to end users in the process of channel allocation. A certain SINR corresponds to certain transmission configurations like modulation type and data rate. The oscillation of SINR resulted from WBS changing the working channel during the convergence process may cause reconfiguration, reduced throughput or delay variance, which is not preferred. We propose a metric *Cost of Oscillation* (COS) to represent the stability of SINR in the converging process. Assuming each update of channel takes 1 time unit, the variance of SINR of end user i at time $t+1$ is

$$\varDelta\gamma_i(t+1) = \left| \frac{\gamma_i(t+1) - \gamma_i(t)}{\gamma_i(t)} \right|$$

.

The COS value for one network applied with a certain channel allocation scheme is,

$$COS = \sum_{t=1}^{T} \sum_{i \in \mathcal{N}} \varDelta\gamma_i(t) \quad (55)$$

$\gamma_i(0)$ is the SINR for i before starting channel allocation. The variance of SINR in channel allocation process is shown in table IV from which we can see WhiteCat achieves only 6% of oscillation on SINR compared with No-regret approach.

Scheme	COS	95% confidence interval
WhiteCat	8850	2984
No-regret	145460	1541
WhiteCase	246790	168050

Table IV. Variance of SINR during the convergence process

6.2. Performance of Joint Power and Channel Allocation

As introduced in section **??**, after channel allocation is conducted, transmission power is adjusted in a distributive manner. In other words, power and channel allocation is executed with two cascaded distributed schemes. As comparisons, we implement two joint power channel allocation schemes. One is centralized optimization introduced in section **??**, which is used as upper bound in the comparison. The other comparison is distributed joint power and channel allocation scheme [41] which is introduced in Section 3, we name it as *potentialGame*. We need to point it out that, scheme *potentialGame* doesn't aim to improve the SINR on end terminals, but on the sum of produced and received interferences. The performance of joint channel and power allocation schemes are presented in Fig. 13, 14 and 15 in terms of total utility, power consumption and achieved SINR on end users respectively.

Figure 13 illustrates the comparison of the cascaded solutions, i.e., channel allocation and the following power control, in terms of the total utility in the network. We can see that our proposed scheme *whiteCat+dpa*, the cascaded channel and power allocation method falls behind the cascaded channel allocation optimization and power allocation, and the joint channel and power allocation optimization, but outperforms all the other distributed solutions. Note that the *potentialGame* method along with power control results the worst performance, the reason is the objective adopted by *potentialGame* is to minimize the sum of received interference in the network, thus the performance on summed utility demonstrates randomness. Figure 15 draws the CDF of SINR on end users when applying different channel allocation and power allocation schemes. it is clear that our proposed approach achieves the best among the distributed schemes, and only worse wore than the schemes which involves centralized optimization.

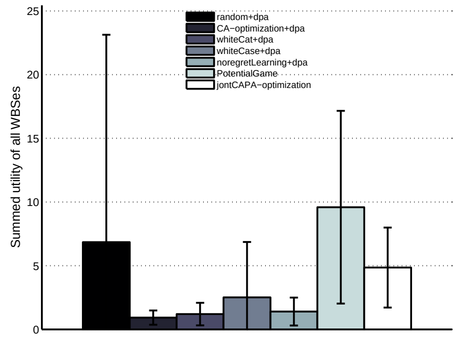

Figure 13. Summed utility of all WBSs, which is the objective in problem 7. dpa in legend represents distributed power allocation

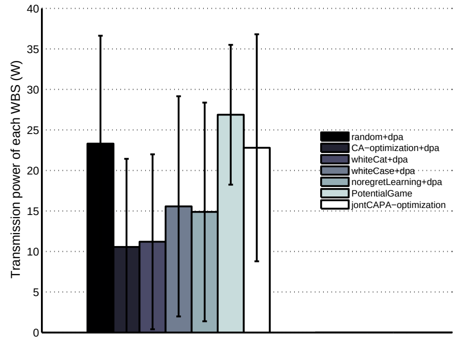

Figure 14. Average transmission power of one WBS

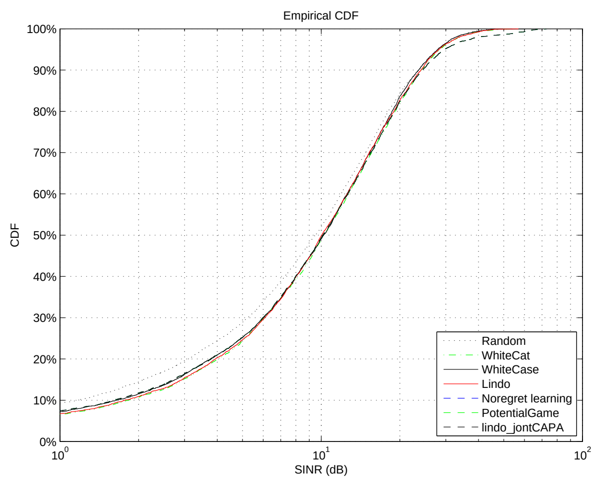

Figure 15. Cumulative distribution of SINR on end users after channel and power allocation

7. CONCLUSIONS

Congestion game is applied to analyse the channel allocation problem, where transmission power is not necessarily identical. The proposed algorithm which is derived from the best response of the congestion game converges quickly, and achieves better performance than other distributed schemes. Without consider the communication latency between WBSs and the database, this distributed scheme executes much faster than the centralized scheme.

In particular, we investigate the channel allocation problem in the context of utilization of TV white space. Except for channel allocation, we also propose solutions for transmission power control for the cellular network which complies with IEEE 802.22 standard.

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