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RESEARCH ARTICLE

Distributed Schemes under Congestion Game framework and Optimization for Spectrum and Power Allocation in TVWS

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ABSTRACT

In this chapter, we will see the application of congestion game in solving the channel allocation problem in the context of TV white space. The channel allocation problem we will address is a general problem, as the transmission power is not identical for every transmitter and on each channel, actually, the transmission power could be unique for each transmitter-channel combination. With the suitable utility function designed for transmitters, the behaviours of the transmitters can be described by a congestion game. The algorithm of channel allocation is derived from the dynamics of the transmitter in the game, which reaches Nash equilibrium quickly.

Furthermore, we provide a complete solution to fully exploit TV white space complying with IEEE 802.22 standard. We propose a centralized methods to regulate the upper bound of transmission power, so that to strictly protect the primary users. The the distributed channel allocation and power control are conducted sequentially.

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1. INTRODUCTION

With the transition from analog TV to digital terrestrial TV, a considerable amount of frequency bands, as shown in Figure 1, become vacant. The spectrum that is left over by digital TV and other incumbent users is referred to as TV white spaces (TVWSs). TVWS can be used for telemedicine, precision agriculture, smart energy and so on by new devices as long as digital terrestrial TV reception is not interfered [1]. These unlicensed devices are called white space devices and their operation should be restricted in terms of location, channel and transmission power, so that no harmful interference is disturbing the incumbents. The utilization of TVWS by the white space devices are regulated by national regulatory authorities and relevant committees, i.e., in the US, the Federal Communications Commission (FCC) has regulated the utilization of TVWS since 2010 [1]. The Electronic Communications Committee (ECC) and Ofcom have released guidelines in EU and UK respectively in terms of the operation of white space devices [2,3]. Although many regulation details from the three institutes are different, white space data base (WSDB) is adopted by all the three institutes.

TV White Space database was first introduced as a way to overcome the technical hurdles faced by spectrum sensing techniques to precisely detect very weak primary signals [4]. white space device should contact a white space data base and provide its location and technical characteristics. The WSDB translates the information on incumbent services and the technical characteristics and location of the white space devices into a list of allowed frequencies and associated transmit powers for devices. [2]. white space device needs to access the WSDB to get the available channels and powers before starting transmission.

white space deviceB plays the key role in the utilization of TVWS, which has the global view of the white space devices in the network and protects the digital terrestrial TV receivers. The resource which is available for the white space devices is decided by the strategy of WSDB, which is as per the regulators. Ofcom and ECC impose restrictions on the aggregated interference caused by white space devices on the digital terrestrial TV receivers, and implicitly allow white space devices to have different transmission powers based on their locations. FCC employs stringent restriction on the universal transmission power for white space devices, but the digital terrestrial TV receivers are not protected from the aggregated interference caused by white space devices which work on the same channel. FCC's stringent regulation on transmission power results in decreased

TVWS as suggested in [5]. Nevertheless, as to the issue of spectrum sharing among the white space devices, the regulator institutes only provide suggestion instead of regulations. In this paper, we will follow the most prevalent regulations on the usage of TVWS and investigate the spectrum sharing issue among the white space devices.

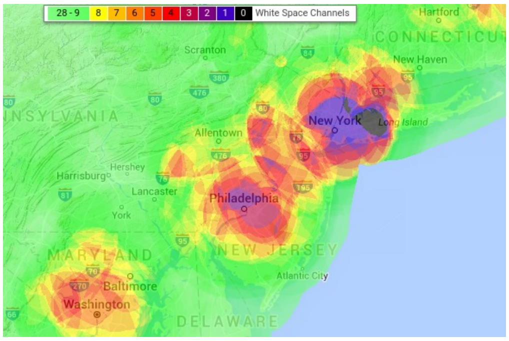

Figure 1. Variability of available channels in a densely populated area. This figure is obtained from [6]

The white space devices are classified into two functional categories: 1) High power, stationary stations such a base stations , 2) lower power personal/portable devices, such as Wi-Fi network interface cards in laptops etc.. The fixed devices and some portable devices should access the WSDB to obtain the available spectrum and permitted transmission. The fixed white space devices work as base stations and provides a backhaul for broadband client access. They are the base stations in the IEEE 802.22 standard which is designed for wireless regional area networks. As the fixed white space devices work with high transmission power, they are the major potential source of interference to the digital terrestrial TV receivers so that their transmission characteristics should be carefully decided by the TWDB. IEEE 802.22 regulates the MAC layer protocol, but it doesn't prevent the co-channel interference caused by the co-channel white space devices which work as white space base stations (WBSs). In this paper, we investigate efficient schemes to exploit the TV spectrum among the fixed white space devices which work as base station in IEEE 802.22 network.

Given all the other WBSs' selection on channel and transmission power, a WBS is interested in choosing the channel which brings it the best performance, i.e., the data rate of its end users. A WBS prefers to choose the channel which experiences the minimum interference, while simultaneously the transmission power can be set as high as possible, so as to obtain better SINR at associated terminals and and generally increase coverage. [7, 8]. Nevertheless, high transmission power causes significant co-channel interference to other secondary users operating on the same channel. Hence, a secondary cell has to balance its transmission power and the caused interference on other cells while simultaneously choosing a working channel that decreases the interference its terminals are exposed to. The existence of WSDB makes it a natural choice to adopt centralized solution for the channel allocation problem, i.e., the WSDB not only provide the WBSs with the available channels and corresponding maximum powers, but also the channels to work on at a given time. But where to make the channel allocation decision is not regulated, thus we will also provide distributed solutions.

The rest of the paper is organized as follows. We elucidate the system model in Section 2, afterwards related work is presented in Section 3. In Section 4, with respect to the regulation from ECC, we present the centralized optimization and game driven distributed scheme in terms of channel allocation. In Section 5, with respect to FCC regulations, both centralized optimization and game driven distributed schemes are introduced. Thereafter performance evaluation is presented in Section 6. Finally, we conclude our work and point out directions of future research in Section 7.

2. SYSTEM MODEL AND PROBLEM STATEMENT

We consider an IEEE 802.22 compliant cellular network where the fixed white space devices work as base station and provide broadband access to their terminals. The network is illustrated in Figure 2. We call the fixed white space devices which work as base stations as White space Base Stations (WBSs), they are located in one area which is surrounded by DTV stations and receivers. Several critical points are deployed in the vicinity of the DTV receivers which are the most venerable to the interference caused by the white space devices. WBSs work in underlay manner and coexist with the DTV stations and receivers, the aggregate interference generated by WBSs should not exceed the threshold on each channel at each critical point. As to the WBSs, the out-of-band emission is regard as trivial, therefore, we only consider co-channel interference among the WBSs. To simplify the analysis, we assume that each DTV station as well as each WBS utilizes exactly one channel.*

As to the notations, the set of critical points is denoted as $\mathcal{K}$ and the set of WBSs is denoted as $\mathcal{N}$ where $|\mathcal{N}| = N$. The TVWS spectrum bands are denoted as set $\mathbb{C}$. We represent the usage of channel for WBS i with a binary vector $X_i^{|\mathbb{C}|\times 1} = \{x_i^1, \cdots, x_i^k, \cdots, x_i^{|\mathbb{C}|}\} \in \{0,1\}^{|\mathbb{C}|}$, where $k \in \mathbb{C}$ and binary variable x_i^k denotes whether channel k is used by user i. All the WBSs work with the channels approved by the WSDB, they operate with a channel from the approved ones after choosing it, thus we omit the time index in the channel usage. As each node can only uses one channel, for X_i, there

*The assumption that one WBS only utilizes one channel is for convenience of analysis. In reality multiple channel usage (channel bonding) is requirement as one single TV channel's bandwidth is 6 MHz which is not adequate for a WBS to fulfill terminal demands.

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is $\sum_{k=1}^{|\mathbb{C}|} x_i^k = 1$. The transmission power of WBS i on channel c is P_i^c. $c(i)$ denotes the channel used by a WBS $i \in \mathcal{N}$.

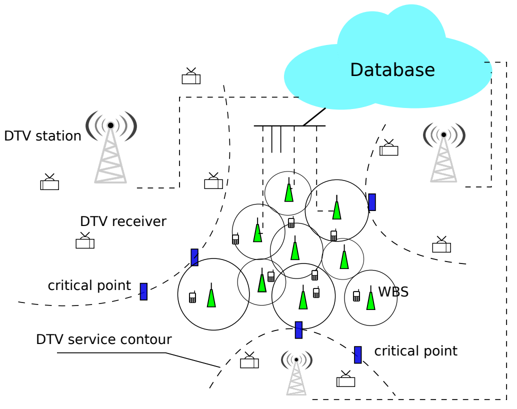

Figure 2. System model: WBS cells and DTV systems

For a terminal m which is associated to WBS i, the attenuation between WBS i and m is denoted as h_{im}. For the attenuation, we only take path loss and shadowing into account in the following. The path loss is dependent on the distance between the corresponding equipment, e.g. $h_{im} = K \cdot d_{im}^{-\alpha}$, where α is the path loss exponent, d_{im} is the distance between i and m, while K is a constant which models the reference loss over a single unit of distance. Shadowing without fading is considered in our model. z_{im} models the zero-mean log-normally distributed shadow fading between i and m, with the standard deviation σ_{SH}. N_0 denotes the thermal noise power. The SINR at end terminal m is,

$$\gamma_m = \frac{P_i^c \cdot h_{im} \cdot z_{im}}{f_m^c} = \frac{P_i^c \cdot h_{im} \cdot z_{im}}{\sum (P_j^c \cdot h_{jm} \cdot z_{jm}) + N_0}, \quad j \in \mathcal{N} \setminus i, c(j) = c \tag{1}$$

where P_j^c denotes the transmission power of interfering WBS j.

In our model, we only assume the WBSs, which work with high transmission power, as the potential interfering unlicensed devices to the DTV service, meanwhile, WBSs are interested in providing broadband access to their associated terminals. Our goal in this paper is to assign TVWS channel to each WBS so as to improve the signal to noise and interference ratio (SINR) of their associated end terminals, meanwhile complying with the prominent regulations i.e., ECC and FCC. The channels in $\mathbb{C}$ are assumed to be identical in terms of attenuation and shadowing on the same path. A WBS's utility is a function of the SINR on all its end terminals, i.e., the average SINR at all its terminals.

2.1. QuasiSINR of WBS

As to WBS's utility, it is not appropriate only to choose one terminal, as done in [9], or even multiple fixed terminals to represent the all the terminals in the same cell, because their locations could diverge greatly with the locations of the other terminals. Thus, we propose a metric *QuasiSINR* to represent WBS's performance in terms of SINR on its end terminals, which is independent on the actual locations of end terminals.

With an auxiliary circle centered at the discussed WBS, which is shown as dashed circle in Figure 3, QuasiSINR is the ratio between the power of signal of interest on the circle and the summation of the strongest power from the interfering WBSs on the auxiliary circle.

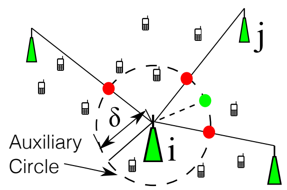

Figure 3. Assuming the radius of the auxiliary circle is δ and all the WBSs work on the a channel, then QuasiSINR is a quotient where the divided is the WBS i's power on the auxiliary circle (i.e., the green pot), and the divisor is the summation of the interfering power on the red pots.

The quasiSINR of WBS i is denoted as γ_i,

$$\begin{aligned}\gamma_i &= \frac{P_i^c \cdot h_{i\rightarrow \text{i's auxiliary circle}} \cdot z_{i\rightarrow \text{i's auxiliary circle}}}{\sum\limits_{\substack{j\neq i, j\in\mathcal{N}\\ c(j)=c(i)}} (P_j^c \cdot h_{j\rightarrow \text{i's auxiliary circle}} \cdot z_{j\rightarrow \text{i's auxiliary circle}}) + N_0} \\ &= \frac{P_i^c \cdot \delta^{-\alpha} \cdot z_{i\rightarrow \text{auxiliary circle}}}{\sum\limits_{\substack{j\neq i, j\in\mathcal{N}\\ c(j)=c(i)}} (P_j^c \cdot (d_{ji} - \delta)^{-\alpha} \cdot z_{j\rightarrow \text{auxiliary circle}}) + N_0}\end{aligned} \tag{2}$$

In the following paper, when we talk about the channel and power allocation with respect to WBS, the notation h_{ij} denotes the attenuation between WBS i to the auxiliary circle of WBS j. h_i denotes the attenuation between WBS i to its own auxiliary circle. Then γ_i becomes,

$$\gamma_i = \frac{P_i^c \cdot h_i \cdot z_i}{\sum\limits_{\substack{j\neq i, j\in\mathcal{N}\\ c(j)=c(i)}} (P_j^c \cdot h_{ji} \cdot z_{ji}) + N_0} \tag{3}$$

The abbreviations and notations used in this paper are summarized in Table I. With auxiliary circle, the decision made by WBSs is independent on the distribution of the end terminals. On the other hand, the radius of the auxiliary circle δ can be adapted to foster better service to the terminals in certain area, i.e., a larger radius δ will take care of the SINR on the terminals reside far away and vice visa.

In our model, WBSs access the WSDB and obtain the transmission parameters, i.e., working channel, transmission power of the other WBSs, the attenuation characteristics between itself and all the other WBSs, and vice visa. WBSs calculate their QuasiSINRs with these information respectively.

Table I. Notations

Abbr. Symbol	Description
TVWS	TV white spaces
WSDB	white space data base
WBS	white space base stations
γ	QuasiSINR
f_{ji}^c	The co-channel interference caused by WBS j on the auxiliary circle of WBS i, c is the working channel for both
f_i^c	The sum of interference caused on the auxiliary circle of WBS i
p_i^c	The Tx power of WBS i on channel c on channel c
P_i^c	The maximal permitted Tx power of WBS i on channel c (ECC solution)
P_μ, P_{op}	The minimum and maximal permitted Tx power of WBS i (FCC solution)
h_{ij}	The attenuation between WBS i to the auxiliary circle of WBS j.
h_i	The attenuation between WBS i to its won auxiliary circle.
z_{ij}	The shadowing from WBS i to the auxiliary circle of WBS j.
z_i	in Section 4.3, the shadowing from WBS i to its own auxiliary circle.
$\alpha_{ij}^k, \beta_{ij}^k$	Binary auxiliary variables in the
y_i, z_i	in Section 5.2, optimization parameters.
cp	Critical point

2.2. Permitted Transmission Power for WBSs

According to regulations, the calculation engine within the WSDB translates the information of incumbent services and technical characteristics and location of the white space devices into a list of allowed frequencies and associated transmit powers for the white space devices [2]. WSDB's decisions on the transmission power are different with respect to different regulations.

2.2.1. ECC Regulations on Permitted Tx Power

ECC and Ofcom propose guidance [10, 11] and focus on the protection of the critical points from the harmful interference, and don't regulate the power limitation. As to decision on the transmission power per ECC, as WBSs work in underlay manner and coexist with digital terrestrial TV receivers, the aggregate generated interference caused on the critical points on each channel should not exceed a threshold. We adopt the interference model and the optimization methodology from the work of [12] to plan the maximum transmission power on each channel for WBSs. For WBS $i \in \mathcal{N}$, the maximum transmission power allowed on channel $c \in \mathbb{C}$ is denoted as P_i^c. As to each channel c, the generated interference on each interference measuring device should be within a predefined interference margin I. The interference margin in a slow fading environment is decided according to [13]. To address this fairness issue, we maximize the sum of the logarithmic value of every WBS's transmission power, and formulate the problem into a convex optimization problem.

$$\begin{aligned} \text{Maximize} \quad & \sum_{i\in\mathcal{N}} \log P_i^c \\ \text{subject to} \quad & \sum_{i\in\mathcal{N}} (P_i^c \cdot h_{i,\text{cp}} \cdot z) < I, \forall \text{cp} \in \mathcal{K} \end{aligned} \tag{4}$$

This optimization is conducted in WSDB for each channel $c \in \mathbb{C}$, and the calculated maximum permitted transmission powers are used by the WBSs afterwards.

2.2.2. FCC Regulations of Permitted Tx Power

FCC initially restricts the maximum transmission power of the fixed devices to be 1 Watt and now relaxes the limitation to 4 Watt. FCC doesn't impose restriction on the aggregated interference caused on primary TV receivers, hence there should be a limit on the number of operating WBSs when the WBSs in the IEEE 802.22 network is dense.

2.3. Problem Formulation

Our goal is to minimize the sum of inverted quasiSINR $\sum_{i\in\mathcal{N}} 1/\gamma_i$ with the WSDB's decisions on WBSs' transmission powers. In order to ensure the fairness among WBSs, we minimize the sum of inverted quasiSINR instead of maximizing the sum of quasiSINR of all WBSs.

3. RELATED WORKS

To exploit TVWS, authors in [14–17] have proposed different approaches for assessment of TVWS capacity under FCC and ECC regulations respectively. Hessar et al. [18] aim to maximize the Shannon capacity of the network which complies with FCC rules. The solution seeks the trade-off between the wide band and co-channel interference, both of which are brought by assigning multiple channels to WBSs. But this scheme doesn't restrict the number of TVWSs and doesn't consider the harmful interference caused on the digital terrestrial TV receivers. Yang et al. [19] and Gopal et al. [20] follow the rules of FCC, and propose throughput maximization of a CSMA/CA based WiFi like network in TVWS under aggregate interference.

Omidvar et al. [21] use potential game to propose a distributed joint power and channel allocation in cognitive radio network. Although the scheme is not tailored for TVWS, protection on the primary users are also considered. Potential game is adopted in work [22] to mitigate the adjacent interference meanwhile bonding multiple channels. In [9], Chen et al. formulate the channel allocation problem TV white space into a potential game where individual WBS's utility is to maximize the capacity of one static terminal.

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With the doctrine of not interfering the primary TV services and regulations imposed by institutes, the problem we solve in this paper is different from the channel allocation problems discussed in the domain of cognitive radio networks. But we think a brief review in terms of the channel allocation techniques in CRN is necessary. With identical transmission power and symetric path attenuation, Nie et al. [23] formulate channel assignment problem in ad-hoc cognitive radio network into a potential game which leads to pure NE, the authors [24, 25] propose algorithms which converge to pure Nash equilibrium (NE) and strongly dominate strategy equilibrium respectively. Simulated annealing is applied to mitigate co-channel interferences in [26]. For the same purpose, no-regret learning [27, 28] is exploit to optimize the choice on channel. in [28], each WBS maps the probability of choosing each strategy to a certain proportion of the regret which the WBS may have if it doesn't choose that strategy, and the WBS choose the strategy with the biggest probability. WBSs update such mapping dynamically and this approach converges to correlated equilibrium.

4. CHANNEL AND POWER ALLOCATION SCHEME COMPLYING WITH ECC RULES

In this section we will discuss the channel and power allocation problem complying with the rules regulated by ECC. In the following subsections, we firstly present the decision on the maximum permitted transmission power on each channel for each WBS, then present how do the WBSs make use of these powers in both centralized and distributed manner respectively.

4.1. The Maximum Permitted Transmission Power

We adopt the interference model and the optimization methodology from the work of [12] to plan the maximum transmission power on each channel for WBSs. Having a global view of the propagation parameters, geolocations of WBSs and interference threshold at the critical points of the DTV receivers, an optimization is implied in the database. By solving the optimization formulation 4, the WSDB obtains the maximum permitted transmission power for every WBS and over every channel. The interference threshold on the critical points will not be exceeded even when all the WBSs work on one same channel.

4.2. Centralized Optimization for Channel Allocation

Given two WBSs i and j, the co-channel interfering relationship is decided as,

$$X_i^T X_j = \sum_{k=1}^{|\mathbb{C}|} x_i^k \cdot x_j^k = \begin{cases} 1 & \text{if } c(i) = c(j) \\ 0 & \text{if } c(i) \neq c(j) \end{cases} \tag{5}$$

The transmission power levels on all channels for WBS i are denoted by a constant vector $\mathbf{P}_i = \{P_i^1, \cdots, P_i^k, \cdots, P_i^{|\mathbb{C}|}\}$. The transmission power adopted by WBS i is $\mathbf{P}_i^T X_i = \sum_{k=1}^{|\mathbb{C}|} P_i^k \cdot x_i^k$.

The problem in Section 2.3 is formulated as a general purpose nonlinear optimization,

$$\begin{aligned} \text{minimize} \quad & \sum_{i=1}^{n} \frac{\sum_{j \in \mathcal{N}, j \neq i} \mathbf{P}_j^T X_j (X_j^T X_i) h_{ji} z_{ji} + N_0}{\mathbf{P}^T X_i h_i z_i} \\ \text{subject to} \quad & \sum_{k=1}^{|\mathbb{C}|} x_i^k = 1, x_i^k \in X_i \in \{0,1\}^{|\mathbb{C}|} \end{aligned} \tag{6}$$

Problem 6 is a non-linear problem with binary variables, but it can be transformed into a quadratic programming problem as follows,

$$\begin{aligned} \text{minimize} & \sum_{i=1}^{n} \Big(\sum_{j \in \mathcal{N}, j \neq i} \sum_{k \in \mathbb{C}} \frac{P_j^k \cdot h_{ji} \cdot z_{ji}}{P_i^k \cdot h_i \cdot z_i} \cdot x_j^k \cdot x_i^k \\ & + \sum_{k \in \mathbb{C}} \frac{N_0}{P_i^k \cdot h_i \cdot z_i} \cdot x_i^k \Big) \\ \text{subject to} & \sum_{k=1}^{|\mathbb{C}|} x_i^k = 1, x_i^k \in X_i \in \{0,1\}^{|\mathbb{C}|} \end{aligned} \tag{7}$$

The reformulation is available in Appendix in our previous work [29]. We use GUROBI [30] which is a state of art non-linear problem solver to solve the problem, which employs Branch-And-Reduce method to get the global optimum for the problem.

4.3. Distributed White Space Channel Allocation (WhiteCat)

In this section a distributed scheme for WBSs to allocate channels meanwhile complying with ECC regulation is proposed, which is named as white space channel allocation technology (WhiteCat).

4.3.1. Algorithm and Protocol

WhiteCat adopts the best response process, where each WBS (referred as i) chooses the channel which brings the largest utility u_i, as shown in Formula 8, as the response of other WBSs' choices on channels. WhiteCat is depicted by Algorithm 1.

$$u_i = \frac{\sum\limits_{\substack{j\in\mathcal{N},j\neq i,\\ c(\sigma_j)=c(\sigma_i)}} f_{ji}}{2\cdot\tilde{P}_i} + \frac{1}{2}\sum_{\substack{j\in\mathcal{N},j\neq i,\\ c(\sigma_j)=c(\sigma_i)}} \frac{f_{ij}}{\tilde{P}_j} + \sum_{\substack{\mathcal{S}:i,j\in\mathcal{S},\\ c(\sigma_j)=c(\sigma_i)}} \frac{N_0}{C\cdot\tilde{P}_i} \tag{8}$$

where $\tilde{P}_i = P_i \cdot h_i \cdot z_i$, similarly $\tilde{P}_j = P_j \cdot h_j \cdot z_j$. Overlooking the constant coefficient 2, the first item of u_i is the inverted QuasiSINR of station i. To minimize the first item, WBS i needs to choose a channel either permits higher transmission power or experiences less interference, whereas the higher power increases the second item which is a part of inverted QuasiSINR of other co-channel WBSs. Hence, the cost function presents a reasonable compromise between the QuasiSINR of one WBS and others.

When WBS only emphasizes on its own utility (e.g. the first part of Formula 8), the best response process doesn't always converge and we have following theorem:

Theorem 4.1. With non-identical transmission power, if every WBS updates its channel based on Algorithm 1 with utility based on its own interests, i.e., the first part of Formula 8, the process doesn't always converge.

The proof is in Appendix 1 in [29].

Algorithm 1: Spectrum selection by WBS i

Input: the distance, path lose and shadowing parameter between WBS i to WBS $j \in \mathcal{N} \setminus i$; radius of auxiliary circle, noise N_0, total number of WBSs N; for $j \in \mathcal{N} \setminus i$, the maximal transmission power $P_j^c, c \in \mathbb{C}$ and the working channel $c(j)$.

```
for c ∈ ℂ \ c(i) do
    calculate u_i(c) based on Formula 8 if u_i(c) < u_i(c(i)) then
        c(i) ← c
    else
        keep c(i) unchanged
    end
end
Notify database of its channel usage, which further notifies the other WBSs
```

WBSs calculate u_i with the information retrieved from WSDB. After executing Algorithm 1, WBSs report to WSDB about their chosen channels if they are updated. As the locations of WBSs and TV stations, along with the transmission channel and power of TV stations are usually static (entries of TV station change averagely once in 2 days [31]), except for the channel usage in the network, the change of the other data stored in WBS is infrequent. We refer [23] to decide WBSs' sequence to update their channels. [23] proposes a method which is akin to the random access mechanism of CSMA/DA, where the access for broadcast medium is changed to get access to the centralized center to retrieve the current channel usage and update its new channel. All WBSs are able to access the database in one round (with random or predetermined sequence). Update of channels can happen in several scenarios, i.e., in boot phase, or when the SINR on end users falling below a threshold, or a fixed time duration coming to end, or new WBSs joining in the network.

4.3.2. Analysis in Game Theoretical Framework

In this section, We give the proof on whiteCat's convergence in the framework of congestion game theory. Formulating a spectrum sharing problem into a congestion game and the concept of *virtual resources* are firstly proposed in [32]. This work reversely engineers the distributed channel allocation schemes proposed in [33, 34], i.e., unifies the algorithms with congestion game. But the problem analysed in [32] assume the transmission power is identical, which is a major difference from the channel allocation problem discussed here.

In congestion game, each player acts selfishly and aims at choosing strategy $\sigma_i \in \Sigma_i$ to minimize their individual cost. The gain (loss) caused by any player's unilateral move is exactly the same as the gain (loss) in the potential, which may be viewed as a global objective function. For problems where the potential of the problem is the same with the summation of the cost of all users, the cost function can be used as a utility function directly. This equivalence doesn't exist in our problem, but by carefully choosing the cost function for players, we can make sure that the change of individuals' cost is in the same direction with that of the global utility.

4.3.3. From WhiteCat to Congestion Game

We utilize the conception of virtual resource which is firstly introduced in [32]. Virtual resource is a triplet $\{i, j, c\}$, where i, j are two WBSs and $c \in \mathbb{C}$ is one channel. This piece of resource is regarded used by i when both i and j use channel c, otherwise, $\{i, j, c\}$ is not used by any WBS.

In the following, we list the element of the congestion game which emulates Algorithm 1. In this section, player and base station are used interchangeably.

- Player i' strategy space is $\Sigma_i = \{(i, j, c), j \in \mathcal{N}, j \neq i, c(\sigma_j) = c, c = 1, 2, \cdots, N\}$, and i has C admissible strategies, one strategy related with channel $c \in \mathbb{C}$ is described by the set of virtual resources it uses: $\sigma_i = \{(i, j, c), j \in \mathcal{N}, j \neq i, c(\sigma_j) = c\}$, note that virtual resource $(i, j, c) \neq (j, i, c)$.

- Under the strategy profile $\sigma = (\sigma_1, \sigma_2, \cdots \sigma_N)$, player i obtains a total cost of

$$g^i(\sigma) = \sum_{\substack{j\in\mathcal{N},j\neq i,\\ c=c(\sigma_i)=c(\sigma_j)}} (g_{(i,j,c)}(n_{(i,j,c)}(\sigma)) + g_{(j,i,c)}(n_{(j,i,c)}(\sigma)) \tag{9}$$

The transmission power over all channels of player i is $\{p_i^1, p_i^2, \cdots, p_i^{|\mathbb{C}|}\}$. We define the cost function for virtual recourses (i, j, c) as follows,

$$g_{(i,j,c)}(k) = \begin{cases} \dfrac{f_{ji}}{2\tilde{P}_i} + \dfrac{f_{ij}}{2\tilde{P}_j} + \dfrac{C \cdot N_0}{N \cdot \tilde{P}_i} & \text{if } k = 2 \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

As resource (i, j, c) only lies in the strategy space of player i and j, thus can only be accessed by this two players. More specifically, according to Formula 10, the cost of resource (i, j, c) is only decided by the number of players using it, which is either 0 or 2. At the first glance, this is a player specific congestion game, as $g_{(i,j,c)}$ is decided by the relevant players' transmission power and inference. But actually the resource (i, j, c) excludes the players except for i and j from using it, thus the cost happened on this resource is only dependant on how many of players from the set $\{i, j\}$ to use it. Hence, the cost is a function of the number of players using the resource, and this is a canonical congestion game.

Now we substitute Formula 10 to Formula 9, the total cost for user i under strategy profile σ .

$$\begin{aligned} g^i(\sigma) &= \sum_{\substack{j\in\mathcal{N}\setminus i,\\ c=c(\sigma_j)=c(\sigma_i)}} (g_{(i,j,c)}(2) + g_{(j,i,c)}(2)) = \sum_{\substack{j\in\mathcal{N}\setminus i,\\ c(\sigma_j)=c(\sigma_i)}} \left(\frac{f_{ji}}{\tilde{P}_i} + \frac{f_{ij}}{\tilde{P}_j} + \frac{C \cdot N_0}{N}\left(\frac{1}{\tilde{P}_i} + \frac{1}{\tilde{P}_j}\right)\right) \\ &= \frac{\sum_{\substack{j\in\mathcal{N}\setminus i,\\ c(\sigma_j)=c(\sigma_i)}} f_{ji}}{\tilde{P}_i} + \sum_{\substack{j\in\mathcal{N}\setminus i,\\ c(\sigma_j)=c(\sigma_i)}} \frac{f_{ij}}{\tilde{P}_j} + \frac{CN_0}{N} \sum_{\substack{j\in\mathcal{N}\setminus i,\\ c(\sigma_j)=c(\sigma_i)}} \left(\frac{1}{\tilde{P}_i} + \frac{1}{\tilde{P}_j}\right) = \frac{\sum_{\substack{j\in\mathcal{N}\setminus i,\\ c(\sigma_j)=c(\sigma_i)}} f_{ji}}{\tilde{P}_i} + \sum_{\substack{j\in\mathcal{N}\setminus i,\\ c(\sigma_j)=c(\sigma_i)}} \frac{f_{ij}}{\tilde{P}_j} + \frac{2CN_0}{N} \sum_{\substack{i\in\mathcal{S}\subset\mathcal{N},\\ \mathcal{S}:\forall i\in\mathcal{S}\\ c(\sigma_i)=c}} \frac{1}{\tilde{P}_i} \end{aligned} \tag{11}$$

where $\mathcal{S}$ denotes the set of WBSs whose working channel is the same with WBS i.

Now we are going to have a look at the *potential* of the network. According to the expression of Rosenthal's potential in Formula 14, the potential is accumulated by adding the players' cost sequentially, in particular, the value which is added is the cost that player experiences when it starts to use the relevant resource, and the value is not changed when other players come to use that resource. Back to our problem, for two WBSs $i, j \in \mathcal{S}$, we assume WBS i's index is smaller than j's index, then the potential increased by i using the resource $\{i, j, c\}$ is 0 according to Formula 9, and the increase brought in by j using the resource $\{i, j, c\}$ is $g_{(i,j,c)}(2) + g_{(j,i,c)}(2)$. In other words, for each interfering pair of WBSs, only the WBS with bigger index contributes to the potential. Note that the summation of one WBS's congestion is related to its index. Then the total potential is,

$$\begin{aligned} G(\sigma) &= \sum_{r\in\mathcal{R}} \sum_{i=1}^{n_r(\sigma)} g_r(i) = \sum_{i\in\mathcal{N}} \sum_{r\in\sigma_i} g_r(n_r^i(\sigma)) \\ &= \sum_{i\in\mathcal{N}} \frac{\sum_{\substack{j\in\mathcal{N},j\neq i,\\ c(\sigma_j)=c(\sigma_i)}} f_{ji}}{\tilde{P}_i} + \frac{CN_0}{N} \sum_{\substack{\mathcal{S}\subset\mathcal{N},\\ \forall i\in\mathcal{S},c(\sigma_i)=c}} |\, \mathcal{S} \,| \sum_{i\in\mathcal{S}} \frac{1}{\tilde{P}_i} \end{aligned} \tag{12}$$

When players minimize their utilities (cost or potential) 11, the total potential 12 in the secondary network sdecreases monotonically before reaching a Nash equilibrium. Players' greedy update in the game to minimize its cost Function 11, which ceases finally in pure Nash Equilibrium. The strategy and cost function of players in the game is transplanted as Algorithm 1 and utility Function 8 respectively.

4.3.4. Potential in the Congestion Game and the sum of Utilities

It is interesting to know, whether the sum of the final utilities of all WBSs is exactly the same with the potential 12 during the convergence process. The answer is, they are identical when N_0 is zero, and there will be a minor difference when N_0 is not zero. Recall the target objective we want to minimize is,

$$\begin{aligned} \sum_{i\in\mathcal{N}} \frac{f_i}{\tilde{P}_i} &= \sum_{i\in\mathcal{N}} \frac{\sum_{\substack{j\in\mathcal{N},j\neq i,\\ c(\sigma_j)=c(\sigma_i)}} f_{ji} + N_0}{\tilde{P}_i} \\ &= \sum_{i\in\mathcal{N}} \frac{\sum_{\substack{j\in\mathcal{N},j\neq i,\\ c(\sigma_j)=c(\sigma_i)}} f_{ji}}{\tilde{P}_i} + \sum_{i\in\mathcal{N}} \left(\frac{N_0}{\tilde{P}_i}\right) \end{aligned} \tag{13}$$

We notice that only the last items of the objective 13 and the potential of the congestion game 12 are different. When $N_0 = 0$, the potential is exactly the same with the object we want to minimize. When $N_0 \neq 0$, if channels are evenly distributed and there is $C/N* \mid \mathcal{S} \mid= 1$, then Formula 13 and 12 are also the same. In both cases, the sum of utilities 13 decreases monotonically with every update of WBSs before the system reaches Nash Equilibrium. When $N_0 \neq 0$ and Formula 13 and 12 are thus different, the monotonicity on the decrease of sum of utilities 13 is not perceived, whereas the system will still cease to NE.

Based on above analysis, we can see the assumption that each WBS only occupies one channel can be easily removed. By assuming multiple WBSs are allocated at one WBS's location while each WBS works on one distinct channel, then the proof on convergence of whiteCat can be applied directly to this case. Note that the convergence of the game is independent on the the concrete form of the cost function. We adopt the function 11 to let the potential of the game be the same with the total utility of all WBSs, so that by executing Algorithm 1, the system objective experiences a monotonic decreasing process before the system reaching NE. The algorithm has potential to solve many other problems, where one user's decision affects others. In this case, the utility of one user can be formulated to incorporate the information of its own utility and others', then the congestion game theory can be used to analogize.

4.3.5. Communication Overhead of WhiteCat

The problem of channel allocation with different and fixed transmission power is NP hard. WhiteCat is a distributed scheme but certain information of the other WBSs is needed. The centralized base station is piggybacked to provided the needed information. As to one WBS, the number of such inquiries is the number of steps before convergence.

In our formulated congestion game, a player i is allowed to access up to $(N-1)$ resources in the same time, i.e., $\{i, j_1, c(i)\}, \{i, j_2, c(i)\} \cdots \{i, j_{N-1}, c(i)\}$, thus the upper bound of converge steps can not be obtained from the conclusion 14 for singleton congestion game. But our problem is special because for each resource, the possible number of players allowed to use each resource is either 2 or 0. Thus we can refer the method used in Section **??** to analyse the update times for our problem. Firstly, we sort the cost values in increasing order. Although a WBS

$$\phi(\sigma) = \sum_{r \in \mathcal{R}} \sum_{i=1}^{n_r(\sigma)} g_r(i) \leq \sum_{r \in \mathcal{R}} \sum_{i=1}^{n_r(\sigma)} n \leq n^2 m \quad (14)$$

The upper bound of total update steps is $2n^2$, thus averagely, the upper bound of update steps for each WBS is $2n$.

5. CHANNEL ALLOCATION SCHEME COMPLYING WITH FCC RULES

FCC allows the WBSs to use the channels which are approved by the WSDB, and operate with the same transmission power P_{op}. This regulation doesn't consider the harmful interference caused by aggregated interference, i.e., when the number of transmitters increases. Authors of [19, 20] proposed solutions to cope this problem, while they focus on the improvement of network throughput. In this Section we look into the problem of channel allocation in order to improve the SINR on end users under the FCC regulations. In this scenario, the number of WBSs operating need to be restricted, and when the number of WBSs is large, some of them will not be allowed to transmit.

5.1. Centralized Optimization for Channel Allocation

Due to the existence of certain WBSs which don't transmit, we can not adopt the problem formulation 15 directly because some denominators in the objective function, which are related with the idle WBSs are zero. In order to solve this problem, we allow the idle WBSs to work with a very low transmission power. That being said, the power used by user i can be presented as $\mathbf{P}_i^T X_i = \sum_{k=1}^{|\mathbb{C}|} p_i^k \cdot x_{ik}$. There is $p_i^k \in \{P_\mu, P_{\text{op}}\}$, where P_μ represents the very weak transmission power. The transmission power levels are the same as for all the WBSs, so we omit the superscript when we refer the transmission power.

$$\begin{aligned}
\text{Minimize} \quad & \sum_{i=1}^{n} \frac{\sum_{j \in \mathcal{N}, j \neq i} \mathbf{P}_j^T X_j (X_j^T X_i) h_{ji} z_{ji} + N_0}{\mathbf{P}_i^T X_i h_i z_i} \\
\text{subject to} \quad & \sum_{k=1}^{|\mathbb{C}|} x_i^k = 1, x_i^k \in X_i \in \{0,1\}^{|\mathbb{C}|} \\
& p_i^k \in \{P_\mu, P_{\text{op}}\}, \texttt{for} \forall i \in \mathcal{N} \\
& \sum_{i \in \mathcal{N}} (p_i^k \cdot h_{i,pt} \cdot z) < I_{pt}^c, \texttt{for} \forall k \in \mathbb{C}
\end{aligned} \quad (15)$$

Problem 15 is a non-linear problem with binary variables, with the technique in Section 4.2, it can be written as,

$$\begin{aligned}
\text{minimize} & \sum_{i=1}^{n} (\sum_{j \in \mathcal{N}, j \neq i} \sum_{k=1}^{|\mathbb{C}|} \frac{p_j \cdot h_{ji} \cdot z_{ji}}{p_i \cdot h_{ii} \cdot z_{ii}} \cdot x_j^k \cdot x_i^k \\
& + \sum_{k=1}^{|\mathbb{C}|} \frac{N_0}{p_i \cdot h_{ii} \cdot z_{ii}} \cdot x_i^k) \\
\text{subject to} & \sum_{k=1}^{|\mathbb{C}|} x_i^k = 1, x_i^k \in X_i \in \{0,1\}^{|\mathbb{C}|} \\
& p_i \in \{P_\mu, P_{\text{op}}\} \texttt{for} \forall i \in \mathcal{N} \\
& \sum_{i=1}^{n} x_i^k \cdot p_i \cdot h_{i,pt} \cdot z \leq I_{pt}^k, \texttt{for} \forall k \in \mathbb{C}
\end{aligned} \quad (16)$$

Formulation 16 is nonlinear but we can transform it into a quadratic optimization with auxiliary variables. We introduce binary number α_{ij}^k, real number β_{ij}^k and q_i, where

$$x_j^k \cdot x_i^k = \alpha_{ij}^k \quad (17)$$

$$\beta_{ij}^k = p_j \cdot \alpha_{ij}^k \quad (18)$$

$$\frac{1}{p_i} = q_i \quad (19)$$

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Now the objective function is is quadratic as shown in Formula 20,

$$\sum_{i=1}^{n}(h_{ji}z_{ji}\sum_{j\in\mathcal{N},j\neq i}\sum_{k=1}^{|\mathbb{C}|}\beta_{ij}^{k}q_i + N_0\sum_{k=1}^{|\mathbb{C}|}x_i^k q_i) \tag{20}$$

Constraint 18 can be transformed as a set of linear constraints:

$$\beta_{ij}^k - p_{\text{op}}\cdot\alpha_{ij}^k \leq 0 \tag{21}$$
$$-\beta_{ij}^k + p_\mu\cdot\alpha_{ij}^k \leq 0 \tag{22}$$
$$\beta_{ij}^k - p_j - p_\mu\cdot\alpha_{ij}^k \leq -p_\mu \tag{23}$$
$$-\beta_{ij}^k + p_j + p_{\text{op}}\cdot\alpha_{ij}^k \leq p_{\text{op}} \tag{24}$$

The constraint 19 is a quadratic equality, which makes the problem to be non-convex. Thus we need to relax it into a linear inequalities. As p_i is either P_{op} or P_μ, we can replace p_i and q_i with the help of two binary variables y_i and z_i respectively:

$$p_i = y_iP_\mu + (1-y_i)P_{\text{op}} \tag{25}$$
$$q_i = z_i\frac{1}{P_{\text{op}}} + (1-z_i)\frac{1}{P_\mu} \tag{26}$$

Here we put a restriction on P_μ and P_{op} with $P_\mu P_{\text{op}} = 1$, Function 26 becomes

$$q_i = P_\mu z_i + (1-z_i)P_{\text{op}} \tag{27}$$

Now we conduct convex relaxation by applying McCormick envelope [35]. As we want to get the lower bound of the objective function, we get the over-estimators so as to expand the search space as follows,

$$1 = p_i\cdot q_i \leq q_iP_{\text{op}} + p_iP_\mu - P_{\text{op}}P_\mu \tag{28}$$
$$1 = p_i\cdot q_i \leq p_iP_{\text{op}} + q_iP_\mu - P_{\text{op}}P_\mu \tag{29}$$
$$1 = p_i\cdot q_i \geq q_iP_\mu + p_iP_\mu - P_\mu^2 \tag{30}$$
$$1 = p_i\cdot q_i \geq q_iP_{\text{op}} + p_iP_{\text{op}} - P_{\text{op}}^2 \tag{31}$$

By bringing Formulas 25, 27 into the Formulas 28 to 31, the binary linear constrains with respect to the transmission powers are obtained, which are inequalities 42 to 45 in the transformed optimization problem 16. Meanwhile, inequalities 36 to 37 are relaxation of Formula 17. Inequalities 38 to 41 are relaxation of Formula 18 when we bring Formula 25 into Formula 23 and 24.

$$\text{minimize} \tag{32}$$
$$\sum_{i=1}^{n}\sum_{j\in\mathcal{N},j\neq i}\sum_{k=1}^{|\mathbb{C}|}((P_\mu - P_{\text{op}})h_{ji}z_{ji}\beta_{ij}^k z_i \tag{33}$$
$$+ (P_\mu - P_{\text{op}})N_0x_i^kz_i + P_{\text{op}}h_{ji}z_{ji}\beta_{ij}^k + P_{\text{op}}N_0x_i^k) \tag{34}$$
$$\text{subject to:} \tag{35}$$
$$x_i^k + x_j^k - \alpha_{ij}^k \leq 1 \tag{36}$$
$$-x_i^k - x_j^k + 2\alpha_{ij}^k \leq 0 \tag{37}$$
$$\beta_{ij}^k - P_{\text{op}}\alpha_{ij}^k \leq 0 \tag{38}$$
$$-\beta_{ij}^k + P_\mu\alpha_{ij}^k \leq 0 \tag{39}$$
$$\beta_{ij}^k + (P_{\text{op}} - P_\mu)y_j - P_\mu\alpha_{ij}^k \leq P_{\text{op}} - P_\mu \tag{40}$$
$$-\beta_{ij}^k + (P_\mu - P_{\text{op}})y_j + P_{\text{op}}\alpha_{ij}^k \leq 0 \tag{41}$$
$$P_\mu y_i + P_{\text{op}}z_i \leq \frac{P_{\text{op}}^2 - 1}{P_{\text{op}} - P_\mu} \tag{42}$$
$$P_{\text{op}}y_i + P_\mu z_i \leq \frac{P_{\text{op}}^2 - 1}{P_{\text{op}} - P_\mu} \tag{43}$$
$$-y_i - z_i \leq \frac{1 + P_\mu^2 - 2P_{\text{op}}P_\mu}{P_\mu(P_{\text{op}} - P_\mu)} \tag{44}$$
$$-y_i - z_i \leq \frac{1 - P_{\text{op}}^2}{P_{\text{op}}(P_{\text{op}} - P_\mu)} \tag{45}$$
$$\sum_{k=1}^{|\mathbb{C}|}x_i^k = 1, x_i^k \in X_i \in \{0,1\}^{|\mathbb{C}|} \tag{46}$$
$$\sum_{i=1}^{n}(((P_\mu - P_{\text{op}})x_i^ky_i + P_{\text{op}}x_i^k)h_{i,pt}z_{i,pt}) \leq I_{pt}^k \tag{47}$$
$$x_i^k, \alpha_{ij}^k, \beta_{ij}^k, y_i, z_i \in \{0,1\} \tag{48}$$
$$\forall i,j \in \mathcal{N}, i\neq j, k\in\mathbb{C} \tag{49}$$

Inequalities 36 and 37 are equivalent to Formula 17. Inequalities 38 - 41 are equivalent to Formula 18. Inequalities 42 - 45 are relaxation of the equality quadratic 19. The objective function is quadratic and there is only linear and quadratic inequality constraints, so we use GUROBI Mixed-Integer Programming solver to solve it get the lower bound of the objective function.

5.2. Distributed White Space Channel Allocation

Being aware of the accumulated interference on the interference measurement devices for the primary users, each WBS can decide whether to switch from idle state to operation state and work on a channel which brings it with the highest utility, or to change its operating channel to get higher utility value. The operation of the WBSs should not result in harmful interference onto the primary system.

The pseudo code is as follows,

When a WBS switches its state from idle to operation, new interference is generated on the other WBSs which work with the same channel. Due to this disturbance , the

Algorithm 2: Spectrum selection by WBS i

Input: For each WBS, the complete graph of the network, which includes: (1) the distance and signal shadowing between it and any other WBS and TV interference measurement device; (2)Transmission power and channel of the WBS $i \in \mathcal{N}$.

while *Operation profile is different from previous round* **do**
  for $i \in \mathcal{N}$ **do**
    if *i is idle* **then**
      if $\exists c$, ***where*** $\sum f_t^c \leq Thold_t^c, \forall t \in \mathcal{T}$, $\gamma_i^c > \gamma_i^{c'}$, $c' \in \mathbb{C} \setminus c$ **then**
        $c(i) = c$
      end
    end
    if *i is operating* **then**
      if $\exists c'$, $f_i^{c'} < f_i^{c(i)}$ ***and*** $\sum f_t^{c'} \leq Thold_t^{c'}, \forall t \in \mathcal{T}$ **then**
        $c(i) = c'$
      end
    end
  end
end

affected WBSs may change their channels to achieve better γ.

This process can be formulated as a congestion game which eventually converges, furthermore, the dynamics initiated by the WBS starting operation eventually ceases. Here we have Lemma 5.1.

Lemma 5.1. The dynamics caused by a WBS changing its state from idle to operation ceases after finite steps.

Proof
After a WBS switches on, all the WBSs will choose the channel which brings the highest γ, i.e., , the lowest co-channel interference considering the transmission power is identical to each WBS. This process is the same as the problem of channel assignment with identical transmission power, which is discussed in [32]. In [32], WBSs change operating channels in order to minimize the received co-channel interference is formulated into a congestion game, then the convergence of the channel assignment is proved. □

Note that according to Algorithm 2, after a WBS starts to operate, it will never change back to idle state. Now we have the following theorem.

Theorem 5.1. *The whole process ends after finite updates.*

Proof
Whenever there is a WBS changes its state from idle to operation, a new congestion game among the WBSs begins. As the convergence is guaranteed as shown in lemma 5.1 and there is only a limited number of WBSs which change their state from idle to operation, the whole process, which consists of a series of concatenated congestion games, will eventually end. □

6. PERFORMANCE EVALUATION

In this section, we will investigate the centralized and distributed schemes proposed to comply with ECC and FCC regulations respectively. The evaluation setting is as follows. A square area which is 60km x 60km is divided evenly into 16 square blocks. There is one WBS which is located in the middle of each block. Same amount of end terminals are distributed in each block, whereas the end terminals don't necessarily choose the WBS which is nearest to them to obtain service, in stead they choose the WBS which provides them with the strongest RSSI. There is a 20km wide rim area around the square area, where the critical points for the DTV receivers are randomly located. One critical point measures the aggregated interference on each channel. The locations of WBSs and TV contours are illustrated in Fig. 4. WBSs' locations are fixed, but the locations of interference measurement devices for the TV receivers, the end terminals, and the sequence for WBS to update are randomly decided in each run. Simulations are conducted for 20 times.

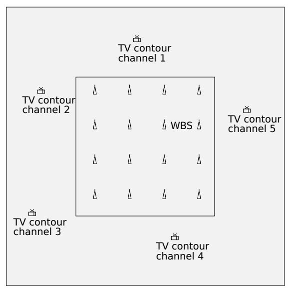

Figure 4. Layout of WBSs and TV contours

Some parameters are listed in Table 9.

Number of channels	5
Number of WBSs	16
Noise	10^{-12}W
Length of side the square to locate WBSs	60km
Distance between quasai terminal and WBS	7km
Interference threshold on TV contour	10^{-7}W
Path loss factor	2
Standard deviation in flat shadowing	8
Number of end terminals in network	800
Min. WBS Tx power † complying with ECC	1W
Max. WBS Tx power complying with ECC	20W
$P_{\rm op}$, Tx power complying with FCC	4W

Table II. Simulation parameters

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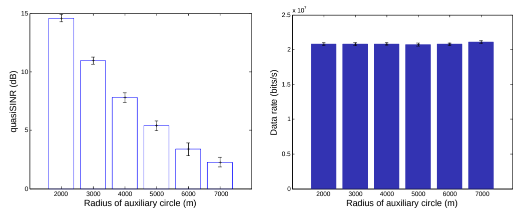
6.1. Schemes Complying with ECC Regulations

We compare *whiteCat* with centralized scheme and three other distributed schemes: the random allocation scheme, *whiteCase* and No-regret learning.

- *Optimization*: centralized quadratic optimization introduced in Section 4.2.
- *WhiteCase*: Whitespace channel allocation selfish, where WBSs minimize only the first item of Formula 8
- *Potential Game*: A potential game based distributed scheme which is introduced in [21].
- *No-Regret Learning*: A distributed scheme introduced in [28].
- *Random Scheme*: WBS choose channel randomly.

6.1.1. The Choice of Radius of Auxiliary Circles, quasiSINR of WBS, and SINR on End Users

The usage of quasiSINR exempts WBSs from taking care the SINR on the end terminals. A WBS's quasiSINR is related with WBS's location and the radius of auxiliary circle. Figure 5 illustrates the effect of using different radii of the auxiliary circle on the data rate can be achieved by end terminals.



(a) quasiSINR of WBSs (b) Data rate of end terminals when applying whiteCat

Figure 5. The effects of different radii of auxiliary circle on end terminals' data rate. Maximum permitted power is obtained by solving convex optimization. WhiteCat is used to assign the channels.

Subfigure 5(a) shows WBSs' quasiSINR decreases when the radii of auxiliary circles increase. Subfigure 5(b) illustrates the choice on radius of auxiliary circle don't influence the performance of whiteCat. In the following simulation, we fixed the radius at 6000 m.

6.1.2. Comparison of Schemes Complying with ECC Regulations

Figure 6 depicts the transmission power consumption of the schemes. whiteCat and the optimization method consume the less energy than the potential game scheme and random scheme. whiteCat and the optimization method consume more energy than No-Regret learning scheme and WhiteCase, but meanwhile the former two schemes output the latter two in terms of SINR which will be shown in Figure 7.

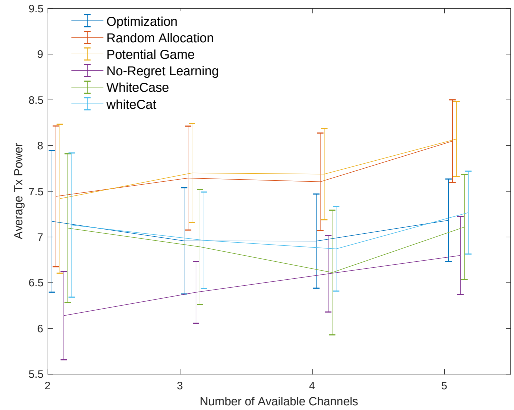


Figure 6. Average Transmission power of WBSs when different number of channels are available

The average SINR on the end terminals is depicted in Figure 7. whiteCat and the centralized optimization achieve similar and the best performance among the schemes. The potential game based scheme and random scheme achieve good SINR with the cost of hight transmission power. WhiteCase and No-Regret learning are at the bottom.

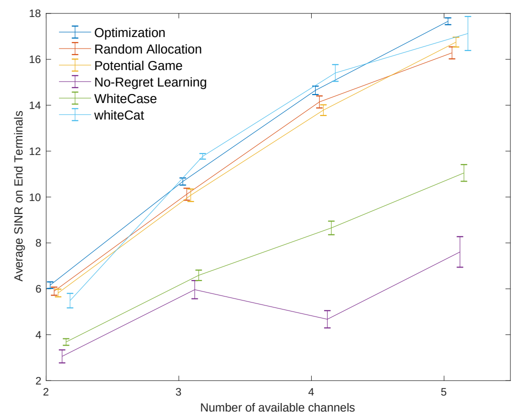


Figure 7. SINR on end terminals

6.1.3. Convergence Speed

In the congestion game where scheme whiteCat is derived, each player (WBS) has at most $(n-1) * |\mathbb{C}|$ resources available for usage, thus there is no polynomial steps converging to NE, while, simulation shows the algorithm can quickly converge to NE when the number of WBS is up to 100. Table III shows the average number of steps needed before convergence in 100 runs

† minimal and maximal power here denote the power level restricted by the specification of hardware.

of simulations. As to whiteCat, we account each WBS accessing the base station (refer to 4.3) as *one step*. We compare the convergence speed of WhteCat with no-regret learning, the scheme derived from potential game [21] and whiteCase. Note that the potential game scheme is to solve joint power and channel allocation problem, as it is developed with game theory, it is reasonable to see its convergence speed. As there is no guarantee for WhiteCase to converge, we stop the channel allocation process after 16000 steps (1000 rounds).

Table III tells that whiteCat is two times faster than the scheme derived from potential game, and 20 times faster than no-regret learning scheme. The relatively smaller confidence interval shows that whiteCat's convergence is not affected by different network configurations. Fast converge is attributed to the working style of WBSs whcih access the database to get the information of other WBSs, thus the distributable decision involves a part of the global information of the network. Thus, we can see that the speed up of convergence is due to the overhead caused by accessing the database.

Figure 8 depicts one instance of the convergence processes of three schemes. The Y axis is the summed utility of all WBSs. We can see whiteCat decreases the summed utility constantly, and the channel allocation process ceases after 38 times of updates. Whereas, noregret learning scheme takes 120 steps before convergence, and whiteCase fails to converge.

Scheme	Average steps	95% CI	Average time (s)
WhiteCat	58	5.6	2
No-Regret Learning	1916	1541	144
Potential Game [21]	120	10	4
Optimization	-	-	40
WhiteCase	4587	2742	50

Table III. Convergence speed of the distributed channel allocation schemes. As to the distributed scheme, the time involved to communicate with database is note considered and included.

6.2. Schemes for FCC Regulations

Figure 9 shows the utilities obtained by optimization is larger than the distributed scheme. The major reason is for the distributed scheme, some WBSs are allowed to be idle, in comparison, every WBS in optimization must operate with either P_μ or P_{op} and the utility becomes large when the transmission power is P_μ.

Figure 10 illustrates the average SINR on end users, where the distributed scheme outperforms the centralized scheme. Figure 11 provides another perspective for the SINR on the end users. From the figure we can see, 95% percent of end terminals have SINR between 1 to 80 dB, and more than 90% have SINR which is higher than 4 dB. In particular, with the centralized scheme 97% of

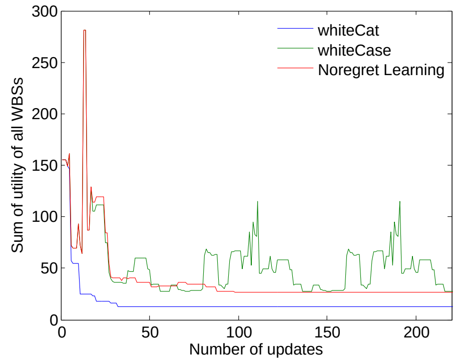


Figure 8. Convergence process of three different schemes in one simulation.

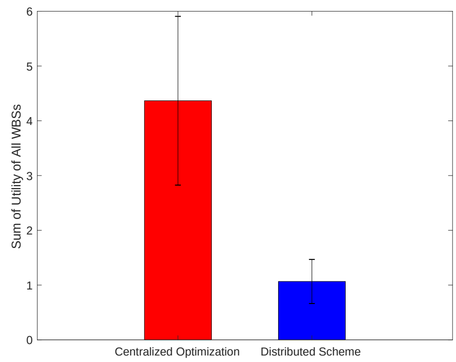


Figure 9. Sum of Utility

end terminals have SINR which is lower than 20 dB, in comparison, with the distributed scheme, this percentage is 87% and the rest of end terminals have SINR which is higher than 20 dB.

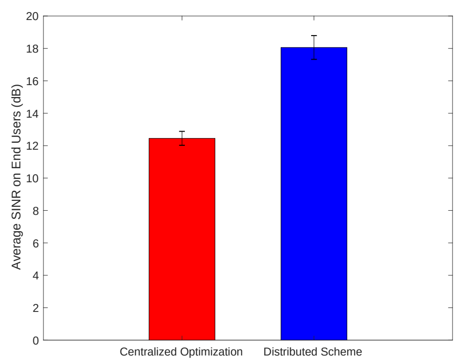


Figure 10. Average SINR on end users

In terms of power consumption as shown in Figure 12, the distributed scheme consumes only 76% of the centralized optimization. The reason is explained in Figure 13 i.e., by executing the former scheme, more WBSs are allowed to be idle than the latter.

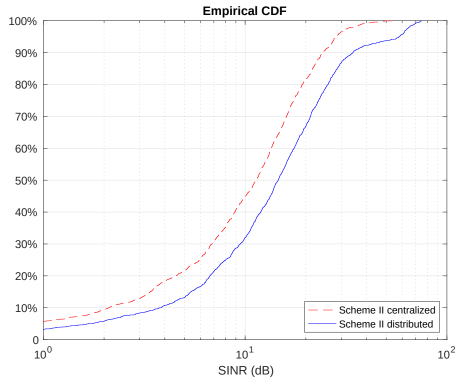


Figure 11. Cumulative distribution of SINR on end users after channel and power allocation

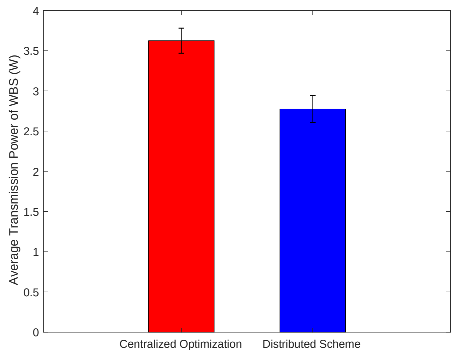


Figure 12. Average transmission power of one WBS

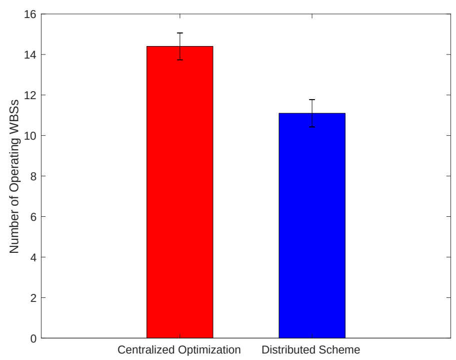


Figure 13. Number of WBSs with P_{op}

7. CONCLUSIONS

In this paper, we look into the problem that how to exploit the TV white space in a IEEE 802.22 like cellular network. Both centralized and distributed solutions for channel allocation problem, with respect to both ECC and FCC regulations, are proposed. In particular, we improve the SINR on the end terminals in the cells. Under the ECC rules, the proposed centralized scheme obtains the global optimal. In comparison, the congestion game based distributed scheme achieves comparable performance in terms of end user SINR and power consumption, but it outperforms other distributed schemes in terms of end user SINR and algorithm execution speed. With respect to the TV spectrum usage under FCC rules, we for the first time investigate the problem of channel allocation in order to improve the SINR on end users, where the TV systems could be interfered by the aggregated interference. Proposed centralized scheme achieves the global optimal after linear relaxizaiton, and the proposed distributed scheme is proved to converge by congestion game theory, and the simulation shows the distributed scheme outperforms the centralized scheme in terms of power consumption and SINR on end users.

REFERENCES

1. "In the matter of unlicensed operation in the TV broadcast bands: second memorandum opinion and order, federal communications commission, fcc 10-174," http://hraunfoss.fcc.gov/edocs_public/attachmatch/DA-11-131A1.pdf, Jan. 2011, visited July 2012.
2. T. E. C. C. (ECC), "guidance for national implementation of a regulatory framework for tv wsd using geo-location databases," May. 2015.
3. Ofcom, "Implementing tv white spaces." [Online]. Available: https://www.ofcom.org.uk/__data/assets/pdf_file/0034/68668/tvws-statement.pdf
4. J. W. Mwangoka, P. Marques, and J. Rodriguez, "Exploiting tv white spaces in europe: The cogeu approach," in *2011 IEEE International Symposium on Dynamic Spectrum Access Networks (DySPAN)*, May 2011, pp. 608–612.
5. K. Harrison and A. Sahai, "Seeing the bigger picture: Context-aware regulations," in *2012 IEEE International Symposium on Dynamic Spectrum Access Networks*, Oct 2012, pp. 21–32.
6. "Google spectrum database," visited Juni 2016. [Online]. Available: https://www.google.com/get/spectrumdatabase/
7. Y. Wu and D. Tsang, "Distributed power allocation algorithm for spectrum sharing cognitive radio networks with qos guarantee," in *Proc. of IEEE INFOCOM 2009*, Apr. 2009.
8. A. Hoang, Y. Liang, and M. Islam, "Power control and channel allocation in cognitive radio networks with primary users' cooperation," *IEEE Transactions on Mobile Computing*, vol. 9, no. 3, pp. 348–360, Mar. 2010. [Online]. Available: http://dx.doi.org/10.1109/TMC.2009.136
9. X. Chen and J. Huang, "Game theoretic analysis of distributed spectrum sharing with database," in *In Proceedings of 2012 IEEE 32nd International Conference on Distributed Computing Systems (ICDCS)*, June 2012, pp. 255–264.

10. T. E. C. C. (ECC), *Technical and operational requirements for the operation of white space devices undergeo-location approach*.
11. E. International, *MAC and PHY for Operation in TVWS*, ser. 1 June 2012.
12. K. Koufos, K. Ruttik, and R. Jantti, “Controlling the interference from multiple secondary systems at the TV cell border,” in *Proc. of IEEE PIMRC 2011*, Sep. 2011.
13. K. Ruttik, K. Koufos, and R. Jantti, “Model for computing aggregate interference from secondary cellular network in presence of correlated shadow fading,” in *In Proceedings of IEEE PIMRC 2011*, Sept 2011, pp. 433–437.
14. K. Harrison, S. Mishra, and A. Sahai, “How much white-space capacity is there?” in *Proc. of DySPAN 2010*, Apr. 2010, pp. 1 –10.
15. F. Hessar and S. Roy, “Capacity considerations for secondary networks in tv white space,” *IEEE Transactions on Mobile Computing*, vol. 14, no. 9, pp. 1780–1793, Sep. 2015.
16. M. Deshmukh, F. B. Frederiksen, and R. Prasad, “A closed form estimate of tvws capacity under the impact of an aggregate interference,” *Wireless Personal Communications*, vol. 82, no. 1, pp. 551–568, May 2015. [Online]. Available: https://doi.org/10.1007/s11277-014-2240-y
17. A. Achtzehn, M. Petrova, and P. Mhnen, “On the performance of cellular network deployments in tv whitespaces,” in *2012 IEEE International Conference on Communications (ICC)*, June 2012, pp. 1789–1794.
18. F. Hessar and S. Roy, “Resource allocation techniques for cellular networks in tv white space spectrum,” in *2014 IEEE International Symposium on Dynamic Spectrum Access Networks (DYSPAN)*, April 2014, pp. 72–81.
19. Y. Yang, L. Shi, and J. Zander, “On the capacity of wi-fi system in tv white space with aggregate interference constraint,” in *8th International Conference on Cognitive Radio Oriented Wireless Networks*, July 2013, pp. 123–128.
20. S. Gopal, S. K. Kaul, and S. Roy, “Optimizing city-wide white-fi networks in tv white spaces,” *IEEE Transactions on Cognitive Communications and Networking*, vol. 4, no. 4, pp. 749–763, Dec 2018.
21. N. Omidvar and B. Khalaj, “A game-theoretic approach for joint channel and power allocation in downlink of cellular cognitive radio networks,” in *Proc. of IEEE PIMRC 2012*, Sept 2012, pp. 1198–1202.
22. J. Elias, F. Martignon, L. Chen, and M. Krunz, “Distributed spectrum management in tv white space networks,” *IEEE Transactions on Vehicular Technology*, vol. 66, no. 5, pp. 4161–4172, May 2017.
23. N. Nie and C. Comaniciu, “Adaptive channel allocation spectrum etiquette for cognitive radio networks,” in *Proc. of IEEE DySpan 2005*, Nov. 2005.
24. M. Felegyhazi, M. Cagalj, S. Bidokhti, and J.-P. Hubaux, “Non-cooperative multi-radio channel allocation in wireless networks,” in *Proc. of IEEE INFOCOM 2007*, May 2007.
25. F. Wu, S. Zhong, and C. Qiao, “Globally optimal channel assignment for non-cooperative wireless networks,” in *Proc. of IEEE INFOCOM 2008*, Apr. 2008.
26. B. Ye, M. Nekovee, A. Pervez, and M. Ghavami, “TV white space channel allocation with simulated annealing as meta algorithm,” in *Proc. of 2012 7th International ICST Conference on Cognitive Radio Oriented Wireless Networks and Communications*, June 2012, pp. 175–179.
27. J. Huang, Q. Zhu, V. Krishnamurthy, and T. Basar, “Distributed correlated q-learning for dynamic transmission control of sensor networks,” in *Proc. of IEEE ICASSP 2010*, Mar. 2010.
28. S. Hart and A. Colell, “A simple adaptive procedure leading to correlated equilibrium,” *Econometrica*, vol. 68, no. 5, pp. 1127–1150, 2000. [Online]. Available: http://dx.doi.org/10.2307/2999445
29. D. Li and J. Gross, “Distributed tv spectrum allocation for cognitive cellular network under game theoretical framework,” *2012 IEEE International Symposium on Dynamic Spectrum Access Networks*, pp. 327–338, 2012.
30. L. Gurobi Optimization, “Gurobi optimizer reference manual,” 2018. [Online]. Available: http://www.gurobi.com
31. R. Murty, R. Chandra, T. Moscibroda, and V. Bahl, “Senseless: A database-driven white spaces network,” in *Proc. of IEEE DySpan 2011*, May 2011.
32. M. Liu and Y. Wu, “Spectum sharing as congestion games,” in *Proc. of Allerton Conference 2008*, Sep. 2008.
33. B. Babadi and V. Tarokh, “Distributed dynamic frequency allocation in wireless networks under time-varying user activities,” in *In Proceedings of IEEE Sarnoff Symposium 2008*, April 2008, pp. 1–5.
34. B. Ko, V. Misra, J. Padhye, and D. Rubenstein, “Distributed channel assignment in multi-radio 802.11 mesh networks,” in *Proc. of IEEE WCNC 2007*, Mar. 2007.
35. A. Mitsos, B. Chachuat, and P. I. Barton, “Mccormick-based relaxations of algorithms,” *SIAM Journal on Optimization*, vol. 20, pp. 573–601, 2009.

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