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RESEARCH ARTICLE

Distributed Schemes under Congestion Game framework and Optimization for Spectrum and Power Allocation in TVWS

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ABSTRACT

In this chapter, we will see the application of congestion game in solving the channel allocation problem in the context of TV white space. The channel allocation problem we will address is a general problem, as the transmission power is not identical for every transmitter and on each channel, actually, the transmission power could be unique for each transmitter-channel combination. With the suitable utility function designed for transmitters, the behaviours of the transmitters can be described by a congestion game. The algorithm of channel allocation is derived from the dynamics of the transmitter in the game, which reaches Nash equilibrium quickly.

Furthermore, we provide a complete solution to fully exploit TV white space complying with IEEE 802.22 standard. We propose a centralized methods to regulate the upper bound of transmission power, so that to strictly protect the primary users. The the distributed channel allocation and power control are conducted sequentially.

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1. INTRODUCTION

With the transition from analog TV to digital terrestrial TV (DTT), a considerable amount of frequency bands, as shown in Figure 1, become vacant. The spectrum that is left over by digital TV and other incumbent users is referred to as TV white spaces (TVWSs). TVWS can be used for telemedicine, precision agriculture, smart energy and so on by new devices as long as DTT reception is not interfered [1]. These unlicensed devices are called white space devices (WSDs) and their operation should be restricted in terms of location, channel and transmission power, so that no harmful interfering is generated to the incumbents. The utilization of TVWS by the WSDs are regulated by national regulatory authorities and relevant committees, i.e., in the US, the Federal Communications Commission (FCC) has regulated the utilization of TVWS since 2010 [1]. The Electronic Communications Committee (ECC) and Ofcom have released guidelines in EU and UK respectively in terms of the operation of WSDs [2,3]. Although many regulation details from the three institutes are different, white space data base (WSDB) is adopted by all the three institutes.

TV White Space database was first brought as a way to overcome the technical hurdles faced by spectrum sensing techniques to precisely detect very weak primary signals. WSD should contact a white space data base and provide its location and technical characteristics. The TVDB translates the information on incumbent services and the technical characteristics and location of the WSDs into a list of allowed frequencies and associated transmit powers for devices. [2]. WSD needs to access the WSDB to get the available channels and powers before starting transmission. WSDB is proposed mainly due to the difficulty to implement spectrum sensing, which is suggested by the regulators.

WSDB lies in the middle of the utilization of TVWS, which has the global view of the WSDs in the network and protects the DTT receivers. The resource which is available for the WSDs is decided by the strategy of WSDB, which is regulated by the regulators. Ofcom and ECC impose restrictions on the resulted aggregated interference caused by TVDBs on the DTT receivers, and implicitly allow TVDBs to have different transmission powers based on their locations. FCC employs stringent restriction on the universal transmission power for WBSs, but the DTT receivers are not protected from the aggregated interference caused by WBSs working on the same channel. The stringent regulation on transmission power results in decreased TVWS as suggested in [4]. Nevertheless, as to the issue of spectrum sharing among the WSDs, the regulator institutes only provide suggestion instead of regulations. In this paper, we will follow the most prevalent regulations on the usage of TVWS and investigate the spectrum sharing issue among among the WSDs.

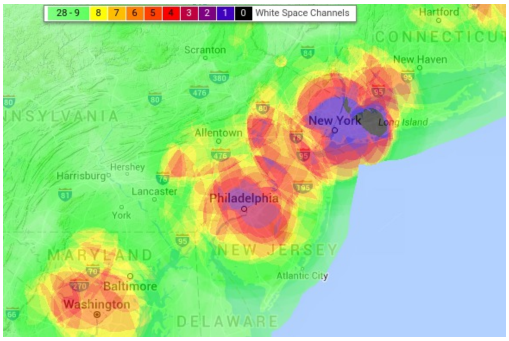

Figure 1. Variability of available channels in a densely populated area. This figure is obtained from [5]

The WSDs are classified into two functional categories: 1) higher power fixed devices , 2) lower power personal/portable devices, such as Wi-Fi like cards in laptop computers. The fixed devices and some portable devices should access the TVDB to obtain the available spectrum and permitted transmission. The fixed WSDs work as base stations and provides a backhaul for broadband client access. They are the base stations in the IEEE 802.22 standard which is designed for wireless regional area networks. As the fixed WSDs work with high transmission power, they are the major thread to the DTT receivers so that their transmission characteristics should be carefully decided by the TWDB. IEEE 802.22 regulates the MAC layer protocol, but it doesn't prevent the co-channel interference caused by the co-channel WSDs which work as base stations. In this paper, we investigate the efficient way to exploit the TV spectrum among the fixed WSDs which work as base station in a IEEE 802.22 network.

Given all the other WBSs' selection on channel and transmission power, a WBS is interested in choosing the channel which brings it the best performance, i.e., the data rate of its end users. A WBS prefers to choose the channel which experiences the minimum interference, and the transmission power allowed on that channel is higher, so as to obtain better SINR on its terminals and meanwhile maximize their coverage [6, 7]. Nevertheless, high transmission power causes significant co-channel interference to other secondary users operating on the same channel. Hence, a secondary cell has to balance its transmission power and the caused interference on other cells, meanwhile to choose working channel to decrease the experienced interference on its terminals. The existence of WSDB makes it a natural choice to adopt centralized decision and the calculation happens only in the WSDB, but considering the provision of WSDB services is still under discussion, we will also provide distributed solutions, which will distribute the work load of the WSDB.

The rest of the paper is organized as follows. We elucidate the system model in Section 2, afterwards related work is presented in Section 3. In Section 4, with respect to the regulation from ECC, we present the centralized optimization and game driven distributed scheme in terms of channel allocation. In Section 5, with respect to FCC regulations, both centralized optimization and game driven distributed schemes are introduced. Thereafter performance evaluation is presented in Section 6. Finally, we conclude our work and point out directions of future research in Section 7.

2. SYSTEM MODEL AND PROBLEM STATEMENT

We look in to a IEEE 802.22 compliant cellular network where the fixed WSDs work as base station and provide broadband access to their terminals. The network is illustrated in Figure 2. We name the fixed WSDs working as base stations as White space Base Stations (WBSs), they locate in one area which is surrounded by DTV stations and receivers. There are several critical points in the vicinity of the DTV receivers and the interference cause by the WSDs should below a certain threshold. In our model, we only see the WBSs, which work with high transmission power, as the possible interfering unlicensed devices to the DTV service. As to the WBSs, the out-of-band emission is regard as trivial and we only consider co-channel interference among the WBSs. To simplify the analysis, we assume that each DTV station as well as each WBS utilizes exactly one channel.*

As to the notations, the set of critical points is denoted as $\mathcal{K}$ and the collection of WBSs is denoted as $\mathcal{N}$ where $|\mathcal{N}| = N$. The TVWS spectrum bands are denoted as set $\mathbb{C}$. The channels in $\mathcal{C}$ are assumed to be identical in terms of attenuation and shadowing on the same path. We represent the usage of channel for WBS i with a binary vector $X_i^{|\mathbb{C}|\times 1} = \{\cdots, x_{ik}, \cdots\} \in \{0,1\}^{|\mathbb{C}|}$, where $k \in \mathbb{C}$ and binary variable x_{ik} denotes whether channel k is used by user i. As each node can only uses one channel, for X_i, there is $\sum_{k=1}^{|\mathbb{C}|} x_{ik} = 1$. The transmission power of WBS i on channel c is P_i^c. $c(i)$ denotes the channel used by a WBS $i \in \mathcal{N}$.

WBSs are interested in providing broad band access to the their associated terminals. As to performance metric for the QoS provisioning, we choose the signal to noise and interference ratio (SINR) on the terminals. For a terminal m which is associated to WBS i, the attenuation between WBS i and m is denoted as h_{im}. The path loss

*The assumption that one WBS only utilizes one channel is for convenience of analysis. In reality multiple channel usage (channel bonding) is requisite as one single TV channel's bandwidth is 6 MHz which is not adequate for a WBS to fulfil system requirement.

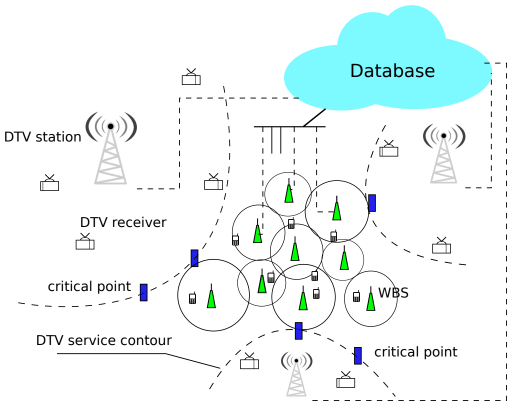

Figure 2. System model: WBS cells and DTV systems

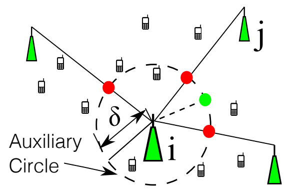

Figure 3. Assuming the radius of the auxiliary circle is δ and all the WBSs work on the a channel, then QuasiSINR is a quotient where the divided is the WBS i's power on the auxiliary circle (i.e., the green pot), and the divisor is the summation of the interfering power on the red pots.

is dependent on the distance between the corresponding equipments, e.g. $h_{im} = K \cdot d_{im}^{-\alpha}$, where α is the path loss exponent, d_{im} is the distance between i and m, K is a constant which models the reference loss over a single unit of distance. N_0 denotes the thermal noise power. Shadowing without fading is considered in our model. z_{im} models the zero-mean log-normally distributed shadow fading between i and m, and the standard deviation is σ_{SH}. The SINR at end terminal m is,

$$\gamma_m = \frac{P_i^c \cdot h_{im} \cdot z_{im}}{f_m^c} = \frac{P_i^c \cdot h_{im} \cdot z_{im}}{\sum (P_j^c \cdot h_{jm} \cdot z_{jm}) + N_0}, \quad j \in \mathcal{N} \setminus i, c(j) = c \tag{1}$$

where P_j^c denotes the transmission power of interfering WBS j.

A WBS's utility is a function of the SINR on all its end terminals, i.e., the average SINR at all its terminals. But as the terminals are mobile and they are influenced by many factors, i.e., the type of service provided to the terminals, the utility may diverge from the real performance of the terminals. Meanwhile, it is not appropriate only to choose one [8] or more fixed terminals, and use their SINRs to represent the SINR for all the other terminals in that cell, because their location could diverge greatly with the locations of the other terminals. Thus, we propose a metric *QuasiSINR* to represent WBS's performance in terms of SINR on its end terminals, which is independent on the actual locations of end terminals.

2.1. QuasiSINR of WBS

With an auxiliary circle centered at the discussed WBS, which is shown as dashed circle in Figure 3, QuasiSINR is the ratio between the power of signal of interest on the circle and the summation of the strongest power from the interfering WBSs on the auxiliary circle.

The quasiSINR of WBS i is denoted as γ_i,

$$\begin{aligned}\gamma_i &= \frac{P_i^c \cdot h_{i\to\text{i's auxiliary circle}} \cdot z_{i\to\text{i's auxiliary circle}}}{\sum\limits_{\substack{j\neq i, j\in\mathcal{N}\\ c(j)=c(i)}} (P_j^c \cdot h_{j\to\text{i's auxiliary circle}} \cdot z_{j\to\text{i's auxiliary circle}}) + N_0} \\ &= \frac{P_i^c \cdot \delta^{-\alpha} \cdot z_{i\to\text{auxiliary circle}}}{\sum\limits_{\substack{j\neq i, j\in\mathcal{N}\\ c(j)=c(i)}} (P_j^c \cdot (d_{ji} - \delta)^{-\alpha} \cdot z_{j\to\text{auxiliary circle}}) + N_0}\end{aligned} \tag{2}$$

In the following paper, when we talk about the channel and power allocation with respect to WBS, the notation h_{ij} denotes the attenuation between WBS i to the auxiliary circle of WBS j. h_i denotes the attenuation between WBS i to its own auxiliary circle. Then γ_i becomes,

$$\gamma_i = \frac{P_i^c \cdot h_i \cdot z_i}{\sum\limits_{\substack{j\neq i, j\in\mathcal{N}\\ c(j)=c(i)}} (P_j^c \cdot h_{ji} \cdot z_{ji}) + N_0} \tag{3}$$

The notations are summarized in Table I. With auxiliary circle, the decision made by WBSs is independent on the distribution of the end terminals. On the other hand, the radius of the auxiliary circle δ can be adapted to foster better service to the terminals in certain area, i.e., a larger radius δ will take care of the SINR on the terminals reside far away and vice visa.

In our model, WBSs access the TVDB and obtain the transmission parameters, i.e., working channel, transmission power of the other WBSs, the attenuation characteristics between itself and all the other WBSs, and vice visa. With these information, WBSs calculate their QuasiSINRs respectively.

2.2. Permitted Transmission Power for WBSs

According to regulations, the calculation engine within the TVDB translates the information on incumbent services and the technical characteristics and location of the WSDs into a list of allowed frequencies and associated transmit powers for the WSDs [2]. ECC and Ofcom propose guidance [9, 10] and focus on the protection of the critical points from the harmful interference, and don't regulate the power limitation. FCC initially restricts the maximum transmission power to under 1 W and some time later relaxes the limitation to 4 W. As a results, TVDB's

Table I. Notations

Symbol	Description
γ	QuasiSINR
f^c_{ji}	The co-channel interference caused by WBS j on the auxiliary circle of WBS i, c is the working channel for both
f^c_i	The sum of interference caused on the auxiliary circle of WBS i
p^c_i	The Tx power of WBS i on channel c on channel c
P^c_i	The maximal permitted Tx power of WBS i on channel c (ECC solution)
P_μ, P_{op}	The minimum and maximal permitted Tx power of WBS i (FCC solution)
h_{ij}	The attenuation between WBS i to the auxiliary circle of WBS j.
z_{ij}	The shadowing from WBS i to the auxiliary circle of WBS j.
$\alpha^k_{ij}, \beta^k_{ij}$ y_i, z_i	Binary auxiliary variables in the optimization in Section 5.
cp	Critical point

decisions on the transmission power are different with respect to different regulations.

2.2.1. ECC Regulations of Permitted Tx Power

As to decision on the transmission power per ECC, as WBSs work in underlay manner and coexist with DTT receivers, the aggregate generated interference caused on the critical points on each channel should not exceed a threshold. We adopt the interference model and the optimization methodology from the work of [11] to plan the maximum transmission power on each channel for WBSs. For WBS $i \in \mathcal{N}$, the maximum transmission power allowed on channel $c \in \mathbb{C}$ is denoted as P^c_i. As to each channel c, the generated interference on each interference measuring device should be within a predefined interference margin I. The interference margin in a slow fading environment is decided according to [12]. To address this fairness issue, we maximize the sum of the logarithmic value of every WBS's transmission power, and formulate the problem into a convex optimization problem.

$$\begin{aligned} \text{Maximize} \quad & \sum_{i \in \mathcal{N}} \log P^c_i \\ \text{subject to} \quad & \sum_{i \in \mathcal{N}} (P^c_i \cdot h_{i,\text{cp}} \cdot z) < I, \forall \text{cp} \in \mathcal{K} \end{aligned} \tag{4}$$

This optimization is conducted in TVDB for each channel $c \in \mathbb{C}$, and the calculated maximum permitted transmission powers are used by the WBSs afterwards.

2.2.2. FCC Regulations of Permitted Tx Power

As to the decision made on the transmission power per FCC, all WBSs work with 4 W transmission power. Obviously there needs a limit on the number of WBSs which operation.

2.3. Problem Formulation

Our goal is to minimize the sum of inverted quasiSINR $\sum_{i\in\mathcal{N}} 1/\gamma_i$ with the WSDB's decisions on WBSs' transmission powers. In order to ensure the fairness among WBSs, we minimize the sum of inverted quasiSINR instead of maximizing the sum of quasiSINR of all WBSs.

3. RELATED WORKS

To exploit TVWS, authors in [13–16] have proposed different approaches for assessment of TVWS capacity under FCC and ECC regulations respectively. Hessar et al. [17] aim to maximize the Shannon capacity of the network which complies with FCC rules. The solution seeks the trade-off between the wide band and co-channel interference, both of which are brought by assigning multiple channels to WBSs. But this scheme doesn't restrict the number of TVWSs and doesn't consider the harmful interference caused on the DTTs. Yang et al. [18] and Gopal et al. [19] propose throughput maximization of a CSMA/CA based WiFi like network in TVWS under aggregate interference.

Omidvar et al. [20] use potential game to propose a distributed joint power and channel allocation in cognitive radio network. Although the scheme is not tailored for TVWS, protection on the primary users are also considered. Potential game is adopted in work [21] to mitigate the adjacent interference. Chen et al. [8] investigate the channel allocation problem in the scenario of TV white space. The channel allocation problem is formulated into a potential game, individual WBS's utility is to maximize the capacity of one single static terminal.

With the doctrine of not interfering the primary TV services and regulations imposed by institutes, the problem we solve in this paper is different from the channel allocation problems discussed in the domain of cognitive radio networks. But we think a brief review in terms of the channel allocation techniques in CRN is necessary. With identical transmission power and symetric path attenuation, Nie et al. [26] formulate channel assignment problem in ad-hoc cognitive radio network into a potential game which leads to pure NE, the authors [27,28] propose algorithms which converge to pure Nash equilibrium (NE) and strongly dominate strategy equilibrium respectively. Simulated annealing is applied to mitigate co-channel interferences in [24]. For the same purpose, no-regret learning [25, 29] is exploit to optimize the choice on channel.

4. CHANNEL AND POWER ALLOCATION SCHEME COMPLYING WITH ECC RULES

In this section we will discuss the channel and power allocation problem complies with the rules regulated by ECC. According to the ECC regulations, the transmission power of the WBSs are strictly regulated, so that the TV receivers are not affected even when all the WBSs operate on the same channel. In the following subsections, we firstly present the decision on the maximum permitted transmission power on each channel for each WBS, then present how do the WBS make use of these powers in centralized and distributed manner respectively.

4.1. The Maximum Permitted Transmission Power

The WBSs work in underlay manner and coexist with primary TV stations and receivers, the aggregate interference generated by WBSs should not exceed the threshold at each critical point and on each channel . We adopt the interference model and the optimization methodology from the work of [11] to plan the maximum transmission power on each channel for WBSs. Having a global view of the propagation parameters, geolocations of WBSs and interference threshold at interference measuring devices which locate on the contour of TV service area, linear programming is implied in the database to calculate the maximum permitted power over each channel.

By solving 4, the WSDB obtains the maximum permitted transmission power for every WBS over each channel. The interference threshold on the critic points will not be exceeded even when all the WBSs work on the same channel.

4.2. Centralized Optimization for Channel Allocation

We formulate the channel allocation problem into a binary quadratic programming problem which can be solved in a centralized way. Let $X_i = \{x_i^1, \cdots, x_i^k, \cdots, x_i^{|\mathbb{C}|}\}$ denote the vector of channel usage, there is $|X_i| = |\mathbb{C}|$ and binary element x_i^k represent whether WBS i occupies channel k. Given two WBSs i and j, there is,

$$X_i^T X_j = \sum_{k=1}^{|\mathbb{C}|} x_i^k \cdot x_j^k = \begin{cases} 1 & \text{if } c(i) = c(j) \\ 0 & \text{if } c(i) \neq c(j) \end{cases} \tag{5}$$

$c(i)$ means the working channel which is chosen by WBS i. The transmission power levels on all channels for WBS i are denoted by a constant vector $\mathbf{P}_i = \{P_i^1, \cdots, x_i^k, \cdots, x_i^{|\mathbb{C}|}\}$. The transmission power adopted by WBS i is $\mathbf{P}_i^T X_i = \sum_{k=1}^{|\mathbb{C}|} P_i^k \cdot x_i^k$.

The problem in Section 2.3 can be modeled via general purpose nonlinear optimization:

$$\begin{aligned} \text{minimize} \quad & \sum_{i=1}^{n} \frac{\sum_{j\in\mathcal{N}, j\neq i} \mathbf{P}_j^T X_j (X_j^T X_i) h_{ji} z_{ji} + N_0}{\mathbf{P}^T X_i h_i z_i} \\ \text{subject to} \quad & \sum_{k=1}^{|\mathbb{C}|} x_i^k = 1, x_i^k \in X_i \in \{0,1\}^{|\mathbb{C}|} \end{aligned} \tag{6}$$

Problem 15 is a non-linear problem with binary variables, but it can be reformulated into a quadratic programming problem as,

$$\begin{aligned} \text{minimize} & \sum_{i=1}^{n} \Big(\sum_{j\in\mathcal{N}, j\neq i} \sum_{k\in\mathbb{C}} \frac{P_j^k \cdot h_{ji} \cdot z_{ji}}{P_i^k \cdot h_i \cdot z_i} \cdot x_j^k \cdot x_i^k \\ & + \sum_{k\in\mathbb{C}} \frac{N_0}{P_i^k \cdot h_i \cdot z_i} \cdot x_i^k \Big) \\ \text{subject to} & \sum_{k=1}^{|\mathbb{C}|} x_i^k = 1, x_i^k \in X_i \in \{0,1\}^{|\mathbb{C}|} \end{aligned} \tag{7}$$

The reformulation is available in Appendix in [30]. We use LINDO [31] which is a state of art non-linear problem solver to solve the problem, which employs Branch-And-Reduce method to get the global optimum for the problem. The result of this centralized channel assignment will be evaluated in the simulation section with other schemes.

4.3. Distributed White Space Channel Allocation (WhiteCat)

In this section a distributed scheme for WBSs to allocate channels is proposed, which is named as white space channel allocation technology (WhiteCat).

4.3.1. Algorithm and Protocol

WhiteCat adopts the best response process, where each WBS (referred as i) chooses the channel which brings the largest utility u_i as the response of other WBSs' choices on channels. WhiteCat is depicted by algorithm 1.

$$u_i = \frac{\sum_{\substack{j\in\mathcal{N}, j\neq i, \\ c(\sigma_j)=c(\sigma_i)}} f_{ji}}{2 \cdot \tilde{P}_i} + \frac{1}{2} \sum_{\substack{j\in\mathcal{N}, j\neq i, \\ c(\sigma_j)=c(\sigma_i)}} \frac{f_{ij}}{\tilde{P}_j} + \sum_{\substack{\mathcal{S}: i,j\in\mathcal{S}, \\ c(\sigma_j)=c(\sigma_i)}} \frac{N_0}{C \cdot \tilde{P}_i} \tag{8}$$

where $\tilde{P}_i = P_i \cdot h_i \cdot z_i$, similarly $\tilde{P}_j = P_j \cdot h_j \cdot z_j$. Overlooking the constant coefficient 2, the first item of u_i is the inverted QuasiSINR of station i. To minimize the first item, WBS i needs to choose a channel either permits higher transmission power or experiences less interference, whereas the higher power increases the second item which is a part of inverted QuasiSINR of other co-channel WBSs. Hence, the cost function presents a reasonable comprise between the QuasiSINR of one WBS and others.

When WBS only emphasizes on its own utility (e.g. the first part of Formula 8), the best response process doesn't converge. We have following theorem:

Theorem 4.1. With non-identical transmission power, if every WBS updates its channel based on Algorithm 1 with utility based on its own interests, i.e., the first part of Formula 8, the process doesn't always converge.

The proof is in Appendix 1 in [30].

Algorithm 1: Spectrum selection by WBS i

Input: the distance, path lose and shadowing parameter between WBS i to WBS $j \in \mathcal{N} \setminus i$;
radius of auxiliary circle, noise N_0, total number of WBSs N;
for $j \in \mathcal{N} \setminus i$, the maximal transmission power $P_j^c, c \in \mathbb{C}$ and the working channel $c(j)$.

for $c \in \mathbb{C} \setminus c(i)$ **do**
  calculate $u_i(c)$ based on Formula 8 **if** $u_i(c) < u_i(c(i))$ **then**
    $c(i) \leftarrow c$
  else
    keep $c(i)$ unchanged
  end
end
Notify database of its channel usage, which further notifies the other WBSs

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WBSs calculate u_i with the information retrieved from TVDB. After executing Algorithm 1, it reports to TVDB about its channel if its working channel is updated. As the location of WBSs and TV stations and the transmission channel and power of TV stations are usually static (entries of TV station change averagely once in 2 days [32]), except for the channel usage in the network, the change of the other data stored in WBS is infrequent. We refer [26] to decide WBSs' sequence to update their channels. [26] proposes a method which is akin to the random access mechanism of CSMA/DA, where the access for broadcast medium is changed to get access to the centralized center to retrieve the current channel usage and update its new channel. All WBSs are able to access the database in one round (with random or predetermined sequence). As WBSs are connected with database, the control messages needed to decide the sequence will not become a burden. Update of channels can happen in the boot phase, or when the quality of services (the SINR on its end users) of WBSs falls below a threshold, or a fixed time duration comes to end, or a new WBS joins in the network.

4.3.2. Analysis in Game Theoretical Framework

In this section, We give the proof on whiteCat's convergence in the framework of congestion game theory. Formulating a spectrum sharing problem into a congestion game and the concept of *virtual resources* are firstly proposed in [33]. This work reversely engineers the distributed channel allocation schemes proposed in [23, 34], i.e., unifies the algorithms with congestion game. But the problem analysed in [33] assume the transmission power is identical, which is a major difference from the channel allocation problem discussed here.

In congestion game, each player acts selfishly and aims at choosing strategy $\sigma_i \in \Sigma_i$ to minimize their individual cost. The gain (loss) caused by any player's unilateral move is exactly the same as the gain (loss) in the potential, which may be viewed as a global objective function. For problems where the potential of the problem is the same with the summation of the cost of all users, the cost function can be used as a utility function directly. This equivalence doesn't exist in our problem, but by carefully choosing the cost function for players, we can make sure that the change of individuals' cost is in the same direction with that of the global utility.

4.3.3. From WhiteCat to Congestion Game

We utilize the conception of virtual resource which is firstly introduced in [33]. Virtual resource is a triplet $\{i, j, c\}$, where i, j are two WBSs and $c \in \mathbb{C}$ is one channel. This piece of resource is regarded used by i when both i and j use channel c, otherwise, $\{i, j, c\}$ is not used by any WBS.

In the following, we list the element of the congestion game which emulates Algorithm 1. In this section, player and base station are used interchangeably.

- Player i' strategy space is $\Sigma_i = \{(i, j, c), j \in \mathcal{N}, j \neq i, c(\sigma_j) = c, c = 1, 2, \cdots, N\}$, and i has C admissible strategies, one strategy related with channel $c \in \mathbb{C}$ is described by the set of virtual resources it uses: $\sigma_i = \{(i, j, c), j \in \mathcal{N}, j \neq i, c(\sigma_j) = c\}$, note that virtual resource $(i, j, c) \neq (j, i, c)$.
- Under the strategy profile $\sigma = (\sigma_1, \sigma_2, \cdots \sigma_N)$, player i obtains a total cost of

$$g^i(\sigma) = \sum_{\substack{j \in \mathcal{N}, j \neq i, \\ c = c(\sigma_i) = c(\sigma_j)}} \left(g_{(i,j,c)}(n_{(i,j,c)}(\sigma)) + g_{(j,i,c)}(n_{(j,i,c)}(\sigma))\right) \tag{9}$$

The transmission power over all channels of player i is $\{p_i^1, p_i^2, \cdots, p_i^{|\mathbb{C}|}\}$. We define the cost function for virtual recourses (i, j, c) as follows,

$$g_{(i,j,c)}(k) = \begin{cases} \dfrac{f_{ji}}{2\tilde{P}_i} + \dfrac{f_{ij}}{2\tilde{P}_j} + \dfrac{C \cdot N_0}{N \cdot \tilde{P}_i} & \text{if } k = 2 \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

As resource (i, j, c) only lies in the strategy space of player i and j, thus can only be accessed by this two players. More specifically, according to Formula 10, the cost of resource (i, j, c) is only decided by the number of players using it, which is either 0 or 2. At the first glance, this is a player specific congestion game, as $g_{(i,j,c)}$ is decided by the relevant players' transmission power and inference. But actually the resource (i, j, c) excludes the players except for i and j from using it, thus the cost happened on this resource is only dependant on how many of players from the set $\{i, j\}$ to use it. Hence, the cost is a function of the number of players using the resource, and this is a canonical congestion game.

Now we substitute Formula 10 to Formula 9, the total cost for user i under strategy profile σ .

$$
\begin{aligned}
g^i(\sigma) &= \sum_{\substack{j\in\mathcal{N}\setminus i,\\ c=c(\sigma_j)=c(\sigma_i)}} (g_{(i,j,c)}(2) + g_{(j,i,c)}(2)) = \sum_{\substack{j\in\mathcal{N}\setminus i,\\ c(\sigma_j)=c(\sigma_i)}} (\frac{f_{ji}}{\tilde{P}_i} + \frac{f_{ij}}{\tilde{P}_j} + \frac{C\cdot N_0}{N}(\frac{1}{\tilde{P}_i} + \frac{1}{\tilde{P}_j})) \\
&= \frac{\sum_{\substack{j\in\mathcal{N}\setminus i,\\ c(\sigma_j)=c(\sigma_i)}} f_{ji}}{\tilde{P}_i} + \sum_{\substack{j\in\mathcal{N}\setminus i,\\ c(\sigma_j)=c(\sigma_i)}} \frac{f_{ij}}{\tilde{P}_j} + \frac{CN_0}{N} \sum_{\substack{j\in\mathcal{N}\setminus i,\\ c(\sigma_j)=c(\sigma_i)}} (\frac{1}{\tilde{P}_i} + \frac{1}{\tilde{P}_j}) = \frac{\sum_{\substack{j\in\mathcal{N}\setminus i,\\ c(\sigma_j)=c(\sigma_i)}} f_{ji}}{\tilde{P}_i} + \sum_{\substack{j\in\mathcal{N}\setminus i,\\ c(\sigma_j)=c(\sigma_i)}} \frac{f_{ij}}{\tilde{P}_j} + \frac{2CN_0}{N} \sum_{\substack{i\in\mathcal{S}\subset\mathcal{N},\\ \mathcal{S}:\forall i\in\mathcal{S}\\ c(\sigma_i)=c}} \frac{1}{\tilde{P}_i}
\end{aligned}
\tag{11}
$$

where $\mathcal{S}$ denotes the set of WBSs whose working channel is the same with WBS i.

Now we are going to have a look at the *potential* of the network. According to the expression of Rosenthal's potential in Formula 14, the potential is accumulated by adding the players' cost sequentially, in particular, the value which is added is the cost that player experiences when it starts to use the relevant resource, and the value is not changed when other players come to use that resource. Back to our problem, for two WBSs $i, j \in \mathcal{S}$, we assume WBS i's index is smaller than j's index, then the potential increased by i using the resource $\{i, j, c\}$ is 0 according to Formula 9, and the increase brought in by j using the resource $\{i, j, c\}$ is $g_{(i,j,c)}(2) + g_{(j,i,c)}(2)$. In other words, for each interfering pair of WBSs, only the WBS with bigger index contributes to the potential. Then the total potential is,

$$
\begin{aligned}
G(\sigma) &= \sum_{r\in\mathcal{R}} \sum_{i=1}^{n_r(\sigma)} g_r(i) = \sum_{i\in\mathcal{N}} \sum_{r\in\sigma_i} g_r(n_r^i(\sigma)) \\
&= \sum_{i\in\mathcal{N}} \frac{\sum_{\substack{j\in\mathcal{N},j\neq i,\\ c(\sigma_j)=c(\sigma_i)}} f_{ji}}{\tilde{P}_i} + \frac{CN_0}{N} \sum_{\substack{\mathcal{S}\subset\mathcal{N},\\ \forall i\in\mathcal{S},c(\sigma_i)=c}} \mid \mathcal{S} \mid \sum_{i\in\mathcal{S}} \frac{1}{\tilde{P}_i}
\end{aligned}
\tag{12}
$$

note that the summation of one WBS's congestion is related to its index.

When players minimize their utilities (cost or potential) illustrated by Formula 11, the total congestion in the secondary network given by Formula 12 decreases monotonically before reaching one Nash equilibrium. Players' greedy update in the game to minimize its cost Function11, which ceases finally in pure Nash Equilibrium. The strategy and cost function of players in the game is transplanted as Algorithm 1 and utility Function 8 respectively.

4.3.4. Comparison between the Potential in the Congestion Game and the Objective of WhiteCat

It is natural to raise the question, is the sum of the final utilities of all WBSs exactly the same with the value of potential when the game converges to a Nash equilibrium, which is represented by 12. The answer is, they are identical when N_0 is zero, and there will be a little difference when N_0 is not zero. Recall the target objective we want to minimize in Problem **??** is,

$$
\begin{aligned}
\sum_{i\in\mathcal{N}} \frac{f_i}{\tilde{P}_i} &= \sum_{i\in\mathcal{N}} \frac{\sum_{\substack{j\in\mathcal{N},j\neq i,\\ c(\sigma_j)=c(\sigma_i)}} f_{ji} + N_0}{\tilde{P}_i} \\
&= \sum_{i\in\mathcal{N}} \frac{\sum_{\substack{j\in\mathcal{N},j\neq i,\\ c(\sigma_j)=c(\sigma_i)}} f_{ji}}{\tilde{P}_i} + \sum_{i\in\mathcal{N}} (\frac{N_0}{\tilde{P}_i})
\end{aligned}
\tag{13}
$$

We notice that only the last items of the objective 13 and the potential of the congestion game 12 are different. When $N_0 = 0$, the potential is exactly the same with the object we want to minimize. When $N_0 \neq 0$, if channels are evenly distributed and there is $C/N* \mid \mathcal{S} \mid = 1$, then Formula 13 and 12 are also the same. In both cases, the sum of utilities 13 decreases monotonically with every update of WBSs before the system reaches Nash Equilibrium. When $N_0 \neq 0$ and Formula 13 and 12 are thus different, the monotonicity on the decrease of sum of utilities 13 is not perceived, whereas the system will still cease to NE.

Based on above analysis, we can see the assumption that each WBS only occupies one channel can be easily removed. If we regard one WBS as multiple ones which locate at the same place, and each WBS works on one distinct channel, then the proof on convergence of whiteCat can be applied directly to this case.

Note that the convergence of the game is independent on the the concrete form of the cost function. We adopt the function 11 to let the potential of the game be the same with the total utility of all WBSs, so that by executing

Algorithm 1, the system objective experiences a monotonic decreasing process before the system reaching NE. The algorithm has potential to solve many other problems, where one user's decision affects others. In this case, the utility of one user can be formulated to incorporate the information of its own utility and others', then the congestion game theory can be used to analogize.

4.3.5. Communication Overhead of WhiteCat

The problem of channel allocation with different and fixed transmission power is NP hard. WhiteCat is a distributed scheme but certain information of the other WBSs is needed. The centralized base station is piggybacked to provided the needed information. As to one WBS, the number of such inquiries is the number of steps before convergence.

In our formulated congestion game, a player i is allowed to access up to $(N-1)$ resources in the same time, i.e., $\{i, j_1, c(i)\}, \{i, j_2, c(i)\} \cdots \{i, j_{N-1}, c(i)\}$, thus the upper bound of converge steps can not be obtained from the conclusion 14 for singleton congestion game. But our problem is special because for each resource, the possible number of players allowed to use each resource is either 2 or 0. Thus we can refer the method used in Section **??** to analyse the update times for our problem. Firstly, we sort the cost values in increasing order. Although a WBS

$$\phi(\sigma) = \sum_{r \in \mathcal{R}} \sum_{i=1}^{n_r(\sigma)} g_r(i) \leq \sum_{r \in \mathcal{R}} \sum_{i=1}^{n_r(\sigma)} n \leq n^2 m \tag{14}$$

The upper bound of total update steps is $2n^2$, thus averagely, the upper bound of update steps for each WBS is $2n$.

5. CHANNEL ALLOCATION SCHEME COMPLYING WITH FCC RULES

The second scheme is in line with the FCC proposals in terms of transmission power, i.e., WBSs operate with the same transmission power P_{op}. When the number of transmitters increases, the accumulated interference may cause interference to TV receivers. In this scenario, the number of WBSs operating need to be restricted. This being said, when the number of WBSs is large, some of them need to stop transmitting.

5.1. Centralized Optimization for Channel Allocation

Due to the existence of certain WBSs which don't transmit, we can not adopt the problem formulation 15 directly because some denominators in the objective function, which are related with the idle WBSs are zero. In order to solve this problem, we allow the idle WBSs to work with a very low transmission power. That being said, the power used by user i can be presented as $\mathbf{P}_i^T X_i = \sum_{k=1}^{|\mathbb{C}|} p_i^k \cdot x_{ik}$. There is $p_i^k \in \{P_\mu, P_{\text{op}}\}$, where P_μ represents the very weak transmission power. The transmission power levels are the same as for all the WBSs, so we omit the superscript when we refer the transmission power.

$$\begin{aligned}
\text{Minimize} \quad & \sum_{i=1}^{n} \frac{\sum_{j \in \mathcal{N}, j \neq i} \mathbf{P}_j^T X_j (X_j^T X_i) h_{ji} z_{ji} + N_0}{\mathbf{P}_i^T X_i h_i z_i} \\
\text{subject to} \quad & \sum_{k=1}^{|\mathbb{C}|} x_i^k = 1, x_i^k \in X_i \in \{0,1\}^{|\mathbb{C}|} \\
& p_i^k \in \{P_\mu, P_{\text{op}}\}, \texttt{for} \forall i \in \mathcal{N} \\
& \sum_{i \in \mathcal{N}} (p_i^k \cdot h_{i,pt} \cdot z) < I_{pt}^c, \texttt{for} \forall k \in \mathbb{C}
\end{aligned} \tag{15}$$

Problem 15 is a non-linear problem with binary variables, with the technique in Section 4.2, it can be written as,

$$\begin{aligned}
\text{minimize} & \sum_{i=1}^{n} \Big(\sum_{j \in \mathcal{N}, j \neq i} \sum_{k=1}^{|\mathbb{C}|} \frac{p_j \cdot h_{ji} \cdot z_{ji}}{p_i \cdot h_{ii} \cdot z_{ii}} \cdot x_j^k \cdot x_i^k \\
& + \sum_{k=1}^{|\mathbb{C}|} \frac{N_0}{p_i \cdot h_{ii} \cdot z_{ii}} \cdot x_i^k \Big) \\
\text{subject to} & \sum_{k=1}^{|\mathbb{C}|} x_i^k = 1, x_i^k \in X_i \in \{0,1\}^{|\mathbb{C}|} \\
& p_i \in \{P_\mu, P_{\text{op}}\} \texttt{for} \forall i \in \mathcal{N} \\
& \sum_{i=1}^{n} x_i^k \cdot p_i \cdot h_{i,pt} \cdot z \leq I_{pt}^k, \texttt{for} \forall k \in \mathbb{C}
\end{aligned} \tag{16}$$

Formulation 16 is nonlinear but we can transform it into a quadratic optimization with auxiliary variables. We introduce binary number α_{ij}^k, real number β_{ij}^k and q_i, where

$$x_j^k \cdot x_i^k = \alpha_{ij}^k \tag{17}$$

$$\beta_{ij}^k = p_j \cdot \alpha_{ij}^k \tag{18}$$

$$\frac{1}{p_i} = q_i \tag{19}$$

Now the objective function is is quadratic as shown in Formula 20,

$$\sum_{i=1}^{n} (h_{ji} z_{ji} \sum_{j \in \mathcal{N}, j \neq i} \sum_{k=1}^{|\mathbb{C}|} \beta_{ij}^k q_i + N_0 \sum_{k=1}^{|\mathbb{C}|} x_i^k q_i) \tag{20}$$

Constraint 18 can be transformed as a set of linear constraints:

$$\beta_{ij}^k - p_{\text{op}} \cdot \alpha_{ij}^k \leq 0 \tag{21}$$

$$-\beta_{ij}^k + p_\mu \cdot \alpha_{ij}^k \leq 0 \tag{22}$$

$$\beta_{ij}^k - p_j - p_\mu \cdot \alpha_{ij}^k \leq -p_\mu \tag{23}$$

$$-\beta_{ij}^k + p_j + p_{\text{op}} \cdot \alpha_{ij}^k \leq p_{\text{op}} \tag{24}$$

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The constraint 19 is a quadratic equality, which makes the problem to be non-convex. Thus we need to relax it into a linear inequalities. As p_i is either P_{op} or P_μ, we can replace p_i and q_i with the help of two binary variables y_i and z_i respectively:

$$p_i = y_i P_\mu + (1 - y_i) P_{\text{op}} \tag{25}$$
$$q_i = z_i \frac{1}{P_{\text{op}}} + (1 - z_i) \frac{1}{P_\mu} \tag{26}$$

Here we put a restriction on P_μ and P_{op} with $P_\mu P_{\text{op}} = 1$, Function 26 becomes

$$q_i = P_\mu z_i + (1 - z_i) P_{\text{op}} \tag{27}$$

Now we conduct convex relaxation by applying McCormick envelope [35]. As we want to get the lower bound of the objective function, we get the over-estimators so as to expand the search space as follows,

$$1 = p_i \cdot q_i \leq q_i P_{\text{op}} + p_i P_\mu - P_{\text{op}} P_\mu \tag{28}$$
$$1 = p_i \cdot q_i \leq p_i P_{\text{op}} + q_i P_\mu - P_{\text{op}} P_\mu \tag{29}$$
$$1 = p_i \cdot q_i \geq q_i P_\mu + p_i P_\mu - P_\mu^2 \tag{30}$$
$$1 = p_i \cdot q_i \geq q_i P_{\text{op}} + p_i P_{\text{op}} - P_{\text{op}}^2 \tag{31}$$

By bringing Formulas 25, 27 into the Formulas 28 to 31, the binary linear constrains with respect to the transmission powers are obtained, which are inequalities 42 to 45 in the transformed optimization problem 16. Meanwhile, inequalities 36 to 37 are relaxation of Formula 17. Inequalities 38 to 41 are relaxation of Formula 18 when we bring Formula 25 into Formula 23 and 24.

$$\text{minimize} \tag{32}$$
$$\sum_{i=1}^{n} \sum_{j \in \mathcal{N}, j \neq i} \sum_{k=1}^{|\mathbb{C}|} ((P_\mu - P_{\text{op}}) h_{ji} z_{ji} \beta_{ij}^k z_i \tag{33}$$
$$+ (P_\mu - P_{\text{op}}) N_0 x_i^k z_i + P_{\text{op}} h_{ji} z_{ji} \beta_{ij}^k + P_{\text{op}} N_0 x_i^k) \tag{34}$$
$$\text{subject to:} \tag{35}$$
$$x_i^k + x_j^k - \alpha_{ij}^k \leq 1 \tag{36}$$
$$- x_i^k - x_j^k + 2\alpha_{ij}^k \leq 0 \tag{37}$$
$$\beta_{ij}^k - P_{\text{op}} \alpha_{ij}^k \leq 0 \tag{38}$$
$$- \beta_{ij}^k + P_\mu \alpha_{ij}^k \leq 0 \tag{39}$$
$$\beta_{ij}^k + (P_{\text{op}} - P_\mu) y_j - P_\mu \alpha_{ij}^k \leq P_{\text{op}} - P_\mu \tag{40}$$
$$- \beta_{ij}^k + (P_\mu - P_{\text{op}}) y_j + P_{\text{op}} \alpha_{ij}^k \leq 0 \tag{41}$$
$$P_\mu y_i + P_{\text{op}} z_i \leq \frac{P_{\text{op}}^2 - 1}{P_{\text{op}} - P_\mu} \tag{42}$$
$$P_{\text{op}} y_i + P_\mu z_i \leq \frac{P_{\text{op}}^2 - 1}{P_{\text{op}} - P_\mu} \tag{43}$$
$$- y_i - z_i \leq \frac{1 + P_\mu^2 - 2 P_{\text{op}} P_\mu}{P_\mu (P_{\text{op}} - P_\mu)} \tag{44}$$
$$- y_i - z_i \leq \frac{1 - P_{\text{op}}^2}{P_{\text{op}} (P_{\text{op}} - P_\mu)} \tag{45}$$
$$\sum_{k=1}^{|\mathbb{C}|} x_i^k = 1, x_i^k \in X_i \in \{0,1\}^{|\mathbb{C}|} \tag{46}$$
$$\sum_{i=1}^{n} (((P_\mu - P_{\text{op}}) x_i^k y_i + P_{\text{op}} x_i^k) h_{i,pt} z_{i,pt}) \leq I_{pt}^k \tag{47}$$
$$x_i^k, \alpha_{ij}^k, \beta_{ij}^k, y_i, z_i \in \{0,1\} \tag{48}$$
$$\forall i, j \in \mathcal{N}, i \neq j, k \in \mathbb{C} \tag{49}$$

Inequalities 36 and 37 are equivalent to Formula 17. Inequalities 38 - 41 are equivalent to Formula 18. Inequalities 42 - 45 are relaxation of the equality quadratic 19. The objective function is quadratic and there is only linear and quadratic inequality constraints, so we use GUROBI Mixed-Integer Programming solver to solve it get the lower bound of the objective function.

5.2. Distributed White Space Channel Allocation (WhiteSussa)

Being aware of the accumulated interference on the interference measurement devices for the primary users, each WBS can decide whether to switch from idle state to operation state and work on a channel which brings it with the highest utility, or to change its operating channel to get higher utility value. The operation of the WBSs should not result in harmful interference onto the primary system.

The pseudo code is as follows,

When a WBS switches its state from idle to operation, new interference is generated on the other WBSs which

Algorithm 2: Spectrum selection by WBS i

Input: For each WBS, the complete graph of the network, which includes: (1) the distance and signal shadowing between it and any other WBS and TV interference measurement device; (2)Transmission power and channel of the WBS $i \in \mathcal{N}$.

while *Operation profile is different from previous round* **do**
    for $i \in \mathcal{N}$ **do**
        if i *is idle* **then**
            if $\exists c$, ***where*** $\sum f_t^c \leq Thold_t^c, \forall t \in \mathcal{T}$, $\gamma_i^c > \gamma_i^{c'}$, $c' \in \mathbb{C} \setminus c$ **then**
                $c(i) = c$
            end
        end
        if i *is operating* **then**
            if $\exists c', f_i^{c'} < f_i^{c(i)}$ ***and*** $\sum f_t^{c'} \leq Thold_t^{c'}, \forall t \in \mathcal{T}$ **then**
                $c(i) = c'$
            end
        end
    end
end

work with the same channel. Due to this disturbance , the affected WBSs may change their channels to achieve better γ.

This process can be formulated as a congestion game which eventually converges, furthermore, the dynamics initiated by the WBS starting operation eventually ceases. Here we have Lemma 5.1.

Lemma 5.1. The dynamics caused by a WBS changing its state from idle to operation ceases after finite steps.

Proof
After a WBS switches on, all the WBSs will choose the channel which brings the highest γ, i.e., , the lowest co-channel interference considering the transmission power is identical to each WBS. This process is the same as the problem of channel assignment with identical transmission power, which is discussed in [33]. In [33], WBSs change operating channels in order to minimize the received co-channel interference is formulated into a congestion game, then the convergence of the channel assignment is proved. □

Note that according to Algorithm 2, after a WBS starts to operate, it will never change back to idle state. Now we have the following theorem.

Theorem 5.1. *The whole process ends after finite updates.*

Proof
Whenever there is a WBS changes its state from idle to operation, a new congestion game among the WBSs begins. As the convergence is guaranteed as shown in lemma 5.1 and there is only a limited number of WBSs which change their state from idle to operation, the whole process, which consists of a series of concatenated congestion games, will eventually end. □

6. PERFORMANCE EVALUATION

In this section, we will investigate the centralized and distributed schemes which are proposed to comply with ECC and FCC regulations respectively. The evaluation setting is as follows. A square area which is 60km x 60km is divided evenly into 16 square blocks. There is one WBS locating in the middle of each block. Same amount of end terminals are distributed in each block, where the end terminals don't necessarily choose the WBS which is nearest to them to obtain service, in stead they choose the WBS which provides them with the strongest RSSI. There is a 20km wide rim area around the square area, where the interference measurement devices for the TV receivers are randomly located. As to each channel there is one interference measurement device. The locations of WBSs and TV contours are illustrated in Fig. 4. WBSs' locations are fixed, but the locations of interference measurement devices for the TV receivers, the end terminals, and the sequence for WBS to update are randomly decided in each run. Simulations are conducted for 50 times.

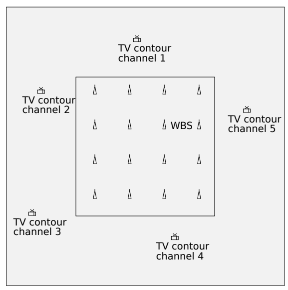


Figure 4. Layout of WBSs and TV contours

Some parameters are listed in Table 9.

6.1. Schemes for ECC Regulations

6.1.1. Maximal Permitted Power Decision and the Distributed Channel Allocation Schemes

We compare *whiteCat* with centralized scheme and three other distributed schemes: the random allocation scheme, *whiteCase* and No-regret learning.

- *Optimization*: centralized quadratic optimization introduced in Section 4.2.

† minimal and maximal power here denote the power level restricted by the specification of hardware.

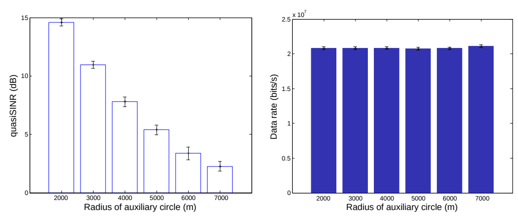
Number of channels	5
Number of WBSs	16
Noise	10^{-12}W
Length of side the square to locate WBSs	60km
Distance between quasai terminal and WBS	7km
Interference threshold on TV contour	10^{-7}W
Path loss factor	2
Standard deviation in flat shadowing	8
Minimal WBS transmission power †	1W
Maximal WBS transmission power	20W
Number of end terminals in network	800
P_{op}, Tx power according to FCC	10W

Table II. Simulation parameters

- *WhiteCase*: Whitespace channel allocation selfish, where WBSs minimize the first part of Formula 8
- *No-Regret Learning*: Each WBS maps the probability of choosing each strategy to a certain proportion of the regret which the WBS may have if it doesn't choose that strategy, and the WBS choose the strategy with the biggest probability. WBSs update such mapping dynamically and this approach converges to correlated equilibrium. Please refer the original paper [29] for details.

6.1.2. The Choice of Radius of Auxiliary Circles, quasiSINR of WBS, and SINR on End Users

The usage of quasiSINR exempts WBSs from taking care the SINR on the end terminals. A WBS's quasiSINR is related with WBS's location and the radius of auxiliary circle. Figure 5 illustrates the effect of using different radii of the auxiliary circle on the data rate can be achieved by end terminals.



(a) quasiSINR of WBSs

(b) Data rate of end terminals when applying whiteCat

Figure 5. The effects of different radii of auxiliary circle on end terminals' data rate. Maximum permitted power is obtained by solving convex optimization. WhiteCat is used to assign the channels.

Subfigure 5(a) shows WBSs' quasiSINR decreases when the radii of auxiliary circles increase. Subfigure 5(b) illustrates the choice on radius of auxiliary circle don't influence the performance of whiteCat. In the following simulation, we fixed the radius at 6000 m.

6.1.3. Performance of Channel Allocation Schemes

In Section 4.1, two different optimization formulations are introduced to obtain the maximum permitted transmission power for WBSs, i.e., convex optimization and linear optimization respectively. In Figure **??**, we have seen that the convex optimization generates power levels which distribute evenly between the minimum and maximum transmission power levels configured by the hardware, while, the majority of the power levels generated by linear optimization are either the minimum or maximum transmission power. In this section we run the channel allocation schemes with the maximum permitted power levels obtained from convex and linear optimization respectively. The simulation in this subsection carries twofold meanings. The first is to see which maximum permitted power decision method outperforms the other, the second is to evaluate the performance of the channel allocation schemes. The adopted metrics are the SINR at end terminals and transmission power consumption.

6.1.4. Comparison of the Methods for Maximum Permitted Transmission Power

Figure 6 depicts the power consumption of the channel allocation schemes which work with the two groups of maximum permitted transmission power decided by linear and convex problems respectively. When given maximum permitted transmission power, whiteCat and the centralized optimization scheme consume the least energy. The schemes utilize less transmission power with the maximum permitted transmission power decided by convex optimization. Figure **??** shows the quasiSINR of WBSs. The centralized optimization scheme achieves the highest quasiSINR, because the optimization formulation 15 obtains the global optima.

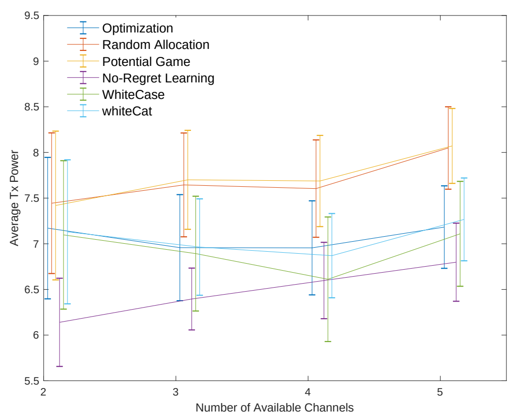


Figure 6. Average Transmission power of WBSs when different number of channels are available

The average SINR on the end terminals is depicted in Figure 7. When the given maximum permitted transmission power, whiteCat and the centralized optimization achieve similar and the best performance among the schemes. It is also noticed that, the maximum permitted transmission power decided by linear optimization helps the channel allocation schemes achieve better SINR.

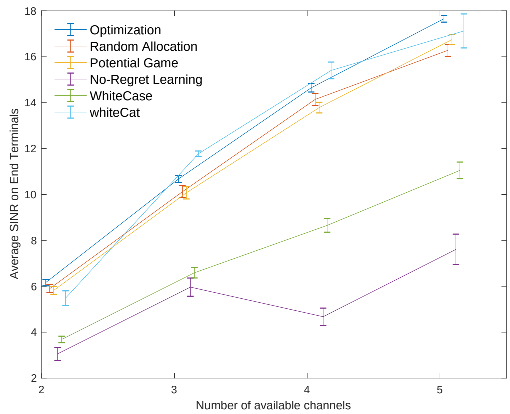


Figure 7. SINR on end terminals

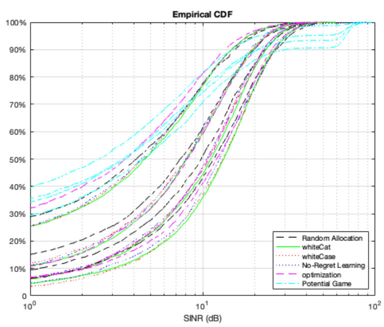


Figure 8. CDF of SINR on end users

The empirical cumulative distribution function curve of SINR on end terminals is drawn in Figure 8. The SINR achieved by WhiteCat and the centralized optimization is stably higher than that obtained from other schemes. For example, the 20% and 80% percentile of the SINR achieved by WhiteCat and the centralized optimization are 0.5 to 1 dB higher than the other channel allocation schemes.

6.1.5. Convergence Speed

In the congestion game where scheme whiteCat is derived, each player (WBS) has at most $(n-1) * |\mathbb{C}|$ resources available for usage, thus there is no polynomial steps converging to NE, while, simulation shows the algorithm can quickly converge to NE when the number of WBS is up to 100. Table III shows the average number of steps needed before convergence in 100 runs of simulations. As to whiteCat, we account each WBS accessing the base station (refer to 4.3) as *one step*. We compare the convergence speed of WhteCat with no-regret learning, the scheme derived from potential game [20] and whiteCase. Note that the potential game scheme is to solve joint power and channel allocation problem, as it is developed with game theory, it is reasonable to see its convergence speed. As there is no guarantee for WhiteCase to converge, we stop the channel allocation process after 16000 steps (1000 rounds).

Table III tells that whiteCat is two times faster than the scheme derived from potential game, and 20 times faster than no-regret learning scheme. The relatively smaller confidence interval shows that whiteCat's convergence is not affected by different network configurations. Fast converge is attributed to the working style of WBSs whcih access the database to get the information of other WBSs, thus the distributable decision involves a part of the global information of the network. Thus, we can see that the speed up of convergence is due to the overhead caused by accessing the database.

Figure 9 depicts one instance of the convergence processes of three schemes. The Y axis is the summed utility of all WBSs. We can see whiteCat decreases the summed utility constantly, and the channel allocation process ceases after 38 times of updates. Whereas, noregret learning scheme takes 120 steps before convergence, and whiteCase fails to converge.

Scheme	Average steps	95% CI	Average time (s)
WhiteCat	58	5.6	2
No-Regret Learning	1916	1541	144
Potential Game [20]	120	10	4
Optimization	-	-	40
WhiteCase	4587	2742	50

Table III. Convergence speed of the distributed channel allocation schemes. As to the distributed scheme, the time involved to communicate with database is note considered and included.

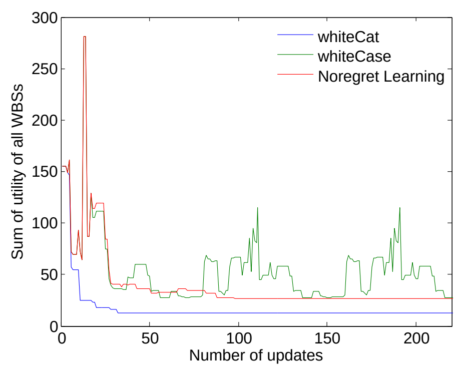


Figure 9. Convergence process of three different schemes in one simulation.

6.1.6. Stability of SINR in Convergence Process

WBS provides service to end users in the process of channel allocation. A certain SINR corresponds to certain transmission configurations like modulation type and data rate. The oscillation of SINR resulted from WBS changing the working channel during the convergence process may cause reconfiguration, reduced throughput or delay variance, which is not preferred. We propose a metric *Cost of Oscillation* (COS) to represent the stability of SINR in the converging process. Assuming each update of channel takes 1 time unit, the variance of SINR of end user i at time $t+1$ is

$$\Delta\gamma_i(t+1) = \left| \frac{\gamma_i(t+1) - \gamma_i(t)}{\gamma_i(t)} \right|$$

.

The COS value for one network applied with a certain channel allocation scheme is,

$$COS = \sum_{t=1}^{T} \sum_{i \in \mathcal{N}} \Delta\gamma_i(t) \tag{50}$$

$\gamma_i(0)$ is the SINR for i before starting channel allocation. The variance of SINR in channel allocation process is shown in table IV from which we can see WhiteCat achieves only 6% of oscillation on SINR compared with No-regret approach.

Scheme	COS	95% confidence interval
WhiteCat	8850	2984
No-Regret Learning	145460	1541
WhiteCase	246790	168050

Table IV. Variance of SINR during the convergence process

6.2. Schemes for FCC Regulations

Figure 10 shows the utilities obtained by optimization is larger than the distributed scheme. Note the distributed scheme takes an advantage that some WBSs are allowed to be idle, in comparison, every WBS in optimization must operate with either P_μ or P_{op}.

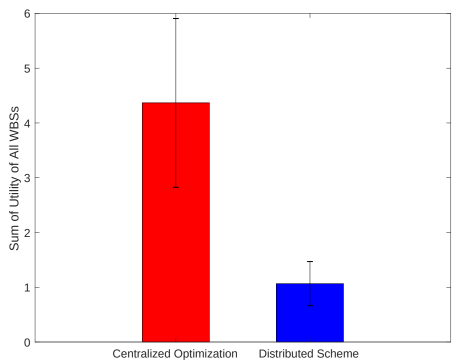


Figure 10. Sum of Utility

Figure 11 shows the distributed scheme outperforms the centralized scheme in terms of SINR on end users.

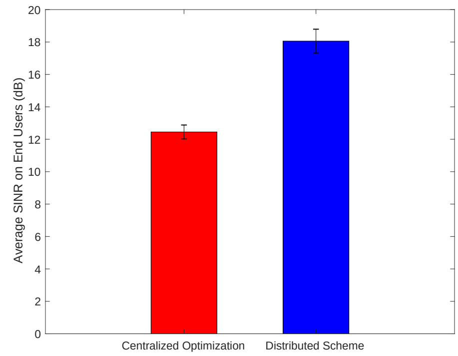


Figure 11. Average SINR on end users

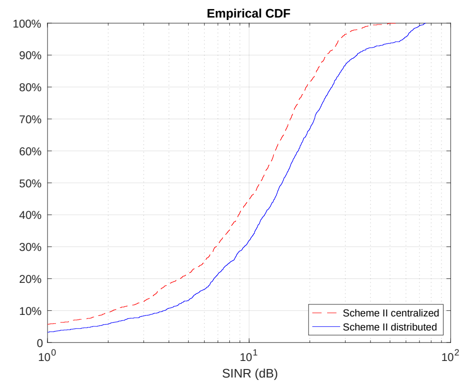


Figure 12. Cumulative distribution of SINR on end users after channel and power allocation

Figure 12 provides another perspective for the SINR on the end users.

In terms of transmission power, the distributed scheme consumes 76% of the centralized optimization, which is shown in Figure 13. The reason is explained in Figure 14 i.e., by executing the former scheme, more WBSs are allowed to be idle than the latter.

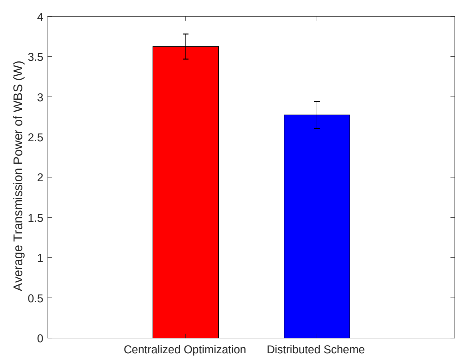


Figure 13. Average transmission power of one WBS

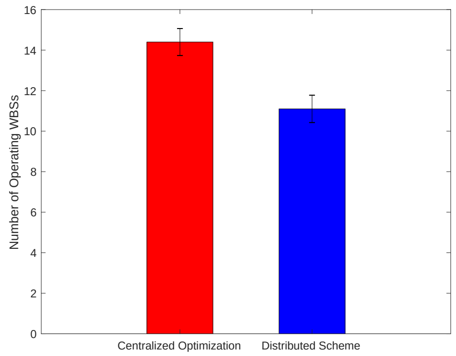


Figure 14. Number of WBSs with P_{op}

7. CONCLUSIONS

In this paper, we propose solutions for spectrum sharing among white space devices with respect to the regulations made by both ECC and FCC respectively. As to distributed solution, congestion game is applied to analyze the channel allocation problems, where transmission powers is identical according to FCC rules and unidentical accordingly to ECC rules. The proposed algorithm which is derived from the best response of the congestion game converges quickly, and achieves better performance than other distributed schemes and centralized scheme in some aspects. Centralized optimization formulations are also proposed for the two regulation scenarios.

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